Algebra

1.1 $k + 7$ 1.2 $k - 2$ 1.3 $k + 5$

2.1 subtract 1 from w **or** 1 less than w

2.2 add 6 to w and multiply the answer by 2

3.1 $-3x^6 + 7x^5 + 5x^4 + 2x^2 - x + 5$ 3.2 6 terms

3.3 -1 3.4 degree 6

3.5 The constant term is 5 . . . *the term with no x's*
$\therefore\ 5 - 2 = 3$

4.
$$\frac{x^2 - 49}{7x^2 + 49x} \div \frac{2x - 14}{21x}$$
$$= \frac{x^2 - 49}{7x^2 + 49x} \times \frac{21x}{2x - 14}$$
$$= \frac{\cancel{(x+7)}\cancel{(x-7)}}{\cancel{7}\cancel{x}\cancel{(x+7)}} \times \frac{{}^{3}\cancel{21}\,\cancel{x}}{2\cancel{(x-7)}}$$
$$= \frac{3}{2}$$

Exponents

1.1
$$\sqrt{1+3+5+7} + \sqrt{10^2 - 6^2} - 2 \times \sqrt{2 \times 18}$$
$$= \sqrt{16} + \sqrt{100 - 36} - 2 \times \sqrt{36}$$
$$= 4 + \sqrt{64} - 2 \times 6$$
$$= 4 + 8 - 12$$
$$= 0$$

1.2
$$(2+3)^2 - (3^2 - 2^2)$$
$$= 5^2 - (9 - 4)$$
$$= 25 - 5$$
$$= 20$$

2.1 $\boxed{\sqrt{49} = 7;\ \sqrt[3]{-27} = -3;\ \left(\sqrt{5}\right)^2 = 5;\ -\sqrt[3]{8} = -2;\ 1^4 = 1}$

$\therefore$ In ascending order: $\sqrt[3]{-27}\,;\ -\sqrt[3]{8}\,;\ 1^4;\ \left(\sqrt{5}\right)^2;\ \sqrt{49}$

2.2 $\boxed{\sqrt[3]{125} = 5;\ -\sqrt{16} = -4;\ \sqrt[3]{8} = 2;\ -1^2 = -1;\ \sqrt{81} = 9}$

$\therefore$ In ascending order: $-\sqrt{16}\,;\ -1^2;\ \sqrt[3]{8};\ \sqrt[3]{125}\,;\ \sqrt{81}$

2.3 $\boxed{4^2 = 16;\ -3^2 = -9;\ (-2)^3 = -8;\ (-5)^2 = 25;\ -(1)^3 = -1}$

$\therefore$ In ascending order: $-3^2;\ (-2)^3;\ (-1)^3;\ 4^2;\ (-5)^2$

3.1
$$\sqrt{3^2 + 3^1 + 3^0 + 3^{-1} + 3^{-2}}$$
$$= \sqrt{9 + 3 + 1 + \frac{1}{3} + \frac{1}{9}}$$
$$= \sqrt{\frac{13}{1} + \frac{1}{3} + \frac{1}{9}}$$
$$= \sqrt{\frac{117 + 3 + 1}{9}}$$
$$= \sqrt{\frac{121}{9}}$$
$$= \frac{11}{3}$$

3.2
$$\frac{\sqrt{49a^6b^{-2}}\,.\,(3a^2b^{-1})^{-2}}{2a^0b^{-3}}$$
$$= \frac{7a^3b^{-1}\,.\,3^{-2}a^{-4}b^2}{2a^0b^{-3}}$$
$$= \frac{7b^{-1+2+3}}{9 \times 2a^{0-3+4}}$$
$$= \frac{7b^4}{18a}$$

OR

$$\frac{\sqrt{49a^6b^{-2}}\,.\,(3a^2b^{-1})^{-2}}{2a^0b^{-3}}$$
$$= \frac{7a^3b^{-1}}{(3a^2b^{-1})^2 \times 2a^0b^{-3}}$$
$$= \frac{7a^3b^{-1}}{9a^4b^{-2} \times 2(1)b^{-3}}$$
$$= \frac{7b^{-1+2+3}}{18a^{4-3}}$$
$$= \frac{7b^4}{18a}$$

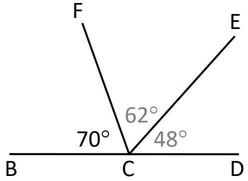
Geometry of Straight lines

1. $v + 62° + 48° = 180°$ ($\angle$s on a str line)

$\therefore\ v + 110° = 180°$

$70° + 110° = 180°$

$\therefore\ v = 70°$



2. $w + w + 90° = 360°$ ($\angle$s in a rev)

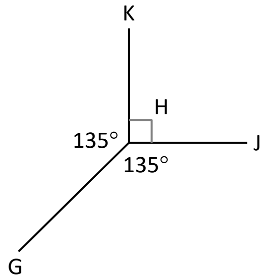
$\therefore\ 2w + 90° = 360°$

$270° + 90° = 360°$

$\therefore\ 2w = 270°$

$2 \times 135° = 270°$

$\therefore\ w = 135°$



3.1 $115° + e + 32° = 180°$ . . . *co-int. $\angle$'s; YZ || AB*

$\therefore\ e + 147°$ **− 147°** $= 180°$ **− 147°**

$\therefore\ e = 33°$

3.2 $\hat{B}_2 + 68° = 180°$ . . . *co-int. $\angle$'s; EF || GH*

$\therefore\ \hat{B}_2 + 68°$ **− 68°** $= 180°$ **− 68°**

$\therefore\ \hat{B}_2 = 112°$

and $f = \hat{B}_2$ . . . *vertically opposite $\angle$'s*

$\therefore\ f = 112°$

OR Find $\hat{B}_3$ using alternate angles or $\hat{B}_1$ using corresponding angles and then use angles on a straight line to find f.

$\hat{D}_1 = 68°$ . . . *corresponding $\angle$'s; GH || JK*

and $g = \hat{D}_1$ . . . *vertically opposite $\angle$'s*

$\therefore\ g = 68°$

Note:
There are often several ways of showing something in geometry.

4. $E\hat{O}A = \mathbf{x}$ . . . *corresponding $\angle$'s ; AB || CD*

$\therefore\ \mathbf{x} + 5\mathbf{x} + 5° + 50° - \mathbf{x} = 180°$ . . . *$\angle$'s on str. line*

$\therefore\ 5\mathbf{x} = 125°$

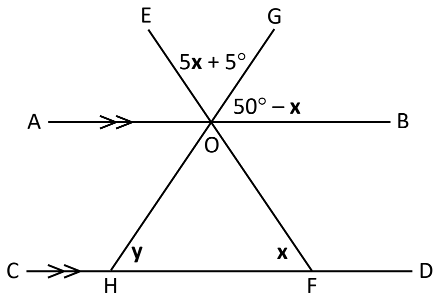
$\therefore\ \mathbf{x} = 25°$

$\therefore\ G\hat{O}B = 50° - 25°$

$= 25°$

$\therefore\ \mathbf{y} = 25°$ . . . *corresponding $\angle$'s ; AB || CD*

(There are alternative solutions)



5. $a = 57°$ . . . *co-interior $\angle$'s ; PQ || RS*

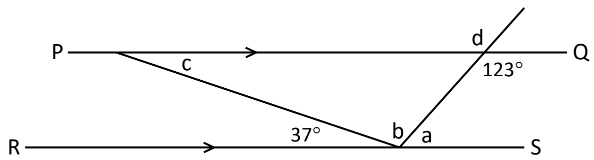
$a + b + 37° = 180°$ . . . *$\angle$'s on straight line*

$\therefore\ b = 180° - 37° - 57°$

$= 86°$

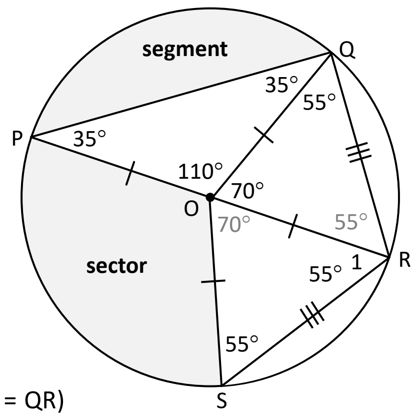
$c = 37°$ . . . *alternate $\angle$'s ; PQ || RS*

$d = 123°$ . . . *vertically opposite $\angle$'s*



Geometry of 2D Shapes

GRADE 7



1.1 OP, OQ, OR, OS

1.2 POR (or PR)

1.3 PQ, QR, SR

1.4 1G 2C 3I 4D
5A 6H 7B 8F 9E

1.5 $P\hat{Q}R = 55° + 35° = 90°$ (refer to 2.4)
$\therefore$ ΔPQR is a right-angled, scalene triangle.
Quadrilateral QOSR is a kite (OQ = OS and SR = QR)

1.6 ΔQOR and ΔSOR are congruent.

Grade 8

2.1 ΔABC is an isosceles triangle.

2.2 $D\hat{B}C + D\hat{C}B + 100° = 180°$ ... *sum of ∠'s in ΔDBC*

$\therefore\ D\hat{B}C + D\hat{C}B + 100° \mathbf{- 100°} = 180° \mathbf{- 100°}$

$\therefore\ D\hat{B}C + D\hat{C}B = 80°$

But $D\hat{B}C = D\hat{C}B$ ... *∠'s opp. = sides BD and CD*

$\therefore\ D\hat{B}C = D\hat{C}B = 40°$

$\therefore\ A\hat{B}C = 30° + 40° = 70°$

$A\hat{C}B = A\hat{B}C = 70°$ ... *∠'s opposite = sides*

$\therefore\ \hat{A} + A\hat{B}C + A\hat{C}B = 180°$ ... *sum of ∠'s in Δ*

$\therefore\ \hat{A} + 70° + 70° = 180°$

$\therefore\ \hat{A} + 140° = 180°$

$\therefore\ \hat{A} + 140° \mathbf{- 140°} = 180° \mathbf{- 140°}$

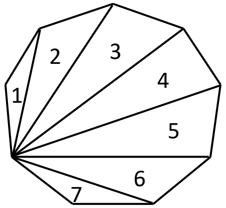
$\therefore\ \hat{A} = 40°$

GRADE 9

3.1 9 vertices

$\therefore$ A total of
$7 \times 180° = 1\,260°$

$\therefore$ Interior ∠
$= 1\,260° \div 9$
$= 140°$

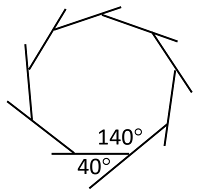


OR 9 vertices

& Sum of ext ∠'s of all polygons = 360°

$\therefore$ Each exterior ∠ = $360° \div 9 = 40°$

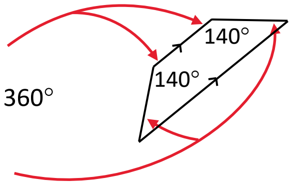
$\therefore$ Each interior ∠
$= 180° - 40°$ ... *∠'s on a*
$= 140°$

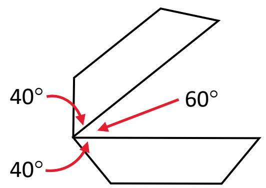


3.2 The interior ∠'s of the nonagon are each 140°

and 4 ∠'s of a quad are 360°

$\therefore$ The base ∠'s of the quadrilateral are 40°





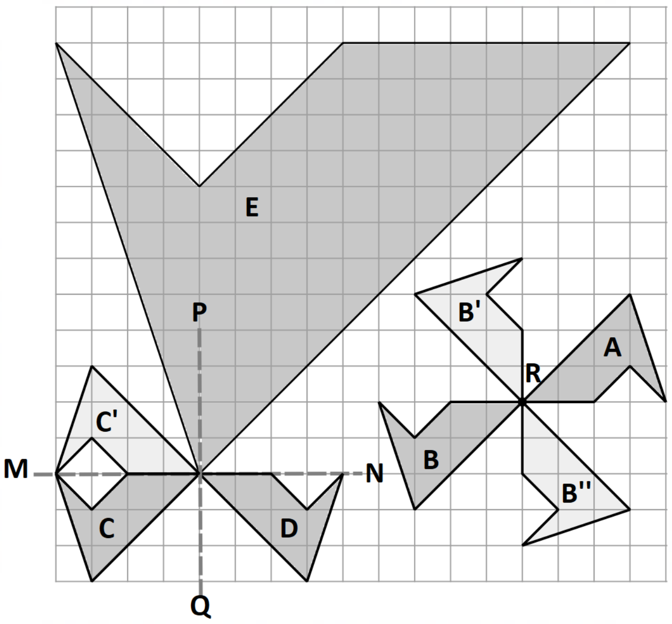
If each base ∠ is 40° then each of the angles of the triangle is:

$140° - (40° + 40°) = 60°$

$\therefore$ The diagonals form an equilateral Δ ... *3 ∠'s = 60°*

Transformations

1.



1.1.1 Shape **A rotates** 180° to form shape **B**.

1.1.2 Shape **B translate**s 9 units left and 2 units down to shape **C**.Z

1.1.3 Shape **C reflects** in line PQ to form shape **D**.

1.1.4 Shape **C enlarges** by a factor of 4 to form shape **E**.

1.2 Shape **E** is **similar**, but not congruent, to shape **A**.

1.3 Shape **B rotates** 90° clockwise to form shape **B'**.

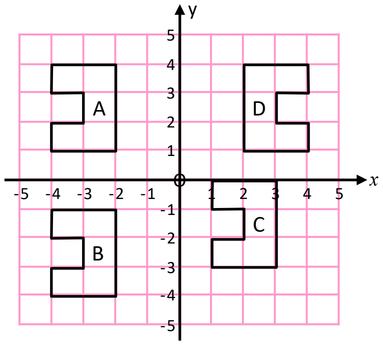
1.4 Shape **B rotates** 270° clockwise to form shape **B''**.

1.5 Shape **C reflects** in the line MN to form shape **C'**.

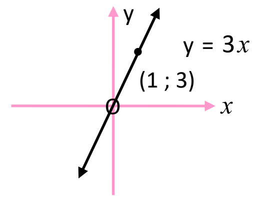
2.1 image B

2.2 image C

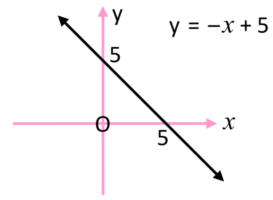
2.3 image D



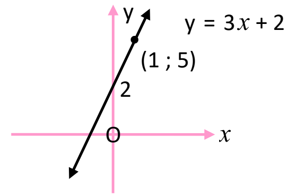
3.1

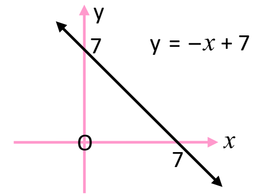


3.2

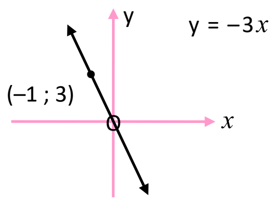


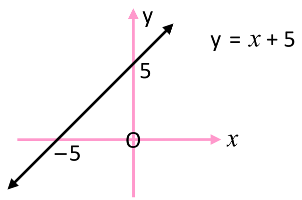
(i)



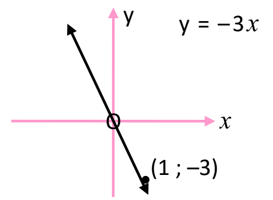


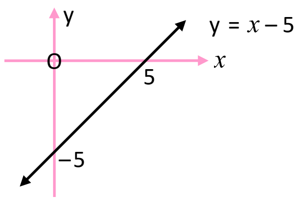
(ii)



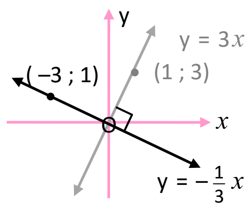


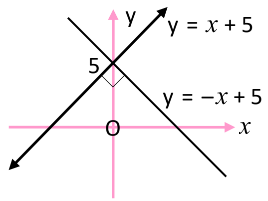
(iii)





(iv)

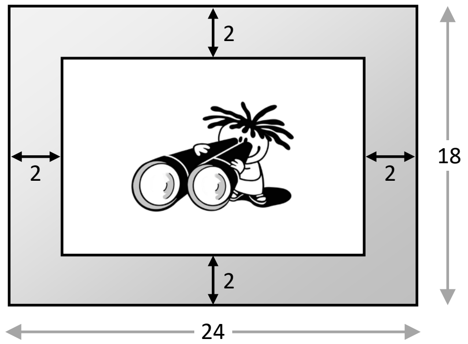




Note:
The gradient of the image is the negative reciprocal of the gradient of the given graph.

Area and Perimeter of 2D Shapes

1.



1.1 The length of the inner edge of the border
$= 24 - 2 - 2$
$= 20$ cm

The breadth of the inner edge of the border
$= 18 - 2 - 2$
$= 14$ cm

1.2 The perimeter of the inner edge of the border
$= 20 + 14 + 20 + 14$
$= 68$ cm

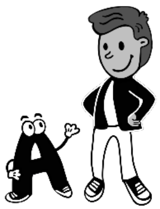
1.3 The outer perimeter of the frame
$= 24 + 18 + 24 + 18$
$= 84$ cm

The sum of the inner and outer perimeters
$= 84 + 68$
$= 152$ cm

Extension answers

1.4 The area of the shaded part
= Area of large rectangle – Area of small rectangle
$= 24 \times 18 - 20 \times 14$
$= 24(20 - 2) - 280$
$= 480 - 48 - 280$
$= 152 \text{ cm}^2$

1.5 Shaded area : unshaded area
$= 152 : 280$
$= 19 : 35$

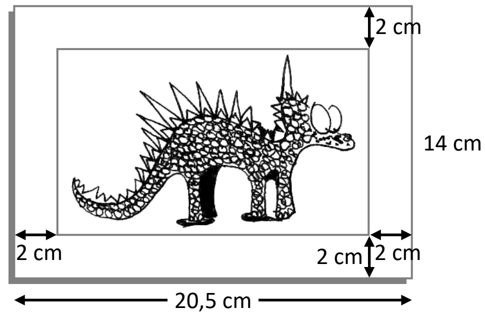


2.1 Frame A:
$P = 2(\ell + b)$
$= 2(20{,}5 + 14)$
$= 69$ cm

Frame B:
$P = 2(\ell + b)$
$= 2(18 + 13)$
$= 62$ cm

$\therefore$ Frame B has the smallest perimeter.

2.2 A:

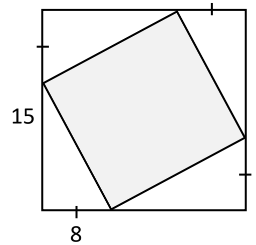


$\therefore$ length of picture
$= 20{,}5 \text{ cm} - 2 \text{ cm} - 2 \text{ cm}$
$= 16{,}5$ cm

breadth of picture
$= 14 \text{ cm} - 2 \text{ cm} - 2 \text{ cm}$
$= 10$ cm

$\therefore$ Area of picture $= \ell \times b$
$= 16{,}5 \text{ cm} \times 10 \text{ cm}$
$= 165 \text{ cm}^2$

3. Lengths of sides



- sides of larger square
  $= 15 + 8 = 23$ units
- sides of shaded square
  $= \sqrt{15^2 + 8^2}$
  $= 17$ units . . . *Pythag*
  . . . *(or Pythag triple 8 : 15 : 17)*

$\therefore$ Area of larger square $= 23^2 = 529 \text{ units}^2$

& Area of shaded square $= 17^2 = 289 \text{ units}^2$

$529 \div 2 = 264{,}5$

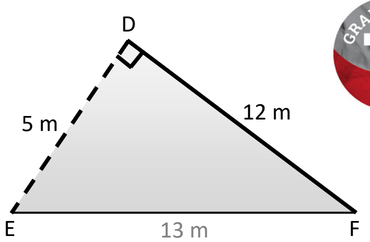
$\therefore$ 289 units^2 is just a bit more than 264,5,
i.e. just a bit more than half the area of the larger square.

Area and Perimeter of 2D Shapes

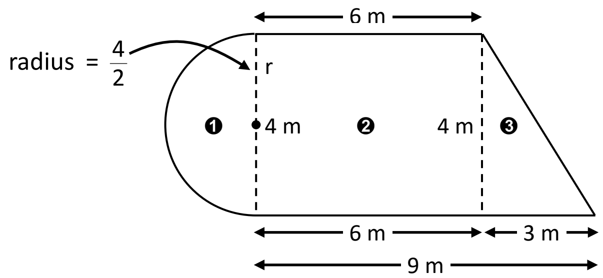
1. Area $\Delta DEF = \frac{1}{2} \times \text{base} \times \perp \text{height}$

$= \frac{1}{2} \times 12 \times 5$

$= 30 \text{ m}^2$



2. Break the complex figure down into basic shapes and fill in any deduced measurements:



Area of semi-circle ❶ $= \frac{1}{2} \times \pi r^2$

$= \frac{1}{2} \times \pi(2)^2$

$= 6{,}28 \text{ m}^2$

Area of rectangle ❷

$= \ell \times b$

$= 6 \times 4$

$= 24 \text{ m}^2$

Area of triangle ❸

$= \frac{b \times h}{2}$

$= \frac{3 \times 4}{2}$

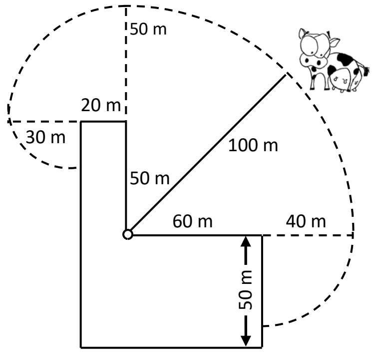
$\therefore$ Total area $= 6{,}28 + 24 + 6$

$= 36{,}28 \text{ m}^2$

$\therefore$ Total cost $= \text{R2 } 100 \times 36{,}28 \text{ m}^2$

$= \text{R76 } 188$

3.



$\frac{1}{4}$ of a circle with r = 100 m

$+ \frac{1}{4}$ of a circle with r = 50 m

$+ \frac{1}{4}$ of a circle with r = 30 m

$+ \frac{1}{4}$ of a circle with r = 40 m

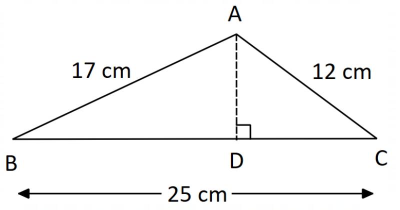
Grazing area

$= \frac{1}{4}\pi(100)^2 + \frac{1}{4}\pi(50)^2 + \frac{1}{4}\pi(30)^2 + \frac{1}{4}\pi(40)^2$

$= 11\ 780{,}97 \text{ m}^2$

Area and Perimeter of 2D Shapes

1.



1.1 Using Heron's formula (no heights are given):

$$s = \frac{25 + 12 + 17}{2} = \frac{54}{2} = 27 \text{ cm}$$

Area of ΔABC

$= \sqrt{27(27-25)(27-12)(27-17)}$

$= \sqrt{27(2)(15)(10)}$

$= \sqrt{27 \times 300}$

$= \sqrt{8\ 100}$

$= 90 \text{ cm}^2$

1.2 $\frac{1}{2} \times AD \times 25 = 90$

$\frac{1}{2} \times 180 = 90$

$\therefore AD \times 25 = 180$

$7{,}2 \times 25 = 180$

$\therefore AD = 7{,}2$ cm

or

$\frac{1}{2} \times AD \times 25 = 90$

$\therefore 12{,}5 \times AD = 90$

$\therefore \frac{12{,}5 \times AD}{\mathbf{12{,}5}} = \frac{90}{\mathbf{12{,}5}}$

$\therefore AD = \frac{900}{125}$

$= \frac{36}{5}$

$= 7{,}2$ cm

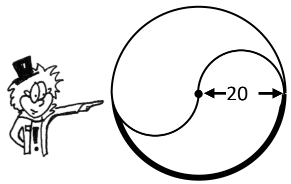
2. Circumference of outer big semi-circle

$= \frac{1}{2} \times 2\pi r$

$= \frac{1}{2} \times 2\pi(20)$

$= \pi \times 20$

$(= 62{,}831\ldots$ mm)



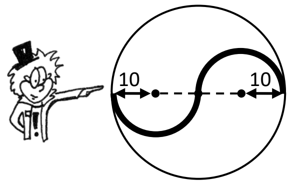
Circumference of 2 small inner semi-circles

$= 2 \times \left(\frac{1}{2} \times 2\pi r\right)$

$= 2 \times \left[\frac{1}{2} \times 2\pi(10)\right]$

$= \pi \times 20$

$(= 62{,}831\ldots$ mm)



$\therefore$ Total perimeter of black teardrop

= circumference of outer big semi-circle + circumference of 2 small inner semi-circles

$= 62{,}831\ldots \times 2$ (or $2 \times \pi \times 20$)

$= 125{,}663\ldots$ mm

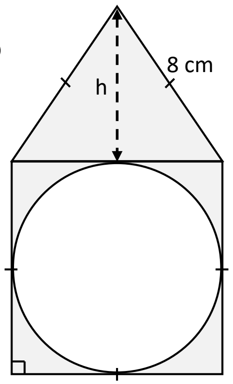
$\approx 125{,}66$ mm . . . *correct to 2 decimal places*

3. Shaded area

= Area of Δ + Area of $\square$ − Area of $\bigcirc$

$= \frac{8 \times 6{,}93}{2} + 8 \times 8 - \pi(4)^2$

$= 41{,}45 \text{ cm}^2$



Volume and Surface Area of 3D Shapes

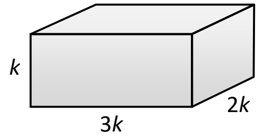
1.1 $3 : 2 : 1 = 3k : 2k : k$

Volume of prism

$= \text{length} \times \text{breadth} \times \text{height}$

$= 3k \times 2k \times k$

$= 6k^3$



$\therefore\ 6k^3 = 750$

$6k^3 \div 6 = 750 \div 6 \quad (6 \times 125 = 750)$

$\therefore\ k^3 = 125$

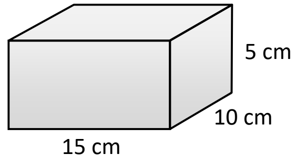
$\therefore\ k = 5 \text{ cm} \quad \left(\sqrt[3]{125} = \sqrt[3]{5^3} = 5\right)$

$2k = 2 \times 5 = 10 \text{ cm}$

$3k = 3 \times 5 = 15 \text{ cm}$

$\therefore$ the dimensions are $15 \text{ cm} \times 10 \text{ cm} \times 5 \text{ cm}$.

1.2

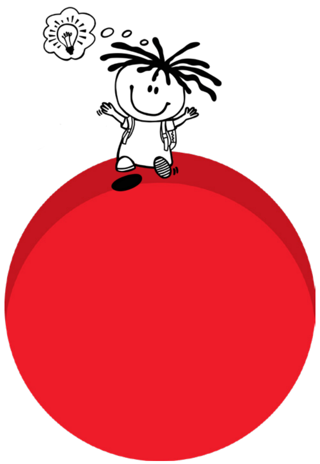


The total surface area of the prism

$= (2 \times 15 \times 5) + (2 \times 10 \times 5) + (2 \times 15 \times 10)$

$= 150 + 100 + 300$

$= 550 \text{ cm}^2$



2.1 Let the height be h.

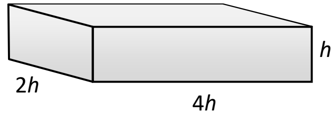
$\therefore$ the breadth $= 2h$ and the length $= 4h$.

The total surface area

$= (2 \times 4h \times h) + (2 \times 2h \times h) + (2 \times 2h \times 4h)$

$= 8h^2 + 4h^2 + 16h^2$

$= 28h^2$



$\therefore\ 28h^2 = 112$

$28 \times 4 = 112$

$\therefore\ h^2 = 4$

$\therefore\ h = 2 \text{ cm}$

$2h = 2 \times 2 = 4 \text{ cm}$

$4h = 4 \times 2 = 8 \text{ cm}$

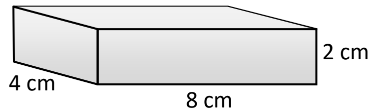
$\therefore$ the dimensions are $2 \text{ cm} \times 4 \text{ cm} \times 8 \text{ cm}$.

2.2 The volume of the prism

$= \text{length} \times \text{breadth} \times \text{height}$

$= 8 \times 4 \times 2$

$= 64 \text{ cm}^3$

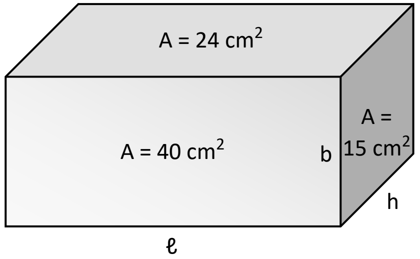


2.3 The capacity of the prism

$= 64 \text{ m}\ell$

$= 0{,}064$ litres

3.



Integers are positive and negative whole numbers; but since we are working with dimensions we are looking for positive integers, i.e. ***whole positive numbers****.*

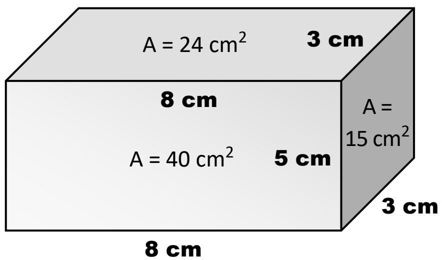
The only pairs of products of factors of 15 are 1×15 and 3×5.

Because 15 is not a factor of 40, it can be concluded that 3 by 5 are the dimensions of the sides ($h \times b$). And because 5 is a factor of 40 (and not 3) or because 3 is a factor of 24 (and not 5), h = 5 cm and b = 3 cm.

To then find ℓ you can either divide 40 by 5 or divide 24 by 3.

$\therefore$ dimensions are ℓ = 8 cm, b = 3 cm, h = 5 cm

$8 \times 3 = 24\text{ cm}^2$
$8 \times 5 = 40\text{ cm}^2$
$3 \times 5 = 15\text{ cm}^2$



4.1 $44 - \frac{1}{2}t = 0$ . . . *W = 0 when the tank is empty.*

$\therefore -\frac{t}{2} = -44$ . . . $\frac{1}{2}t = \frac{1}{2} \times \frac{t}{1} = \frac{t}{2}$

$\therefore t = 88$

$\therefore$ 88 minutes till the tank is empty

OR by inspection:

$\frac{1}{2}\,\ell$ per 1 min

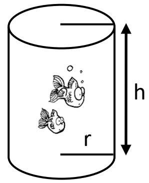
$\therefore$ 44 ℓ per 88 min

4.2 $W = 44 - \frac{1}{2}(62)$

$= 44 - 31$

$= 13$

$\therefore$ 13 litres will be in the tank.



4.3 Height = 35 cm & Vol = area of base $\times$ height

1 ℓ : 1 000 cm³

$\therefore$ 44 ℓ : 44 000 cm³

$44\,000 = \pi r^2 \times 35$

$\therefore r^2 = \frac{44\,000}{35\pi}$ . . . *÷ by 35π to isolate r^2*

$\therefore r^2 = 400{,}160\ldots$

$\therefore r = 20{,}004\ldots \approx 20$

$\therefore r \approx 20{,}00$

$\therefore$ radius is 20 cm.