

# Algebra Solutions

## Non-negotiable

1. Solve for  $x$ :  $\sqrt{5-x}-1=x$  (5)

$$\sqrt{5-x}=x+1 \quad \checkmark$$

$$\therefore 5-x=x^2+2x+1 \quad \checkmark$$

$$\therefore x^2+3x-4=0 \quad \checkmark$$

$$\therefore (x+4)(x-1)=0$$

$$\therefore x=-4 \text{ or } x=1 \quad \checkmark$$

If  $x=-4$ :  $LHS = \sqrt{5-(-4)} = 3$  and  $RHS = -4+1 = -3 \therefore LHS \neq RHS$

If  $x=1$ :  $LHS = \sqrt{5-1} = 2$  and  $RHS = 1+1 = 2 \therefore LHS = RHS$

$$\therefore x=1 \quad \checkmark$$



## Take it up a notch

2. Given  $A = \frac{\sqrt{x+4} \cdot (x-3)}{(x-1)^2}$

For what value(s) of  $x$  is:

2.1  $A = 0$ ?

$$x + 4 = 0 \text{ or } x - 3 = 0$$
$$\therefore x = -4 \checkmark \text{ or } x = 3 \checkmark$$

2.2  $A$  undefined?

$$x - 1 = 0$$
$$\therefore x = 1 \checkmark$$

2.3  $A$  non-real?

$$x + 4 < 0 \checkmark$$
$$\therefore x < -4 \checkmark$$

2.4  $A \leq 0$ ?

$$\sqrt{x+4} \cdot (x-3) \leq 0 \quad \dots \text{ since } (x-1)^2 \geq 0 \text{ for all } x$$

$$\therefore -4 \leq x \leq 3 \checkmark\checkmark; x \neq 1 \checkmark$$

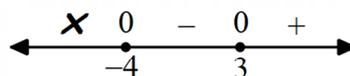


(2)

(1)

(2)

(3)



## Reach for the stars



<https://www.theanswer.co.za/maths-grade-12-revision-algebra-2022/>

3. A closed box has the shape of a rectangular prism with a square base. The sides of the base are  $x$  cm long. The height is  $y$  cm. The surface area of the box is  $288 \text{ cm}^2$ . The lengths of the edges are such that  $2x + y = 21$ . Determine the values of  $x$  and  $y$ .

(7)

$$SA = 2x^2 + 4xy = 288 \quad \checkmark$$

$$\therefore x^2 + 2xy = 144$$

$$y = 21 - 2x \quad \checkmark$$

$$\therefore x^2 + 2x(21 - 2x) = 144 \quad \checkmark$$

$$\therefore x^2 + 42x - 4x^2 = 144 \quad \checkmark$$

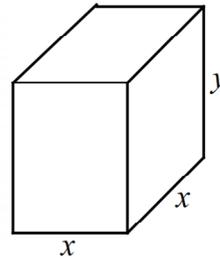
$$\therefore 3x^2 - 42x + 144 = 0 \quad \checkmark$$

$$\therefore x^2 - 14x + 48 = 0$$

$$\therefore (x - 6)(x - 8) = 0$$

$$\therefore x = 6 \text{ or } x = 8 \quad \checkmark$$

$$\therefore y = 9 \text{ or } y = 5 \quad \checkmark$$



# Sequences and Series Solutions

## Non-negotiable

1. The first three terms of a convergent geometric sequence are  $7x+1$ ;  $2x+2$ ;  $x-1$ . Determine the value of  $x$ . (7)

$$\frac{2x+2}{7x+1} = \frac{x-1}{2x+2} \quad \checkmark$$

$$\therefore (2x+2)^2 = (7x+1)(x-1) \quad \checkmark$$

$$\therefore 4x^2 + 8x + 4 = 7x^2 - 6x - 1 \quad \checkmark$$

$$\therefore 3x^2 - 14x - 5 = 0 \quad \checkmark$$

$$\therefore (3x+1)(x-5) = 0$$

$$\therefore x = -\frac{1}{3} \text{ or } x = 5 \quad \checkmark$$

If  $x = -\frac{1}{3}$ : series is  $-\frac{4}{3}$ ;  $\frac{4}{3}$ ;  $-\frac{4}{3}$  which has  $r = -1$ ,  $\therefore$  not convergent

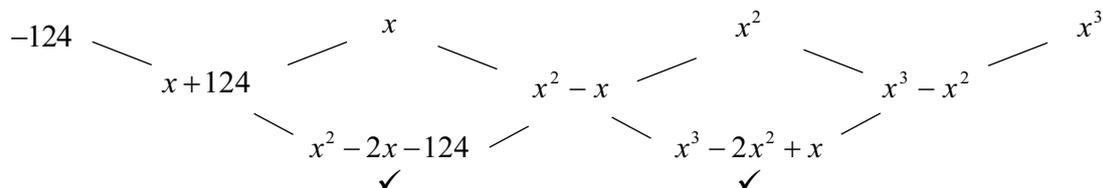
If  $x = 5$ : series is 36; 12; 4 which has  $r = \frac{1}{3}$ ,  $\therefore$  convergent since  $-1 < r < 1$

$$\therefore x = 5 \quad \checkmark \checkmark$$



## Take it up a notch

2. The first four terms of a quadratic pattern are  $-124; x; x^2; x^3$ . Determine  $T_n$  of the sequence. (8)

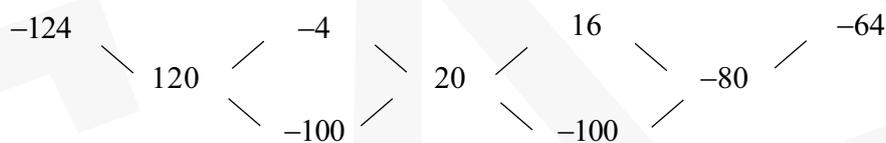


$$x^2 - 2x - 124 = x^3 - 2x^2 + x$$

$$\therefore x^3 - 3x^2 + 3x + 124 = 0 \quad \checkmark$$

$$\therefore (x+4)(x^2 - 7x + 31) = 0 \quad \checkmark$$

$$\therefore x = -4 \text{ only } \checkmark \quad \dots \text{If } x^2 - 7x + 31 = 0 \quad \therefore \Delta = (-7)^2 - 4(1)(31) = -75 \quad \therefore \text{imag roots}$$



$$2a = -100$$

$$\therefore a = -50 \quad \checkmark$$

$$3a + b = 120$$

$$\therefore b = 270 \quad \checkmark$$

$$a + b + c = -124$$

$$\therefore c = -344 \quad \checkmark$$

$$\therefore T_n = -50n^2 + 270n - 344$$



## Reach for the stars



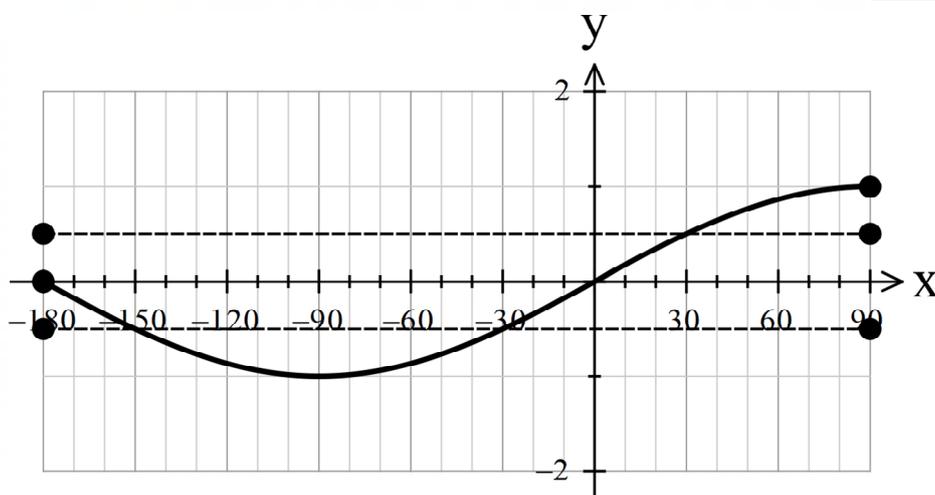
<https://www.theanswer.co.za/maths-grade-12-revision-patterns-and-sequences-2022/>

3. Given  $\sin x - 2 \sin^2 x + 4 \sin^3 x - 8 \sin^4 x + \dots$   
For what values of  $x$ , with  $x \in [-180^\circ; 90^\circ]$  will the series converge? (6)

$$r = -2 \sin x \quad \checkmark$$

$$\therefore -1 < -2 \sin x < 1 \quad \checkmark$$

$$\therefore \frac{1}{2} > \sin x > -\frac{1}{2} \quad \checkmark$$



$$\therefore -180^\circ < x < -150^\circ \checkmark \text{ or } -30^\circ < x < 30^\circ \checkmark \text{ with } x \neq 0^\circ \checkmark$$

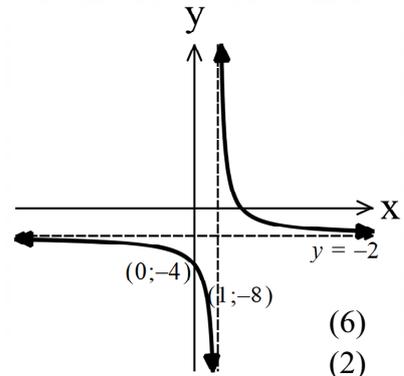


# Functions Solutions

## Non-negotiable

1. The graph of  $f(x) = \frac{k}{x+r} + d$  is sketched below.

The graph passes through the points  $(0; -4)$  and  $(1; -8)$ , and has a horizontal asymptote of  $y = -2$ .



- 1.1 Determine the values of  $k$ ,  $r$  and  $d$ . (6)

- 1.2 Write down the range of  $f$ . (2)

- 1.3 Write down an equation for the axis of symmetry of  $f$  that has a negative gradient. (3)

1.1  $y = \frac{k}{x+r} - 2$  ✓ ... horizontal asymptote at  $-2 \therefore d = -2$

$-4 = \frac{k}{0+r} - 2$  ✓ subs.  $(0; -4)$

$\therefore -2 = \frac{k}{r}$

$\therefore k = -2r$

$-8 = \frac{k}{1+r} - 2$  ✓ subs.  $(1; -8)$

$\therefore -6 = \frac{k}{1+r}$

$\therefore k = -6 - 6r$

$\therefore -2r = -6 - 6r$  ✓

$\therefore 4r = -6$

$\therefore r = -\frac{3}{2}$  ✓

$\therefore k = -2r = -2\left(-\frac{3}{2}\right) = 3$  ✓

$\therefore k = 3; r = -\frac{3}{2}; d = -2$

- 1.2  $y \in \mathbb{R}$  ✓;  $y \neq -2$  ✓

- 1.3 Asymptotes cut at  $\left(\frac{3}{2}; -2\right)$

$\therefore y + 2 = -\left(x - \frac{3}{2}\right)$  ✓✓

$\therefore y = -x - \frac{1}{2}$  ✓



## Take it up a notch

2. Given:  $f(x) = \frac{8}{x} + 2$ . Determine the value of  $f(4) + f'(4) + f^{-1}(4)$ . (7)

$$x = \frac{8}{y} + 2 \quad \checkmark$$

$$\therefore xy = 8 + 2y$$

$$\therefore y(x-2) = 8$$

$$\therefore f^{-1}(x) = \frac{8}{x-2} \quad \checkmark$$

$$\therefore f(4) + f'(4) + f^{-1}(4)$$

$$= \left[ \frac{8}{4} + 2 \right] \checkmark + \left[ -\frac{8}{4^2} \right] \checkmark + \left[ \frac{8}{4-2} \right] \checkmark$$

$$= 4 + \left( -\frac{1}{2} \right) + 4$$

$$= 7\frac{1}{2} \quad \checkmark$$

$$f(x) = 8x^{-1} + 2$$

$$\therefore f'(x) = -8x^{-2}$$

$$\therefore f'(x) = -\frac{8}{x^2} \quad \checkmark$$



## Reach for the stars

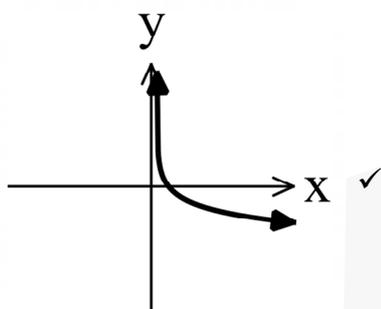
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3. Given:  $f(x) = a^x$  with  $0 < a < 1$ , and  $h(x) = \frac{k}{x}$  with  $k > 0$ .

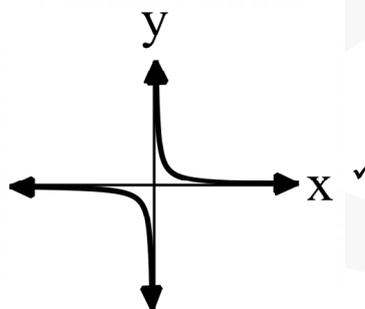
Determine the values of  $x$  which are common to the domains of  $f^{-1}(x)$  and  $h(x)$ . (4)

$$f^{-1}(x) = \log_a x \checkmark \text{ with } 0 < a < 1$$



Domain:  $x > 0$   
 $\therefore x > 0 \checkmark$

$$h(x) = \frac{k}{x} \text{ with } k > 0$$



Domain:  $x \in \mathbb{R}; x \neq 0$



# Finance Solutions

## Non-negotiable

1. On 30 June 2013 and at the end of each month thereafter, Asif deposited R2 500 into a bank account that pays interest at 6% per annum, compounded monthly. He wants to continue to deposit this amount until 31 May 2018. Calculate how much money Asif will have in this account immediately after depositing R2 500 on 31 May 2018. (5)

$$F = \frac{2500 \left[ \left( 1 + \frac{0,06}{12} \right)^{60} - 1 \right]}{\frac{0,06}{12}} \quad \checkmark \checkmark \checkmark$$
$$\therefore F = R174\,425,08 \quad \checkmark \checkmark$$



## Take it up a notch

2. A woman invests R108 706,86 in a bank account and begins withdrawing R6 000 per quarter after three months. How many withdrawals can she make before all the money is withdrawn, if interest is calculated at 15% p.a. compounded quarterly? (5)

$$108\,706,86 = \frac{6000 \left[ 1 - \left( 1 + \frac{0,15}{4} \right)^{-n} \right]}{\frac{0,15}{4}} \quad \checkmark$$

$$\therefore \left( \frac{83}{80} \right)^{-n} = 0,3205... \quad \checkmark$$

$$\therefore -n = \log_{\frac{83}{80}} 0,3205... \quad \checkmark$$

$$\therefore -n = -30,9017... \quad \checkmark$$

$\therefore$  she can make 31 withdrawals where the last one will be less than R6 000  $\checkmark$





## Reach for the stars

<https://www.theanswer.co.za/maths-grade-12-revision-finance-2022/>

3. The Bradford's were granted a bond of R2 million at a rate of 13,5% p.a., compounded monthly, to be amortised in 20 years. Consider the implications of the interest rate changing to 14% p.a., compounded monthly, two years after the bond was granted.

3.1 If the bond is still to be amortised in 20 years, determine the increased monthly payments. (7)

3.2 If the monthly payments remain the same, determine how much longer it will take to pay off the bond. (4)

$$3.1 \quad 2\,000\,000 = \frac{x \left[ 1 - \left( 1 + \frac{0,135}{12} \right)^{-240} \right]}{\frac{0,135}{12}} \quad \checkmark$$

$$\therefore x = 24\,147,49 \quad \checkmark$$

$$BO = 2\,000\,000 \left( 1 + \frac{0,135}{12} \right)^{24} - \frac{24\,147,49 \left[ \left( 1 + \frac{0,135}{12} \right)^{24} - 1 \right]}{\frac{0,135}{12}} \quad \checkmark \checkmark$$

$$\therefore BO = 1\,954\,896,67 \quad \checkmark$$

$$\therefore 1\,954\,896,67 = \frac{x \left[ 1 - \left( 1 + \frac{0,14}{12} \right)^{-216} \right]}{\frac{0,14}{12}} \quad \checkmark$$

$$\therefore x = R24\,834,68 \quad \checkmark$$

$$3.2 \quad 1\,954\,896,67 = \frac{24\,147,49 \left[ 1 - \left( 1 + \frac{0,14}{12} \right)^{-n} \right]}{\frac{0,14}{12}} \quad \checkmark$$

$$\therefore \left( \frac{607}{600} \right)^{-n} = 0,0555... \quad \checkmark$$

$$\therefore -n = \log_{\frac{607}{600}} 0,0555...$$

$$\therefore n = 249,26 \quad \checkmark$$

$$\therefore n = 250$$

18 years from the time the interest rate changed gives 216 months.

$\therefore$  it will take 34 more months to pay off the bond  $\checkmark$



# Calculus Solutions

## Non-negotiable

1. Given  $f(x) = -3x^2$ . Determine  $f'(x)$  using first principles. (5)

$$\begin{aligned}f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ \therefore f'(x) &= \lim_{h \rightarrow 0} \frac{-3(x+h)^2 - (-3x^2)}{h} \checkmark \\ \therefore f'(x) &= \lim_{h \rightarrow 0} \frac{-3x^2 - 6xh - 3h^2 + 3x^2}{h} \checkmark \\ \therefore f'(x) &= \lim_{h \rightarrow 0} \frac{-6xh - 3h^2}{h} \checkmark \\ \therefore f'(x) &= \lim_{h \rightarrow 0} (-6x - 3h) \checkmark \\ \therefore f'(x) &= -6x \checkmark\end{aligned}$$



## Take it up a notch

2. The line  $g(x) = 5x + 1$  is a tangent to the curve of a function  $f$  at the point where  $x = 2$ . Calculate the value of  $f(2) + f'(2)$ . (4)

Since  $f$  and  $g$  meet at the point where  $x = 2$ :

$$\therefore f(2) = g(2) = 5(2) + 1 = 11 \checkmark \checkmark$$

The gradient of  $f$  and  $g$  when  $x = 2$  is the same:

$$\therefore f'(2) = g'(2) = 5 \checkmark$$

$$\therefore f(2) + f'(2) = 11 + 5 = 16 \checkmark$$



## Reach for the stars



<https://www.theanswer.co.za/maths-grade-12-revision-calculus-2022/>

3. Given:

- $f(x) = x^3 + bx^2 + cx + d$
- $f'(x) > 0$  for  $x < -1$  or  $x > 2$
- $f'(x) < 0$  for  $-1 < x < 2$

Determine the values of  $b$  and  $c$ .

(5)

$$f'(x) = 3x^2 + 2bx + c \quad \checkmark$$

$$f'(-1) = f'(2) = 0$$

$$3(-1)^2 + 2b(-1) + c = 0 \quad \checkmark$$

$$\therefore -2b + c = -3 \quad \textcircled{1}$$

$$3(2)^2 + 2b(2) + c = 0 \quad \checkmark$$

$$\therefore 4b + c = -12 \quad \textcircled{2}$$

$$\therefore 6b = -9 \quad \text{Eqn } \textcircled{2} - \textcircled{1}$$

$$\therefore b = -\frac{3}{2} \quad \checkmark \text{ and } c = -6 \quad \checkmark$$



# Probability Solutions

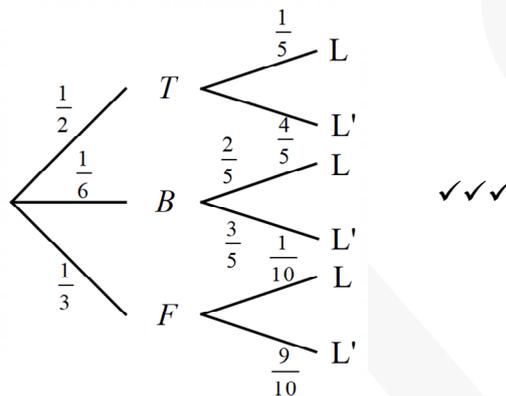
## Non-negotiable

1. On a randomly chosen day, the probability that James travels to school by train, by bus or on foot is  $\frac{1}{2}$ ,  $\frac{1}{6}$  and  $\frac{1}{3}$  respectively. The probability of being late

when using these methods of travel is  $\frac{1}{5}$ ,  $\frac{2}{5}$  and  $\frac{1}{10}$  respectively.

- 1.1 Draw a tree diagram to represent this information. (3)  
1.2 Find the probability that on a randomly chosen day:  
1.2.1 James travels by foot and is late. (2)  
1.2.2 James is not late. (4)

1.1



1.2.1  $P(\text{foot and late}) = \frac{1}{3} \times \frac{1}{10} \checkmark = \frac{1}{30} \checkmark$

1.2.2  $P(\text{not late}) = \frac{1}{2} \times \frac{4}{5} \checkmark + \frac{1}{6} \times \frac{3}{5} \checkmark + \frac{1}{3} \times \frac{9}{10} \checkmark = \frac{4}{5} \checkmark$

## Take it up a notch

2. Given that  $P(A) = 0,35$ ,  $P(B) = 0,45$  and  $P(A \text{ and } B) = 0,1$ .

2.1 Find  $P(A \text{ or } B)$ . (2)

2.2 It is further given that  $P(C) = 0,2$ . The events  $A$  and  $C$  are mutually exclusive and events  $B$  and  $C$  are independent.

2.2.1 Determine  $P(B \text{ and } C)$ . (2)

2.2.2 Draw a Venn diagram showing  $A$ ,  $B$  and  $C$ , and all probabilities. (4)

2.2.3 Determine  $P[(B \text{ and } C)']$ . (2)

2.1  $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$$\therefore P(A \text{ or } B) = 0,35 + 0,45 - 0,1 \checkmark$$

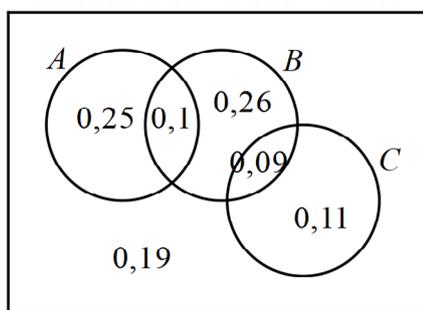
$$\therefore P(A \text{ or } B) = 0,7 \checkmark$$

2.2.1  $P(B \text{ and } C) = P(B) \times P(C)$

$$\therefore P(B \text{ and } C) = 0,45 \times 0,2 \checkmark$$

$$\therefore P(B \text{ and } C) = 0,09 \checkmark$$

2.2.2



2.2.3  $P[(B \text{ and } C)'] = 1 - 0,09 = 0,91 \checkmark \checkmark$



## Reach for the stars



<https://www.theanswer.co.za/maths-grade-12-revision-probability-2022/>

3. There are 11 players in a cricket team. They are asked to stand in a straight line for a photo. Three of the players, Andrew, Bobby, and Cuan refuse to stand next to each other. The other team members do not mind where they stand. Determine the number of ways in which the 11 players can be positioned for the photo. (5)

Total number of ways the players can stand together  
 $= 11! = 39\,916\,800$  ✓

Number of ways all three of A, B and C can stand together  
 $= 9! \times 3! = 2\,177\,280$  ✓

Number of ways exactly two of A, B and C can stand together  
 $= 8! \times 3 \times 2! \times 9 \times 8 = 17\,418\,240$  ✓✓

Number of ways they can be positioned for the photo  
 $= 39\,916\,800 - 2\,177\,280 - 17\,418\,240$   
 $= 20\,321\,280$  ✓



# Data Handling Solutions

## Non-negotiable

1. Various Grade 3 classes were tested for their reading ability. The following information was gathered.

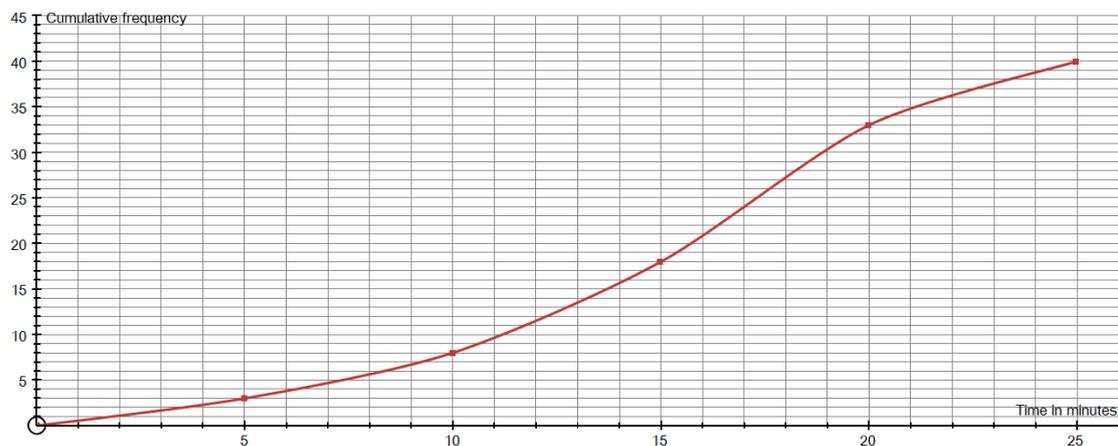
Class size ( $x$ )	Mean reading score ( $y$ )
35	70
30	80
35	60
35	72
40	58
33	71
38	68
30	75
29	72
39	62

- 1.1 Determine the equation of the least squares regression line. (3)
- 1.2 Write down the correlation coefficient. (1)
- 1.3 Comment on the correlation coefficient, and explain how class size affects the mean reading score. (2)
- 1.1  $a = 117,78$  ✓  
 $b = -1,42$  ✓  
 $\therefore y = 117,78 - 1,42x$  ✓
- 1.2  $r = -0,80$  ✓
- 1.3 There is a strong, negative correlation coefficient. ✓ This means the bigger the class, the lower the reading score. ✓

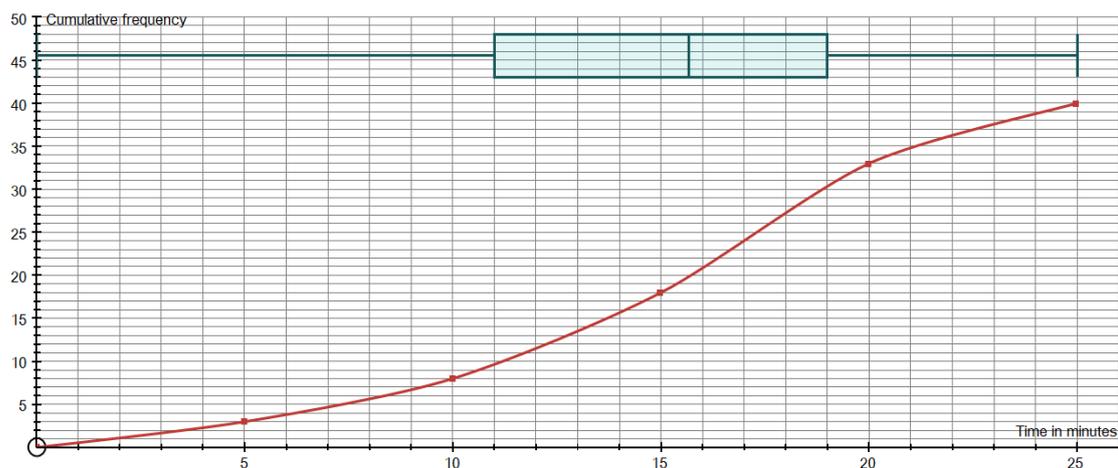


## Take it up a notch

2. The length of time, in minutes, of a certain number of cell phone calls was recorded. No call lasted longer than 25 minutes. A cumulative frequency diagram of this data is shown below.



Draw a box and whisker plot from the cumulative frequency curve. (5)



✓ minimum and maximum ✓ lower quartile ✓ median ✓ upper quartile ✓ shape



## Reach for the stars



<https://www.theanswer.co.za/maths-grade-12-revision-data-2022/>

3. Five numbers,  $a$ ,  $b$ ,  $c$ ,  $d$  and  $e$ , are written in ascending order. Determine the upper quartile if the largest number is 42, the range is 17, the lower quartile is 27,  $c$  and  $d$  are equal, and the mean of the five numbers is 34. (4)

$$e = 42$$

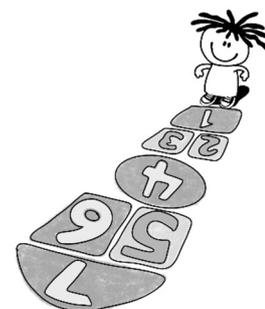
$$a = 42 - 17 = 25 \checkmark$$

$$\frac{a+b}{2} = 27 \therefore b = 29 \checkmark \dots \text{the lower quartile is halfway between } a \text{ and } b$$

$$\frac{a+b+c+d+e}{5} = 34 \therefore c = d = 37 \checkmark$$

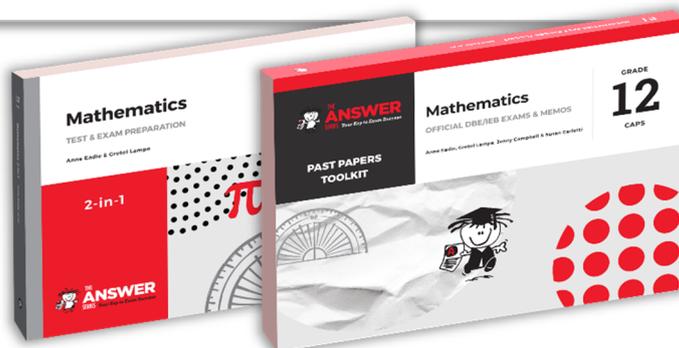
The five numbers are 25; 29; 37; 37; 42

$$\therefore \text{upper quartile} = \frac{37+42}{2} = 39\frac{1}{2} \checkmark$$



For more examples, see page 44 to 49 in the **Grade 12 Maths 2-in-1 Study Guide**, as well as page 42 in the Challenging Questions booklet.

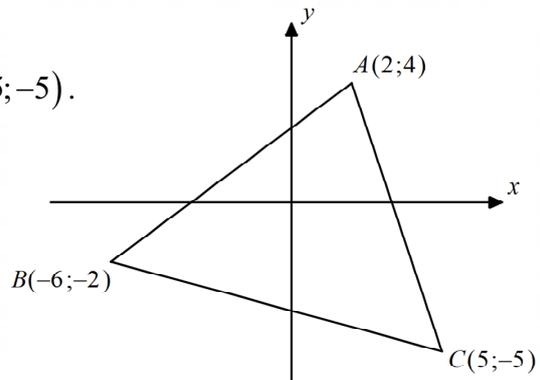
More examples can be found in the **Grade 12 Maths Past Papers Toolkit**.



# Analytical Geometry Solutions

## Non-negotiable

1.  $\triangle ABC$  has  $A(2;4)$ ,  $B(-6;-2)$  and  $C(5;-5)$ .



- 1.1 Determine the co-ordinates of  $M$ , the midpoint of  $AB$ . (2)  
1.2 Determine the gradient of  $AC$ . (2)  
1.3 Determine the equation of the line passing through  $M$  parallel to  $AC$ . (3)  
1.4 Determine the length of  $AC$ . (2)  
1.5 Write down, with a reason and correct to two decimal places, the length of  $MN$  if  $MN \parallel AC$  with  $N$  on  $BC$ . (2)

1.1  $M\left(\frac{-6+2}{2}; \frac{-2+4}{2}\right)$

$\therefore M(-2; 1)$

1.2  $m_{AC} = \frac{4 - (-5)}{2 - 5} = -3$

1.3  $y = -3x + c$

$\therefore 1 = -3(-2) + c$

$\therefore c = -5$

$\therefore y = -3x - 5$

1.4  $AC = \sqrt{(2-5)^2 + (4-(-5))^2}$

$\therefore AC = 3\sqrt{10}$  (or 9,49)

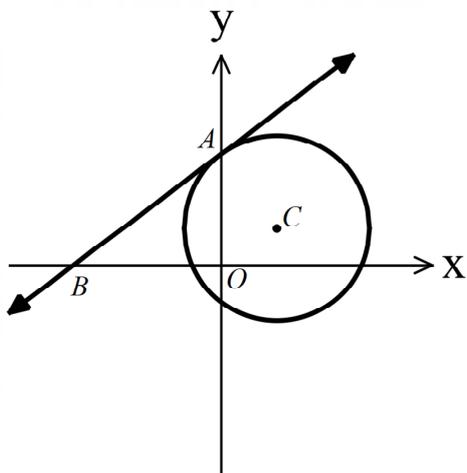
1.5  $MN = \frac{1}{2} AC$  (conv midpt thm)

$\therefore MN = 4,74$



## Take it up a notch

2. In the diagram, the circle with centre  $C$  and with equation  $x^2 - 6x + y^2 - 4y = 12$  cuts the  $y$ -axis at  $A$ .  $BA$  is a tangent to the circle with  $B$  on the  $x$ -axis.



- 2.1 Determine the co-ordinates of  $C$ . (4)  
2.2 Determine the equation of  $BA$ . (6)

2.1  $x^2 - 6x + y^2 - 4y = 12$   
 $\therefore (x-3)^2 + (y-2)^2 = 12 + 9 + 4 \checkmark$   
 $\therefore (x-3)^2 + (y-2)^2 = 25$   
 $\therefore C(3;2) \checkmark$

2.2  $(0)^2 - 6(0) + y^2 - 4y = 12 \checkmark$   
 $\therefore y^2 - 4y - 12 = 0$   
 $\therefore (y-6)(y+2) = 0$   
 $\therefore y = 6 \text{ or } y = -2 \checkmark$   
 $\therefore A(0;6) \checkmark$

$$m_{AC} = \frac{6-2}{0-3} = -\frac{4}{3} \checkmark$$

$$\therefore m_{\text{tan}} = \frac{3}{4} \checkmark$$

$$\therefore BA: y = \frac{3}{4}x + 6 \checkmark$$

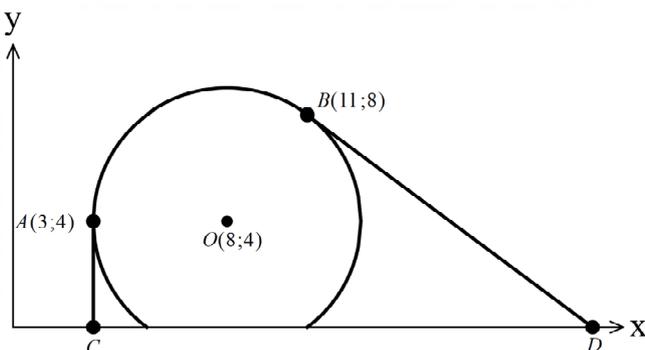


## Reach for the stars



<https://www.theanswer.co.za/maths-grade-12-revision-analytical-2022/>

3. A circle centred at  $O(8;4)$  passes through  $A(3;4)$  and  $B(11;8)$ .  $AC$  is a vertical line and  $BD$  is a tangent to the circle at  $B$ . A piece of wire is stretched from  $C$  to  $A$ , round the circle to  $B$ , then to  $D$  on the  $x$  axis.



Determine the length of the wire.

(10)

$$AC: AC = 4 \checkmark$$

$$AB: m_{OB} = \frac{8-4}{11-8} = \frac{4}{3} \checkmark$$

$$\therefore \angle \text{ of incl} = \tan^{-1} \frac{4}{3} = 53,13^\circ$$

$$\therefore \widehat{AOB} = 180^\circ - 53,13^\circ = 126,87^\circ \checkmark$$

$$\therefore \text{arc } AB = \frac{126,87^\circ}{360^\circ} \times 2\pi(5) \checkmark = 11,07 \checkmark$$

$$BD: m_{BD} = -\frac{3}{4} \checkmark$$

$$\therefore 8 = -\frac{3}{4}(11) + c$$

$$\therefore c = \frac{65}{4}$$

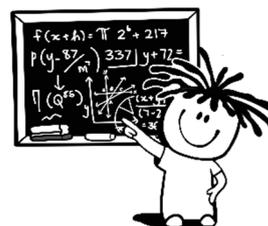
$$\therefore y = -\frac{3}{4}x + \frac{65}{4} \checkmark$$

$$\text{At } D: y = 0$$

$$\therefore D\left(\frac{65}{3}; 0\right) \checkmark$$

$$\therefore BD = \sqrt{\left(11 - \frac{65}{3}\right)^2 + (8 - 0)^2} = \frac{40}{3} \checkmark$$

$$\therefore \text{length of wire} = 4 + 11,07 + \frac{40}{3} = 28,40 \text{ units } \checkmark$$



# Trigonometry Solutions

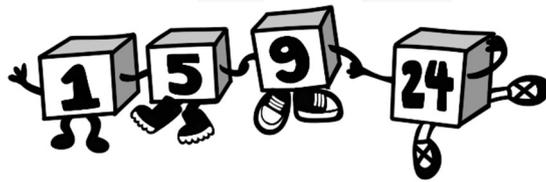
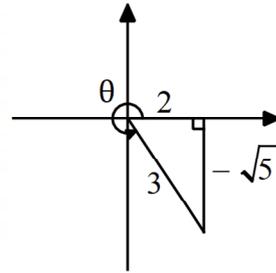
## Non-negotiable

1. If  $\cos \theta = \frac{2}{3}$  and  $\theta > 90^\circ$ , determine the value of  $\cos(\theta + 45^\circ)$  without the use of a calculator. Leave your answer in surd form. (4)

$$\cos(\theta + 45^\circ) = \cos \theta \cos 45^\circ - \sin \theta \sin 45^\circ \quad \checkmark$$

$$\therefore \cos(\theta + 45^\circ) = \frac{2}{3} \times \frac{1}{\sqrt{2}} \checkmark - \left( -\frac{\sqrt{5} \checkmark}{3} \right) \times \frac{1}{\sqrt{2}}$$

$$\therefore \cos(\theta + 45^\circ) = \frac{2 + \sqrt{5}}{3\sqrt{2}} \quad \checkmark$$



## Take it up a notch

2. Prove the identity:

$$2 \cos \theta \cdot \cos 2\theta + \frac{\sin^2 2\theta}{\cos \theta} = 2 \cos \theta \quad (4)$$

$$\begin{aligned} LHS &= 2 \cos \theta (1 - 2 \sin^2 \theta) + \frac{(2 \sin \theta \cos \theta)^2}{\cos \theta} \\ &= 2 \cos \theta - 4 \sin^2 \theta \cos \theta + \frac{4 \sin^2 \theta \cos^2 \theta}{\cos \theta} \checkmark \\ &= 2 \cos \theta - 4 \sin^2 \theta \cos \theta + 4 \sin^2 \theta \cos \theta \checkmark \\ &= 2 \cos \theta \\ &= RHS \end{aligned}$$

OR

$$\begin{aligned} LHS &= 2 \cos \theta (\cos^2 \theta - \sin^2 \theta) + \frac{(2 \sin \theta \cos \theta)^2}{\cos \theta} \\ &= 2 \cos^3 \theta - 2 \cos \theta \sin^2 \theta + \frac{4 \sin^2 \theta \cos^2 \theta}{\cos \theta} \checkmark \\ &= \frac{2 \cos^4 \theta - 2 \cos^2 \theta \sin^2 \theta + 4 \sin^2 \theta \cos^2 \theta}{\cos \theta} \\ &= \frac{2 \cos^4 \theta + 2 \cos^2 \theta \sin^2 \theta}{\cos \theta} \\ &= \frac{2 \cos^2 \theta (\cos^2 \theta + \sin^2 \theta)}{\cos \theta} \checkmark \\ &= \frac{2 \cos^2 \theta (1)}{\cos \theta} \\ &= 2 \cos \theta \\ &= RHS \end{aligned}$$

OR

$$\begin{aligned} LHS &= 2 \cos \theta (2 \cos^2 \theta - 1) + \frac{(2 \sin \theta \cos \theta)^2}{\cos \theta} \\ &= 4 \cos^3 \theta - 2 \cos \theta + \frac{4 \sin^2 \theta \cos^2 \theta}{\cos \theta} \checkmark \\ &= 4 \cos^3 \theta - 2 \cos \theta + 4 \sin^2 \theta \cos \theta \\ &= 4 \cos \theta (\cos^2 \theta + \sin^2 \theta) - 2 \cos \theta \checkmark \\ &= 4 \cos \theta (1) - 2 \cos \theta \\ &= 4 \cos \theta - 2 \cos \theta \\ &= 2 \cos \theta \\ &= RHS \end{aligned}$$

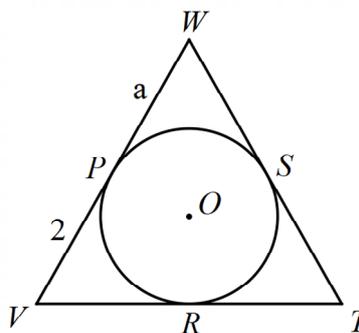


## Reach for the stars



<https://www.theanswer.co.za/maths-grade-12-revision-trig-2022/>

3. In the diagram, circle PSR with centre O is drawn inside  $\Delta WVT$ , as shown.  
 $WV = WT$   
 $WP = a$  units  
 $PV = 2$  units



Determine  $\cos V$  in terms of  $a$ .

(7)

$$WS = a \text{ (tans from same pt) } \checkmark$$

$$ST = 2 \text{ (} WV = WT \text{)} \checkmark$$

$$VR = 2 \text{ (tans from same pt) } \checkmark$$

$$TR = 2 \text{ (tans from same pt) } \checkmark$$

$$\therefore \cos V = \frac{(a+2)^2 + 4^2 - (a+2)^2}{2(a+2)(4)} \checkmark$$

$$\therefore \cos V = \frac{16}{8(a+2)} \checkmark$$

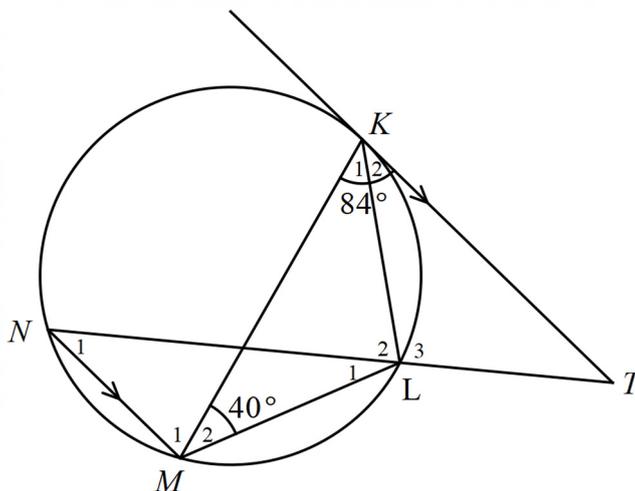
$$\therefore \cos V = \frac{2}{a+2} \checkmark$$



# Euclidean Geometry Solutions

## Non-negotiable

1. In the diagram, tangent  $KT$  to the circle at  $K$  is parallel to the chord  $NM$ .  
 $NT$  cuts the circle at  $L$ .  $\Delta KML$  is drawn.  $\widehat{M}_2 = 40^\circ$  and  $\widehat{M\hat{K}T} = 84^\circ$ .



Determine, giving reasons, the size of:

- |     |                 |     |
|-----|-----------------|-----|
| 1.1 | $\widehat{K}_2$ | (2) |
| 1.2 | $\widehat{N}_1$ | (3) |
| 1.3 | $\widehat{T}$   | (2) |
| 1.4 | $\widehat{L}_2$ | (2) |
| 1.5 | $\widehat{L}_1$ | (1) |

- 1.1  $\widehat{K}_2 = 40^\circ$  ✓ (tan chord thm) ✓  
 1.2  $\widehat{K}_1 = 44^\circ$  ✓  
 $\therefore \widehat{N}_1 = 44^\circ$  ✓ ( $\angle$ 's in same seg) ✓  
 1.3  $\widehat{T} = 44^\circ$  ✓ (alt  $\angle$ 's;  $KT \parallel NM$ ) ✓  
 1.4  $\widehat{L}_2 = 84^\circ$  ✓ (ext  $\angle$  of  $\Delta KLT$ ) ✓  
 1.5  $\widehat{L}_1 = 12^\circ$  ( $\angle$  sum of  $\Delta KLM$ ) ✓



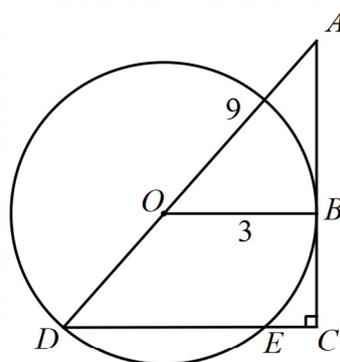
## Take it up a notch

2.  $O$  is the centre of the circle. Tangent  $ABC$  meets chord  $DE$  produced to  $C$  such that  $\widehat{C} = 90^\circ$ .  $DO$  produced meets the tangent at  $A$ .  $AO = 9$  units and  $OB = 3$  units.

Calculate, giving reasons:

2.1  $AB$

2.2  $BC$



(4)

(5)

2.1  $\widehat{OBA} = 90^\circ$  ✓ (tan  $\perp$  rad) ✓  
 $\therefore AB = \sqrt{9^2 - 3^2} = 6\sqrt{2}$  ✓ units (Pythag) ✓

2.2  $OB \parallel DC$  (corres  $\angle$ s =) ✓  
 $OD = 3$  (radii) ✓  
 $\frac{AO}{OD} = \frac{AB}{BC}$  (prop thm;  $OB \parallel DC$ ) ✓  
 $\therefore \frac{9}{3} = \frac{6\sqrt{2}}{BC}$  ✓  
 $\therefore BC = 2\sqrt{2}$  units ✓

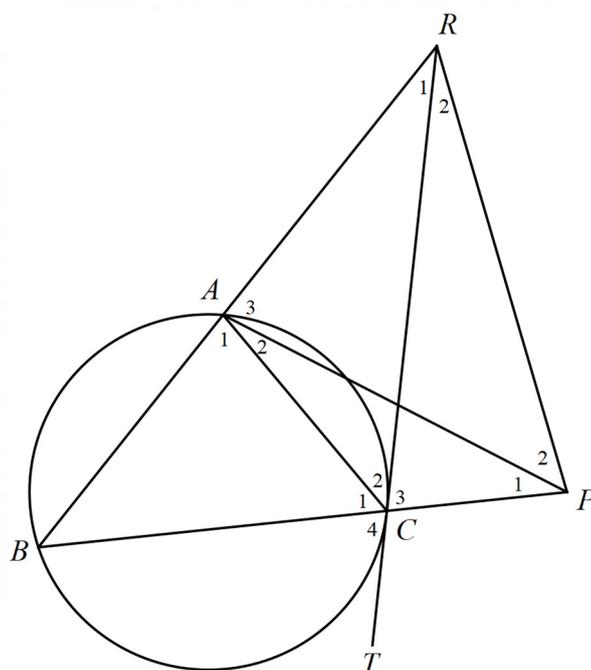


## Reach for the stars



<https://www.theanswer.co.za/maths-grade-12-revision-euclidean-2022/>

3. In the diagram, chord BA and tangent TC of circle ABC are produced to meet at R. BC is produced to P with  $RC = RP$ . AP is not a tangent.



- 3.1 Prove that:
- 3.1.1 ACPR is a cyclic quadrilateral. (5)
- 3.1.2  $\triangle CBA \parallel \triangle RPA$  (4)
- 3.1.3  $RC = \frac{CB \cdot RA}{AC}$  (2)
- 3.1.4  $RB \cdot AC = RC \cdot CB$  (4)
- 3.2 Hence, prove that  $RC^2 = RA \cdot RB$  (3)



3.1.1 Let  $\widehat{C}_3 = x$   
 $\therefore \widehat{C}_4 = x$  (vert opp  $\angle$ 's) ✓  
 $\therefore \widehat{A}_1 = x$  (tan chord thm) ✓  
 $R\widehat{P}C = x$  ( $\angle$ 's opp equal sides) ✓  
 $\therefore \widehat{A}_1 = R\widehat{P}C$  ✓  
 $\therefore ACPR$  is a cyclic quadrilateral (ext  $\angle =$  int opp  $\angle$ ) ✓

3.1.2 In  $\triangle CBA$  and  $\triangle RPA$

- $\widehat{C}_1 = \widehat{A}RP$  (ext  $\angle$  of cyc quad) ✓
- $\widehat{A}_3 = x$  ( $\angle$ 's in same seg) ✓  
 $\therefore \widehat{A}_1 = \widehat{A}_3$  ✓  
 $\therefore \triangle CBA \parallel \triangle RPA$  (AAA) ✓

3.1.3  $\frac{CB}{RP} = \frac{BA}{PA} = \frac{CA}{RA}$  ( $\triangle CBA \parallel \triangle RPA$ ) ✓  
 $\therefore RP = \frac{CB \cdot RA}{CA}$  ✓  
 $\therefore RC = \frac{CB \cdot RA}{CA}$  ( $RP = RC$ )

3.1.4 In  $\triangle RBP$  and  $\triangle CBA$

- $\widehat{B}$  is common ✓
- $\widehat{B}PR = \widehat{A}_1$  (ext  $\angle$  of cyc quad) ✓  
 $\therefore \triangle RBP \parallel \triangle CBA$  (AAA) ✓  
 $\therefore \frac{RB}{CB} = \frac{BP}{BA} = \frac{RP}{CA}$  ( $\triangle RBP \parallel \triangle CBA$ )  
 $\therefore RB \cdot CA = RP \cdot CB$  ✓  
 $\therefore RB \cdot CA = RC \cdot CB$  ( $RP = RC$ )

3.2  $RC = \frac{RB \cdot AC}{CB}$  ✓ (from 3.1.4)  
 $\therefore RC^2 = \frac{\cancel{CB} \cdot RA}{\cancel{AC}} \times \frac{RB \cdot \cancel{AC}}{\cancel{CB}}$  ✓✓  
 $\therefore RC^2 = RA \cdot RB$

