Please paste the barcoded label here	TOTAL MARKS

INTERNATIONAL SECONDARY CERTIFICATE EXAMINATION
MAY 2025

FURTHER STUDIES MATHEMATICS (STANDARD): PAPER I

EXAMINATION NUMBER

Time: 2 hours 200 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 24 pages and an Information Booklet of 4 pages (i–iv). Please check that your question paper is complete.
2. **Answer ALL the questions on the question paper and hand it in at the end of the examination. Remember to write your examination number in the space provided.**
3. Non-programmable and non-graphical calculators may be used, unless otherwise indicated.
4. All necessary calculations must be clearly shown and writing must be legible.
5. Diagrams have not been drawn to scale.
6. Round off your answers to 2 decimal digits, unless otherwise indicated.
7. One blank page (page 24) is included at the end of the question paper. If you run out of space for a question, use this page. Clearly indicate the number of the question you are answering should you use this additional space.

	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Total
Mark												
Marker initial												
Moderated Mark												
Question Total	**13**	**35**	**15**	**16**	**39**	**12**	**9**	**19**	**14**	**14**	**14**	**/200**

FS12MATC-1-E

QUESTION 1

Prove by mathematical induction that $10^{2n-1} + 1$ is divisible by 11 for all $n \in \mathbb{N}$.

[13]

QUESTION 2

2.1 Solve for $x \in \mathbb{R}$.

(a) $2^{\ln|x|} = 0,25^{y}$ in terms of y, and simplify your answer.

(4)

(b) $x^2 - 3|x| = 10$

(7)

(c) $\dfrac{x^2}{x-3} \leq x-3$

(6)

2.2 If In(2) = A and $\log_2(3) = B$, write the following in terms of A and B.

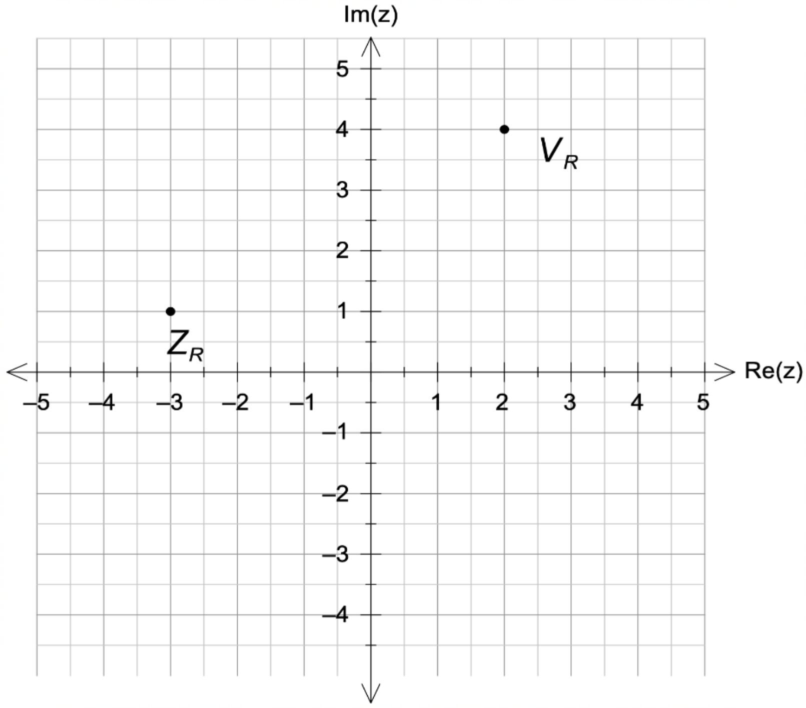
(a) $In\left(\dfrac{1}{4}\right) - \log_{\frac{1}{2}}(3)$

(4)

(b) $\log_2(e) + \log_2(12) - 2$

(6)

2.3 The diagram below shows two sets of coordinates V_R and Z_R.



(a) Given that the complex number $V_R = 2 + 4i$ has been represented on the axes above as (2; 4), write the co-ordinates of point Z_R in the form $a + bi$.

(2)

(b) In alternating current circuits V_R represents the voltage and Z_R the impedance while I_R represents the current. To calculate the voltage in a circuit the formula $V_R = I_R \times Z_R$ is used.

Determine the current in the circuit, in the form $a + bi$, with the values of V_R and Z_R represented on the axes above.

(6)
[35]

QUESTION 3

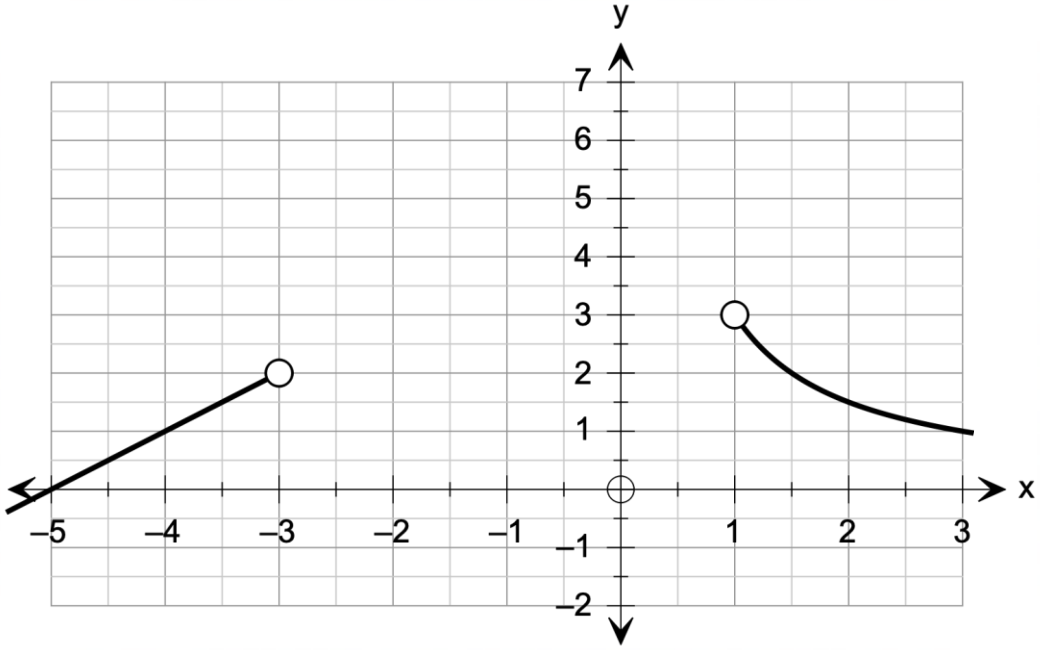
3.1 Paying careful attention to notation in justifying your reasoning, discuss, on the given domain, the continuity of

$$f(x) = \begin{cases} \dfrac{2x+3}{x+1} & x < -1 \\ 2|x-1| + 3 & -1 \leq x \leq 2 \\ 3^x - 4 & x > 2 \end{cases}$$

(7)

3.2 Complete the diagram below such that the function has the following properties:

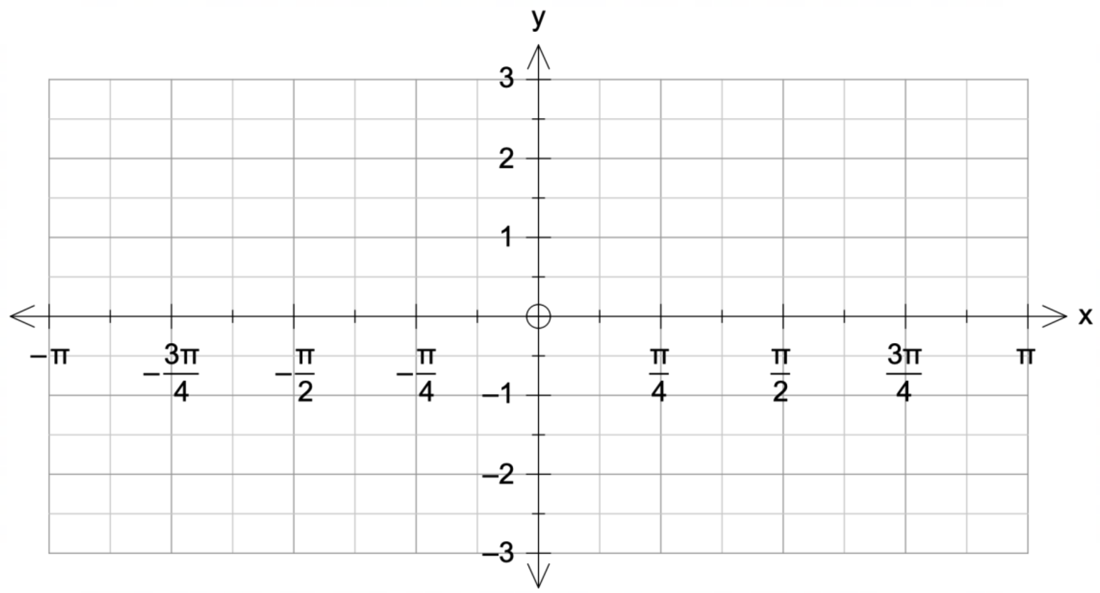
- Differentiable at $x = -3$.
- Not differentiable at $x = -1$.
- Continuous for $x \in \mathbb{R}$, $x \neq 1$.
- Removable discontinuity at $x = 1$.



(8)
[15]

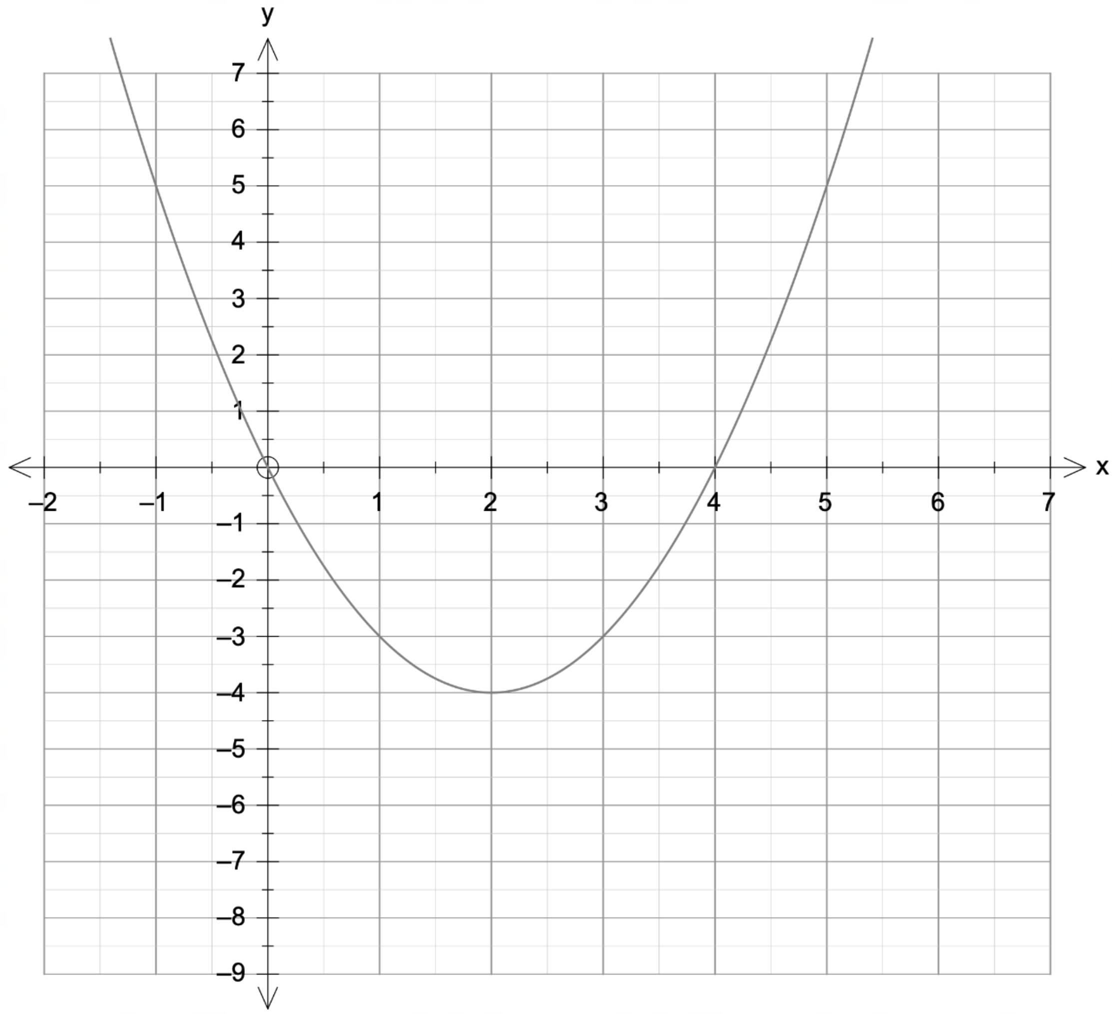
QUESTION 4

4.1 Make a neat sketch of $y = 2\sin|x|$ for $x \in [-\pi; \pi]$



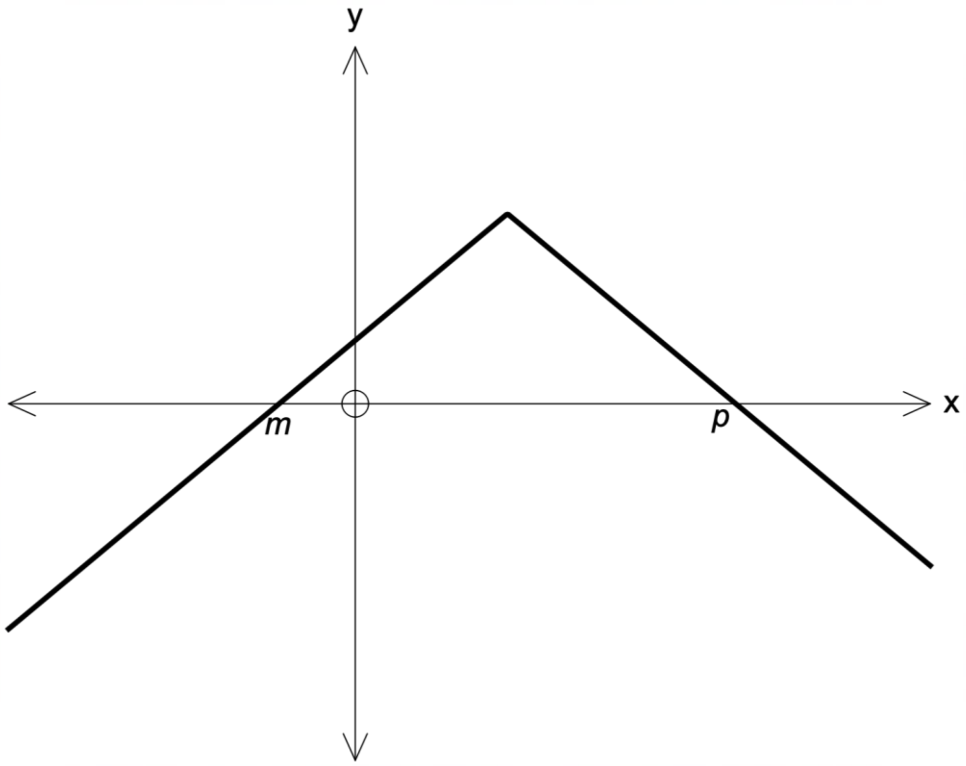
(4)

4.2 Drawn below is $g(x) = x^2 - 4x$. On the same axes, make a neat sketch of $y = |x^2 - 4x| - 5$.



(6)

4.3 Drawn below is the graph of $f(x) = -|2x - k| + 3$ with x–intercepts m and p.



Give the values of x, in terms of k, m and p, for which $f(x).f'(x) < 0$.

(6)
[16]

QUESTION 5

5.1 Given: $f(x) = \sqrt{4x - 8}$

Determine $f'(x)$ by using first principles.

(10)

5.2 Determine $\frac{dy}{dx}$ for the following:

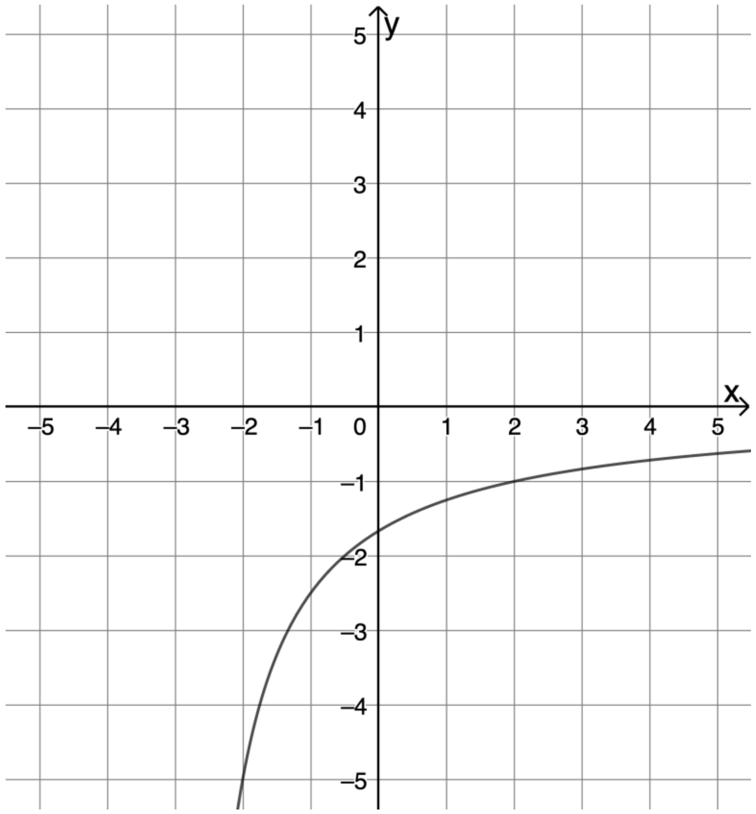
(a) $y = \frac{\cos^2(x)}{3x-1}$

(6)

(b) $e^{3y} - 3yx^2 = 4x^2 - \cot(x)$

(9)

5.3 A portion of the graph of $f(x) = -\dfrac{5}{x+3}$ is drawn below:



(a) Make a neat sketch of $g(x) = ln(x + 3)$ on the diagram above. Clearly indicate the intercepts with the axes and the asymptote on your diagram.

(4)

(b) Lou-Anne wants to determine the minimum vertical distance between the graphs of *f(x)* and *g(x)*.

She calculates the x-value where the minimum distance occurs as $x = –8$, but when she wants to calculate the actual distance, it gives her a 'MATH ERROR' on her calculator. Explain why her answer can't be correct.

(2)

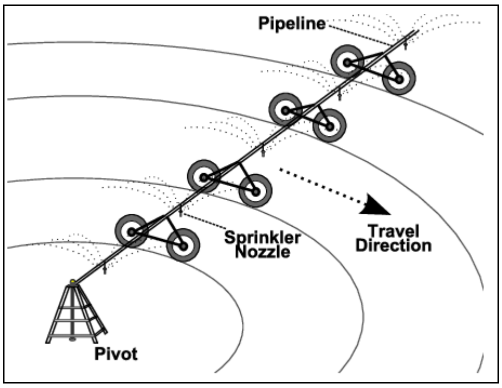
(c) Determine the correct x-value where the vertical distance between the graphs is at a minimum.

(8)

[39]

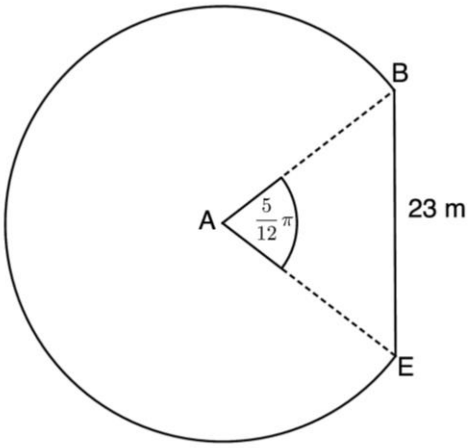
QUESTION 6

As the name suggests, centre pivots irrigate in a circular pattern around a central pivot point. Pivots can apply water, fertiliser, chemicals, and herbicides.



[Source: <https://www.researchgate.net/profile/Agnelo-Silva/publication/227350590/figure/fig1/AS :668998614142992@1536513107740/Basic-components-of-a-center-pivot-CP-system.png>]

At times a field is not big enough to have a full circular path. The farmer will then cut the circular path of the pivot with a chord. In this case, the irrigated field has a segment with a 23-metre chord cut from a circle. The chord subtends an angle of $\frac{5}{12}\pi$ between the radii.



6.1 Calculate the length of the radius of the field.

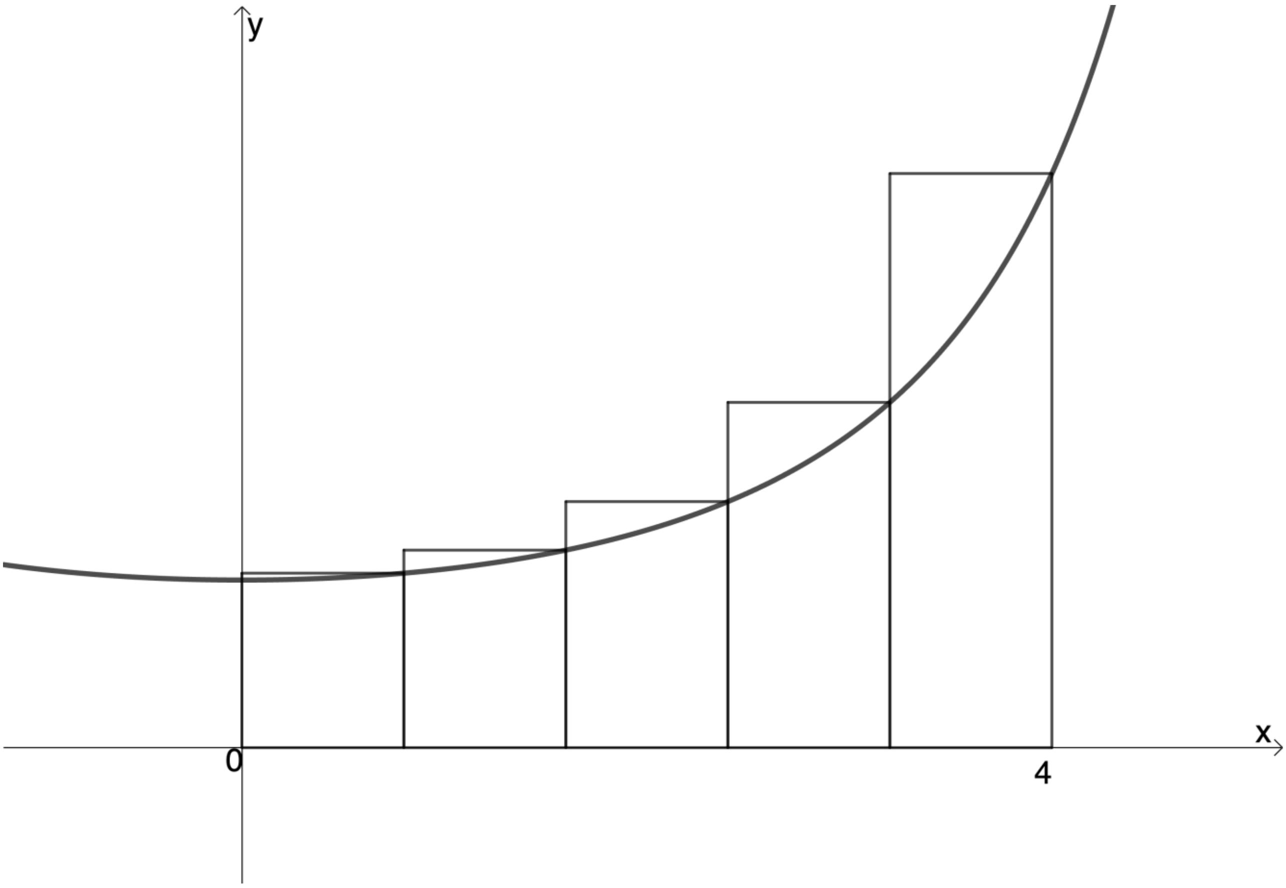
(4)

6.2 Determine the area of the field.

(8)
[12]

QUESTION 7

The graph below shows a portion of $f(x) = \sec^2\left(\frac{1}{4}x\right)$.



The first steps to determine the area between the *x*-axis and the graph, between $x = 0$ and $x = 4$, is to do a Riemann-sum.

7.1 Would the above Riemann-sum for the estimated area be an over- or underestimation of the actual area?

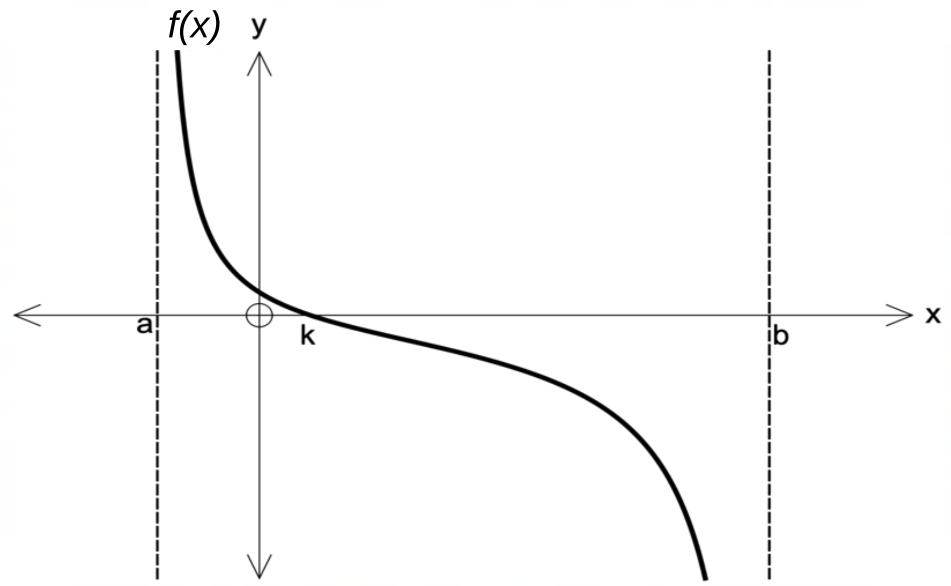
(1)

7.2 Calculate the value of the $\frac{\textit{Estimated area}}{\textit{Actual area}}$ between the x-axis and the graph of f, between $x = 0$ and $x = 4$ using five intervals as indicated on the sketch on the previous page.

(8)
[9]

QUESTION 8

The graph below represents a part of the graph of $f(x) = \dfrac{4x-2}{x^2-4x-5}$.



8.1 Determine the values of a, b and k.

(5)

8.2 Evaluate $\int \frac{4x-2}{x^2-4x-5}dx$.

(10)

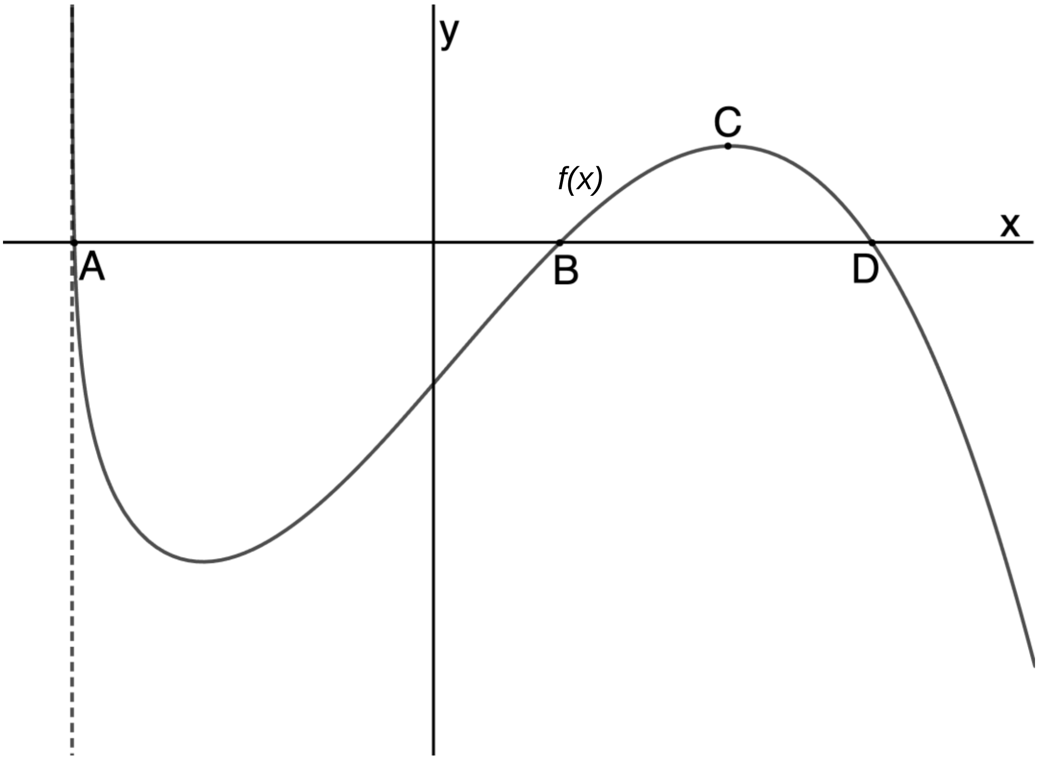
8.3 A new graph is formed by taking the product of *f(x)* and a monomial. This new graph has a horizontal asymptote of $y = 8$.

Give an expression for the monomial. Justify your choice of monomial using a limit.

(4)
[19]

QUESTION 9

The graph shows $f(x) = -\ln(x+2) - \frac{1}{3}x^3 + 3x - 1$ with the x-intercepts at A, B and D. C's coordinates are at approximately (1,65; 1,16)



9.1 To calculate the coordinates of B, should you choose $x = 0$ or $x = 3$ as the initial approximation for the Newton-Raphson method? Justify your answer.

(2)

9.2 Give a reason you cannot choose the x-coordinate of C as an initial approximation for the Newton-Raphson method.

(2)

9.3 Determine the coordinates of B, correct to 4 decimal places, using the Newton-Raphson method.

- Use $x_0 = 0{,}5$ as the initial approximation.
- Show the answer of your first iteration accurate to 4 decimal places.

(10)
[14]

QUESTION 10

Evaluate the following integrals:

10.1 $\int \sin(4x)\sin(2x)\,dx$

(6)

10.2 $\int x^2 \ln x\,dx$

(8)
[14]

QUESTION 11

Given: $f(x) = \dfrac{2x^2 - x - 7}{x - 1}$ has an oblique asymptote of $y = ax + b$.

11.1 Determine the values of a and b.

(4)

11.2 The oblique asymptote is rotated about the x-axis between $x = 1$ and $x = k$. This solid has a volume of 200π units cubed.

Determine the value of k, correct to 1 decimal place.

(10)
[14]

Total: 200 marks

ADDITIONAL SPACE (ALL QUESTIONS)

REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU HAVE USED THE ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.

Please paste the barcoded label here

TOTAL MARKS

INTERNATIONAL SECONDARY CERTIFICATE EXAMINATION
MAY 2025

FURTHER STUDIES MATHEMATICS (EXTENDED): PAPER II

EXAMINATION NUMBER

Time: 1 hour

100 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 36 pages and an Information Booklet of 4 pages (i–iv). Please check that your question paper is complete.
2. This question paper consists of THREE modules. Choose **ONE** of the **THREE** modules and tick (✓) the one you have chosen.

   MODULE 2: STATISTICS (100 marks) OR
   MODULE 3: FINANCE AND MODELLING (100 marks) OR
   MODULE 4: MATRICES AND GRAPH THEORY (100 marks)
3. **Answer the questions on the question paper and hand it in at the end of the examination. Remember to write your examination number in the space provided.**
4. Non-programmable and non-graphical calculators may be used, unless otherwise indicated.
5. All necessary calculations must be clearly shown and writing must be legible.
6. Diagrams have not been drawn to scale.
7. **Rounding of final answers.**
   MODULE 2: Four decimal places, unless otherwise stated.
   MODULE 3: Two decimal places, unless otherwise stated.
   MODULE 4: Two decimal places, unless otherwise stated.
8. Four blank pages (pages 33–36) are included at the end of the question paper. If you run out of space for an answer, use these pages. Clearly indicate the number of the question you are answering should you use this extra space.

FOR MARKER'S USE ONLY

Module 2	Q1	Q2	Q3	Q4	Q5	Total
Marks	**16**	**20**	**20**	**25**	**19**	**100**

Module 3	Q1	Q2	Q3	Q4	Q5	Q6	Total
Marks	**13**	**16**	**25**	**15**	**21**	**10**	**100**

Module 4	Q1	Q2	Q3	Q4	Q5	Q6	Total
Marks	**24**	**16**	**10**	**20**	**22**	**8**	**100**

MODULE 2 STATISTICS

QUESTION 1

When Juliana rides her horse, she successfully jumps a certain height 3 out of every 5 attempts.

(a) Determine the probability that in 8 jumps she will be successful 6 times.

(6)

(b) Juliana has 40 attempted jumps.

(1) Explain why it is important for both $np > 5$ and $nq > 5$, in order to use the normal distribution as a suitable approximation.

(2)

(2) Hence, determine the probability that she is successful in more than 30 jumps.

(8)
[16]

QUESTION 2

Jamie works in an airconditioned office where the temperature of the office is normally distributed with a mean of 20 °C and a standard deviation of 0,8 °C.

(a) Calculate, showing all working, the probability that on a random day the temperature will be below 21 °C.

(6)

(b) During a random week, find the probability that 2 of the days will have an office temperature that is **above** 21 °C.

(6)

(c) Jamie finds that the temperature in the office is comfortable for 70% of the days.

If the upper limit of a comfortable temperature is 21 °C, calculate the lower limit of a comfortable temperature (correct to 1 decimal place).

(8)
[20]

QUESTION 3

3.1 A research study showed that out of a sample of 140 teenagers, 49 of them will download a series and pre-watch it before it airs on television.

(a) Calculate a 96% confidence interval for the proportion of all teenagers who pre-watch a series.

(6)

(b) The researcher finds that the interval is too wide. Suggest one way for her to narrow the confidence interval.

(2)

3.2 Michael claims that swimming a 50-metre freestyle race will be faster by at least 1 second when swimming with goggles. He swam thirty 50-metre races without goggles and thirty 50-metre races with goggles. A summary of his times (in seconds) are as follows:

With goggles	$n = 30$	Mean swim time: $(\bar{x} = 24{,}69)$	Standard deviation: $\sigma_x = 0{,}49$
Without goggles	$n = 30$	Mean swim time: $(\bar{y} = 25{,}99)$	Standard deviation: $\sigma_y = 0{,}8$

(a) State the null and alternate hypotheses used to conduct this test.

(2)

(b) The hypothesis test at the $\alpha\%$ significance level suggested that there is sufficient evidence to support Michael's claim. Find α, correct to the nearest percentage.

(10)
[20]

QUESTION 4

4.1 Bucket A contains a large number of red circular discs.
Each disc is numbered 1, 2, 3 or 4.

A disc from the bucket is selected at random and has the following probability distribution:

x	1	2	3	4
$P(X = x)$	m	m	$0{,}3 - m^2$	$0{,}4 - 0{,}7m$

(a) Show that $m = 0{,}3$.

(5)

(b) Hence, find the value of $E[X]$.

(4)

(c) Bucket B contains a large number of white circular discs, each numbered 1, 2, 3, or 4.
A disc is selected at random and has the following probability distribution.

y	1	2	3	4
$P(Y=y)$	p	p	q	q

A disc is simultaneously selected from bucket A and bucket B. The probability of obtaining discs showing the same number is 0,24.

Find the values of p and q.

(8)

4.2 A random variable X has probability density function

$$f(x) = \begin{cases} ax & 0 < x < b \\ 0 & \text{otherwise} \end{cases}$$

where a and b are constants. Given that the median of X is 4, find a and b.

(8)
[25]

QUESTION 5

5.1 Determine the number of times the 5-letter word ADMIN appears when the letters of the 14-letter word **ADMINISTRATION** are arranged.

(5)

5.2 The members of a particular bridge club are made up of married couples. For any married couple in the club:

- The probability that the husband is retired is 0,7.
- The probability that the wife is retired is 0,4.
- Given that the wife is retired, the probability that the husband is retired is 0,8.

(a) Draw a Venn diagram of the information presented above.

(8)

(b) Two married couples are chosen at random.

Find the probability that only one of the two husbands and only one of the two wives is retired.

(6)
[19]

Total for Module 2: 100 marks

MODULE 3 FINANCE AND MODELLING

QUESTION 1

Mr Liakos invests an amount x rands now and a further $2x$ rands in 5 years' time. After a further 3 years, he withdraws R25 000. Initially, the interest rate is 8% per annum, compounded annually. After 4 years, this changes to 12% per annum, compounded quarterly, and after a further 2 years it changes to 10% per annum compounded monthly. After 10 years the balance is R89 357,16.

(a) Set up a time-line with the relevant amounts indicated.

(4)

(b) Write down an equation that can be used to solve for x.

(7)

(c) Solve for x, correct to the nearest rand.

(2)
[13]

QUESTION 2

Tsholo's car is currently worth R320 000, and depreciating on the reducing balance method. The car she intends to purchase in 5 years' time currently costs R750 000 and the price is rising with inflation at 9% per annum. Tsholo makes 60 monthly deposits of R14 154,85 in a savings account, starting in one month, to have exactly the right amount to purchase the car in 5 years' time, taking into account that she will trade in the old car. The interest on her investment is 6% per annum compounded monthly for the first three years, and then 8% per annum compounded monthly for a further 2 years.

(a) Calculate the cost of the car, in 5 years' time, that Tsholo would like to purchase.

(2)

(b) Determine the amount that Tsholo will have saved in 5 years' time.

(9)

(c) Hence, calculate the rate of depreciation on Tsholo's current car.

(5)
[16]

QUESTION 3

Modiehi takes out a loan to buy a new building for her successful fashion business. The loan is described by the following recursive rule, where the unit of *n* is months:

$P_{n+1} = 1{,}01P_n - x$; $P_0 = 15\ 000\ 000$ and $P_{240} = 0$.

(a) What does P_n represent?

(2)

(b) Convert the recursive rule into an explicit formula to calculate the value of *x*.

(6)

After 4 years, the recursive rule becomes.

$Q_{m+1} = 1{,}012Q_m - y$ and $Q_{192} = 0$.

(c) Calculate Q_0.

(7)

(d) Calculate y.

(5)

(e) Calculate the effective annual interest rate over the course of the loan, as a percentage.

(5)
[25]

QUESTION 4

Some information about a population, whose growth is thought to be logistical, is given below:

Year	Population (P)	$\frac{\Delta P}{P}$
2022	2 200	
2023	2 352	x
2024	2 525	y
2025	2 614	

(a) Calculate the values of x and y, correct to five decimal places.

(6)

(b) Assuming that (2 352; x) and (2 525; y) lie on the line of best fit, write the equation of the function of $\frac{\Delta P}{P}$ against P, giving parameters to four decimal places.

(5)

(c) Estimate the intrinsic growth rate, correct to 4 decimal places.

(2)

(d) Estimate the carrying capacity of the population, correct to the nearest whole number.

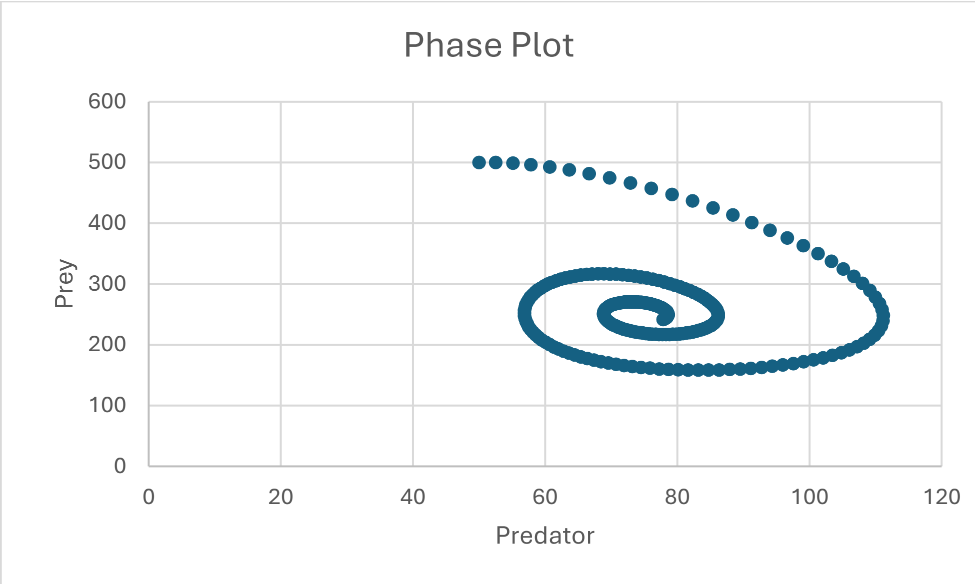
(2)
[15]

QUESTION 5

For a particular predator-prey model, the following system of equations apply:

$$R_{n+1} = 1{,}1R_n - 0{,}0001R_n^2 - 0{,}001R_nF_n$$
$$F_{n+1} = 0{,}95F_n + 0{,}0002F_nR_n$$

Furthermore, a phase plot is as follows:



(a) Write down the initial populations of predator and prey.

(2)

(b) Explain why the predator population initially increases while the prey decreases.

(2)

(c) State the intrinsic growth rate of the prey.

(2)

(d) Find the carrying capacity of the environment.

(3)

(e) Determine the reproductive growth of the predator in the first cycle.

(4)

(f) Calculate the equilibrium values of the predator and prey, showing all working.

(8)
[21]

QUESTION 6

Tsibela owns a large pond in which he breeds koi fish. The koi population is increasing exponentially. To estimate the number of koi, he captures and tags 50 of them. (Tagging involves fastening a tiny ring to a lip of the fish. It is not harmful or painful). After 1 month, he randomly captures a sample of 460 fish and notes that 10 of these are tagged. After a further month, he randomly samples 529 fish and 10 of these are tagged.

(a) Determine the rate of increase of fish per month.

(4)

(b) Determine the initial number of koi in the pond.

(6)
[10]

Total for Module 3: 100 marks

MODULE 4 MATRICES AND GRAPH THEORY

QUESTION 1

1.1 Solve for *x*, *y* and *z* using matrix algebra

$$\begin{bmatrix} -3 & 2 & 7 \\ 8 & 3 & -2 \\ -2 & -1 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -8 \\ 38 \\ -10 \end{bmatrix}$$

(10)

1.2 If matrix A = $\begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 2 \\ -1 & 2 & 1 \end{bmatrix}$

(a) Find the adjoint of matrix A.

(6)

(b) Show that $\det(adj\ A) = [\det A]^{n-1}$ for the given square matrix $A_{n\times n}$.

(8)
[24]

QUESTION 2

If matrix B = $\begin{bmatrix} 3 & 4 \\ 6 & 2 \end{bmatrix}$

2.1 Write down the resultant matrix when the points represented in matrix B have been reflected about $y = -x$.

(3)

2.2 Write down the resultant matrix when the points represented in matrix B have been sheared by a factor of 2, parallel to the y-axis with the y-axis invariant.

(4)

2.3 Write down the resultant matrix when the points represented in matrix B have first been reflected about the x-axis, and then enlarged by $\frac{1}{2}$.

(5)

2.4 The image of B is B' = $\begin{pmatrix} 3 & 4 \\ -12 & -4 \end{pmatrix}$. State in words the transformation(s) performed on B.

(4)
[16]

QUESTION 3

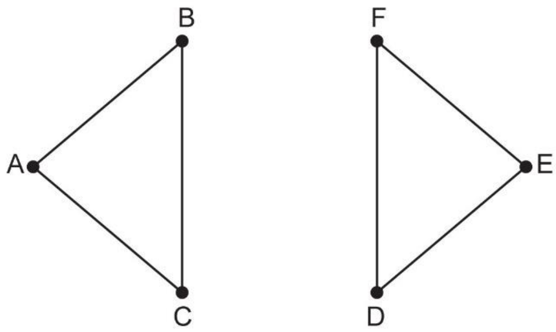
A matrix R = $\begin{bmatrix} 7 & 3 \\ 5 & 2 \end{bmatrix}$ rotated yields an image, $R' = \begin{bmatrix} 5{,}112 & 2{,}235 \\ 6{,}918 & 2{,}829 \end{bmatrix}$

By what degree, accurate to 1 decimal place, were the points represented in matrix R rotated?

[10]

QUESTION 4

4.1



(a) What edges need to be added to the graph, to make it a complete graph?

(6)

(b) What edge or edges need to be added or removed to make the original graph a connected graph?

(2)

(c) What edges can be added so that a Hamiltonian circuit is possible?

(3)

4.2 A *simply connected graph*, is a graph that is both simple and connected. Given a simply connected graph, A, with 6 vertices and 9 edges.

(a) What is the sum of the degrees of the vertices?

(2)

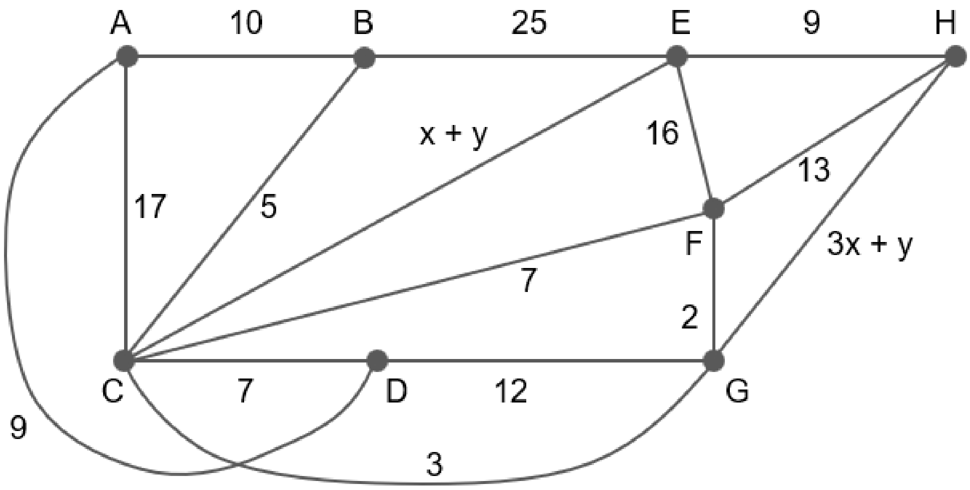
(b) What is the maximum degree any one vertex can have?

(2)

(c) Draw an example of the graph A such that the graph contains an Eulerian circuit.

(5)
[20]

QUESTION 5

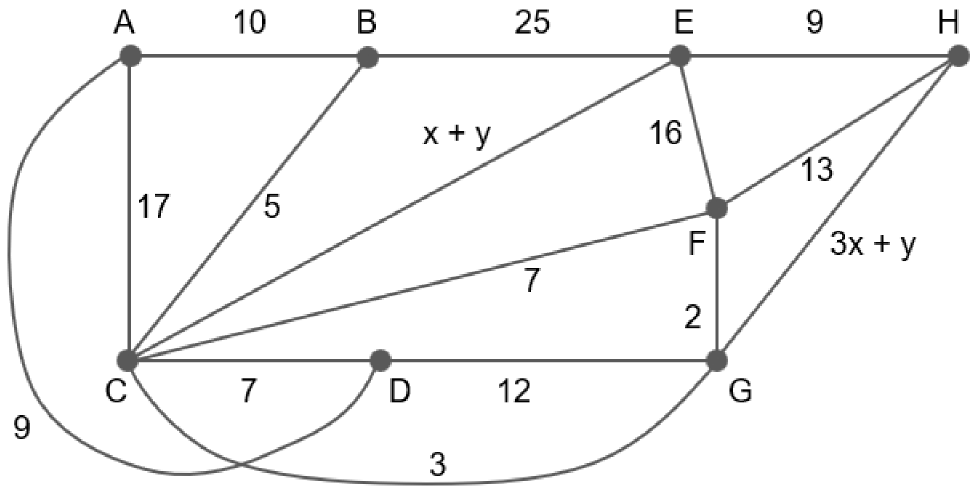


The weights in the above graph, are positive distances.

5.1 Use Dijkstra's Algorithm to solve x and y, if there are three shortest routes, all equal in weight, between A and H.

Show clear evidence of your working, including the termination of non-viable routes.

(14)

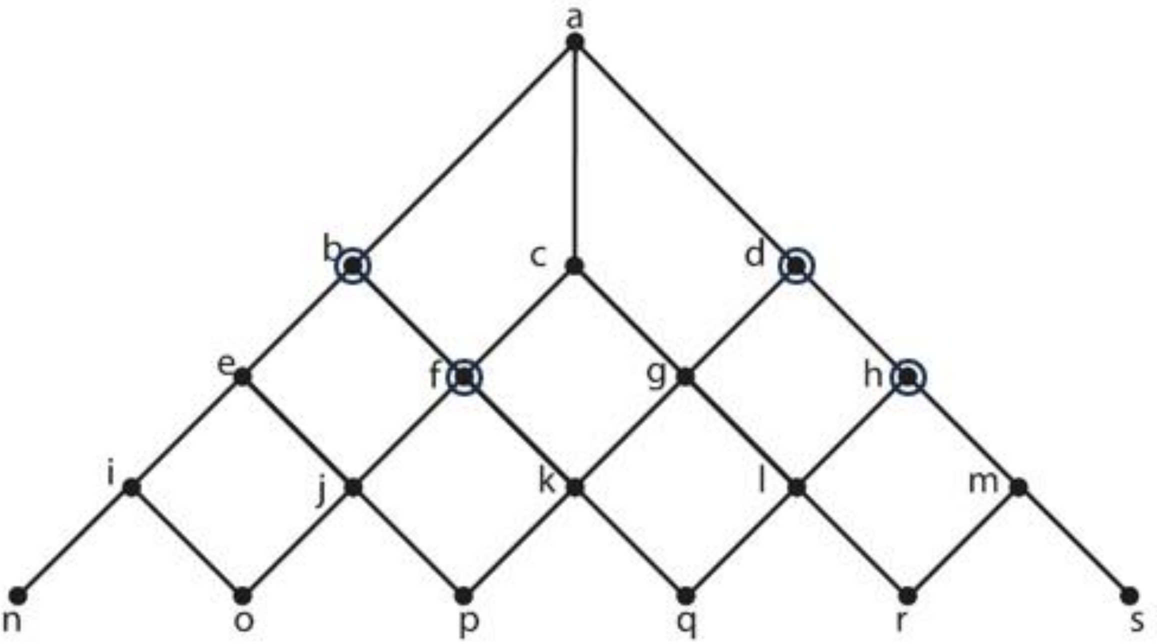


5.2 Use Primm's algorithm, starting at A, to find a minimum spanning tree if $x = 3$ and $y = 6$.

Clearly state the order in which you choose the edges, as well as the length of the minimum spanning tree.

(8)
[22]

QUESTION 6



A wedding planner arranges glasses in a pyramid.

The glasses labelled b, f, d and h have been prefilled with liquid.

Liquid is only poured into glass a, at the top of the pyramid. Liquid distributes evenly as it flows down the pyramid to lower glasses.

6.1 Ten glasses of liquid are poured into glass a. How many glasses of liquid overflow from glass l and glass j?

(5)

6.2 Hence, which glass(es) on the bottom row will have the most liquid in them?

(3)
[8]

Total for Module 4: 100 marks

ADDITIONAL SPACE (ALL QUESTIONS)

REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.