Problem Solving in FET Solutions

Grade 10

1. $a = b - 1$ and $c = b + 1$

$$\begin{aligned} \text{LHS} &= (b-1)^2 + (b+1)^2 \\ &= b^2 - 2b + 1 + b^2 + 2b + 1 \\ &= 2b^2 + 2 \\ &\neq \text{RHS} \end{aligned}$$

It is impossible for $2b^2 = 2b^2 + 2$.

$\therefore$ it is not possible to find three consecutive integers that satisfy $a^2 + c^2 = 2b^2$.

2. $$T_n = \frac{n}{x^2 + (n+2)x}$$

$$\therefore \frac{30}{x^2 + 32x} = \frac{10}{91}$$

$$\therefore 10x^2 + 320x = 2\ 730$$

$$\therefore x^2 + 32x - 273 = 0$$

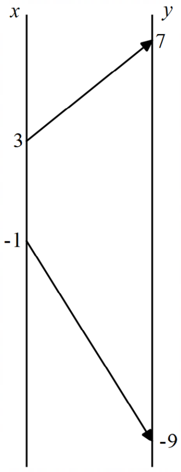
$$\therefore (x-7)(x+39) = 0$$

$$\therefore x = 7 \quad \text{or} \quad x = -39$$

3. 3.1 $3a + b = 7$ ❶ and $-a + b = -9$ ❷

$\therefore 4a = 16$ ❶ – ❷

$\therefore a = 4$ and $b = -5$



3.2 $y = 4x - 5$

For the shortest arrow, $x = y$

$$\therefore 4x - 5 = x$$

$$\therefore 3x = 5$$

$$\therefore x = \frac{5}{3}$$

$\therefore$ the end-points are $x = \frac{5}{3}$ and $y = \frac{5}{3}$.

4. Distance between –3 and 9 is 12, therefore the amplitude is 6.

Midway between –3 and 9 is 3, therefore the graph is moved 3 units up.

$\therefore \ a = 6$ and $b = 3$.

5. $m_{AB} = -\frac{1}{4}$

$\therefore y - 3 = -\frac{1}{4}\left(x + 4\right)$

$\therefore y = -\frac{1}{4}x + 2$

$4x - 15 = -\frac{1}{4}x + 2$

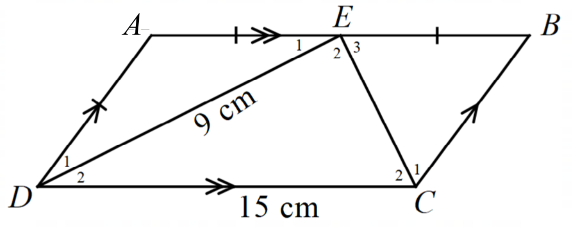
$\therefore \frac{17}{4}x = 17$

$\therefore x = 4$

$\therefore$ the midpoint of AB is $\left(4; 1\right)$.

$\therefore B\left(12;\ -1\right)$

6. Let $\hat{D}_1 = x$



$\therefore \hat{E}_1 = x$ ($\angle$s opp = sides)

$\therefore \hat{A} = 180° - 2x$ ($\angle$ sum of ΔADE)

$\therefore \hat{B} = 2x$ (coint $\angle$s; $AD \parallel BC$)

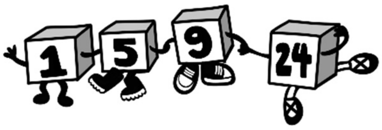
$AD = BC$ (opp sides of parm)

$\therefore \hat{E}_3 = \hat{C}_1$ ($\angle$s opp = sides)

$\therefore \hat{E}_3 = 90° - x$ ($\angle$ sum of ΔEBC)

$\therefore \hat{E}_2 = 90°$ ($\angle$s on a str line)

$\therefore EC = 12$ cm (Pythag in ΔDEC)

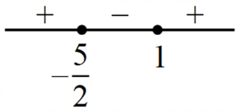


Grade 11

1. $$2x^2+3x-5>0$$

$$\therefore (2x+5)(x-1)>0$$

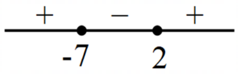
$$\therefore x<-\frac{5}{2} \quad \text{or} \quad x>1$$



$$x^2+5x-14\le 0$$

$$\therefore (x+7)(x-2)\le 0$$

$$\therefore -7\le x\le 2$$



$$\therefore -7\le x<-\frac{5}{2} \quad \text{or} \quad 1<x\le 2$$

2. Odd terms: $13;\ 7;\ 1;\ -5;\ -11;\ \dots$

$$\therefore T_n=-6n+19$$

$$\therefore T_{25}=-6(25)+19=-131$$

Even terms:

$$\begin{array}{ccccccccccc} 7 & & 5 & & -1 & & -11 & & -25 & & \dots \\ & -2 & & -6 & & -10 & & -14 & & & \\ & & -4 & & -4 & & -4 & & & & \end{array}$$

$$2a=-4 \qquad \therefore a=-2$$

$$3a+b=-2 \qquad \therefore b=4$$

$$a+b+c=7 \qquad \therefore c=5$$

$$\therefore T_n=-2n^2+4n+5$$

$$T_{25}=-2(25)^2+4(25)+5=-1\ 145$$

$$\therefore T_{49}+T_{50}=-131+(-1\ 145)=-1\ 276$$

3. $$x^2+xy+12=0$$

$$\therefore x^2+x(mx+8)+12=0$$

$$\therefore x^2+mx^2+8x+12=0$$

$$\Delta=0$$

$$\therefore (8)^2-4(1+m)(12)=0$$

$$\therefore 64-48-48m=0$$

$$\therefore -48m=-16$$

$$\therefore m=\frac{1}{3}$$

4. $m_{AB} = m_{AC}$

$$\therefore \frac{3-5p-3}{2+4p-2} = \frac{13-3}{2q+20-2}$$

$$\therefore \frac{-5p}{4p} = \frac{10}{2q+18}$$

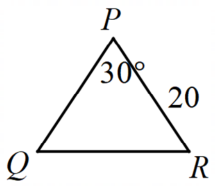
$$\therefore \frac{-5}{4} = \frac{10}{2q+18}$$

$$\therefore -10q - 90 = 40$$

$$\therefore -10q = 130$$

$$\therefore q = -13$$

5. $$\frac{QR}{\sin 30^\circ} = \frac{20}{\sin Q}$$



$$\therefore QR = \frac{20 \sin 30^\circ}{\sin Q}$$

$$\therefore QR = \frac{20 \times \frac{1}{2}}{\sin Q}$$

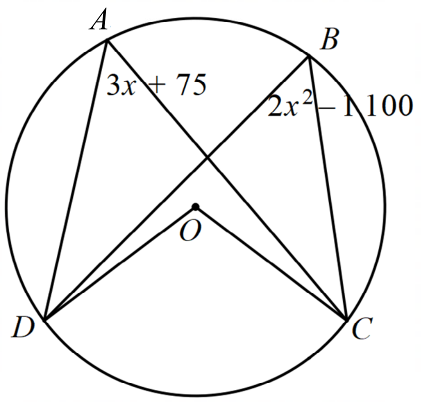
$$\therefore QR = \frac{10}{\sin Q}$$

For a minimum value of QR, $\sin Q$ must be a maximum.

$\therefore \sin Q = 1$

$\therefore$ the minimum length of QR is 10 units.

6. $3x + 75 = 2x^2 - 1100$ ($\angle$s in the same seg)



$$\therefore 2x^2 - 3x - 1175 = 0$$

$$\therefore x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-1175)}}{2(2)}$$

$$\therefore x = 25 \quad \text{or} \quad x = -23\frac{1}{2}$$

$$\therefore \hat{A} = 150^\circ \quad \text{or} \quad \hat{A} = 4\frac{1}{2}^\circ$$

$\therefore D\hat{O}C = 300^\circ$ or $D\hat{O}C = 9^\circ$ ($\angle$ at centre = 2 × $\angle$ at circum)

Grade 12

1.

$$3^p = 9 \times 27^q$$
$$\therefore 3^p = 3^2 \times 3^{3q}$$
$$\therefore 3^p = 3^{2+3q}$$
$$\therefore p = 2 + 3q$$

$$\sum_{k=1}^{5}\left(kp^2 + q\right) = 230$$
$$\therefore \left(p^2 + q\right) + \left(2p^2 + q\right) + \ldots + \left(5p^2 + q\right) = 230$$
$$\therefore 15p^2 + 5q = 230$$
$$\therefore 3p^2 + q = 46$$

$$\therefore 3\left(2 + 3q\right)^2 + q = 46$$
$$\therefore 12 + 36q + 27q^2 + q = 46$$
$$\therefore 27q^2 + 37q - 34 = 0$$
$$\therefore \left(q + 2\right)\left(27q - 17\right) = 0$$
$$\therefore q = -2 \quad \text{or} \quad q = \frac{17}{27}$$
$$\therefore p = -4 \quad \text{or} \quad p = \frac{35}{9}$$
$$\therefore p = -4 \quad \text{and} \quad q = -2$$

2. Horizontal moves: 12; – 6; 3; ...

$$S_\infty = \frac{12}{1 - \left(-\frac{1}{2}\right)} = 8$$

Vertical moves: 4; – 2; 1; ...

$$S_\infty = \frac{4}{1 - \left(-\frac{1}{2}\right)} = \frac{8}{3}$$

$\therefore$ the robot will end up at $\left(8 + 8; -5 + \frac{8}{3}\right) = \left(16; -2\frac{1}{3}\right)$

3.

$$f\left(4\right) = 0$$
$$\therefore 64 + 4p + q = 0$$
$$\therefore 4p + q = -64 \quad ①$$

$$f\left(-1\right) = 0$$
$$\therefore -1 - p + q = 0$$
$$\therefore -p + q = 1 \quad ②$$

$$\therefore 5p = -65 \qquad ① - ②$$
$$\therefore p = -13$$
$$\therefore q = -12$$

$$\therefore f\left(x\right) = x^3 - 13x - 12$$
$$\therefore f\left(x\right) = \left(x - 4\right)\left(x^2 + 4x + 3\right)$$
$$\therefore f\left(x\right) = \left(x - 4\right)\left(x + 1\right)\left(x + 3\right)$$

$\therefore$ the third root is –3

4. $$y = \frac{x^3}{3} - ax^2 - \frac{5}{4}x$$

$$\therefore \frac{dy}{dx} = x^2 - 2ax - \frac{5}{4}$$

At $x = \frac{3}{2}$: $\quad \frac{dy}{dx} = \left(\frac{3}{2}\right)^2 - 2a\left(\frac{3}{2}\right) - \frac{5}{4} = -3a + 1$

At $x = -\frac{1}{2}$: $\quad \frac{dy}{dx} = \left(-\frac{1}{2}\right)^2 - 2a\left(-\frac{1}{2}\right) - \frac{5}{4} = a - 1$

$$(-3a+1)(a-1) = -1$$

$$\therefore -3a^2 + 4a - 1 = -1$$

$$\therefore -3a^2 + 4a = 0$$

$$\therefore -a(3a-4) = 0$$

$$\therefore a = 0 \quad \text{or} \quad a = \frac{4}{3}$$

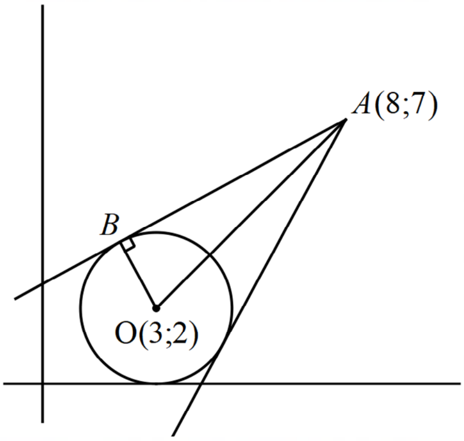
5. Centre of circle is $O(3;2)$ and radius $= 2$

$$OA = \sqrt{(8-3)^2 + (7-2)^2} = 5\sqrt{2}$$

$O\hat{B}A = 90°$ (tan ⊥ rad)

$$\therefore AB = \sqrt{\left(5\sqrt{2}\right)^2 - 2^2} = \sqrt{46}$$

$\therefore$ the lengths of the tangents are $\sqrt{46}$



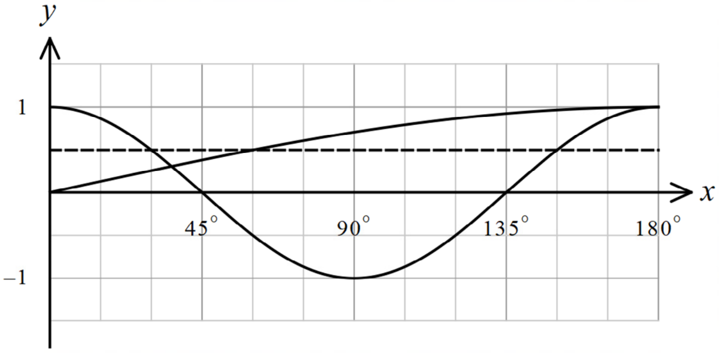
6. $$\cos 2x < 0$$

$$\therefore 45° < x < 135°$$

$$\sin \frac{1}{2}x \geq 0{,}5$$

$$\therefore 60° \leq x \leq 180°$$

$$\therefore 60° \leq x < 135°$$



7. Let radius $= r$

$$\therefore OC = 25 - r$$

$O\hat{D}C = 90°$ (tan ⊥ rad)

$$(25-r)^2 = r^2 + 10^2 \text{ (Pythag)}$$

$$\therefore 625 - 50r + r^2 = r^2 + 100$$

$$\therefore -50r = -525$$

$$\therefore r = 10\frac{1}{2} \text{ units}$$