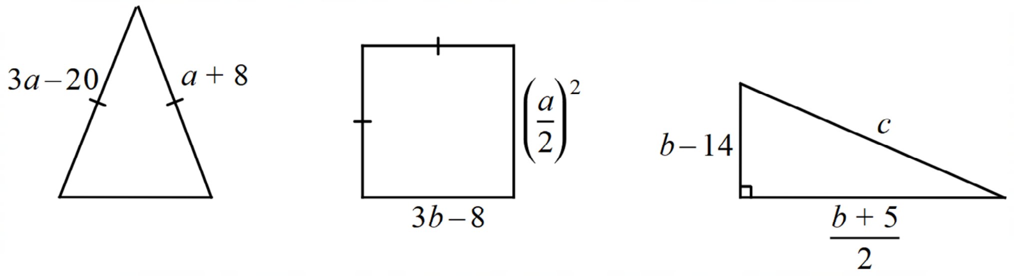
Problem Solving in Grades 8 and 9

Solutions

Syllabus Based Problem Solving

1.
$$2(a+b)=5(a-b)$$
$$\therefore 2a+2b=5a-5b$$
$$\therefore 7b=3a$$
$$\therefore \frac{a}{b}=\frac{7}{3}$$

2.



$$3a-20=a+8 \qquad \therefore 2a=28 \qquad \therefore a=14$$

$$3b-8=\left(\frac{a}{2}\right)^2$$
$$\therefore 3b-8=\left(\frac{14}{2}\right)^2$$
$$\therefore 3b-8=49$$
$$\therefore 3b=57$$
$$\therefore b=19$$

$$c^2=(b-14)^2+\left(\frac{b+5}{2}\right)^2$$
$$\therefore c^2=(19-14)^2+\left(\frac{19+5}{2}\right)^2$$
$$\therefore c^2=5^2+12^2$$
$$\therefore c^2=169$$
$$\therefore c=13$$

3. The factors of 54 are 1; 2; 3; 6; 9; 18; 27; 54.
The perimeter of the base must be 20, so the length and breadth must add up to 10.
$\therefore$ the length and breadth must be 9 cm and 1cm.
$\therefore$ the height $54\div 9=6$ cm.

4.
$$\frac{x(y+2)\times y(x+2)}{xy}=\frac{\frac{4}{5}\times\frac{15}{8}}{\frac{9}{10}}$$

$$\therefore (y+2)(x+2)=\frac{\cancel{4}^{1}}{\cancel{5}_{1}}\times\frac{\cancel{15}^{\cancel{3}\,1}}{\cancel{8}_{\cancel{2}\,1}}\times\frac{\cancel{10}^{5}}{\cancel{9}_{3}}$$

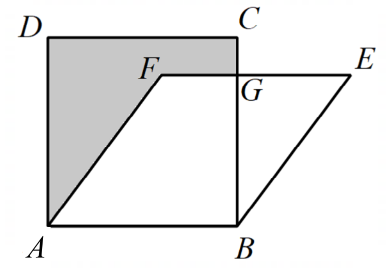
$$\therefore (x+2)(y+2)=\frac{5}{3}$$

5. $AB = 5$ cm (area of square $ABCD$)

$\therefore$ $BG = 4$ cm (area of rhombus $ABEF$)

$BE = 5$ cm (side of rhombus)

$\therefore$ $GE = 3$ cm (Pythag)



$\therefore$ Area of $\Delta BGE = \frac{1}{2} \times 4 \times 3 = 6 \text{ cm}^2$

$\therefore$ Area of $AFGB = 20 - 6 = 14 \text{ cm}^2$

$\therefore$ Shaded area $= 25 - 14 = 11 \text{ cm}^2$

6.

5	4 675
5	935
11	187
17	17
	1

$\therefore$ $4\ 675 = 5^2 \times 11 \times 17$

$\therefore$ the numbers are $5 \times 11 = 55$ and $5 \times 17 = 85$

$\therefore$ the sum is $55 + 85 = 140$.

7. Let the exterior angle be x

$\therefore$ each interior angle is $180° - x$

$\therefore (180° - x) - x = 140°$

$\therefore 2x = 40°$

$\therefore x = 20°$

$\therefore$ no. of sides $= \frac{360°}{20°} = 18$

or

Each interior angle $= \frac{(n-2) \times 180°}{n}$

Each exterior angle $= \frac{360°}{n}$

$\therefore \frac{(n-2) \times 180°}{n} - \frac{360°}{n} = 140°$

$\therefore (n-2) \times 180° - 360° = 140° \times n$

$\therefore 180°n - 360° - 360° = 140°n$

$\therefore 40°n = 720°$

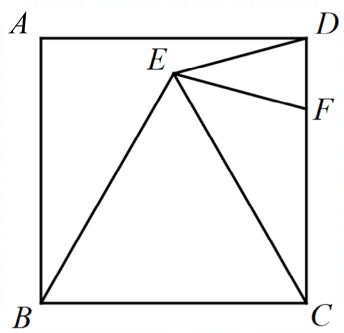
$\therefore n = 18$ sides

8. $E\hat{C}B = 60°$ (ΔEBC is equilateral)

$\therefore E\hat{C}D = 30°$ ($\angle$ of square)

$BC = CD$ (side of square) and $BC = EC$ (ΔEBC is equilateral)

$\therefore CD = EC$



$D\hat{E}C + E\hat{D}C = 150°$ ($\angle$ sum in ΔEDC)

$\therefore D\hat{E}C = E\hat{D}C = 75°$ ($\angle$s opposite = sides)

$\therefore E\hat{F}D = 75°$ ($\angle$s opposite = sides in ΔEFD)

$\therefore D\hat{E}F = 30°$ ($\angle$ sum in ΔEFD)

$\therefore F\hat{E}C = 75° - 30° = 45°$

9.

	Number of boys	Number of girls
Original	x	$4x$
Went on outing	$x+7$	$4x-2$

$$\frac{x+7}{4x-2}=\frac{2}{3}$$
$$\therefore 3x+21=8x-4$$
$$\therefore -5x=-25$$
$$\therefore x=5$$

$\therefore$ total number of children on the outing

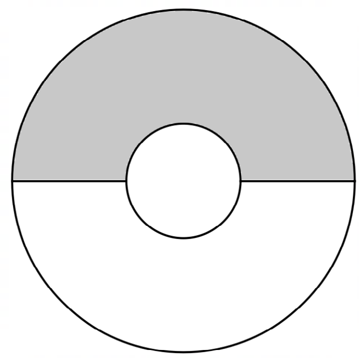
$=(5+7)+(4\times5-2)$

$=30$

10. Shaded region: $A=\frac{\pi(3r)^2}{2}-\frac{\pi r^2}{2}$

$\therefore A=\frac{9\pi r^2}{2}-\frac{\pi r^2}{2}$

$\therefore A=4\pi r^2$



Small circle: $A=\pi r^2$

$\therefore$ ratio $=\frac{4\pi r^2}{\pi r^2}=4$

Non-Syllabus Based Problem Solving

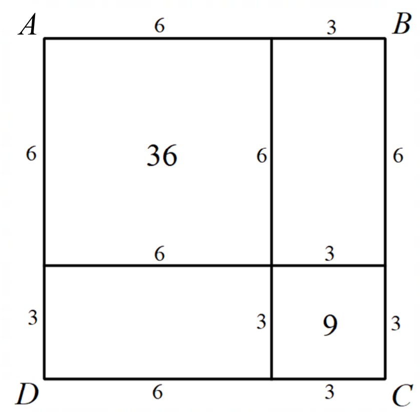
1. $12 = 2 \times 2 \times 3$

   $\therefore$ the numbers must be $1;\ -1;\ 2;\ -2;\ 3$

   $\therefore$ the smallest integer is -2.

2. The sides of the small squares are 6 cm and 3 cm.

   $\therefore$ the area of $ABCD = 9 \times 9 = 81\ \text{cm}^2$



3. The numbers that use 2 and / or 3 are:

   2; 3; 12; 13; 20; 21; 22; 23; 24; 25; 26; 27; 28; 29; 30; 31; 32; 33; 34; 35; 36; 37; 38; 39; …

   Up to this point we have 14 2's, and 14 3's.

   We need six more 2's and five more 3's.

   These come from:
   42; 43; 52; 53; 62; 63; 72; 73; 82; 83; 92

   $\therefore N = 92$

4. 25 heads where each animal has 2 legs, equals 50 legs.
   But there are 60 legs, so the extra 10 legs belong to the pigs.
   $\therefore$ there are 5 pigs.

5. C has to be 5, as this is the only number that adds three times and ends in the same number.
   So we have:

$$\begin{array}{cccc} & A & {}^{1}B & 5 \\ & A & B & 5 \\ + & A & B & 5 \\ \hline & 5 & 5 & 5 \\ \hline \end{array}$$

   This means that B has to be 8, as if I add 8 three times, and the 1 that is carried, I end in a 5.
   Now we have:

$$\begin{array}{cccc} & {}^{2}A & {}^{1}8 & 5 \\ & A & 8 & 5 \\ + & A & 8 & 5 \\ \hline & 5 & 5 & 5 \\ \hline \end{array}$$

   $\therefore$ A has to be 1.

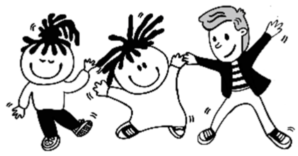
   $\therefore$ A + B + C = 1 + 8 + 5 = 14

6. The mother had her children at age 25; 28 and 31.

Ages now

Child 3	Child 2	Child 1	Mother
x	$x+3$	$x+6$	$x+31$

$$x+x+3+x+6=x+31$$
$$\therefore 2x=22$$
$$\therefore x=11$$



$\therefore$ the youngest child is 11 years old.

7. The cheapest way to buy the caps is to buy them in batches of 6.

$2025 \div 6 = 337$ batches of 6, with 3 caps remaining.

$\therefore$ the cost $= 337 \times 100 + 3 \times 20 = \text{R}33\ 760$

8. Start with Eric, as he has to shake all five hands.

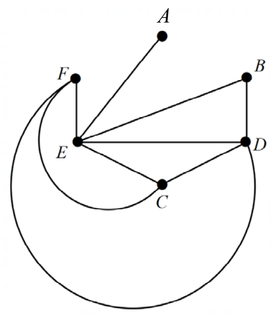
Angie cannot shake hands with anyone else.

Dene must now shake hands with all the others that are left.

So Bonnie is now finished.

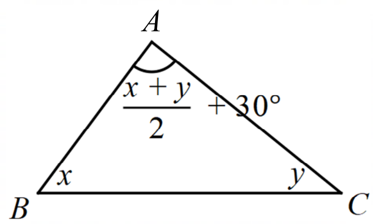
All that remains is for Clive to get one more handshake, which must be with Frank.

$\therefore$ Frank shook hands with three people.



9. Let two of the angles be x and y.

$\therefore$ the third angle is $\dfrac{x+y}{2}+30°$



$$x+y+\frac{x+y}{2}+30°=180°$$
$$\therefore \frac{3}{2}x+\frac{3}{2}y=150°$$
$$\therefore x+y=100°$$
$$\therefore \hat{A}=80°$$

To get the largest possible angle, the two remaining angles must be 99° and 1°.

$\therefore$ the largest possible angle is 99°.

10. $16-x+20-x=13$

$$\therefore x=23$$
$$\therefore x=11\frac{1}{2}$$

$\therefore$ radii are $11\frac{1}{2};\ 4\frac{1}{2};\ 8\frac{1}{2}$

$\therefore$ the smallest radius is $4\frac{1}{2}$ units.