MOMENTUM AND IMPULSE

MOMENTUM ($\vec{p}$)

A moving object has momentum. The momentum of the object at any given instant depends on the mass and velocity of the object at that instant.

The **momentum** ($\vec{p}$) of an object is the product of the mass (m) and the velocity ($\vec{v}$) of the object.

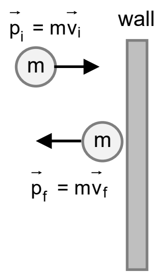
momentum = mass × velocity

$$\vec{p} = m\vec{v}$$

Symbol: $\vec{p}$ SI unit: **kg·m·s^{-1}** Vector nature: **vector**
(magnitude: **mv** direction: **same direction as $\vec{v}$**)

Change in momentum ($\Delta\vec{p}$)

A net force ($\vec{F}_{net}$) acting on an object, for a specific time interval (Δt), can change the momentum of the object.



Change in momentum: $\Delta\vec{p} = \vec{p}_f - \vec{p}_i$
$= m\vec{v}_f - m\vec{v}_i$
$= m(\vec{v}_f - \vec{v}_i)$
$= m\Delta\vec{v}$

f = final; i = initial

Rate of change in momentum (Newton II in terms of momentum)

The net force ($\vec{F}_{net}$) acting on an object is equal to the rate of change in its momentum.

$$\vec{F}_{net} = \frac{\Delta\vec{p}}{\Delta t}$$

Newton II

$$\vec{F}_{net} = m\vec{a} = \frac{m\Delta\vec{v}}{\Delta t} = \frac{\Delta\vec{p}}{\Delta t} \qquad \vec{a} = \frac{\Delta\vec{v}}{\Delta t}$$

Practical application: Safety in everyday situations

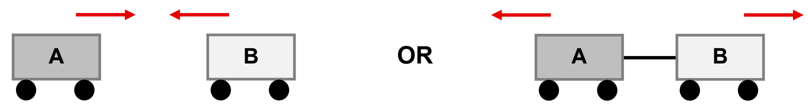
Consider situations where the momentum of an object changes due to a collision ($\Delta\vec{p} = \vec{p}_f - \vec{p}_i$), e.g. a collision of cars, a person jumping onto the floor, a ball being caught by a cricket player.

❶ The net force on the object is given by: $\vec{F}_{net} = \frac{\Delta\vec{p}}{\Delta t}$.

❷ For the same $\Delta\vec{p}$, the longer the time interval (Δt), the smaller is $\vec{F}_{net}$ on the object, and the lower the chance of injuries. $\vec{F}_{net}\,(\downarrow) = \frac{\Delta\vec{p}}{\Delta t\,(\uparrow)}$

The Law of conservation of momentum

Consider two objects in a straight line in an isolated system:



- During collisions and explosions, both objects have the same change in momentum, but in opposite directions ($\Delta\vec{p}_A = -\Delta\vec{p}_B$).
- Thus the total linear momentum of the system remains constant ($\Delta\vec{p}_A + \Delta\vec{p}_B = 0$).

An **isolated system** is a system on which the resultant/net external force is zero.

An isolated system thus *excludes* any external forces acting on the system of colliding objects, e.g. an *applied force* or *force of friction*.

Law of conservation of momentum:
The total linear momentum in an isolated system is constant in magnitude and direction.

$$\Sigma\vec{p}_i = \Sigma\vec{p}_f$$

$$\vec{p}_{i1} + \vec{p}_{i2} = \vec{p}_{f1} + \vec{p}_{f2}$$

❶ $m_1\vec{v}_{i1} + m_2\vec{v}_{i2} = m_1\vec{v}_{f1} + m_2\vec{v}_{f2}$ — *If the objects remain apart before and after the collision.*

❷ $m_1\vec{v}_{i1} + m_2\vec{v}_{i2} = (m_1 + m_2)\vec{v}_{f2}$ — *If the objects join during the collision and move further as a unit.*

or ❸ $(m_1 + m_2)\vec{v}_{i2} = m_1\vec{v}_{f1} + m_2\vec{v}_{f2}$ — *If the objects separate during an explosion.*

Important:
Remember to indicate the direction of motion and of the momentum of each object with the correct sign.

Elastic and inelastic collisions

- During **elastic** collisions in an isolated system, the **kinetic energy** of the system of objects is **conserved**.

$$\Sigma E_{k_i} = \Sigma E_{k_f}$$

$$\frac{1}{2}m_1 v_{i1}^2 + \frac{1}{2}m_2 v_{i2}^2 = \frac{1}{2}m_1 v_{f1}^2 + \frac{1}{2}m_2 v_{f2}^2$$

Remember: *$E_k = \frac{1}{2}mv^2$*

- During **inelastic** collisions in an isolated system, the **kinetic energy** of the system of objects is **not conserved**, but is converted to other forms of energy, e.g. heat or sound energy:

$$\Sigma E_{k_i} \neq \Sigma E_{k_f}$$

$$\frac{1}{2}m_1 v_{i1}^2 + \frac{1}{2}m_2 v_{i2}^2 \neq \frac{1}{2}(m_1 + m_2)v_f^2$$

If you have to prove that a collision is inelastic, calculate ΣE_{k_i} and ΣE_{k_f} separately and show that they are not equal.

- During both **elastic** and **inelastic** collisions in an isolated system, the **momentum** of the system of objects is **conserved**:

$$\Sigma\vec{p}_i = \Sigma\vec{p}_f$$

IMPULSE

- **Definition:**

Impulse is the product of the net force $(\vec{F}_{net})$ acting on an object, and the time interval (Δt) the force is acting.

$$\textbf{Impulse} = \vec{F}_{net}\,\Delta t$$

Unit: **N·s** or **kg·m·s^{-1}** Vector nature: **vector**
(magnitude: **$F_{net}\Delta t$** direction: **same direction as that of F_{net}**)

Impulse-momentum theory

- **Definition:**

Impulse = change in momentum

$$\textbf{Impulse} = \vec{F}_{net}\,\Delta t = \Delta\vec{p}$$

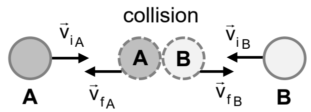
$$\therefore \vec{F}_{net}\,\Delta t = m\Delta\vec{v}$$

NB: *$\vec{F}_{net}$, $\Delta\vec{p}$ and impulse have the same direction.*

MOMENTUM AND IMPULSE GRAPHS

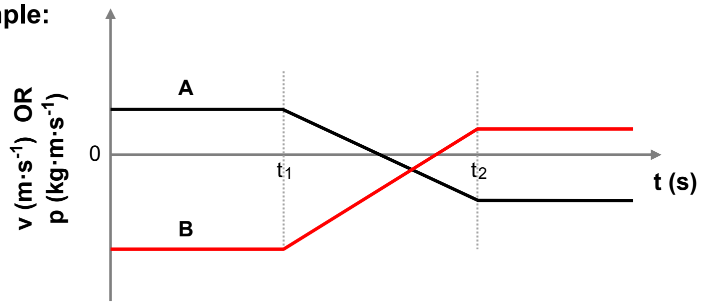
Different types of graphs can be drawn to illustrate the concept of momentum and impulse, e.g. a **velocity-time** graph or a **momentum-time** graph or an **F_{net}-time** graph.

- **Velocity-time graph or momentum-time graph**



Both objects A and B had an initial velocity $(\vec{v}_i)$ and momentum $(\vec{p}_i)$ before the collision, and a final velocity $(\vec{v}_f)$ and momentum $(\vec{p}_f)$ after the collision. So the values of $\vec{v}_i$ or $\vec{p}_i$ and $\vec{v}_f$ or $\vec{p}_f$ can be obtained from the graph and Δp for one or both objects can be calculated. The duration of the collision (Δt) is from t_1 to t_2.

Example:



The **gradient** of the straight-line graph between t_1 and t_2 for the graph:

❶ p versus t is: $\vec{F}_{net} = \frac{\Delta\vec{p}}{\Delta t}$

❷ v versus t is: $\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$

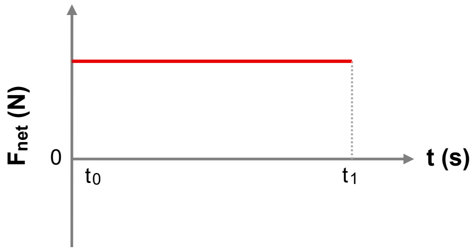
- **F_{net}-time graph**

  The average force (F_{net}) on an object can be obtained from the graph and the impulse can be obtained by calculating the area underneath the graph.

  Impulse $= \vec{F}_{net}\Delta t = \Delta\vec{p}$.

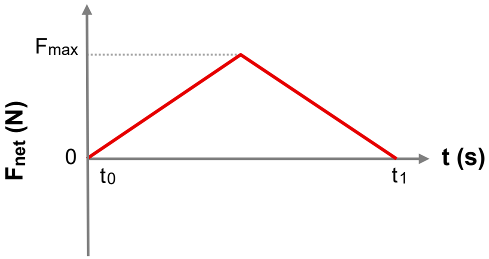
  The duration of the collision (Δt) is from t_0 to t_1.

Examples:



Impulse $= F_{net}\Delta t$
$= \ell \times b$

OR



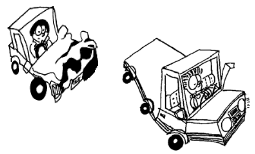
Impulse $= F_{net(ave)}\Delta t$
$= \frac{1}{2}F_{max}\Delta t$
$= \frac{1}{2}bh$

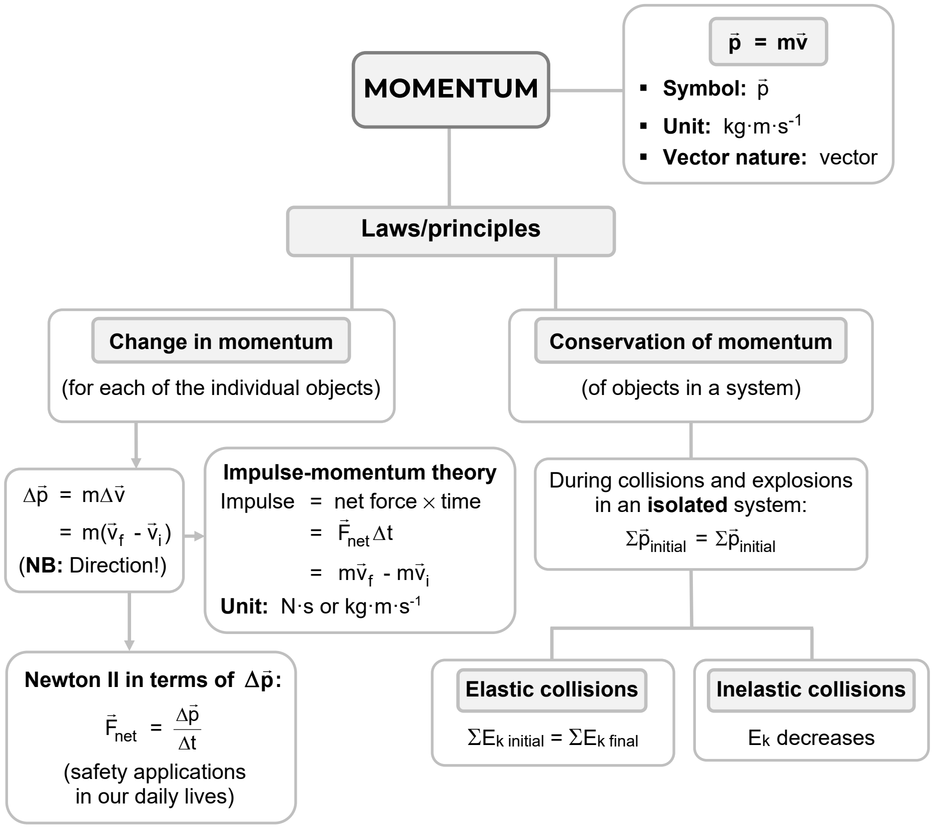
Important notes

- Learn to identify a problem as a **momentum** problem. Usually you would choose between (1) using the impulse-momentum theory, in which you consider one object and work with the change in momentum of the object, or (2) you would use the Law of Conservation of Momentum of all the objects in the system.
- Take note of the difference between momentum $(\vec{p})$, change in momentum $(\Delta\vec{p})$ and rate of change in momentum $\left(\frac{\Delta\vec{p}}{\Delta t}\right)$.
- Always consider the vector nature of momentum, impulse and F_{net}, and use the correct sign in any calculations to indicate their directions.
- Learn to read, analyse and interpret graphs, e.g. to find the initial and final values of velocity or momentum from a graph.

Definitions

- An **isolated system** is a system on which the resultant/net external force is zero.
- The **momentum** $(\vec{p})$ of an object is the product of the mass (m) and the velocity $(\vec{v})$ of the object.
- **Impulse** is the product of the net force $(\vec{F}_{net})$ acting on an object, and the time interval (Δt) the force is acting.
- **Law of conservation of momentum:** The total linear momentum in an isolated system is constant in magnitude and direction.





EXAMPLES

Question 1

1.1 Define momentum.

1.2 Explain what the vector nature of momentum is attributed to.

ANSWER

1.1 Momentum is defined as the product of the mass and velocity of an object.

1.2 Momentum is the product of a scalar (mass) and a vector (velocity). The vector nature is ascribed to velocity, which is a vector, and thus it is a quantity that has both magnitude and direction. (The direction of the momentum is the same as the direction of the velocity of an object.)

Question 2

Calculate the momentum of a cricket ball with a mass of 165 g that is bowled eastwards at 120 km·h^{-1}.

ANSWER

$p = mv$
$= (0{,}165)(33{,}33...)$
$= 5{,}5\ \text{kg·m·s}^{-1}$ east

$m = 165\ \text{g} = 0{,}165\ \text{kg}$

$v = 120\ \text{km·h}^{-1} = \frac{(120)(1\ 000)}{3\ 600} = 33{,}33\ \text{m·s}^{-1}$

OR $v = \frac{120\ \text{km·h}^{-1}}{3{,}6} = 33{,}33\ \text{m·s}^{-1}$

Question 3

Calculate the initial momentum and the change in momentum of the following objects:

3.1 A tennis ball, with a mass of 60 g, hits the racquet of a player at a velocity of 12 m·s^{-1} to the right and is hit back in the opposite direction at a velocity of 15 m·s^{-1}.

3.2 A car, with a mass of 800 kg, travels southwards at a constant velocity of 120 km·h^{-1}. The driver suddenly applies the brakes and brings the car to a standstill in 30 s.

ANSWER

3.1 $p_i = mv_i$
$= (0{,}06)(12) = 0{,}72\ \text{kg·m·s}^{-1}$ right

$\Delta p = m(v_f - v_i)$
$= 0{,}06(-15 - 12)$
$= 0{,}06(-27) = -1{,}62$

$\therefore \Delta p = 1{,}62\ \text{kg·m·s}^{-1}$ away from racquet (left)

OR $p_f = mv_f = (0{,}06)(-15) = -0{,}9\ \text{kg·m·s}^{-1}$
$\therefore \Delta p = p_f - p_i = -0{,}9 - 0{,}72 = -1{,}62\ \text{kg·m·s}^{-1}$
$= 1{,}62\ \text{kg·m·s}^{-1}$ left

Assume direction of racquet (right) is **+**
$m = 60\ \text{g} = 0{,}06\ \text{kg}$
$v_i = 12\ \text{m·s}^{-1}$
$v_f = -15\ \text{m·s}^{-1}$

3.2 $p_i = m_i v_i$
$= (800)(33{,}33)$
$= 26\ 666{,}67\ \text{kg·m·s}^{-1}$ south

$\Delta p = m(v_f - v_i)$
$= (800)(0 - 33{,}33)$
$= -26\ 666{,}67 = 26\ 666{,}67\ \text{kg·m·s}^{-1}$ north

Take south as **+**
$m = 800\ \text{kg}$
$v_i = 120\ \text{km·h}^{-1} = 33{,}33\ \text{m·s}^{-1}$
$v_f = 0\ \text{m·s}^{-1}$

Question 4

A hockey ball, with a mass of 150 g, rolls towards a player at a velocity of 15 $m\cdot s^{-1}$. The player hits the ball in the opposite direction so that its momentum changes by 3 $kg\cdot m\cdot s^{-1}$.

4.1 At what velocity does the hockey ball move away from the player's stick?

4.2 Calculate the net force exerted on the ball if the contact time between the stick and the ball is 0,2 s.

ANSWER

4.1
$\Delta p = m(v_f - v_i)$
$\therefore -3 = (0,15)(v_f - 15)$
$\therefore -\frac{3}{0,15} = v_f - 15$
$\therefore -20 = v_f - 15$
$\therefore v_f = 15 - 20$
$= -5$
$= 5\ m\cdot s^{-1}$ away from player

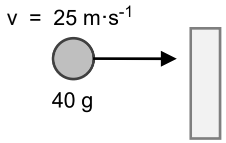
Towards the player **+**
$m = 0,15$ kg
$v_i = 15\ m\cdot s^{-1}$
$v_f = ?$
$\Delta p = -3\ kg\cdot m\cdot s^{-1}$

4.2
$F_{net}\Delta t = \Delta p$
$\therefore F_{net} = \frac{-3}{0,2}$
$= -15$ N
$= 15$ N away from player

Towards player **+**

Question 5

A tennis ball with a mass of 40 g hits a wall perpendicularly at 25 $m\cdot s^{-1}$ to the right. An average force of 8 N is exerted on the ball during the contact time of 0,2 s with the wall. (Ignore air resistance.)



5.1 Calculate the impulse of the wall on the ball.

5.2 Calculate the change in momentum experienced by the ball.

5.3 Give the rate of change of momentum of the ball.

5.4 Calculate the velocity at which the ball bounces off the wall.

ANSWER

5.1 impulse $= F_{net}\Delta t$
$= (-8)(0,2)$
$= -1,6\ N\cdot s$
$= 1,6\ N\cdot s$ away from wall

5.2 $F_{net}\Delta t = \Delta p$
$\therefore \Delta p = 1,6\ kg\cdot m\cdot s^{-1}$ away from wall

Assume towards wall (right) is **+**
$F_{net} = F_{(wall\ on\ ball)}$
$= -8$ N (the force is away from the wall (left))
$\Delta t = 0,2$ s

5.3 $\frac{\Delta p}{\Delta t} = F_{net}$
$= -8\ kg\cdot m\cdot s^{-1}$ (given)

NB: *The rate of change of momentum = net force. This is derived from the units as follows:*
$\frac{kg\cdot m\cdot s^{-1}}{s} = kg\cdot m\cdot s^{-2} = N$ ($ma = F_{net}$)

$\therefore$ rate of change of momentum $\left(\frac{\Delta p}{\Delta t}\right) = 8$ N away from wall

5.4 $F_{net}\Delta t = \Delta p$
$= mv_f - mv_i$
$\therefore -1,6 = 0,04v_f - (0,04)(25)$
$\therefore 0,04v_f = -0,6$
$\therefore v_f = -15\ m\cdot s^{-1} = 15\ m\cdot s^{-1}$ away from the wall

Question 6

State the law of conservation of linear momentum.

ANSWER

The total linear momentum in an isolated system remains constant in magnitude and direction.

Question 7

A car with a mass of 650 kg travels at a velocity of 60 $km\cdot h^{-1}$ and crashes into the back of a truck with a mass of 1 000 kg that is travelling in the same direction at 30 $km\cdot h^{-1}$. Immediately after the collision, the truck travels at 36 $km\cdot h^{-1}$ in the original direction of motion. (Ignore the effects of friction.)

7.1 Calculate the velocity of the car immediately after the collision.

7.2 What was the change in momentum of the car during the collision?

7.3 What should the change in momentum of the truck be? Give a reason for your answer.

7.4 Use a calculation to confirm your answer in Question 7.3.

ANSWER

7.1

Take original direction as **+**

car:

m_c = 650 kg

v_{ic} = 60 km·h^{-1} = $\frac{(60)(1\,000)}{3\,600}$ = 16,67 m·s^{-1}

v_{fc} = ?

truck:

m_t = 1 000 kg

v_{it} = 30 km·h^{-1} = 8,33 m·s^{-1}

v_{ft} = 36 km·h^{-1} = 10 m·s^{-1}

$$\Sigma p_{initial} = \Sigma p_{final}$$
$$\therefore m_c v_{ic} + m_t v_{it} = m_c v_{fc} + m_i v_{ft}$$
$$\therefore (650)(16{,}67) + (1\,000)(8{,}33) = 650 v_{fc} + 1\,000(10)$$
$$10\,833{,}33 + 8\,333{,}33 = 650 v_{fc} + 10\,000$$
$$\therefore v_{fc} = \frac{9\,166{,}67}{650}$$
$$= 14{,}10 \text{ m·s}^{-1}$$
$$= 50{,}77 \text{ km·h}^{-1} \text{ in the original direction}$$

(14,10)(3,6) = 50,76 km·h^{-1}

7.2 Δp (car) = $p_f - p_i$
= $mv_{fc} - mv_{ic}$
= (650)(14,10) - (650)(16,67)
= -1 666,67
= 1 666,67 kg·m·s^{-1} in opposite direction of motion

7.3 1 666,67 kg·m·s^{-1} in the original direction of motion because the total momentum in the system remains constant.

$\therefore$ Δp (system) = Δp (car) + Δp (truck) = 0

$\therefore$ Δp (car) = $-\Delta p$ (truck)

7.4 Δp (truck) = $p_f - p_i$ = $mv_{ft} - mv_{it}$
= (1 000)(10) - (1 000)(8,33)
= 1 666,67 kg·m·s^{-1} in the original direction

Question 8

8.1 Define an elastic collision.

8.2 If the car and truck in Question 3 travel at these same speeds and collide, but are joined together after the collision, calculate the velocity of the car-truck system immediately after the collision.

8.3 Do the necessary calculations to determine whether the collision was elastic or not.

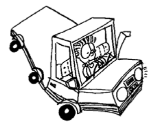
ANSWER

8.1 In an elastic collision both the total linear momentum and the total kinetic energy of the system are conserved.

8.2
$$\Sigma p_{initial} = \Sigma p_{final}$$
$$m_c v_{ic} + m_t v_{it} = (m_c + m_t) v_f$$
$$(650)(16{,}67) + (1\,000)(8{,}33) = (650 + 1\,000) v_f$$
$$19\,166{,}67 = 1\,650 v_f$$
$$v_f = 11{,}62 \text{ m·s}^{-1} \text{ in the original direction}$$

8.3 total E_k (before collision) = E_k (car) + E_k (truck)
$$= \frac{1}{2} m_m v_{im}^2 + \frac{1}{2} m_t v_{it}^2$$
$$= \frac{1}{2}(650)(16{,}67)^2 + \frac{1}{2}(1\,000)(8{,}33)^2$$
$$= 90\,277{,}78 + 34\,722{,}22$$
$$= 125\,000 \text{ J}$$

total E_k (after collision) $= \frac{1}{2}(m_c + m_t)v_f^2$
$$= \frac{1}{2}(650 + 1\,000)(11{,}62)^2$$
$$= 111\,321{,}55 \text{ kJ}$$



total E_k (before collision) > total E_k (after collision)

$\therefore$ collision is inelastic

The unrounded-off answers of the calculated velocities are used throughout for more accuracy.