AMESA 2026
WORKSHOP

An investigation into different ways of finding AREAS

Senior Phase

SCAN ME

for questions

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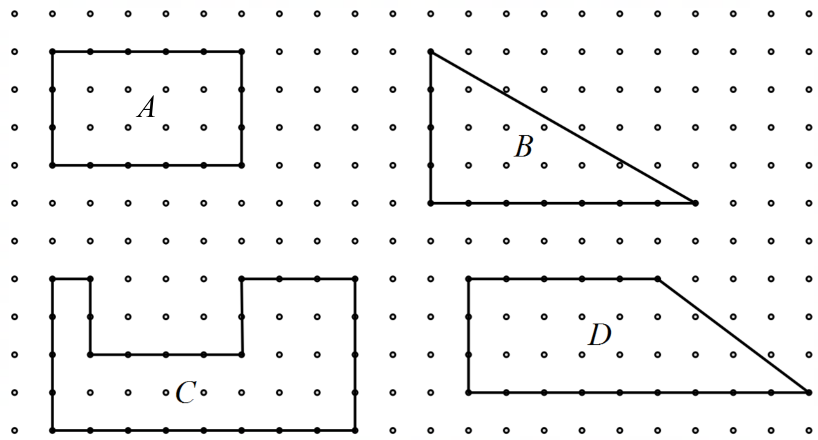
Your Key to Exam Success

Pick's Theorem

If you are asked to find the area of a shape on grid paper, Pick's theorem is very useful.

In all the grids, each block represents 1 unit^2.

1. Count the number of blocks in order to calculate the area of each shape.



2. Complete the table.

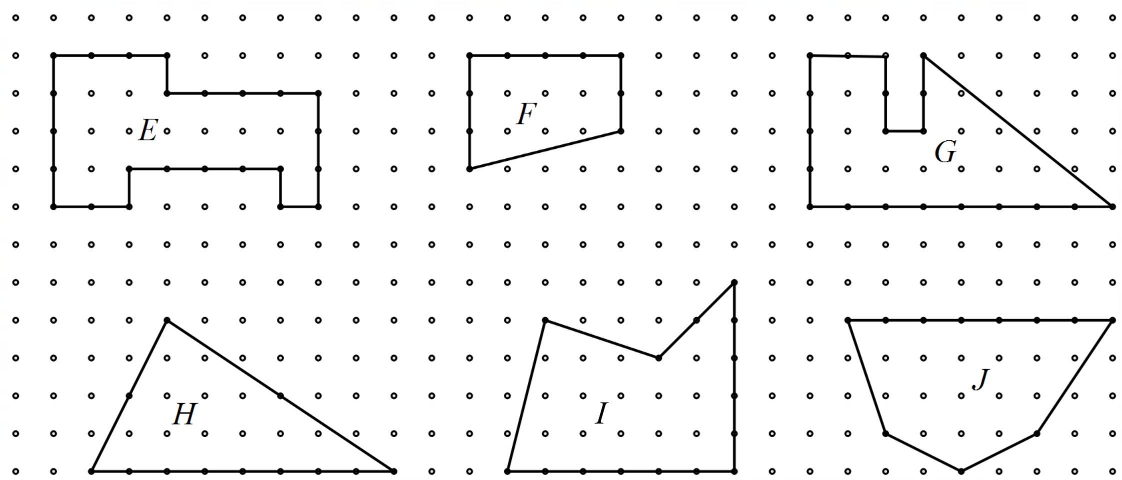
Shape	Area
A	15
B	14
C	24
D	21

3. Count the number of dots on the boundary of each shape and the number of dots on the inside. The first one has been done for you. Complete the table.

Shape	No. of dots on boundary	No. of dots inside
A	16	8
B	12	9
C	28	11
D	18	13

4. Count the number of dots on the boundary of each shape and the number of dots on the inside.

Then use the blocks to determine the area of each shape. Complete the table.

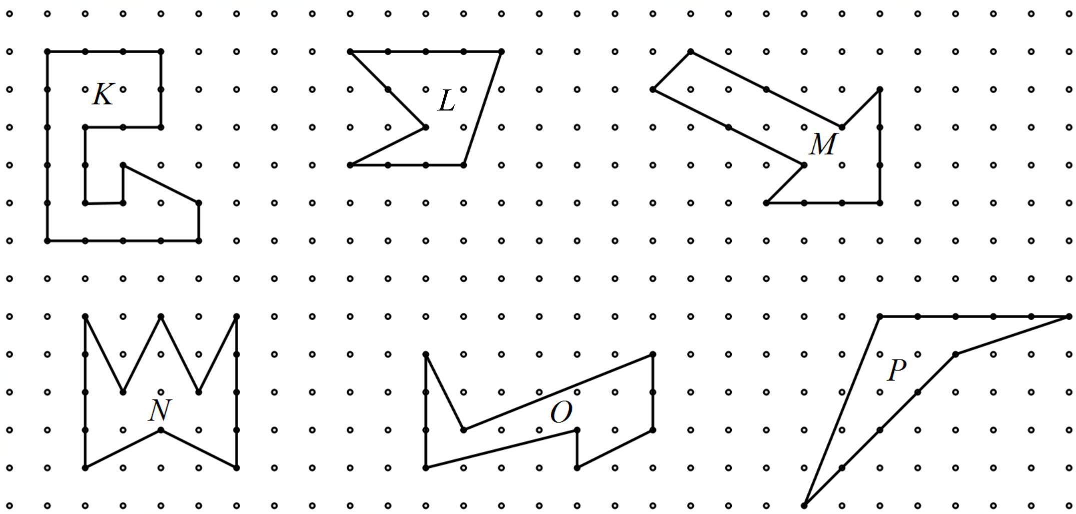


Shape	No. of dots on boundary	No. of dots inside	Area
E	24	9	20
F	10	6	10
G	20	11	20
H	12	11	16
I	15	14	20 ½
J	11	14	18 ½

5. Look at the ten shapes above and see if you can find a relationship between the number of dots on the boundary of the shape (B), the number of dots inside the shape (I), and the area (A).

$$A = \frac{B}{2} + I - 1$$

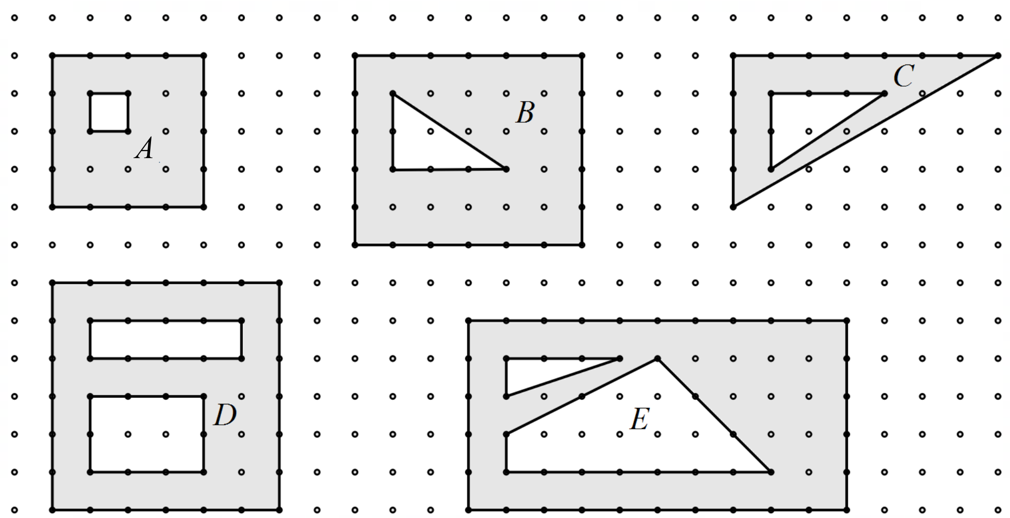
6. Either by using your formula, or by counting the number of blocks, determine the area of the following shapes.



Shape	Area
K	13
L	7 ½
M	10 ½
N	10
O	9
P	8 ½

7. What happens now if we have shapes that have "holes" in them?

7.1 We want to calculate the shaded area.



When counting, only consider the shaded shape. Count the number of dots on any boundary of each shape. Then count the number of dots within the shape. Use the blocks to determine the area of each shape. Complete the table.

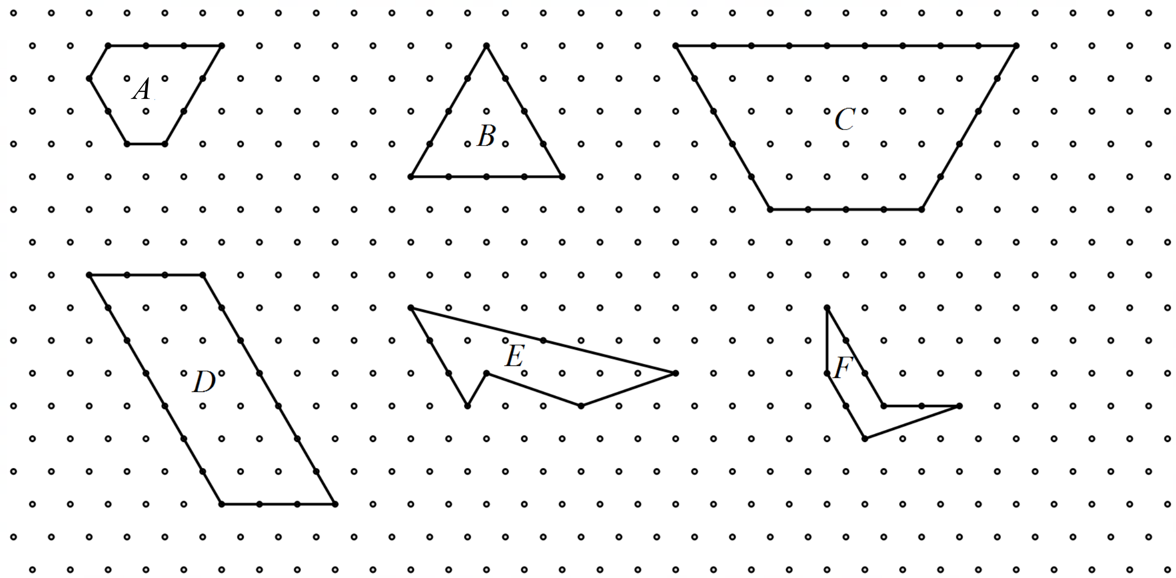
Shape	No. of dots on boundary	No. of dots inside	No. of holes	Area
A	20	5	1	15
B	28	13	1	27
C	18	2	1	11
D	44	3	2	26
E	48	11	2	36

7.2 Determine a relationship between the number of dots on the boundary of the shape (B), the number of dots inside the shape (I), the number of holes (H), and the area (A) for shapes that have holes in them.

$$A = \frac{B}{2} + I + H - 1$$

8. Let us now consider a triangular grid. Each small triangle represents 1 square unit.

8.1 Count the number of dots on the boundary of each shape and the number of dots on the inside. The first one has been done for you. Complete the table.



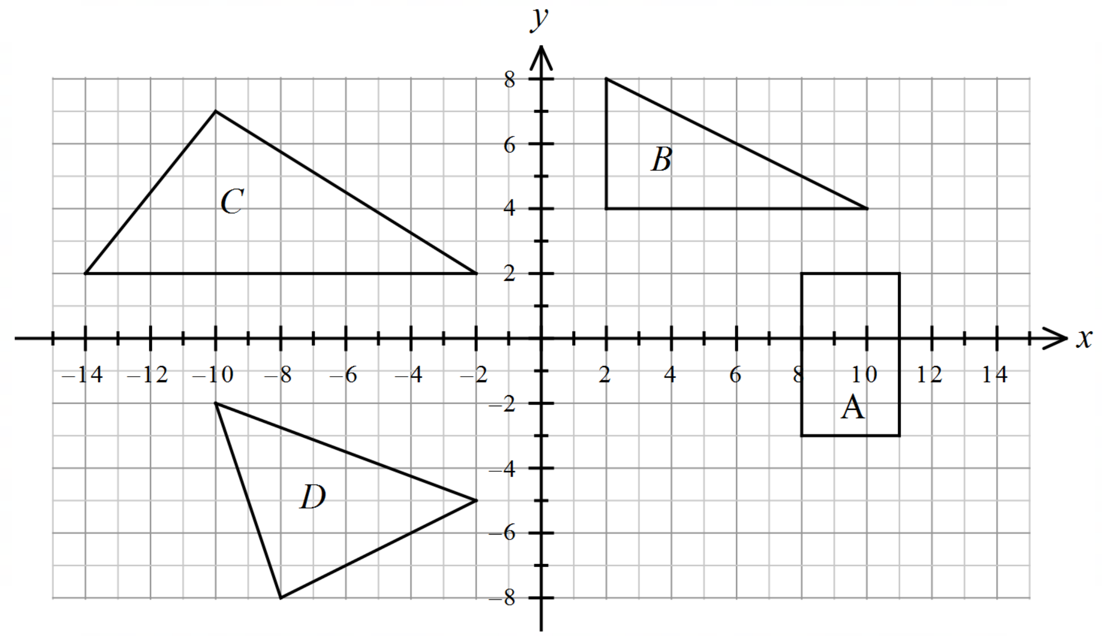
Shape	No. of dots on boundary	No. of dots inside	Area
A	10	3	14
B	12	3	16
C	23	22	65
D	20	12	42
E	8	6	18
F	9	0	7

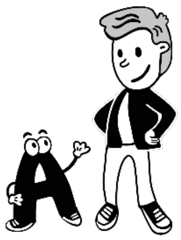
8.2 Determine a relationship between the number of dots on the boundary of the shape (B), the number of dots inside the shape (I), and the area (A) for shapes that have holes in them.

$A = B + 2I - 2$

Shoelace Theorem

1. Determine the area of each shape below. Complete the table.





Shape	Area
A	15
B	16
C	30
D	21

2. Now determine the area of shape D in question 1. Use the following to start off.

$$\left|\begin{array}{rr} -10 & -2 \\ -2 & -5 \\ -8 & -8 \\ -10 & -2 \end{array}\right|$$

$(-10 \times -5) + (-2 \times -8) + (-8 \times -2) \ = \ 82$

$$\left|\begin{array}{rr} -10 & -2 \\ -2 & -5 \\ -8 & -8 \\ -10 & -2 \end{array}\right|$$

$(-2 \times -2) + (-5 \times -8) + (-8 \times -10) \ = \ 124$

$$\therefore \ \text{Area} \ = \ \frac{1}{2}(124 - 82) \ = \ 21 \text{ units}^2$$

3. Determine, using the Shoelace Theorem, the area of the polygon with co-ordinates $(3; 5)$, $(6; 1)$, $(5; -3)$, $(-1; -2)$ and $(-7; 0)$. Don't forget to repeat the first co-ordinate.

$$\left|\begin{array}{rr} 3 & 5 \\ 6 & 1 \\ 5 & -3 \\ -1 & -2 \\ -7 & 0 \\ 3 & 5 \end{array}\right|$$

$3 \times 1 + 6 \times -3 + 5 \times -2 + (-1 \times 0) + (-7 \times 5) \ = \ -60$

$$\left|\begin{array}{rr} 3 & 5 \\ 6 & 1 \\ 5 & -3 \\ -1 & -2 \\ -7 & 0 \\ 3 & 5 \end{array}\right|$$

$5 \times 6 + 1 \times 5 + (-3 \times -1) + (-2 \times -7) + 0 \times 3 \ = \ 52$

$$\therefore \ \text{Area} \ = \ \frac{1}{2}\big(52 - (-60)\big) \ = \ 56 \text{ units}^2$$

4. A triangle with co-ordinates $(a; 5)$, $(-2a; 3)$ and $(0; a+1)$ has an area of $28\frac{1}{2}$ units^2. Determine the value(s) of a.

$$\begin{vmatrix} a & 5 \\ -2a & 3 \\ 0 & a+1 \\ a & 5 \end{vmatrix}$$

$$a \times 3 + [-2a(a+1)] + 0 \times 5 \;=\; 3a - 2a^2 - 2a \;=\; -2a^2 + a$$

$$5 \times -2a + 3 \times 0 + a(a+1) \;=\; -10a + a^2 + a \;=\; a^2 - 9a$$

If $\;-2a^2 + a > a^2 - 9a$

$$\frac{1}{2}\Big[\left(-2a^2 + a\right) - \left(a^2 - 9a\right)\Big] \;=\; 28\frac{1}{2}$$

$$\therefore\; -2a^2 + a - a^2 + 9a \;=\; 57$$

$$\therefore\; 3a^2 - 10a + 57 \;=\; 0$$

which has no real solution

If $\;a^2 - 9a > -2a^2 + a$

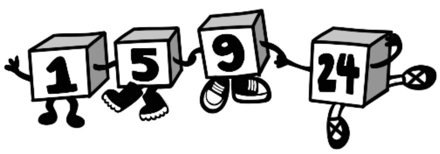
$$\frac{1}{2}\Big[\left(a^2 - 9a\right) - \left(-2a^2 + a\right)\Big] \;=\; 28\frac{1}{2}$$

$$\therefore\; a^2 - 9a + 2a^2 - a \;=\; 57$$

$$\therefore\; 3a^2 - 10a - 57 \;=\; 0$$

$$\therefore\; (3a - 19)(a + 3) \;=\; 0$$

$$\therefore\; a = \frac{19}{3} \quad \text{or} \quad a = -3$$



Heron's Formula

This is useful if you know, or can calculate, the three sides of a triangle.

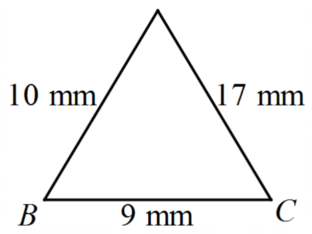
Heron's Formula:

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where a, b and c are the sides of a triangle and s is the semi-perimeter, i.e. $s = \frac{1}{2}(a+b+c)$

1. Determine the area of the following triangles. They are not drawn to scale.

1.1

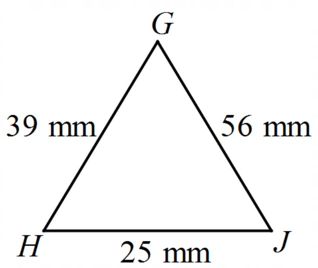


$$s = \frac{1}{2}(10+17+9) = 18$$

$$\therefore \text{ Area } = \sqrt{18(18-10)(18-17)(18-9)}$$

$$= 36 \text{ mm}^2$$

1.2

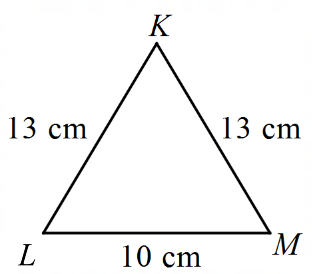


$$s = \frac{1}{2}(39+56+25) = 60$$

$$\therefore \text{ Area } = \sqrt{60(60-39)(60-56)(60-25)}$$

$$= 420 \text{ mm}^2$$

1.3

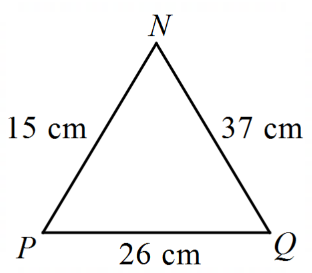


$$s = \frac{1}{2}(13+13+10) = 18$$

$$\therefore \text{ Area } = \sqrt{18(18-13)(18-13)(18-10)}$$

$$= 60 \text{ cm}^2$$

1.4

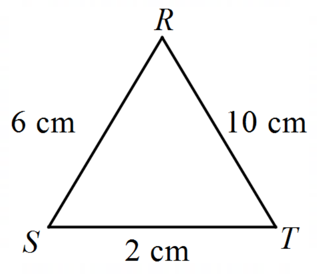


$$s = \frac{1}{2}(15+37+26) = 39$$

$$\therefore \text{ Area } = \sqrt{39(39-15)(39-37)(39-26)}$$

$$= 156 \text{ cm}^2$$

2.



If you use Heron's formula to calculate the area you will notice there is a problem.

Explain why this problem exists.

$$s = \frac{1}{2}(6+10+2) = 9$$

$$\therefore \text{ Area } = \sqrt{9(9-6)(9-10)(9-2)}$$

$$= \sqrt{-189}$$

The triangle does not exist since the two shorter sides add up to less than the longest side.

3. An equilateral triangle has sides of length a cm.

3.1 Using Heron's formula, determine a formula for the area of the triangle.

$$s = \frac{1}{2}(a+a+a) = \frac{3a}{2}$$

$$\therefore \text{ Area } = \sqrt{\frac{3a}{2}\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)}$$

$$= \sqrt{\frac{3a}{2} \times \frac{a}{2} \times \frac{a}{2} \times \frac{a}{2}}$$

$$= \sqrt{\frac{3a^4}{16}}$$

$$= \frac{\sqrt{3}}{4}a^2$$

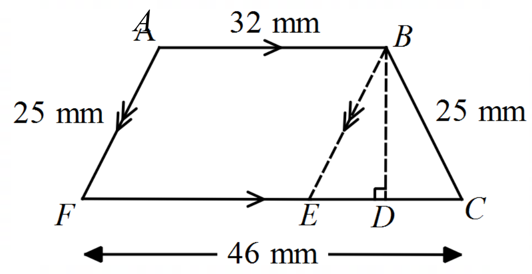
3.2 Determine the area if $a = 12$ mm, correct to 2 decimal places.

$$\text{Area } = \frac{\sqrt{3}}{4}(12)^2$$

$$= 36\sqrt{3}$$

$$= 62{,}35 \text{ mm}^2$$

4. Given trapezium $ABCF$ with $AF = BC = 25$ mm, $AB = 32$ mm and $FC = 46$ mm.



4.1 Write down the lengths of FE, EC and BE.

$FE = 32$ mm; $EC = 46 - 32 = 14$ mm; $BE = 25$ mm

4.2 Using Heron's formula, determine the area of ΔBEC.

$$s = \frac{1}{2}(25 + 25 + 14) = 32$$

$$\therefore \text{ Area } = \sqrt{32(32-25)(32-25)(32-14)}$$
$$= 168 \text{ mm}^2$$

4.3 Hence determine the length of BD.

$$\frac{1}{2} \times 14 \times BD = 168$$
$$\therefore 7 \times BD = 168$$
$$\therefore BD = 24 \text{ mm}$$

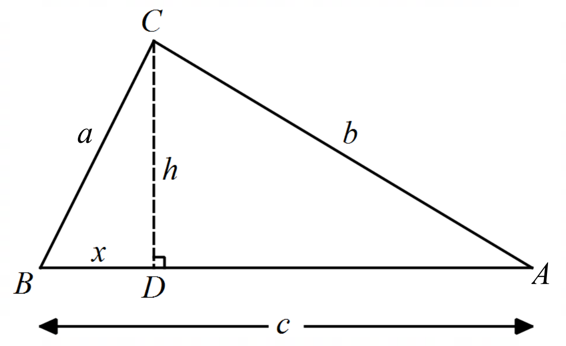
4.4 Hence determine the area of trapezium $ABCF$.

$$\text{Area} = \frac{1}{2}(32 + 46)(24)$$
$$= 936 \text{ mm}^2$$

For Enrichment

Why does Heron's formula work?

Given ΔABC with height CD being h.

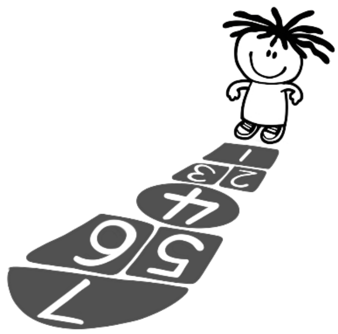


Complete the following proof.

$$\text{Let } BD = x$$
$$\therefore\ AD = c - x$$

$$a^2 = x^2 + h^2 \quad (\text{Pythag in } \Delta BCD)$$
$$\therefore\ h^2 = a^2 - x^2 \quad ❶$$

$$b^2 = h^2 + (c - x)^2 \quad (\text{Pythag in } \Delta ACD)$$
$$= h^2 + c^2 - 2cx + x^2$$
$$\therefore\ h^2 = b^2 - c^2 + 2cx - x^2 \quad ❷$$



Equate ❶ and ❷

$$a^2 - x^2 = b^2 - c^2 + 2cx - x^2$$
$$\therefore\ 2cx = a^2 - b^2 + c^2$$
$$\therefore\ x = \frac{a^2 - b^2 + c^2}{2c}$$

$$h^2 = a^2 - x^2$$
$$\therefore\ h^2 = a^2 - \left(\frac{a^2 - b^2 + c^2}{2c}\right)^2$$
$$= \left[a - \frac{a^2 - b^2 + c^2}{2c}\right]\left[a + \frac{a^2 - b^2 + c^2}{2c}\right]$$
$$= \left[\frac{2ac - a^2 + b^2 - c^2}{2c}\right]\left[\frac{2ac + a^2 - b^2 + c^2}{2c}\right]$$
$$= \left[\frac{b^2 - (a - c)^2}{2c}\right]\left[\frac{(a + c)^2 - b^2}{2c}\right]$$
$$= \left[\frac{(b - (a - c))(b + (a - c))}{2c}\right]\left[\frac{(a + c - b)(a + c + b)}{2c}\right]$$
$$= \frac{(b - a + c)(b + a - c)(a + c - b)(a + c + b)}{4c^2}$$

Let $a + b + c = 2s$

$$\therefore \ h^2 = \frac{(2s-2a)(2s-2c)(2s-2b)(2s)}{4c^2}$$

$$= \frac{16(s-a)(s-c)(s-b)(s)}{4c^2}$$

$$= \frac{4(s-a)(s-c)(s-b)(s)}{c^2}$$

$$\text{Area} = \frac{1}{2}ch$$

$$= \frac{1}{2}c \times \sqrt{\frac{4(s-a)(s-c)(s-b)(s)}{c^2}}$$

$$= \frac{1}{2}c \times \frac{2}{c}\sqrt{(s-a)(s-c)(s-b)(s)}$$

$$= \sqrt{(s-a)(s-c)(s-b)(s)}$$