1: MECHANICS

NEWTON'S LAWS

FORCE

Definition:

A **force** can be defined as any kind of **push** or **pull** on an object in an attempt to change the object's state of rest or motion.

Symbol: $\vec{F}$ SI Unit: **Newton (N)** Vector nature: **Vector**

KINDS OF FORCES

Non-contact forces/field forces

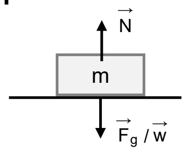
Non-contact forces *are the forces that objects exert on each other over a distance, without the objects touching each other.*

Examples of non-contact forces

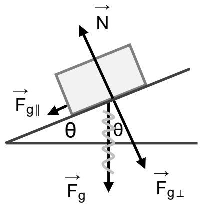
Gravitational force ($\vec{F}_g / \vec{w}$)

- This is a force of attraction between objects on any other object with mass.
  - The earth exerts a **gravitational force** on any object with mass on or near its surface. It is also called its **weight**, and is calculated as follows:

    $\vec{F}_g / \vec{w} = m\vec{g}$ [$\vec{g}$ = 9,8 m·s^{-2} (gravitational acceleration)]
  - The gravitational **pull** of the earth on an object:
    - is directed directly **downwards** to the **centre** of the earth
    - **on a horizontal plane:** is ⊥ to the plane



on an inclined plane: is two dimensional (between the plane and ⊥ direction)



- On an inclined plane the weight can be resolved into two useful components, i.e.:
  - the component **parallel** to the inclined plane $(\vec{F}_{g\parallel})$
  - the component perpendicular to the inclined plane $(\vec{F}_{g\perp})$

Parallel component ($F_{g\parallel}$)	**Perpendicular component ($F_{g\perp}$)**
$\frac{F_{g\parallel}}{F_g} = \sin\theta$ $F_{g\parallel} = F_g \sin\theta$ $= mg\sin\theta$	$\frac{F_{g\perp}}{F_g} = \cos\theta$ $F_{g\perp} = F_g\cos\theta$ $= mg\cos\theta$

Magnetic force

Attractive or **repulsive** force between magnetic objects.

Electrostatic force $(\vec{F}_E)$

Attractive or **repulsive** force between charges (charged objects).

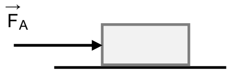
Contact forces

Most forces are contact forces, i.e. forces that objects exert on each other when they touch or are connected to one another.

Examples of contact forces

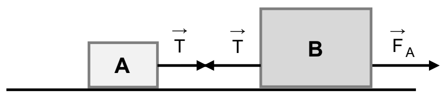
An applied force $(\vec{F}_A)$

A force that is externally applied to an object, e.g. push or pull or engine force.

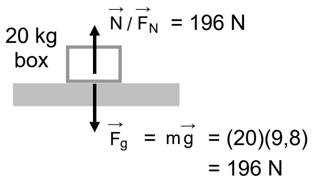


A tensile force / tension $(\vec{T})$

If a pull force $\vec{F}_A$ is exerted on B (or A), the string tightens and a tensile force $\vec{T}$ is present at both ends of the string.



Normal force ($\vec{N}$ or $\vec{F}_N$)



Normal force:

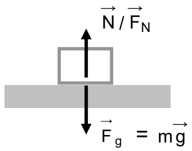
- is the force exerted by a flat surface (plane) on an object with which it is in contact.
- always acts at right angles (perpendicular (⊥)) to the surface.

> A **normal force** is the force which a surface exerts on an object with which it is in contact, and which is perpendicular to the surface.

Calculation of normal forces:

1 Calculate the normal force acting on an object (if **no other perpendicular forces** are present, except $\vec{F}_N$ and $\vec{F}_g/\vec{F}_{g\perp}$):

On a horizontal plane:



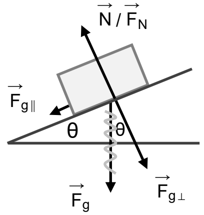
$\vec{F}_N + \vec{F}_g = 0$

$\therefore \vec{F}_N = -\vec{F}_g$

$\therefore \vec{F}_N = m\vec{g}$ (magnitudes)

On an inclined plane:

θ = the angle of the inclined plane



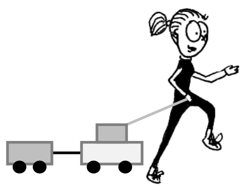
$\vec{F}_N + \vec{F}_{g\perp} = 0$

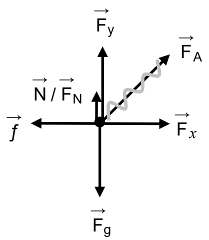
$\therefore \vec{F}_N = -\vec{F}_{g\perp}$

$\therefore \vec{F}_N = m\vec{g}\cos\theta$ (magnitude)

2 Calculate the normal force acting on an object (if **a 2D-applied force** acts on the object), e.g.:

2D pulling force:

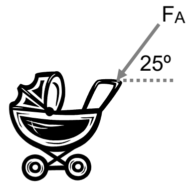


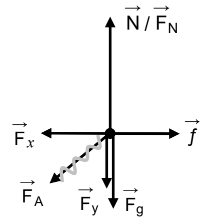


$\vec{F}_g = \vec{F}_N + \vec{F}_y$

$\therefore \vec{F}_N = \vec{F}_g - \vec{F}_y$

2D pushing force:



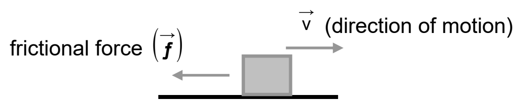


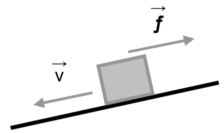
$\vec{F}_N = \vec{F}_g + \vec{F}_y$

Frictional force ($\vec{f}$ or $\vec{F}_{fr}$)

> A **frictional force** is a contact force that opposes the motion of an object and which acts parallel to the surface.

A frictional force is parallel to the surface but is in the **opposite direction** to that in which the object moves/tends to move.





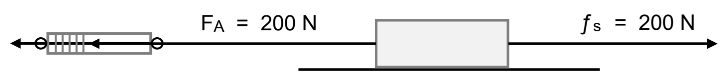
- ### Static frictional force ($\vec{f}_s$)

> The **static frictional force** is the force that opposes the tendency of motion of a stationary object relative to a surface.

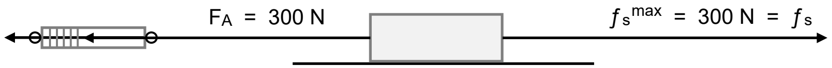
The magnitude of the static frictional force (f_s)

1 = 0 N if no applied force acts on a stationary object

2 = an applied force F_A that acts on an object, but does not bring the object into motion



③ = a maximum value $f_{s\,(max)}$ if the applied force F_A is on the point of bringing the object into motion



The magnitude of the static frictional force increases to a maximum of f_s^{max} as the forces trying to set the object in motion increase.

Calculate the maximum static frictional force (f_s^{max})

$$f_s^{max} \propto N$$

$$\boldsymbol{f_s^{max} = \mu_s N}$$

where: f_s^{max} = maximum static frictional force in newton (N)

μ_s = coefficient of static friction

N = normal force in newton (N)

μ_s is a proportionality constant for the ratio $\dfrac{f_s^{max}}{N}$. It has no unit.

The greater the weight of the object, the greater the normal force of the plane on the object (the harder the surfaces press against each other).

- **Kinetic frictional force $\left(\vec{f}_k\right)$**

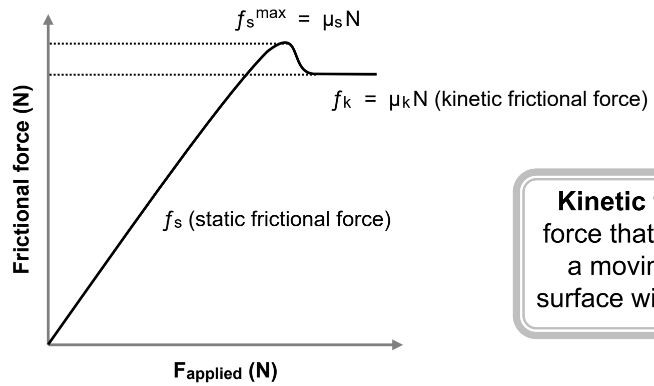
As soon as an object is set in motion, it no longer experiences static frictional force, but rather a kinetic frictional force counteracting its motion.

$$\boldsymbol{f_k < f_s^{max}}$$

$$f_k \propto N$$

$$\boldsymbol{f_k = \mu_k N}$$

where: μ_k = coefficient of kinetic friction



Kinetic frictional force is the force that opposes the motion of a moving object relative to a surface with which it is in contact.

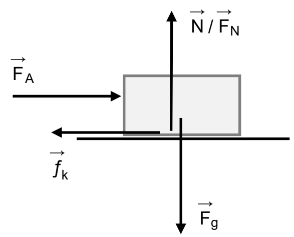
- *Kinetic frictional force remains constant during the motion of an object.*
- *Frictional force is independent of (1) the size of the contact surface and (2) the velocity of motion.*
- *Coefficients of friction are usually smaller than 1 and depend on the two surfaces in contact with each other.*

FORCE AND FREE-BODY DIAGRAMS

Force diagram

The object/s of interest are drawn and all the forces to and from the object are indicated by arrows. The force arrows point in the direction of the force.

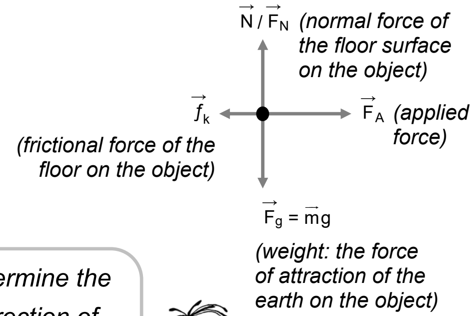
Force diagram



Free-body diagram

The object is represented by a dot and all the forces that act on the object are indicated by arrows pointing away from the dot.

Free-body diagram



Use a free-body or force diagram to determine the net force $\left(\vec{F}_{net}\right)$ parallel to the plane/direction of motion. Use Newton I and II to determine the type of movement (constant velocity or accelerated motion).

Definitions

- A **force** can be defined as any kind of **push** or **pull** on an object in an attempt to change the object's state of rest or motion.
- **Normal force**, $\vec{N}/\vec{F}_N$, is the force or the component of a force which a surface exerts on an object with which it is in contact, and which is perpendicular to the surface.
- **Frictional force**, $\vec{f}$, is the force that opposes the motion of an object and which acts parallel to the surface.
- **Static frictional force**, $\vec{f}_s$, is the force that opposes the tendency of motion of a stationary object relative to a surface.
- **Kinetic frictional force**, $\vec{f}_k$, is the force that opposes the motion of a moving object relative to a surface.

NEWTON'S FIRST LAW OF MOTION

Newton I: A body will remain in its state of rest or motion at constant velocity, unless a non-zero resultant/net force acts on it.

Newton I

↓

Two or more constant forces acting on an object are in equilibrium

↓

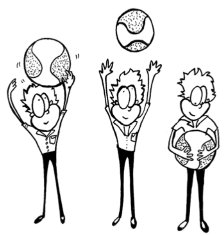
No net force
$\vec{F}_{net} = 0$

↓

$\vec{a} = 0\ m{\cdot}s^{-2}$

↙ Object remains at **rest**

↘ Object **moves** at a constant velocity in a straight line



- The property of a body that enables it to resist change in its state of motion is called **inertia**.
- The greater the **mass** of a body, the greater its inertia.

  Example: It is more difficult for a bus full of passengers to come to a stop than an empty bus.

NEWTON'S SECOND LAW OF MOTION

- From Newton **I**, it follows that if the resultant/net force on an object is zero $(\vec{F}_{net} = 0\ N)$ / forces on the object balance each other, its **state** of rest or motion remains **unchanged**.
- However, if a net force is exerted on an object $(\vec{F}_{net} \neq 0)$, the **state** of rest or motion of the object will **change**.

Newton II

↓

Constant force(s) acting on an object not in equilibrium

↓

$\vec{F}_{net} \neq 0$

Stationary object:
- object starts moving
- and accelerates uniformly

$\vec{a} \neq 0$

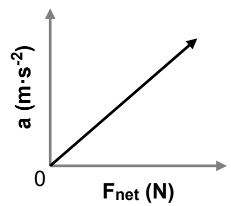
Moving object:
- velocity $(\vec{v})$ increases uniformly (it accelerates in the direction of motion)

OR

- velocity $(\vec{v})$ decreases uniformly (it accelerates in the direction opposite to the direction of motion) and comes to a standstill

Relationship between F_{net} and a (m constant)

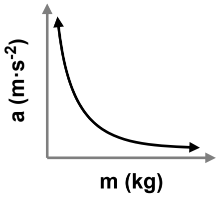
- **Graphic representation**
  - Graph of **a** versus $\mathbf{F_{net}}$ (**m** constant):



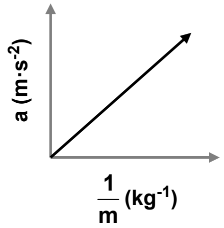
$\therefore a \propto F_{net}$ (directly proportional)

Relationship between m and a (F_{net} constant)

- **Graphic representation**
  - Graph of **a** versus **m** (F_{net} constant):



OR



$a \propto \frac{1}{m}$ (inversely proportional)

Combined: $\mathbf{a} \propto \frac{\mathbf{F_{net}}}{\mathbf{m}}$

$\therefore\ \vec{a} = \frac{\vec{F}_{net}}{m} \rightarrow \vec{F}_{net} = m\vec{a}$

Thus, according to Newton II:

Resultant force = mass × acceleration

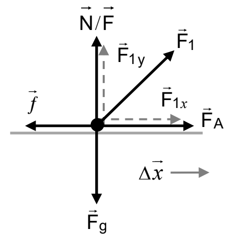
$\mathbf{\vec{F}_{net} = m\vec{a}}$

Newton II: When a resultant force $(\vec{F}_{net}/\vec{F}_{res} \neq 0)$ is applied to an object, the object will accelerate in the direction of the force. The acceleration is directly proportional to the force ($a \propto F_{net}$) and inversely proportional to the mass of the object $\left(a \propto \frac{1}{m}\right)$.

Problems involving a single object

- Consider a **single object** on which **different forces** are acting, so that the object remains **at rest** or move at a **constant velocity** or **accelerate**:

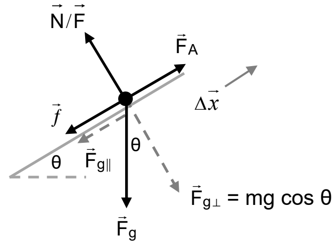
Horizontal plane



$\vec{F}_{net} = m\vec{a}$

$\therefore\ \vec{f} + \vec{F}_{1x} + \vec{F}_A = m\vec{a}$

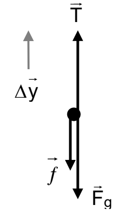
Inclined plane



$\vec{F}_{net} = m\vec{a}$

$\therefore\ \underbrace{\vec{f}}_{\mu_k N} + \underbrace{\vec{F}_{g\parallel}}_{mg \sin\theta} + \vec{F}_A = m\vec{a}$

Vertical plane



$\vec{F}_{net} = m\vec{a}$

$\therefore\ \vec{T} + \vec{F}_g + \vec{f} = m\vec{a}$

Perpendicular forces are in equilibrium:
$\therefore\ \vec{N} + \vec{F}_{1y} + \vec{F}_g = 0$

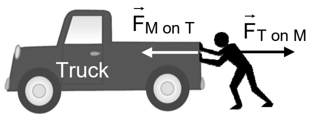
Perpendicular forces are in equilibrium:
$\therefore\ \vec{N} + \vec{F}_{g\perp} = 0$

Note: No normal force

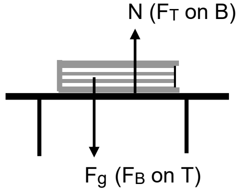
Application (steps in problem solving)

- *Draw a free-body diagram for the object.*
- *Use* $\vec{F}_{net} = m\vec{a}$ *or* $\vec{a} = \frac{F_{net}}{m}$
- *Substitute* $\vec{F}_{net}$ *with the vector sum of the forces acting on the object (e.g.* $\vec{F}_A + \vec{f}$*).*
- *Determine the resultant force* $(\vec{F}_{net})$ *by using vector addition of the forces according to magnitude and direction.*
- *If* $\vec{F}_{net} = 0$*, then* $\vec{a} = 0$ *and the object moves at a* ***constant*** *velocity.* ***OR*** *if stated that the object moves at a constant velocity, set* $\vec{F}_{net} = 0$
- *If* $\vec{F}_{net} \neq 0$*, the object will experience an acceleration in the direction of the resultant force.*
- *Determine any missing values, e.g. one of the forces or a.*

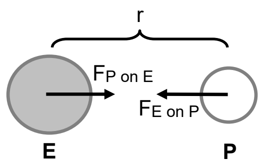
NEWTON'S THIRD LAW OF MOTION



$\vec{F}_{M\ on\ T} = -\vec{F}_{T\ on\ M}$



$\vec{F}_{T\ on\ B} = -\vec{F}_{B\ on\ T}$



$\vec{F}_{P\ on\ E} =$

Newton III: When object A exerts a force on object B, object B **simultaneously** exerts an oppositely directed force of equal magnitude on object A.

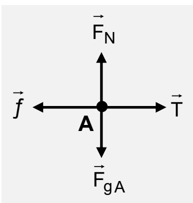
Remember:

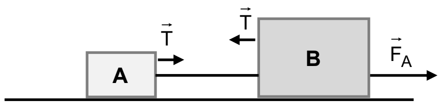
According to Newton III, forces always work in pairs, and the two forces:

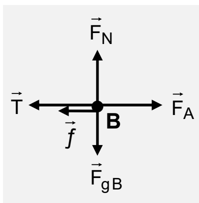
- *are of equal magnitude*
- *act in opposite directions*
- *act on **different** objects and therefore **don't balance** each other out.*

Problems involving two-body systems (joined by a light, inelastic rope)

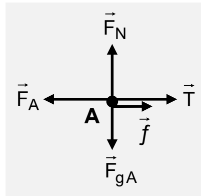
- Consider a system of two bodies that are in contact or connected to each other and move or accelerate together, for example:
  - **Both bodies A and B are on a flat horizontal plane, with or without friction.**

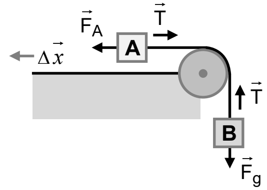


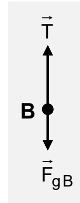




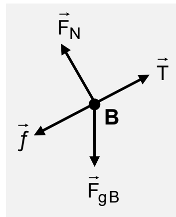
  - **One body A on a horizontal plane, with or without friction, and a second body B, hanging vertically from a string over a frictionless pulley.**

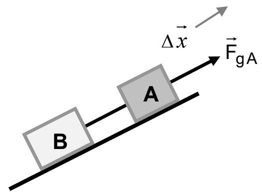


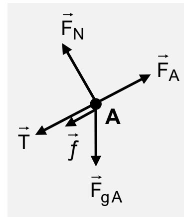




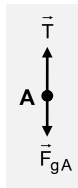
  - **Both bodies A and B are on an inclined plane, with or without friction.**

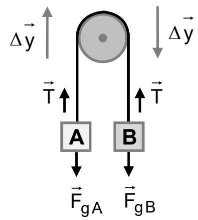


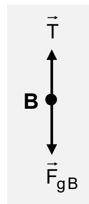




  - **Both bodies A and B are hanging vertically from a string over a frictionless pulley.**







- Identify the forces on each of the bodies:
  - If a pull force $\vec{F}_A$ is exerted on B (or A), the string tightens and a **tensile force** $\vec{T}$ is present at both ends of the string.
  - A and B experience an equal tensile force $(\vec{T})$ in opposite directions.
  - While the length of the string between them is constant, they have the same acceleration.
  - The gravitational force on objects moving vertically forms part of $\vec{F}_{net}$ and accelerates the system.

Application:

❶ Draw a force or free-body diagram for each of the objects.

❷ State a separate equation, $\vec{F}_{net} = m\vec{a}$, for both objects. (Remember, $\vec{a}$ is the same for both.)

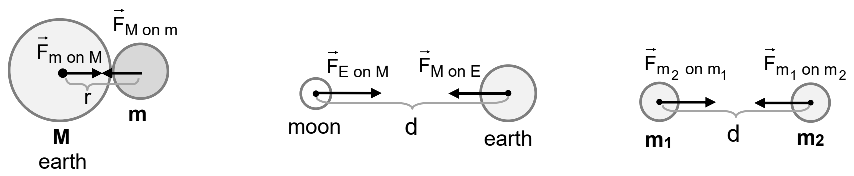
❸ Determine $\vec{F}_{net}$ for each object (use the vector sum of the applicable forces).

❹ If both the acceleration $\vec{a}$ and one of the forces are unknown, $\vec{a}$ can be calculated by simultaneous solving of the equations.

❺ Substitute $\vec{a}$ back into one of the equations to calculate the contact force between the objects or the tensile force in the string.

NEWTON'S LAW OF UNIVERSAL GRAVITATION

- There exists a force of attraction between any two objects with mass in the universe.



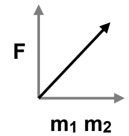
- Newton called this force **gravitational force**. Gravitational force has the following properties:
  - It is an attractive force.
  - It is a **non-contact force**.
- The force exerted by each object on another is of equal magnitude, but opposite in direction. $F_{m_2\ on\ m_1} = -F_{m_1\ on\ m_2}$ (Newton III)

Calculation of gravitational force

Through scientific investigations it has been determined that the gravitational force between objects is:

- directly proportional to the product of their masses

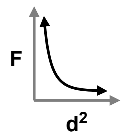
$$F_g \propto m_1 m_2$$

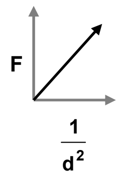


- inversely proportional to the square of the distance between their centres (inverse square law)

$$F_g \propto \frac{1}{d^2}$$

$$\therefore F_g \propto \frac{m_1 m_2}{d^2}$$





This can be expressed as a mathematical equation, i.e.:

$$\mathbf{F_g = G\frac{m_1 m_2}{d^2}}$$

where: F_g = gravitational pull between two objects in newton (N)

m_1 ; m_2 = masses of the objects in kilogram (kg)

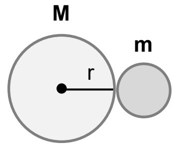
d = distance between their centres in metre (m)

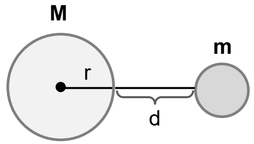
G = universal gravitational constant

= $6{,}67 \times 10^{-11}\ N{\cdot}m^2{\cdot}kg^{-2}$

Newton's Law of Universal Gravitation:
Every body in the universe attracts every other body with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres.

Calculate the **gravitational force** between the **earth** (or another **planet**) and an object, on its surface or at a distance **d** near its surface, as follows:





$$F_g = G\frac{Mm}{r^2} \quad ...❶$$

OR $$F_g = mg \quad ...❷$$

$$F_g = G\frac{Mm}{(r+d)^2}$$

g = gravitational acceleration near the earth
= $9{,}8\ m\cdot s^{-2}$

Set ❶ = ❷:

$$\therefore \cancel{m}g = G\frac{M\cancel{m}}{r^2}$$

$$\therefore g = G\frac{M}{r^2}$$

Application

❶ **Solve problems** where a substitution must be done into Newton's Gravitational Force equation.

❷ **Make predictions** by which factor the gravitational force between two objects will change if the mass of one or both objects changes and/or the distance between their centers change, i.e.:

Question:

Suppose the gravitational force of attraction between m_1 and m_2 is F, what will the force between them be in terms of F, if m_2's mass is doubled and the distance between their centers is halved?

$$\text{if } m_2 \to 2m_2 : F \to 2F$$

$$\text{if } d \to \frac{1}{2}d : F \to \left(\frac{2}{1}\right)^2 F \to 4F$$

$$(2 \times 4)F = 8F$$

OR $$F = G\frac{m_1 m_2}{d^2}$$

$$= G\frac{m_1(2m_2)}{\left(\frac{1}{2}d\right)^2} = G\frac{2m_1 m_2}{\frac{1}{4}d^2} = 8\frac{Gm_1 m_2}{d^2} = 8F$$

Definitions

- **Newton's first law of motion:** A body will remain in its state of rest or motion at constant velocity unless a non-zero resultant/net force acts on it.
- **Newton's second law of motion:** When a net force acts on an object, the object will accelerate in the direction of the force and the acceleration is directly proportional to the force and inversely proportional to the mass of the object.
- **Newton's third law of motion:** When object A exerts a force on object B, object B **simultaneously** exerts an oppositely directed force of equal magnitude on object A.
- **Newton's Law of Universal Gravitation:** Each body in the universe attracts every other body with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres.

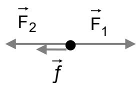
FORCE

- **Symbol:** $\vec{F}$
- **Unit:** N ($1\ N = 1\ kg\cdot m\cdot s^{-2}$)
- **Vector nature:** vector

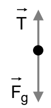
Contact and Non-contact forces

Force and Free-body diagrams

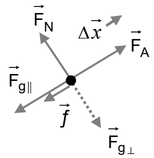
- **Horizontal forces**



- **Vertical forces**



- **Inclined-plane forces**



$F_{g\parallel} = F_g \sin\theta$

$F_{g\perp} = F_g \cos\theta$

Resultant/Net force $(\vec{F}_{net} / \vec{F}_{res})$

$\vec{F}_{net}$ = vector sum of the forces exerted on an object

Laws/Principles

- **Newton I** – principle of inertia
- **Newton II** – $\vec{F}_{net} = m\vec{a}$
- **Newton III** – If object A and object B interact, $\vec{F}_{A\ on\ B} = -\vec{F}_{B\ on\ A}$
- **Newton's Law of Gravitation** – $F_g = G\frac{m_1 m_2}{d^2}$

1 **NOTES**

MECHANICS