INTERNATIONAL SECONDARY CERTIFICATE EXAMINATION
MAY 2025

FURTHER STUDIES MATHEMATICS (STANDARD): PAPER I

MARKING GUIDELINES

Time: 2 hours | 200 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

QUESTION 1

Prove by mathematical induction that $10^{2n-1} + 1$ is divisible by 11 for all $n \in \mathbb{N}$.

For $k = 1$:	k=1 conclusion
$10^{2(1)-1} + 1 = 11$	Correct sub
The statement holds true for $n = 1$.	
Assume true for $n = k$ that $10^{2k-1} + 1$ is divisible by 11.	Assumption
Let $10^{2k-1} + 1 = M$ where $M \in \mathbb{N}$ is divisible by 11.	Introducing M
$10^{2k-1} = M - 1$	$M \in \mathbb{N}$ Manipulation
For $n = k + 1$: $10^{2(k+1)-1} + 1$	
$10^{2(k+1)-1} + 1$	n=k+1
$= 10^{2k+2-1} + 1$	Sub
$= 100.10^{2k-1} + 1$	Manipulation
$= 100.(M-1) + 1$	Answer divisible by 11
$= 100M - 100 + 1$	
$= 100M - 99$	
This is divisible by 11.	
Hence, the statement holds true for $n = k + 1$.	
By PMI $10^{2n-1} + 1$ is divisible by 11 for $n \in \mathbb{N}$	Conclusion

[13]

QUESTION 2

2.1 Solve for $x \in \mathbb{R}$.

(a) $2^{\ln|x|} = 0{,}25^y$ in terms of y, and simplify your answer.

$2^{\ln\lvert x\rvert} = 2^{-2y}$	Getting bases =
$\ln\lvert x\rvert = -2y$	Equating exponents
$\lvert x\rvert = e^{-2y}$	Writing in exponential form
$x = \pm e^{-2y}$	
$x = \pm\dfrac{1}{e^{2y}}$	Both answers

(4)

(b) $x^2 - 3|x| = 10$

$\lvert x\rvert^2 - 3\lvert x\rvert = 10$	Std form
$\lvert x\rvert^2 - 3\lvert x\rvert - 10 = 0$	Factors
$(\lvert x\rvert - 5)(\lvert x\rvert + 2) = 0$	Two equations
$\lvert x\rvert = 5$ or $\lvert x\rvert \neq -2$	disqualifying -2
$x = \pm 5$	Ans

Alternative:

$x^2 - 10 = 3\lvert x\rvert$	Isolating abs value
For $x \geq 0$:	Two options
$x^2 - 10 = 3x$	Factorisation
$x^2 - 3x - 10 = 0$	
$x = 5$; $x \neq -2$	+5
For $x < 0$	
$x^2 - 10 = -3x$	
$x^2 + 3x - 10 = 0$	-5
$x \neq 2$ or $x = -5$	Not equal +-2

(7)

(c) $\frac{x^2}{x-3} \le x-3$

$\frac{x^2}{x-3}-(x-3) \le 0$	Standard form
$\frac{x^2-(x-3)^2}{x-3} \le 0$	LCD Last term correctly on LCD
$\frac{x^2-x^2+6x-9}{x-3} \le 0$	
$\frac{6x-9}{x-3} \le 0$	Correct expression
$x \in \left[\frac{3}{2};3\right)$	Critical Values Inequality

(6)

2.2 If ln(2) = A and $\log_2(3) = B$, write the following in terms of A and B.

(a) $In\left(\frac{1}{4}\right)-\log_{\frac{1}{2}}(3)$

$= \ln 2^{-2}-(-\log_2(3))$	log law with base
$= -2\ln(2) + \log_2(3)$	log law with exp
$= -2A + B$	Ans

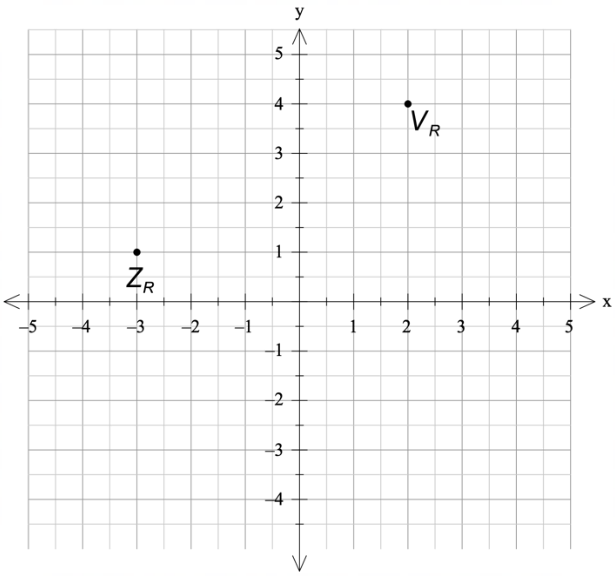
(4)

(b) $\log_2(e) + \log_2(12)-2$

$= \frac{1}{\ln(2)} + \log_2(4) + \log_2(3)-2$	log law base log law product log law exp log law =1 Ans
$= \frac{1}{A} + 2\log_2(2) + B-2$	
$= \frac{1}{A} + 2 + B-2$	
$= \frac{1}{A} + B$	

(6)

2.3 The diagram below shows two sets of coordinates V_R and Z_R.



(a) Given that the complex number $V_R = 2 + 4i$ has been represented on the axes above as (2 ; 4), write the co-ordinates of point Z_R in the form a + bi.

$Z_R = -3 + 1i$ — Z_R (2)

(b) In alternating current circuits V_R represents the voltage and Z_R the impedance while I_R represents the current. To calculate the voltage in a circuit the formula $V_R = I_R \times Z_R$ is used.

Determine the current in the circuit, in the form $a + bi$, with the values of V_R and Z_R represented on the axes above.

$$I = \frac{V_R}{Z_R}$$ Writing as quotient
Sub

$$\frac{2+4i}{-3+i}$$

$$= \frac{(2+4i)}{(-3+i)} \times \frac{-3-i}{-3-i}$$ x by conjugate

$$= \frac{-6-14i+4i^2}{9-i^2}$$

$i^2 = -1$

$$= \frac{-6-14i+4}{9+1}$$

Numerator

$$= \frac{-2-14i}{10}$$

Ans

$$= -\frac{1}{5} - \frac{7}{5}i$$ (6)

[35]

QUESTION 3

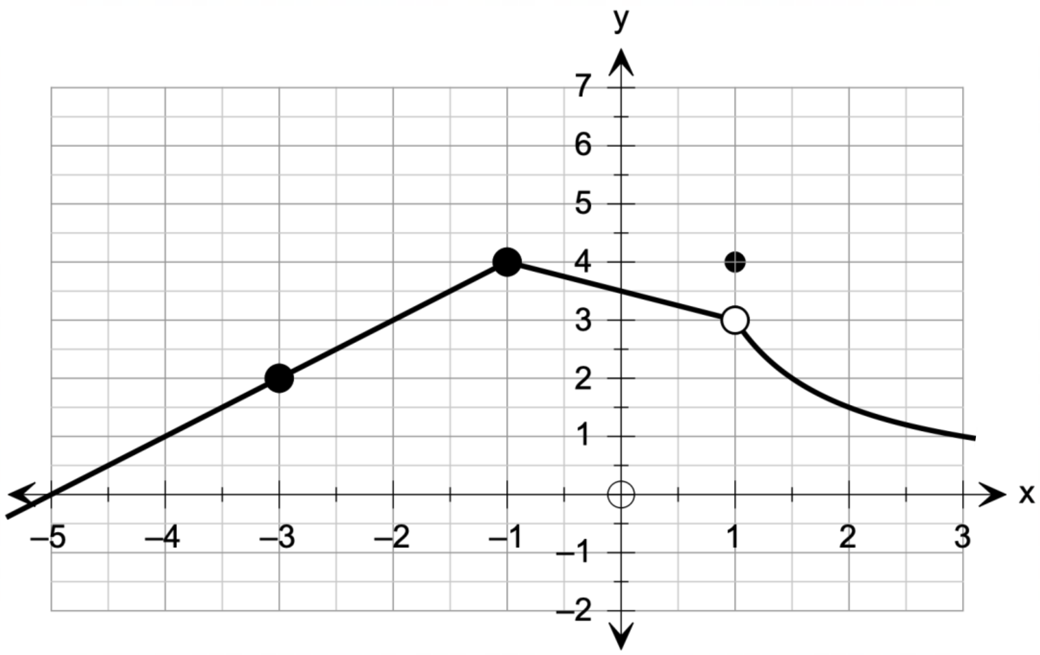
3.1 Paying careful attention to notation in justifying your reasoning, discuss, on the domain, the continuity of

$$f(x) = \begin{cases} \dfrac{2x+3}{x+1} & x < -1 \\ 2|x-1| + 3 & -1 \le x \le 2 \\ 3^x - 4 & x > 2 \end{cases}$$

$\lim\limits_{x \to -1} \dfrac{2x+3}{x+1}$ does not exist	estimate limit DNE conclusion
$\lim\limits_{x \to 2^-} (2\|x-1\|+3) = 5$ $\lim\limits_{x \to 2^+} (3^x - 4) = 5$ $f(2) = 5$ Continuous at $x = 2$	$\lim\limits_{x \to 2^-} (2\|x-1\| + 3)$ $\lim\limits_{x \to 2^+} (3^x - 4) = 5$ Answers to limits Conclusion (7)

3.2 Complete the diagram below such that the function has the following properties:

- Differentiable at $x = -3$.
- Not differentiable at $x = -1$.
- Continuous for $x \in \mathbb{R}, x \neq 1$.
- Removable discontinuity at $x = 1$.



Continuity at x = –3

Same gradient on right of –3

Sharp edge at x = –1

Function

Connection at x = 1

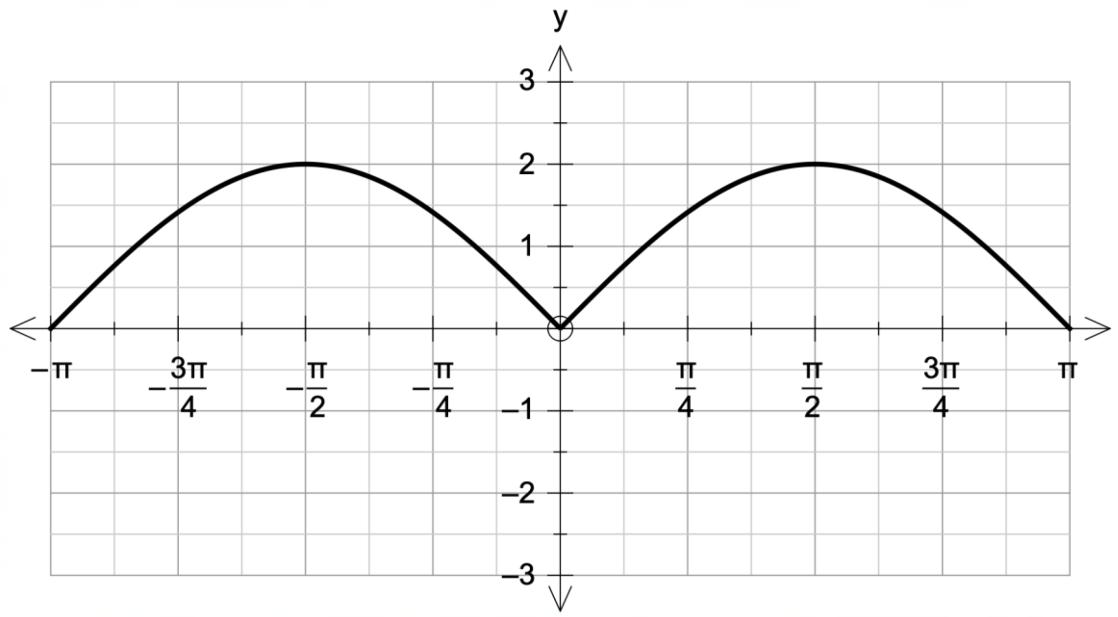
Removable discontinuity at x = 1.

(8)

[15]

QUESTION 4

4.1 Make a neat sketch of $y = 2\sin|x|$ for $x \in [-\pi;\pi]$

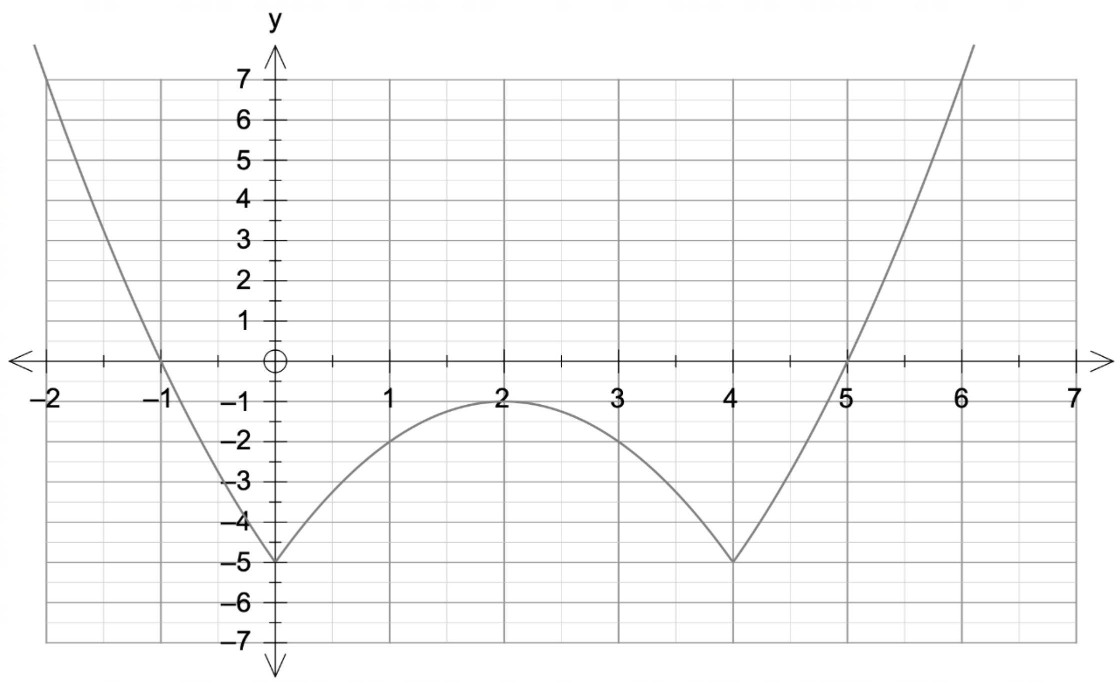


sin(x) graph
Amplitude =2
absolute value parts as reflection up

(4)

4.2 Drawn below is $g(x) = x^2 - 4x$. On the same axes, make a neat sketch of

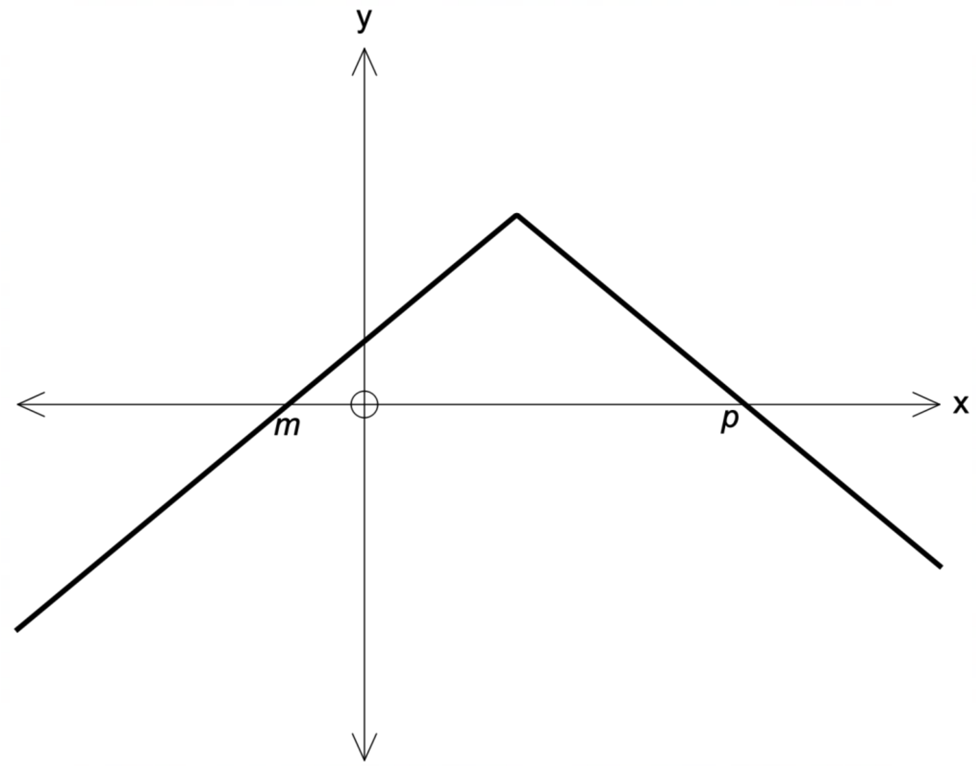
$y = |x^2 - 4x| - 5$



x-intercepts
y-intercept
turning point
upwards reflections of bottom part of parabola
shape of graph

(6)

4.3 Drawn below is the graph $f(x) = -|2x - k| + 3$ with x–intercepts m and p.



Give the values of x, in terms of k, m and p, for which $f(x).f'(x) < 0$.

Salient point: $\left(\frac{k}{2};3\right)$ — $x=k/2$

$x \in (-\infty;m)$ OR $\left(\frac{k}{2};p\right)$ — $(-\infty;m)$ — $\left(\frac{k}{2};p\right)$

(6)

[16]

QUESTION 5

5.1 Given: $f(x) = \sqrt{4x-8}$

Determine $f'(x)$ by using first principles.

$f(x+h) = \sqrt{4(x+h)-8}$	f(x+h)
$= \sqrt{4x+4h-8}$	
$f'(x) = \lim_{h \to 0} \dfrac{\sqrt{4x+4h-8}-\sqrt{4x-8}}{h}$	notation sub
$f'(x) = \lim_{h \to 0} \dfrac{\sqrt{4x+4h-8}-\sqrt{4x-8}}{h} \times \dfrac{\sqrt{4x+4h-8}+\sqrt{4x-8}}{\sqrt{4x+4h-8}+\sqrt{4x-8}}$	x by conjugate
$f'(x) = \lim_{h \to 0} \dfrac{(4x+4h-8)-(4x-8)}{h \times \left(\sqrt{4x+4h-8}+\sqrt{4x-8}\right)}$	Numerator Denominator
$f'(x) = \lim_{h \to 0} \dfrac{4h}{h \times \left(\sqrt{4x+4h-8}+\sqrt{4x-8}\right)}$	notation
$f'(x) = \lim_{h \to 0} \dfrac{4}{\sqrt{4x+4h-8}+\sqrt{4x-8}}$	form for sub
$f'(x) = \dfrac{4}{2\sqrt{4x-8}}$	sub
$f'(x) = \dfrac{2}{\sqrt{4x-8}}$	Ans

(10)

5.2 Determine $\frac{dy}{dx}$ for the following:

(a) $y = \frac{\cos^2(x)}{3x-1}$

$y = \frac{\cos^2(x)}{3x-1}$	quotient rule
$\frac{dy}{dx} = \frac{2\cos(x).(-\sin(x)).(3x-1)-3(\cos^2(x))}{(3x-1)^2}$	denominator
	Chain rule (m)
	$\frac{d}{dx}(\cos x) = \sin x$
	$\frac{d}{dx}(\cos^2(x)) = \mathbf{2}(\cos(x))$
	$\frac{d}{dx}(3x-1) = 3$

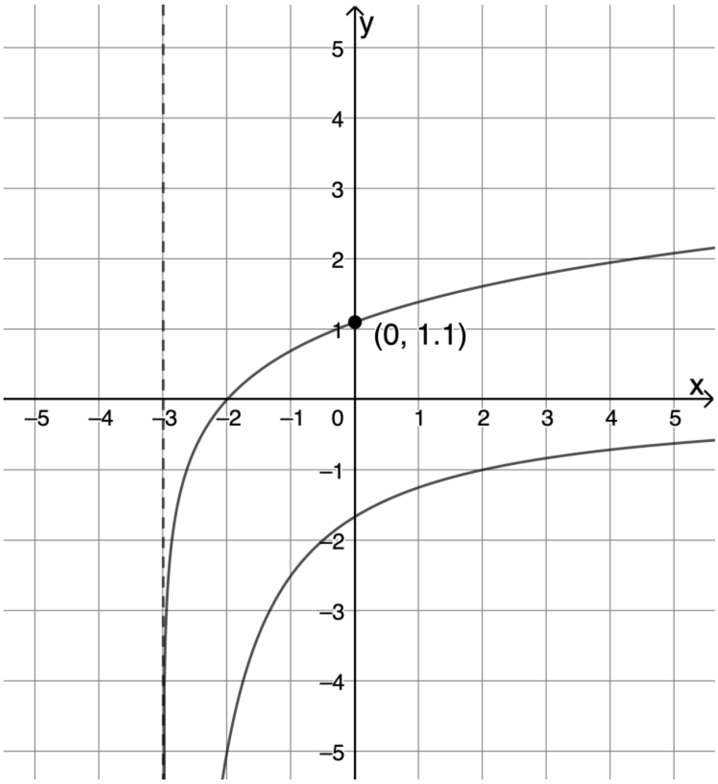
(6)

(b) $e^{3y} - 3yx^2 = 4x^2 - \cot(x)$

$\frac{d}{dx}(e^{3y} - 3yx^2) = \frac{d}{dx}(4x^2 - \cot(x))$	Taking derivative both sides
$3 \times \frac{d}{dx}e^{3y} - \left(3x^2 \times \frac{d}{dx} + 6yx\right) = 8x + \cos ec^2 x$	Right side
$3 \times \frac{d}{dx}e^{3y} - 3x^2 \times \frac{d}{dx} + 6yx = 8x + \cos ec^2 x$	$\frac{d}{dx}(e^{3y}) = 3 \times e^{3y}$
$3 \times \frac{d}{dx}e^{3y} - 3x^2 \times \frac{d}{dx} = 8x + \cos ec^2 x + 6yx$	$\frac{d}{dx}(3x^2 y) = \frac{3dy}{dx}.x^2 + 6xy$
$\frac{d}{dx}(3e^{3y} - 3x^2) = 8x + \cos ec^2 x + 6yx$	isolating dy/dx
	Taking out common
$\frac{d}{dx} = \frac{8x + \cos ec^2 x + 6yx}{3e^{3y} - 3x^2}$	Ans

(9)

5.3 The graph of $f(x) = -\dfrac{5}{x+3}$ is drawn below:



(a) Make a neat sketch of *g(x)* = *In(x + 3)* on the diagram above. Clearly indicate the intercepts with the axes and the asymptote on your diagram.
Space for working.

y-int (0;1,1)

x-intercept (-2;0)

Asymptote

Shape and graph (4)

(b) Lou-Anne wants to determine the minimum vertical distance between the graphs of *f(x)* and *g(x)*.

She calculates the x-value where the maximum distance occurs as $x = -8$, but when she wants to calculate the actual distance, it gives her a 'MATH ERROR' on her calculator. Explain why her answer can't be correct.

The graph of $g(x)$ is defined for $x > -3$, therefore the graph is not defined at $x = -8$.

(2)

(c) Determine the correct x-value where the vertical distance between the graphs is at a minimum.

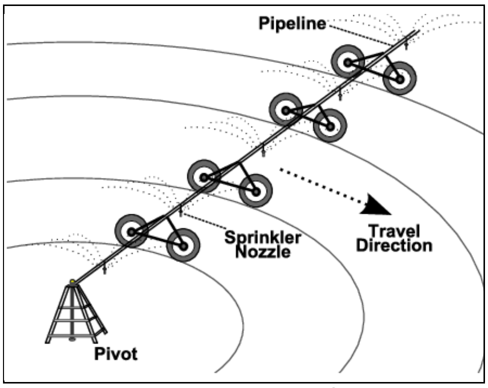
Distance=g(x)-f(x)	Vertical Difference
$= \ln(x+3) - \left(-\dfrac{5}{x+3}\right)$	Substitute
$= \ln(x+3) + \dfrac{5}{x+3}$	Derivative (m)
$D_x\left(\ln(x+3) + \dfrac{5}{x+3}\right)$	$\dfrac{d}{dx}(\ln(x+3)) = \dfrac{1}{x+3}$
$= \dfrac{1}{x+3} - \dfrac{5}{(x+3)^2}$	$\dfrac{d}{dx}\left(\dfrac{1}{x+3}\right) = -\dfrac{1}{(x+3)^2}$
$0 = \dfrac{1}{x+3} - \dfrac{5}{(x+3)^2}$	= 0
$0 = x + 3 - 5$	Simplification
$x = 2$	ans

(8)

[37]

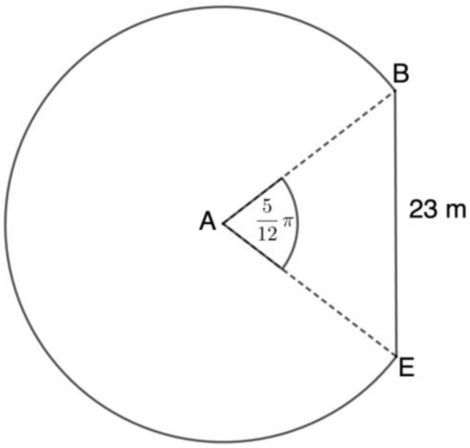
QUESTION 6

As the name suggests, centre pivots irrigate in a circular pattern around a central pivot point. Pivots can apply water, fertiliser, chemicals, and herbicides.



[Source: <https://www.researchgate.net/profile/Agnelo-Silva/publication/227350590/figure/fig1/AS:668998614142992@1536513107740/Basic-components-of-a-center-pivot-CP-system.png>]

At times a field is not big enough to have a full circular path. The farmer will then cut the circular path of the pivot with a chord. In this case, the irrigated field has a segment with a 23-metre chord cut from a circle. The chord subtends an angle of $\frac{5}{12}\pi$ between the radii.



6.1 Calculate the length of the radius of the field.

$$\frac{11,5}{r} = \sin\left(\frac{5\pi}{24}\right)$$

$$\frac{11,5}{\sin\left(\frac{5\pi}{24}\right)} = r$$

$$18,89 = r$$

Trig ratio
Halve angle
halve side
Ans

(4)

Alternative:

$$A\hat{E}B = \frac{\pi - \frac{5}{12}\pi}{2} = \frac{7}{24}\pi$$

$$\frac{r}{\sin\left(\frac{7}{24}\pi\right)} = \frac{23}{\sin\left(\frac{5}{12}\pi\right)}$$

$$r = \frac{23\sin\left(\frac{7}{24}\pi\right)}{\sin\left(\frac{5}{12}\pi\right)}$$

$$r = 18{,}89$$

Angle
Sine rule
Sub correct
Ans

6.2 Determine the area of the field.

Area of full circle – Area of cut off segment

$$= \pi r^2 - \left(\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin(\theta)\right)$$

$$= \pi(18{,}89)^2 - \left(\frac{1}{2}(18.89)^2 \times \frac{5\pi}{12} - \frac{1}{2}(18.89)^2\sin\left(\frac{5\pi}{12}\right)\right)$$

$$= 1\,059{,}81m^2$$

subtract
Full circle
Area of cut of seg
Sub
Ans

Alternative:

Angle in major sector $= 2\pi - \frac{5\pi}{12}$

$$= \frac{19\pi}{12}$$

$$Area = \frac{1}{2} \times 18.89^2 \times \frac{19\pi}{12} + \frac{1}{2} \times 18.89^2 \times \sin\left(\frac{5\pi}{12}\right)$$

$1\,059{,}81m^2$

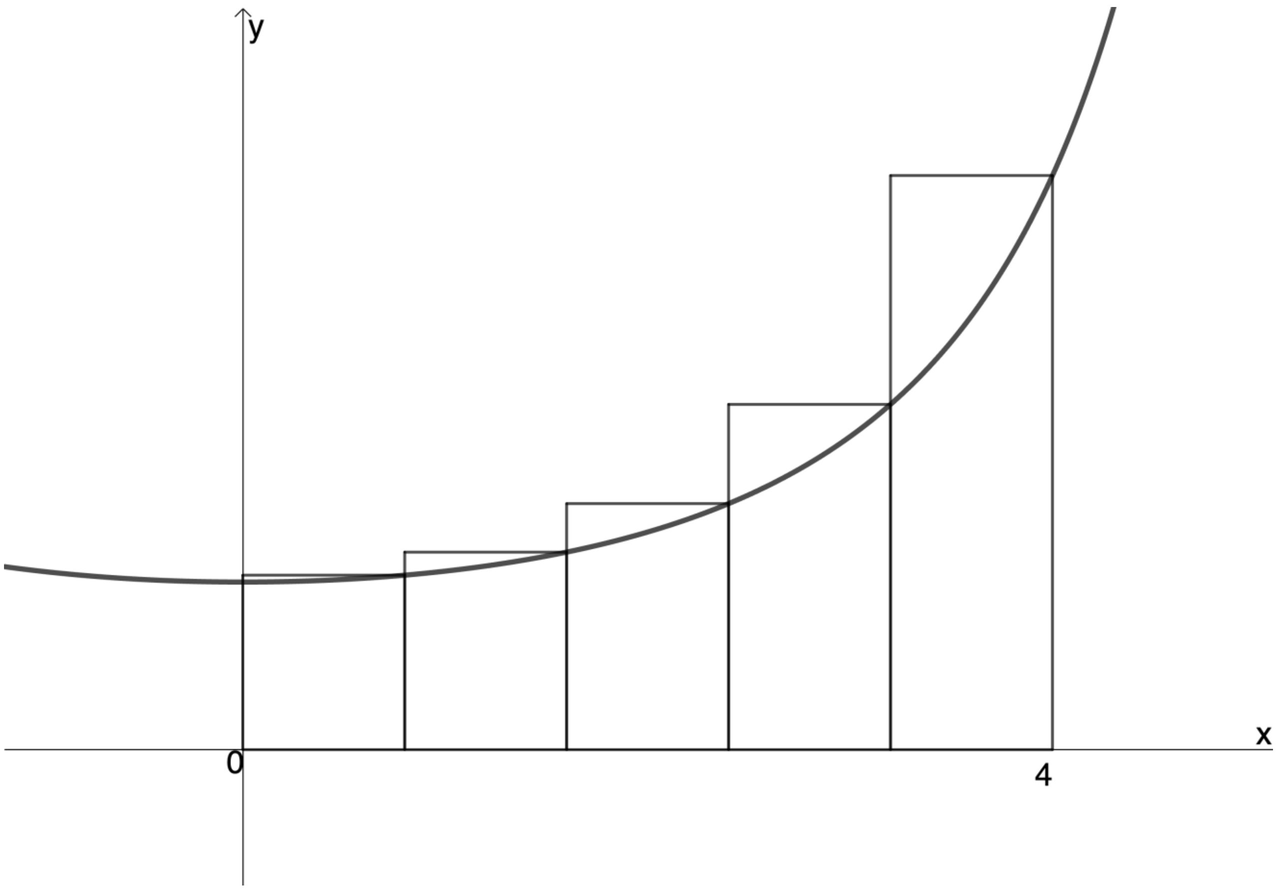
subtract angle
Angle answer
Area of sector
Area of cut of triangle
Sub
Ans

(8)

[12]

QUESTION 7

The graph below shows a portion of $f(x) = \sec^2\left(\frac{1}{4}x\right)$.



The first steps to determine the area between the x-axis and the graph, between x = 0 and x = 4, is to do a Riemann-sum.

7.1 Would the above Riemann-sum for the estimated area be an over- or underestimation of the actual area?

Overestimation. Ans (1)

7.2 Calculate the value of the $\dfrac{\textit{Estimated area}}{\textit{Actual area}}$ between the x-axis and the graph of *f*, between *x* = 0 and 4 using five intervals as indicated on the sketch on the previous page.

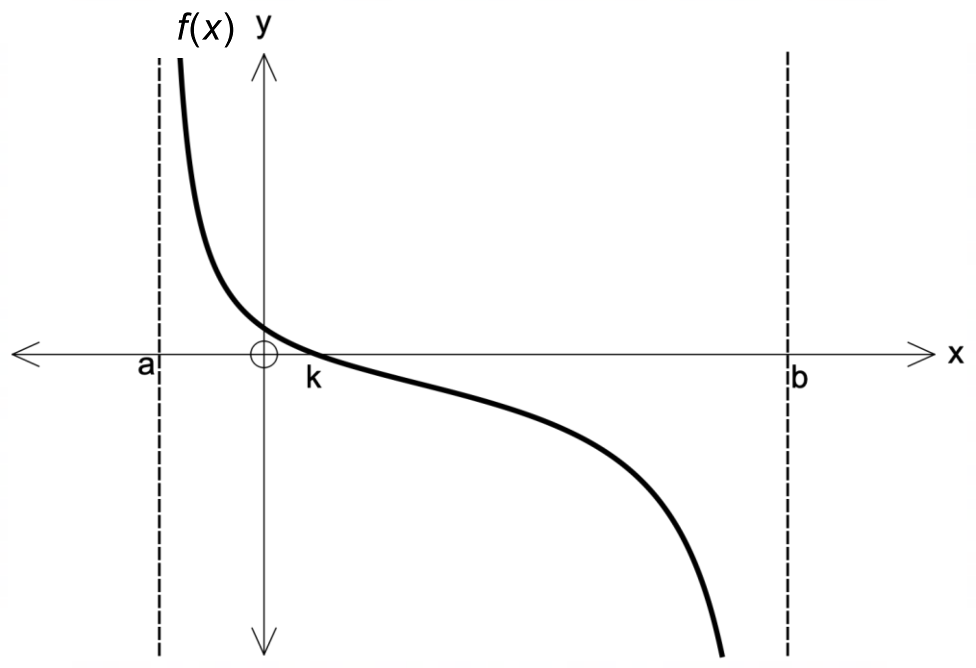
$=\left(f\left(\frac{4}{5}\right)+f\left(\frac{8}{5}\right)+f\left(\frac{12}{5}\right)+f\left(\frac{16}{5}\right)+f(4)\right)\frac{4}{5}$	x-values $\sec^2(\ )$ correct use
$=\left(\sec^2\left(\frac{1}{4}\left(\frac{4}{5}\right)\right)+\sec^2\left(\frac{1}{4}\left(\frac{8}{5}\right)\right)+\sec^2\left(\frac{1}{4}\left(\frac{12}{5}\right)\right)+\ldots+\sec^2\left(\frac{1}{4}(4)\right)\right)\times\frac{4}{5}$	Rectangle width
$=7{,}33$	Summation Ans
$f(x)=\sec^2\left(\frac{1}{4}x\right)$	Integral setup
$\textit{Actual Area}=\int_0^4\left(\sec^2\left(\frac{1}{4}x\right)\right)dx$	Ans
$=6{,}23\ \textit{units}^2$	
$\dfrac{\textit{Estimated Area}}{\textit{Actual Area}}$	
$=\dfrac{7{,}33}{6{,}22}=1{,}18$	Ans

(8)

[9]

QUESTION 8

The graph below represents a part of the graph of $f(x) = \frac{4x-2}{x^2-4x-5}$.



8.1 Determine the values of *a*, *b* and *k*.

$$f(x) = \frac{4x-2}{x^2-4x-5}$$
$$f(x) = 0$$
$$4x-2=0$$
$$= \frac{1}{2}$$
$$\therefore k = \frac{1}{2}$$

$$x^2-4x-5=0$$
$$(x-5)(x+1)=0$$
$$x=5 \ or \ x=-1$$
$$a=-1 \ and \ b=5$$

4x–2=0

k = ½

$x^2-4x-5 = 0$

a=–1

b=5

(5)

8.2 Evaluate $\int \frac{4x-2}{x^2-4x-5}dx$.

$\frac{4x-2}{x^2-4x-5} = \frac{4x-2}{(x-5)(x+1)}$ factorise

$\frac{4x-2}{x^2-4x-5} = \frac{A}{x-5} + \frac{B}{x+1}$ Std form

Mult by LCD

$4x-2 = A(x+1) + B(x-5)$

$\boldsymbol{x = -1}$

$-6 = -6B$ x-values(m)

$1 = B$

$\boldsymbol{x = 5}$ B=1

$18 = 6A$

$3 = A$ A=3

$\frac{4x-2}{x^2-4x-5} = \frac{3}{x-5} + \frac{1}{x+1}$

$\int \frac{4x-2}{x^2-4x-5}dx = \int \left(\frac{3}{x-5} + \frac{1}{x+1}\right)dx$ Sub P.F.

Integration

$= 3\ln|x-5| + \ln|x+1| + c$ +c (10)

8.3 A new graph is formed by taking the product of *f(x)* and a monomial. This new graph has a horizontal asymptote of $y = 8$.

Give an expression for the monomial. Justify your choice of monomial using a limit.

$y = ax\left(\frac{4x-2}{x^2-4x-5}\right)$ ax

limit to infinity

$y = \frac{4ax^2-2ax}{x^2-4x-5}$

horizontal asymptote:

$y = \lim_{x \to \infty}\left(\frac{4ax^2-2ax}{x^2-4x-5}\right) = 8$

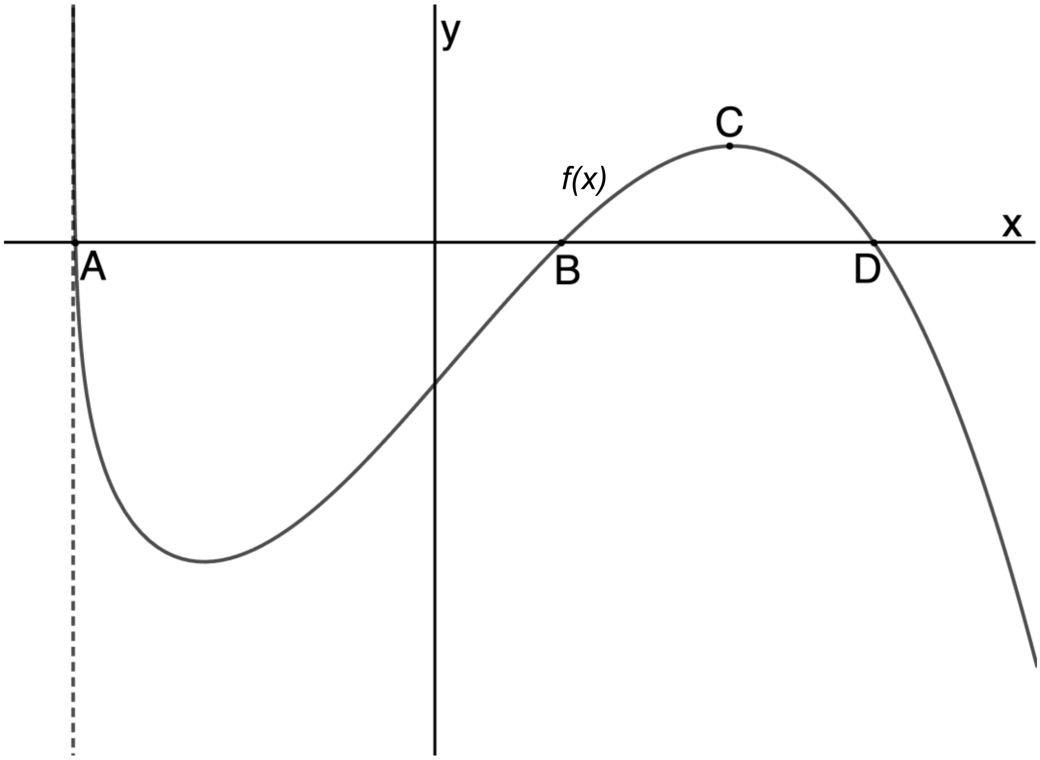
$4a = 8$ =8

$a = 2$ Ans (4)

[19]

QUESTION 9

The graph shows $f(x) = -\ln(x+2) - \frac{1}{3}x^3 + 3x - 1$ with the x-intercepts at A, B and D. C's coordinates are at approximately (1,65; 1,16)



9.1 To calculate the coordinates of B, should you choose $x = 0$ or $x = 3$ as the initial approximation for the Newton-Raphson method? Justify your answer.

$x = 0$ must be chosen. By choosing $x = 3$ will converge to D. — x = 0; Justification

(2)

9.2 Give a reason you cannot choose the x-coordinate of C as an initial approximation for the Newton-Raphson-method.

At x 1,65, the Newton-Raphson method will not converge to any value. — Justification

(2)

9.3 Determine the coordinates of B, correct to 4 decimal places, using the Newton-Raphson method.

- Use x_0 = 0,5 as the initial approximation.
- Show the answer of your first iteration accurate to 4 decimal places.

$-ln(x+2)-\frac{1}{3}x^3+3x-1=0$	=0
$f(x)=-ln(x+2)-\frac{1}{3}x^3+3x-1$	$-x^2+3$
	$-\frac{1}{x+2}$
$f'(x)=-\frac{1}{x+2}-x^2+3$	
$x_1=x_0-\frac{f(x)}{f'(x)}$	Formula (implied)
$x_1=0,5-\frac{-ln((0,5)+2)-\frac{1}{3}(0,5)^3+3(0,5)-1}{-\frac{1}{(0,5)+2}-(0,5)^2+3}$	Substitution starting point x = 0,5
$x_1=0,6949$	First iteration
$\vdots$	Ans
$x=0,7035$	

(10)

[14]

QUESTION 10

Evaluate the following integrals:

10.1 $\int \sin(4x)\sin(2x)\ dx$

$$\frac{1}{2}\int(\cos(4x-2x)-\cos(4x+2x))\ dx$$

$$=\frac{1}{2}\int(\cos(2x-\cos 6x)\ dx$$

$$=\frac{1}{2}\left[\int\frac{1}{2}(2\cos 2x)\ dx-\int\frac{1}{6}(6\cos 6x)\ dx\right]$$

$$=\frac{1}{4}\sin 2x-\frac{1}{12}\sin 6x+c$$

sub into formula
a
2 and 1/2
6 and 1/6
integrals

(6)

10.2 $\int x^2 \ln x\ dx$

$$g'(x) = x^2$$

$$g(x) = \frac{1}{3}x^3$$

$$f(x) = \ln(x)$$

$$f'(x) = \frac{1}{x}$$

$$\int x^2 \ln x\ dx = \frac{1}{3}x^3\ln x - \int \frac{1}{3}x^3 \times \frac{1}{x}\ dx + c$$

$$\therefore \quad = \frac{1}{3}x^3\ln x - \frac{1}{3}\int x^2\ dx + c$$

$$\therefore \quad = \frac{1}{3}x^3\ln x - \frac{1}{9}x^3 + c$$

g'(x)
g(x)
f(x)
f'(x)
Integration by parts (m)
x^2
form of integral
ans

(8)

[14]

QUESTION 11

Given: $f(x) = \dfrac{2x^2 - x - 7}{x - 1}$ has an oblique asymptote of $y = ax + b$.

11.1 Determine the values of *a* and *b*.

$$f(x) = \frac{2x^2 - x - 7}{x-1}$$

$$\therefore \quad = \frac{2x(x-1) + 2x - x - 7}{x-1}$$ Manipulation

$$\therefore \quad = 2x + \frac{x-7}{x-1}$$

$$\therefore \quad = 2x + \frac{x - 1 + 1 - 7}{x-1}$$

$$\therefore \quad = 2x + 1 - \frac{6}{x-1}$$ –6

Horizontal asymptote:

$\therefore \quad y = 2x + 1$ a

$a = 2$ and $b = 1$ b (4)

11.2 The oblique asymptote is rotated about the *x*-axis between $x = 1$ and $x = k$. This solid has a volume of 200π units cubed.

Determine the value of *k*, correct to 1 decimal place.

$$200\pi = \int_1^k \pi\,(2x+1)^2\,dx$$

$$200\pi = \frac{1}{3} \times \frac{1}{2} \int_1^k 3 \times 2\,(2x+1)^2\,dx$$

$$200\pi = \frac{1}{6}\pi \times \left((2x+1)^3\right)_1^k$$

$$200\pi = \frac{1}{6}\pi\,\left((2k+1)^3 - (2(1)+1)^3\right)$$

Using the solve the function:

k = 4,9

Formula
= 200π
sub into formula
start and end values
3 and 1/3
2 and 1/2
integral
sub in of k and 1
ans (10)

[14]

TOTAL: 200

INTERNATIONAL SECONDARY CERTIFICATE EXAMINATION
MAY 2025

FURTHER STUDIES MATHEMATICS (EXTENDED): PAPER II

MARKING GUIDELINES

Time: 1 hour 100 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

MODULE 2 STATISTICS

QUESTION 1

1.1 (a) $X \sim B(8;0,6)$

$$P(X=6)=\binom{8}{6}(0,6)^6(0,4)^2$$
$$=0,2090$$

(6)

(b) (1) $np>5$ prevents the distribution being positively skewed and $nq>5$ prevents the distribution being negatively skewed and hence the binomial distribution resembles the normal distribution. (2)

(2) $X \sim B(40;0,6)$ approx to $N\left(24;\sqrt{9,6}^2\right)$

$$P(X>30)\rightarrow P(X>30,5)$$
$$=P\left(Z>\frac{30,5-24}{\sqrt{9,6}}\right)$$
$$=P(Z>2,1)$$
$$=0,5-0,4821$$
$$=0,0179$$

(8)
[16]

QUESTION 2

(a) $X \sim N(20;0,8^2)$

$$P(X<21)=P\left(Z<\frac{21-20}{0,8}\right)$$
$$=P(Z<1,25)$$
$$=0,5+0,3944$$
$$=0,8944$$

(6)

(b) $X \sim B(7\,;0,1056)$

$$P(X=2)=\binom{7}{2}(0,1056)^2(0,8944)^5$$
$$=0,134$$

(6)

(c) $P(k < X < 21) = 0,7$ OR $P(X < k) = 0,8944 - 0,7$
$= 0,1944$

$\therefore P(k < X < 20) = 0,7 - 0,3944$

$P(k < X < 20) = 0,3056$

$-0,86 = \dfrac{k - 20}{0,8}$

$k = 19,3°$

(8)
[20]

QUESTION 3

3.1 (a) A 96% CI for p is

$0,35 \pm 2,05\sqrt{\dfrac{(0,35)(0,65)}{140}}$ or $0,35 \pm 2,06\sqrt{\dfrac{(0,35)(0,65)}{140}}$

$(0,2674; 0,4326)$ $(0,2670; 0,4330)$ (6)

(b) Increase the sample size or decrease the confidence interval. (2)

3.2 (a) $H_0 : \mu_Y - \mu_X = 1$

$H_1 : \mu_Y - \mu_X > 1$

(2)

(b) $Z = \dfrac{(25,99 - 24,69) - 1}{\sqrt{\dfrac{0,8^2}{30} + \dfrac{0,49^2}{30}}}$

$= 1,75$

$\therefore P(Z > 1,75) = 0,5 - 0,4599$

$= 0,0401$

$\therefore \alpha = 4\%$

(10)
[20]

QUESTION 4

4.1 (a) $2m + 0,3 - m^2 + 0,4 - 0,7m = 1$

$m^2 - 1,3m + 0,3 = 0$

$m \neq 1$ or $m = 0,3$

(5)

(b) $E[X] = 0,3 + 2(0,3) + 3(0,21) + 4(0,19)$

$= 2,29$

(4)

(c) $2p + 2q = 1 \quad \therefore p + q = 0,5$

$2(0,3p) + 0,21q + 0,19q = 0,24$

$0,6p + 0,4q = 0,24$

$\therefore p = 0,2 \quad q = 0,3$

(8)

4.2 $\int_0^4 ax\,dx = 0,5$

$\left[\frac{ax^2}{2}\right]_0^4 = \frac{1}{2}$

$8a = \frac{1}{2}$

$a = \frac{1}{16}$

$\therefore \left[\frac{x^2}{32}\right]_0^b = 1$

$b^2 = 32$

$b = 4\sqrt{2}$

(8)

[25]

QUESTION 5

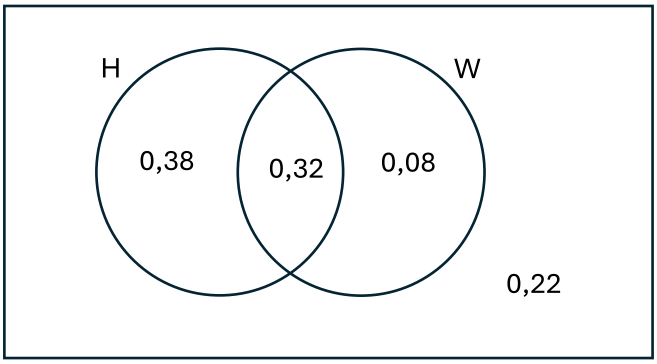
5.1 (ADMIN) ISTRATION

$$\frac{10!}{2!2!} = 907200$$ (5)

5.2 (a) $P(H|W) = 0{,}8$

$$\frac{P(H \cap W)}{0{,}4} = 0{,}8$$

$$\therefore P(H \cap W) = 0{,}32$$



(8)

(b)

Couple 1	Prob	Couple 2	prob
$H \cap W'$	0,38	$H' \cap W$	0,08
$H' \cap W$	0,08	$H \cap W'$	0,38
$H \cap W$	0,32	$H' \cap W'$	0,22
$H' \cap W'$	0,22	$H \cap W$	0,32

P(only 1 H and only 1 W) = $(0{,}38)(0{,}08) \times 2 + (0{,}32)(0{,}22) \times 2$

$= 0{,}2016$ (6)

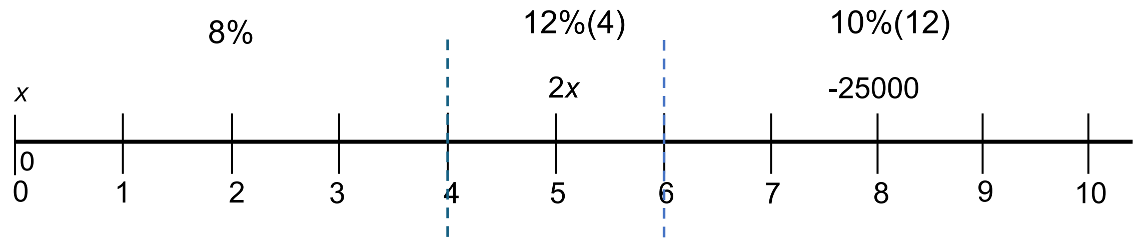
[19]

Total for Module 2: 100 marks

MODULE 3 FINANCE AND MODELLING

QUESTION 1

(a)



(4)

(b)

$$x(1+0{,}08)^4\left(1+\frac{0{,}12}{4}\right)^8\left(1+\frac{0{,}1}{12}\right)^{48}+2x\left(1+\frac{0{,}12}{4}\right)^4\left(1+\frac{0{,}1}{12}\right)^{48}-25\,000\left(1+\frac{0{,}1}{12}\right)^{24}$$
$$=89\,357{,}16$$

(7)

(c) x = R20 250 (2)

[13]

QUESTION 2

(a) 750 000 (1 + 0,09)^5 = R1 153 967,97 (2)

(b)

$$\frac{14154{,}85\left[\left(1+\frac{0{,}06}{12}\right)^{36}-1\right]}{\frac{0{,}06}{12}}\cdot\left(1+\frac{0{,}08}{12}\right)^{24}$$
$$+\frac{14154{,}85\left[\left(1+\frac{0{,}08}{12}\right)^{24}-1\right]}{\frac{0{,}08}{12}}$$
$$= \text{R}1\,020\,140{,}50$$

(9)

(c) 1 153 967,97 – 320 000 (1 – i)^5 = 1 020 140,50
i = 16%

(5)

[16]

QUESTION 3

(a) P_n is the outstanding balance after n months. (2)

(b) $$15\ 000\ 000 = \frac{x\left[1-(1+0{,}01)^{-240}\right]}{0{,}01}$$

x = R165 162,92 (6)

(c) $Q_0 = P_{48}$

$$P_{48} = 15\ 000\ 000(1+0{,}01)^{48}$$

$$-\frac{165\ 162{,}92[(1+0{,}01)^{48}-1]}{0{,}01}$$

$$= R14\ 071\ 686{,}50$$

(7)

(d) $$14\ 071\ 686{,}50 = \frac{y[1-(1+0{,}012)^{-192}]}{0{,}012}$$

$y = 187880{,}83$ (5)

(e) $(1+i)^{20} = (1+0{,}01)^{48}\ (1+0{,}012)^{192}$

$i = 14{,}84\%$ (5)

[25]

QUESTION 4

(a) $$x = \frac{2\ 525 - 2\ 200}{2\times 2\ 352} = 0{,}06909$$

$$y = \frac{2\ 614 - 2\ 352}{2\times 2\ 525} = 0{,}05188$$

(6)

(b) $$m = \frac{0{,}05188 - 0{,}06909}{2\ 525 - 2\ 352}$$

$$= -0{,}0001$$

$$\therefore \frac{\Delta P}{P} = -0{,}0001P + c$$

$$\therefore 0{,}06909 = -0{,}0001(2\ 352) + c$$

$$\therefore c = 0{,}3043$$

(5)

(c) $r = 0{,}3043$ (2)

(d) $\pi = \dfrac{0{,}3043}{0{,}0001}$

$= 3\ 043$ (2)

[15]

QUESTION 5

(a) $F_0 = 50$ $R_0 = 500$ (2)

(b) Initial pred numbers are less than equilibrium while initial prey numbers are greater than equilibrium. (2)

(c) $r = 0{,}1$ (2)

(d) $R_n + 0{,}1 R_n\left(1 - \dfrac{R_n}{1000}\right) - 0{,}001 R_n F_n$

$K = 1000$

(3)

(e) $0{,}0002 \times 50 \times 500$

$= 5$ (4)

(f) $F_e = 0{,}95 F_e + 0{,}0002 F_e R_e$

$\therefore R_e = \dfrac{0{,}05}{0{,}0002} = 250$

$R_e = 1{,}1 R_e - 0{,}0001 R_e^2 - 0{,}001 R_e F_e$

$1 = 1{,}1 - 0{,}0001 R_e - 0{,}001 F_e$

$\therefore 0{,}001 F_e = 0{,}1 - 0{,}0001 \times 250$

$\therefore F_e = 75$ (8)

[21]

QUESTION 6

(a) $529 = 460 (1 + i)^1$

$\therefore$ i = 0,15 or 15% (4)

(b) $460 (1 + i)^{-1}$

$= 400$

$\therefore$ 400 : 10 tagged

But since 50 were tagged

There are 2 000 koi initially (6)

[10]

Total for Module 3: 100 marks

MODULE 4

QUESTION 1

1.1

$$\left[\begin{array}{ccc|c} -3 & 2 & 7 & -8 \\ 8 & 3 & -2 & 38 \\ -2 & -1 & 5 & -10 \end{array}\right]$$

$$\frac{-1}{125}\begin{bmatrix} 13 & -17 & -25 \\ -36 & -1 & 50 \\ -2 & -7 & -25 \end{bmatrix} \overset{\text{m}}{\begin{bmatrix} -3 & 2 & 7 \\ 8 & 3 & -2 \\ -2 & -1 & 5 \end{bmatrix}}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{-1}{125}\overset{\text{m}}{\begin{bmatrix} 13 & -17 & -25 \\ -36 & -1 & 50 \\ -2 & -7 & -25 \end{bmatrix}}\begin{bmatrix} -8 \\ 38 \\ -10 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix} \quad \therefore x = 4, \; y = 2, \; z = 0 \quad \text{m}$$

Alternative

$3R_2 + 8R_1$

$3R_3 - 2R_1$

$$\left[\begin{array}{ccc|c} -3 & 2 & 7 & -8 \\ 0 & 25 & 50 & 50 \\ 0 & -7 & 1 & -14 \end{array}\right]$$

$\frac{1}{25}R_2$

$$\left[\begin{array}{ccc|c} -3 & 2 & 7 & -8 \\ 0 & 1 & 2 & 2 \\ 0 & -7 & 1 & -14 \end{array}\right]$$

$R_3 + 7R_2$

$$\left[\begin{array}{ccc|c} -3 & 2 & 7 & -8 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 15 & 0 \end{array}\right]$$

$z = 0 \quad y = 2 \quad x = 4$

OR

$25R_3 + 7R_2$

$$\left[\begin{array}{ccc|c} -3 & 2 & 7 & -8 \\ 0 & 25 & 50 & 50 \\ 0 & 0 & 375 & 0 \end{array}\right]$$

(10)

1.2 Adjoint A

(a)

$$\begin{bmatrix} -3 & -4 & 5 \\ -4 & 0 & -4 \\ 5 & -4 & -3 \end{bmatrix}$$

sign

values

(6)

(b) det (adj A) = +256

det A = – 16 $\therefore (\det A)^{3-1} = \det(adj\ A)$

Proof:

$$\det A\,(A^{-1}) = \text{adj } A$$
$$A\,(\det A)\,(A^{-1}) = A\,(\text{adj } A)$$
$$I\,(\det A) = A\,(\text{adj } A)$$
$$\det(I.\det A) = \det(A \text{ adj } A)$$
$$(\det A)^n = \det A.\ \det(\text{adj } A)$$
$$(\det A)^{n-1} = \det(\text{adj } A)$$

(8)

[24]

QUESTION 2

2.1 $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 3 & 4 \\ 6 & 2 \end{pmatrix} = \begin{pmatrix} -6 & -2 \\ -3 & -4 \end{pmatrix}$ (3)

2.2 $\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 3 & 4 \\ 6 & 2 \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ 12 & 10 \end{pmatrix}$ (4)

2.3 $\frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 3 & 4 \\ 6 & 2 \end{pmatrix} = \begin{pmatrix} \frac{3}{2} & 2 \\ -3 & -1 \end{pmatrix}$

(5)

2.4 $\begin{pmatrix} 1 & 0 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} 3 & 4 \\ 6 & 2 \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ -12 & -4 \end{pmatrix}$

Reflection about *x* axis, stretch by k = 2 *x*-axis invariant
OR
Stretch with a factor of –2 and the *x*-axis invariant
[The marks are allocated to the words]

(4)

[16]

QUESTION 3

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 7 & 3 \\ 5 & 2 \end{bmatrix}=\begin{bmatrix} 5{,}112 & 2{,}235 \\ 6{,}918 & 2{,}829 \end{bmatrix}$$

(1) $7\cos\theta - 5\sin\theta = 5{,}112$
(2) $3\cos\theta - 2\sin\theta = 2{,}235$
(3) $7\sin\theta + 5\cos\theta = 6{,}918$
(4) $3\sin\theta + 2\cos\theta = 2{,}829$

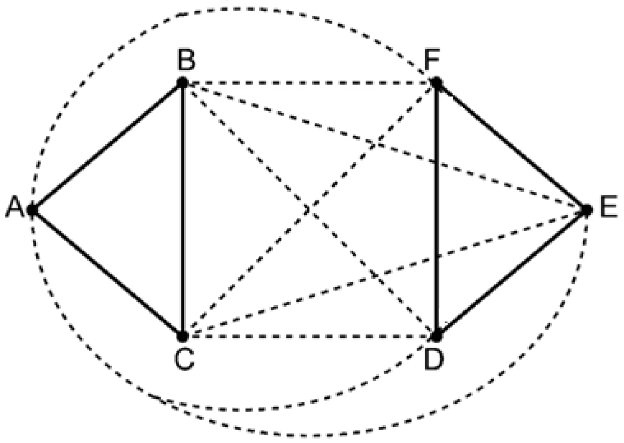
$\cos\theta = \dfrac{951}{1000}$ $\qquad$ $\sin\theta = \dfrac{309}{1000}$

$\theta = 18$ $\qquad$ $\theta = 18$

[10]

QUESTION 4

4.1 (a)



AF ; AE ; AD ; BD ; BE ; BF ; CD ; CF ; CE

(6)

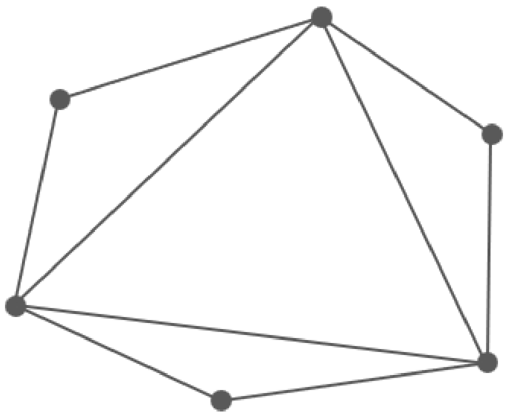
(b) BF or CD (2)

(c) Any 2 edges that produce a Hamiltonian graph.
marks per edge & Hamiltonian. (3)

4.2 (a) $2 \times 9 = 18$ (2)

(b) Half of all the edges to one vertex; 5 (2)

(c)

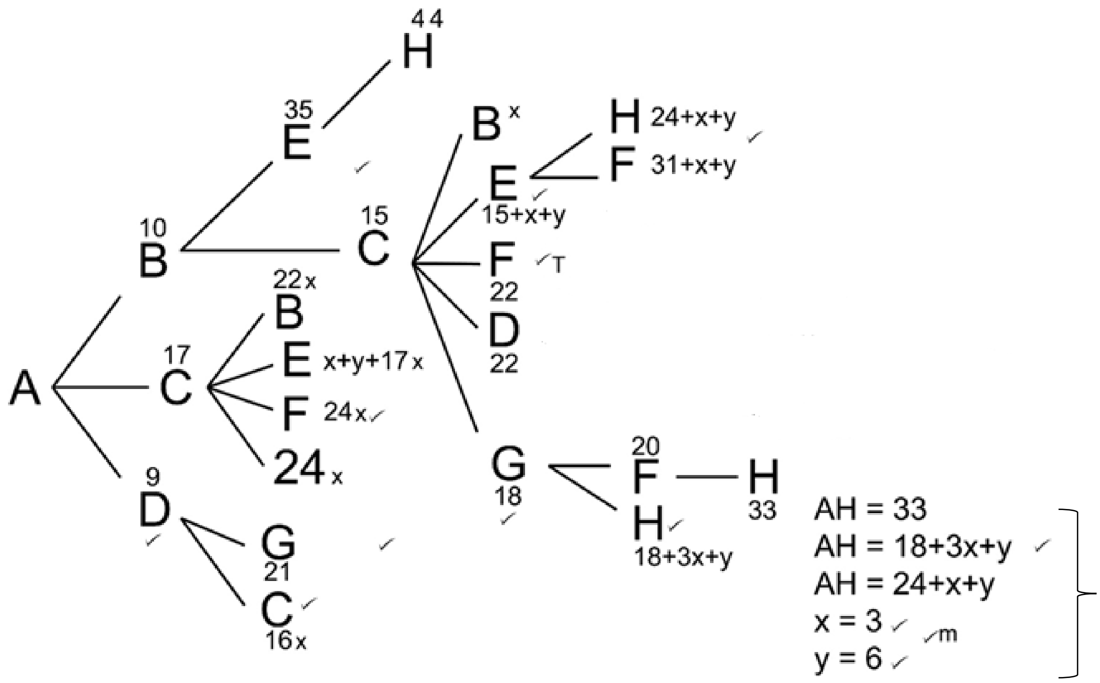


18 = 3 order 4 and 3 order 2 vertices

(5)
[20]

QUESTION 5

5.1



(14)

5.2 A $\underline{9}$ D
D $\underline{7}$ C
C $\underline{3}$ G

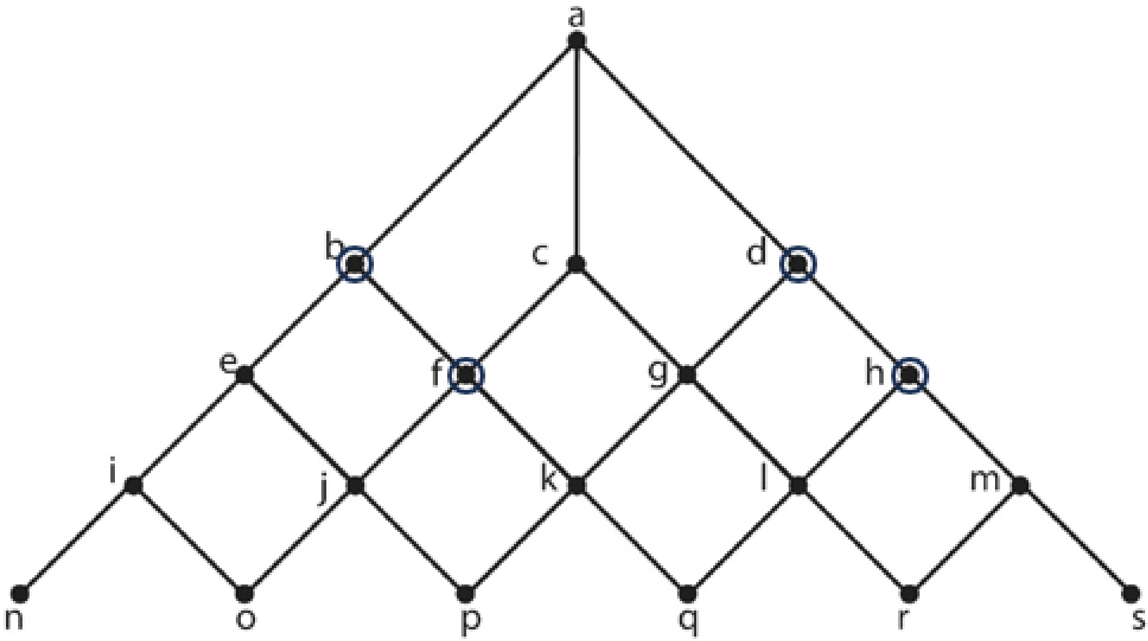
G $\underline{2}$ F
C $\underline{5}$ B
C $\underline{9}$ E
E $\underline{9}$ H

Total weight = 44

(8)
[22]

QUESTION 6

6.1



F = 1,5 + 1 = 2,5	1,25 → J	1,25 → K
G = 1,5 + 1 – 1 = 1,5	0,75 → L	0,75 → K
J = 0,25 + 1,25 – 1 = 0,5	0,25 → P	0,25 → O
K = 1,25 + 0,75 – 1 = 1	0,5 → P	0,5 → Q
L = 0,75 + 0,75 – 1 = 0,5	0,25 → R	0,25 → Q

a b, c, d each get 3 glasses flowing in
ca 1.5 glasses flow from b to each of e and f (OR from d to g and h)
ca e and g each use a glass to fill up first
ca j receives $\frac{5}{4}$ glasses from f OR l receives $\frac{3}{4}$ glass from h
answer = $\frac{1}{2}$ glass flows from each

A half a glass flows from each of L and J into the glasses below. (5)

6.2 p and q (3)
[8]

Total for Module 4: 100 marks