DBO NOV 2025 VRAESTEL 1

ALGEBRA & VERGELYKINGS & ONGELYKHEDE [25]

1.1.1 $(x+5)(x-2) = 0$

$\therefore\ \boldsymbol{x = -5}$ **of** $\boldsymbol{x = 2}$ ◄

1.1.2 $5x^2 + 2 = -9x$

$\therefore\ 5x^2 + 9x + 2 = 0 \qquad (a = 5,\ b = 9,\ c = 2)$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-9 \pm \sqrt{(9)^2 - 4(5)(2)}}{2(5)}$

$= \dfrac{-9 \pm \sqrt{41}}{10}$

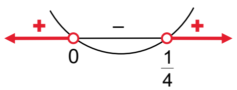
$= \mathbf{-0{,}26}$ **of** $\mathbf{-1{,}54}$ ◄

1.1.3 $8x^2 > 2x$

$\therefore\ 8x^2 - 2x > 0$

$\therefore\ 2x(4x - 1) > 0$

$\therefore\ \boldsymbol{x < 0}$ **of** $\boldsymbol{x > \frac{1}{4}}$ ◄



1.1.4 $2.2^{2x} - 9.2^x + 4 = 0$

$(2.2^x - 1)(2^x - 4) = 0$

$\therefore\ 2.2^x - 1 = 0$ of $2^x - 4 = 0$

$\therefore\ 2^x = \frac{1}{2} \qquad \therefore\ 2^x = 4$

$\therefore\ 2^x = 2^{-1} \qquad \therefore\ 2^x = 2^2$

$\therefore\ \boldsymbol{x = -1} \qquad \therefore\ \boldsymbol{x = 2}$ ◄

OF: $2.2^{2x} - 9.2^x + 4 = 0$

Laat $2^x = k$

$\therefore\ 2k^2 - 9k + 4 = 0$

$\therefore\ (2k - 1)(k - 4) = 0$

$\therefore\ k = \frac{1}{2}$ of $k = 4$

$\therefore\ 2^x = 2^{-1} \qquad \therefore\ 2^x = 2^2$

$\therefore\ \boldsymbol{x = -1} \qquad \therefore\ \boldsymbol{x = 2}$ ◄

1.1.5 $\left(\sqrt{\sqrt{\frac{1}{x}} + 2}\right)^2 = \left(\frac{1}{\sqrt{x}}\right)^2$

$\therefore\ \sqrt{\frac{1}{x}} + 2 = \frac{1}{x}$

$\therefore\ \left(\sqrt{\frac{1}{x}}\right)^2 = \left(\frac{1}{x} - 2\right)^2$

$\therefore\ \frac{1}{x} = \frac{1}{x^2} - \frac{4}{x} + 4$

$\therefore\ x = 1 - 4x + 4x^2 \qquad (\times \text{ met } x^2)$

$\therefore\ 4x^2 - 5x + 1 = 0$

$\therefore\ (4x - 1)(x - 1) = 0$

$\therefore\ x = \frac{1}{4}$ of $x \neq 1$

$\therefore$ **slegs** $\boldsymbol{x = \frac{1}{4}}$ ◄

Toets

$x = 1$: LK $= \sqrt{3}$ maar RK $= 1$ ✗

$x = \frac{1}{4}$: LK $= 2 =$ RK ✓

OF:

$\sqrt{\sqrt{\frac{1}{x}} + 2} = \frac{1}{\sqrt{x}}$

Laat $\frac{1}{\sqrt{x}} = k \qquad \left(\sqrt{\frac{1}{x}} = \frac{1}{\sqrt{x}}\right)$

$\therefore\ \left(\sqrt{k + 2}\right)^2 = (k)^2$

$\therefore\ k + 2 = k^2$

$\therefore\ k^2 - k - 2 = 0$

$\therefore\ (k - 2)(k + 1) = 0$

$\therefore\ k = 2$ of $k \neq -1 \qquad \left(\sqrt{\frac{1}{x}} \neq -1\right)$

$\therefore\ \left(\sqrt{\frac{1}{x}}\right)^2 = (2)^2$

$\therefore\ \frac{1}{x} = 4$

$\therefore\ \boldsymbol{x = \frac{1}{4}}$ ◄

1.2 $x = 2 + y \quad \ldots$ ❶

$5xy = x^2 + 6 \quad \ldots$ ❷

Stel ❶ in ❷ in:

$5(2 + y)y = (2 + y)^2 + 6$

$\therefore\ 10y + 5y^2 = 4 + 4y + y^2 + 6$

$\therefore\ 4y^2 + 6y - 10 = 0$

$\therefore\ 2y^2 + 3y - 5 = 0$

$\therefore\ (2y + 5)(y - 1) = 0$

$\therefore\ y = -\frac{5}{2}$ of $y = 1$

As $y = -\frac{5}{2}$, dan is $x = 2 + \left(-\frac{5}{2}\right) = -\frac{1}{2}$

As $y = 1$, dan is $x = 2 + (1) = 3$

$\therefore$ **Oplossing:** $\left(-\frac{1}{2}; -\frac{5}{2}\right)$ **of** $(3; 1)$ ◄

OF:

$x = 2 + y$

$\therefore\ y = x - 2 \quad \ldots$ ❶

$5xy = x^2 + 6 \quad \ldots$ ❷

Stel ❶ in ❷ in:

$5x(x - 2) = x^2 + 6$

$\therefore\ 5x^2 - 10x = x^2 + 6$

$\therefore\ 4x^2 - 10x - 6 = 0$

$\therefore\ 2x^2 - 5x - 3 = 0$

$\therefore\ (2x + 1)(x - 3) = 0$

$\therefore\ x = -\frac{1}{2}$ of $x = 3$

As $x = -\frac{1}{2}$, dan is $y = -\frac{1}{2} - 2 = -\frac{5}{2}$

As $x = 3$, dan is $y = 3 - 2 = 1$

$\therefore$ **Oplossing:** $\left(-\frac{1}{2}; -\frac{5}{2}\right)$ **of** $(3; 1)$ ◄

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PATRONE & RYE [25]

2.1.1 $(t + 10) + (t - 2) + (t + 4) + \ldots$ is 'n oneindige MR

$$\therefore \frac{t-2}{t+10} = \frac{t+4}{t-2} \quad \ldots$$

In 'n meetkundige reeks $\frac{T_2}{T_1} = \frac{T_3}{T_2}$

$$\therefore (t+10)(t+4) = (t-2)^2$$
$$\therefore t^2 + 14t + 40 = t^2 - 4t + 4$$
$$\therefore 18t = -36$$
$$\therefore \mathbf{t = -2}$$

2.1.2 Die eerste drie terme is: 8; −4; 2

$a = 8$ en $r = -\frac{1}{2}$

$$\therefore T_{25} = (8)\left(-\frac{1}{2}\right)^{24}$$
$$= (2^3)\left(\frac{1}{2}\right)^{24}$$
$$= (2^3)(2^{-24})$$
$$= 2^{-21}$$
$$= \left(\frac{1}{2}\right)^{21}$$

2.1.3 $S_\infty = \frac{a}{1-r} = \frac{8}{1-\left(-\frac{1}{2}\right)} = \frac{\mathbf{16}}{\mathbf{3}} \quad \ldots \quad -1 < r < 1$

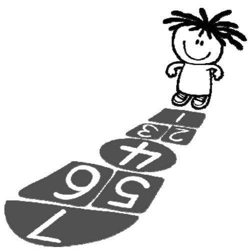
2.2.1 $T_{14} = 4(k+13) - 1 = 4k + 51$

$T_6 = 4(k+5) - 1 = 4k + 19$

$$\therefore \mathbf{T_{14} - T_6 = 32}$$

OF:

$$T_{14} - T_6 = 8d$$
$$= 8(4)$$
$$= \mathbf{32}$$



2.2.2 $\sum_{p=k}^{117}(4p - 1) = 26\,675$

$n = 117 - k + 1 = 118 - k$

$a = 4k - 1$

$d = 4$

$S_n = \frac{n}{2}[2a + (n-1)d] = 26\,675$

$$\therefore \frac{118-k}{2}[2(4k-1) + (117-k)(4)] = 26\,675$$
$$\therefore (118-k)[(4k-1) + 2(117-k)] = 26\,675$$
$$\therefore (118-k)[4k - 1 + 234 - 2k] = 26\,675$$
$$\therefore (118-k)(2k + 233) = 26\,675$$
$$\therefore 27\,494 + 3k - 2k^2 - 26\,675 = 0$$
$$\therefore 2k^2 - 3k - 819 = 0$$
$$\therefore (2k+39)(k-21) = 0$$
$$\therefore \textbf{slegs } \mathbf{k = 21} \quad \left(k \neq -\frac{39}{2}\right)$$

OF:

$n = 117 - k + 1 = 118 - k$

$a = 4k - 1$

$d = 4$

$l = 4(117) - 1 = 467$

$S_n = \frac{n}{2}[a + l] = 26\,675$

$$\therefore \frac{118-k}{2}[(4k-1) + (467)] = 26\,675$$
$$\therefore \frac{118-k}{2}[4k + 466] = 26\,675$$
$$\therefore (118-k)[2k + 233] = 26\,675$$
$$\therefore -2k^2 + 3k + 27\,494 - 26\,675 = 0$$
$$\therefore 2k^2 - 3k - 819 = 0$$
$$\therefore (2k+39)(k-21) = 0$$
$$\therefore \textbf{slegs } \mathbf{k = 21} \quad \left(k \neq -\frac{39}{2}\right)$$

(Jy kan ook die kwadratiese formule gebruik as jy dit verkies.)

OF:

Hierdie metode los vir k op deur die som van 117 terme te bepaal en die voorafgaande terme af te trek.

$n = 117 \quad a = 3 \quad d = 4$

$l = T_{117} = 467$

$$\therefore S_{117} = \frac{117}{2}[3 + 467] = 27\,495$$

$n = k - 1 \quad a = 3 \quad d = 4$

$l = T_{k-1} = 4(k-1) - 1 = 4k - 5$

$$\therefore S_{k-1} = \frac{k-1}{2}[3 + 4k - 5]$$
$$= \frac{k-1}{2}[4k - 2]$$
$$= (k-1)(2k-1)$$
$$= 2k^2 - 3k + 1$$

$$\therefore S_{117} - S_{k-1} = 26\,675$$
$$\therefore 27\,495 - (2k^2 - 3k + 1) = 26\,675$$
$$\therefore 2k^2 - 3k - 819 = 0$$
$$\therefore (2k+39)(k-21) = 0$$
$$\therefore \textbf{slegs } \mathbf{k = 21} \quad \left(k \neq -\frac{39}{2}\right)$$

3.

36 71 104 135 164

35 33 31 29

–2 –2 –2

Vir 'n kwadratiese patroon:

a + b + c 4a + 2b + c 9a + 3b + c

3a + b 5a + b

2a

3.1 **Die diepte van die torpedo na 5 sekondes is 164 m.** ◄

3.2 $2a = -2$ $\quad 3(-1) + b = 35$ $\quad -1 + 38 + c = 36$

$\therefore a = -1$ $\quad \therefore b = 38$ $\quad \therefore c = -1$

$\therefore \mathbf{T_n = -n^2 + 38n - 1}$ ◄

3.3 $-2n + 38 = 0 \quad \therefore n = 19 \quad \therefore T_{19} = -19^2 + 38(19) - 1 = 360$

OF:

$-\frac{b}{2a} = -\frac{38}{2(-1)} = 19$ en $T_{19} = 360$ (sien hierbo)

OF:

$T_n = -(n^2 - 38n + 1) = -[(n-19)^2 - 360] = -(n-19)^2 + 360$

$\therefore$ **die maksimum diepte wat die torpedo bereik het is 360 m.** ◄

3.4 $T_3 = T_{19-16} = T_{19+16} = T_{35}$ (gebruik simmetrie)

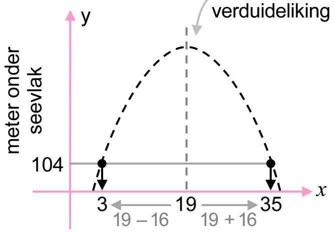
OF:

$-n^2 + 38n - 1 = 104$

$\therefore n^2 - 38n + 105 = 0$

$\therefore (n-3)(n-35) = 0$

$\therefore n = 3$ of $n = 35$



$\therefore$ **die torpedo was na 35s vir die 2de keer 104 m onder seevlak** ◄

FUNKSIES & GRAFIEKE [36]

4.1 $f(x) = \log_{\frac{1}{3}} x$

$\therefore f(3) = \log_{\frac{1}{3}} 3 = -1$

$\therefore \mathbf{t = -1}$ ◄

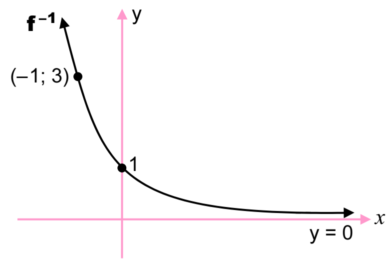
4.2 **A(1; 0)** $\left(\log_{\frac{1}{3}} 1 = 0\right)$ ◄

4.3 $x = \log_{\frac{1}{3}} y$

$\therefore \mathbf{f^{-1}(x) = y = \left(\frac{1}{3}\right)^x}$ ◄ . . . $c = \log_b a \Leftrightarrow a = b^c$

4.4 **y = 0** ◄

4.5

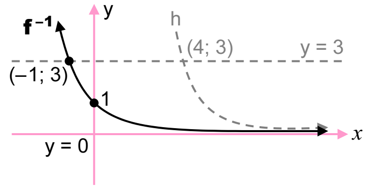


4.6 $h(x) = \left(\frac{1}{3}\right)^{x-5}$

$h(x) > 0$ vir alle waardes van x

$h(4) = \left(\frac{1}{3}\right)^{-1} = 3$

$\therefore \mathbf{0 < y < 3}$ ◄

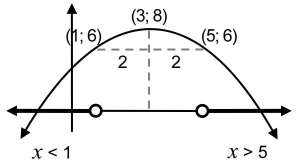


5.1 $\boldsymbol{x \in \mathbb{R},\ x \neq 3}$ ◄

5.2 $\mathbf{y \leq 8}$ of $\mathbf{y \in (-\infty; 8]}$ ◄

5.3.1 $\mathbf{3 < x \leq 5}$ of $\mathbf{x \in (3; 5]}$ ◄

5.3.2 $\mathbf{x < 1}$ of $\mathbf{x > 5}$ of $\mathbf{x \in (-\infty; 1) \cup (5; \infty)}$ ◄



5.4 $y = a(x-3)^2 + 8$

Stel D(5; 6) in

$\therefore 6 = a(5-3)^2 + 8$

$\therefore 6 = a(2)^2 + 8$

$\therefore 4a = -2$ en $a = -\frac{1}{2}$

$\therefore y = -\frac{1}{2}(x-3)^2 + 8$

$= -\frac{1}{2}(x^2 - 6x + 9) + 8$

$= \mathbf{-\frac{1}{2}x^2 + 3x + \frac{7}{2}}$ ◄

5.5 By M, $f(x) = 0$

$\therefore -\frac{1}{2}x^2 + 3x + \frac{7}{2} = 0$

$\therefore x^2 - 6x - 7 = 0$

$\therefore (x+1)(x-7) = 0$

$\therefore x = -1$ of $x = 7$

$\therefore \mathbf{x_M = -1}$ ◄

OF: By M, $f(x) = 0$

$\therefore 0 = -\frac{1}{2}(x-3)^2 + 8$

$\therefore (x-3)^2 = 16$

$\therefore x - 3 = \pm 4$

$\therefore x = 3 - 4 = -1$ of $x = 3 + 4 = 7$

$\therefore \mathbf{x_M = -1}$ ◄

By T, $g(x) = 0$

$\therefore \frac{-4}{x-3} + 8 = 0$

$\therefore -4 + 8(x-3) = 0$

$\therefore -4 + 8x - 24 = 0$

$\therefore 8x = 28$

$\therefore x_T = \frac{7}{2}$

$\therefore \mathbf{MT = \frac{7}{2} - (-1) = \frac{9}{2} = 4{,}5}$ **eenhede** ◄

5.6 $f'(x) = -x + 3$

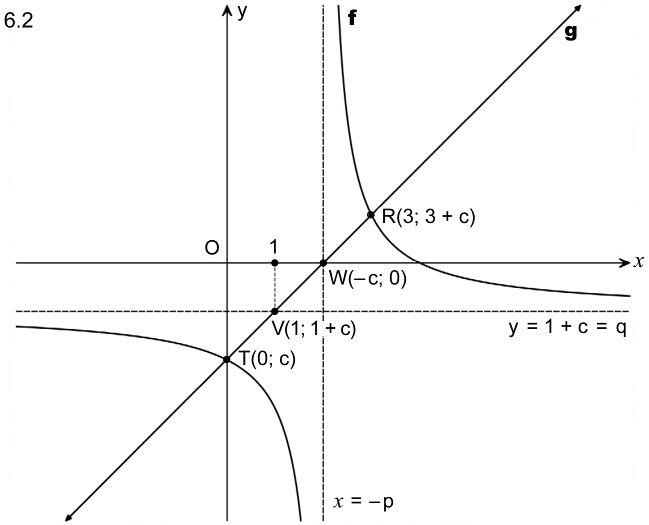
$\therefore m = f'(5) = -5 + 3 = -2$

$\therefore y - 6 = -2(x-5)$ . . . $y - y_1 = m(x - x_1)$

$\therefore \mathbf{y = -2x + 16}$ ◄

6.1 $(-p;\ 0)$ ◄

6.2



R(3; 3 + c) [g(3) = 3 + c] W(–c; 0) [g(–c) = 0]

V(1; 1 + c) [g(1) = 1 + c] T(0; c) [g(0) = c]

$\therefore\ p = c$ en $q = 1 + c$

$\therefore\ f(0) = \frac{a}{0+c} + 1 + c = c$

$\therefore\ \frac{a}{c} + 1 = 0$

$\therefore\ \frac{a}{c} = -1$

$\therefore\ a = -c$

$\therefore\ f(3) = \frac{-c}{3+c} + 1 + c = 3 + c$

$\therefore\ \frac{-c}{3+c} = 2$

$\therefore\ 2c + 6 = -c$

$\therefore\ 3c = -6$

$\therefore\ c = -2$

$\therefore\ a = -c = 2$

$\therefore\ p = c = -2$

$\therefore\ q = c + 1 = -1$

$\therefore\ \mathbf{f(x) = \frac{2}{x-2} - 1}$ ◄

OF:

y-afsnit: $c = \frac{a}{p} + q$ . . . ❶

By $x = 1$: $1 + c = q$ . . . ❷

By $x = 3$: $3 + c = \frac{a}{3+p} + q$ . . . ❸

Stel ❷ in ❶ in:

$c = \frac{a}{p} + 1 + c$

$\therefore\ \frac{a}{p} = -1$

$\therefore\ a = -p$

Stel ❷ en $a = -p$ in ❸ in:

$3 + c = \frac{-p}{3+p} + 1 + c$

$\therefore\ 2 = \frac{-p}{3+p}$

$\therefore\ 6 + 2p = -p$

$\therefore\ 3p = -6$

$\therefore\ p = -2$

$\therefore\ c = -2$ ($m_g = 1$ dus is OT = OW & $-c = -p$)

$\therefore\ a = -(-2) = 2$

$\therefore\ q = 1 + (-2) = -1$

$\therefore\ \mathbf{f(x) = \frac{2}{x-2} - 1}$ ◄

6.3 $g(x) = x - 2$

Skuif g 1 eenheid regs: $\mathbf{y = (x-1) - 2 = x - 3}$ ◄

of

Skuif g 1 eenheid af: $\mathbf{y = (x-2) - 1 = x - 3}$ ◄

FINANSIES, GROEI & VERVAL [15]

7.1 $A = P(1 + i)^n$

$= 40\ 000\left(1 + \frac{7,8}{100}\right)^5$

$= \mathbf{R58\ 230,94}$ ◄

7.2 Sarah het 24 kwartaallikse deposito's van R2 300 gemaak en die geld toe vir nog 'n kwartaal daar gelaat.

$\mathbf{F_v = \frac{x\left[(1+i)^n - 1\right]}{i}}$ . . . met $x = R2\ 300$

$n = 6 \times 4 = 24$

$i = \frac{5,8\%}{4} = \frac{5,8}{400} = \frac{0,058}{4}$

$$F_v = \frac{2\ 300\left[\left(1 + \frac{5,8}{400}\right)^{24} - 1\right]}{\frac{5,8}{400}} \times \left(1 + \frac{5,8}{400}\right)$$

$= \mathbf{R66\ 411,60}$ ◄

7.3.1 $900\ 000\left(1 + \frac{6,8}{1\ 200}\right)^3 = \frac{10\ 000\left[1 - \left(1 + \frac{6,8}{1\ 200}\right)^{-n}\right]}{\frac{6,8}{1\ 200}}$ *

$\therefore\ \left(1 + \frac{6,8}{1\ 200}\right)^{-n} = 0,4812\ldots$

$\therefore\ -n = \log_{\left(1+\frac{6,8}{1\ 200}\right)}(0,4812\ldots)$

$\therefore\ -n = -129,419\ldots$

$\therefore\ n = 129,419\ldots$

Rajesh het in totaal 130 betalings gemaak en het dus sy lening 133 maande nadat dit goedgekeur is, terugbetaal. ◄

* $\mathbf{P_v = \frac{x\left[1 - (1+i)^{-n}\right]}{i}}$

OF:

$$900\,000 = \frac{10\,000\left[1-\left(1+\frac{6,8}{1\,200}\right)^{-n}\right]}{\frac{6,8}{1\,200}}\left(1+\frac{6,8}{1\,200}\right)^{-3}$$

$$\therefore \left(1+\frac{6,8}{1\,200}\right)^{-n} = 0,4812\ldots$$

$$\therefore -n = \log_{\left(1+\frac{6,8}{1\,200}\right)}(0,4812\ldots)$$

$$\therefore -n = -129,419\ldots$$

$$\therefore n = 129,419\ldots$$

Rajesh het in totaal 130 betalings gemaak en het dus sy lening 133 maande nadat dit goedgekeur is, terugbetaal. ◄

7.3.2 Finale betaling:

$$\left[900\,000\left(1+\frac{6,8}{1\,200}\right)^{132} - 10\,000 \times \frac{\left(1+\frac{6,8}{1\,200}\right)^{129}-1}{\frac{6,8}{1\,200}}\right]\left(1+\frac{6,8}{1\,200}\right)$$

$= \textbf{R4 197,21}$ ◄

DIFFERENSIAALREKENE [33]

8.1

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0}\frac{[-2(x+h)+3]-[-2x+3]}{h}$$
$$= \lim_{h\to 0}\frac{[-2x-2h+3]-[-2x+3]}{h}$$
$$= \lim_{h\to 0}\frac{-2h}{h}$$
$$= \lim_{h\to 0} -2$$
$$= \mathbf{-2}$$ ◄

8.2.1 $g(x) = -3x^4 + 2x$

$\therefore g'(x) = \mathbf{-12x^3 + 2}$ ◄

8.2.2

$$y = \frac{2x^4+1}{x^2}$$
$$= \frac{2x^4}{x^2} + \frac{1}{x^2}$$
$$= 2x^2 + x^{-2}$$
$$\therefore \frac{dy}{dx} = 4x - 2x^{-3}$$
$$= \mathbf{4x - \frac{2}{x^3}}$$ ◄

9.1 $f(x) = x^3 - 8x^2 + 5x + 14$

$f'(x) = 3x^2 - 16x + 5 = 0$ by D en E

$\therefore (3x-1)(x-5) = 0$

$\therefore x = \frac{1}{3}$ of $x = 5$

$f(5) = 5^3 - 8(5)^2 + 5(5) + 14 = -36$

$\therefore$ **E(5; –36)** ◄

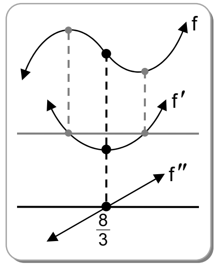
9.2 $f''(x) = 6x - 16$

f is konkaaf na onder wanneer $f''(x) < 0$.

$f''(x) < 0$

$\therefore 6x - 16 < 0$

$\therefore \boldsymbol{x < \frac{8}{3}}$ ◄



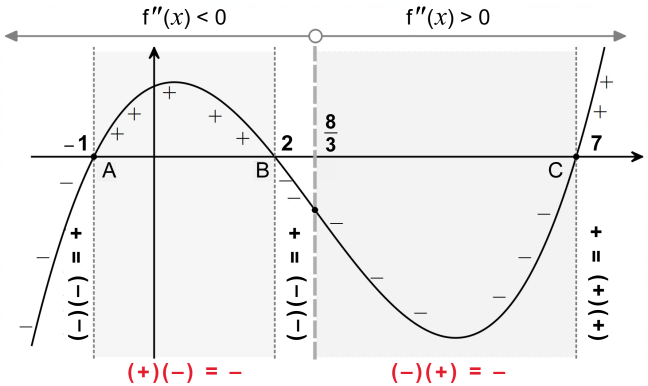
9.3 $f(x) = x^3 - 8x^2 + 5x + 14$

$\therefore f(2) = 0$

$\therefore f(x) = (x-2)(x^2-6x-7)$

$= (x-2)(x-7)(x+1)$

A(–1; 0), B(2; 0) en C(7; 0) word op die grafiek hieronder getoon.



$f(x)$ en $f''(x)$ moet teenoorgestelde tekens hê vir $f(x).f''(x) < 0$.

$\mathbf{-1 < x < 2}$ **of** $\mathbf{\frac{8}{3} < x < 7}$ **of** $\mathbf{x \in (-1; 2) \cup \left(\frac{8}{3}; 7\right)}$ ◄

Die + en – tekens op die grafiek toon waar $y = f(x)$ + of – is.

$f''(x) < 0$ vir $x < \frac{8}{3}$ en $f''(x) > 0$ vir $x > \frac{8}{3}$

9.4 $f'(x) = 3x^2 - 16x + 5 = -11$

$\therefore\ 3x^2 - 16x + 16 = 0$

$\therefore\ (3x-4)(x-4) = 0$

$\therefore\ x = \frac{4}{3}$ of $x = 4$

$f\left(\frac{4}{3}\right) = \frac{238}{27}$

$\therefore\ y - \frac{238}{27} = -11\left(x - \frac{4}{3}\right)$

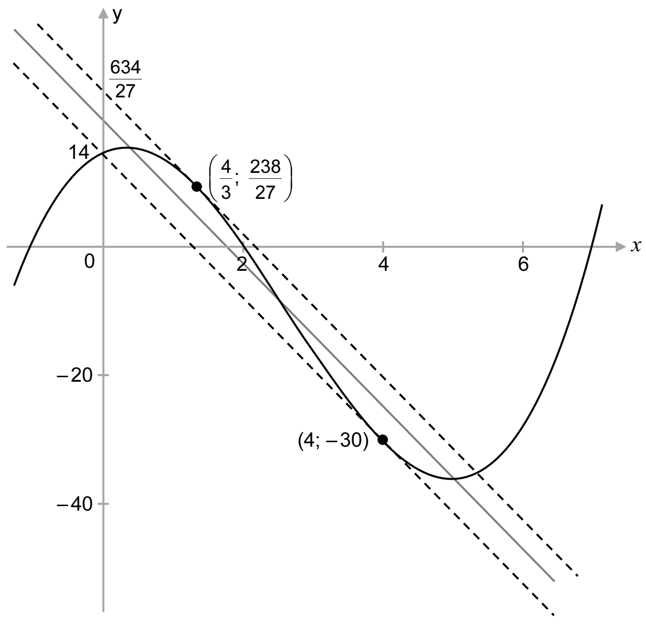
$\therefore\ y = -11x + \frac{634}{27}$ is 'n raaklyn aan f

& $f(4) = -30$

$\therefore\ y - (-30) = -11(x-4)$

$\therefore\ y = -11x + 14$ is 'n raaklyn aan f

$\therefore$ **$14 < t < \frac{634}{27}$** $\left(\frac{634}{27} = 23\frac{13}{27} = 23{,}\dot{4}8\dot{1}\right)$ ◄



https://www.desmos.com/calculator/oyerfq4oof

10.1 $2x + 2h = 50$

$\therefore\ 2h = 50 - 2x$

$\therefore\ h = 25 - x$

$2\pi r = x$

$\therefore\ r = \frac{x}{2\pi}$

$V = \pi r^2 h$

$= \pi\left(\frac{x}{2\pi}\right)^2(25 - x)$

$= \left(\frac{\pi x^2}{4\pi^2}\right)(25 - x)$

$= \left(\frac{x^2}{4\pi}\right)(25 - x)$

$= \mathbf{\frac{25x^2}{4\pi} - \frac{x^3}{4\pi}}$ ◄

10.2 $V = \frac{25x^2}{4\pi} - \frac{x^3}{4\pi}$

$\therefore\ V' = \frac{50x}{4\pi} - \frac{3x^2}{4\pi}$

Maksimum volume wanneer $\frac{50x}{4\pi} - \frac{3x^2}{4\pi} = 0$

$\therefore\ 50x - 3x^2 = 0$

$\therefore\ x(50 - 3x) = 0$

$x \neq 0 \quad \therefore\ 3x = 50$

$\mathbf{x = \frac{50}{3}}$ **eenhede** ◄ $\left(\frac{50}{3} = 16\frac{2}{3} = 16{,}\dot{6} \approx 16{,}67\right)$

WAARSKYNLIKHEID [16]

11.1

	SAP	ENERGIEDRANKIES	**TOTAAL**
Vroulik	a = 48	b = 72	c = 120
Manlik	36	54	f = 90
Totaal	e = 84	d = 126	210

11.1.1 P(manlik) × P(verkies sap) = P(manlik en verkies sap)

$\therefore\ \frac{90}{210} \times \frac{e}{210} = \frac{36}{210}$ . . . P(A) . P(B) = P(A en B) vir onafhanklike gebeurtenisse

$\therefore\ \frac{3e}{7} = 36$

$\therefore$ **e = 7 × 12 = 84** ◄

11.1.2 Hierdie vraag kan op twee verskillende maniere geïnterpreteer word daarom is daar twee moontlike uitkomste.

P(leerder, willekeurig gekies, is vroulik en hou van energiedrankies)

$= \frac{72}{210}$

$= \frac{12}{35}$

of, die ander interpretasie:

P(vroulik, willekeurig gekies, hou van energiedrankies)

$= \frac{72}{120}$

$= \mathbf{\frac{3}{5}}$ ◄

11.2 P(persoon koop 'n koppie koffie op 'n nie-reënerige dag) = x.

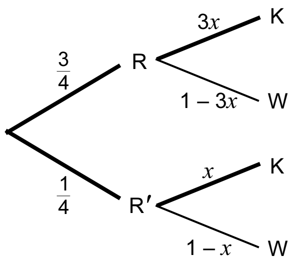
P(persoon koop 'n koppie koffie op enige gegewe dag) = $\frac{7}{12}$

$\therefore\ \left(\frac{3}{4}\right)(3x) + \left(\frac{1}{4}\right)(x) = \frac{7}{12}$

$\therefore\ \frac{9x}{4} + \frac{x}{4} = \frac{7}{12}$

$\therefore\ \frac{10x}{4} = \frac{7}{12}$

$\therefore\ x = \frac{7}{30}$



$\frac{7}{30} \times 120 = 28$

$\therefore$ **Op 'n nie-reënerige dag sal 28 koppies koffie verkoop word.** ◄

OF:

Laat x die aantal koppies koffie op 'n nie-reënerige dag verkoop, wees.

P(persoon koop koffie op enige gegewe dag)

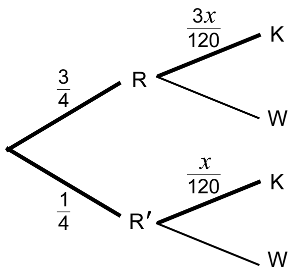
$= \frac{9x}{480} + \frac{x}{480}$

$= \frac{10x}{480}$

$= \frac{x}{48}$

$\therefore\ \frac{x}{48} = \frac{7}{12}$

$\therefore\ x = 28$



$\therefore$ **Op 'n nie-reënerige dag sal 28 koppies koffie verkoop word.** ◄

11.3.1

	1	2	3	4	5	6	7	8
1	A	B	–	–	–	–	–	–
2	–	A	B	–	–	–	–	–
3	–	–	A	B	–	–	–	–
4	–	–	–	A	B	–	–	–
5	–	–	–	–	A	B	–	–
6	–	–	–	–	–	A	B	–
7	–	–	–	–	–	–	A	B

Die 8 deelnemers kan op 7! = 5 040 maniere eindig met Bongi (B) direk na Andrew (A).

11.3.2 Andrew eerste . . . $5 \times 6!$

	1	2	3	4	5	6	7	8
1	A	–	–	B	–	–	–	–
2	A	–	–	–	B	–	–	–
3	A	–	–	–	–	B	–	–
4	A	–	–	–	–	–	B	–
5	A	–	–	–	–	–	–	B

Andrew tweede . . . $4 \times 6!$

	1	2	3	4	5	6	7	8
1	–	A	–	–	B	–	–	–
2	–	A	–	–	–	B	–	–
3	–	A	–	–	–	–	B	–
4	–	A	–	–	–	–	–	B

Andrew derde . . . $3 \times 6!$

	1	2	3	4	5	6	7	8
1	–	–	A	–	–	B	–	–
2	–	–	A	–	–	–	B	–
3	–	–	A	–	–	–	–	B

Andrew vierde . . . $2 \times 6!$

	1	2	3	4	5	6	7	8
1	–	–	–	A	–	–	B	–
2	–	–	–	A	–	–	–	B

Andrew vyfde . . . $1 \times 6!$

	1	2	3	4	5	6	7	8
1	–	–	–	–	A	–	–	B

$(1 + 2 + 3 + 4 + 5)(6!) = 15 \times 6!$

P(TWEE of MEER deelnemers eindig ná Andrew en voor Bongi)

$= \dfrac{15 \times 6!}{8!}$

$= \mathbf{\dfrac{15}{56}}$ ◄

OF, deur die komplement te gebruik:

8 deelnemers kan die wedloop op 8! maniere voltooi.

In die helfte van hierdie maniere, eindig Bongi voor Andrew.

$\therefore$ P(Bongi eindig voor Andrew) $= \dfrac{1}{2}$

Andrew eindig op 7! maniere direk voor Bongi. (vanaf 11.3.1)

$\therefore$ P(Andrew eindig direk voor Bongi) $= \dfrac{7!}{8!} = \dfrac{1}{8}$

Andrew eindig op $6 \times 6!$ maniere voor Bongi met EEN deelnemer tussen hulle.

	1	2	3	4	5	6	7	8
1	A	–	B	–	–	–	–	–
2	–	A	–	B	–	–	–	–
3	–	–	A	–	B	–	–	–
4	–	–	–	A	–	B	–	–
5	–	–	–	–	A	–	B	–
6	–	–	–	–	–	A	–	B

$\therefore$ P(Andrew eindig voor Bongi met EEN deelnemer tussen hulle)

$= \dfrac{6 \times 6!}{8!} = \dfrac{3}{28}$

P(TWEE of MEER deelnemers eindig die wedloop ná Andrew en voor Bongi)

$= 1 - \dfrac{1}{2} - \dfrac{1}{8} - \dfrac{3}{28}$

$= \mathbf{\dfrac{15}{56}}$ ◄