GRADE
12

THE ANSWER SERIES

MATHEMATICS

TAS-style Diagnostic Report

Each Question + its Memo, each with

- **Common Errors & Misconceptions**
- **Suggestions to Teachers for Improvement**



November Exam
Paper 1
2024

ALGEBRA (69%): DBE NOVEMBER 2024

QUESTION 1 69%

74%

1.1 Solve for x:

1.1.1 $(x-3) = 0$ (2)

1.1.2 $2x^2 + 1 = 4x$ *(correct to TWO decimal places)* (4)

1.1.3 $x^2 - 2x - 3 > 0$ (4)

Memo

1.1.1 $x(x-3) = 0$

$\therefore x = 0$ ◄ or $x - 3 = 0$

$\therefore x = 3$ ◄

1.1.2 $2x^2 + 1 = 4x$

$2x^2 - 4x + 1 = 0$

$\therefore x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(1)}}{2(2)}$ ...

$= \frac{4 \pm \sqrt{8}}{4}$

$\approx 1{,}71$ or $0{,}29$ ◄

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

1.1.3 $x^2 - 2x - 3 > 0$

$\therefore (x+1)(x-3) > 0$

Critical values: -1 & 3

$\therefore x < -1$ or $x > 3$ ◄



Common Errors and Misconceptions

(a) In **Q1.1.2** some candidates did not write the equation correctly in standard form, so they substituted incorrectly for the values of 'b' and 'c'. The candidates arrived at the standard form of $2x^2 + 1 - 4x = 0$ and then substituted into the quadratic formula to get $x = \frac{-(1) \pm \sqrt{(1)^2 - 4(2)(-4)}}{2(2)}$.

If candidates wrote the equation in the correct standard form, another common error was substituting for 'b' as 4 and not -4.

(b) In **Q1.1.3** many candidates treated the inequality as an equation and then struggled to interpret the answer to the question. Other candidates used the word 'and' in the solution which was incorrect. Notation still seems to a problem as some candidates left their answer as $-1 > x > 3$.

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1.1.4 $2^{2x} - 2^{x+2} - 32 = 0$ (5)

1.1.5 $\sqrt{-2x+4} - x = 2$ (4)

Memo

1.1.4 $2^{2x} - 2^{x+2} - 32 = 0$

$\therefore (2^x)^2 - 2^x \cdot 2^2 - 32 = 0$

$\therefore (2^x)^2 - 4(2^x) - 32 = 0$

$\therefore (2^x + 4)(2^x - 8) = 0$

$2^x \neq -4$ ... $2^x > 0$ *for all* $x \in \mathbb{R}$

$\therefore 2^x = 8$

$\therefore x = 3$ only ◄

OR: Let $2^x = k$,

$k > 0 \quad \because 2^x > 0$

$\therefore k^2 - 4k - 32 = 0$

$\therefore (k-8)(k+4) = 0$

$\therefore k = 8$, etc.

1.1.5 $\sqrt{-2x+4} - x = 2$

$\therefore \sqrt{-2x+4} = x + 2$ ... ❶

$\therefore \left(\sqrt{-2x+4}\right)^2 = (x+2)^2$

$\therefore -2x + 4 = x^2 + 4x + 4$

$\therefore 0 = x^2 + 6x$

$\therefore x(x+6) = 0$

$\therefore x = 0$ ◄ $x \neq -6 \quad \because \sqrt{\ } \geq 0$ in ❶



Common Errors and Misconceptions

(c) In the exponential equation in **Q1.1.4**, many candidates applied exponential laws incorrectly, arriving at $2^{2x} - 2^{x+2} - 2^5 = 0$ which led them incorrectly to the equation $2x - x + 2 - 5 = 0$, but then to the correct answer of $x = 3$.



Another common error was when candidates did not distinguish between real and invalid solutions:

$2^x = 8$ or $2^x = -4$ leading to $x = 3$ or $x = -2$.

(d) In **Q1.1.4**, there were still a large number of candidates who did not check the validity of their solutions. $x = -6$ needed to be rejected in the answer.

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1.2 Solve for x and y simultaneously:

90%

$2x + y = 3 \qquad y^2 + xy = 2$ (5)

1.3 Consider the product $\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\ldots$

6%

Each factor in the product is of the form

$\left(1+\frac{1}{n}\right)$ for $n \geq 2$.

Determine ALL the values of n for which the product will be an integer value. (3)

[27]

Memo

1.2 $2x + y = 3$

$\therefore y = 3 - 2x \quad \ldots$ ❶

❷: $y^2 + xy = 2$

❶ in ❷: $\therefore (3-2x)^2 + x(3-2x) - 2 = 0$

$\therefore 9 - 12x + 4x^2 + 3x - 2x^2 - 2 = 0$

$\therefore 2x^2 - 9x + 7 = 0$

$\therefore (2x-7)(x-1) = 0$

$\therefore x = \frac{7}{2}$ or 1

❶: For $x = \frac{7}{2}$: $y = 3 - 2\left(\frac{7}{2}\right) = -4$

& For $x = 1$: $y = 3 - 2(1) = 1$

$\therefore$ Solution: $\left(\frac{7}{2}; -4\right)$ **or** **(1; 1)** ◄

1.3 $\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\left(1+\frac{1}{5}\right) \ldots \left(1+\frac{1}{n}\right)$

$= \left(\frac{3}{2}\right)\left(\frac{4}{3}\right)\left(\frac{5}{4}\right)\left(\frac{6}{5}\right) \ldots \left(\frac{n+1}{n}\right) \ldots$



Note the cancelling

$= \frac{n+1}{2}$

So, n + 1 must be any even number ≥ 4

$\therefore$ **n must be any odd number greater than 2** ◄

i.e. n = 3 ; 5 ; 7 ; 9 ; … ◄

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Common Errors and Misconceptions

(e) Most candidates were unable to establish the pattern generated in **Q1.3**. Candidates who were able to generate a pattern did not articulate themselves well in drawing the conclusion required.



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QUESTION 1: Suggestions for Improvement



(a) Learners must ensure that they understand what correct *standard form* is in a quadratic equation.

(b) Learners must be taught to check their solutions when using the squaring technique to solve an equation that is not originally quadratic, as well as to check the validity of solutions generated in exponential equations. Teachers must emphasise that implicit restrictions are placed on *surd* equations.

(c) Notation in quadratic inequalities should be emphasised. Graphical tools should be shown to the learners to ensure they understand what the mathematical notation means on a graph.

(d) Exponential rules, manipulation and equations should be practised. Learners should understand when to use the technique of equating exponents because the bases are the same, and when to factorise. These are techniques that are taught in earlier grades and should be revised from Grade 9 through to Grade 12. The importance of language used by teachers is emphasised here. Many understanding misconceptions can be attributed to language used by teachers in the classroom. An example of this is 'drop the bases' in an exponential equation.

(e) Regular revision and emphasis on working with *prime bases* in *exponents* is important.

(f) Problem-solving must be practised. Utilise Olympiad-style questions for practice. Learners should be taught to reason mathematically through argument either in words or in symbols.

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(g) Teachers must make time to revise algebraic and exponential law work with learners.

(h) Teachers should place more emphasis on understanding of concept/s being taught and reduce the emphasis on how learners can use the calculator to obtain answers. The answers become meaningful when learners understand the concept and this allows them to interpret their answers.

(i) As suggested in previous reports:

- Teachers should not take for granted that learners know how to round off a number to the required number of places. Where necessary, this skill should be retaught in Grades 11 and 12. Teachers should penalise learners in class work and SBA tasks when they do not round off to the correct number of places.
- Teachers should take some time, preferably in Grade 10, to focus on teaching learners how to represent inequalities (e.g. $-3 < x < 5$; $x < -3$ or $x > 5$) on a number line and how to also write an inequality from the illustration on a number line. This will benefit learners as they are required to write inequality solutions for a number of questions in both examination papers. Emphasis on the correct notation is essential when writing down the solutions to inequalities.
- Linked to this, teachers should explain the difference between 'and' and 'or' in the context of inequalities. Learners cannot use these words interchangeably as they have different meanings.

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PATTERNS (56%; 40%): DBE NOVEMBER 2024

QUESTION 2 56%

47%

2.1 The first term of an arithmetic series is 7. The common difference of this series is 5 and the series contains 20 terms.

2.1.1 Calculate the sum of this series. (2)

2.1.2 The original arithmetic series is extended to 75 terms. The sum of these 75 terms is 14 400. Using sigma notation, write down an equation for the sum of the terms added to the original series. (4)

Memo

2.1 **A.S:** $a = 7$; $d = 5$; $n = 20$

2.1.1 $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore S_{20} = \frac{20}{2}[2(7) + (20-1)(5)]$

$= 10(14 + 95)$

$= \mathbf{1\,090}$ ◄

2.1.2 $n = 75$; $S_{75} = 14\,400$ (& $a = 7$)

$T_n = a + (n-1)d$

$= 7 + (n-1)(5)$

$= 7 + 5n - 5$

$= 5n + 2$

$\therefore$ Sum of the terms added, T_{21} to T_{75},

$\therefore \sum_{n=21}^{75}(5n + 2) = S_{75} - S_{20} = 14\,400 - 1\,090 = \mathbf{13\,310}$ ◄

Common Errors and Misconceptions

(a) In **Q2.1.2** many candidates had difficulty understanding what needed to be done to arrive at an answer. Added to this, they had difficulty writing the answer in the correct *sigma notation*. Common erroneous answers were $\sum_{n=1}^{75}(5n + 2) = 14\,400$; $\sum_{n=21}^{75}(5n + 2)$ which did not equate sigma to the required sum of $\sum_{n=1}^{55}(5n + 2) = 14\,400$.



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64%

2.2 The sequence of the first differences of a quadratic pattern is: 1 ; 3 ; 5 ; ...

2.2.1 If T_{99} of this quadratic pattern is 9 632, calculate the value of T_{98}. (3)

Memo

2.2.1 T_1 T_2 T_3 T_4 ... T_{98} T_{99}

1 3 5 *

1st differences terms: $2n - 1$ ... *odd numbers*

$\therefore T_{99} - T_{98} = 2(98) - 1 = 195$ ... *

$\therefore \mathbf{T_{98}} = T_{99} - 195$

$= 9\,632 - 195$

$= \mathbf{9\,437}$ ◄

OR: T_1 T_2 T_3 T_4 T_5 ... T_{98} 9 632

1 3 5 195

$\therefore T_{99} = 9\,632 - 195 = 9\,437$

Explanation:

The 1st difference lies between T_1 and T_2.

The 2nd difference lies between T_2 and T_3.

The 98th difference lies between T_{98} and T_{99}.

The general term for 1; 3; 5; ... is given by $T_k = 2k - 1$

$\therefore T_{98} = 2(98) - 1 = 195$ (98th first difference)



2.2.2 If it is further given that the third term of the quadratic pattern is 32, determine the general term, T_n, of the quadratic pattern. (5) **[14]**

Memo

2.2.2 If $T_3 = 32$, then $T_2 = 32 - 3 = 29$

& $T_1 = 29 - 1 = 28$



1st differences: (28) 29 32

1 3 5

2nd differences: (1) 2

2 2

The general term, $\mathbf{T_n = an^2 + bn + c}$

$2a = 2$	$3a + b = 1$	$(T_1 =)\ a + b + c = 28$
$\therefore a = 1$	$\therefore 3 + b = 1$	$\therefore 1 - 2 + c = 28$
(common diff.)	$\therefore b = -2$	$\therefore c = 29$
	(1st 1st difference)	*(1st term)*

$\therefore \mathbf{T_n = n^2 - 2n + 29}$ ◄

Check: $T_{98} = 98^2 - 2(98) + 29 = 9\,437$



Common Errors and Misconceptions

(b) Many candidates did not recognise that the *linear pattern* given in **Q2.2.1** was the first difference of the *quadratic pattern* which would help them determine the required term value in the *quadratic pattern*. A large number of candidates used their answer in **Q2.2.2** to calculate the answer for **Q2.2.1**; or they calculated the *general term* of the *quadratic pattern* in both **Q2.2.1** and **Q2.2.2**.

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QUESTION 2: Suggestions for Improvement



(a) A solid foundation of *sigma notation* should be emphasised in the classroom. Learners should be exposed to both calculating from given *sigma notation*, and writing information from a given pattern into *sigma notation*.

(b) Teaching of *patterns* needs to include exposing learners to questions that require reading, understanding and interpretation, not just calculations. Even though time in the syllabus is limited, it is vital for learners to be able to answer questions of a different nature. Added to this, questions must be read carefully so that learners know what is required of them.

(c) As mentioned in previous reports, learners should be discouraged from using information provided in later questions to answer earlier questions in an examination.

(d) Teachers need to teach the relationships specifically between a *quadratic pattern* and a *linear pattern*, making particular reference to how terms are calculated by using the *sum* of *linear pattern terms* to generate a term in the *quadratic pattern*.

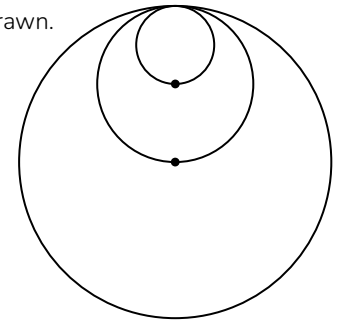
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QUESTION 3 40%

A circle with radius 6 cm is drawn.

A second, smaller circle is drawn through the centre of the first circle and also touches the first circle internally, as shown in the diagram.



A third, smaller circle is drawn through the centre of the second circle and touches the second circle internally.

The process of drawing circles continues and forms a geometric pattern.

3.1 Write down the radius of the 3rd circle. (2)

58%

3.2 Calculate the sum of the areas of the first 10 circles. (4)

35%

Memo

3.1 **G.S.:** The radii: 6 ; 3 ; $\frac{3}{2}$; … The radii halve each time

∴ **The radius of the 3rd circle is $\frac{3}{2}$ cm** ◄

3.2 $S_n = \frac{a(1-r^n)}{1-r}$

The areas of the circles are: 36π ; 9π ; $2{,}25\pi$; etc.

S_{10}? ; $n = 10$; $a = 36\pi$; $r = \frac{1}{4}$

$$\therefore S_{10} = \frac{36\pi\left[1-\left(\frac{1}{4}\right)^{10}\right]}{1-\frac{1}{4}} \simeq 150{,}80 \text{ cm}^2 \text{ ◄}$$

Common Errors and Misconceptions

(a) A large majority of the candidates interpreted **Q3.2** as requiring the sum of the first 10 radii and not the areas of the first 10 circles as required by the question. Those who did understand that the sum of the areas of the first 10 circles was required, omitted π in their solution or did not calculate the new ratio of the *geometric sequence* formed by the areas of the circles to be $\frac{1}{4}$.



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3.3 Which circle has a diameter of $\frac{3}{128}$ cm? (4)

36% **[10]**

Memo

3.3 Use $T_n = ar^{n-1}$ for the radii:

n? ; a = 6 ; r = $\frac{1}{2}$; $T_n = \frac{1}{2}$ of $\frac{3}{128} = \frac{3}{256}$

$\therefore \frac{3}{256} = 6.\left(\frac{1}{2}\right)^{n-1}$

$\therefore \left(\frac{1}{2}\right)^{n-1} = \frac{\cancel{3}}{256} \times \frac{1}{\cancel{6}_2}$ ...

$\therefore \left(\frac{1}{2}\right)^{n-1} = \frac{1}{2^8 \times 2}$

$\therefore \left(\frac{1}{2}\right)^{n-1} = \left(\frac{1}{2}\right)^9$

$\therefore n - 1 = 9$

$\therefore n = 10$ $\therefore$ **The 10th circle** ◄

OR: Use logs

$n - 1 = \log_{\frac{1}{2}}\left(\frac{1}{512}\right)$

$\therefore n - 1 = 9$

$\therefore$ **n = 10** ◄

Common Errors and Misconceptions

(b) The lack of understanding when reading contributed to many candidates not answering **Q3.3** well. Candidates incorrectly equated the general term of the radii with the diameter $\left[6\left(\frac{1}{2}\right)^{n-1} = \frac{3}{128}\right]$. Added to this, exponential laws were incorrectly applied. This meant candidates simplified $12\left(\frac{1}{2}\right)^{n-1} = \frac{3}{128}$ to $6^{n-1} = \frac{3}{128}$.



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QUESTION 3: Suggestions for Improvement



(a) Teachers should expose learners to real-life and mensuration problems in *patterns*, as well as other topics in the Mathematics curriculum.

(b) Learners should be encouraged to write out the first three terms of a *pattern* so that they are able to correctly identify the first term and the constant *ratio* or *first difference* in the *pattern*.

(c) Learners should be taught that when working with constants, such as π, they should not convert them to a decimal format as this can hinder the recognition of a possible pattern.

(d) Teachers should encourage learners to work with fractions, particularly in *geometric patterns*, *exponential graphs* and *hyperbolic functions*. Added to this, correct language should be used at all times by the teacher to ensure understanding of the algebraic methods are transferred to learners in the classroom. 'Tip and times' is a phrase that should be replaced with 'multiply by the reciprocal'.

(e) As suggested in previous reports:

- The inclusion of word problems in the *patterns* section is important. Teachers need to emphasise how to take the words of a problem and write them in symbolic form to solve an *equation* or *inequality*.
- Constant revision of *exponential laws* to solve equations correctly is pivotal to candidates' success. Teachers need to emphasise this and revise this thoroughly in all grades.

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FUNCTIONS (60%; 42%; 44%): DBE NOVEMBER 2024

QUESTION 4 60%

Given: $f(x) = a^x - 1$ for $a > 0$. B$\left(2; \frac{-5}{9}\right)$ is a point on f.

4.1 Calculate the value of a. (2)

82%

4.2 Write down the range of f. (1)

45%

Memo

4.1 Subst. B$\left(2; \frac{-5}{9}\right)$ in $y = a^x - 1$

$$\therefore \frac{-5}{9} = a^2 - 1$$

$$\therefore \frac{4}{9} = a^2$$

$$\therefore a = \frac{2}{3} \blacktriangleleft \quad \ldots a > 0$$

4.2 $f(x) = \left(\frac{2}{3}\right)^x - 1$

Range of f: $y > -1,\ y \in \mathbb{R}$ ◄

Common Errors and Misconceptions

(a) Candidates struggled to correctly answer the *range* of the *exponential function* in **Q4.2**, which meant their answer was incorrectly written as $y \in \mathbb{R}; y \neq -1$.

4.3 Sketch the graph of f. Clearly show the intercepts with the axes and asymptotes, if any. (3)

62%

4.4 It is further given that C is a point on f at $y = \frac{19}{8}$. Determine the coordinates of C', the image of C, when C is reflected about the line $y = x$. (3) **[9]**

48%

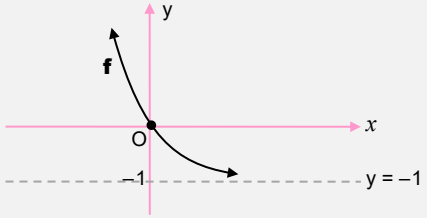
Memo

4.3 X-intercept ($y = 0$): $\left(\frac{2}{3}\right)^x - 1 = 0$

$$\left(\frac{2}{3}\right)^x = 1$$

$$\therefore x = 0 \blacktriangleleft$$

Y-intercept ($x = 0$): $y = \left(\frac{2}{3}\right)^0 - 1 = 1 - 1 = 0$



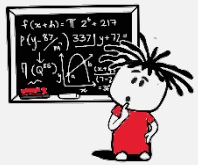
4.4 Subst. C$\left(x; \frac{19}{8}\right)$ in $y = \left(\frac{2}{3}\right)^x - 1$

$$\therefore \frac{19}{8} = \left(\frac{2}{3}\right)^x - 1$$

$$\therefore \frac{27}{8} = \left(\frac{2}{3}\right)^x$$

$$\therefore \left(\frac{3}{2}\right)^3 = \left(\frac{3}{2}\right)^{-x}$$

$$\therefore x = -3$$



$\therefore$ Pt C $\left(-3; \frac{19}{8}\right)$

$\therefore$ Pt C' $\left(\frac{19}{8}; -3\right)$ ◄ … *refl about* $y = x$ $(x; y) \rightarrow (y; x)$

Common Errors and Misconceptions

(b) A fair number of candidates drew a *hyperbola* in **Q4.3**. This showed a lack of understanding of the format of the equation of the function provided in the stem of **Q4**. The *asymptote* was not necessarily labelled or the drawn graph crossed through its *asymptote*.

(c) Many candidates did not solve for 'x' when $y = \frac{19}{8}$ which led them to assume that $x = 0$. Secondly, the coordinates of the image of C were not answered well as candidates reflected the point over the x-axis, y-axis or the line $y = -x$.

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QUESTION 4: Suggestions for Improvement



(a) Learners should be taught to calculate x- and y-intercepts when they are required to draw a graph. If the learner is still unsure, their default method should be to draw the graph via point-by-point plotting.

(b) The general form of functions and their shapes should be emphasised from Grade 10 and revised every time a functions question is worked through in class. In addition, the effects of the parameters a, b, p and q should be thoroughly taught, revised and practised by learners from the introduction of functions in Grade 10 through to the end of the Grade 12 academic year.

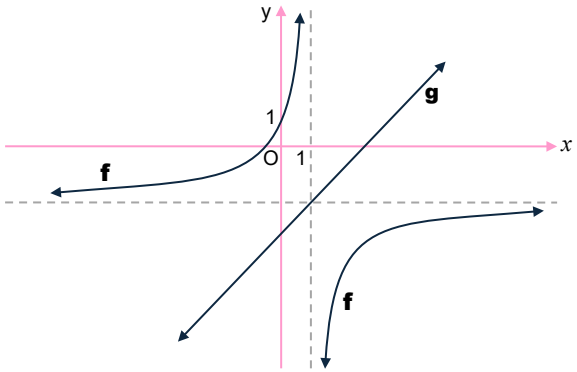
(c) Teachers should ensure that learners work with transformations in functions from Grade 10. This should include point transformation, recognition of the transformation applied to one function's equation to result in another and the rules of reflection, horizontal and vertical shifts on the equation of a function.



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QUESTION 5 42%

Sketched below is the graph of $f(x) = \frac{a}{x+p} + q$ having the domain $(-\infty; 1) \cup (1; \infty)$. The graph of f cuts the y-axis at (0; 1). A line of symmetry of f is given by $g(x) = x - 3$.



5.1 Write down the value of p. (1)

62%

5.2 Determine the equation of the horizontal asymptote of f. (2)

50%

5.3 Calculate the value of a. (2)

69%

Memo

5.1 **p = –1** ➤ ... *a shift 1 unit to the right*

5.2 Subst. $x = 1$ in $y = x - 3$ ... *(1; y) on g*

$\therefore y = 1 - 3$

$= -2$

$\therefore$ Eqn of horizontal asymptote: **y = –2** ➤

5.3 Subst. (0; 1), p = –1 & q = –2:

$f(x) = \frac{a}{x+p} + q$: $1 = \frac{a}{0-1} - 2$

$\therefore 3 = -a$

$\therefore$ **a = –3** ➤

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Common Errors and Misconceptions

(a) In **Q5.1**, many candidates wrote p = 1 which could be attributed to the form of the *hyperbola* being given as $y = \frac{a}{x+p} + q$.

The other incorrect answer from candidates was $x = -1$.

(b) **Q5.2** required candidates to calculate the value of the *horizontal asymptote*. Candidates who incorrectly answered this question wrote down the answer of y = –3, taking the value of y from the equation of the straight line. Candidates did not realise that the y-value of the point of intersection of the *vertical asymptote* and the straight line, g, was the equation of the *horizontal asymptote*.

(c) In **Q5.3**, some candidates arrived at a positive answer for a.

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5.4 For which values of x is $f(x) \geq 0$? (3)

30%

5.5 Graph f undergoes a transformation to h where:

17%

- The domain and range of h are the same as that of f
- $h'(x)$, the derivative of h, is negative on its domain

Describe a possible transformation that f could have undergone to result in h. (2) **[10]**

Memo

5.4 **X-int.** $(y = 0)$: $0 = \frac{-3}{x-1} - 2$

$\therefore\ 2 = \frac{-3}{x-1}$

$\therefore\ 2(x-1) = -3$

$\therefore\ x - 1 = -\frac{3}{2}$

$\therefore\ x = -\frac{1}{2}$

$\therefore\ f(x) \geq 0$ for: $-\frac{1}{2} \leq x < 1$ ◄

5.5 **A reflection in the line $x = 1$**

or A reflection in the line $y = -2$ ◄



OR: Rotation 90° anticlockwise about $(1; -2)$

OR: Rotation 90° clockwise about $(1; -2)$

Common Errors and Misconceptions

(d) **Q5.4** was poorly answered by many candidates because they tried to solve a rational inequality, or they omitted the *vertical asymptote* from their solution.

(e) **Q5.5**, a problem-solving question, was not well answered by the majority of candidates. Many candidates had difficulty in interpreting the question and indicated 'reflection over the line $y = x$' as the answer. This was incorrect.

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QUESTION 5: Suggestions for Improvement



(a) Teachers must refer to the *Curriculum Assessment and Policy Statement* for the correct general form of the functions to be used in class. For the *hyperbola*, this is $y = \frac{a}{x+p} + q$. A 'recipe' for learners to calculate the values of p and q leads to a lack of understanding for the learner. It is suggested that teachers emphasise that a *vertical asymptote* on a *function* is created when the *denominator* is *undefined*. This will lead to a better understanding by the learners and a more accurate calculation of the parameter p in the *hyperbola* equation.

(b) Learners should be taught to work with what is presented to them in each diagram. Before answering any question on a *function*, learners should ideally recognise what each point on the diagram represents and what their properties are. For example: the value 1 on the x-axis in this graph indicates the x-intercept which means the y-value is 0 and the coordinate is (1; 0). This process should be followed for each point on a given graph.

(c) Teachers need to help learners 'error check' their solutions. For a positive 'a'-value in a *hyperbola*, the graph needs to be decreasing and for a negative 'a'-value, the graph is increasing.

(d) It should be emphasised to learners that when answering graphical interpretation questions on graphs that have a *vertical asymptote*, this value should always be considered in their answers to questions of the nature '*For what values of* x *...*'.

(e) Transformation geometry should be taught as an integral part of functions.

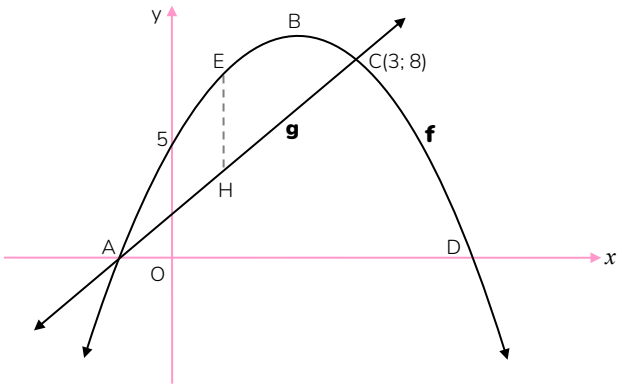
(f) Teachers need to include functions practice for the learner throughout the year as a means of regular revision of the concepts. The functions from Grade 11 are not retaught in Grade 12 which means that the concepts need to be taught thoroughly in Grade 11 and then revised thoroughly throughout Grade 12.

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QUESTION 6 44%

In the diagram below, the graphs of $f(x) = -x^2 + 4x + 5$ and g, a straight line, are drawn. C(3; 8) is a point of intersection of f and g. EH is drawn parallel to the y-axis, with E a point on f and H a point on g.



6.1 Calculate the coordinates of B, the turning point of f. (3)

84%

Memo

6.1 $y = -x^2 + 4x + 5$



Axis of Sym.: $x = -\frac{b}{2a} = -\frac{4}{2(-1)} = 2$

$\therefore$ Max $y = -2^2 + 4(2) + 5$

$= -4 + 8 + 5$

$= 9$

$\therefore$ B(2; 9) ◄

OR: $f'(x) = -2x + 4 = 0$ at the turning point

$\therefore -2x = -4$

$\therefore x = 2$, etc.

6.2 Show that the equation of the line through A and C is given by $g(x) = 2x + 2$. (3)

68%

6.3 Calculate the maximum length of EH for f > g. (4)

35%

Memo

6.2 Eqn of f: $y = -(x^2 - 4x - 5)$

$\therefore y = -(x + 1)(x - 5)$

$\therefore x = -1$ at A (& $x = 5$ at D)

$\therefore m_{AC} = \frac{8-0}{3-(-1)} = \frac{8}{4} = 2$

Subst. (3; 8) & m = 2 in

$y - y_1 = m(x - x_1)$

$y - 8 = 2(x - 3)$

$\therefore y = 2x - 6 + 8$

$\therefore g(x) = 2x + 2$ ◄

OR: $y = mx + c$

$\therefore 8 = (2)(3) + c$

$\therefore c = 2$, etc.

6.3 EH $= f(x) - g(x)$ ... *vertical length*

$= (-x^2 + 4x + 5) - (2x + 2)$

$= -x^2 + 2x + 3$

Max when $x = -\frac{2}{2(-1)} = 1$

$\therefore$ Max length of EH

$= -1^2 + 2(1) + 3$

$=$ 4 units ◄

OR: $D_x[-x^2 + 2x + 3] = 0$

$\therefore -2x + 2 = 0$

$\therefore -2x = -2$

$\therefore x = 1$, etc.



Common Errors and Misconceptions

(a) It was a common error for candidates to use the equation given in the question to 'prove' that this was the equation of the *straight line* in **Q6.2**.

(b) In **Q6.3** many candidates failed to create an expression for the *vertical length* EH. They attempted to solve the equation $f(x) - g(x) = 0$ as a minimum and maximum equation. Conceptually, candidates did not realise this calculated the points of intersection of the graphs f and g.

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6.4 Given: $k(x) = f(x + m) = -x^2 - 2mx - m^2 + 4x + 4m + 5$

13% Determine the value of m such that g is a tangent to k. (5)

[15]

Memo

6.4

g a tangent to k
➡ The equation $k(x) = g(x)$ has equal roots. So, $\Delta = 0$

$$-x^2 - 2mx - m^2 + 4x + 4m + 5 = 2x + 2$$
$$\therefore -x^2 - 2mx + 2x - m^2 + 4m + 3 = 0$$
$$\therefore -x^2 - (2m - 2)x - (m^2 - 4m - 3) = 0$$
$$\therefore x^2 + (2m - 2)x + (m^2 - 4m - 3) = 0$$

$$\Delta = b^2 - 4ac$$
$$= (2m - 2)^2 - 4(1)(m^2 - 4m - 3)$$
$$= 4m^2 - 8m + 4 - 4m^2 + 16m + 12$$
$$= 8m + 16$$

$\Delta = 0$ ➡ $8m + 16 = 0$
$\therefore 8m = -16$
$\therefore$ **m = −2** ◄

OR:

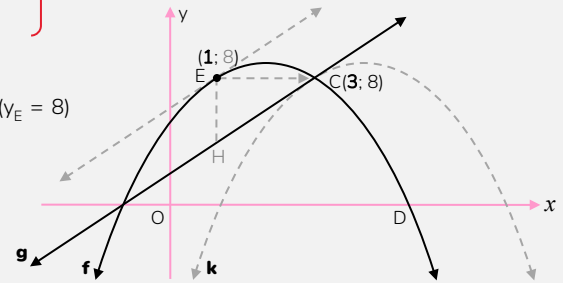
$k(x) = f(x + m)$ means that k is f shifted horizontally by m units. (f would have to shift to the right)



A tangent to f (with gradient, 2) would touch f at E, when EH is at maximum length, when $x = 1$ (See 6.3).

The gradient of g is 2
& The gradient of $f = f'(x) = -2x + 4$
$\therefore$ At E: $-2x + 4 = 2$
$\therefore -2x = -2$
$\therefore x_E = 1$

E must shift horizontally to C ... ($y_E = 8$)
$\therefore$ by 2 units
$\therefore$ **m = −2** ◄



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Common Errors and Misconceptions

(c) **Q6.4** was poorly answered by the majority of the candidates as it integrated concepts of functions, calculus and algebraic manipulation. Added to this, candidates incorrectly assumed that the point E will be the point of contact of the tangent g and the function k. Candidates also assumed that C and E lie on the same horizontal line. Even though this is a fact, it was an assumption as this information was not provided in the question.

(d) Candidates who did not know that they should use *gradient* to answer **Q6.4**, incorrectly differentiated each term that had a variable, irrespective of whether that variable was x or m. They failed to realise that m should have been treated as a constant in this instance.

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QUESTION 6: Suggestions for Improvement



(a) Learners need to be taught the difference between 'showing' and 'proving' and using the given equation. The best approach to questions like Q6.2 is to teach learners to read it as 'work out the equation of the line through A and C.'

(b) Teachers need to emphasise that learners cannot assume information in a given scenario. Learners need to work from the given information to correctly deduce that a point is a point of contact or prove this through calculation.

(c) It is imperative that learners are exposed to questions that integrate functions, calculus and transformations. When faced with problem-solving questions, teachers need to help learners to decode what the words mean (for example: *tangent* indicates that the gradients of the two functions are equal at the point of contact) and then guide learners to use correct algebraic methods to answer the given question.

(d) Learners should practise differentiating expressions that have more than one variable so that they become familiar with differentiating the expression with respect to the required variable.

(e) Nature of the roots should not be ignored as an alternate method of dealing with graphs that intersect, are tangential or do not intersect. This requires teachers to incorporate the concepts taught in nature of the roots when teaching quadratic functions.

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FINANCE (40%): DBE NOVEMBER 2024

QUESTION 7 40%

77% 7.1 Mary's grandparents deposited R5 000 into a savings account on the day that she was born. The account pays interest at a rate of 6,8% p.a., compounded quarterly. Calculate the accumulated amount in this account on Mary's 16th birthday. (3)

53% 7.2 After 4 years, the value of a printer was half of its original value. Determine the rate at which the value of the printer depreciated over this period, if depreciation was calculated according to a straight-line method. (2)

25% 7.3 Tshepo was granted a loan of R100 000 on 1 March 2022 at an interest rate of 13,5% p.a., compounded monthly. Tshepo agreed to repay the loan over 5 years in monthly instalments of R2 300,98, starting on 1 April 2022.

7.3.1 Calculate the total interest that he will pay over the 5 years. (2)

Memo

7.1 $P = R5\,000$; $i = 6{,}8\% \div 4$; $n = 16 \times 4$; A?

$A = P(1+i)^n \Rightarrow A = 5\,000(1+0{,}017)^{64}$

$\simeq$ R14 706,56 ◄

7.2 $n = 4$; P = original value ; $A = \frac{1}{2}P$; i?

$A = P(1-in)$

$\therefore \frac{1}{2}P = P(1-i.4)$



(÷P) $\therefore \frac{1}{2} = 1-4i$

$\therefore 4i = \frac{1}{2}$

$\therefore i = \frac{1}{8}$ (= 0,125)

$\therefore$ The rate of depreciation was **12,5% per annum** ◄

7.3.1 The value of the payments

$= 5 \times 12 \times R2\,300{,}98$

= R138 058,80

The value of the loan = R100 000

$\therefore$ The interest = R138 058,80 – R100 000 ... *the difference!*

= **R38 058,80** ◄

Common Errors and Misconceptions

(a) In **Q7.1** the common errors were $A = 5\,000\left(1+\frac{6{,}8}{100}\right)^{16}$ which omitted the understanding of quarterly compounding or $A = 5\,000\left(1+\frac{6{,}8}{400}\right)^{16}$ which failed to account for the number of quarters in 16 years.

(b) A common error in **Q7.2** was that candidates used the *reducing-balance depreciation* method as they did not realise that the *straight-line depreciation* was linked to the *simple interest* formula.

(c) A number of candidates over-complicated **Q7.3.1** and used an *annuity formula*, which showed a lack of understanding of the difference between interest on a loan and the interest rate charged on a loan. Their responses highlighted a lack of understanding of the total monies paid out when paying back a loan.

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7.3.2 Tshepo paid R22 300,98 (his monthly instalment and an additional R20 000) on 1 March 2024 into the loan account. He continues to pay the original monthly instalment thereafter. How many months earlier will Tshepo repay the loan? (7)
[14]

Memo

7.3.2 Up to 1 March 2024, before the R20 000 payment:

$x = \text{R}2\,300{,}98$; $n = 2 \times 12 = 24$; $i = \frac{13{,}5\%}{12} = \frac{0{,}135}{12}$

12 monthly payments for each year



After 2 years:

The balance of the loan (before the payment)

$= A - F_v$ A: the accrued value of the loan

F_v: the accrued value of the payments

$$= P(1+i)^n - \frac{x\left[1-(1+i)^{-n}\right]}{i}$$

$$= 100\,000\left(1+\frac{0{,}135}{12}\right)^{24} - \frac{2\,300{,}98\left[\left(1+\frac{0{,}135}{12}\right)^{24}-1\right]}{\frac{0{,}135}{12}}$$

$\simeq$ R67 805,20 ... *Now subtract the payment*

Subtract the payment of R20 000, then the balance of the loan

= R47 805,20

= the Present value (P_v) of the remaining n payments

$$P_v = \frac{x\left[1-(1+i)^{-n}\right]}{i}$$

$P_v = 47\,805{,}20$; $x = 2\,300{,}98$; n? ; $i = \frac{0{,}135}{12}$

$$\therefore \frac{2\,300{,}98\left[1-\left(1+\frac{0{,}135}{12}\right)^{-n}\right]}{\frac{0{,}135}{12}} = 47\,805{,}20$$

$$\therefore \left[1-\left(1+\frac{0{,}135}{12}\right)^{-n}\right] = 0{,}2337...$$

$$\therefore 0{,}76626... = \left(1+\frac{0{,}135}{12}\right)^{-n}$$

$$\therefore -n = \log_{\left(1+\frac{0{,}135}{12}\right)} 0{,}76626...$$

$$= -23{,}796...$$

$\therefore$ n $\simeq$ 24 months ... *The 24th payment would be a lesser payment*

$\therefore$ **12 months earlier** ... *It would have been another 3 years, i.e. 36 months*

OR:

Using Present values: $P_v = A(1+i)^{-n}$ and $P_v = \frac{x\left[1-(1+i)^{-n}\right]}{i}$

Let n be the total number of payments made.

$$100\,000 = 20\,000\left(1+\frac{0{,}135}{12}\right)^{-24} + \frac{2\,300{,}98\left[1-\left(1+\frac{0{,}135}{12}\right)^{-n}\right]}{\frac{0{,}135}{12}}$$

$$\therefore 1-\left(1+\frac{0{,}135}{12}\right)^{-n} = 0{,}41416...$$

$$\therefore \left(1+\frac{0{,}135}{12}\right)^{-n} = 0{,}585...$$

n = the total number of payments made on the original loan

$$\therefore -n = \log_{\left(1+\frac{0{,}135}{12}\right)} 0{,}585... = -47{,}796...$$

$$\therefore n = 47{,}796...$$

He needs to make 48 payments to pay off the loan.

$\therefore$ he will pay off the loan 12 months earlier than originally planned.

Finance Formulae

$A = P(1 \pm in)$ $\quad$ $F_v = \frac{x\left[(1+i)^n - 1\right]}{i}$

$A = P(1 \pm i)^n$ $\quad$ $P_v = \frac{x\left[1-(1+i)^{-n}\right]}{i}$

$$1 + i_{eff} = \left(1+\frac{i_{nom}}{m}\right)^m$$

Common Errors and Misconceptions

(d) **Q7.3.2** was poorly answered by the majority of the candidates.

The question involved reading for understanding and multiple steps to the solution. A large number of candidates left out this question.

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QUESTION 7: Suggestions for Improvement



(a) Drills and practice should be undertaken on different compounding periods and the compound interest formulae so that learners can familiarise themselves with the number of times that interest is compounded in a specified time frame.

(b) Teachers should teach Financial Mathematics with conceptual understanding and real-life problems. It may help to have learners draw timelines and identify what happens at different stages of an annuity (either present or future valued). Visual representation helps to break up a problem that requires reading for understanding.

(c) Learners should be taught to read for understanding in all problems, especially in Financial Mathematics problems. Teachers need to emphasise the importance of this skill.

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QUESTION 8 68%

8.1 Determine:

83% 8.1.1 $\frac{d}{dx}[3x - 5x^2]$ (2)

8.1.2 $g'(x)$ if $g(x) = \frac{2}{x^2} - \sqrt[3]{x^7}$ (4)

8.2 Determine the equation of the tangent to

56% $f(x) = x^3 - 4x^2 + 2x + 3$ at $x = 2$. (3)

Memo

8.1.1 $\frac{d}{dx}[3x - 5x^2] = \mathbf{3 - 10x}$ ◄

8.1.2 $g(x) = 2x^{-2} - x^{\frac{7}{3}}$

$\therefore g'(x) = \mathbf{-4x^{-3} - \frac{7}{3}x^{\frac{4}{3}}}$ ◄

$\left(= -\frac{4}{x^3} - \frac{7}{3}\sqrt[3]{x^4} \text{ ◄} \right)$

8.2 $f(x) = x^3 - 4x^2 + 2x + 3$

$\therefore f'(x) = 3x^2 - 8x + 2$ … *the gradient of the tangent to f*

$\therefore f'(2) = 3(2)^2 - 8(2) + 2$ … *the gradient of the tangent to f at x = 2*

$= 12 - 16 + 2$

$= -2$

& $f(2) = 2^3 - 4(2)^2 + 2(2) + 3$

$= 8 - 16 + 4 + 3$

$= -1$

$\therefore$ Point of contact is $(2; -1)$

Subst. $m = -2$ & pt $(2; -1)$ in

$y - y_1 = m(x - x_1)$

$\therefore y + 1 = -2(x - 2)$

$\therefore \mathbf{y = -2x + 3}$ ◄

OR: $y = mx + c$

$\therefore -1 = (-2)(2) + c$

$\therefore c = 3$, etc.

Common Errors and Misconceptions

(a) A common error by some candidates in **Q8.1.2** was not working with the negative and rational exponent correctly. Candidates incorrectly changed the *surd* term to $-x^{\frac{3}{7}}$.

(b) In **Q8.2** many candidates were unable to calculate the gradient of the function at the point where $x = 2$ or they did not calculate the point of contact of the function and the tangent, they used the y-intercept instead of f.



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62%

8.3 Given: $f(x) = -6x^2$

8.3.1 Determine $f'(x)$ from first principles. (5)

8.3.2 Write down how you will restrict the domain of f such that f^{-1}, the inverse of f, is a function. (1)

8.3.3 Determine the equation of f^{-1} for $f^{-1}(x) \leq 0$. Write your answer in the form $y = \ldots$ (3) **[18]**

Memo

8.3.1 $f(x) = -6x^2$

$\therefore f(x+h) = -6(x+h)^2$

$= -6(x^2 + 2xh + h^2)$

$= -6x^2 - 12xh - 6h^2$

$\therefore f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ ← def of a derivative

$= \lim_{h\to 0} \frac{-6x^2 - 12xh - 6h^2 - (-6x^2)}{h}$

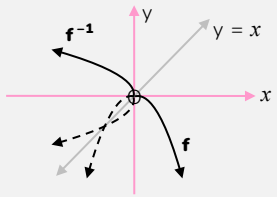
$= \lim_{h\to 0} \frac{12xh - 6h^2}{h}$

$= \lim_{h\to 0} (-12x - 6h)$

$= -12x$ ◄

8.3.2 $x \geq 0;\ x \in \mathbb{R}$ ◄

or $x \leq 0;\ x \in \mathbb{R}$ ◄



8.3.3 Equation of f: $y = -6x^2$

$\therefore$ Equation of f^{-1}: $x = -6y^2$

$\therefore 6y^2 = -x$

$\therefore y^2 = -\frac{x}{6}$

$\therefore y = -\sqrt{-\frac{x}{6}}$ where $x \leq 0$ ◄ ... $f^{-1}(x) \leq 0$

Common Errors and Misconceptions

(c) As mentioned in previous reports, the most common errors in the *first principles* question **Q8.3.1** were incorrect notations.

(d) Many candidates merely found the equation of the *inverse function* of f instead of writing down the restriction in **Q8.3.2**. Those who did write down a restriction indicated $x < 0$ or $x > 0$ where the equality was omitted from the restriction.

(e) In **Q8.3.3** many candidates were unaware that there were two possible solutions of y when taking the square root of the equation $y^2 = -\frac{6}{x}$, i.e. $y = \pm\sqrt{-\frac{6}{x}}$. Many just wrote down $y = \sqrt{-\frac{6}{x}}$ as the solution instead of choosing the negative root.

(f) Another common error made by some candidates when solving for y in **Q8.3.3**, was to incorrectly write their answer as $y = -\sqrt{\frac{6}{x}}$ instead of $y = \sqrt{-\frac{6}{x}}$. These candidates were confused about the placement of the negative sign because they were taught that you cannot determine the square root of a negative number.

(g) In **Q8.3.3** some candidates, after swapping the x and y in the equation, incorrectly used the *logarithms* to isolate y. This led them to $x = -6y^2$ then $\frac{y}{6} = \log x^2$.

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QUESTION 8: Suggestions for Improvement

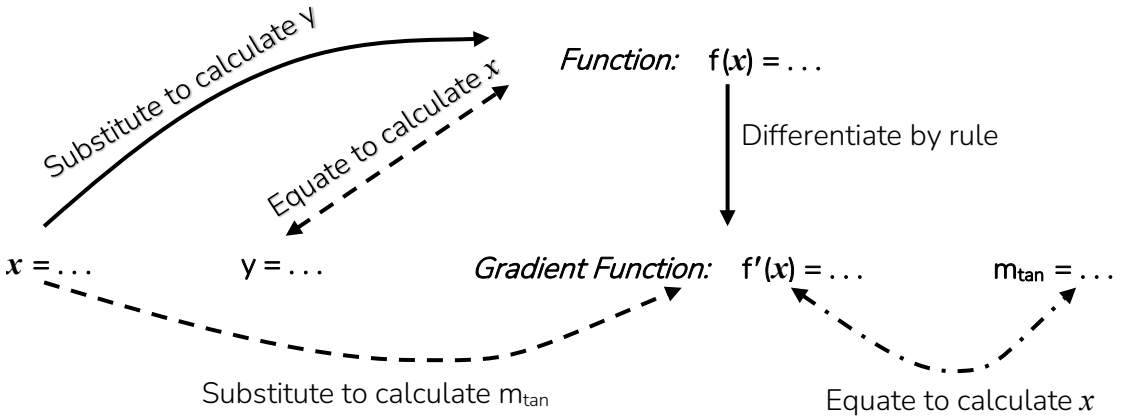


(a) As mentioned in previous reports:

- Emphasis should be placed on the use of the correct notation when determining the derivative, either when using first principles or the rules.
- Teachers should revise the rules of *exponents* and *surds* when changing an expression into differentiable format.
- Integration and re-emphasis of algebraic concepts, viz. fractions, factorising, inequalities and exponential rules, should be undertaken when working with Calculus.

(b) Teachers must teach all *inverse functions* and not just the *logarithm*. Learners must practise the inverse of the parabola and straight line as part of this section of work.

(c) The derivative needs to be taught in context with the graph and conceptual understanding needs to be the fundamental of how Calculus is taught. A map of how the function, x- and y-values of a point, the derivative (gradient function) and the gradient of the tangent needs to be linked in a visual representation for learners. The diagram below could help learners understand how these are all integrated and what to do in different situations.

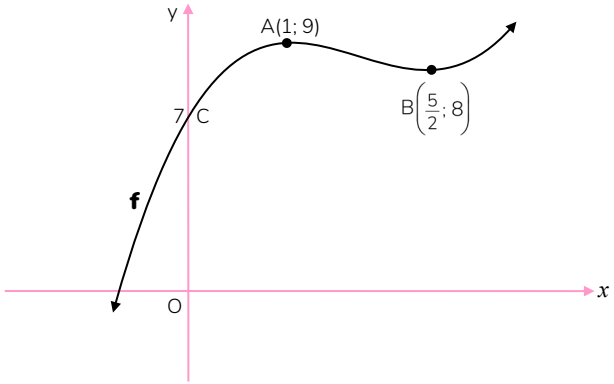


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QUESTION 9 34%

A(1; 9) and B$\left(\frac{5}{2}; 8\right)$ are the turning points of graph f below.
C(0; 7) is the y-intercept of f.



9.1 For which values of x is f decreasing? (2)

53%

9.2 Write down the x-intercepts of f', the derivative of f. (2)

50%

9.3 For which values of x is f concave up? (2)

27%

9.4 Determine the values of k for which $y = f(x) + k$ will have THREE positive x-intercepts. (2)

6%

[8]

Memo

9.1 $1 < x < \frac{5}{2}$ ◄ ... $1 \leq x \leq \frac{5}{2}$ *will also be accepted*

9.2 x-intercepts of f':

$(1; 0)$ & $\left(\frac{5}{2}; 0\right)$ ◄

9.3 At the point of inflection:

$x = \frac{1 + \frac{5}{2}}{2} = 1\frac{3}{4}$

$\therefore$ f is concave up for $x > 1\frac{3}{4}$ ◄



9.4 $-9 < k < -8$ ◄

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Common Errors and Misconceptions

(a) Few candidates realised that the x-values of the x-intercepts of the *derivative* were the x-values of the turning points given in **Q9.2**.

(b) In the absence of the equation of the *cubic function*, many candidates were unable to determine the x-value of the *point of inflection* of the graph. A fair number of candidates attempted to determine the equation of f so that they could determine the *second derivative* of f to answer the question.

(c) Most candidates struggled to interpret **Q9.4**. Some tried to use the discriminant because they saw k in the question. Others tried to calculate the equation of f.

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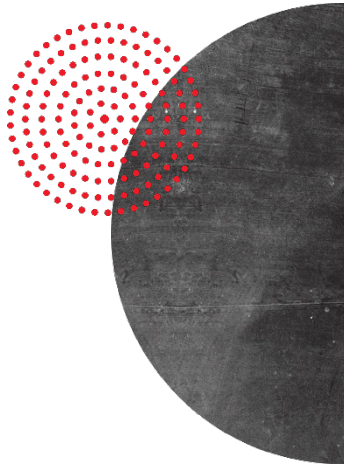
QUESTION 9: Suggestions for Improvement



(a) The concept of the point of inflection needs to be taught explicitly by explaining both its link to the second derivative and the conceptual understanding of how it links to the concavity of the cubic function. Learners must understand that the x-value of the point of inflection lies halfway between the x-values of the turning points of a cubic function.

(b) Learners need to be exposed to graphical interpretation questions where they apply their understanding of *gradient*, *concavity*, *positive* and *negative* values of a function as examples.

(c) As mentioned in previous reports:



- Teachers should ensure that there is enough time for learners to understand the application of Calculus fully.
- Learners should be taught to determine the properties of a graph from Grade 10 to 12 in a progressive manner. The defining property of a turning point having a zero gradient is a way to describe a turning point. Teachers need to prove this link between the definition of a turning point and the Grade 11 concept of determining the axis of symmetry to calculate the x-coordinate of the turning point.
- Teachers should continue to teach graphical interpretation in cubic graphs as a follow on from the interpretation taught in Grade 10 and 11.

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QUESTION 10 23%

A cyclist rode from town P and stopped at town T. The speed (in km/h) at which this cyclist rode, is represented by the equation $s'(t) = -3t^2 + 18t$.

NOTE: Speed is the rate of change in distance with respect to time.

10.1 Calculate the maximum speed that the cyclist reached on this ride. (3)

40%

Memo

10.1 The speed $= s'(t) = -3t^2 + 18t$

$\therefore$ The maximum speed occurs when $s''(t) = 0$

$\therefore -6t + 18 = 0$

$\therefore -6t = -18$

$\therefore t = 3$

$\therefore$ The maximum speed $= -3(3)^2 + 18(3)$

$= -27 + 54$

$= 27$ km/h ◄

Common Errors and Misconceptions

(a) In **Q10.1** most candidates equated the speed to 0 and solved. This led to

$-3t^2 + 18t = 0$

$-3t(t - 6) = 0$

$t = 0$ or $t = 6$

(b) Some candidates calculated the second derivative but did not explicitly set it equal to zero in **Q10.1**.

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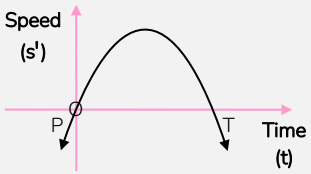
10.2 Calculate the distance between town P and town T. (5)

12%

[8]

Memo

10.2 At the start and the end
i.e. at town P and town T,
the speed = 0



$\therefore\ -3t^2 + 18t = 0$

$\therefore\ -3t(t-6) = 0$

$\therefore\ t = 0$ (at P) or $t = 6$ (at T)

$\therefore$ The time taken is 6 hours

The distance:

$s(t) = -t^3 + 9t^2$ ... *by inspection*, Note: s'(0) = 0*

$\therefore\ s(6) = -(6)^3 + 9(6)^2$

$= 108$ km ◄

*OR:

$s(t) = at^3 + bt^2 + ct + d$

$s'(t) = 3at^2 + 2bt + c$

We have: $s'(t) = -3t^2 + 18t$ ... *s'(0) = c = 0*

$\therefore\ 3a = -3$ $\quad 2b = 18$ $\quad c = 0$ $\quad d = 0$

$\therefore\ a = -1$ $\quad \therefore\ b = 9$ $\quad \because\ s(0) = 0$

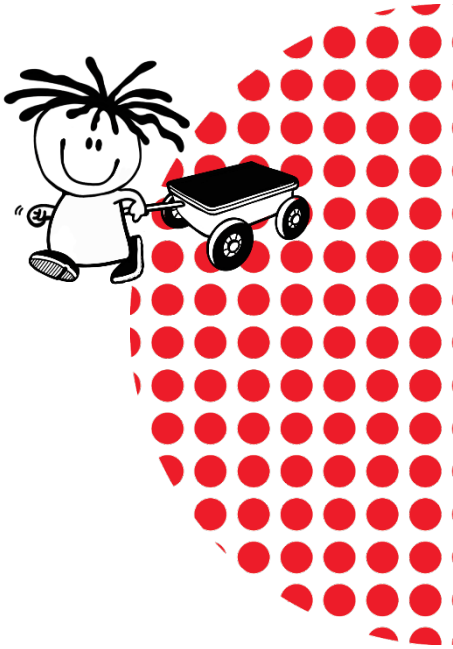
$\therefore\ s(t) = -t^3 + 9t^2$, etc.

Common Errors and Misconceptions

(c) In **Q10.2** most candidates did not recognise that the distance was represented by s(t) which is the function from which s'(t) is derived.

(d) In answering **Q10.2** many candidates tried to use the concept of

$$\text{speed} = \frac{\text{distance}}{\text{time}} \quad \text{or} \quad \Delta s = \frac{\Delta d}{\Delta t}.$$



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QUESTION 10: Suggestions for Improvement



(a) Calculus lends itself to many applications in optimisation and rates of change. Teachers need to expose learners to a wide variety of questions, which include integration of topics including *rate of change*, *analytical geometry*, *measurement* and *trigonometry*.

(b) *Rate of change* must be taught explicitly as part of the optimisation and application of Calculus.

(c) In all topics, reading for understanding should be ongoing if learners are to improve their responses to problems.

(d) Learners need to be exposed to working from the derivative function to the original function using equating of coefficients.

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PROBABILITY (40%): DBE NOVEMBER 2024

QUESTION 11 40%

A certain number of learners are sitting for examinations in Mathematics, Tourism and Geography.

- All these learners sit for at least one of these examinations.
- The total number of learners who sit for Mathematics (**M**), is 22.
- The total number of learners sitting for Tourism (**T**), is 16.
- The total number of learners sitting for Geography (**G**), is 18.
- 5 learners sit for Mathematics and Tourism, but not Geography.
- 4 learners sit for Mathematics and Geography, but not Tourism.
- 3 learners sit for Tourism and Geography, but not Mathematics.
- 6 learners sit for only Tourism.

11.1 Draw a Venn diagram to represent ALL the learners sitting for these examinations. (3)

59%

Memo

11. All sit for at least one examination.

n(M) = 22 ; n(T) = 16 ; n(G) = 18

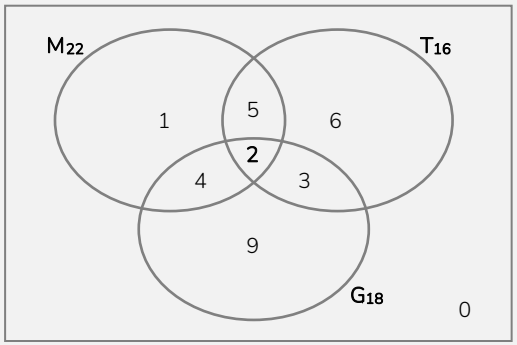
$n(M \cap T \cap G') = 5$

$n(M \cap G \cap T') = 4$

$n(T \cap G \cap M') = 3$

n(T only) = 6

11.1



$n(T) = 16 \Rightarrow n(M \cap T \cap G) = 2$

Common Errors and Misconceptions

(a) In **Q11.1** many candidates failed to calculate the intersection of the three overlapping sets in the *Venn diagram* correctly.

Many candidates used a sample space of 56 for the *Venn diagram*.

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11.2 Calculate the probability that a learner, chosen at random, will sit for examinations in at least TWO of the subjects. (2)

37%

11.3 Determine if the events: sitting for examinations in Mathematics and sitting for examinations in Tourism are independent. Support your answer with the necessary calculations. (4)

29%

[9]

Memo

11.2 The total number of learners = 40

The number taking at least 2 subjects

$= 4 + 2 + 3 + 5 = 14$

$\therefore$ P(at least 2 subjects) $= \frac{14}{40} = \frac{7}{20}$ ... **(= 0,35 = 35%)** ➤

11.3 $P(M) = \frac{22}{40}$ & $P(T) = \frac{16}{40}$

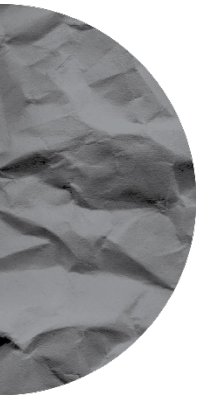
$\therefore P(M) \times P(T) = \frac{22}{40} \times \frac{16}{40} = \frac{11}{50}$

whereas $P(M \text{ and } T) = \frac{7}{40}$

$\therefore$ **P(M) × P(T) ≠ P(M and T)**

$\therefore$ **The events are not independent** ➤





Common Errors and Misconceptions

(b) Most candidates did not understand the term *'at least'* in the context of probability in **Q11.2**. This led these candidates to adding P(M) + P(T) + P(G) instead of using their *Venn diagram* to answer the question.

(c) In answering **Q11.3** many candidates did not realise that this problem was similar to working with a contingency table. The necessary values could be found from the *Venn diagram*. A large number of candidates used the probability rule or used P(M only) × P(T only) in their calculation rather than the entire set for M and T.

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QUESTION 11: Suggestions for Improvement



(a) In the teaching of probability, teachers need to emphasise the terminology of *'at least'*. Teachers must also describe and show learners what is meant by P(A), P(B) and P(A and B).

(b) Learners need to be taught the difference between mutually exclusive and independent events and the rules that pertain to each of these concepts. This should be thoroughly drilled in Grades 10 and 11.

(c) Teachers need to expose learners to different contexts, diagrams and problems in which independent events can be tested.

(d) When working with *Venn diagrams*, learners need to start with the intersection of all the sets first.

(e) As mentioned in previous reports:

- Teaching basic concepts cannot be overlooked. When learners understand the basic concepts well enough, then the more complex concepts are easier to grasp.
- Reading for understanding must be a regular practice in the classroom. This should equip learners with the skills to deal with word problems in assessment tasks.
- Teachers need to teach both tree diagrams and Venn diagrams thoroughly. These concepts should be examined in school-based assessment tasks throughout the FET phase.

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COUNTING PRINCIPLES (33%): DBE NOVEMBER 2024

QUESTION 12 33%

A company generates a 4-character code using the 26 letters of the alphabet and the 10 digits, from 0 to 9.

The code is in the form:

letter	digit	letter	digit

12.1 Determine how many different codes can be formed if letters and digits may be repeated. (2)

48%

Memo

12.1 26 10 26 10

$\therefore$ No of different codes $= 26 \times 10 \times 26 \times 10$

$= 67\,600$ ◄

Common Errors and Misconceptions

(a) In **Q12.1** some candidates included the factorial in their answer to arrive incorrectly at $26! \times 10! \times 26! \times 10!$

12.2 Determine how many different codes can be formed if: 32%

- The letters D, F, I, Q, U and V may NOT be used
- The code may NOT start with a W or a Z
- Letters or digits may NOT be repeated
- The code ends with an odd digit (4)

Memo

12.2 20 letters are used
Choice of 18 for 1st spot
No repeats of letters or digits
Last slot must be odd

18 9 19 5

$\therefore$ No of different codes $= 18 \times 9 \times 19 \times 5$

$= 15\,390$ ◄

Common Errors and Misconceptions

(b) Most candidates were unable to work with the constraints placed on the calculation in **Q12.2**. Responses from candidates included $(26 \times 26 \times 26) \times (10 \times 10 \times 10)$; 26×10; $26 \times 10 \times 19 \times 5$ or they included the use of factorials in their answers.

12.3 The company wishes to increase the number of 4-character codes formed in Question 12.2 by allowing the letters D, F, I, Q, U and V to be used. 19%

Calculate the percentage increase in the number of different codes that can now be formed. (2)

[8]

TOTAL: 150

Memo

12.3 24 9 25 5

$\therefore$ No of different codes $= 27\,000$

$$\% \text{ increase} = \frac{27\,000 - 15\,390}{15\,390} \%$$

$= 75{,}44\%$ ◄

Common Errors and Misconceptions

(c) Many candidates were unable to calculate the percentage increase correctly in **Q12.3**. Other candidates did not know how to deal with the code formed being odd.

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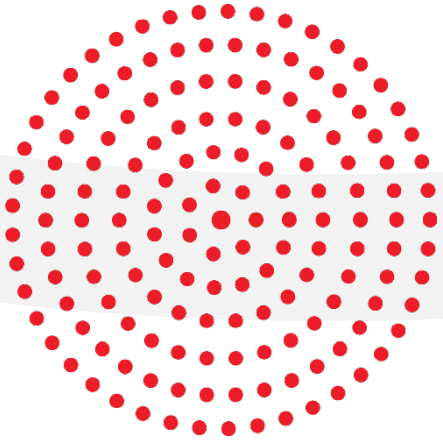
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QUESTION 12: Suggestions for Improvement



(a) Teach learners the *Fundamental Counting Principle* in such a way that they will be able to base their answers on their reasoning, rather than on any rule. The concept of the *factorial* needs to be explained thoroughly.

(b) When teaching learners about the number of options available for a code or set of items, it is a good idea for learners to draw lines or boxes to represent each space that is available. Thereafter, learners need to be taught to recognise how many options are available for each position in the code or list of items. It is important to stress to learners that they should put a '×' between these numbers so that they can arrive at the correct solution using the *Fundamental Counting Principle*.



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