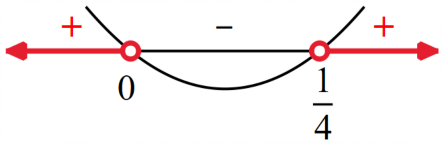
2025 Grade 12 DBE Maths Paper 1 November Possible Memo

1.1.1	$(x+5)(x-2)=0$ $\therefore x=-5 \text{ or } x=2$	(2)
1.1.2	$5x^2+2=-9x$ $\therefore 5x^2+9x+2=0 \qquad (a=5,\ b=9,\ c=2)$ $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ $=\dfrac{-9\pm\sqrt{(9)^2-4(5)(2)}}{2(5)}$ $=\dfrac{-9\pm\sqrt{41}}{10}$ $=-0,26 \text{ or } -1,54$	(4)
1.1.3	$8x^2>2x$ $\therefore 8x^2-2x>0$ $\therefore 2x(4x-1)>0$ $\therefore x<0 \text{ or } x>\dfrac{1}{4}$  	(4)
1.1.4	$2.2^{2x}-9.2^x+4=0$ $(2.2^x-1)(2^x-4)=0$ $\therefore 2.2^x-1=0 \text{ or } 2^x-4=0$ $\therefore 2^x=\dfrac{1}{2} \text{ or } 2^x=4$ $\therefore 2^x=2^{-1} \text{ or } 2^x=2^2$ $\therefore x=-1 \text{ or } x=2$ $2.2^{2x}-9.2^x+4=0$ Let $2^x=k$ $\therefore 2k^2-9k+4=0$ $\therefore (2k-1)(k-4)=0$ $\therefore k=\dfrac{1}{2} \text{ or } k=4$ $\therefore 2^x=2^{-1} \text{ or } 2^x=2^2$ $\therefore x=-1 \text{ or } x=2$	(4)

1.1.5	$\left(\sqrt{\sqrt{\frac{1}{x}}+2}\right)^2=\left(\frac{1}{\sqrt{x}}\right)^2$ $\therefore\sqrt{\frac{1}{x}}+2=\frac{1}{x}$ $\therefore\left(\sqrt{\frac{1}{x}}\right)^2=\left(\frac{1}{x}-2\right)^2$ $\therefore\frac{1}{x}=\frac{1}{x^2}-\frac{4}{x}+4$ $\therefore x=1-4x+4x^2 \quad \left(\times \text{ by } x^2\right)$ $\therefore 4x^2-5x+1=0$ $\therefore(4x-1)(x-1)=0$ $\therefore x=\frac{1}{4} \text{ or } x\neq 1$ $\therefore x=\frac{1}{4}$ only	$\sqrt{\sqrt{\frac{1}{x}}+2}=\frac{1}{\sqrt{x}}$ Let $\frac{1}{\sqrt{x}}=k$ $\therefore\left(\sqrt{k+2}\right)^2=(k)^2 \quad \left(\sqrt{\frac{1}{x}}=\frac{1}{\sqrt{x}}\right)$ $\therefore k+2=k^2$ $\therefore k^2-k-2=0$ $\therefore(k-2)(k+1)=0$ $\therefore k=2 \text{ or } k=-1$ $\therefore\left(\sqrt{\frac{1}{x}}\right)^2=(2)^2 \quad \left(\sqrt{\frac{1}{x}}\neq -1\right)$ $\therefore\frac{1}{x}=4$ $\therefore x=\frac{1}{4}$	
	Check $x=1$: LHS $=\sqrt{3}$ but RHS $=1$ ✗ $x=\frac{1}{4}$: LHS = 2 = RHS ✓		(5)

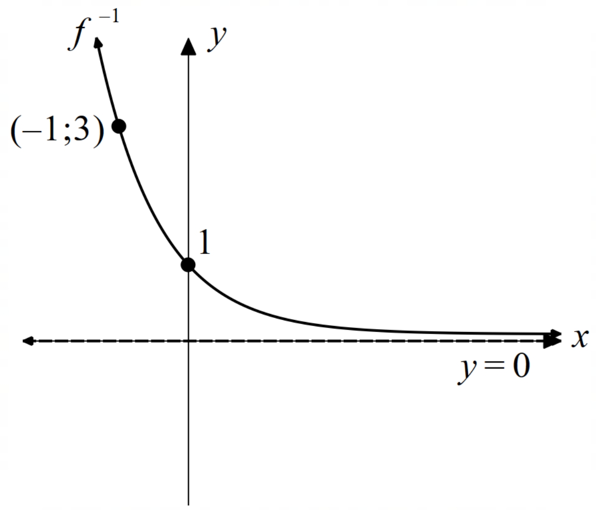
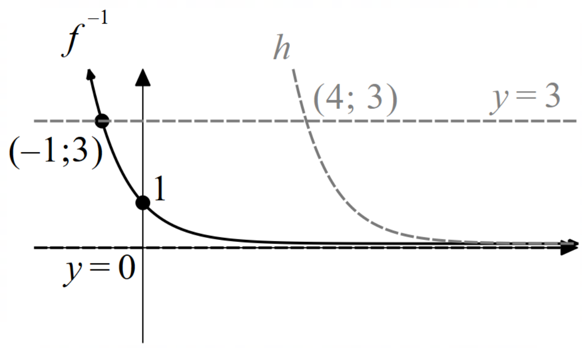
1.2	$x = 2 + y$ ① $5xy = x^2 + 6$ ② Substitute ① into ②: $5(2+y)y = (2+y)^2 + 6$ $\therefore 10y + 5y^2 = 4 + 4y + y^2 + 6$ $\therefore 4y^2 + 6y - 10 = 0$ $\therefore 2y^2 + 3y - 5 = 0$ $\therefore (2y+5)(y-1) = 0$ $\therefore y = -\frac{5}{2}$ or $y = 1$ If $y = -\frac{5}{2},\ x = -\frac{1}{2}$ If $y = 1,\ x = 3$ $\therefore x = -\frac{1}{2}$ and $y = -\frac{5}{2}$ or $x = 3$ and $y = 1$	$x = 2 + y$ $\therefore y = x - 2$ ① $5xy = x^2 + 6$ ② Substitute ① into ②: $5x(x-2) = x^2 + 6$ $\therefore 5x^2 - 10x = x^2 + 6$ $\therefore 4x^2 - 10x - 6 = 0$ $\therefore 2x^2 - 5x - 3 = 0$ $\therefore (2x+1)(x-3) = 0$ $\therefore x = -\frac{1}{2}$ or $x = 3$ If $x = -\frac{1}{2},\ y = -\frac{5}{2}$ If $x = 3,\ y = 1$ $\therefore x = -\frac{1}{2}$ and $y = -\frac{5}{2}$ or $x = 3$ and $y = 1$	(6)
			[25]

2.1.1	$(t+10)+(t-2)+(t+4)+\dots$ is an infinite GS $\therefore \frac{t-2}{t+10}=\frac{t+4}{t-2}$ $\therefore (t+10)(t+4)=(t-2)^2$ $\therefore t^2+14t+40=t^2-4t+4$ $\therefore 18t=-36$ $\therefore t=-2$ $\left(\text{When } t=-2,\ r=-\frac{1}{2} \text{ and } -1<-\frac{1}{2}<1\right)$	(3)
2.1.2	$a=8$ and $r=-\frac{1}{2}$ $\therefore \mathrm{T}_{25}=(8)\left(-\frac{1}{2}\right)^{24}$ $=(2^3)(2^{-24})$ $=2^{-21}$ $=\left(\frac{1}{2}\right)^{21}$	(3)
2.1.3	$\mathrm{S}_\infty=\frac{a}{1-r}=\frac{8}{1-\left(-\frac{1}{2}\right)}=\frac{16}{3}$	(2)

2.2.1	$\therefore T_{14} - T_6 = [4(k+13)-1] - [4(k+5)-1]$ $= [4k+51] - [4k+19]$ $= 32$	$T_{14} - T_6 = 8d$ $= 8(4)$ $= 32$	(2)
2.2.2	$\sum_{p=k}^{117}(4p-1) = 26\ 675$ $n = 117 - k + 1 = 118 - k$ $a = 4k - 1$ $d = 4$ $S_n = 26\ 675$ $s_{n=}\frac{n}{2}(2a + (n-1)d)$ $\therefore 26\ 675 = \frac{118-k}{2}[2(4k-1) + (117-k)(4)]$ $\therefore 26\ 675 = (118-k)[(4k-1) + 2(117-k)]$ $\therefore 26\ 675 = (118-k)[(4k-1) + 234 - 2k]$ $\therefore 26\ 675 = (118-k)(2k+233)$ $\therefore 26\ 675 = -2k^2 + 3k + 27\ 494$ $\therefore 27\ 494 + 3k - 2k^2 - 26\ 675 = 0$ $\therefore 2k^2 - 3k - 819 = 0$ $\therefore (2k+39)(k-21) = 0$ $\therefore k = 21$ only $\left(k \neq -\frac{39}{2}\right)$		

	$n = 117 - k + 1 = 118 - k$ $a = 4k - 1$ $d = 4$ $l = (4k-1) + (117-k)(4) = 467$ $S_n = \frac{n}{2}[a + l] = 26\ 675$ $\therefore \frac{118-k}{2}[(4k-1)+(467)] = 26\ 675$ $\therefore \frac{118-k}{2}[4k+466] = 26\ 675$ $\therefore (118-k)[2k+233] = 26\ 675$ $\therefore -2k^2 + 3k + 27\ 494 - 26\ 675 = 0$ $\therefore 2k^2 - 3k - 819 = 0$ $\therefore (2k+39)(k-21) = 0$ $\therefore k = 21$ only $\left(k \neq -\frac{39}{2}\right)$	
	$n = 117 \quad a = 3 \quad d = 4$ $l = T_{117} = [3 + 116(4)] = 467$ $S_{117} = \frac{117}{2}[3 + 467] = 27\ 495$ $n = k - 1 \quad a = 3 \quad d = 4$ $l = T_{k-1} = [3 + (k-2)(4)] = 4k - 5$ $S_{k-1} = \frac{k-1}{2}[3 + 4k - 5]$ $= \frac{k-1}{2}[4k-2]$ $= (k-1)(2k-1)$ $= 2k^2 - 3k + 1$ $S_{117} - S_{k-1} = 27\ 495 - (2k^2 - 3k + 1) = -2k^2 + 3k + 27\ 494 = 26\ 675$ $\therefore 2k^2 - 3k - 819 = 0$ $\therefore (2k+39)(k-21) = 0$ $\therefore k \neq -\frac{39}{2}$ or $k = 21$	(5)
		[15]

3.	$a+b+c$ 36 71 104 135 164 $3a+b$ 35 33 31 29 $2a$ -2 -2 -2	
3.1	The depth of the torpedo after 5 seconds is 164 m.	(2)
3.2	$2a=-2$ $\therefore a=-1$ $3(-1)+b=35$ $\therefore b=38$ $-1+38+c=36$ $\therefore c=-1$ $\therefore T_n=-n^2+38n-1$	(3)
3.3	$-\frac{b}{2a}=-\frac{38}{2(-1)}=19$ and $T_{19}=360$ $\therefore$ the maximum depth the torpedo reached is 360 m.	
	$-2n+38=0$ $\therefore n=19$ $\therefore T_{19}=-19^2+38(19)-1=360$ $\therefore$ the maximum depth the torpedo reached is 360 m.	
	$T_n=-(n^2-38n+1)=-\left[(n-19)^2-360\right]=-(n-19)^2+360$ $\therefore$ the maximum depth the torpedo reached is 360 m.	
	depth $=-\frac{\Delta}{4a}=-\frac{38^2-4(-1)(-1)}{4(-1)}=360$ $\therefore$ the maximum depth the torpedo reached is 360 m.	(3)
3.4	$T_3=T_{19-16}=T_{19+16}=T_{35}$ (using symmetry)	
	$-n^2+38n-1=104$ $\therefore n^2-38n+105=0$ $\therefore (n-3)(n-35)=0$ $\therefore n=3$ or $n=35$	
	$\therefore$ the torpedo was 104 m below sea level for the 2nd time after 35s.	(2)
		[10]

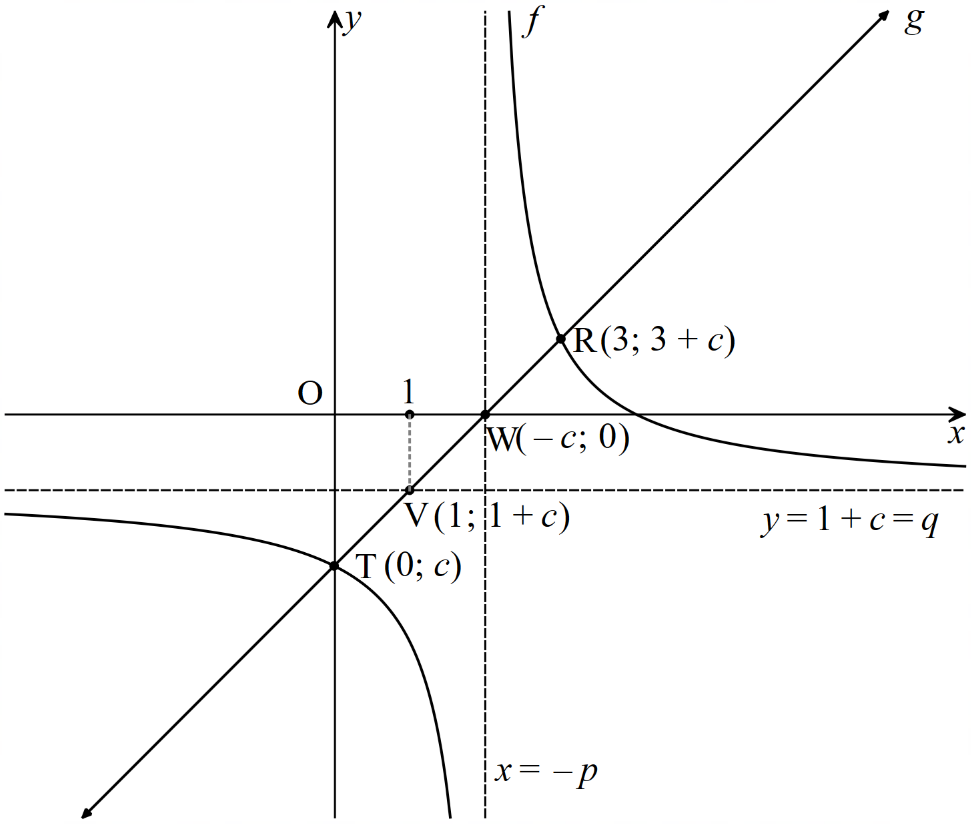
4.1	$f(x)=\log_{\frac{1}{3}} x$ $\therefore f(3)=\log_{\frac{1}{3}} 3=-1$ $\therefore t=-1$	(1)
4.2	A(1; 0) $\left(\log_{\frac{1}{3}} 1=0\right)$	(1)
4.3	$x=\log_{\frac{1}{3}} y$ $\therefore f^{-1}(x)=y=\left(\frac{1}{3}\right)^{x}$	(2)
4.4	$y=0$	(1)
4.5		(3)
4.6	$h(x)=\left(\frac{1}{3}\right)^{x-5}$ $h(x)>0$ for all values of x $h(4)=\left(\frac{1}{3}\right)^{-1}=3$ $\therefore 0<y<3$ 	(2)
		[10]

5.1	$x \in \mathbb{R},\ x \neq 3$	(1)
5.2	$y \leq 8,\ y \in \mathbb{R}$ or $y \in (-\infty; 8]$	(1)
5.3.1	$3 < x \leq 5,\ y \in \mathbb{R}$ or $x \in (3;\ 5]$	(2)
5.3.2	$x < 1$ or $x > 5,\ y \in \mathbb{R}$ or $x \in (-\infty; 1) \cup (5; \infty)$	(2)
5.4	$y = a(x-3)^2 + 8$ Substitute $D(5;\ 6)$ $\therefore 6 = a(5-3)^2 + 8$ $\therefore 6 = a(2)^2 + 8$ $\therefore 4a = -2$ and $a = -\frac{1}{2}$ $\therefore y = -\frac{1}{2}(x-3)^2 + 8$ $= -\frac{1}{2}(x^2 - 6x + 9) + 8$ $= -\frac{1}{2}x^2 + 3x + \frac{7}{2}$	(3)

5.5	At M, $f(x)=0$ $\therefore -\frac{1}{2}x^2+3x+\frac{7}{2}=0$ $\therefore x^2-6x-7=0$ $\therefore (x+1)(x-7)=0$ $\therefore x=-1$ or $x=7$ $\therefore x_{\text{M}}=-1$ At T, $g(x)=0$ $\therefore \frac{-4}{x-3}+8=0$ $\therefore -4+8(x-3)=0$ $\therefore -4+8x-24=0$ $\therefore 8x=28$ $\therefore x_{\text{T}}=\frac{7}{2}$ $\therefore \text{MT}=\frac{7}{2}-(-1)=\frac{9}{2}=4{,}5$ units	At M, $f(x)=0$ $\therefore 0=-\frac{1}{2}(x-3)^2+8$ $\therefore (x-3)^2=16$ $\therefore x-3=\pm 4$ $\therefore x=3-4=-1$ or $x=3+4=7$ $\therefore x_{\text{M}}=-1$	(6)
5.6	$f'(x)=-x+3$ $\therefore m=f'(5)=-5+3=-2$ $\therefore y-6=-2(x-5)$ $\therefore y=-2x+16$		(3)
			[18]

6.1 $\left(-p;0\right)$ **(1)**

6.2



$\mathrm{R}\left(3;3+c\right)$ $\left[g(3)=3+c\right]$ $\mathrm{W}\left(-c;0\right)$ $\left[g(-c)=0\right]$

$\mathrm{V}\left(1;1+c\right)$ $\left[g(1)=1+c\right]$ $\mathrm{T}\left(0;c\right)$ $\left[g(0)=c\right]$

$\therefore\ p=c$ and $q=1+c$

$$\therefore f(0)=\frac{a}{0+c}+1+c=c$$

$$\therefore \frac{a}{c}+1=0$$

$$\therefore \frac{a}{c}=-1$$

$$\therefore a=-c$$

$$\therefore f(3)=\frac{-c}{3+c}+1+c=3+c$$

$$\therefore \frac{-c}{3+c}=2$$

$$\therefore 2c+6=-c$$

$$\therefore 3c=-6$$

$$\therefore c=-2$$

$$\therefore a=-c=2$$

$$\therefore p=c=-2$$

$$\therefore\ q=c+1=-1$$

$$\therefore f(x)=\frac{2}{x-2}-1$$

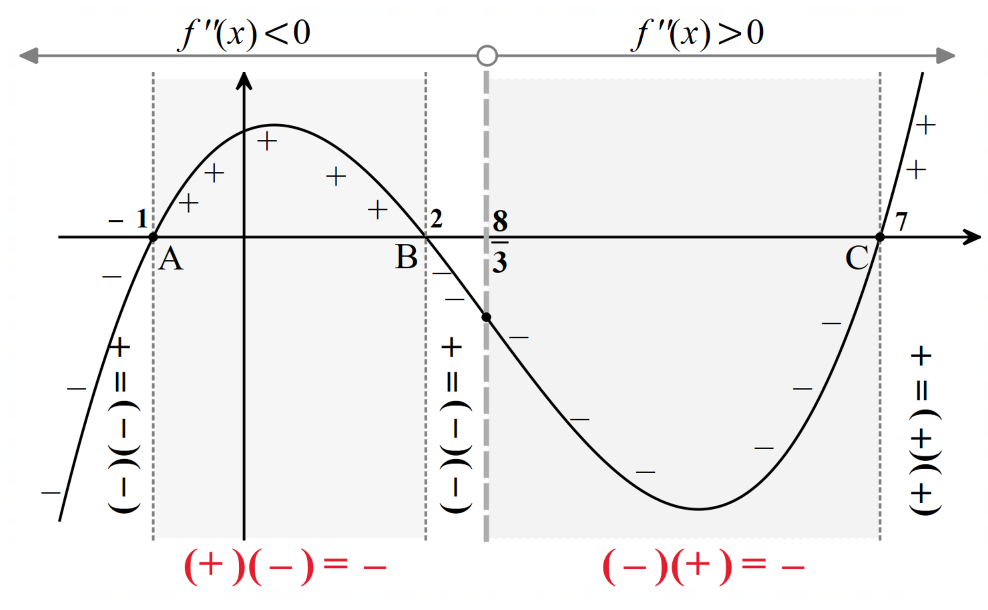
(5)

<table>
<tr><td>6.2</td><td colspan="2">y-intercept: $c=\frac{a}{p}+q$ ①
At $x=1$: $1+c=q$ ②
At $x=3$: $3+c=\frac{a}{3+p}+q$ ③</td><td></td></tr>
<tr><td></td><td>Substitute ② into ①:
$c=\frac{a}{p}+1+c$
$\therefore \frac{a}{p}=-1$
$\therefore a=-p$</td><td>② and ③:
$3+c=\frac{-p}{3+p}+1+c$
$\therefore 2=\frac{-p}{3+p}$
$\therefore 6+2p=-p$
$\therefore 3p=-6$
$\therefore p=-2$
$\therefore c=-2$ $\left(m_g=1 \text{ so OT = OW \& } -c=-p\right)$
$\therefore a=-(-2)=2$
$\therefore q=1+(-2)=-1$</td><td></td></tr>
<tr><td></td><td colspan="2">$\therefore f(x)=\frac{2}{x-2}-1$</td><td>(5)</td></tr>
<tr><td>6.3</td><td colspan="2">$g(x)=x-2$
Shift g 1 unit right: $y=(x-1)-2=x-3$
or
Shift g 1 unit down: $y=(x-2)-1=x-3$</td><td>(2)</td></tr>
<tr><td></td><td colspan="2"></td><td>[8]</td></tr>
</table>

7.1	$A = P(1+i)^n$ $= 40\ 000\left(1+\frac{7,8}{100}\right)^5$ $= \text{R}\,58\ 230,94$	(2)
7.2	Sarah made 24 quarterly deposits of R2 300 and then left the money in for an extra quarter. $F_v = \frac{2\ 300\left[\left(1+\frac{5,8}{400}\right)^{24}-1\right]}{\frac{5,8}{400}} \times \left(1+\frac{5,8}{400}\right)$ $= \text{R}\,66\ 411,60$	
	Sarah made 24 quarterly deposits of R2 300 and then left the money in for an extra quarter. Assume that she paid at the end of the final quarter as well and then subtract the first payment. $F_v = \frac{2\ 300\left[\left(1+\frac{5,8}{400}\right)^{25}-1\right]}{\frac{5,8}{400}} - 2\ 300$ $= \text{R}\,66\ 411,60$	(4)
7.3.1	$900\ 000\left(1+\frac{6,8}{1200}\right)^3 = \frac{10\ 000\left[1-\left(1+\frac{6,8}{1200}\right)^{-n}\right]}{\frac{6,8}{1200}}$ $\therefore \left(1+\frac{6,8}{1200}\right)^{-n} = 0,4812...$ $\therefore -n = \log_{\left(1+\frac{6,8}{1200}\right)}(0,4812...)$ $\therefore -n = -129,419...$ $\therefore n = 129,419...$ Rajesh made 130 payments in total, so he paid off his loan 133 months after he took it out.	

7.3.1	$$900\ 000=\left(\frac{10\ 000\left[1-\left(1+\frac{6,8}{1200}\right)^{-n}\right]}{\frac{6,8}{1200}}\right)\left(1+\frac{6,8}{1200}\right)^{-3}$$ $$\therefore\left(1+\frac{6,8}{1200}\right)^{-n}=0,4812...$$ $$\therefore -n=\log_{\left(1+\frac{6,8}{1200}\right)}(0,4812...)$$ $$\therefore -n=-129,419...$$ $$\therefore n=129,419...$$ Rajesh made 130 payments in total, so he paid off his loan 133 months after he took it out.	(5)
7.3.2	Final payment: $$\left[900\ 000\left(1+\frac{6,8}{1200}\right)^{132}-\frac{10\ 000\left[\left(1+\frac{6,8}{1200}\right)^{129}-1\right]}{\frac{6,8}{1200}}\right]\left(1+\frac{6,8}{1200}\right)$$ $=\text{R}\,4\ 197,21$	(4)
		[15]

8.1	$f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\dfrac{[-2(x+h)+3]-[-2x+3]}{h}$ $=\lim_{h\to 0}\dfrac{[-2x-2h+3]-[-2x+3]}{h}$ $=\lim_{h\to 0}\dfrac{-2h}{h}$ $=\lim_{h\to 0}-2$ $=-2$	(4)
8.2.1	$g(x)=-3x^4+2x$ $\therefore g'(x)=-12x^3+2$	(2)
8.2.2	$y=\dfrac{2x^4+1}{x^2}$ $=\dfrac{2x^4}{x^2}+\dfrac{1}{x^2}$ $=2x^2+x^{-2}$ $\therefore \dfrac{dy}{dx}=4x-2x^{-3}$ $=4x-\dfrac{2}{x^3}$	(4)
		[10]

9.1	$f(x) = x^3 - 8x^2 + 5x + 14$ $f'(x) = 3x^2 - 16x + 5 = 0$ at D and E $\therefore (3x-1)(x-5) = 0$ $\therefore x = \frac{1}{3}$ or $x = 5$ $f(5) = 5^3 - 8(5)^2 + 5(5) + 14 = -36$ $\therefore \text{E}(5; -36)$	(4)
9.2	f is concave down when $f''(x) < 0$. $f''(x) < 0$ $\therefore 6x - 16 < 0$ $\therefore x < \frac{8}{3}$	(3)
9.3	$f(x) = x^3 - 8x^2 + 5x + 14$ $\therefore f(2) = 0$ $\therefore f(x) = (x-2)(x^2 - 6x - 7)$ $= (x-2)(x-7)(x+1)$ A(– 1; 0), B(2; 0) and C (7; 0) are shown on the graph below.  $f(x)$ and $f''(x)$ must have opposite signs for $f(x) \cdot f''(x) < 0$. $-1 < x < 2$ or $\frac{8}{3} < x < 7$ or $x \in (-1;2) \cup \left(\frac{8}{3};7\right)$ The + and – signs on the graph show where $y = f(x)$ is + or – . $f''(x) < 0$ for $x < \frac{8}{3}$ and $f''(x) > 0$ for $x > \frac{8}{3}$	(4)

9.4	$f'(x)=3x^2-16x+5=-11$ $\therefore 3x^2-16x+16=0$ $\therefore (3x-4)(x-4)=0$ $\therefore x=\frac{4}{3}$ or $x=4$ $f\left(\frac{4}{3}\right)=\frac{238}{27}=8,\dot{8}1\dot{4}$ and $y=-11x+\frac{634}{27}$ is a tangent to *f*. $f(4)=-30$ and $y=-11x+14$ $\therefore 14<t<23\frac{13}{27}$ $\left(\frac{634}{27}=23\frac{13}{27}=23,\dot{4}8\dot{1}\right)$	(6)
	$x^3-8x^2+5x+14=-11x+t$ $x^3-8x^2+16x+14=t$ 3 distinct points lie between the turning points $\therefore 3x^2-16x+16=0$ $\therefore (3x-4)(x-4)=0$ $\therefore x=\frac{4}{3}$ or $x=4$ $t=f\left(\frac{4}{3}\right)=\frac{238}{27}=8,\dot{8}1\dot{4}$ $t=f(4)=-30$ $\therefore 14<t<23\frac{13}{27}$ $\left(\frac{634}{27}=23\frac{13}{27}=23,\dot{4}8\dot{1}\right)$	
		[17]

10.1	$2x+2h=50$ $\therefore 2h=50-2x$ $\therefore h=25-x$ $2\pi r=x$ $\therefore r=\dfrac{x}{2\pi}$ $\mathrm{V}=\pi r^2 h$ $=\pi\left(\dfrac{x}{2\pi}\right)^2(25-x)$ $=\left(\dfrac{\pi x^2}{4\pi^2}\right)(25-x)$ $=\left(\dfrac{x^2}{4\pi}\right)(25-x)$ $=\dfrac{25x^2}{4\pi}-\dfrac{x^3}{4\pi}$	(3)
10.2	$\mathrm{V}=\dfrac{25x^2}{4\pi}-\dfrac{x^3}{4\pi}$ $\therefore \mathrm{V}'=\dfrac{50x}{4\pi}-\dfrac{3x^2}{4\pi}$ Maximum volume when $\dfrac{50x}{4\pi}-\dfrac{3x^2}{4\pi}=0$ $\therefore 50x-3x^2=0$ $\therefore x(50-3x)=0$ $x\neq 0 \quad \therefore 3x=50$ $x=\dfrac{50}{3}$ units $\quad \left(\dfrac{50}{3}=16\dfrac{2}{3}=16,\dot{6}\approx 16,67\right)$	(3)
		[6]

11.1		JUICE	ENERGY DRINKS	**TOTAL**	
	Female	$a = 48$	$b = 72$	$c = 120$	
	Male	36	54	$f = 90$	
	Total	$\boldsymbol{e = 84}$	$\boldsymbol{d = 126}$	**210**	
11.1.1	$P(\text{Male}) \times P(\text{preferring juice}) = P(\text{Male and preferring juice})$ $\therefore \frac{90}{210} \times \frac{e}{210} = \frac{36}{210}$ $\therefore \frac{3e}{7} = 36$ $\therefore e = 7 \times 12 = 84$				(3)
11.1.2	This question is potentially ambiguous. P(learner, chosen at random, is female and likes energy drinks) $= \frac{72}{210}$ $= \frac{12}{35}$ or P(female, chosen at random, likes energy drinks) $= \frac{72}{120}$ $= \frac{3}{5}$ $= 0,6$				(3)

<table>
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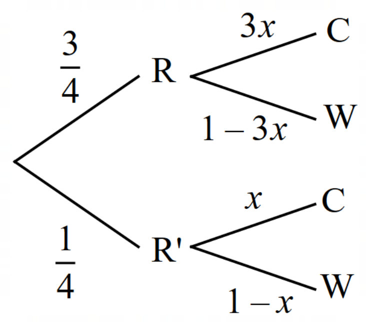
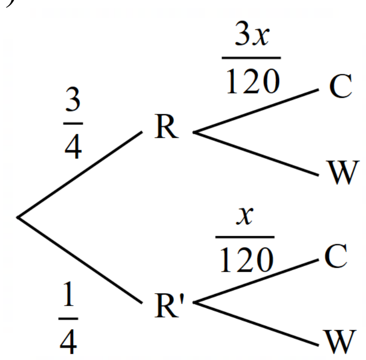
<td>11.2</td>
<td>
P(a person buys a cup of coffee on a rainy day) = x.
P(person buys a cup of coffee on any given day) $= \frac{7}{12}$
$\therefore \left(\frac{3}{4}\right)(3x) + \left(\frac{1}{4}\right)(x) = \frac{7}{12}$
$\therefore \frac{9x}{4} + \frac{x}{4} = \frac{7}{12}$
$\therefore \frac{10x}{4} = \frac{7}{12}$
$\therefore x = \frac{7}{30}$


$\frac{7}{30} \times 120 = 28$
$\therefore$ 28 cups of coffee will be sold on a non- rainy day.

or
Let x be the number of cups of coffee bought on a rainy day.

P(person buys coffee on any given day)
$= \frac{9x}{480} + \frac{x}{480}$
$= \frac{10x}{480}$
$= \frac{x}{48}$
$\therefore \frac{x}{48} = \frac{7}{12} = \frac{28}{48}$


$\therefore$ 28 cups of coffee will be sold on a non-rainy day.
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<td>11.3.1</td>
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<tr><td>1</td><td>A</td><td>B</td><td>–</td><td>–</td><td>–</td><td>–</td><td>–</td><td>–</td></tr>
<tr><td>2</td><td>–</td><td>A</td><td>B</td><td>–</td><td>–</td><td>–</td><td>–</td><td>–</td></tr>
<tr><td>3</td><td>–</td><td>–</td><td>A</td><td>B</td><td>–</td><td>–</td><td>–</td><td>–</td></tr>
<tr><td>4</td><td>–</td><td>–</td><td>–</td><td>A</td><td>B</td><td>–</td><td>–</td><td>–</td></tr>
<tr><td>5</td><td>–</td><td>–</td><td>–</td><td>–</td><td>A</td><td>B</td><td>–</td><td>–</td></tr>
<tr><td>6</td><td>–</td><td>–</td><td>–</td><td>–</td><td>–</td><td>A</td><td>B</td><td>–</td></tr>
<tr><td>7</td><td>–</td><td>–</td><td>–</td><td>–</td><td>–</td><td>–</td><td>A</td><td>B</td></tr>
</table>
Andrew (A) and Bongi (B) can finish with Andrew immediately in front of Bongi in $7! = 5\ 040$ different ways.
</td>
<td>(2)</td>
</tr>
<tr>
<td>11.3.2</td>
<td>
Andrew, followed by TWO runners, then Bongi ... $5 \times 6!$
<table>
<tr><th></th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th></tr>
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</table>
Andrew, followed by THREE runners, then Bongi ... $4 \times 6!$
<table>
<tr><th></th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th></tr>
<tr><td>1</td><td>A</td><td>–</td><td>–</td><td>–</td><td>B</td><td>–</td><td>–</td><td>–</td></tr>
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</table>
Andrew, followed by FOUR runners, then Bongi ... $3 \times 6!$
<table>
<tr><th></th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th></tr>
<tr><td>1</td><td>A</td><td>–</td><td>–</td><td>–</td><td>–</td><td>B</td><td>–</td><td>–</td></tr>
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<tr><td>3</td><td>–</td><td>–</td><td>A</td><td>–</td><td>–</td><td>–</td><td>–</td><td>B</td></tr>
</table>
Andrew, followed by FIVE runners, then Bongi ... $2 \times 6!$
<table>
<tr><th></th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th></tr>
<tr><td>1</td><td>A</td><td>–</td><td>–</td><td>–</td><td>–</td><td>–</td><td>B</td><td>–</td></tr>
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</table>
Andrew, followed by SIX runners, then Bongi ... $1 \times 6!$
<table>
<tr><th></th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th></tr>
<tr><td>1</td><td>A</td><td>–</td><td>–</td><td>–</td><td>–</td><td>–</td><td>–</td><td>B</td></tr>
</table>
$(1+2+3+4+5)(6!) = 15 \times 6!$
P(TWO or MORE runners finish after Andrew and before Bongi)
$= \dfrac{15 \times 6!}{8!}$
$= \dfrac{15}{56}$
</td>
<td></td>
</tr>
</table>

Andrew first . . . $5 \times 6!$

	1	2	3	4	5	6	7	8
1	A	–	–	B	–	–	–	–
2	A		–	–	B	–	–	–
3	A	–		–	–	B	–	–
4	A	–	–		–	–	B	–
5	A	–	–	–		–	–	B

Andrew second . . . $4 \times 6!$

	1	2	3	4	5	6	7	8
1	–	A	–	–	B	–	–	–
2	–	A	–	–	–	B	–	–
3	–	A	–	–	–	–	B	–
4	–	A	–	–	–	–	–	B

Andrew third . . . $3 \times 6!$

	1	2	3	4	5	6	7	8
1	–	–	A	–	–	B	–	–
2	–	–	A	–	–	–	B	–
3	–	–	A	–	–	–	–	B

Andrew fourth . . . $2 \times 6!$

	1	2	3	4	5	6	7	8
1	–	–	–	A	–	–	B	–
2	–	–	–	A	–	–	–	B

Andrew fifth . . . $1 \times 6!$

	1	2	3	4	5	6	7	8
1	–	–	–	–	A	–	–	B

$(1+2+3+4+5)(6!) = 15 \times 6!$

P(TWO or MORE runners finish after Andrew and before Bongi)

$= \dfrac{15 \times 6!}{8!}$

$= \dfrac{15}{56}$

Andrew FIRST, two runners other than Bongi, then anyone:

$= 1 \times 6 \times 5 \times 5!$

$= 5 \times 6!$

Anyone other than Andrew or Bongi, Andrew SECOND, two runners other than Bongi, then anyone:

$= 6 \times 1 \times 5 \times 4 \times 4!$

$= 4 \times 6!$

Any two other than Andrew or Bongi, Andrew THIRD, two runners other than Bongi, then anyone:

$= 6 \times 5 \times 1 \times 4 \times 3 \times 3!$

$= 3 \times 6!$

Any three other than Andrew or Bongi, Andrew FOURTH, two runners other than Bongi, then anyone:

$= 6 \times 5 \times 4 \times 1 \times 3 \times 2 \times 2!$

$= 2 \times 6!$

Any four other than Andrew or Bongi, Andrew FIFTH, two runners other than Bongi, then Bongi:

$= 6 \times 5 \times 4 \times 3 \times 1 \times 2 \times 1 \times 1$

$= 6!$

P(TWO or MORE runners finish after Andrew and before Bongi)

$= \frac{15 \times 6!}{8!}$

$= \frac{15}{56}$

Both options below make use of the complement.		
11.3.2	Andrew, followed by ONE runner, then Bongi . . . $6\times 6!$ P(Bongi finishing before Andrew, with all scenarios) = $\frac{1}{2}$ $\therefore$ P(Andrew followed by TWO or MORE runners, then Bongi) $=1-\frac{1}{2}-\frac{6\times 6!+7!}{8!}$ $=\frac{1}{2}-\frac{13\times 6!}{8!}$ $=$ $=\frac{28-13}{56}$ $=\frac{15}{56}$	
	Bongi first, followed by Andrew: 7 ways Bongi second, followed by Andrew: 6 ways Bongi third, followed by Andrew: 5 ways Bongi fourth, followed by Andrew: 4 ways Bongi fifth, followed by Andrew: 3 ways Bongi sixth, followed by Andrew: 2 ways Bongi seventh, followed by Andrew: 1 way $\therefore$ Bongi finishes before Andrew in 28 ways Andrew first, followed immediately by Bongi: 7 ways Andrew before Bongi, with 1 runner between them: 6 ways $\therefore 28+7+6=41$ ways that don't work for Andrew and Bongi Total number of ways they can finish in any order is 8! P(Andrew before Bongi with TWO or MORE runners between) $=\frac{8!-41\times 6!}{8!}$ $=\frac{15}{56}$	

Possible solution to question if interpreted differently		
	Andrew finishes in positions 1- 6 only (2 or more runners finish after him) Bongi finishes in positions 3 – 8 only (2 runners finish before him) A in position 1 or 2, B has 6 options . . . $2 \times 6 \times 6!$ A __ B __ __ __ __ __ __ A B __ __ __ __ __ A in position 3 – 6, B has 5 options . . . $4 \times 5 \times 6!$ __ __ A B __ __ __ __ $\therefore \frac{12 \times 6! + 20 \times 6!}{8!} = \frac{32}{56} = \frac{4}{7}$	**(4)**
		[16]