GRADE
12

THE ANSWER SERIES

MATHEMATICS

TAS-style Diagnostic Report

Each Question + its Memo, each with

- **Common Errors & Misconceptions**
- **Suggestions to Teachers for Improvement**



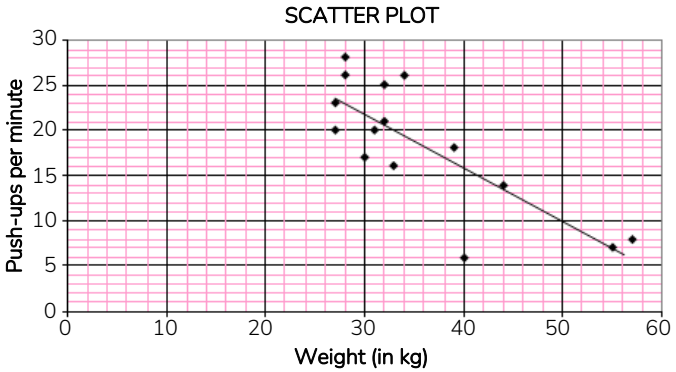
November Exam
Paper 2
2024

DATA HANDLING (61%; 41%): DBE NOVEMBER 2024

QUESTION 1 61%

At the beginning of a season, the coach of a junior boys' rugby team recorded the weight (in kg) of the 15 players in his team and the number of push-ups that each player was able to do in one minute. The data is represented in the table and scatter plot below. The least squares regression line for the data is drawn.

Weight (in kg) (x)	34	32	40	27	33	28	27	55	39	44	30	57	28	32	31
Number of push-ups per minute (y)	26	21	6	20	16	26	23	7	18	14	17	8	28	25	20



1.1 Determine the equation of the least squares regression line for the data. (3)

94%

Memo

Refer to Calculator Instructions in 'documents for teachers'

1.1 $A \simeq 39{,}46$; $B \simeq -0{,}59$

$\therefore$ Eqn: $y = 39{,}46 - 0{,}59x$ ◄ ... $y = A + Bx$

Common Errors and Misconceptions

(a) In **Q1.1** to **Q1.4** many candidates failed to round off their answers to the required number of places.

(b) When writing the equation in **Q1.1**, many candidates interchanged the values of a and b. Another common error in this question was that candidates failed to round off their answers for a and b correctly to two decimal places. Some candidates did not enter the data correctly into the calculator. They arrived at answers that were close to the correct answers for a and b.

Copyright © The Answer Series: Photocopying of this material is illegal

THE ANSWER SERIES Your Key to Exam Success

1.2 Write down the correlation coefficient. (1)

79%

1.3 The coach uses the least squares regression line to set the target for the minimum number of push-ups by each team member according to their weight. Predict the number of push-ups that a member of the team, who weighs 29 kg, should do to meet the target. (2)

92%

1.4 Write down the mean number of push-ups for the given data. (1)

69%

1.5 The players trained hard during the season. At the end of the season, the coach reported that each player was able to do 5 more push-ups per minute than they did at the beginning of the season. How does the increase in the number of push-ups influence the standard deviation of the data? (1)

46%

1.6 At the beginning of the season, the coach used the least squares regression line as the minimum target for a player to aim for. Determine the maximum possible increase in the number of push-ups that a team member must obtain to reach the minimum target. (2)

12%

[10]

Memo

1.2 $r \simeq -0{,}80$ ◄

1.3 Subst. $x = 29$: $y = 39{,}46 - 0{,}59(29)$
$= 22{,}35$

$\therefore$ 22 or 23 push-ups ◄



1.4 $\bar{y} \simeq 18{,}33$ ◄

1.5 Not at all ◄ . . . *ALL of the players did 5 more. The mean will increase by 5, but the σ will remain the same.*

1.6 Maximum possible increase = 16 – 6 = 10 push-ups ◄

. . . *the boy weighing 40 kg needed to increase the most, from 6 to 16 push-ups.*

Common Errors and Misconceptions

(c) Many candidates did not round off, or rounded off incorrectly, when answering **Q1.2**. They gave an answer of $-0{,}7979453714\ldots$ or $-0{,}79$ instead of $0{,}8$.

(d) While several candidates substituted $x = 29$ correctly into the least-squares regression equation, they did not arrive at the correct answer. This was only possible if they had made mistakes when using the calculator.

(e) Many candidates showed that they lacked the skill of reading for understanding when answering **Q1.4**. They calculated the mean of the x-values instead of the mean of the y-values.

(f) Many candidates did not attempt **Q1.5**. Some candidates re-calculated the standard deviation but did not state how the increase in the number of push-ups influenced the standard deviation. These candidates failed to respond to the question, which pointed to candidates not being able to read for understanding.

(g) **Q1.6** was answered very poorly. Many calculated the predicted value when $x = 40$ instead of the increase in the number of push-ups. These candidates failed to subtract 6 from the predicted value.

Copyright © The Answer Series: Photocopying of this material is illegal

THE ANSWER SERIES *Your Key to Exam Success*

QUESTION 1: Suggestions for Improvement



(a) It should not be taken for granted that learners are able to round off correctly to two decimal places. In this regard, an exercise on rounding can help correct any misconceptions.

(b) Teachers should link the equation of the least-squares regression line ($y = a + bx$) with the equation of the straight line and emphasise that 'a' refers to the *y-intercept* and 'b' refers to the *gradient*.

(c) As stated in previous reports, when determining the equation of the least-squares regression line, it is advisable that learners write down the values of a and b and then write down the equation of the regression line. In this way, they can get the CA mark for the equation.

(d) When analysing bivariate data, both the x- and y-values are considered in determining the equation of the least-squares regression line and the value of the *correlation coefficient*. However, the x- and y-values in a bivariate data set can also be considered as two separate sets of data. Learners need to be made aware that they can be asked to calculate the *mean* and *standard deviation* of the x- and/or y-variables of a bivariate set of data.

(e) Learners should be proficient in using their calculators in STAT mode. They should be familiar with what the symbols on the calculator represent, for example σ_x represents *population standard deviation* and r represents *correlation coefficient*.

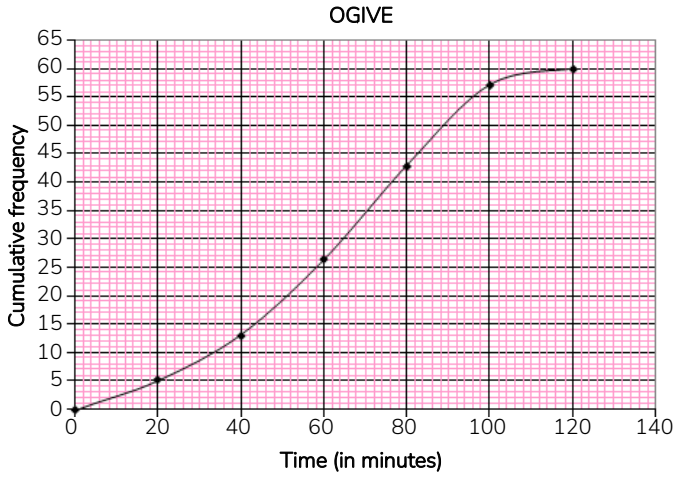
(f) As mentioned in previous reports, learners should be able to use the values of their calculations to make predictions and comments about the data. Time should be devoted to interpretation questions.

Copyright © The Answer Series: Photocopying of this material is illegal

THE ANSWER SERIES Your Key to Exam Success

QUESTION 2 41%

The cumulative frequency graph (ogive) shows the time taken (in minutes) for 60 employees to travel to work each morning.



2.1 Estimate the median travel time. (1)

44%

2.2 Estimate the lower quartile. (1)

49%

2.3 Estimate the interquartile range. (2)

51%

Memo

2.1 Estimated median (Q_2) travel time = 66 minutes ◄

. . . read from 30 on the y-axis

2.2 Estimated lower quartile (Q_1) = 44 minutes ◄

. . . read from 15 on the y-axis

2.3 Estimated upper quartile (Q_3) = 84 minutes ◄

∴ Estimated IQR = $Q_3 - Q_1$ = 40 minutes ◄

Common Errors and Misconceptions

(a) Many candidates gave the position of the median and not its value when answering **Q2.1**. Some candidates did not realise that the minor gridlines represented two units and not one unit. This led to them reading off the answers incorrectly. Some candidates calculated the position of the median incorrectly by using the formula for ungrouped data, i.e. $\frac{1}{2}(n + 1)$. Instead, they should have used the formula $\frac{1}{2}n$. A few candidates incorrectly indicated the interval in which the median lay.

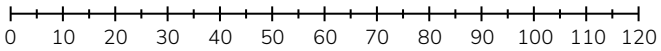
(b) The errors listed in (a) were made when determining the *lower quartile* (in **Q2.2**) and the *interquartile range* (in **Q2.3**). In **Q2.3** some candidates subtracted the positions instead of the values of the upper and lower quartiles. A few candidates calculated the *range* or *semi-interquartile range*; neither of these were correct.

Copyright © The Answer Series: Photocopying of this material is illegal

THE ANSWER SERIES Your Key to Exam Success

2.4 The minimum and maximum times taken for an employee to travel to work are 5 and 120 minutes respectively. On the scaled line below, draw a box and whisker diagram to indicate the distribution of the data as represented in the ogive above. (2)

73%



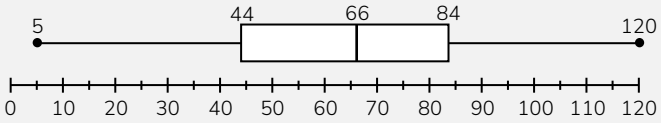
2.5 The company manager decided that all employees who travel for an hour or more will be allowed to work from home for part of the day.

47%

What percentage of the employees will be allowed to work from home for part of the day? (2)

Memo

2.4



2.5 There are 60 employees.

25 employees travel for less than 60 minutes.

∴ The % allowed to work from home

$= \frac{60-25}{60}\% = \frac{35}{60}\% \simeq$ **58,33%** ◄

Common Errors and Misconceptions

(c) Many candidates knew how to draw the box-and-whisker diagram required in **Q2.4**. Some of them used values of the median and quartiles that were different to the ones that they had calculated in **Q2.1** to **Q2.3**. These candidates were not awarded any marks.

(d) Candidates were able to correctly read the value at one hour from the *ogive*. However, they were unable to use this information to answer the question correctly. They failed to subtract 26 from 60 because they did not realise that more than one hour was to the right of 26. A few candidates incorrectly took the total number of workers to be 65 and subtracted 26 from 65. They were not awarded any marks.

Copyright © The Answer Series: Photocopying of this material is illegal

THE ANSWER SERIES *Your Key to Exam Success*

12%

2.6 Employees work 8 hours in a normal working day. The manager decided on the following rule for time to work from home:

- An employee is allowed to work half an hour from home for each time interval of 20 minutes, or part thereof, above an hour taken to travel to work.

On a certain day, an employee takes 110 minutes to travel to work. Calculate the number of minutes that this employee will be allowed to work from home on this day. (2)

[10]

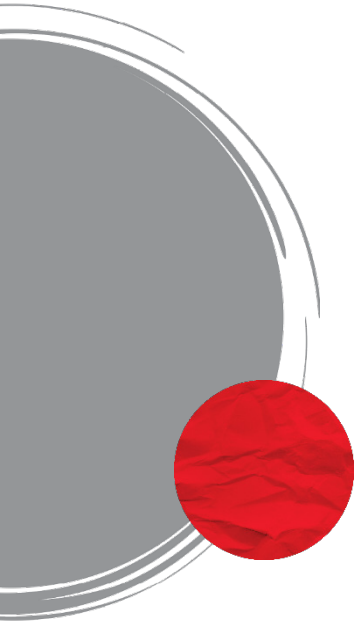
Memo

2.6 The employee takes (60 + 50) minutes to travel

$\therefore$ $2\frac{1}{2}$ intervals of 20 minutes, taken as 3

$\therefore$ The number of minutes allowed to work from home

$= 3 \times 30 =$ **90 minutes** ✓



Common Errors and Misconceptions

(e) Many candidates did not attempt **Q2.6**. This question required candidates to read with understanding and extract the relevant information from the *ogive*. Many candidates were unable to interpret 'part thereof' correctly. The common incorrect response was that workers who travelled for 110 minutes will be allowed 2,5 × 30 minutes = 75 minutes instead of 3 x 30 minutes = 90 minutes. A fair number of candidates had no understanding of the question and responded in the following way: 110 – 20 = 90 minutes. Although they arrived at the correct answer, they were not awarded any marks.

Copyright © The Answer Series: Photocopying of this material is illegal

THE ANSWER SERIES *Your Key to Exam Success*

QUESTION 2: Suggestions for Improvement



(a) Reading for understanding is a fundamental requirement in the *Statistics* section and must be developed in classroom activities.

(b) Questions on *Statistics* are normally set in context. Learners should be taught to answer the questions within the context of the question.

(c) Teachers need to explain the difference between the position of the quartiles and the value of the quartiles. The values of the quartiles are the statistics that describe various points in the set of data and this information is required in data analysis.

(d) In Grade 11, teachers should take time to explain the concept of *cumulative frequency* and how to read and interpret the cumulative frequency from an *ogive*. For example, the value of 26 does not represent the number of workers who took 60 minutes to travel to work. Instead, 26 workers took up to 60 minutes to travel to work, i.e. these workers travelled between 0 and 60 minutes to get to work.

(e) Much of this question was based on reading off the *ogive*, which is done in Grade 11. Revision of Grade 11 work in Grade 12 will assist learners prepare for the examinations.

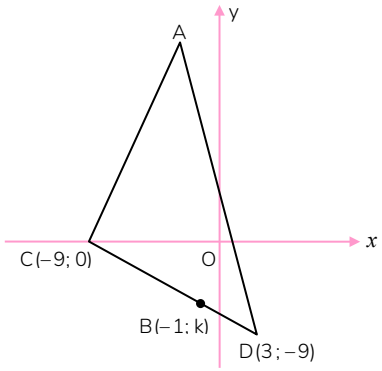
Copyright © The Answer Series: Photocopying of this material is illegal

ANALYTICAL GEOMETRY (57%; 62%): DBE NOVEMBER 2024

QUESTION 3 57%

In the diagram below, ΔACD has vertices A, D(3; –9) and C(–9; 0), where A is a point in the second quadrant.

B(–1; k) lies on side DC.



98% 3.1 Calculate the gradient of DC. (2)

96% 3.2 Determine the equation of DC in the form $y = mx + c$. (2)

90% 3.3 Show that $k = -6$. (1)

Memo

3.1 $M_{DC} = \frac{0-(-9)}{-9-3} = \frac{9}{-12} = -\frac{3}{4}$ ◄

3.2 Subst. $m = -\frac{3}{4}$ & (–9; 0) in

$y - y_1 = m(x - x_1)$ OR $y = mx + c$

$\therefore y - 0 = -\frac{3}{4}(x + 9)$ $\therefore 0 = \left(-\frac{3}{4}\right)(-9) + c$

$\therefore y = -\frac{3}{4}x - \frac{27}{4}$ ◄ $\therefore -6\frac{3}{4} = c$, etc

3.3 Subst. (–1; k) in $y = -\frac{3}{4}x - \frac{27}{4}$

$\therefore k = -\frac{3}{4}(-1) - 6\frac{3}{4}$

$\therefore k = -6$ ◄

Common Errors and Misconceptions

(a) Some candidates were unable to substitute correctly into the gradient formula when answering **Q3.1**.

(b) In **Q3.2** some candidates swopped the x- and y-values around when substituting into the equation of the straight line. Other candidates substituted correctly but failed to simplify the equation correctly.

(c) Some candidates incorrectly assumed that B was the midpoint of CD and used the midpoint formula to calculate the value of k when answering **Q3.3**.

Copyright © The Answer Series: Photocopying of this material is illegal

THE ANSWER SERIES Your Key to Exam Success

3.4 Calculate the length of DC. (2)

97%

3.5 Calculate the ratio of $\frac{DB}{DC}$. (2)

81%

3.6 If M is a point on AD such that AC || MB, calculate the ratio of $\frac{\text{Area } \Delta MBD}{\text{Area } \Delta ACD}$. (4)

17%

Memo

3.4 $DC^2 = (-9-3)^2 + (0+9)^2$

$= 144 + 81$

$= 225$

$\therefore DC = \sqrt{225} = 15$ units ✓

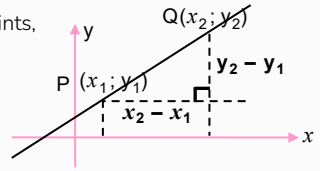


The Distance Formula

The distance between 2 points, $P(x_1; y_1)$ and $Q(x_2; y_2)$

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

... *Theorem of Pythag*



3.5 $DB^2 = (-1-3)^2 + (-6+9)^2$

$= 16 + 9$

$= 25$

$\therefore DB = \sqrt{25} = 5$ units

Applying the Distance Formula



$\therefore \frac{DB}{DC} = \frac{5}{15} = \frac{1}{3}$ ✓

3.6 $\frac{DM}{DA} = \frac{DB}{DC} \left(= \frac{1}{3}\right)$ ... *prop theorem; AC || MB*

$\therefore \frac{\text{Area } \Delta MBD}{\text{Area } \Delta ACD} = \frac{\frac{1}{2} DB \cdot DM \sin D}{\frac{1}{2} DC \cdot DA \sin D}$

$= \frac{DB}{DC} \cdot \frac{DM}{DA}$

$= \frac{1}{3} \times \frac{1}{3}$

$= \frac{1}{9}$ ✓

Copyright © The Answer Series: Photocopying of this material is illegal

Common Errors and Misconceptions

(d) Candidates who failed to answer **Q3.4** correctly used the coordinates of points other than C and D in their working or they failed to use their calculators correctly to calculate the length of CD.

(e) Although **Q3.5** was well answered by most candidates, some candidates calculated the ratio of the gradients of DB and DC instead of calculating the ratio of the lengths of DB and DC. Some candidates incorrectly assumed that this question required Euclidean Geometry knowledge.

(f) **Q3.6** was not well answered by most of the candidates. Very few realised that they had to use the *proportionality theorem* to answer the question. Many incorrectly calculated the areas by using the area of triangle formula: $\frac{1}{2}$base × height although the triangles chosen were not right-angled. Some candidates wrote down the area rule incorrectly. A few candidates only substituted the length of one side in the area rule instead of two.

THE ANSWER SERIES *Your Key to Exam Success*

52%

3.7 If it is further given that the gradient of AD is -4 and the length of AD is $\sqrt{612}$ units, calculate the coordinates of A. (6)

[19]

Memo

3.7 Let the pt A be (p; q)

$$\therefore \frac{q+9}{p-3} = -4$$

$$\therefore q + 9 = -4p + 12$$

$$\therefore q + 4p = 3$$

$$\therefore q = 3 - 4p \quad \ldots \text{❶}$$

$$AD^2 = (p-3)^2 + (q+9)^2 = 612 \quad \ldots \text{❷}$$

$$\therefore p^2 - 6p + 9 + (3 - 4p + 9)^2 = 612$$

$$\therefore p^2 - 6p + 9 + (12 - 4p)^2 = 612$$

$$\therefore p^2 - 6p + 9 + 144 - 96p + 16p^2 = 612$$

$$\therefore 17p^2 - 102p - 459 = 0$$

$$(\div 17) \quad \therefore p^2 - 6p - 27 = 0$$

$$\therefore (p+3)(p-9) = 0$$

$$\therefore p = -3 \quad p \neq 9 \quad \because p < 0$$

❶: $q = 3 - 4(-3)$

$= 3 + 12$

$= 15$

$\therefore$ A(–3; 15) ◄

Common Errors and Misconceptions

(g) In **Q3.7** many candidates were able to establish two equations from the information given in the question but did not realise that they needed to solve these equations simultaneously. Some who attempted to solve the equations simultaneously made errors in calculations that resulted in them not being able to solve for x. A fair number of candidates assumed that the y-intercept of AD was the midpoint of AD without first proving this fact. They were not credited for making this assumption.

Copyright © The Answer Series: Photocopying of this material is illegal

THE ANSWER SERIES Your Key to Exam Success

QUESTION 3: Suggestions for Improvement



(a) If learners are not sure, they should consult the information sheet for the correct formula.

(b) Teachers should ensure that learners are able to use calculators correctly. It might be useful to call out the procedure step-by-step as learners perform the calculations.

(c) As stated in previous reports, it is important that learners realise that it is not acceptable to make any assumptions, e.g. that a certain point is the midpoint of a line. Even if it looks as if the point is the midpoint, it may not just be assumed and used. These need to be proven first before the results can be used in an answer.

(d) Teach learners to analyse diagrams in Analytical Geometry and to use relevant properties to respond to questions.

(e) Learners should be advised that they need to fill in the calculated values and additional information on their sketch in the answer book as they proceed with subquestions. This helps them visualise what information is at their disposal when answering the next subquestion.

(f) Teach learners how to identify when to use which formula: area $= \frac{1}{2}$base $\times$ height or area $= \frac{1}{2}a.b.\sin C$ when calculating the area of a triangle.

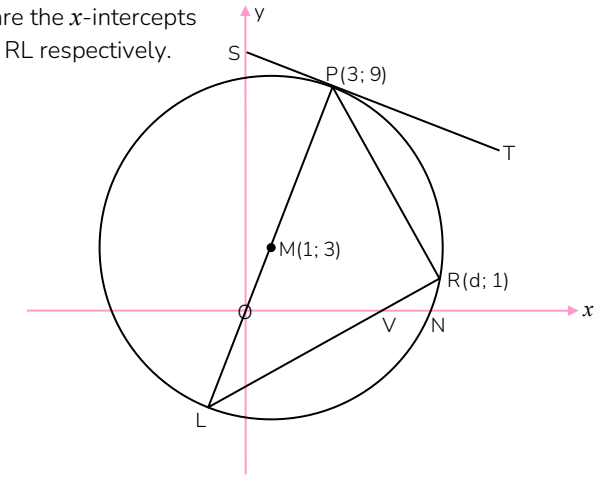
(g) Teach learners to expect that Euclidean Geometry facts will be integrated into Analytical Geometry and will be needed in the answering of some Analytical Geometry questions.

Copyright © The Answer Series: Photocopying of this material is illegal

THE ANSWER SERIES *Your Key to Exam Success*

QUESTION 4 62%

In the diagram, M(1; 3) is the centre of the circle.
The circle cuts the x-axis at N.
ST is a tangent to the circle at P(3; 9).
R(d; 1), with d > 0, and L lie on the circle.
O and V are the x-intercepts of PL and RL respectively.



4.1 Write down the coordinates of L. (2)

88%

4.2 Determine the equation of tangent ST to the circle at P. (4)

82%

Memo

4.1 L(−1; −3) ◄

4.2 $m_{MP} = \frac{9-3}{3-1} = \frac{6}{2} = 3$

$\therefore m_{ST} = -\frac{1}{3}$



Subst. $m = -\frac{1}{3}$ & (3; 9) in

$y - y_1 = m(x - x_1)$

$\therefore y - 9 = -\frac{1}{3}(x - 3)$

$\therefore y = -\frac{1}{3}x + 1 + 9$

$\therefore y = -\frac{1}{3}x + 10$ ◄

OR

$y = mx + c$

$\therefore 9 = \left(-\frac{1}{3}\right)(3) + c$

$\therefore 10 = c$, etc

Copyright © The Answer Series: Photocopying of this material is illegal

Common Errors and Misconceptions

(a) Few candidates attempted to answer **Q4.1** by using the gradient formula instead of the midpoint formula.

(b) While many candidates answered **Q4.2** correctly, some candidates incorrectly used the gradient of the radius when determining the equation of the tangent. They failed to realise that the tangent was perpendicular to the radius.

Some incorrectly used the gradient of $\frac{1}{3}$ instead of $-\frac{1}{3}$.

THE ANSWER SERIES Your Key to Exam Success

4.3 Show that the equation of the circle with centre M is
74% $x^2 + y^2 - 2x - 6y - 30 = 0$. (4)

4.4 Show that d = 7. (2)

70%

4.5 Calculate the size of $\hat{L}$. (5)

63%

Memo

4.3 $r^2 = (3-1)^2 + (9-3)^2$
$= 4 + 36$
$= 40$

& Centre M(1; 3)

$\therefore$ Eqn of ⊙M: $(x-1)^2 + (y-3)^2 = 40$
$\therefore x^2 - 2x + 1 + y^2 - 6y + 9 = 40$
$\therefore x^2 + y^2 - 2x - 6y - 30 = 0$ ◄

4.4 Subst. (d; 1): $\therefore d^2 + 1 - 2d - 6 - 30 = 0$
$\therefore d^2 - 2d - 35 = 0$
$\therefore (d+5)(d-7) = 0$
$\therefore$ **d = 7** ◄ ... $d \neq -5$ $\because d > 0$

4.5 A set of mini-axes is useful



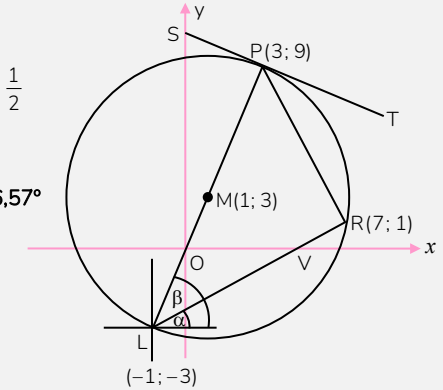
$m_{LP} = \frac{9+3}{3+1} = 3$ $\hat{L} = \beta - \alpha$

$\therefore \beta = 71{,}57°$

& $m_{LR} = \frac{1+3}{7+1} = \frac{1}{2}$

$\therefore \alpha = 26{,}57°$

$\therefore \hat{L} = 71{,}57° - 26{,}57°$
$= 45°$ ◄



Copyright © The Answer Series: Photocopying of this material is illegal

Common Errors and Misconceptions

(c) Many candidates answered **Q4.3** correctly. Some worked backwards from the equation that they were required to prove. This was unacceptable and they were not awarded any marks.

(d) In **Q4.4** some candidates incorrectly substituted the value of y into the equation of the tangent calculated in **Q4.3** instead of the equation of the circle.

(e) In **Q4.5** many candidates did not label the angles correctly, for example, they wrote $\hat{P} = \hat{L}$, instead of $L\hat{P}R = \hat{L}$. Some candidates assumed, without proof, that PR = LR and therefore concluded that $\hat{L} = 45°$. Other candidates also assumed that $\hat{R} = 90°$ without first proving it. These candidates were not awarded any marks in both instances.

THE ANSWER SERIES Your Key to Exam Success

4.6 TR is a tangent to the circle at R. Prove that PT ⊥ RT. (3)

43%

[20]

Memo

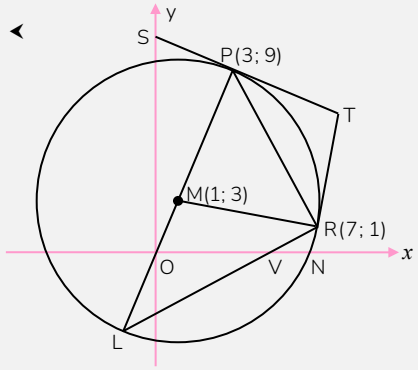
4.6 $m_{MR} = \frac{3-1}{1-7} = -\frac{1}{3}$

$\therefore m_{TR} = 3$ ... *tang TR ⊥ rad MR*

$\therefore m_{PT} \times m_{TR} = -\frac{1}{3} \times 3 = -1$ ... $m_{ST} = -\frac{1}{3}$ *in 4.2*

$\therefore$ PT ⊥ RT ◄



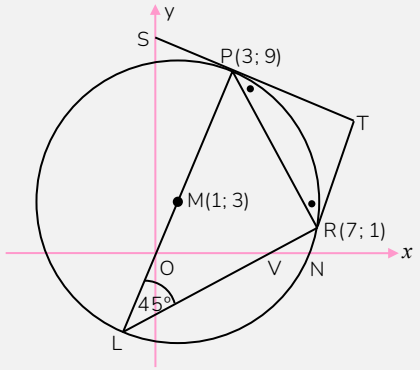


OR: $P\hat{R}T = \hat{L}$ (= 45° in 4.5) ... *tan chord thm*

& TP = TR ... *tans from common point*

$\therefore$ TPR = $P\hat{R}T$ = 45° ... $\angle^s$ *opp = sides (in Δ)*

$\therefore$ PT ⊥ RT ◄ ... *sum of* $\angle^s$ *in Δ*



OR: $P\hat{R}L$ = 90° ... $\angle$ *in semi-⊙*

& $\hat{L}$ = 45° ... *proved in 4.5*

$\therefore$ $L\hat{P}R$ = 45° ... *sum of* $\angle^s$ *in Δ*

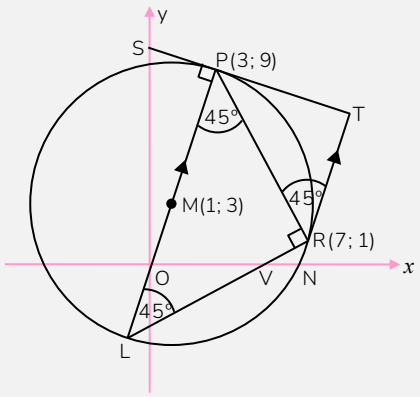
& $P\hat{R}T$ = 45° ... *tan chord thm*

$\therefore$ PL || TN ... *alt* $\angle^s$ =

$S\hat{P}L$ = 90° ... *tan ⊥ diameter*

$\therefore$ $\hat{T}$ = 90° ... *corresp* $\angle^s$; *PL || TN*

$\therefore$ PT ⊥ RT ◄





Common Errors and Misconceptions

(f) **Q4.6** was poorly answered by many candidates. Many candidates presented arguments that were constructed on circular reasoning. They started their responses by stating that $m_{PT} \times m_{RT} = -1$ and used the value of m_{PT} from **Q4.2** to calculate the value of m_{RT}. Thereafter, they showed that the product of the same two gradients, m_{PT} and m_{RT}, was equal to −1.

Copyright © The Answer Series: Photocopying of this material is illegal

THE ANSWER SERIES Your Key to Exam Success

QUESTION 4: Suggestions for Improvement



(a) As mentioned in previous reports, teachers need to revise the concepts of *perpendicular lines* and *gradients*, particularly that the tangent is perpendicular to the radius at the point of contact.

(b) Learners should practise using a formula to get an answer (e.g. using the formula to calculate the coordinates of the midpoint), as well as to calculate an unknown variable if the answer has been given (e.g. calculate the coordinates of an endpoint if one endpoint and the midpoint are given).

(c) Teachers should develop learners' ability to reason logically and to write down the steps in their reasoning.

(d) For learners to be able to reason and answer even more complex questions, they need a very good understanding of basic concepts, including those from lower grades. Regular revision of these concepts can help consolidate understanding them.

Copyright © The Answer Series: Photocopying of this material is illegal

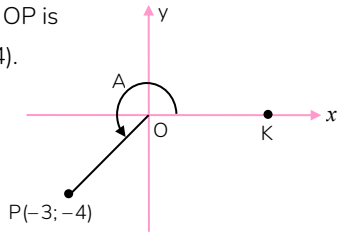
THE ANSWER SERIES *Your Key to Exam Success*

TRIGONOMETRY (48%; 45%; 35%; 40%): DBE NOV 2024

QUESTION 5 48%

69%

5.1 In the diagram, line OP is given with P(–3; –4). $K\hat{O}P = A$.



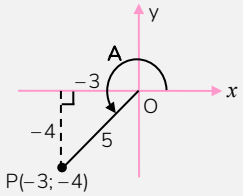
Determine, **without using a calculator**, the value of:

5.1.1 cos A (2)

5.1.2 cos 2A (2)

Memo

5.1.1 $\cos A = \frac{-3}{5} = -\frac{3}{5}$ ◄



5.1.2 $\cos 2A = 2\cos^2 A - 1 = 2\left(-\frac{3}{5}\right)^2 - 1 = 2\left(\frac{9}{25}\right) - 1 = -\frac{7}{25}$ ◄

Common Errors and Misconceptions

(a) In **Q5.1.1** some candidates arrived at $r = -5$ and did not realise that there was a mistake in their answer as r is a distance and its value cannot be negative. Other candidates gave the answer as $\cos A = \frac{3}{4}$. This was incorrect.

(b) In **Q5.1.2** some candidates were unable to write the expansion for $\cos 2A$ correctly despite it being given in the information sheet. Instead, they incorrectly wrote the expansion for $\cos 2A$ as $2\cos A$. Other candidates substituted a ratio into the place of the angle, i.e. $\cos\left(\frac{-3}{5}\right)$.

Copyright © The Answer Series: Photocopying of this material is illegal

THE ANSWER SERIES Your Key to Exam Success

5.1.3 $\sin(A - B)$, if it is further given that

$\sin B = \frac{4}{5}$ and $90° < B < 360°$ (4)

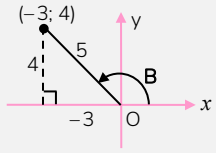
Memo

5.1.3 $\sin(A - B) = \sin A \cos B - \cos A \sin B$

$= \left(-\frac{4}{5}\right)\left(-\frac{3}{5}\right) - \left(-\frac{3}{5}\right)\left(\frac{4}{5}\right)$

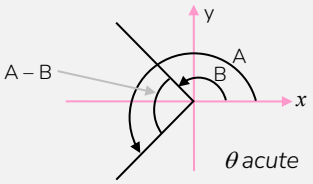
$= \frac{12}{25} + \frac{12}{25}$

$= \mathbf{\frac{24}{25}}$ ◄



OR:

The Δ^s are reflections of each other.

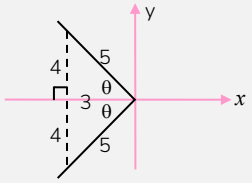


$\sin(A - B) = \sin 2\theta$

$= 2 \sin \theta \cos \theta$

$= 2\left(\frac{4}{5}\right)\left(\frac{3}{5}\right)$

$= \mathbf{\frac{24}{25}}$ ◄



Common Errors and Misconceptions

(c) **Q5.1.3** was not well answered as many candidates did not realise that they had to draw another sketch for $\hat{B}$. Some candidates incorrectly placed $\hat{B}$ in either the first or third quadrants instead of the second quadrant. Some candidates made the assumption that $\hat{A} + \hat{B} = 90°$ and used co-ratios to solve the question.

Copyright © The Answer Series: Photocopying of this material is illegal

THE ANSWER SERIES Your Key to Exam Success

24%

5.2 If $\cos\alpha = p$, express the following expression in terms of p:

$$\frac{\cos\left(\frac{\alpha}{2} - 45°\right)\sin\left(\frac{\alpha}{2} - 45°\right)}{2}$$

(4)

[12]

Memo

5.2

$$\frac{\cos\left(\frac{\alpha}{2} - 45°\right)\sin\left(\frac{\alpha}{2} - 45°\right)}{2} \quad \ldots \cos x \sin x$$

$$= \frac{\frac{1}{2}\left[2\cos\left(\frac{\alpha}{2} - 45°\right)\sin\left(\frac{\alpha}{2} - 45°\right)\right]}{2} \quad \ldots \frac{1}{2}(2\cos x \sin x)$$

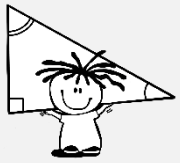
$$= \frac{\frac{1}{2}\cdot\sin 2\left(\frac{\alpha}{2} - 45°\right)}{2} \quad \ldots \frac{1}{2}\sin 2x$$

$$= \frac{1}{4}\sin(\alpha - 90°)$$

$$= -\frac{1}{4}\sin(90° - \alpha)$$

$$= -\frac{1}{4}\cos\alpha$$

$$= -\frac{1}{4}p \checkmark$$



Common Errors and Misconceptions

(d) **Q5.2** was poorly answered by many candidates as they failed to multiply the numerator and denominator by 2. In doing so, they would have created the expansion of the sine double angle in the numerator which would have eased their simplification. Instead, many candidates expanded the numerator by using the sine and cosine compound angle expansions. They made many algebraic mistakes in the process. Some candidates incorrectly replaced a with p instead of cos a with p. They did not know the difference between the angle and the value of the ratio.

Copyright © The Answer Series: Photocopying of this material is illegal

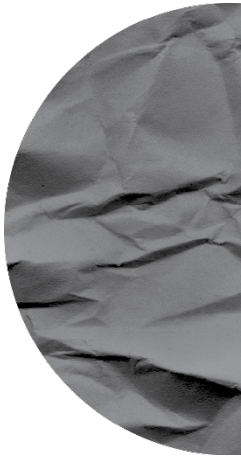
THE ANSWER SERIES Your Key to Exam Success

QUESTION 5: Suggestions for Improvement



(a) Teachers should ensure that all learners are able to select the relevant quadrant when drawing sketches in the Cartesian plane to calculate trigonometric ratios. Regular revision of Grade 10 and 11 Trigonometric concepts can help consolidate this work.

(b) As stated in previous reports, teachers must remind learners that the same simplification skills used in Algebra also apply to Trigonometry. Revise addition, subtraction and simplification of algebraic fractions with learners before teaching simplification of trigonometric expressions and proving trigonometric identities.



(c) Teachers should emphasise the use of the information sheet when working with compound angles.

(d) Expose learners to questions on trigonometric ratios involving combinations of compound angles, angles greater than 360° and co-ratios.

(e) As mentioned in previous reports, teachers must discuss the difference between an angle and a trigonometric ratio at the beginning of the study of Trigonometry in Grade 10. The relevance of an angle in the trigonometric ratio must be emphasised.

(f) Teachers should expose learners to a few methods of simplifying trigonometric expressions. However, they should develop skills in learners that allow them to answer questions in the most efficient way.

Copyright © The Answer Series: Photocopying of this material is illegal

THE ANSWER SERIES *Your Key to Exam Success*

QUESTION 6 45%

67%

6.1 Given the identity: $\cos(x-y) = \cos x \cos y + \sin x \sin y$

6.1.1 Use the compound angle identity given above to derive a formula for $\cos(x+y)$. (2)

6.1.2 Hence, or otherwise, show that:

$$\frac{\cos(90°-x)\cos y + \sin(-y)\cos(180°+x)}{\cos x \cos(360°+y) + \sin(360°-x)\sin y} = \tan(x+y) \quad (6)$$

Memo

6.1.1 Bookwork

6.1.2 LHS $= \dfrac{\sin x . \cos y + (-\sin y)(-\cos x)}{\cos x \cos y + (-\sin x)\sin y}$

$= \dfrac{\sin x \cos y + \sin y \cos x}{\cos x \cos y - \sin x \sin y}$

$= \dfrac{\sin(x+y)}{\cos(x+y)}$

$= \tan(x+y)$

= RHS ✓



Common Errors and Misconceptions

(a) **Q6.1.1** tested bookwork. However, many candidates were unable to derive the expression for $\cos(x+y)$ using the identity for $\cos(x-y)$. Instead, candidates merely wrote down the expansion as given in the information sheet. They did not receive any marks for their effort.

(b) Although **Q6.1.2** was well answered by many candidates, some candidates used the compound angle formulae to expand $\cos(90° - x)$ or $\sin(360° - x)$ instead of using reduction formulae. Some learners failed to show all the steps in their working. They went directly from $\sin(-y).\cos(180° + x)$ to $\sin y.\cos x$ instead of first writing $(-\sin y)(-\cos x)$. These candidates were penalised as they did not clearly demonstrate full understanding of the reduction formulae.

Copyright © The Answer Series: Photocopying of this material is illegal

THE ANSWER SERIES Your Key to Exam Success

6.2 Given: $f(x) = \sqrt{6\sin^2 x - 11\cos(90° + x) + 7}$

52% Solve for x in the interval $x \in (0°; 360°)$ if $f(x) = 2$. (6)

Memo

6.2 $f(x) = \sqrt{6\sin^2 x - 11(-\sin x) + 7} = 2$

$\therefore\ 6\sin^2 x + 11\sin x + 7 = 4$

$\therefore\ 6\sin^2 x + 11\sin x + 3 = 0$

$\therefore\ (2\sin x + 3)(3\sin x + 1) = 0$

$\therefore\ \sin x \neq -\frac{3}{2}$ ... not possible $\because$ $-1 \leq \sin\theta \leq 1$ for all θ

or $\sin x = -\frac{1}{3}$

$\therefore\ x = 180° + 19{,}47°$ or $360° - 19{,}47$

$= \mathbf{199{,}47°}$ or $\mathbf{340{,}53°}$ ◄

Common Errors and Misconceptions

(c) The following were noted in **Q6.2**: some candidates equated $f(x)$ to 0 instead of 2 as given in the question. Many candidates incorrectly reduced $-11\cos(90° + x)$ to $-11\sin x$ instead of $11\sin x$.

Some candidates failed to factorise the quadratic expression in $\sin x$ correctly. A fair number of candidates were able to factorise $6\sin^2 x + 11\sin x + 3$ correctly but then incorrectly stated that $x = -\frac{1}{3}$ or $x = -\frac{3}{2}$ instead of $\sin x = -\frac{1}{3}$ or $\sin x = -\frac{3}{2}$. Some candidates left their answers as general solutions and not answers in the third and fourth quadrants as required.

Copyright © The Answer Series: Photocopying of this material is illegal

THE ANSWER SERIES *Your Key to Exam Success*

14%

6.3 Consider the function: $g(x) = \frac{4 - 8\sin^2 x}{3}$

6.3.1 Calculate the maximum value of g. (3)

6.3.2 Write down the smallest possible value of x for which g will have a maximum value in the interval $x \in (0°; 360°]$. (1)

[18]

Memo

6.3.1 Maximum of $g(x) = \frac{4 - 8(0)}{3} = \frac{4}{3}$ ◄

... *when* $\sin^2 x$ *is as small as possible*

6.3.2 $\sin^2 x = 0$ ... *see 6.3.1*

$\therefore \sin x = 0$

$\therefore x = 0° + n(180°), n \in \mathbb{Z}$

$\therefore$ **Smallest value of x = 180°** ◄ ... $x \in (0°; 360°]$

OR: 6.3.1 $g(x) = \frac{4(1 - 2\sin^2 x)}{3} = \frac{4\cos 2x}{3}$

Maximum of $g(x) = \frac{4(1)}{3} = \frac{4}{3}$ ◄

... *when* $\cos 2x$ *is as big as possible*

6.3.2 $\cos 2x = 1$

$\therefore 2x = 0° + n(360°), n \in \mathbb{Z}$

$\therefore x = 0° + n(180°), n \in \mathbb{Z}$

$\therefore$ **Smallest value of x = 180°** ◄ ... $x \in (0°; 360°]$

Common Errors and Misconceptions



(d) In **Q6.3.1** many candidates substituted values of x into the given expression and then concluded that the maximum value of the expression occurred at $x = 0°$. Some candidates incorrectly stated that the expression will have a maximum when $x = \frac{-b}{2a}$ instead of when $\sin x = \frac{-b}{2a}$. These candidates also arrived at the answer $x = 0$. Many candidates were unsuccessful in their attempt to calculate the derivative of the given expression. They did not apply the Chain rule correctly and omitted $\cos x$ from their derivative.

(e) The answer to **Q6.3.2** depended on calculating the answer to **Q6.3.1** correctly. As a result, many candidates were unable to answer **Q6.3.2** correctly. Candidates failed to notice that the value of 0° was excluded from the domain. Some candidates gave the answer as an interval without realising that the function will attain a maximum value at a single point.

Copyright © The Answer Series: Photocopying of this material is illegal

THE ANSWER SERIES *Your Key to Exam Success*

QUESTION 6: Suggestions for Improvement



(a) *CAPS* indicates that the derivation of the compound angle formulae is examinable. Teachers should teach the derivation of these formulae.

(b) Teachers should remind learners that they must still use the reduction formulae together with the compound angle formulae when answering questions in Grade 12.

(c) Teachers should stress the importance of showing the signs when reducing trigonometric ratios.

(d) Teachers should advocate the use of the k-method when dealing with quadratic equations involving trigonometric ratios. A simplified quadratic equation may be easier to solve.

(e) Teachers should explain the difference between the general solution and the specific solutions within an interval. This point was covered in previous reports.

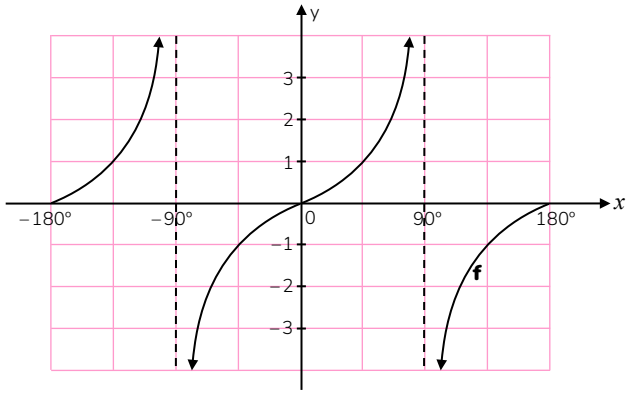
(f) Expose learners to different types of exercises involving inequalities, writing and interpreting intervals and working with angles in different quadrants.

Copyright © The Answer Series: Photocopying of this material is illegal

THE ANSWER SERIES Your Key to Exam Success

QUESTION 7 35%

In the diagram below, the graph of $f(x) = \tan x$ is drawn for the interval $x \in [-180°; 180°]$.



7.1 Write down the equation of the asymptote of f in the interval $x \in [0°; 180°]$. (1)

70%

7.2 Write down the values of x in the interval $x \in [-180°; 0°]$ for which $f(x) \leq 0$. (2)

23%

Memo

7.1 $x = 90°$ ◄

7.2 $x = -180°$ or $-90° < x \leq 0°$ ◄

Common Errors and Misconceptions

(a) In **Q7.1** some candidates incorrectly gave the answer as y = 90° or just 90° instead of $x = 90°$. Many candidates ignored the domain specified and gave $x = -90°$ as an answer. Other candidates gave the answer as an interval. This was incorrect.

(b) Many candidates did not attempt **Q7.2**. Many of those who attempted this question failed to realise that $x = -180°$ was also a solution. Some candidates confused which of the endpoints to include. They gave the answer as $x \in [-90° ; 0°]$ instead of $x \in (-90° ; 0°]$.

Copyright © The Answer Series: Photocopying of this material is illegal

THE ANSWER SERIES Your Key to Exam Success

63%

7.3 Given: $g(x) = \cos 2x + 1$

7.3.1 Write down the period of g. (1)

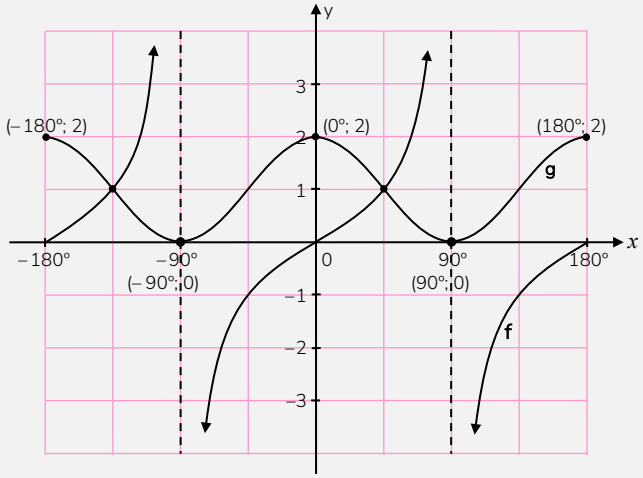
7.3.2 On the grid given above, draw the graph of $g(x) = \cos 2x + 1$ for the interval $x \in [-180°; 180°]$.

Clearly show the intercepts with the axes as well as the coordinates of the turning points. (3)

Memo

7.3.1 **180°** ➤

7.3.2



Common Errors and Misconceptions

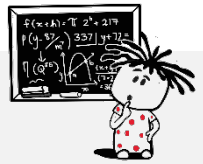
(b) While **Q7.3.1** was well answered by most candidates, some incorrectly gave the answer as 90° or 360° while others incorrectly gave the answer as an interval $x \in [-180° ; 180°]$.

(c) When answering **Q7.3.2** some candidates overlooked the vertical translation of the graph. They sketched the graph of $y = \cos 2x$ and left it at that.

15%

7.4 Use the graphs to determine the general solution of $2\cos^3 x - \sin x = 0$. (4)

[11]



Memo

7.4

$2\cos^3 x - \sin x = 0$

$(\div \cos x) \quad \therefore\ 2\cos^2 x - \tan x = 0$

$\therefore\ 2\cos^2 x = \tan x$

$\therefore\ 2\cos^2 x - 1 + 1 = \tan x$

$\therefore\ \cos 2x + 1 = \tan x$

$\therefore\ g(x) = f(x)$

$\tan x = 1 \Rightarrow \cos 2x + 1 = 1$ ... *f & g intersect*

$\therefore\ \cos 2x = 0$

$\therefore\ 2x = 90° + n(360°)$

$\therefore\ x = 45° + n(180°); n \in \mathbb{Z}$ ➤

(OR: By inspection, from the graphs)

Common Errors and Misconceptions

(d) Most candidates could not link the equation given in **Q7.4** to the equations of the graphs. Hence, they were unable to use the graphs to establish the answers. Other candidates made the link between the given equation and the graphs. However, instead of giving the general solution, they only gave the x-values for which the graphs intersected as the solution.

Copyright © The Answer Series: Photocopying of this material is illegal

THE ANSWER SERIES *Your Key to Exam Success*

QUESTION 7: Suggestions for Improvement



(a) It is necessary for learners to be reminded constantly of the meaning of concepts like *period*, *domain*, *amplitude* and *range*.

(b) As mentioned in previous reports, learners should be told that the period of a trigonometric function is the length of a function's cycle. Since this value is a length, it is a single number and not an interval of values.

(c) Learners should be shown how to write intervals, using both inequality and interval notations. Teachers are encouraged to use both forms of notations in class. It is good practice to write an interval in one form and then ask learners to write the same answer in the other form.

(d) Teachers should teach the 'mother graphs' well so that learners can develop insight into their characteristics. Thereafter, learners must be exposed to how the change in the different parameters affect the 'mother graphs'.

Copyright © The Answer Series: Photocopying of this material is illegal

THE ANSWER SERIES *Your Key to Exam Success*

QUESTION 8 40%

In the diagram, C is the foot of a vertical building and D is the top of the same building.

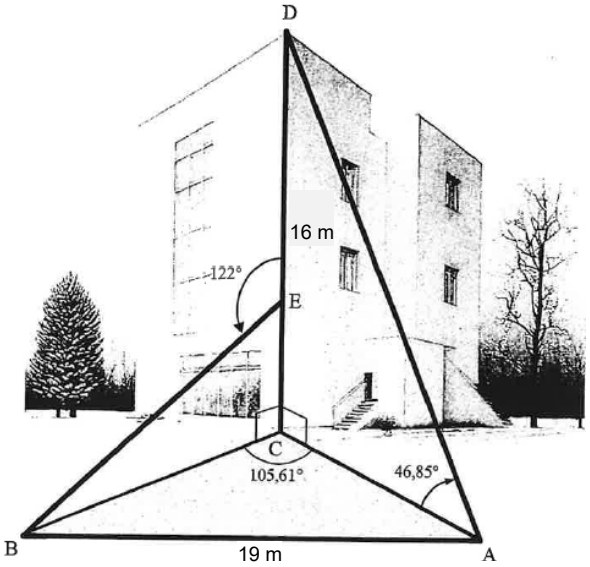
The height of the building, CD, is 16 m.

Two observers are standing 19 m apart at points A and B, where A, B and C lie in the same horizontal plane.

A painter is working at point E on the building.

The angle of elevation of D from A is 46,85°.

$D\hat{E}B = 122°$ and $B\hat{C}A = 105{,}61°$.



8.1 Calculate the length of AC, the distance between the observer at A and the foot of the building. (2)

62%

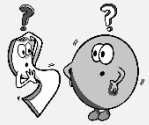
Memo

8.1 In ΔDCA: $\frac{16}{AC} = \tan 46{,}85°$

$\therefore AC = \frac{16}{\tan 46{,}85°}$

$= 14{,}998...$

$\simeq$ **15,00 m** ✓



Common Errors and Misconceptions

(a) In **Q8.1** some candidates chose the incorrect trigonometric ratio to calculate AC. They used sin 46,85° or cos 46,85° instead of tan 46,85°. Other candidates failed to identify the opposite and adjacent sides with reference to 46,85°. They wrote $\tan 46{,}85° = \frac{AC}{16}$ instead of $\tan 46{,}85° = \frac{16}{AC}$. Some candidates made mistakes when making AC the subject of the formula. They rewrote $\tan 46{,}85° = \frac{16}{AC}$ incorrectly as $AC = 16 \tan 46{,}85°$.

Copyright © The Answer Series: Photocopying of this material is illegal

8.2 Calculate how far the painter at E is from the top of the building. (7)
[9]

40%

Memo

8.2 In ΔABC: $\frac{\sin C\hat{B}A}{15} = \frac{\sin 105{,}61°}{19}$

$\therefore \sin C\hat{B}A = \frac{15 \sin 105{,}61°}{19}$

$= 0{,}76....$

$\therefore C\hat{B}A = 49{,}50°$

Then: $C\hat{A}B = 180° - (49{,}50 + 105{,}61°)$ ... $\angle^s$ *of* Δ

$= 24{,}89°$

$\therefore BC^2 = 15^2 + 19^2 - 2(15)(19) \cos 24{,}89°$

$= 68{,}94$

$\therefore BC \simeq 8{,}30$ m



In ΔEBC: $B\hat{E}C = 180° - 122° = 58°$

$\therefore E\hat{B}C = 32°$ ... $\angle^s$ *on a str line*

$\therefore \frac{EC}{BC} = \tan 32°$

$\therefore EC = BC \tan 32° = 8{,}30 \times \tan 32°$

$\simeq 5{,}19$ m

$\therefore$ **DE $\simeq$ 16 – 5,19 $\simeq$ 10,81 m** ◄

Common Errors and Misconceptions

(b) **Q8.2** required candidates to perform several calculations to arrive at the answer. This proved to be beyond many candidates. Some candidates assumed that AC = BC. This led to inappropriate values for EC. Some candidates made errors when substituting into the cosine rule, especially since BC appeared in two places in the formula.

Other candidates made errors in their calculations that led to distances obtained that did not match the given information.

Copyright © The Answer Series: Photocopying of this material is illegal

THE ANSWER SERIES *Your Key to Exam Success*

QUESTION 8: Suggestions for Improvement



(a) Teachers should devote the appropriate amount of time to this section. This should allow learners to score the accessible marks in this section of work.

(b) Learners need to develop strategies to be used when solving right-angled triangles and triangles that are not right-angled. Teach learners when to use basic trigonometric ratios and which basic ratio is appropriate for a given context.

(c) As mentioned previously, it might be a good idea to give learners an exercise in which they identify which rule is to be used to solve the question. The learners must also substantiate why they think that the rule that they have selected applies to the question.

(d) Remind learners that the sine and cosine rules are applicable to a single triangle. Learners may not create a proportion by using the sides and angles of two different triangles using the sine and cosine rules.

(e) Learners need to be reminded constantly that they may not make assumptions about the lengths of sides and the sizes of angles based on the diagram. Learners must work with the information that they are given in the question.

Copyright © The Answer Series: Photocopying of this material is illegal

THE ANSWER SERIES Your Key to Exam Success

EUCLIDEAN GEOMETRY (51%; 32%; 33%): DBE NOV 2024

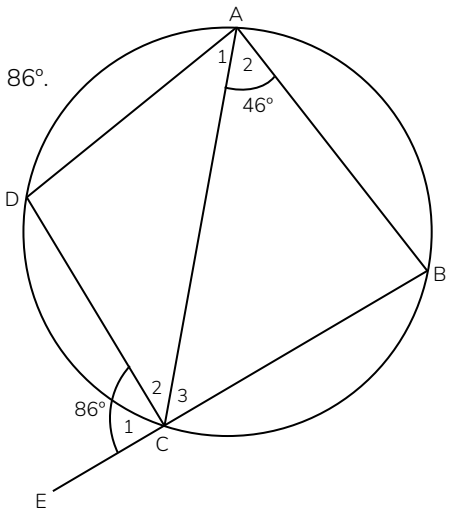
QUESTION 9 51%

In the diagram, ABCD is a cyclic quadrilateral.

BC is produced to E.

AC is drawn.

$\hat{A}_1 = \frac{1}{2}\hat{B}$, $\hat{A}_2 = 46°$ and $\hat{C}_1 = 86°$.



9.1 Calculate, with a reason, the value of $\hat{A}_1$. (2)

67%

9.2 Hence, prove that AD = DC. (4)

51%

[6]



Memo

9.1 $\hat{C}_1 = \hat{A}_1 + \hat{A}_2$ ... *ext ∠ of c.q.*

$\therefore 86° = \hat{A}_1 + 46°$

$\therefore \hat{A}_1 = 40°$ ✓

9.2 $A\hat{C}E = \hat{A}_2 + \hat{B}$ ... *ext ∠ of Δ*

$\therefore \hat{C}_2 + 86° = 46° + 2\hat{A}_1$

$\therefore \hat{C}_2 = -40° + 2(40°)$

$\therefore \hat{C}_2 = 40°$

$\therefore \hat{C}_2 = \hat{A}_1$ ... *both = 40°*

$\therefore$ **AD = DC** ✓ ... *sides opp = ∠s (in Δ)*

(or) *equal chords; equal ∠s*

Common Errors and Misconceptions

(a) In **Q9.1** many candidates assumed that ECB was a tangent to the circle. Hence, they stated that $\hat{A}_1 = 86°$ instead of 40°. Some candidates confused the cyclic quadrilateral theorems and regarded the opposite angles of the cyclic quadrilateral to be equal instead of being supplementary. Other incorrect assumptions made were that $\hat{C}_1$ was the exterior angle of triangle ADC and that AB || DC and therefore the corresponding angles were equal.

(b) Some candidates, in answering **Q9.2**, stated that $\hat{C}_2 = \hat{A}_1$ without proving it. Many candidates incorrectly used $\hat{B} = \frac{1}{2}\hat{A}_1$ instead of $\hat{A}_1 = \frac{1}{2}\hat{B}$. Some candidates incorrectly assumed that AC was the diameter of the circle and therefore incorrectly stated that $\hat{D} = 90°$.

Copyright © The Answer Series: Photocopying of this material is illegal

THE ANSWER SERIES Your Key to Exam Success

QUESTION 9: Suggestions for Improvement



(a) As mentioned in previous reports, teachers are encouraged to use the 'Acceptable Reasons' in the *Examination Guidelines* when teaching. This should start from as early as Grade 8. Learners should be issued with a copy of the 'Acceptable Reasons'.

(b) Teachers should make use of a diagram with annotations to explain a theorem. Illustrate which information is given and what conclusions can be made from this given information.

(c) Teachers must insist that learners read the information given in the question. This information contains key words that direct learners to the theory required to solve the question.

(d) Teach learners to identify all the theorems that are applicable to a question and how to select which ones can be used to answer the question.

(e) Teachers must make learners aware that they are not allowed to draw additional lines on a diagram. In doing so, they are changing the question, and this is not acceptable.

Copyright © The Answer Series: Photocopying of this material is illegal

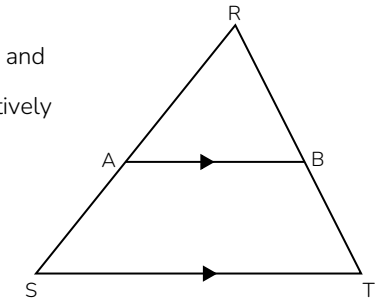
THE ANSWER SERIES Your Key to Exam Success

QUESTION 10 32%

48%

10.1 In the diagram, ΔRST is drawn.

Line AB intersects RS and RT at A and B respectively such that AB || ST.



Prove the theorem which states that a line drawn parallel to one side of a triangle divides the other two sides proportionally, i.e. $\frac{RA}{AS} = \frac{RB}{BT}$ (6)

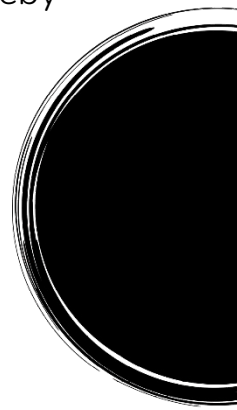
Memo

10.1 Theorem



Common Errors and Misconceptions

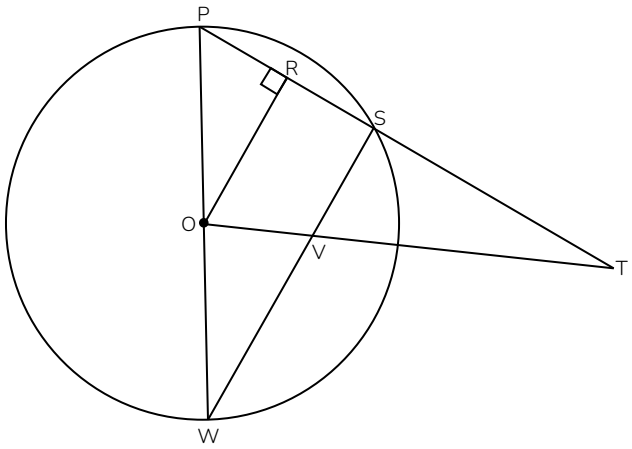
(a) In **Q10.1** some candidates did not draw the constructions nor did they state the constructions necessary to prove the theorem. Some omitted the perpendicular symbols for the heights of the triangles. These candidates were not awarded any marks. Some candidates used the incorrect heights for the triangles they were working with. Some candidates chose the incorrect triangles and therefore were unable to prove the theorem. Some learners renamed the given triangle, thereby changing the question. This was not accepted.



Copyright © The Answer Series: Photocopying of this material is illegal

THE ANSWER SERIES Your Key to Exam Success

10.2 In the diagram, O is the centre of the circle.
27% ΔPWS is drawn with P, W and S on the circle.
OR ⊥ PS. PRS is produced to T.
SW and OT intersect at V. OV:OT = 1:4



10.2.1 Prove, with reasons, that OR : WS = 1 : 2 (5)

Memo

10.2.1 In ΔPWS: O midpoint PW
& R midpt PS ... *line from centre ⊥ to chord*
∴ OR = $\frac{1}{2}$ WS ... *Midpt Thm*
i.e. **OR : WS = 1 : 2** ◄


Always remember the Midpt Thm!

OR: $P\hat{S}W$ = 90° ... ∠ *in semi-⊙*
∴ $P\hat{S}W$ = $P\hat{R}O$ (= 90°)
∴ In ΔPOR & ΔPWS
(1) $\hat{P}$ is common
(2) $P\hat{R}O$ = $P\hat{S}W$ ... *both = 90°*
∴ ΔPOR ||| ΔPWS ... ∠∠∠
∴ OR:WS = PR:PS ... ||| Δ^s
But R midpt PS ... *OR ⊥ chord PS*
∴ PR:PS = 1:2
∴ **OR:WS = 1:2** ◄

Copyright © The Answer Series: Photocopying of this material is illegal

Common Errors and Misconceptions

(b) When answering **Q10.2.1**, some candidates applied the midpoint theorem without first showing that both O and R were midpoints. Some candidates assumed that lines OR and WS were parallel in the diagram without proving that they were. The solution of many candidates lacked logic in their presentation. They presented the necessary information, but the sequence of the steps created gaps in the logic. These candidates were not awarded full marks.

THE ANSWER SERIES Your Key to Exam Success

10.2.2 Calculate the length of PT if ST = 15 units. (4)
[15]

Memo

10.2.2 $\hat{PSW} = 90°$ ... $\angle$ *in semi* $\odot$

$\therefore \hat{PRO} = \hat{PSW}$

$\therefore$ (W)VS || OR ... *corresp* $\angle^s$ *equal*

$\therefore$ In ΔROT: $\frac{RS}{RT} = \frac{OV}{OT}$ ... *prop thm*

$= 1:4$

$\therefore \frac{RS}{RT} = \frac{1}{3}$

& $ST = 15$



$\therefore RS = \frac{15}{3} = 5$ units

$\therefore PT = 2RS + ST = 2(5) + 15 = \mathbf{25\ units}$ ◄

Common Errors and Misconceptions

(c) **Q10.2.2** was poorly answered by many candidates. Some candidates applied the proportionality theorem without first proving that OR || WS. This was considered a breakdown. Many candidates lost a mark for not stating the parallel lines in the reason 'proportionality theorem'. Some candidates chose the incorrect sides when using the proportionality theorem to calculate the length of PT. For example, they stated that $\frac{PT}{PS} = \frac{OT}{OV}$ but PT was not a side of triangle ROT.

THE ANSWER SERIES *Your Key to Exam Success*

QUESTION 10: Suggestions for Improvement



(a) Learners should be taught that a construction is required to prove a theorem. If the construction is not shown, then the proof is regarded as a breakdown, and they receive no marks.

(b) Learners should be made to prove theorems as part of their informal tasks. A good strategy is to expect learners to write the proof of a theorem as a task the day after the theorem was explained in class. Teachers should also choose random letters to label the triangles and not stick to the conventional A, B and C.

(c) As mentioned in previous reports, learners should be taught that all statements must be accompanied by reasons. It is essential that the parallel lines be mentioned when stating that corresponding angles are equal, alternate angles are equal, and the sum of the co-interior angles is 180° or when stating the proportional intercept theorem.

(d) It is advisable to train learners to reason logically and to write corresponding statements and reasons when teaching Euclidean Geometry in Grade 8. This should enable learners to present coherent proofs or solutions in Grade 12.

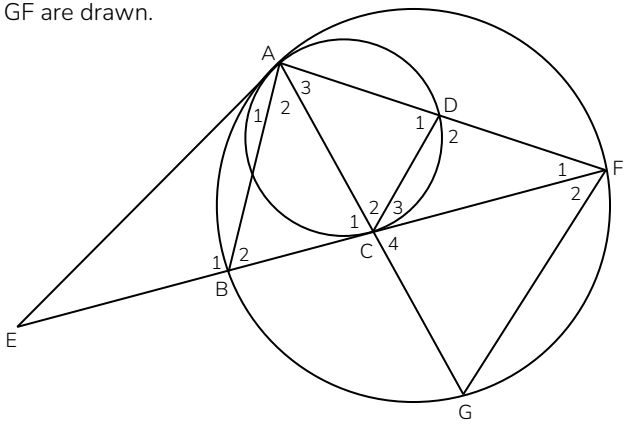
(e) Teachers should point out to learners that the *midpoint theorem* is a special case of the *proportionality theorem*.

Copyright © The Answer Series: Photocopying of this material is illegal

THE ANSWER SERIES *Your Key to Exam Success*

QUESTION 11 33%

In the diagram, A, B, G and F lie on the larger circle.
A smaller circle is drawn to touch the larger circle internally at A.
EA is a common tangent to both circles.
EBCF is a tangent to the smaller circle at C.
AC is produced to G. AF cuts the smaller circle at D.
AB, CD and GF are drawn.



11.1 If $E\hat{A}G = x$, determine with reasons, FOUR other angles that are equal to x. (6)

51%

11.2 Prove that AG.AD = AC.AF (4)

41%

Memo

11.1 [$E\hat{A}G = x$ ... *given*]

$A\hat{F}G = x$ ◄ ... *tan chord thm (larger ⊙)*
& $\hat{D}_1 = x$ ◄ ... *tan chord thm (small ⊙)*
$\hat{C}_1$ ◄ = $\hat{D}_1$ (= x) ◄ ... *tan chord thm*
$\hat{C}_4$ ◄ = vert opp $\hat{C}_1$ (= x) ◄

11.2 ∴ $\hat{D}_1 = A\hat{F}G$ ... *both = x in 11.1*
∴ CD || GF ... *corresp ∠s =*
∴ In ΔAGF: $\frac{AG}{AC} = \frac{AF}{AD}$ ... *prop thm; CD || GF*
∴ AG.AD = AC.AF ◄

Copyright © The Answer Series: Photocopying of this material is illegal

Common Errors and Misconceptions

(a) Many candidates did not name angles in a precise and correct way when answering **Q11.1**. For example, they wrote $\hat{F}$. In the context of this question, $\hat{F}$ could refer to three different angles: $\hat{F}_1$ or $\hat{F}_2$ or $A\hat{F}G$. Some candidates stated that $E\hat{A}G = \hat{C}_1$ and provided the reason 'tangents from a common point'. These candidates combined two steps into one. They were not credited with any marks because the reason did not correspond to the statement. Some candidates assumed that if $E\hat{A}G = x$, then $\hat{A}_1 = x$ or $\hat{A}_2 = x$.



(b) In **Q11.2** many candidates assumed that CD || GF and immediately started with a ratio of sides. This was not accepted as candidates had to first prove that CD || GF. When proving that CD || GF, many provided the incorrect reasons: 'corresponding angles', 'converse corresponding angles' or 'corresponding angles present' instead of corresponding angles are equal.

THE ANSWER SERIES Your Key to Exam Success

11.3 Prove that $\Delta AGF \,|||\, \Delta ABC$ (4)

50%

11.4 Prove that $GF^2 = \frac{BC.FC.AF}{AD}$ (6)

10%

[20]

TOTAL: 150

Memo

11.3 In Δ^s AGF & ABC

(1) $\hat{G} = \hat{B}_2$ (= y) ... $\angle^s$ *in the same segment*

(2) $A\hat{F}G = \hat{C}_1$ ... *both = x in 11.1*

$\therefore \Delta AGF \,|||\, \Delta ABC$ ◄ ... $\angle\angle\angle$

11.4 $\therefore \frac{GF}{BC} = \frac{AF}{AC} \left(= \frac{AG}{AB}\right)$

$\therefore GF = \frac{BC.AF}{AC}$ ... ❶

Note

Mark the sides you **still** need: GF.FC, AD and, also AC (to cancel)



But, in Δ^s GFC and CAD

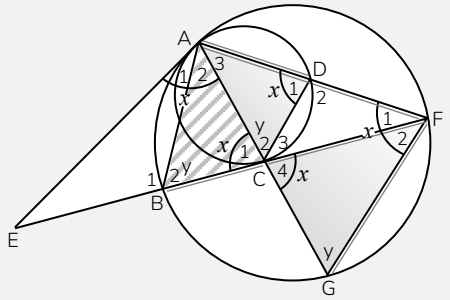
1. $\hat{G} = \hat{C}_2$ (= y, say) ... *corresp* $\angle^s$*; CD || GF in 11.2*
2. $\hat{D}_1 = \hat{C}_4$ (= x, in 11.1)

$\therefore \Delta GFC \,|||\, \Delta CAD$ ... $\angle\angle\angle$

$\therefore \frac{GF}{AC} = \frac{FC}{AD}$

$\therefore GF = \frac{FC.AC}{AD}$ ... ❷

❶ × ❷: $\therefore GF^2 = \frac{BC.AF}{AC} \times \frac{FC.AC}{AD} = \frac{BC.FC.AF}{AD}$ ◄



Common Errors and Misconceptions

(c) When answering **Q11.3** many candidates stated that $\hat{A}$ was common. Some even stated that $\hat{A}_2 = \hat{A}_3$ and gave the reason that they were common even though these were angles in two different triangles. When proving the two triangles similar, some candidates used the reason 'remaining angles' or 'sum of angles in triangle' for the first pair of equal angles. They failed to understand that remaining angles is a consequence of two other pairs of angles being equal in both triangles.

(d) In **Q11.4** most of the candidates found it challenging to correctly identify which pair of triangles they should prove similar. Some candidates were able to write down the correct proportions from the pairs of similar triangles but were unable to link these proportions to prove that $GF^2 = \frac{BC.FC.AF}{AD}$.

Copyright © The Answer Series: Photocopying of this material is illegal

THE ANSWER SERIES Your Key to Exam Success

QUESTION 11: Suggestions for Improvement



(a) As mentioned in previous reports, more time needs to be spent on the teaching of Euclidean Geometry in all grades. Learners should read the given information carefully without making any assumptions. Exercises on Grade 11 and 12 Euclidean Geometry must include different activities and all levels of the taxonomy.

(b) The need for learners to name the angles correctly has been mentioned in several reports previously. Teachers should not credit learners with marks in school-based assessment tasks if the angles are not named correctly.

(c) Teach learners not to assume any facts in a geometry sketch but to use only what was given and that which had already been proven in earlier questions.

(d) Learners need to be made aware that writing correct statements that are irrelevant to the answer in Euclidean Geometry will not earn them any marks in an examination.

(e) Consider teaching the approach of 'angle chasing' where you label one angle as x and then relate other angles to x. In this way, learners should find it easy to identify angles that are equal but moreover, they should find it easier to establish the reasons for the relationships between the angles.

(f) Learners need to be told that success in answering Euclidean Geometry comes from regular practice, starting off with the easy and progressing to the difficult.

Copyright © The Answer Series: Photocopying of this material is illegal

THE ANSWER SERIES *Your Key to Exam Success*