DBO NOV 2024 VRAESTEL 2

STATISTIEK [20]

Verwys na Sakrekenaar-instruksies in 'dokumente vir onderwysers'

1.1 **A** $\simeq$ 39,46 ; **B** $\simeq$ –0,59

$\therefore$ Vgl: $\mathbf{y = 39{,}46 - 0{,}59}x$ ◄ . . . $y = A + Bx$

1.2 $\mathbf{r \simeq -0{,}80}$ ◄

1.3 Stel $x = 29$ in: $y = 39{,}46 - 0{,}59(29)$

$= 22{,}35$

$\therefore$ **22 of 23 opstote** ◄

1.4 $\bar{y} \simeq \mathbf{18{,}33}$ ◄

1.5 **Geensins** ◄ . . .*AL die spelers het 5 meer gedoen. Die gemiddelde sal met 5 toeneem, maar die σ sal dieselfde bly.*

1.6 **Maksimum moontlike toename = 16 – 6 = 10 opstote** ◄

. . . *Die seun wat 40kg weeg se toename moes die meeste wees, vanaf 6 tot 16 opstote.*

2.1 Geskatte mediaan-reistyd **(K_2)** = 66 minute ◄

. . . *lees vanaf 30 op die y-as*

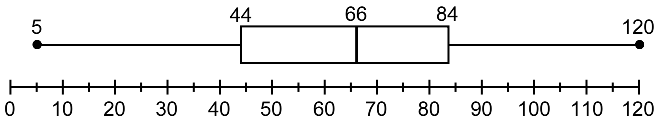
2.2 Geskatte onderste kwartiel **(K_1)** = 44 minute ◄

. . . *lees vanaf 15 op die y-as*

2.3 Geskatte boonste kwartiel **(K_3)** = 84 minute ◄

$\therefore$ Geskatte **IKV** = **$K_3 - K_1$** = 40 minute ◄

2.4



2.5 Daar is 60 werknemers.

25 werknemers reis vir minder as 60 minute.

$\therefore$ Die % toegelaat om van die huis af te werk

$= \frac{60-25}{60}\ \% = \frac{35}{60}\ \% \simeq \mathbf{58{,}33\%}$ ◄

2.6 Die werknemer neem (60 + 50) minute om te reis

$\therefore$ $2\frac{1}{2}$ intervalle van 20 minute, geneem as 3

$\therefore$ Die aantal minute toegelaat om van die huis af te werk

$= 3 \times 30 = \mathbf{90\ minute}$ ◄

ANALITIESE MEETKUNDE [39]

3.1 $M_{DC} = \frac{0-(-9)}{-9-3} = \frac{9}{-12} = \mathbf{-\frac{3}{4}}$ ◄

3.2 Stel $m = -\frac{3}{4}$ & (–9; 0) in, in

$\mathbf{y - y_1 = m(x - x_1)}$ OF $\mathbf{y = m}x\mathbf{ + c}$

$\therefore\ y - 0 = -\frac{3}{4}(x+9)$ $\therefore\ 0 = \left(-\frac{3}{4}\right)(-9) + c$

$\therefore\ y = \mathbf{-\frac{3}{4}}x\mathbf{ - \frac{27}{4}}$ ◄ $\therefore\ -6\frac{3}{4} = c$, ens.

3.3 Stel (–1; k) in, in $y = -\frac{3}{4}x - \frac{27}{4}$

$\therefore\ k = -\frac{3}{4}(-1) - 6\frac{3}{4}$

$\therefore\ \mathbf{k = -6}$ ◄

3.4 $DC^2 = (-9-3)^2 + (0+9)^2$

$= 144 + 81$

$= 225$

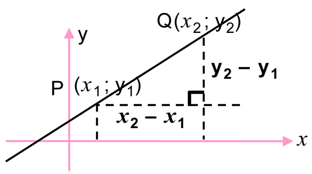
$\therefore\ \mathbf{DC} = \sqrt{\mathbf{225}} = \mathbf{15\ eenhede}$ ◄

Die Afstandformule

Die afstand tussen 2 punte, P(x_1; y_1) en Q(x_2; y_2)

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

. . . *Stelling van Pythag*



3.5 $DB^2 = (-1-3)^2 + (-6+9)^2$

$= 16 + 9$

$= 25$

$\therefore\ DB = \sqrt{25} = 5$ eenhede

Toepassing van die Afstandformule

$\therefore\ \frac{DB}{DC} = \frac{5}{15} = \mathbf{\frac{1}{3}}$ ◄

3.6 $\frac{DM}{DA} = \frac{DB}{DC}\left(= \frac{1}{3}\right)$ . . . *eweredigheidstelling; AC || MB*

$\therefore\ \frac{\text{Oppv. }\Delta MBD}{\text{Oppv. }\Delta ACD} = \frac{\frac{1}{2}\,DB.DM\ \sin D}{\frac{1}{2}\,DC.DA\ \sin D}$

$= \frac{DB}{DC} \cdot \frac{DM}{DA}$

$= \frac{1}{3} \times \frac{1}{3}$

$= \mathbf{\frac{1}{9}}$ ◄

3.7 Laat die pt. A (p; q) wees

$\therefore\ \frac{q+9}{p-3} = -4$

$\therefore\ q + 9 = -4p + 12$

$\therefore\ q + 4p = 3$

$\therefore\ q = 3 - 4p$ . . . ❶

$AD^2 = (p-3)^2 + (q+9)^2 = 612$ . . . ❷

$\therefore\ p^2 - 6p + 9 + (3 - 4p + 9)^2 = 612$

$\therefore\ p^2 - 6p + 9 + (12 - 4p)^2 = 612$

$\therefore\ p^2 - 6p + 9 + 144 - 96p + 16p^2 = 612$

$\therefore\ 17p^2 - 102p - 459 = 0$

$(\div 17)$ $\therefore\ p^2 - 6p - 27 = 0$

$\therefore\ (p+3)(p-9) = 0$

$\therefore\ p = -3$ $p \neq 9$ $\because\ p < 0$

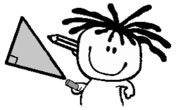
❶: $q = 3 - 4(-3)$

$= 3 + 12$

$= 15$

$\therefore$ **A(–3; 15)** ◄

4.1 $\mathbf{L(-1;\ -3)}$ ◄



4.2 $m_{MP} = \frac{9-3}{3-1} = \frac{6}{2} = 3$

$\therefore\ m_{ST} = -\frac{1}{3}$

Stel $m = -\frac{1}{3}$ & (3; 9) in, in

$\mathbf{y - y_1 = m(x - x_1)}$ OF $\mathbf{y = mx + c}$

$\therefore\ y - 9 = -\frac{1}{3}(x - 3)$ $\therefore\ 9 = \left(-\frac{1}{3}\right)(3) + c$

$\therefore\ y = -\frac{1}{3}x + 1 + 9$ $\therefore\ 10 = c$, ens.

$\therefore\ \mathbf{y = -\frac{1}{3}x + 10}$ ◄

4.3 $r^2 = (3-1)^2 + (9-3)^2$

$= 4 + 36$

$= 40$

& Middelpunt M(1; 3)

$\therefore$ Vgl van ⊙M: $(x-1)^2 + (y-3)^2 = 40$

$\therefore\ x^2 - 2x + 1 + y^2 - 6y + 9 = 40$

$\therefore\ \mathbf{x^2 + y^2 - 2x - 6y - 30 = 0}$ ◄

4.4 Stel (d; 1) in: $\therefore\ d^2 + 1 - 2d - 6 - 30 = 0$

$\therefore\ d^2 - 2d - 35 = 0$

$\therefore\ (d+5)(d-7) = 0$

$\therefore\ \mathbf{d = 7}$ ◄ . . . $d \neq -5\ \because\ d > 0$

4.5 'n Stel mini-asse is nuttig

$m_{LP} = \frac{9+3}{3+1} = 3$ $\hat{L} = \beta - \alpha$

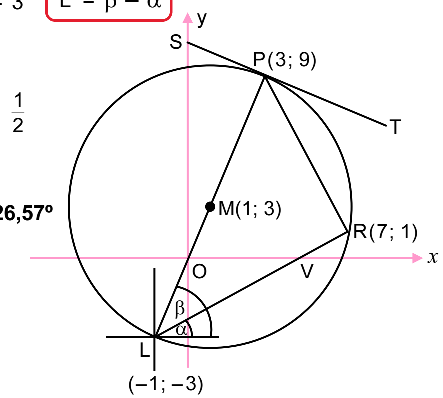
$\therefore\ \beta = 71{,}57°$

& $m_{LR} = \frac{1+3}{7+1} = \frac{1}{2}$

$\therefore\ \alpha = 26{,}57°$

$\therefore\ \mathbf{\hat{L} = 71{,}57° - 26{,}57°}$

$\mathbf{= 45°}$ ◄

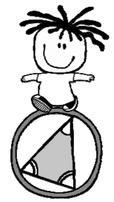


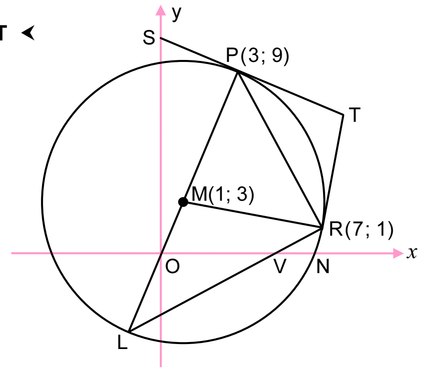
4.6 $m_{MR} = \frac{3-1}{1-7} = -\frac{1}{3}$

$\therefore\ m_{TR} = 3$ . . . *raaklyn TR ⊥ rad MR*

$\therefore\ m_{PT} \times m_{TR} = -\frac{1}{3} \times 3 = -1$ . . . *$m_{ST} = -\frac{1}{3}$ in 4.2*

$\therefore$ **PT ⊥ RT** ◄



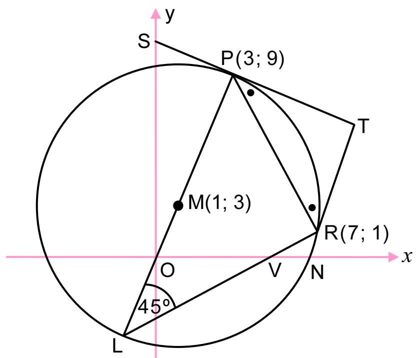


OF: $P\hat{R}T = \hat{L}$ (= 45° in 4.5) . . . *raaklyn-koord stelling*

& TP = TR . . . *raaklyne vanaf gemene punt*

$\therefore\ T\hat{P}R = P\hat{R}T = 45°$ . . . *∠^{e} teenoor = sye (in Δ)*

$\therefore$ **PT ⊥ RT** ◄ . . . *som van ∠^{e} in Δ*



OF: $P\hat{R}L = 90°$ . . . *∠ in semi-⊙*

& $\hat{L} = 45°$ . . . *bewys in 4.5*

$\therefore\ L\hat{P}R = 45°$ . . . *som van ∠^{e} in Δ*

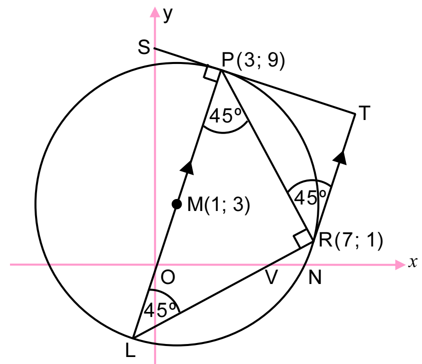
& $P\hat{R}T = 45°$ . . . *raaklyn-koord stelling*

$\therefore$ PL || TN . . . *verw. ∠^{e} =*

$S\hat{P}L = 90°$ . . . *raaklyn ⊥ middellyn*

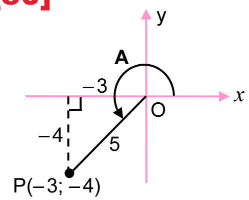
$\therefore\ \hat{T} = 90°$ . . . *ooreenk. ∠^{e}; PL || TN*

$\therefore$ **PT ⊥ RT** ◄



TRIGONOMETRIE [50]

5.1.1 $\cos A = \frac{-3}{5} = -\frac{\mathbf{3}}{\mathbf{5}}$ ◄



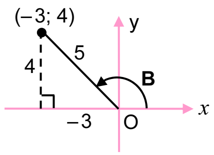
5.1.2 $\cos 2A = 2\cos^2 A - 1 = 2\left(-\frac{3}{5}\right)^2 - 1 = 2\left(\frac{9}{25}\right) - 1 = -\frac{\mathbf{7}}{\mathbf{25}}$ ◄

5.1.3 $\sin(A - B) = \sin A \cos B - \cos A \sin B$

$= \left(-\frac{4}{5}\right)\left(-\frac{3}{5}\right) - \left(-\frac{3}{5}\right)\left(\frac{4}{5}\right)$

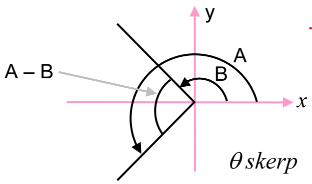
$= \frac{12}{25} + \frac{12}{25}$

$= \mathbf{\frac{24}{25}}$ ◄



OF:

Die Δ^e is refleksies van mekaar.

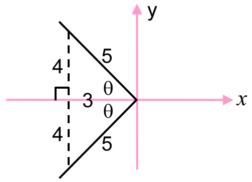


$\sin(A - B) = \sin 2\theta$

$= 2 \sin\theta \cos\theta$

$= 2\left(\frac{4}{5}\right)\left(\frac{3}{5}\right)$

$= \mathbf{\frac{24}{25}}$ ◄



5.2 $\dfrac{\cos\left(\frac{\alpha}{2} - 45^\circ\right)\sin\left(\frac{\alpha}{2} - 45^\circ\right)}{2}$ . . . $\cos x \sin x$

$= \dfrac{\frac{1}{2}\left[2\cos\left(\frac{\alpha}{2} - 45^\circ\right)\sin\left(\frac{\alpha}{2} - 45^\circ\right)\right]}{2}$ . . . $\frac{1}{2}(2\cos x \sin x)$

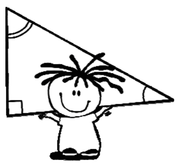
$= \dfrac{\frac{1}{2}\cdot\sin 2\left(\frac{\alpha}{2} - 45^\circ\right)}{2}$ . . . $\frac{1}{2}\sin 2x$

$= \frac{1}{4}\sin(\alpha - 90^\circ)$

$= -\frac{1}{4}\sin(90^\circ - \alpha)$

$= -\frac{1}{4}\cos\alpha$

$= \mathbf{-\frac{1}{4}\,p}$ ◄



6.1.1 Boekwerk

6.1.2 **LK** $= \dfrac{\sin x \cdot \cos y + (-\sin y)(-\cos x)}{\cos x \cos y + (-\sin x)\sin y}$

$= \dfrac{\sin x \cos y + \sin y \cos x}{\cos x \cos y - \sin x \sin y}$

$= \dfrac{\sin(x + y)}{\cos(x + y)}$

$= \tan(x + y)$

$=$ **RK** ◄

6.2 $f(x) = \sqrt{6\sin^2 x - 11(-\sin x) + 7} = 2$

$\therefore\ 6\sin^2 x + 11\sin x + 7 = 4$

$\therefore\ 6\sin^2 x + 11\sin x + 3 = 0$

$\therefore\ (2\sin x + 3)(3\sin x + 1) = 0$

$\therefore\ \sin x \neq -\frac{3}{2}$ . . . nie moontlik nie $\because\ -1 \leq \sin\theta \leq 1$ vir alle θ

of $\sin x = -\frac{1}{3}$

$\therefore\ x = 180^\circ + 19{,}47^\circ$ of $360^\circ - 19{,}47$

$= \mathbf{199{,}47^\circ}$ of $\mathbf{340{,}53^\circ}$ ◄

6.3.1 Maksimum van $g(x) = \frac{4 - 8\mathbf{(0)}}{3} = \mathbf{\frac{4}{3}}$ ◄

. . . *wanneer* $\sin^2 x$ *so klein as moontlik is*

6.3.2 $\sin^2 x = 0$ . . . *sien 6.3.1*

$\therefore\ \sin x = 0$

$\therefore\ x = 0^\circ + n(180^\circ),\ n \in \mathbb{Z}$

$\therefore$ **Kleinste waarde van $x = 180^\circ$** ◄ . . . $x \in (0^\circ; 360^\circ]$

OF: 6.3.1 $g(x) = \frac{4(1 - 2\sin^2 x)}{3} = \frac{4\cos 2x}{3}$

Maksimum van $g(x) = \frac{4\mathbf{(1)}}{3} = \mathbf{\frac{4}{3}}$ ◄

. . . *wanneer* $\cos 2x$ *so groot as moontlik is*

6.3.2 $\cos 2x = 1$

$\therefore\ 2x = 0^\circ + n(360^\circ),\ n \in \mathbb{Z}$

$\therefore\ x = 0^\circ + n(180^\circ),\ n \in \mathbb{Z}$

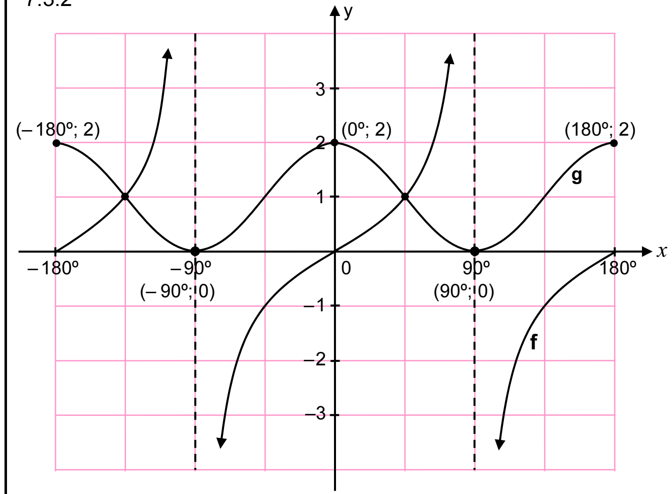
$\therefore$ **Kleinste waarde van $x = 180^\circ$** ◄ . . . $x \in (0^\circ; 360^\circ]$

7.1 $\mathbf{x = 90^\circ}$ ◄

7.2 $\mathbf{x = -180^\circ}$ **of** $\mathbf{-90^\circ < x \leq 0^\circ}$ ◄

7.3.1 **180º** ◄

7.3.2



7.4 $2\cos^3 x - \sin x = 0$

$(\div \cos x)$ $\therefore\ 2\cos^2 x - \tan x = 0$

$\therefore\ 2\cos^2 x = \tan x$

$\therefore\ 2\cos^2 x - 1 + 1 = \tan x$

$\therefore\ \cos 2x + 1 = \tan x$

$\therefore\ g(x) = f(x)$

$\tan x = 1$ ➡ $\cos 2x + 1 = 1$ . . . *f & g sny*

$\therefore\ \cos 2x = 0$

$\therefore\ 2x = 90^\circ + n(360^\circ)$

$\therefore\ x = 45^\circ + n(180^\circ);\ n \in \mathbb{Z}$ ◄

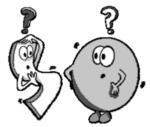
OF: Deur inspeksie, vanaf die grafieke

8.1 In ΔDCA: $\frac{16}{AC} = \tan 46{,}85^\circ$

$\therefore\ AC = \frac{16}{\tan 46{,}85^\circ}$

$= 14{,}998\ldots$

$\simeq$ **15,00 m** ◄



8.2 In ΔABC: $\frac{\sin C\hat{B}A}{15} = \frac{\sin 105{,}61^\circ}{19}$

$\therefore \sin C\hat{B}A = \frac{15 \sin 105{,}61^\circ}{19}$

$= 0{,}76....$

$\therefore C\hat{B}A = 49{,}50^\circ$

Dan is: $C\hat{A}B = 180^\circ - (49{,}50 + 105{,}61^\circ)$ . . . *∠e van Δ*

$= 24{,}89^\circ$

$\therefore BC^2 = 15^2 + 19^2 - 2(15)(19) \cos 24{,}89^\circ$

$= 68{,}94$

$\therefore BC \simeq 8{,}30$ m

In ΔEBC: $B\hat{E}C = 180^\circ - 122^\circ = 58^\circ$

$\therefore E\hat{B}C = 32^\circ$ . . . *∠e op 'n reguitlyn*

$\therefore \frac{EC}{BC} = \tan 32^\circ$

$\therefore EC = BC \tan 32^\circ = 8{,}30 \times \tan 32^\circ$

$\simeq 5{,}19$ m

$\therefore$ **DE ≃ 16 – 5,19 ≃ 10,81 m** ◄

▸ EUKLIDIESE MEETKUNDE [41]

9.1 $\hat{C}_1 = \hat{A}_1 + \hat{A}_2$ . . . *buite∠ van kvh.*

$\therefore 86^\circ = \hat{A}_1 + 46^\circ$

$\therefore$ **$\hat{A}_1 = 40^\circ$** ◄

9.2 $A\hat{C}E = \hat{A}_2 + \hat{B}$ . . . *buite∠ van Δ*

$\therefore \hat{C}_2 + 86^\circ = 46^\circ + 2\hat{A}_1$

$\therefore \hat{C}_2 = -40^\circ + 2(40^\circ)$

$\therefore \hat{C}_2 = 40^\circ$

$\therefore \hat{C}_2 = \hat{A}_1$ . . . *albei = 40°*

$\therefore$ **AD = DC** ◄ . . . *sye teenoor = ∠e (in Δ)*

(of) gelyke koorde; gelyke ∠e

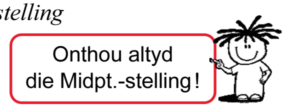
10.1 Stelling

10.2.1 In ΔPWS: O middelpunt PW

& R midpt. PS . . . *lyn vanuit middelpt. ⊥ op koord*

$\therefore OR = \frac{1}{2} WS$ . . . *Midpt.-stelling*

d.w.s. **OR : WS = 1 : 2** ◄



OF: $P\hat{S}W = 90^\circ$ . . . *∠ in semi-⊙*

$\therefore P\hat{S}W = P\hat{R}O$ (= 90°)

$\therefore$ In ΔPOR & ΔPWS

(1) $\hat{P}$ is gemeen

(2) $P\hat{R}O = P\hat{S}W$ . . . *albei = 90°*

$\therefore \Delta POR \mid\mid\mid \Delta PWS$ . . . ∠∠∠

$\therefore OR : WS = PR : PS$ . . . ||| Δe

Maar R midpt. PS . . . *OR ⊥ koord PS*

$\therefore PR : PS = 1 : 2$

$\therefore$ **OR : WS = 1 : 2** ◄

10.2.2 $P\hat{S}W = 90^\circ$ . . . *∠ in semi-⊙*

$\therefore P\hat{R}O = P\hat{S}W$

$\therefore$ (W)VS || OR . . . *ooreenk. ∠e gelyk*

$\therefore$ In ΔROT: $\frac{RS}{RT} = \frac{OV}{OT}$ . . . *eweredigh.stell.*

$= 1 : 4$

$\therefore \frac{RS}{RT} = \frac{1}{3}$

& ST = 15

$\therefore RS = \frac{15}{3} = 5$ eenhede

$\therefore$ PT = 2RS + ST = 2(5) + 15 = **25 eenhede** ◄

11.1 [$E\hat{A}G = x$ . . . *gegee*]

$A\hat{F}G = x$ ◄ . . . *raaklyn-koord stell. (groter ⊙)*

& **$\hat{D}_1 = x$** ◄ . . . *raaklyn-koord stell. (klein ⊙)*

$\hat{C}_1$ ◄ **= $\hat{D}_1$ (= x)** ◄ . . . *raaklyn-koord stell.*

$\hat{C}_4$ ◄ **= regoorst. $\hat{C}_1$ (= x)** ◄

11.2 $\therefore \hat{D}_1 = A\hat{F}G$ . . . *albei = x in 11.1*

$\therefore$ CD || GF . . . *ooreenk. ∠e =*

$\therefore$ In ΔAGF: $\frac{AG}{AC} = \frac{AF}{AD}$ . . . *eweredigh.stell.; CD || GF*

$\therefore$ **AG . AD = AC . AF** ◄

11.3 In Δe AGF & ABC

(1) $\hat{G} = \hat{B}_2$ (= y) . . . *∠e in dieselfde segment*

(2) $A\hat{F}G = \hat{C}_1$ . . . *albei = x in 11.1*

$\therefore$ **ΔAGF ||| ΔABC** ◄ . . . ∠∠∠

11.4 $\therefore \frac{GF}{BC} = \frac{AF}{AC} \left(= \frac{AG}{AB}\right)$

$\therefore GF = \frac{BC . AF}{AC}$ . . . ❶

Let wel
Merk die sye wat jy **nog moet** kry:
GF . FC, AD
en, ook AC
(om te kanselleer)

Maar, in Δe GFC en CAD

1. $\hat{G} = \hat{C}_2$ (= y, gestel) . . . *ooreenk. ∠e; CD || GF in 11.2*

2. $\hat{D}_1 = \hat{C}_4$ (= x, in 11.1)

$\therefore \Delta GFC \mid\mid\mid \Delta CAD$ . . . ∠∠∠

$\therefore \frac{GF}{AC} = \frac{FC}{AD}$

$\therefore GF = \frac{FC . AC}{AD}$ . . . ❷

❶ × ❷: $\therefore GF^2 = \frac{BC . AF}{AC} \times \frac{FC . AC}{AD} =$ **$\frac{BC . FC . AF}{AD}$** ◄