DBO NOV 2025 VRAESTEL 2

STATISTIEK [20]

1.1 A $\simeq$ 331 397,20 ; B $\simeq$ –22 988,32

$\therefore$ **y = 331 397,20 – 22 988,32x** ◄

Die vgl.: **y = A + Bx**

1.2 Voorspelde verkoopprys: Stel x = 5 in

R216 455,60 ◄

1.3 **r $\simeq$ –0,95** ◄

Die voorspelling is geldig omdat die korrelasiekoëffisiënt sterk is.

1.4 Die beraamde gemiddelde jaarlikse verandering

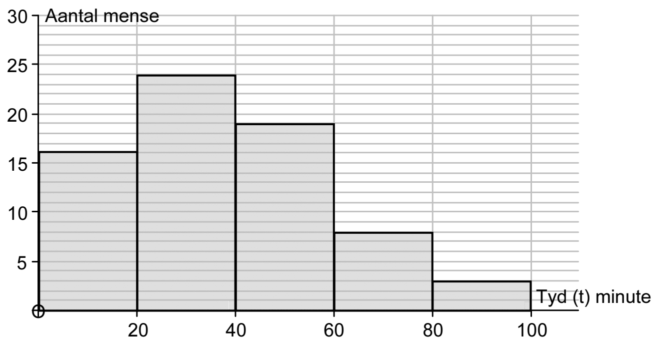
$= \frac{dy}{dx} = -$R22 988,32

$\therefore$ Die beraamde gemiddelde jaarlikse verlaging = **R22 988,32** ◄

2.1.1 **70 mense** ◄

2.1.2 67 – 40 = **27** ◄

2.1.3



2.1.4 Die data is regs (positief) skeef ◄

2.2 Totale aantal punte (9 spelers) = 108

Totale aantal punte aangeteken deur 8 spelers

= 11 + 14 + 19 + ... + 14

= 98

$\therefore$ Die 9[de] speler het 10 punte aangeteken . . . *108 – 98*

Die gemiddelde, $\bar{x}$ = 12 & die standaardafwyking, $\sigma \simeq$ 5,23

Die aantal spelers wat

óf minder as $\bar{x} - \sigma$, 6,77, dus 6 en minder

of meer as $\bar{x} + \sigma$, 17,23, dus 18 of meer aangeteken het

$\therefore$ d.w.s. Diegene wat 2, 19 en 20 aangeteken het

$\therefore$ **3 Spelers** ◄

ANALITIESE MEETKUNDE [39]

3.1 $QR^2 = (12 + 4)^2 + (2 + 6)^2$

$= 256 + 64$

$= 320$

$\therefore$ QR = $\sqrt{320} = \sqrt{64 \times 5}$ = **$8\sqrt{5}$ eenhede** ◄

3.2 $m_{QR} = \frac{2+6}{12+4} = \frac{8}{16} = \mathbf{\frac{1}{2}}$ ◄ . . . *= tan θ*

3.3 $\therefore \theta \simeq$ **26,57°** ◄

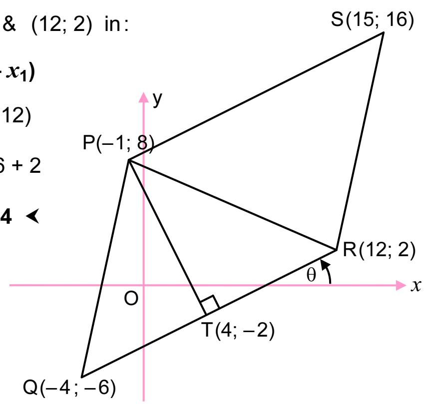
3.4 Stel $m_{QR} = \frac{1}{2}$ & (12; 2) in:

$y - y_1 = m(x - x_1)$

$\therefore y - 2 = \frac{1}{2}(x - 12)$

$\therefore y = \frac{1}{2}x - 6 + 2$

$\therefore$ **$y = \frac{1}{2}x - 4$** ◄



3.5 Deur inspeksie: **S(15; 16)** ◄

3.6 Vgl. van QR: $y = \frac{1}{2}x - 4$

Vgl. van PT: m = –2 & pt (–1; 8)

$\therefore y - 8 = -2(x + 1)$

$\therefore y = -2x + 6$

By T: $\frac{1}{2}x - 4 = -2x + 6$

$\therefore \frac{5}{2}x = 10$

$\left(\times\frac{2}{5}\right) \quad \therefore x = 4$

& $y = \frac{1}{2}(4) - 4 = -2$

$\therefore$ **T(4; –2)** ◄

Let wel:
Dit gebeur so dat T die middelpunt van QR is!
(Maar, dit sal nie reg wees om die middelpuntformule te gebruik nie)

3.7 Oppervlakte van ||[m] PQRS = QR × PT

$PT^2 = (4 + 1)^2 + (-2 - 8)^2$

$= 25 + 100$

$\therefore$ PT = $\sqrt{125} = 5\sqrt{5}$ eenhede

$\therefore$ Oppervlakte van PQRS = $8\sqrt{5} \times 5\sqrt{5}$

= **200 eenhede^2** ◄

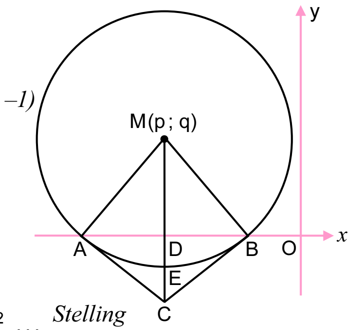
(OF: Bepaal die oppervlakte van ΔPQR, en verdubbel dit.)

4.1 p = **–6** ◄

4.2 DE = 1 eenheid . . . *pt E(6; –1)*

$\therefore$ ME = q + 1

$\therefore$ MA = q + 1 . . . *gelyke radiusse*



In reg∠[ige] ΔMDA:

$MA^2 = AD^2 + MD^2$ . . . *Stelling van Pyth*

$\therefore (q + 1)^2 = (q - 1)^2 + q^2$

$\therefore \cancel{q^2} + 2q + \cancel{1} = q^2 - 2q + \cancel{1} + \cancel{q^2}$

$\therefore 0 = q^2 - 4q$

$\therefore q(q - 4) = 0$

$\therefore$ (q = 0 of) **q = 4** ◄

4.3 Middelpunt $(a;\ b)$ is $(-6;\ 4)$

& radius $= q + 1 = 5$

$\therefore$ Vgl. van ⊙M: $(x+6)^2 + (y-4)^2 = \mathbf{25}$ ◂

4.4 Die radius, r = 5 eenhede

& M is 6 eenhede van die y-as $\quad \ldots\ p = x_E = -6$

$\therefore$ Tans is die afstand = 1 eenheid

$\therefore$ **Indien getransleer, is die minimum afstand = 3 eenhede** ◂

4.5 By A & B, is y = 0

& Vgl. van ⊙M: $(x+6)^2 + 16 = 25 \qquad$ OF: $(x+6)^2 = 9$

$\therefore\ (x+6)^2 = 9$

$\therefore\ x + 6 = \pm 3$

$\therefore\ x = -3$ of $-9 \qquad$ OF: 'Op die skets'

$\therefore$ **A(–9; 0) & B(–3; 0)** ◂

4.6 $m_{MB} = \dfrac{0-4}{-3+6} \qquad$ M(–6; 4) & B(–3; 0)

$= -\dfrac{4}{3}$

$\therefore\ m_{BC} = +\dfrac{3}{4} \qquad \ldots$ *raaklyn BC ⊥ rad MB*

& pt. B(–3; 0):

$\therefore$ Vgl. van BC: $y - 0 = \dfrac{3}{4}(x+3)$

$\therefore\ \mathbf{y = \dfrac{3}{4}x + \dfrac{9}{4}}$ ◂

4.7 $\mathbf{C\left(-6;\ -\dfrac{9}{4}\right)}$ ◂ $\quad \ldots\ x_E = -6$

& $y_C = \dfrac{3}{4}(-6) + \dfrac{9}{4} = -\dfrac{9}{2} + \dfrac{9}{4} = -\dfrac{9}{4}$

4.8 In ΔDBC: DB = 3 & DC = $\dfrac{9}{4}$ eenhede

$\therefore\ \tan D\hat{C}B = \dfrac{3}{\frac{9}{4}}$

$= \dfrac{4}{3}$

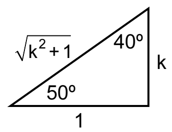
Daar is verskeie moontlike metodes.

$\therefore\ D\hat{C}B = 53{,}13^\circ$

$\therefore\ A\hat{C}B = \mathbf{106{,}26^\circ}$ ◂

TRIGONOMETRIE [50]

5.1 $\tan 50^\circ = \dfrac{k}{1}$



5.1.1 $\cos 40^\circ = \dfrac{\mathbf{k}}{\mathbf{\sqrt{k^2+1}}}$ ◂

5.1.2 $\dfrac{2\sin 25^\circ \cos 25^\circ}{-2 + 4\sin^2 25^\circ}$

$= \dfrac{\sin 2(25^\circ)}{-2(1 - 2\sin^2 25^\circ)} \qquad \ldots\ \sin 2A = 2\sin A \cos A$

$= \dfrac{\sin(50^\circ)}{-2\cos 2(25^\circ)} \qquad \ldots\ \cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A$

$= -\dfrac{1}{2} \cdot \dfrac{\sin 50^\circ}{\cos 50^\circ}$

$= -\dfrac{1}{2}\tan 50^\circ = \mathbf{-\dfrac{1}{2}k}$ ◂

5.1.3 $\sin 10^\circ = \sin(50^\circ - 40^\circ)$

$= \sin 50^\circ \cos 40^\circ - \cos 50^\circ \sin 40^\circ$

$= \sin 50^\circ \sin 50^\circ - \cos 50^\circ \cos 50^\circ$

$= \sin^2 50^\circ - \cos^2 50^\circ$

$= \left(\dfrac{k}{\sqrt{k^2+1}}\right)^2 - \left(\dfrac{1}{\sqrt{k^2+1}}\right)^2$

$= \dfrac{k^2}{k^2+1} - \dfrac{1}{k^2+1}$

$= \mathbf{\dfrac{k^2-1}{k^2+1}}$ ◂

5.2.1 $\dfrac{\sin(540^\circ + x) \cdot \cos(90^\circ + x)}{\sin(-x)}$

$= \dfrac{\sin(180^\circ + x) \cdot (\cancel{-\sin x})}{(\cancel{-\sin x})}$

$= \mathbf{-\sin x}$ ◂

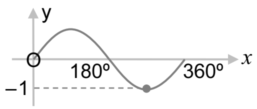
5.2.2 Stel vas wanneer $\sqrt{-\sin x}$ reëel sal wees (Sien 5.2.1):

Wanneer $0 < -\sin x \leq 1 \qquad -\sin x \neq 0$ want dit is in die noemer

$\times(-1)\quad \therefore\ 0 > \sin x \geq -1$

i.e. $\quad -1 \leq \sin x < 0$

$\therefore\ \mathbf{180^\circ < x < 360^\circ}$ ◂



6.1 **LK**

$= [\tan(180^\circ - x)](1 - \cos^2 x) + \cos^2 x = \dfrac{(\sin x - \cos x)(1 + \sin x \cdot \cos x)}{-\cos x}$

$= (-\tan x)(\sin^2 x) + \cos^2 x$

$= -\dfrac{\sin x}{\cos x}(\sin^2 x) + \cos^2 x$

$= \dfrac{-\sin^3 x + \cos^3 x}{\cos x}$

$= \dfrac{-(\sin^3 x - \cos^3 x)}{\cos x}$

$= \dfrac{(\sin x - \cos x)(\sin^2 x + \sin x \cos x + \cos^2 x)}{-\cos x}$

$= \dfrac{(\sin x - \cos x)(1 + \sin x \cos x)}{-\cos x}$

= **RK** ◂

6.2 $\cos^2 x - \sin^2 x = \dfrac{1}{2}\sin 2x - \cos^2 x \qquad \ldots$ *= d, die gemene verskil*

$\therefore\ 2\cos^2 x - \sin^2 x = \dfrac{1}{2}(2\sin x \cos x)$

$\therefore\ 2\cos^2 x - \sin^2 x = \sin x \cos x$

$\therefore\ 2\cos^2 x - \sin x \cos x - \sin^2 x = 0$

$\therefore\ (2\cos x + \sin x)(\cos x - \sin x) = 0$

$\therefore\ 2\cos x = -\sin x \quad$ or $\quad \cos x = \sin x$

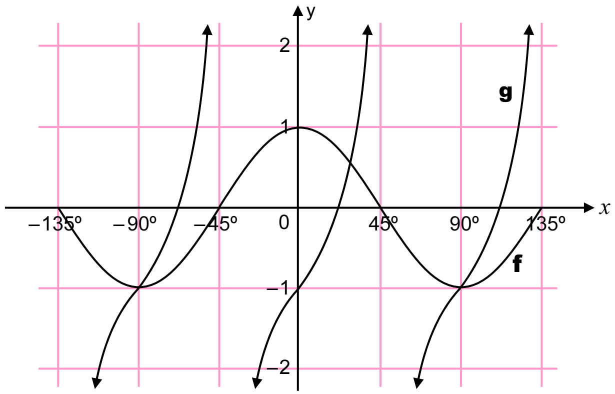
$(\div \cos x)\quad \therefore\ 2 = -\tan x \qquad$ Maar die konstante verskil = 0 $\therefore$ nie van toepassing nie

$\therefore\ \tan x = -2$

$\therefore\ \mathbf{x = 116{,}57^\circ + n(180^\circ),\ n \in \mathbb{Z}}$ ◂

7.1 **180º** ◂

7.2



7.3 $\quad f(x) = \cos 2x$

$\therefore\ h(x) = \cos 2(x + 45^\circ)$

$\therefore\ h(x) = \cos(2x + 90^\circ)$

$\therefore\ \mathbf{h(x) = -\sin 2x}$ ◀

7.4 $\quad \mathbf{y \in [-1;\ 1]}$ **of** $\mathbf{-1 \le y \le 1}$ ◀

7.5 $\quad (1 - \tan 2x)(\cos 2x) \ge 0$

$\therefore\ (\tan 2x - 1)(\cos 2x) \le 0$

$\therefore\ g(x).f(x) \le 0$

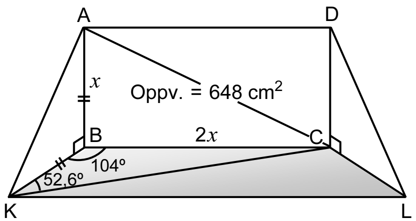
$\therefore\ \mathbf{0^\circ \le x \le 22\tfrac{1}{2}^\circ}$ **of** $\mathbf{112{,}5^\circ \le x \le 135^\circ}$ ◀

8.1 $\quad$ Laat AB = x cm; dan is BC = $2x$ cm

Oppv. van ABCD $= x \times 2x = 648\ \text{cm}^2$ $\quad \boxed{\mathbf{A = b \times \ell}}$

$\therefore\ x^2 = 324\ \text{cm}^2$

$\therefore\ \mathbf{AB = x = 18\ cm}$ ◀



8.2 $\quad$ In ΔABC: $AC^2 = 18^2 + 36^2 = 1\,620$ $\quad$ . . . *(Pyth)*

$\therefore\ \mathbf{AC \simeq 40{,}25\ cm}$ ◀

8.3 $\quad$ In ΔBKC: $\dfrac{KC}{\sin 104^\circ} = \dfrac{36}{\sin 52{,}6^\circ}$

$\therefore\ KC = \dfrac{36 \sin 104^\circ}{\sin 52{,}6^\circ}$

$\simeq \mathbf{43{,}97\ cm}$ ◀

8.4 $\quad$ In reg$\angle^{ige}$ ΔABK: $AK^2 = AB^2 + BK^2$ $\quad$ . . . *(Pyth)*

$= 18^2 + 18^2$ $\quad$ . . . *BK = AB = 18*

$= 648$

$\therefore$ In ΔAKC: $KC^2 = AK^2 + AC^2 - 2AK\ AC \cos K\hat{A}C$

$\therefore\ 43{,}97^2 = 648 + 1\,620 - 2.\sqrt{648}\sqrt{1\,620}\ \cos K\hat{A}C$

$\therefore\ 2\sqrt{648 \times 1\,620}\ \cos K\hat{A}C = 334{,}6391\ldots$

$= 0{,}1633\ldots$

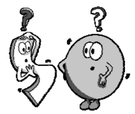
$\therefore\ \mathbf{K\hat{A}C = 80{,}60^\circ}$ ◀

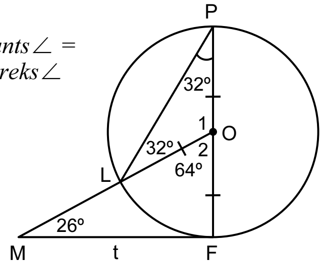
EUKLIDIESE MEETKUNDE [41]

9.1 $\quad$ Stelling

9.2.1 $\quad \hat{O}_2 = 2\hat{P}$ $\quad$ . . . *middelpunts$\angle$ = 2 $\times$ omtreks$\angle$*

$= \mathbf{64^\circ}$ ◀





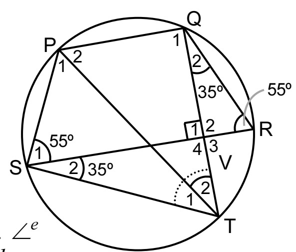
9.2.2 $\quad O\hat{F}M = 90^\circ$ $\quad$ . . . *middellyn $\perp$ raaklyn*

$\therefore\ \mathbf{\hat{M} = 26^\circ}$ ◀ $\quad$ . . . *som van $\angle^e$ in Δ*

10.1 $\quad Q\hat{T}S = Q\hat{R}S$ $\quad$ . . . *$\overset{\frown}{QS}$ onderspan*

$= 90^\circ - 35^\circ$ $\quad$ . . . *buite$\angle$ van Δ of som van $\angle^e$ van Δ*

$= \mathbf{55^\circ}$ ◀



10.2 $\quad \hat{S}_1 = \hat{R} = 55^\circ$

& $\ P\hat{Q}R = \hat{Q}_1 + 35^\circ$

$= 180^\circ - \hat{S}_1$

$= 125^\circ$ $\quad$ . . . *teenoorst. $\angle^e$ van kvh*

$\therefore\ P\hat{Q}R + \hat{R} = 180^\circ$

$\therefore$ **PQ || SR** ◀ $\quad$ . . . *ko-binne $\angle^e$ suppl.*

10.3 $\quad \hat{Q}_1 + \hat{V}_1 = 180^\circ$ $\quad$ . . . *ko-binne $\angle^e$; PQ || SR*

$\therefore\ \hat{Q}_1 = 90^\circ$

$\therefore$ **PT is 'n middellyn van die sirkel** ◀ $\quad$ . . . *omgekeerde $\angle$ in semi$\odot$*

11.1.1 $\quad$ In ΔCDA: $\dfrac{FD}{CF} = \dfrac{GA}{CG}$ $\quad$ . . . *eweredigh.stell.; AD || GF*

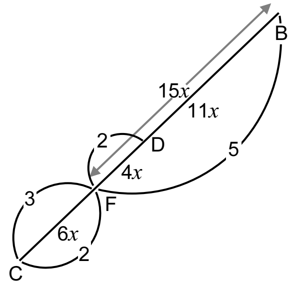
$= \mathbf{\dfrac{2}{3}}$ ◀

11.1.2 $\quad \Delta$BEF: $\dfrac{BA}{EA} = \dfrac{BD}{FD}$ $\quad$ . . . *eweredigh.stell.; AD || EF*

Laat CF = $6x$; dan is FD = $4x$ & BD = $11x$

$\therefore\ \dfrac{BA}{EA} = \dfrac{11x}{4x}$

$= \mathbf{\dfrac{11}{4}}$ ◀



11.1.3 $\quad \dfrac{\text{Oppv. van } \Delta GCF}{\text{Oppv. van } \Delta ACD} = \dfrac{\cancel{\tfrac{1}{2}} GC.CF \sin \cancel{\hat{C}}}{\cancel{\tfrac{1}{2}} AC.CD \sin \cancel{\hat{C}}}$

$= \dfrac{GC}{AC} \cdot \dfrac{CF}{CD}$

$= \dfrac{3}{5} \cdot \dfrac{3}{5}$

$= \dfrac{9}{25}$

$\therefore\ \dfrac{\text{Oppv. van } \Delta GCF}{\text{Oppv. van GFDA}} = \dfrac{9}{25 - 9} = \mathbf{\dfrac{9}{16}}$ ◀

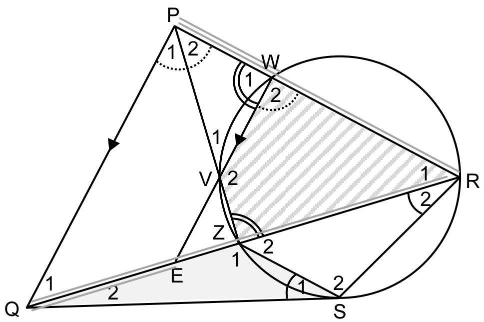
11.2.1

Let wel: Merk die 4 sye op die figuur; dit sal duidelik wees wat om te doen . . .

In ΔRPQ, $WE \parallel PQ$

$\therefore \dfrac{PR}{PW} = \dfrac{QR}{QE}$ . . . *eweredigh.stell.*

$\therefore \mathbf{PR = \dfrac{PW.QR}{QE}}$ ◂



11.2.2 $\Delta PQZ \ ||| \ \Delta RQP$

➧ $\dfrac{PQ}{RQ} = \dfrac{QZ}{PQ} \left(= \dfrac{PZ}{PR} = \dfrac{PQ}{QR} \right)$ . . . *ooreenstemmende sye*

$\therefore \mathbf{PQ^2 = RQ.QZ}$ ◂

11.2.3 In Δ^e QSZ & QRS

(1) $\hat{Q}_2$ is gemeen

(2) $\hat{S}_1 = \hat{R}_2$ . . . *raaklyn-koord stelling*

$\therefore \mathbf{\Delta QSZ \ ||| \ \Delta QRS}$ . . . $\angle\angle\angle$ ◂

11.2.4 Vanaf 11.2.2: $PQ^2 = RQ.QZ$ . . . ❶

& Vanaf 11.2.3: $\dfrac{QS}{QR} = \dfrac{QZ}{QS}$ . . . *ooreenstemmende sye*

$\therefore QS^2 = QR.QZ$ . . . ❷

❶ & ❷ $\therefore PQ^2 = QS^2$

$\therefore \mathbf{PQ = QS}$ ◂

11.2.5 Vanaf 11.2.1: $\dfrac{PW.QR}{QE} = PR$

$\therefore PW = \dfrac{QE.PR}{QR}$ . . . ❶

Vanaf 11.2.2: $\dfrac{PR}{QR} = \dfrac{PZ}{PQ}$ . . . ❷

❷ in ❶: $\therefore PW = \dfrac{QE.PZ}{PQ}$

Vanaf 11.2.4: $PQ^2 = QS^2 = QR.QZ$

$\therefore PQ = \sqrt{QR.QZ}$

$\therefore \mathbf{PW = \dfrac{QE.PZ}{\sqrt{QR.QZ}}}$ ◂