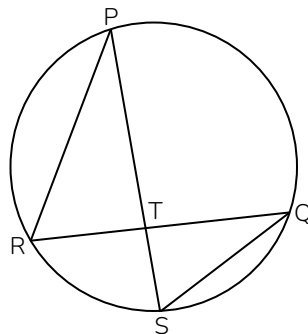


Cross-Topic Solutions

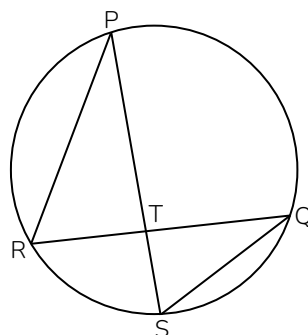
Question 1

$$\begin{aligned}
 3a^2 - 8a &= 4a + 15 \quad \dots \angle^s \text{ in the same seg} \\
 \therefore 3a^2 - 12a - 15 &= 0 \\
 \therefore a^2 - 4a - 5 &= 0 \\
 \therefore (a-5)(a+1) &= 0 \\
 \therefore a &= 5 \text{ or } a = -1 \\
 \therefore \hat{Q} &= 3(5)^2 - 8(5) \quad \text{or} \quad \hat{Q} = 3(-1)^2 - 8(-1) \\
 &= 35^\circ \quad \text{or} \quad = 11^\circ \quad \blacktriangleleft
 \end{aligned}$$



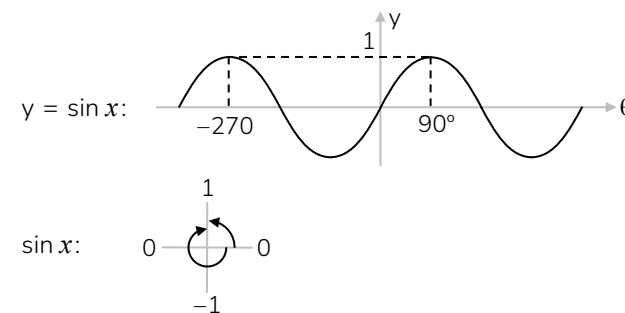
Question 2

$$\begin{aligned}
 \hat{S} &= \hat{R} \quad \dots \angle^s \text{ in the same seg} \\
 \hat{S} &= \sum_{i=1}^4 2 \cdot 3^{i-1} \\
 &= 2 \cdot 3^0 + 2 \cdot 3 + 2 \cdot 3^2 + 2 \cdot 3^3 \\
 &= 2 + 6 + 18 + 54 \\
 &= 80 \\
 \hat{R} &= \sum_{k=1}^n (8k - 80) \\
 &= (-72) + (-64) + (-56) + \dots \\
 a &= -72; d = 8; n?; S_n = 80 \quad \dots \hat{S} = \hat{R} \\
 \therefore \frac{n}{2} [2(-72) + (n-1)(8)] &= 80 \\
 \therefore \frac{n}{2} [-144 + 8n - 8] &= 80 \\
 \therefore n(8n - 152) &= 160 \\
 \therefore 8n^2 - 152n - 160 &= 0 \\
 \therefore n^2 - 19n - 20 &= 0 \\
 \therefore (n-20)(n+1) &= 0 \\
 \therefore n &= 20 \text{ or } n = -1 \\
 \therefore n &= 20 \quad \blacktriangleleft \quad \dots n \in \mathbb{N}
 \end{aligned}$$



Question 3

$$\begin{aligned}
 10^{\sin x} + 10^{\sin x} \cdot 10 &= 110 \\
 \therefore 10^{\sin x} (1 + 10) &= 110 \\
 (\div 11) \quad \therefore 10^{\sin x} &= 10 \\
 \therefore \sin x &= 1 \\
 \therefore x &= 90^\circ \text{ or } -270^\circ \quad \blacktriangleleft
 \end{aligned}$$



Question 4

$$\begin{aligned}
 \frac{2^3 \sin^2 x}{4 \cdot 2^{\sin x}} &= 1 \\
 (\times 4) \quad \frac{2^3 \sin^2 x}{2^{\sin x}} &= 4 \\
 \therefore 2^{3 \sin^2 x - \sin x} &= 2^2 \\
 \therefore 3 \sin^2 x - \sin x &= 2 \\
 \therefore 3 \sin^2 x - \sin x - 2 &= 0 \\
 \therefore (3 \sin x + 2)(\sin x - 1) &= 0 \\
 \therefore \sin x &= -\frac{2}{3} \quad \text{or} \quad \sin x \neq 1 \quad (\because \sin x < 0 \text{ is given}) \\
 \therefore x &= 180^\circ + 41,81^\circ + k(360^\circ) \quad \text{or} \quad 360^\circ - 41,81^\circ + k(360^\circ), k \in \mathbb{Z} \\
 &= 221,81^\circ + k(360^\circ) \quad \text{or} \quad 318,19^\circ + k(360^\circ), k \in \mathbb{Z} \quad \blacktriangleleft
 \end{aligned}$$



Question 5

$$5.1 \quad 3^x(x-5) \leq 0 \quad \left[\text{Note: } 3^x > 0 \text{ for all values of } x. \right]$$

$$\therefore x-5 \leq 0$$

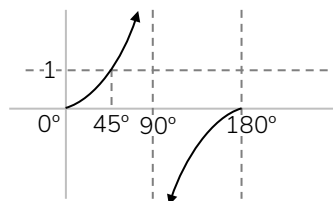
$$\therefore x \leq 5 \quad \blacktriangleleft$$

$$5.2 \quad 3^x(\tan x - 1) \leq 0$$

$$\therefore \tan x - 1 \leq 0$$

$$\therefore \tan x \leq 1$$

$$\therefore 0^\circ \leq x \leq 45^\circ \text{ or } 90^\circ < x \leq 180^\circ \quad \blacktriangleleft$$



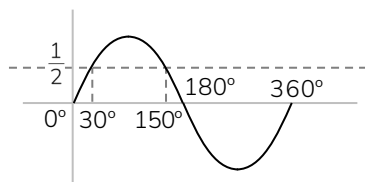
Note: $\tan x$ is undefined for $x = 90^\circ$

$$5.3 \quad 3^x(2 \sin x - 1) \leq 0$$

$$\therefore 2 \sin x - 1 \leq 0$$

$$\therefore \sin x \leq \frac{1}{2}$$

$$\therefore 0^\circ \leq x \leq 30^\circ \text{ or } 150^\circ \leq x \leq 360^\circ \quad \blacktriangleleft$$



Question 6

$$\text{For all } x: -1 \leq \cos x \leq 1$$

$$(\times 2) \quad \therefore -2 \leq 2 \cos x \leq 2$$

$$(+2) \quad \therefore 0 \leq 2 \cos x + 2 \leq 4$$

$$\therefore \text{The required range: } 2^\circ \leq y \leq 2^\circ$$

$$\therefore 1 \leq y \leq 16 \quad \blacktriangleleft$$



Question 7

$$2 \sin x = a - \frac{1}{a}$$

$$\therefore 4 \sin^2 x = \left(a - \frac{1}{a}\right)^2 \quad \left[\text{Square both sides} \right]$$

$$= a^2 - 2 + \frac{1}{a^2}$$

$$= 3 - 2 \quad \dots \quad a^2 + \frac{1}{a^2} = 3$$

$$= 1$$

$$\therefore \sin^2 x = \frac{1}{4}$$

$$\therefore \sin x = \pm \frac{1}{2}$$

$$\therefore x = \pm 30^\circ \quad \blacktriangleleft$$



Question 8

$$\cos^2 x - \sin^2 x = \frac{1}{2} \sin 2x - \cos^2 x \quad \dots = d$$

$$\therefore 2 \cos^2 x - \frac{1}{2} \cdot 2 \sin x \cos x - \sin^2 x = 0$$

$$\therefore 2 \cos^2 x - \sin x \cos x - \sin^2 x = 0$$

$$\therefore (2 \cos x + \sin x)(\cos x - \sin x) = 0$$

$$\therefore 2 \cos x = -\sin x \quad \text{or} \quad \cos x = \sin x$$

$$\div (-\cos x) \quad -2 = \tan x \quad \left[\begin{array}{l} \text{Note: In this case, } \cos x \neq \sin x. \\ \therefore \text{The constant difference cannot be zero.} \end{array} \right]$$

$$\therefore x = 180^\circ - 63.43^\circ + k(180^\circ), k \in \mathbb{Z}$$

$$= 116.57^\circ + k(180^\circ), k \in \mathbb{Z} \quad \blacktriangleleft$$

Question 9

$$\frac{\sin x}{\cos x} = \frac{\sqrt{3} \sin x}{\sin x}$$

$$\therefore \tan x = \sqrt{3}$$

$$\therefore x = 60^\circ + k \cdot 180^\circ, k \in \mathbb{Z}$$

$$\therefore x = 60^\circ \blacktriangleleft$$



Question 10

$$\sum_{y=43^\circ}^{47^\circ} \sin^2 y = \sin^2 43^\circ + \sin^2 44^\circ + \sin^2 45^\circ + \sin^2 46^\circ + \sin^2 47^\circ$$

$$= \sin^2 43^\circ + \sin^2 44^\circ + \left(\frac{1}{\sqrt{2}}\right)^2 + \cos^2 44^\circ + \cos^2 43^\circ$$

$$= \sin^2 43^\circ + \cos^2 43^\circ + \frac{1}{2} + \sin^2 44^\circ + \cos^2 44^\circ$$

$$= 1 + \frac{1}{2} + 1$$

$$= 2\frac{1}{2} \blacktriangleleft$$

Question 11

$$\hat{D} = 180^\circ - \theta \quad \dots \text{ opp. } \angle^s \text{ of cyclic quad}$$

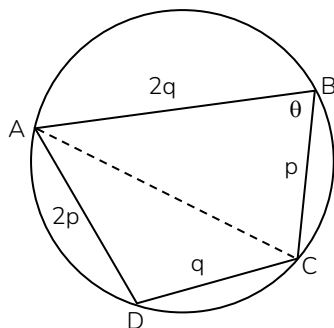
Area of c.q. ABCD

$$= \text{Area of } \triangle ADC + \text{Area of } \triangle ABC$$

$$= \frac{1}{2}(2p)(q) \sin(180^\circ - \theta) + \frac{1}{2}(p)(2q) \sin \theta$$

$$= pq \sin \theta + pq \sin \theta$$

$$= 2pq \sin \theta \blacktriangleleft$$



Question 12

$$12.1 \quad f(x) = x^2 - 2x \cos \theta - \sin^2 \theta$$

$$\Delta = (-2 \cos \theta)^2 - 4(1)(-\sin^2 \theta)$$

$$= 4 \cos^2 \theta + 4 \sin^2 \theta$$

$$= 4(\cos^2 \theta + \sin^2 \theta)$$

$$= 4$$

$\therefore$ roots are real, rational and unequal. $\blacktriangleleft$

$$12.2 \quad f(x) \text{ is a quadratic function.}$$

$$\text{Axis of symmetry: } x = -\frac{b}{2a} = -\frac{-2 \cos \theta}{2} = \cos \theta$$

$$\therefore \text{Min. value} = (\cos \theta)^2 - 2(\cos \theta)(\cos \theta) - \sin^2 \theta$$

$$= \cos^2 \theta - 2 \cos^2 \theta - \sin^2 \theta$$

$$= -\cos^2 \theta - \sin^2 \theta$$

$$= -(\cos^2 \theta + \sin^2 \theta)$$

$$= -1$$

$\therefore$ range: $y \geq -1 \blacktriangleleft$



Question 13

$$CD = \sqrt{(6-10)^2 + (6-3)^2} = 5$$

$\therefore CE = 5$... *tans from same pt*

$\therefore E(5; 3)$

$\therefore M(5; p)$... $ME \perp BC$

Now $MD = ME$

$$\therefore \sqrt{(6-5)^2 + (6-p)^2} = \sqrt{(5-5)^2 + (p-3)^2}$$

$$\therefore 1 + 36 - 12p + p^2 = p^2 - 6p + 9$$

$$\therefore 6p = 28$$

$$\therefore p = \frac{14}{3}$$

$$\therefore \text{radius} = \frac{14}{3} - 3 = \frac{5}{3}$$

$$\text{Area}_{\text{triangle}} = \frac{1}{2} \times 15 \times 9 = \frac{135}{2}$$

$$\text{Area}_{\text{circle}} = \pi \times \left(\frac{5}{3}\right)^2 = \frac{25}{9}\pi$$

$$\therefore \text{Probability} = \frac{\frac{25}{9}\pi}{\frac{135}{2}} = 12,9\% \blacktriangleleft$$



Question 14

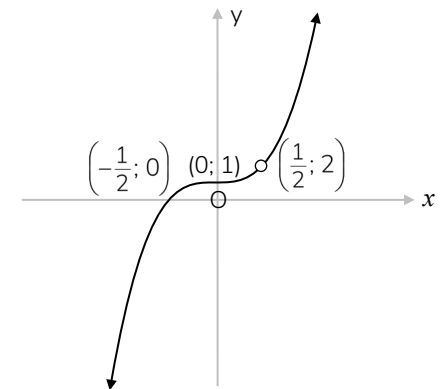
$$y = \left(\frac{2x+1}{8x^3-1}\right)(32x^5 - 16x^4 + 8x^3 - 4x^2 + 2x - 1)$$

$$= \left(\frac{2x+1}{8x^3-1}\right)[8x^3(4x^2 - 2x + 1) - (4x^2 - 2x + 1)]$$

$$= \left(\frac{2x+1}{8x^3-1}\right)[(8x^3-1)(4x^2-2x+1)]$$

$$= (2x+1)(4x^2-2x+1)$$

$$= 8x^3 + 1 \text{ with } x \neq \frac{1}{2} \blacktriangleleft$$

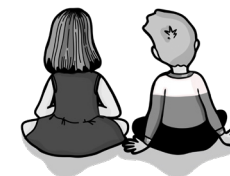


Undefined:

$$\text{If } 8x^3 = 1$$

$$x^3 = \frac{1}{8}$$

$$x = \frac{1}{2}$$



Question 15

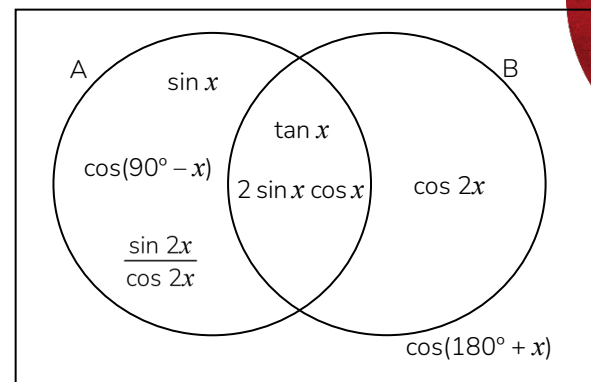
15.1.1 $f(-x) = \cos(-x)$
 $= \cos x$
 $= f(x)$
 $\therefore f(x) = \cos x$ is an **even** function. ◀



15.1.2 $f(-x) = \tan(-x)$
 $= -\tan x$
 $= -f(x)$
 $\therefore f(x) = \tan x$ is an **odd** function. ◀

15.2

$f(x)$	$f(-x)$	Odd or Even	Period
$\sin x$	$\sin(-x) = -\sin x$	Odd	360°
$\tan x$	$\tan(-x) = -\tan x$	Odd	180°
$\cos 2x$	$\cos(-2x) = \cos 2x$	Even	180°
$\cos(90^\circ - x) = \sin x$	$\sin(-x) = -\sin x$	Odd	360°
$2 \sin x \cos x = \sin 2x$	$\sin(-2x) = -\sin 2x$	Odd	180°
$\frac{\sin 2x}{\cos 2x} = \tan 2x$	$\tan(-2x) = -\tan 2x$	Odd	90°
$\cos(180^\circ + x) = -\cos x$	$-\cos(-x) = -\cos x$	Even	360°



$\therefore P(B' \text{ and } A) = \frac{3}{7}$ ◀

