DBO NOV 2025 VRAESTEL 2

STATISTIEK [20]

1.1 **A** ≃ 331 397,20 ; **B** ≃ −22 988,32
∴ **y = 331 397,20 – 22 988,32*x*** ◄

Die vgl.:
y = A + B*x*

1.2 Voorspelde verkoopprys: Stel $x = 5$ in
R216 455,60 ◄

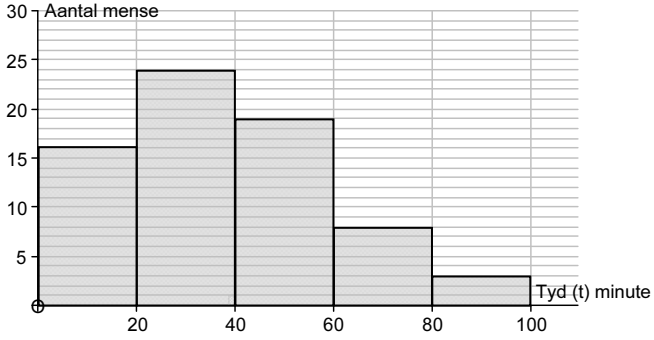
1.3 **r ≃ −0,95** ◄
Die voorspelling is geldig omdat die korrelasiekoëffisiënt sterk is.

1.4 Die beraamde gemiddelde jaarlikse verandering
$= \frac{dy}{dx} = -R22\,988,32$
∴ Die beraamde gemiddelde jaarlikse verlaging = **R22 988,32** ◄

2.1.1 **70 mense** ◄

2.1.2 67 – 40 = **27** ◄

2.1.3



2.1.4 **Die data is regs (positief) skeef.** ◄

2.2 Totale aantal punte (9 spelers) = 108

Totale aantal punte aangeteken deur 8 spelers
= 11 + 14 + 19 + ... + 14
= 98

∴ Die 9de speler het 10 punte aangeteken . . . *108 – 98*

Die gemiddelde, $\bar{x} = 12$ & die standaardafwyking, $\sigma \simeq 5,23$

Die aantal spelers wat
óf minder as $\bar{x} - \sigma$, 6,77, dus 6 en minder
of meer as $\bar{x} + \sigma$, 17,23, dus 18 of meer aangeteken het

∴ d.w.s. diegene wat 2, 19 en 20 aangeteken het

∴ **3 Spelers** ◄

ANALITIESE MEETKUNDE [39]

3.1 $QR^2 = (12 + 4)^2 + (2 + 6)^2$
$= 256 + 64$
$= 320$

∴ QR $= \sqrt{320} = \sqrt{64 \times 5} =$ **$8\sqrt{5}$ eenhede** ◄

3.2 $m_{QR} = \frac{2+6}{12+4} = \frac{8}{16} =$ **$\frac{1}{2}$** ◄ . . . $= \tan\theta$

3.3 ∴ θ ≃ **26,57°** ◄

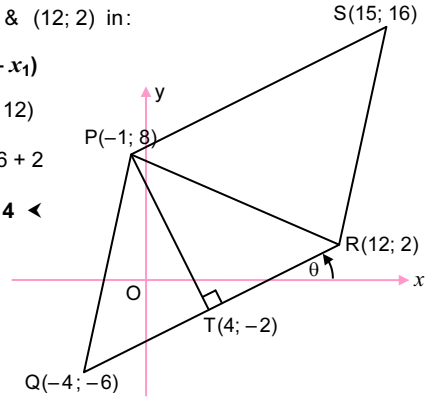
3.4 Stel $m_{QR} = \frac{1}{2}$ & (12; 2) in:

$y - y_1 = m(x - x_1)$

∴ $y - 2 = \frac{1}{2}(x - 12)$

∴ $y = \frac{1}{2}x - 6 + 2$

∴ **$y = \frac{1}{2}x - 4$** ◄



3.5 Deur inspeksie: **S(15; 16)** ◄

3.6 Vgl. van QR: $y = \frac{1}{2}x - 4$

Vgl. van PT: m = −2 & pt (−1; 8)

∴ $y - 8 = -2(x + 1)$

∴ $y = -2x + 6$

By T: $\frac{1}{2}x - 4 = -2x + 6$

∴ $\frac{5}{2}x = 10$

$\left(\times\frac{2}{5}\right)$ ∴ $x = 4$

& $y = \frac{1}{2}(4) - 4 = -2$

∴ **T(4; −2)** ◄

Let wel:
Dit gebeur so dat
T die middelpunt van QR is!
(Maar, dit sal nie reg wees
om die middelpuntformule
te gebruik nie)

3.7 Oppervlakte van ||m PQRS = QR × PT

$PT^2 = (4 + 1)^2 + (-2 - 8)^2$
$= 25 + 100$

∴ PT $= \sqrt{125} = 5\sqrt{5}$ eenhede

∴ Oppervlakte van PQRS $= 8\sqrt{5} \times 5\sqrt{5}$
$=$ **200 eenhede²** ◄

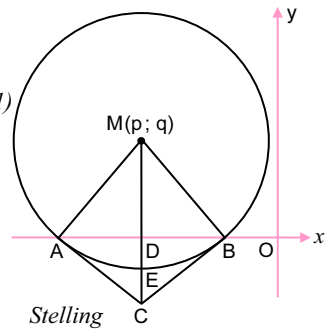
OF: Bepaal die oppervlakte van ΔPQR, en verdubbel dit.

4.1 p = **−6** ◄

4.2 DE = 1 eenheid … *pt E(6; −1)*

∴ ME = q + 1

∴ MA = q + 1 … *gelyke radiusse*



In reg∠ige ΔMDA:

$MA^2 = AD^2 + MD^2$ … *Stelling van Pyth*

∴ $(q + 1)^2 = (q - 1)^2 + q^2$

∴ $q^2 + 2q + 1 = q^2 - 2q + 1 + q^2$

∴ $0 = q^2 - 4q$

∴ $q(q - 4) = 0$

∴ (q = 0 of) **q = 4** ◄

4.3 Middelpunt (a; b) is (−6; 4)
& radius = q + 1 = 5
∴ Vgl. van ⊙M: $(x+6)^2 + (y-4)^2 = 25$ ◄

4.4 Die radius, r = 5 eenhede
& M is 6 eenhede van die y-as ... $p = x_E = -6$
∴ Tans is die afstand = 1 eenheid
∴ **Indien getransleer, is die minimum afstand = 3 eenhede** ◄

4.5 By A & B, is y = 0
& Vgl. van ⊙M:
$(x+6)^2 + 16 = 25$
∴ $(x+6)^2 = 9$
∴ $x + 6 = \pm 3$
∴ $x = -3$ of -9
∴ **A(−9; 0) & B(−3; 0)** ◄

OF:
AD = q − 1 = 4 − 1 = 3
en MD ⊥ AB
∴ AD = DB
MC: $x = -6$ ... $MC \parallel y\text{-as}$
∴ 3 links aan −6 is −9
∴ A(−9; 0)
& 3 regs aan −6 is −3
∴ B(−3; 0)

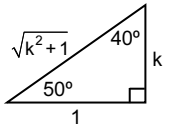
4.6 $m_{MB} = \frac{0-4}{-3+6}$ M(−6; 4) & B(−3; 0)
$= -\frac{4}{3}$
∴ $m_{BC} = +\frac{3}{4}$ ... *raaklyn BC ⊥ rad MB*
& pt. B(−3; 0):
∴ Vgl. van BC: $y - 0 = \frac{3}{4}(x+3)$
∴ $\mathbf{y = \frac{3}{4}x + \frac{9}{4}}$ ◄

4.7 $\mathbf{C\left(-6; -\frac{9}{4}\right)}$ ◄ ... $x_E = -6$
& $y_C = \frac{3}{4}(-6) + \frac{9}{4} = -\frac{9}{2} + \frac{9}{4} = -\frac{9}{4}$

4.8 In ΔDBC: DB = 3 & DC = $\frac{9}{4}$ eenhede
∴ $\tan D\hat{C}B = \frac{3}{\frac{9}{4}}$
$= \frac{4}{3}$
∴ $D\hat{C}B = 53{,}13°$
∴ $A\hat{C}B = \mathbf{106{,}26°}$ ◄

TRIGONOMETRIE [50]

5.1 $\tan 50° = \frac{k}{1}$



5.1.1 $\cos 40° = \frac{\mathbf{k}}{\sqrt{\mathbf{k^2+1}}}$ ◄

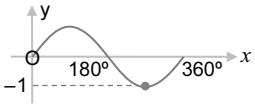
5.1.2 $\frac{2\sin 25°\cos 25°}{-2 + 4\sin^2 25°}$
$= \frac{\sin 2(25°)}{-2(1 - 2\sin^2 25°)}$ ... $\sin 2A = 2\sin A\cos A$
$= \frac{\sin(50°)}{-2\cos 2(25°)}$ ... $\cos 2A = \cos^2 A - \sin^2 A$
$= 1 - 2\sin^2 A$
$= -\frac{1}{2} \cdot \frac{\sin 50°}{\cos 50°}$
$= -\frac{1}{2}\tan 50° = \mathbf{-\frac{1}{2}k}$ ◄

5.1.3 $\sin 10° = \sin(50° - 40°)$
$= \sin 50°\cos 40° - \cos 50°\sin 40°$
$= \sin 50°\sin 50° - \cos 50°\cos 50°$
$= \sin^2 50° - \cos^2 50°$
$= \left(\frac{k}{\sqrt{k^2+1}}\right)^2 - \left(\frac{1}{\sqrt{k^2+1}}\right)^2$
$= \frac{k^2}{k^2+1} - \frac{1}{k^2+1}$
$= \mathbf{\frac{k^2-1}{k^2+1}}$ ◄

5.2.1 $\frac{\sin(540° + x)\cdot\cos(90° + x)}{\sin(-x)}$
$= \frac{\sin(180° + x)\cdot(-\sin x)}{(-\sin x)}$
$= \mathbf{-\sin x}$ ◄

5.2.2 Stel vas wanneer $\sqrt{-\sin x}$ reëel sal wees (Sien 5.2.1):
Wanneer $0 < -\sin x \leq 1$
$\times(-1)$ ∴ $0 > \sin x \geq -1$
i.e. $-1 \leq \sin x < 0$
∴ **$180° < x < 360°$** ◄

$-\sin x \neq 0$ want dit is in die noemer

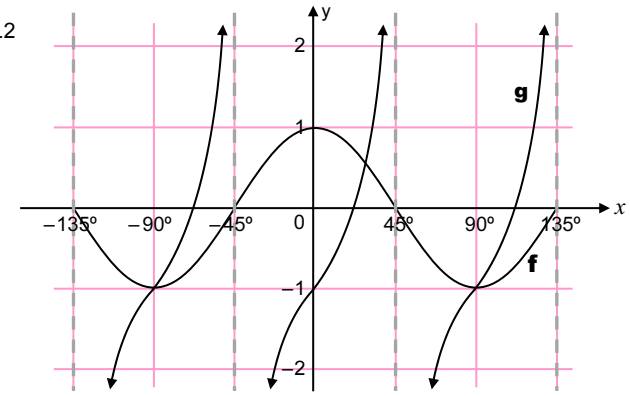


6.1 **LK**
$= [\tan(180° - x)](1 - \cos^2 x) + \cos^2 x = \frac{(\sin x - \cos x)(1 + \sin x \cdot \cos x)}{-\cos x}$
$= (-\tan x)(\sin^2 x) + \cos^2 x$
$= -\frac{\sin x}{\cos x}(\sin^2 x) + \cos^2 x$
$= \frac{-\sin^3 x + \cos^3 x}{\cos x}$
$= \frac{-(\sin^3 x - \cos^3 x)}{\cos x}$
$= \frac{(\sin x - \cos x)(\sin^2 x + \sin x\cos x + \cos^2 x)}{-\cos x}$
$= \frac{(\sin x - \cos x)(1 + \sin x\cos x)}{-\cos x}$
$= \mathbf{RK}$ ◄

6.2 $\cos^2 x - \sin^2 x = \frac{1}{2}\sin 2x - \cos^2 x$ ... *= d, die gemene verskil*
∴ $2\cos^2 x - \sin^2 x = \frac{1}{2}(2\sin x\cos x)$
∴ $2\cos^2 x - \sin^2 x = \sin x\cos x$
∴ $2\cos^2 x - \sin x\cos x - \sin^2 x = 0$
∴ $(2\cos x + \sin x)(\cos x - \sin x) = 0$
∴ $2\cos x = -\sin x$ or $\cos x = \sin x$
(÷ cos x) ∴ $2 = -\tan x$ Maar die konstante verskil = 0
∴ $\tan x = -2$ ∴ nie van toepassing nie
∴ **$x = 116{,}57° + n(180°), n \in \mathbb{Z}$** ◄

7.1 **180°** ◄

7.2



7.3 $f(x) = \cos 2x$

$\therefore h(x) = \cos 2(x + 45°)$

$\therefore h(x) = \cos(2x + 90°)$

$\therefore$ **$h(x) = -\sin 2x$** ◄

7.4 **$y \in [-1; 1]$ of $-1 \leq y \leq 1$** ◄

7.5 $(1 - \tan 2x)(\cos 2x) \geq 0$

$\therefore (\tan 2x - 1)(\cos 2x) \leq 0$

$\therefore g(x).f(x) \leq 0$

$\therefore$ **$0° \leq x \leq 22\frac{1}{2}°$ of $112{,}5° \leq x < 135°$** ◄

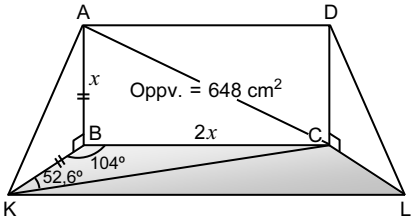
8.1 Laat AB = x cm; dan is BC = $2x$ cm

Oppv. van ABCD = $x \times 2x = 648$ cm^2

$\therefore x^2 = 324$ cm^2

$\therefore$ **AB = x = 18 cm** ◄

A = b × ℓ



8.2 In ΔABC: $AC^2 = 18^2 + 36^2 = 1\,620$ ... *(Pyth)*

$\therefore$ **AC $\simeq$ 40,25 cm** ◄

8.3 In ΔBKC: $\frac{KC}{\sin 104°} = \frac{36}{\sin 52{,}6°}$

$\therefore KC = \frac{36 \sin 104°}{\sin 52{,}6°}$

$\simeq$ **43,97 cm** ◄

8.4 In reg$\angle^{ige}$ ΔABK: $AK^2 = AB^2 + BK^2$ ... *(Pyth)*

$= 18^2 + 18^2$ ... *BK = AB = 18*

$= 648$

$\therefore$ In ΔAKC: $KC^2 = AK^2 + AC^2 - 2AK.AC \cos K\hat{A}C$

$\therefore 43{,}97^2 = 648 + 1\,620 - 2.\sqrt{648}\sqrt{1\,620} \cos K\hat{A}C$

$\therefore 2\sqrt{648 \times 1\,620} \cos K\hat{A}C = 334{,}6391$

$= 0{,}1633...$

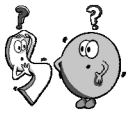
$\therefore$ **$K\hat{A}C = 80{,}60°$** ◄

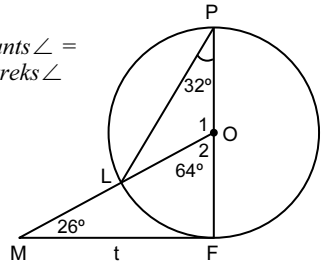
EUKLIDIESE MEETKUNDE [41]

9.1 Stelling

9.2.1 $\hat{O}_2 = 2\hat{P}$ ... *middelpunts$\angle$ = 2 × omtreks$\angle$*

= 64° ◄





9.2.2 $O\hat{F}M = 90°$ ... *middellyn ⊥ raaklyn*

$\therefore$ **$\hat{M} = 26°$** ◄ ... *som van $\angle^e$ in Δ*

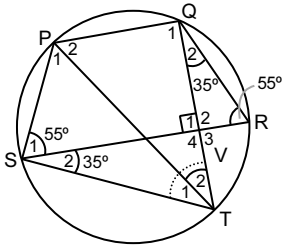
10.1 $V_2 = 90°$ ... *QT ⊥ SR*

$Q\hat{R}S$ = **55°** ◄ ... *som van $\angle^e$ in Δ*

$\therefore Q\hat{T}S$ = **55°** ◄ ... *$\angle^e$ in dieselfde seg.*

10.2 $\hat{S}_1 = \hat{R} = 55°$

& $P\hat{Q}R = 125°$ ... *teenoorst. $\angle^e$ van kvh*



10.3 $\hat{Q}_1 = 125° - 35° = 90°$

$\therefore$ **PT is 'n middellyn van die sirkel** ◄ ... *omgekeerde $\angle$ in semi⊙*

11.1.1 In ΔCDA: $\frac{FD}{CF} = \frac{GA}{CG}$ ... *eweredigh.stell.; AD || GF*

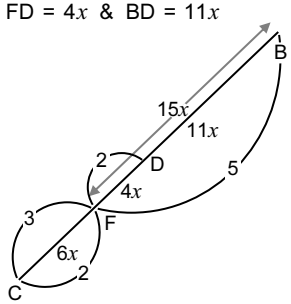
$= \frac{2}{3}$ ◄

11.1.2 In ΔBEF: $\frac{BA}{EA} = \frac{BD}{FD}$ ... *eweredigh.stell.; AD || EF*

Laat CF = $6x$; dan is FD = $4x$ & BD = $11x$

$\therefore \frac{BA}{EA} = \frac{11x}{4x}$

$= \frac{11}{4}$ ◄



11.1.3 $\frac{\text{Oppv. van } \Delta GCF}{\text{Oppv. van } \Delta ACD} = \frac{\frac{1}{2}GC.CF \sin C}{\frac{1}{2}AC.CD \sin C}$

$= \frac{GC}{AC} \cdot \frac{CF}{CD}$

$= \frac{3}{5} \cdot \frac{3}{5}$

$= \frac{9}{25}$

$\therefore \frac{\text{Oppv. van } \Delta GCF}{\text{Oppv. van GFDA}} = \frac{9}{25 - 9} = \frac{9}{16}$ ◄

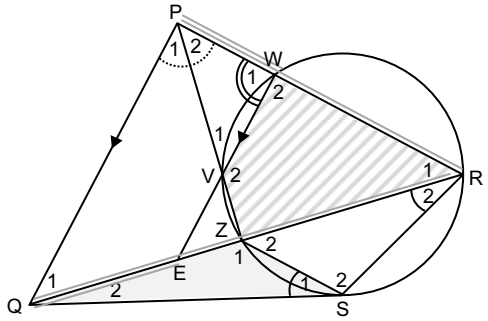
11.2.1

Let wel: Merk die 4 sye op die figuur;
dit sal duidelik wees wat om te doen . . .

In ΔRPQ

$\frac{PR}{PW} = \frac{QR}{QE}$ . . . *eweredigh.stell; WE || PQ.*

$\therefore$ **$PR = \frac{PW.QR}{QE}$** ◄



11.2.2 $\Delta PQZ \,|||\, \Delta RQP$

➡ $\frac{PQ}{RQ} = \frac{QZ}{PQ} \left(= \frac{PZ}{PR}\right)$ . . . *ooreenstemmende sye*

$\therefore$ **$PQ^2 = RQ.QZ$** ◄

11.2.3 In Δ^e QSZ & QRS

(1) $\hat{Q}_2$ is gemeen

(2) $\hat{S}_1 = \hat{R}_2$ . . . *raaklyn-koord stelling*

$\therefore$ **$\Delta QSZ \,|||\, \Delta QRS$** . . . $\angle\angle\angle$ ◄

11.2.4 Vanaf 11.2.2: $PQ^2 = RQ.QZ$ . . . ❶

& Vanaf 11.2.3: $\frac{QS}{QR} = \frac{QZ}{QS}$ . . . *ooreenstemmende sye*

$\therefore QS^2 = QR.QZ$ . . . ❷

❶ & ❷ $\therefore PQ^2 = QS^2$

$\therefore$ **$PQ = QS$** ◄

11.2.5 Afgelei vanaf 11.2.1: $\frac{PW.QR}{QE} = PR$

$\therefore PW = \frac{QE.PR}{QR}$ . . . ❶

Vanaf soorgelyke Δ^e 11.2.2: $\frac{PR}{QR} = \frac{PZ}{PQ}$ . . . ❷

❷ in ❶: $\therefore PW = \frac{QE.PZ}{PQ}$

Vanaf 11.2.2: $PQ^2 = QR.QZ$

$\therefore PQ = \sqrt{QR.QZ}$

$\therefore$ **$PW = \frac{QE.PZ}{\sqrt{QR.QZ}}$** ◄

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