DBE NOV 2025 PAPER 1

ALGEBRA & EQUATIONS & INEQUALITIES [25]

1.1.1 $(x + 5)(x - 2) = 0$

$\therefore x = -5$ or $x = 2$ ◄

1.1.2 $5x^2 + 2 = -9x$

$\therefore 5x^2 + 9x + 2 = 0 \quad (a = 5,\ b = 9,\ c = 2)$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-9 \pm \sqrt{(9)^2 - 4(5)(2)}}{2(5)}$$

$$= \frac{-9 \pm \sqrt{41}}{10}$$

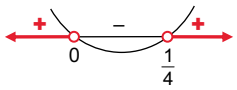
$= -0{,}26$ or $-1{,}54$ ◄

1.1.3 $8x^2 > 2x$

$\therefore 8x^2 - 2x > 0$

$\therefore 2x(4x - 1) > 0$

$\therefore x < 0$ or $x > \frac{1}{4}$ ◄



1.1.4 $2.2^{2x} - 9.2^x + 4 = 0$

$(2.2^x - 1)(2^x - 4) = 0$

$\therefore 2.2^x - 1 = 0$ or $2^x - 4 = 0$

$\therefore 2^x = \frac{1}{2}$ $\quad \therefore 2^x = 4$

$\therefore 2^x = 2^{-1}$ $\quad \therefore 2^x = 2^2$

$\therefore x = -1$ $\quad \therefore x = 2$ ◄

OR: $2.2^{2x} - 9.2^x + 4 = 0$

Let $2^x = k$

$\therefore 2k^2 - 9k + 4 = 0$

$\therefore (2k - 1)(k - 4) = 0$

$\therefore k = \frac{1}{2}$ or $k = 4$

$\therefore 2^x = 2^{-1}$ $\quad \therefore 2^x = 2^2$

$\therefore x = -1$ $\quad \therefore x = 2$ ◄

1.1.5 $\left(\sqrt{\sqrt{\frac{1}{x}} + 2}\right)^2 = \left(\frac{1}{\sqrt{x}}\right)^2$

$\therefore \sqrt{\frac{1}{x}} + 2 = \frac{1}{x}$

$\therefore \left(\sqrt{\frac{1}{x}}\right)^2 = \left(\frac{1}{x} - 2\right)^2$

$\therefore \frac{1}{x} = \frac{1}{x^2} - \frac{4}{x} + 4$

$\therefore x = 1 - 4x + 4x^2 \quad (\times \text{ by } x^2)$

$\therefore 4x^2 - 5x + 1 = 0$

$\therefore (4x - 1)(x - 1) = 0$

$\therefore x = \frac{1}{4}$ or $x \neq 1$

$\therefore x = \frac{1}{4}$ only ◄

Check

$x = 1$: LHS $= \sqrt{3}$ but RHS $= 1$ ✗

$x = \frac{1}{4}$: LHS $= 2 =$ RHS ✓

OR:

$\sqrt{\sqrt{\frac{1}{x}} + 2} = \frac{1}{\sqrt{x}}$

Let $\frac{1}{\sqrt{x}} = k \quad \left(\sqrt{\frac{1}{x}} = \frac{1}{\sqrt{x}}\right)$

$\therefore \left(\sqrt{k + 2}\right)^2 = (k)^2$

$\therefore k + 2 = k^2$

$\therefore k^2 - k - 2 = 0$

$\therefore (k - 2)(k + 1) = 0$

$\therefore k = 2$ or $k \neq -1 \quad \left(\sqrt{\frac{1}{x}} \neq -1\right)$

$\therefore \left(\sqrt{\frac{1}{x}}\right)^2 = (2)^2$

$\therefore \frac{1}{x} = 4$

$\therefore x = \frac{1}{4}$ ◄

1.2 $x = 2 + y \quad \ldots$ ❶

$5xy = x^2 + 6 \quad \ldots$ ❷

Substitute ❶ into ❷:

$5(2 + y)y = (2 + y)^2 + 6$

$\therefore 10y + 5y^2 = 4 + 4y + y^2 + 6$

$\therefore 4y^2 + 6y - 10 = 0$

$\therefore 2y^2 + 3y - 5 = 0$

$\therefore (2y + 5)(y - 1) = 0$

$\therefore y = -\frac{5}{2}$ or $y = 1$

If $y = -\frac{5}{2}$, then $x = 2 + \left(-\frac{5}{2}\right) = -\frac{1}{2}$

If $y = 1$, then $x = 2 + (1) = 3$

$\therefore$ **Solution:** $\left(-\frac{1}{2}; -\frac{5}{2}\right)$ or $(3; 1)$ ◄

OR:

$x = 2 + y$

$\therefore y = x - 2 \quad \ldots$ ❶

$5xy = x^2 + 6 \quad \ldots$ ❷

Substitute ❶ into ❷:

$5x(x - 2) = x^2 + 6$

$\therefore 5x^2 - 10x = x^2 + 6$

$\therefore 4x^2 - 10x - 6 = 0$

$\therefore 2x^2 - 5x - 3 = 0$

$\therefore (2x + 1)(x - 3) = 0$

$\therefore x = -\frac{1}{2}$ or $x = 3$

If $x = -\frac{1}{2}$, then $y = -\frac{1}{2} - 2 = -\frac{5}{2}$

If $x = 3$, then $y = 3 - 2 = 1$

$\therefore$ **Solution:** $\left(-\frac{1}{2}; -\frac{5}{2}\right)$ or $(3; 1)$ ◄

PATTERNS & SEQUENCES [25]

2.1.1 $(t + 10) + (t - 2) + (t + 4) + \ldots$ is an infinite GS

$\therefore \frac{t-2}{t+10} = \frac{t+4}{t-2}$ ...

In a geometric series $\frac{T_2}{T_1} = \frac{T_3}{T_2}$

$\therefore (t + 10)(t + 4) = (t - 2)^2$

$\therefore t^2 + 14t + 40 = t^2 - 4t + 4$

$\therefore 18t = -36$

$\therefore \mathbf{t = -2}$ ◄

2.1.2 The first three terms are: $8; -4; 2$

$a = 8$ and $r = -\frac{1}{2}$

$\therefore T_{25} = (8)\left(-\frac{1}{2}\right)^{24}$

$= (2^3)\left(\frac{1}{2}\right)^{24}$

$= (2^3)(2^{-24})$

$= 2^{-21}$

$= \left(\frac{1}{2}\right)^{21}$ ◄

2.1.3 $S_\infty = \frac{a}{1-r} = \frac{8}{1-\left(-\frac{1}{2}\right)} = \mathbf{\frac{16}{3}}$ ◄ ... $-1 < r < 1$

2.2.1 $T_{14} = 4(k + 13) - 1 = 4k + 51$

$T_6 = 4(k + 5) - 1 = 4k + 19$

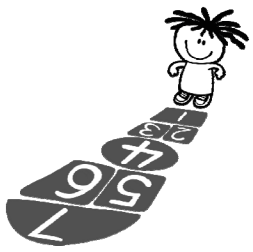
$\therefore \mathbf{T_{14} - T_6 = 32}$ ◄

OR:

$T_{14} - T_6 = 8d$

$= 8(4)$

$= \mathbf{32}$ ◄



2.2.2 $\sum_{p=k}^{117}(4p - 1) = 26\,675$

$n = 117 - k + 1 = 118 - k$

$a = 4k - 1$

$d = 4$

$S_n = \frac{n}{2}[2a + (n - 1)d] = 26\,675$

$\therefore \frac{118-k}{2}[2(4k - 1) + (117 - k)(4)] = 26\,675$

$\therefore (118 - k)[(4k - 1) + 2(117 - k)] = 26\,675$

$\therefore (118 - k)[4k - 1 + 234 - 2k] = 26\,675$

$\therefore (118 - k)(2k + 233) = 26\,675$

$\therefore 27\,494 + 3k - 2k^2 - 26\,675 = 0$

$\therefore 2k^2 - 3k - 819 = 0$

$\therefore (2k + 39)(k - 21) = 0$

$\therefore \mathbf{k = 21 \text{ only}} \left(k \neq -\frac{39}{2}\right)$ ◄

OR:

$n = 117 - k + 1 = 118 - k$

$a = 4k - 1$

$d = 4$

$l = 4(117) - 1 = 467$

$S_n = \frac{n}{2}[a + l] = 26\,675$

$\therefore \frac{118-k}{2}[(4k - 1) + (467)] = 26\,675$

$\therefore \frac{118-k}{2}[4k + 466] = 26\,675$

$\therefore (118 - k)[2k + 233] = 26\,675$

$\therefore -2k^2 + 3k + 27\,494 - 26\,675 = 0$

$\therefore 2k^2 - 3k - 819 = 0$

$\therefore (2k + 39)(k - 21) = 0$

$\therefore \mathbf{k = 21 \text{ only}} \left(k \neq -\frac{39}{2}\right)$ ◄

(You can also use the quadratic formula if you prefer.)

OR:

This method solves for k by finding the sum of 117 terms and subtracting the preceding terms.

$n = 117 \quad a = 3 \quad d = 4$

$l = T_{117} = 467$

$\therefore S_{117} = \frac{117}{2}[3 + 467] = 27\,495$

$n = k - 1 \quad a = 3 \quad d = 4$

$l = T_{k-1} = 4(k - 1) - 1 = 4k - 5$

$\therefore S_{k-1} = \frac{k-1}{2}[3 + 4k - 5]$

$= \frac{k-1}{2}[4k - 2]$

$= (k - 1)(2k - 1)$

$= 2k^2 - 3k + 1$

$\therefore S_{117} - S_{k-1} = 26\,675$

$\therefore 27\,495 - (2k^2 - 3k + 1) = 26\,675$

$\therefore 2k^2 - 3k - 819 = 0$

$\therefore (2k + 39)(k - 21) = 0$

$\therefore \mathbf{k = 21 \text{ only}} \left(k \neq -\frac{39}{2}\right)$ ◄

3.

36	71	104	135	164
35	33	31	29	
−2	−2	−2		

For a quadratic pattern:

a + b + c	4a + 2b + c	9a + 3b + c
	3a + b	5a + b
		2a

3.1 **The depth of the torpedo after 5 seconds is 164 m.** ◄

3.2 $2a = -2$ $\quad 3(-1) + b = 35$ $\quad -1 + 38 + c = 36$

$\therefore a = -1$ $\quad \therefore b = 38$ $\quad \therefore c = -1$

$\therefore$ **$T_n = -n^2 + 38n - 1$** ◄

3.3 $-2n + 38 = 0 \quad \therefore n = 19 \quad \therefore T_{19} = -19^2 + 38(19) - 1 = 360$

OR:

$-\frac{b}{2a} = -\frac{38}{2(-1)} = 19$ and $T_{19} = 360$ (see above)

OR:

$T_n = -(n^2 - 38n + 1) = -[(n - 19)^2 - 360] = -(n - 19)^2 + 360$

$\therefore$ **the maximum depth the torpedo reached is 360 m.** ◄

3.4 $T_3 = T_{19-16} = T_{19+16} = T_{35}$ (using symmetry)

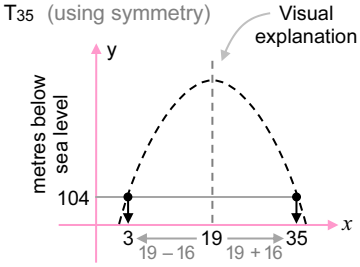
OR:

$-n^2 + 38n - 1 = 104$

$\therefore n^2 - 38n + 105 = 0$

$\therefore (n - 3)(n - 35) = 0$

$\therefore n = 3$ or $n = 35$



$\therefore$ **the torpedo was 104 m below sea level for the 2nd time after 35s** ◄

FUNCTIONS & GRAPHS [36]

4.1 $f(x) = \log_{\frac{1}{3}} x$

$\therefore f(3) = \log_{\frac{1}{3}} 3 = -1$

$\therefore$ **$t = -1$** ◄

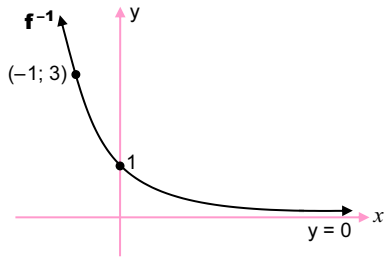
4.2 **A(1; 0)** $\left(\log_{\frac{1}{3}} 1 = 0\right)$ ◄

4.3 $x = \log_{\frac{1}{3}} y$

$\therefore$ **$f^{-1}(x) = y = \left(\frac{1}{3}\right)^x$** ◄ ... $c = \log_b a \Leftrightarrow a = b^c$

4.4 **$y = 0$** ◄

4.5

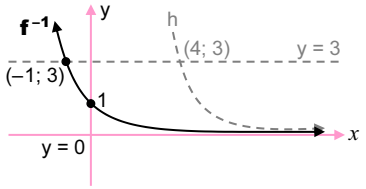


4.6 $h(x) = \left(\frac{1}{3}\right)^{x-5}$

$h(x) > 0$ for all values of x

$h(4) = \left(\frac{1}{3}\right)^{-1} = 3$

$\therefore$ **$0 < y < 3$** ◄

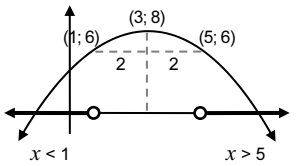


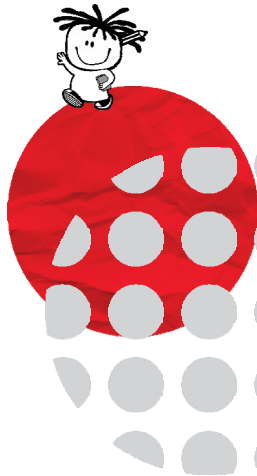
5.1 **$x \in \mathbb{R}, x \neq 3$** ◄

5.2 **$y \leq 8$** or **$y \in (-\infty; 8]$** ◄

5.3.1 **$3 < x \leq 5$** or **$x \in (3; 5]$** ◄

5.3.2 **$x < 1$** or **$x > 5$** or **$x \in (-\infty; 1) \cup (5; \infty)$** ◄





5.4 $y = a(x - 3)^2 + 8$

Substitute D(5; 6)

$\therefore 6 = a(5 - 3)^2 + 8$

$\therefore 6 = a(2)^2 + 8$

$\therefore 4a = -2$ and $a = -\frac{1}{2}$

$\therefore y = -\frac{1}{2}(x - 3)^2 + 8$

$= -\frac{1}{2}(x^2 - 6x + 9) + 8$

$= -\frac{1}{2}x^2 + 3x + \frac{7}{2}$ ◄

5.5 At M, $f(x) = 0$

$\therefore -\frac{1}{2}x^2 + 3x + \frac{7}{2} = 0$

$\therefore x^2 - 6x - 7 = 0$

$\therefore (x + 1)(x - 7) = 0$

$\therefore x = -1$ or $x = 7$

$\therefore$ **$x_M = -1$** ◄

OR: At M, $f(x) = 0$

$\therefore 0 = -\frac{1}{2}(x - 3)^2 + 8$

$\therefore (x - 3)^2 = 16$

$\therefore x - 3 = \pm 4$

$\therefore x = 3 - 4 = -1$ or $x = 3 + 4 = 7$

$\therefore$ **$x_M = -1$** ◄

At T, $g(x) = 0$

$\therefore \frac{-4}{x - 3} + 8 = 0$

$\therefore -4 + 8(x - 3) = 0$

$\therefore -4 + 8x - 24 = 0$

$\therefore 8x = 28$

$\therefore x_T = \frac{7}{2}$

$\therefore$ **MT $= \frac{7}{2} - (-1) = \frac{9}{2} = 4{,}5$ units** ◄

5.6 $f'(x) = -x + 3$

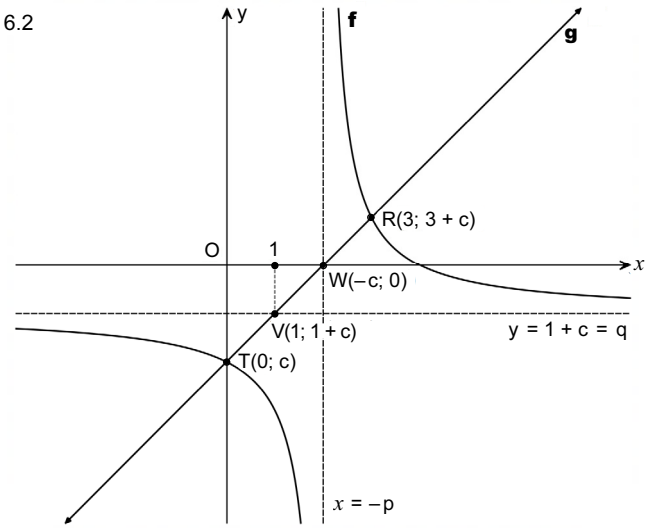
$\therefore m = f'(5) = -5 + 3 = -2$

$\therefore y - 6 = -2(x - 5)$ ... $y - y_1 = m(x - x_1)$

$\therefore$ **$y = -2x + 16$** ◄

6.1 **(–p; 0)** ◄

6.2



R(3; 3 + c) [g(3) = 3 + c] W(–c; 0) [g(–c) = 0]

V(1; 1 + c) [g(1) = 1 + c] T(0; c) [g(0) = c]

$\therefore$ p = c and q = 1 + c

$\therefore f(0) = \frac{a}{0+c} + 1 + c = c$

$\therefore \frac{a}{c} + 1 = 0$

$\therefore \frac{a}{c} = -1$

$\therefore a = -c$

$\therefore f(3) = \frac{-c}{3+c} + 1 + c = 3 + c$

$\therefore \frac{-c}{3+c} = 2$

$\therefore 2c + 6 = -c$

$\therefore 3c = -6$

$\therefore c = -2$

$\therefore a = -c = 2$

$\therefore p = c = -2$

$\therefore q = c + 1 = -1$

$\therefore \mathbf{f(x) = \frac{2}{x-2} - 1}$ ◄

OR:

y-intercept: $c = \frac{a}{p} + q$ . . . ❶

At $x = 1$: $1 + c = q$ . . . ❷

At $x = 3$: $3 + c = \frac{a}{3+p} + q$ . . . ❸

Subst. ❷ into ❶:

$c = \frac{a}{p} + 1 + c$

$\therefore \frac{a}{p} = -1$

$\therefore a = -p$

Substitute ❷ and a = –p into ❸:

$3 + c = \frac{-p}{3+p} + 1 + c$

$\therefore 2 = \frac{-p}{3+p}$

$\therefore 6 + 2p = -p$

$\therefore 3p = -6$

$\therefore p = -2$

$\therefore c = -2$ ($m_g = 1$ so OT = OW & $-c = -p$)

$\therefore a = -(-2) = 2$

$\therefore q = 1 + (-2) = -1$

$\therefore \mathbf{f(x) = \frac{2}{x-2} - 1}$ ◄

6.3 $g(x) = x - 2$

Shift g 1 unit right: $y = (x - 1) - 2 = x - 3$ ◄

or

Shift g 1 unit down: $y = (x - 2) - 1 = x - 3$ ◄

FINANCE, GROWTH & DECAY [15]

7.1 $A = P(1 + i)^n$

$= 40\,000\left(1 + \frac{7{,}8}{100}\right)^5$

= R58 230,94 ◄

7.2 Sarah made 24 quarterly deposits of R2 300 and then left the money in for an extra quarter.

$\mathbf{F_v = \frac{x\left[(1+i)^n - 1\right]}{i}}$ . . . with x = R2 300

n = 6 × 4 = 24

$i = \frac{5{,}8\%}{4} = \frac{5{,}8}{400} = \frac{0{,}058}{4}$

$F_v = \frac{2\,300\left[\left(1 + \frac{5{,}8}{400}\right)^{24} - 1\right]}{\frac{5{,}8}{400}} \times \left(1 + \frac{5{,}8}{400}\right)$

= R66 411,60 ◄

7.3.1 $900\,000\left(1 + \frac{6{,}8}{1\,200}\right)^3 = \frac{10\,000\left[1 - \left(1 + \frac{6{,}8}{1\,200}\right)^{-n}\right]}{\frac{6{,}8}{1\,200}}$ *

$\therefore \left(1 + \frac{6{,}8}{1\,200}\right)^{-n} = 0{,}4812\ldots$

$\therefore -n = \log_{\left(1 + \frac{6{,}8}{1\,200}\right)}(0{,}4812\ldots)$

$\therefore -n = -129{,}419\ldots$

$\therefore n = 129{,}419\ldots$

Rajesh made 130 payments in total, so he paid off his loan 133 months after the loan was granted. ◄

* $\mathbf{P_v = \frac{x\left[1 - (1+i)^{-n}\right]}{i}}$

OR:

$$900\,000 = \frac{10\,000\left[1-\left(1+\frac{6{,}8}{1\,200}\right)^{-n}\right]}{\frac{6{,}8}{1\,200}}\left(1+\frac{6{,}8}{1\,200}\right)^{-3}$$

$$\therefore \left(1+\frac{6{,}8}{1\,200}\right)^{-n} = 0{,}4812\ldots$$

$$\therefore -n = \log_{\left(1+\frac{6{,}8}{1\,200}\right)}(0{,}4812\ldots)$$

$$\therefore -n = -129{,}419\ldots$$

$$\therefore n = 129{,}419\ldots$$

Rajesh made 130 payments in total, so he paid off his loan 133 months after the loan was granted. ◄

7.3.2 Final payment:

$$\left[900\,000\left(1+\frac{6{,}8}{1\,200}\right)^{132} - 10\,000 \times \frac{\left(1+\frac{6{,}8}{1\,200}\right)^{129}-1}{\frac{6{,}8}{1\,200}}\right]\left(1+\frac{6{,}8}{1\,200}\right)$$

= R4 197,21 ◄

DIFFERENTIAL CALCULUS [33]

8.1 $$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{[-2(x+h)+3]-[-2x+3]}{h}$$

$$= \lim_{h\to 0}\frac{[-2x-2h+3]-[-2x+3]}{h}$$

$$= \lim_{h\to 0}\frac{-2h}{h}$$

$$= \lim_{h\to 0}-2$$

$$= -2$$ ◄

8.2.1 $$g(x) = -3x^4 + 2x$$

$$\therefore g'(x) = \mathbf{-12x^3 + 2}$$ ◄

8.2.2 $$y = \frac{2x^4+1}{x^2}$$

$$= \frac{2x^4}{x^2} + \frac{1}{x^2}$$

$$= 2x^2 + x^{-2}$$

$$\therefore \frac{dy}{dx} = 4x - 2x^{-3}$$

$$= \mathbf{4x - \frac{2}{x^3}}$$ ◄

9.1 $$f(x) = x^3 - 8x^2 + 5x + 14$$

$$f'(x) = 3x^2 - 16x + 5 = 0 \text{ at D and E}$$

$$\therefore (3x-1)(x-5) = 0$$

$$\therefore x = \frac{1}{3} \text{ or } x = 5$$

$$f(5) = 5^3 - 8(5)^2 + 5(5) + 14 = -36$$

$$\therefore \mathbf{E(5; -36)}$$ ◄

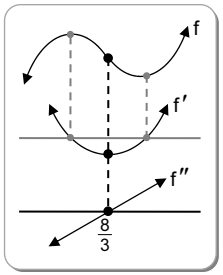
9.2 $$f''(x) = 6x - 16$$

f is concave down when $f''(x) < 0$.

$$f''(x) < 0$$

$$\therefore 6x - 16 < 0$$

$$\therefore \mathbf{x < \frac{8}{3}}$$ ◄



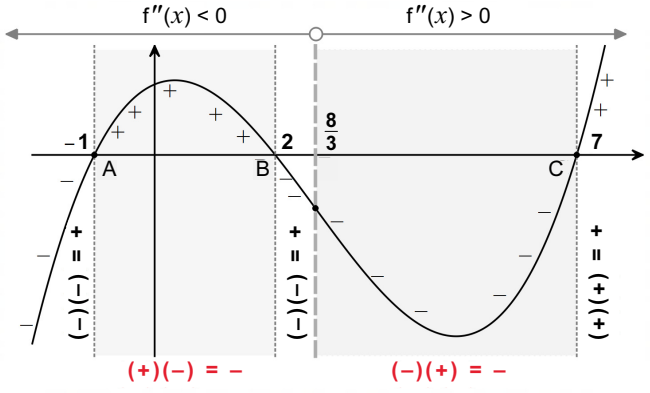
9.3 $$f(x) = x^3 - 8x^2 + 5x + 14$$

$$\therefore f(2) = 0$$

$$\therefore f(x) = (x-2)(x^2-6x-7)$$

$$= (x-2)(x-7)(x+1)$$

A(−1; 0), B(2; 0) and C(7; 0) are shown on the graph below.



$f(x)$ and $f''(x)$ must have opposite signs for $f(x).f''(x) < 0$.

$$\mathbf{-1 < x < 2 \text{ or } \frac{8}{3} < x < 7 \text{ or } x \in (-1; 2) \cup \left(\frac{8}{3}; 7\right)}$$ ◄

The + and − signs on the graph show where $y = f(x)$ is + or −.

$f''(x) < 0$ for $x < \frac{8}{3}$ and $f''(x) > 0$ for $x > \frac{8}{3}$

9.4 $f'(x) = 3x^2 - 16x + 5 = -11$

$\therefore 3x^2 - 16x + 16 = 0$

$\therefore (3x - 4)(x - 4) = 0$

$\therefore x = \frac{4}{3}$ or $x = 4$

$f\left(\frac{4}{3}\right) = \frac{238}{27}$

$\therefore y - \frac{238}{27} = -11\left(x - \frac{4}{3}\right)$

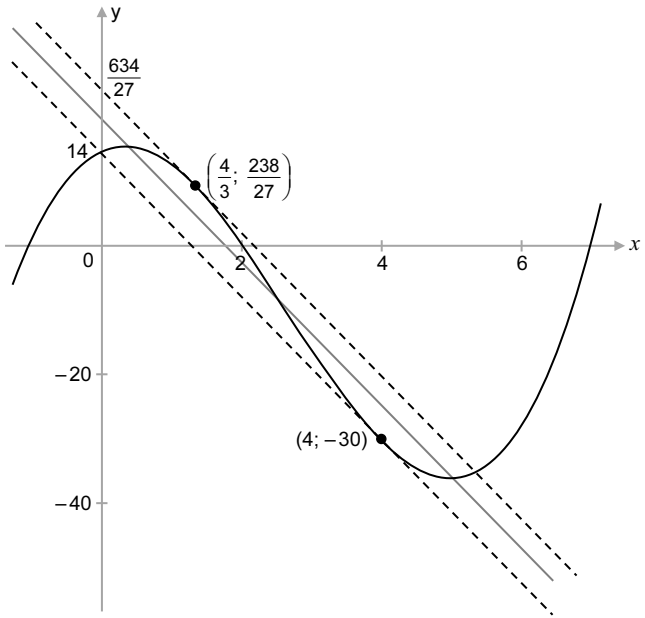
$\therefore y = -11x + \frac{634}{27}$ is a tangent to f

& $f(4) = -30$

$\therefore y - (-30) = -11(x - 4)$

$\therefore y = -11x + 14$ is a tangent to f

$\therefore$ **$14 < t < \frac{634}{27}$** $\left(\frac{634}{27} = 23\frac{13}{27} = 23,\dot{4}8\dot{1}\right)$ ◄



https://www.desmos.com/calculator/oyerfq4oof

10.1 $2x + 2h = 50$

$\therefore 2h = 50 - 2x$

$\therefore h = 25 - x$

$2\pi r = x$

$\therefore r = \frac{x}{2\pi}$

$V = \pi r^2 h$

$= \pi\left(\frac{x}{2\pi}\right)^2(25 - x)$

$= \left(\frac{\pi x^2}{4\pi^2}\right)(25 - x)$

$= \left(\frac{x^2}{4\pi}\right)(25 - x)$

$= \mathbf{\frac{25x^2}{4\pi} - \frac{x^3}{4\pi}}$ ◄

10.2 $V = \frac{25x^2}{4\pi} - \frac{x^3}{4\pi}$

$\therefore V' = \frac{50x}{4\pi} - \frac{3x^2}{4\pi}$

Maximum volume when $\frac{50x}{4\pi} - \frac{3x^2}{4\pi} = 0$

$\therefore 50x - 3x^2 = 0$

$\therefore x(50 - 3x) = 0$

$x \neq 0 \quad \therefore 3x = 50$

$\mathbf{x = \frac{50}{3}}$ **units** ◄ $\left(\frac{50}{3} = 16\frac{2}{3} = 16,\dot{6} \approx 16,67\right)$

PROBABILITY [16]

11.1

	JUICE	ENERGY DRINKS	TOTAL
Female	a = 48	b = 72	c = 120
Male	36	54	f = 90
Total	e = 84	d = 126	210

11.1.1 P(male) × P(preferring juice) = P(male and preferring juice)

$\therefore \frac{90}{210} \times \frac{e}{210} = \frac{36}{210}$ ...

P(A) . P(B) = P(A and B) for independent events

$\therefore \frac{3e}{7} = 36$

$\therefore \mathbf{e = 7 \times 12 = 84}$ ◄

11.1.2 This question can be interpreted in two different ways, so there are two possible solutions.

P(learner, chosen at random, is female and likes energy drinks)

$= \frac{72}{210}$

$= \frac{12}{35}$

or, the other interpretation:

P(female, chosen at random, likes energy drinks)

$= \frac{72}{120}$

$= \mathbf{\frac{3}{5}}$ ◄

11.2 P(person buys a cup of coffee on a non-rainy day) = x.

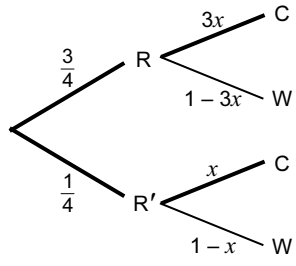
P(person buys a cup of coffee on any given day) = $\frac{7}{12}$

$\therefore \left(\frac{3}{4}\right)(3x) + \left(\frac{1}{4}\right)(x) = \frac{7}{12}$

$\therefore \frac{9x}{4} + \frac{x}{4} = \frac{7}{12}$

$\therefore \frac{10x}{4} = \frac{7}{12}$

$\therefore x = \frac{7}{30}$



$\frac{7}{30} \times 120 = 28$

$\therefore$ **28 cups of coffee will be sold on a non-rainy day.** ◄

OR:

Let x be the no. of cups of coffee bought on a non-rainy day.

P(person buys coffee on any given day)

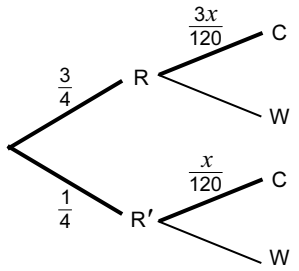
$= \frac{9x}{480} + \frac{x}{480}$

$= \frac{10x}{480}$

$= \frac{x}{48}$

$\therefore \frac{x}{48} = \frac{7}{12}$

$\therefore x = 28$



$\therefore$ **28 cups of coffee will be sold on a non-rainy day.** ◄

11.3.1

	1	2	3	4	5	6	7	8
1	A	B	–	–	–	–	–	–
2	–	A	B	–	–	–	–	–
3	–	–	A	B	–	–	–	–
4	–	–	–	A	B	–	–	–
5	–	–	–	–	A	B	–	–
6	–	–	–	–	–	A	B	–
7	–	–	–	–	–	–	A	B

The 8 runners can finish in 7! = 5 040 ways with Bongi (B) immediately after Andrew (A). ➤

11.3.2 Andrew first ... $5 \times 6!$

	1	2	3	4	5	6	7	8
1	A	–	–	B	–	–	–	–
2	A	–	–	–	B	–	–	–
3	A	–	–	–	–	B	–	–
4	A	–	–	–	–	–	B	–
5	A	–	–	–	–	–	–	B

Andrew second ... $4 \times 6!$

	1	2	3	4	5	6	7	8
1	–	A	–	–	B	–	–	–
2	–	A	–	–	–	B	–	–
3	–	A	–	–	–	–	B	–
4	–	A	–	–	–	–	–	B

Andrew third ... $3 \times 6!$

	1	2	3	4	5	6	7	8
1	–	–	A	–	–	B	–	–
2	–	–	A	–	–	–	B	–
3	–	–	A	–	–	–	–	B

Andrew fourth ... $2 \times 6!$

	1	2	3	4	5	6	7	8
1	–	–	–	A	–	–	B	–
2	–	–	–	A	–	–	–	B

Andrew fifth ... $1 \times 6!$

	1	2	3	4	5	6	7	8
1	–	–	–	–	A	–	–	B

$(1 + 2 + 3 + 4 + 5)(6!) = 15 \times 6!$

P(TWO or MORE runners finish after Andrew and before Bongi)

$= \frac{15 \times 6!}{8!}$

$= \mathbf{\frac{15}{56}}$ ➤

OR, using the complement:

8 runners can finish the race in 8! ways.

In half of these ways, Bongi finishes before Andrew.

$\therefore$ P(Bongi finishes before Andrew) $= \frac{1}{2}$

Andrew finishes immediately before Bongi in 7! ways. (from 11.3.1)

$\therefore$ P(Andrew finishes immediately before Bongi) $= \frac{7!}{8!} = \frac{1}{8}$

Andrew finishes before Bongi with ONE runner between them in $6 \times 6!$ ways.

	1	2	3	4	5	6	7	8
1	A	–	B	–	–	–	–	–
2	–	A	–	B	–	–	–	–
3	–	–	A	–	B	–	–	–
4	–	–	–	A	–	B	–	–
5	–	–	–	–	A	–	B	–
6	–	–	–	–	–	A	–	B

$\therefore$ P(Andrew finishes before Bongi with ONE runner between them) $= \frac{6 \times 6!}{8!} = \frac{3}{28}$

P(TWO or MORE runners finish the race after Andrew and before Bongi)

$= 1 - \frac{1}{2} - \frac{1}{8} - \frac{3}{28}$

$= \mathbf{\frac{15}{56}}$ ➤