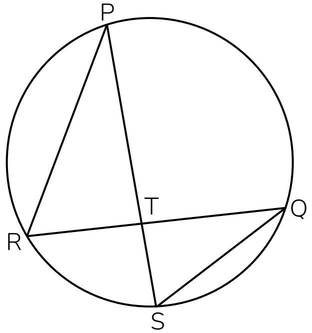
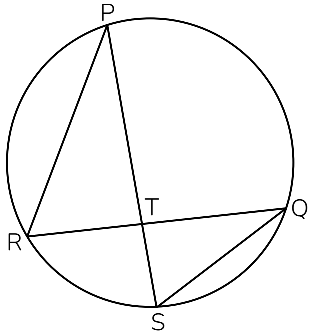
Cross-Topic Solutions

Question 1



$$3a^2 - 8a = 4a + 15 \quad \ldots \angle^s \textit{ in the same seg}$$
$$\therefore\ 3a^2 - 12a - 15 = 0$$
$$\therefore\ a^2 - 4a - 5 = 0$$
$$\therefore\ (a-5)(a+1) = 0$$
$$\therefore\ a = 5 \text{ or } a = -1$$
$$\therefore\ \hat{Q} = 3(5)^2 - 8(5) \quad \text{or} \quad \hat{Q} = 3(-1)^2 - 8(-1)$$
$$= 35° \quad \text{or} \quad = 11°$$

Question 2



$$\hat{S} = \hat{R} \quad \ldots \angle^s \textit{ in the same seg}$$
$$\hat{S} = \sum_{i=1}^{4} 2.3^{i-1}$$
$$= 2.3^0 + 2.3 + 2.3^2 + 2.3^3$$
$$= 2 + 6 + 18 + 54$$
$$= 80$$
$$\hat{R} = \sum_{k=1}^{n} (8k - 80)$$
$$= (-72) + (-64) + (-56) + \ldots$$

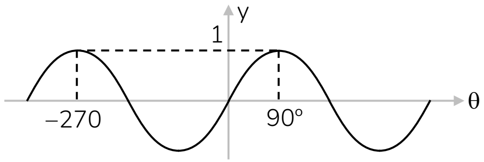
$a = -72$; $d = 8$; n?; $S_n = 80$ $\quad \ldots \hat{S} = \hat{R}$

$$\therefore\ \frac{n}{2}\left[2(-72) + (n-1)(8)\right] = 80$$
$$\therefore\ \frac{n}{2}\left[-144 + 8n - 8\right] = 80$$
$$\therefore\ n(8n - 152) = 160$$
$$\therefore\ 8n^2 - 152n - 160 = 0$$
$$\therefore\ n^2 - 19n - 20 = 0$$
$$\therefore\ (n-20)(n+1) = 0$$
$$\therefore\ n = 20 \text{ or } n = -1$$
$$\therefore\ n = 20 \quad \ldots n \in \mathbb{N}$$

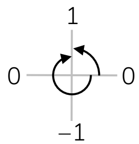
Question 3

$$10^{\sin x} + 10^{\sin x}.10 = 110$$
$$\therefore\ 10^{\sin x}(1 + 10) = 110$$
$$(\div 11) \quad \therefore\ 10^{\sin x} = 10$$
$$\therefore\ \sin x = 1$$
$$\therefore\ \boldsymbol{x = 90°} \text{ or } \boldsymbol{-270°}$$

$y = \sin x$:



$\sin x$:



Question 4

$$\frac{2^{3\sin^2 x}}{4.2^{\sin x}} = 1$$
$$(\times 4) \quad \frac{2^{3\sin^2 x}}{2^{\sin x}} = 4$$
$$\therefore\ 2^{3\sin^2 x - \sin x} = 2^2$$
$$\therefore\ 3\sin^2 x - \sin x = 2$$
$$\therefore\ 3\sin^2 x - \sin x - 2 = 0$$
$$\therefore\ (3\sin x + 2)(\sin x - 1) = 0$$
$$\therefore\ \sin x = -\frac{2}{3} \quad \text{or} \quad \sin x \neq 1 \quad (\because\ \sin x < 0 \text{ is given})$$
$$\therefore\ x = 180° + 41{,}81° + k(360°) \quad \text{or} \quad 360° - 41{,}81° + k(360°),\ k \in \mathbb{Z}$$
$$= 221{,}81° + k(360°) \quad \text{or} \quad 318{,}19° + k(360°),\ k \in \mathbb{Z}$$

Question 5

5.1 $3^x(x-5) \leq 0$ Note: $3^x > 0$ for all values of x.

$\therefore\ x - 5 \leq 0$

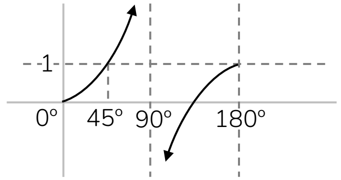
$\therefore\ \boldsymbol{x \leq 5}$ ◄

5.2 $3^x(\tan x - 1) \leq 0$

$\therefore\ \tan x - 1 \leq 0$

$\therefore\ \tan x \leq 1$

$\therefore\ 0° \leq x \leq 45°$ or $90° < x \leq 180°$ ◄



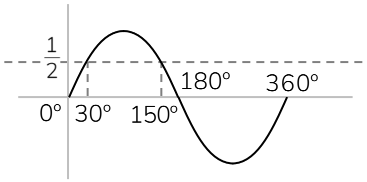
Note: $\tan x$ is undefined for $x = 90°$

5.3 $3^x(2\sin x - 1) \leq 0$

$\therefore\ 2\sin x - 1 \leq 0$

$\therefore\ \sin x \leq \frac{1}{2}$

$\therefore\ 0° \leq x \leq 30°$ or $150° \leq x \leq 360°$ ◄



Question 6

For all x: $\quad -1 \leq \cos x \leq 1$

$(\times 2)\quad \therefore\ -2 \leq 2\cos x \leq 2$

$(+2)\quad \therefore\ 0 \leq 2\cos x + 2 \leq 4$

$\therefore$ The required range: $2^0 \leq y \leq 2^4$

$\therefore\ 1 \leq y \leq 16$ ◄

Question 7

$$2\sin x = a - \frac{1}{a}$$

$$\therefore\ 4\sin^2 x = \left(a - \frac{1}{a}\right)^2 \qquad \text{Square both sides}$$

$$= a^2 - 2 + \frac{1}{a^2}$$

$$= 3 - 2 \qquad \ldots\ a^2 + \frac{1}{a^2} = 3$$

$$= 1$$

$$\therefore\ \sin^2 x = \frac{1}{4}$$

$$\therefore\ \sin x = \pm\frac{1}{2}$$

$$\therefore\ \boldsymbol{x} = \pm 30°$$ ◄

Question 8

$$\cos^2 x - \sin^2 x = \frac{1}{2}\sin 2x - \cos^2 x \qquad \ldots = d$$

$$\therefore\ 2\cos^2 x - \frac{1}{\cancel{2}} \cdot \cancel{2}\sin x\ \cos x - \sin^2 x = 0$$

$$\therefore\ 2\cos^2 x - \sin x\cos x - \sin^2 x = 0$$

$$\therefore\ (2\cos x + \sin x)(\cos x - \sin x) = 0$$

$$\therefore\ 2\cos x = -\sin x \quad \text{or} \quad \cos x = \sin x$$

$\div(-\cos x)\quad -2 = \tan x$

Note: In this case, $\cos x \neq \sin x$.
$\because$ The constant difference cannot be zero.

$$\therefore\ x = 180° - 63{,}43° + k(180°),\ k \in \mathbb{Z}$$

$$= 116{,}57° + k(180°),\ k \in \mathbb{Z}$$ ◄

Question 9

$$\frac{\sin x}{\cos x} = \frac{\sqrt{3}\sin x}{\sin x}$$

$$\therefore \tan x = \sqrt{3}$$

$$\therefore x = 60° + k.180°, k \in \mathbb{Z}$$

$$\therefore \boldsymbol{x} = 60° \; ◀$$

Question 10

$$\sum_{y=43°}^{47°} \sin^2 y = \sin^2 43° + \sin^2 44° + \sin^2 45° + \sin^2 46° + \sin^2 47°$$

$$= \sin^2 43° + \sin^2 44° + \left(\frac{1}{\sqrt{2}}\right)^2 + \cos^2 44 + \cos^2 43°$$

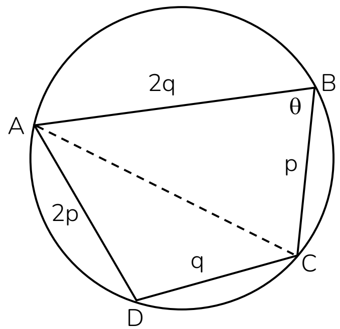
$$= \sin^2 43° + \cos^2 43° + \frac{1}{2} + \sin^2 44 + \cos^2 44°$$

$$= 1 + \frac{1}{2} + 1$$

$$= 2\frac{1}{2} \; ◀$$

Question 11

$\hat{D} = 180° - \theta$ . . . *opp.* $\angle^s$ *of cyclic quad*



Area of c.q. ABCD

$= $ Area of ΔADC + Area of ΔABC

$= \frac{1}{2}(2p)(q)\sin(180° - \theta) + \frac{1}{2}(p)(2q)\sin\theta$

$= pq\sin\theta + pq\sin\theta$

$= \mathbf{2pq\sin\theta}$ ◀

Question 12

12.1 $f(x) = x^2 - 2x\cos\theta - \sin^2\theta$

$$\Delta = (-2\cos\theta)^2 - 4(1)(-\sin^2\theta)$$

$$= 4\cos^2\theta + 4\sin^2\theta$$

$$= 4(\cos^2\theta + \sin^2\theta)$$

$$= 4$$

$\therefore$ roots are real, rational and unequal. ◀

12.2 $f(x)$ is a quadratic function.

Axis of symmetry: $x = -\frac{b}{2a} = -\frac{-2\cos\theta}{2} = \cos\theta$

$\therefore$ Min. value $= (\cos\theta)^2 - 2(\cos\theta)(\cos\theta) - \sin^2\theta$

$$= \cos^2\theta - 2\cos^2\theta - \sin^2\theta$$

$$= -\cos^2\theta - \sin^2\theta$$

$$= -(\cos^2\theta + \sin^2\theta)$$

$$= -1$$

$\therefore$ range: $y \geq -1$ ◀

Question 13

$$CD = \sqrt{(6-10)^2 + (6-3)^2} = 5$$

$\therefore$ CE $= 5$ ... *tans from same pt*

$\therefore$ E(5; 3)

$\therefore$ M(5; p) ... $ME \perp BC$

Now MD = ME

$$\therefore \sqrt{(6-5)^2 + (6-p)^2} = \sqrt{(5-5)^2 + (p-3)^2}$$

$$\therefore 1 + 36 - 12p + p^2 = p^2 - 6p + 9$$

$$\therefore 6p = 28$$

$$\therefore p = \frac{14}{3}$$

$$\therefore \text{radius} = \frac{14}{3} - 3 = \frac{5}{3}$$

$$\text{Area}_{\text{triangle}} = \frac{1}{2} \times 15 \times 9 = \frac{135}{2}$$

$$\text{Area}_{\text{circle}} = \pi \times \left(\frac{5}{3}\right)^2 = \frac{25}{9}\pi$$

$$\therefore \text{Probability} = \frac{\frac{25}{9}\pi}{\frac{135}{2}} = 12{,}9\%$$ ◂

Question 14

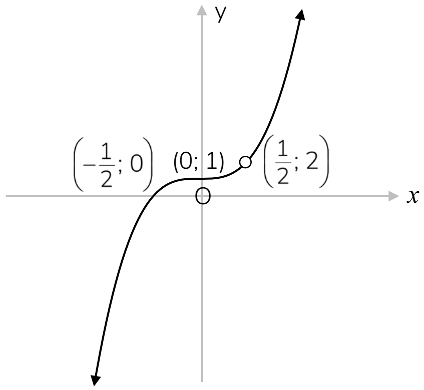
$$y = \left(\frac{2x+1}{8x^3-1}\right)\left(32x^5 - 16x^4 + 8x^3 - 4x^2 + 2x - 1\right)$$

$$= \left(\frac{2x+1}{8x^3-1}\right)\left[8x^3\left(4x^2 - 2x + 1\right) - \left(4x^2 - 2x + 1\right)\right]$$

$$= \left(\frac{2x+1}{8x^3-1}\right)\left[\left(8x^3 - 1\right)\left(4x^2 - 2x + 1\right)\right]$$

$$= (2x+1)(4x^2 - 2x + 1)$$

$$= 8x^3 + 1 \text{ with } x \neq \frac{1}{2}$$ ◂



Undefined:

$$\text{If } 8x^3 = 1$$

$$x^3 = \frac{1}{8}$$

$$x = \frac{1}{2}$$

Question 15

15.1.1 $\quad f(-x) = \cos(-x)$

$= \cos x$

$= f(x)$

$\therefore\ f(x) = \cos x$ is an **even** function. ◂

15.1.2 $\quad f(-x) = \tan(-x)$

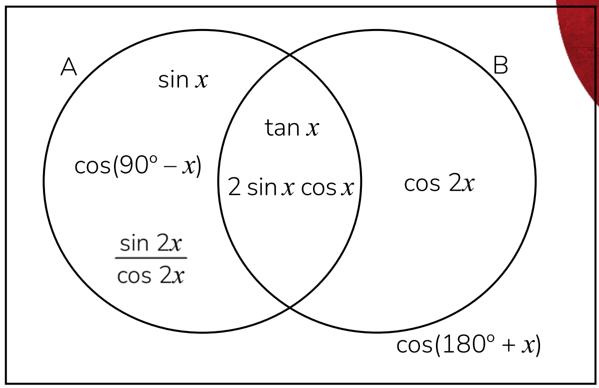
$= -\tan x$

$= -f(x)$

$\therefore\ f(x) = \tan x$ is an **odd** function. ◂

15.2

$f(x)$	$f(-x)$	Odd or Even	Period
$\sin x$	$\sin(-x) = -\sin x$	Odd	360°
$\tan x$	$\tan(-x) = -\tan x$	Odd	180°
$\cos 2x$	$\cos(-2x) = \cos 2x$	Even	180°
$\cos(90° - x) = \sin x$	$\sin(-x) = -\sin x$	Odd	360°
$2\sin x\cos x = \sin 2x$	$\sin(-2x) = -\sin 2x$	Odd	180°
$\dfrac{\sin 2x}{\cos 2x} = \tan 2x$	$\tan(-2x) = -\tan 2x$	Odd	90°
$\cos(180° + x) = -\cos x$	$-\cos(-x) = -\cos x$	Even	360°



$\therefore\ P(B' \text{ and } A) = \dfrac{3}{7}$ ◂