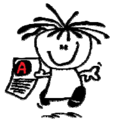


DBE NOV 2024 PAPER 2

STATISTICS [20]

Refer to Calculator Instructions in 'documents for teachers'

1.1 **A** ≃ 39,46 ; **B** ≃ –0,59

∴ Eqn: **y = 39,46 – 0,59x** ◄ . . . *$y = A + Bx$*

1.2 **r ≃ –0,80** ◄

1.3 Subst. x = 29: y = 39,46 – 0,59(29)
= 22,35

∴ **22 or 23 push-ups** ◄

1.4 **$\overline{y}$ ≃ 18,33** ◄

1.5 **Not at all** ◄ . . . *ALL of the players did 5 more. The mean will increase by 5, but the σ will remain the same.*

1.6 **Maximum possible increase = 16 – 6 = 10 push-ups** ◄

. . . *the boy weighing 40 kg needed to increase the most, from 6 to 16 push-ups.*

2.1 Estimated median **(Q₂)** travel time = 66 minutes ◄

. . . *read from 30 on the y-axis*

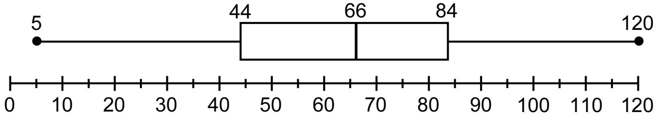
2.2 Estimated lower quartile **(Q₁)** = 44 minutes ◄

. . . *read from 15 on the y-axis*

2.3 Estimated upper quartile **(Q₃)** = 84 minutes ◄

∴ Estimated **IQR** = **Q₃ – Q₁** = 40 minutes ◄

2.4



2.5 There are 60 employees.

25 employees travel for less than 60 minutes.

∴ The % allowed to work from home

$= \frac{60-25}{60}\ \% = \frac{35}{60}\ \% \simeq$ **58,33%** ◄

2.6 The employee takes (60 + 50) minutes to travel

∴ $2\frac{1}{2}$ intervals of 20 minutes, taken as 3

∴ The number of minutes allowed to work from home

= 3 × 30 = **90 minutes** ◄

ANALYTICAL GEOMETRY [39]

3.1 $M_{DC} = \frac{0-(-9)}{-9-3} = \frac{9}{-12} = \mathbf{-\frac{3}{4}}$ ◄

3.2 Subst. m = $-\frac{3}{4}$ & (–9; 0) in

$y - y_1 = m(x - x_1)$

$\therefore\ y - 0 = -\frac{3}{4}(x+9)$

$\therefore\ y = \mathbf{-\frac{3}{4}x - \frac{27}{4}}$ ◄

OR

$y = mx + c$

$\therefore\ 0 = \left(-\frac{3}{4}\right)(-9) + c$

$\therefore\ -6\frac{3}{4} = c$, etc

3.3 Subst. (–1; k) in $y = -\frac{3}{4}x - \frac{27}{4}$

$\therefore\ k = -\frac{3}{4}(-1) - 6\frac{3}{4}$

∴ **k = –6** ◄

3.4 $DC^2 = (-9-3)^2 + (0+9)^2$
= 144 + 81
= 225

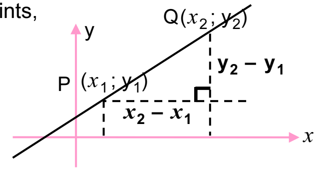
∴ **DC = $\sqrt{225}$ = 15 units** ◄

The Distance Formula

The distance between 2 points, P(x_1; y_1) and Q(x_2; y_2)

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

. . . *Theorem of Pythag*



3.5 $DB^2 = (-1-3)^2 + (-6+9)^2$
= 16 + 9
= 25

∴ DB = $\sqrt{25}$ = 5 units

$\therefore\ \frac{DB}{DC} = \frac{5}{15} = \mathbf{\frac{1}{3}}$ ◄

Applying the Distance Formula

3.6 $\frac{DM}{DA} = \frac{DB}{DC}\left(=\frac{1}{3}\right)$ . . . *prop theorem; AC || MB*

$$\therefore\ \frac{\text{Area } \Delta MBD}{\text{Area } \Delta ACD} = \frac{\cancel{\frac{1}{2}}\,DB\,.\,DM\ \cancel{\sin D}}{\cancel{\frac{1}{2}}\,DC\,.\,DA\ \cancel{\sin D}}$$

$$= \frac{DB}{DC}\,.\,\frac{DM}{DA}$$

$$= \frac{1}{3} \times \frac{1}{3}$$

$= \mathbf{\frac{1}{9}}$ ◄

3.7 Let the pt A be (p; q)

$\therefore\ \frac{q+9}{p-3} = -4$

∴ q + 9 = –4p + 12

∴ q + 4p = 3

∴ q = 3 – 4p . . . ❶

$AD^2 = (p-3)^2 + (q+9)^2 = 612$ . . . ❷

$\therefore\ p^2 - 6p + 9 + (3 - 4p + 9)^2 = 612$

$\therefore\ p^2 - 6p + 9 + (12 - 4p)^2 = 612$

$\therefore\ p^2 - 6p + 9 + 144 - 96p + 16p^2 = 612$

$\therefore\ 17p^2 - 102p - 459 = 0$

(÷17) $\therefore\ p^2 - 6p - 27 = 0$

∴ (p + 3)(p – 9) = 0

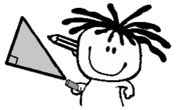
∴ p = –3 p ≠ 9 ∵ p < 0

❶: q = 3 – 4(–3)
= 3 + 12
= 15

∴ **A(–3; 15)** ◄

4.1 $\quad$ **L(–1; –3)** ◄

4.2 $\quad m_{MP} = \frac{9-3}{3-1} = \frac{6}{2} = 3$



$\therefore\ m_{ST} = -\frac{1}{3}$

Subst. $m = -\frac{1}{3}$ & (3; 9) in

$\mathbf{y - y_1 = m(x - x_1)}$

$\therefore\ y - 9 = -\frac{1}{3}(x - 3)$

$\therefore\ y = -\frac{1}{3}x + 1 + 9$

$\therefore\ \mathbf{y = -\frac{1}{3}x + 10}$ ◄

OR $\quad \mathbf{y = mx + c}$

$\therefore\ 9 = \left(-\frac{1}{3}\right)(3) + c$

$\therefore\ 10 = c$, etc

4.3 $\quad r^2 = (3-1)^2 + (9-3)^2$

$= \quad 4 \quad + \quad 36$

$= 40$

& Centre M(1; 3)

$\therefore$ Eqn of ⊙M: $\quad (x-1)^2 + (y-3)^2 = 40$

$\therefore\ x^2 - 2x + 1 + y^2 - 6y + 9 = 40$

$\therefore\ \mathbf{x^2 + y^2 - 2x - 6y - 30 = 0}$ ◄

4.4 $\quad$ Subst. (d; 1): $\quad \therefore\ d^2 + 1 - 2d - 6 - 30 = 0$

$\therefore\ d^2 - 2d - 35 = 0$

$\therefore\ (d + 5)(d - 7) = 0$

$\therefore\ \mathbf{d = 7}$ ◄ $\quad \ldots\ d \neq -5 \quad \because\ d > 0$

4.5 $\quad$ A set of mini-axes is useful

$m_{LP} = \frac{9+3}{3+1} = 3 \qquad \boxed{\hat{L} = \beta - \alpha}$

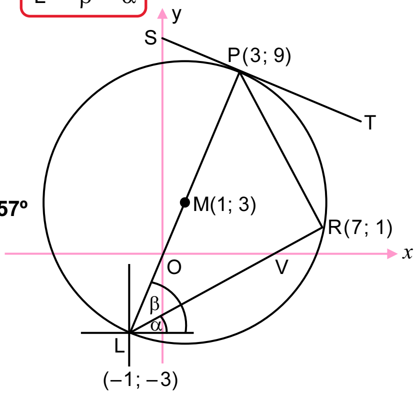
$\therefore\ \beta = 71{,}57°$

& $m_{LR} = \frac{1+3}{7+1} = \frac{1}{2}$

$\therefore\ \alpha = 26{,}57°$

$\therefore\ \mathbf{\hat{L} = 71{,}57° - 26{,}57°}$

$\mathbf{= 45°}$ ◄

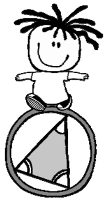


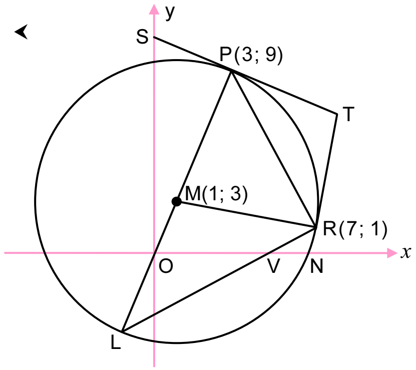
4.6 $\quad m_{MR} = \frac{3-1}{1-7} = -\frac{1}{3}$

$\therefore\ m_{TR} = 3 \quad \ldots$ *tang TR ⊥ rad MR*

$\therefore\ m_{PT} \times m_{TR} = -\frac{1}{3} \times 3 = -1 \quad \ldots\ m_{ST} = -\frac{1}{3}$ *in 4.2*

$\therefore$ **PT ⊥ RT** ◄



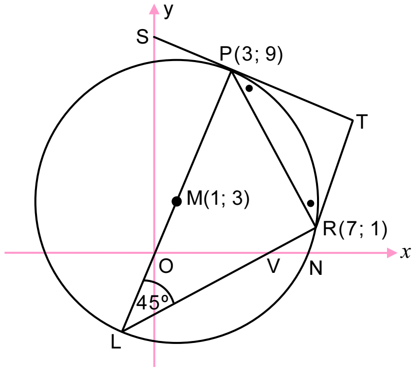


OR: $\quad P\hat{R}T = \hat{L}$ (= 45° in 4.5) $\quad \ldots$ *tan chord thm*

& TP = TR $\quad \ldots$ *tans from common point*

$\therefore$ TPR = $P\hat{R}T$ = 45° $\quad \ldots$ *∠^s opp = sides (in Δ)*

$\therefore$ **PT ⊥ RT** ◄ $\quad \ldots$ *sum of ∠^s in Δ*



OR: $\quad P\hat{R}L = 90°$ $\quad \ldots$ *∠ in semi-⊙*

& $\hat{L} = 45°$ $\quad \ldots$ *proved in 4.5*

$\therefore\ L\hat{P}R = 45°$ $\quad \ldots$ *sum of ∠^s in Δ*

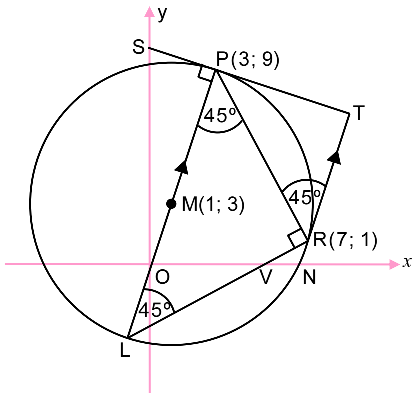
& $P\hat{R}T = 45°$ $\quad \ldots$ *tan chord thm*

$\therefore$ PL || TN $\quad \ldots$ *alt ∠^s =*

$S\hat{P}L = 90°$ $\quad \ldots$ *tan ⊥ diameter*

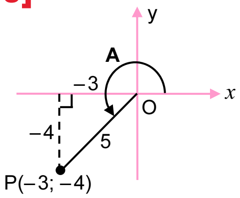
$\therefore\ \hat{T} = 90°$ $\quad \ldots$ *corresp ∠^s; PL || TN*

$\therefore$ **PT ⊥ RT** ◄



TRIGONOMETRY [50]

5.1.1 $\quad \cos A = \frac{-3}{5} = -\mathbf{\frac{3}{5}}$ ◄



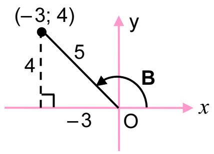
5.1.2 $\quad \cos 2A = 2\cos^2 A - 1 = 2\left(-\frac{3}{5}\right)^2 - 1 = 2\left(\frac{9}{25}\right) - 1 = -\mathbf{\frac{7}{25}}$ ◄

5.1.3 $\sin(A-B) = \sin A\cos B - \cos A\sin B$

$= \left(-\frac{4}{5}\right)\left(-\frac{3}{5}\right) - \left(-\frac{3}{5}\right)\left(\frac{4}{5}\right)$

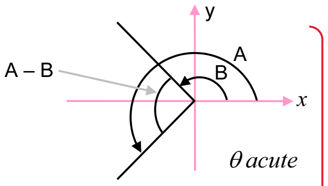
$= \frac{12}{25} + \frac{12}{25}$

$= \mathbf{\frac{24}{25}}$ ◄



OR:

The Δ^s are reflections of each other.

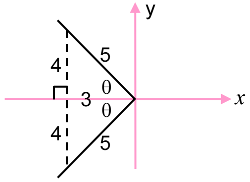


$\sin(A-B) = \sin 2\theta$

$= 2\sin\theta\cos\theta$

$= 2\left(\frac{4}{5}\right)\left(\frac{3}{5}\right)$

$= \mathbf{\frac{24}{25}}$ ◄



5.2 $\dfrac{\cos\left(\frac{\alpha}{2}-45^\circ\right)\sin\left(\frac{\alpha}{2}-45^\circ\right)}{2}$ . . . $\cos x \sin x$

$= \dfrac{\frac{1}{2}\left[2\cos\left(\frac{\alpha}{2}-45^\circ\right)\sin\left(\frac{\alpha}{2}-45^\circ\right)\right]}{2}$ . . . $\frac{1}{2}(2\cos x\sin x)$

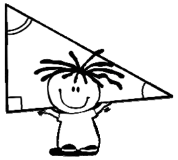
$= \dfrac{\frac{1}{2}\,.\,\sin 2\left(\frac{\alpha}{2}-45^\circ\right)}{2}$ . . . $\frac{1}{2}\sin 2x$

$= \frac{1}{4}\sin(\alpha - 90^\circ)$

$= -\frac{1}{4}\sin(90^\circ - \alpha)$

$= -\frac{1}{4}\cos\alpha$

$= \mathbf{-\frac{1}{4}p}$ ◄



6.1.1 Bookwork

6.1.2 **LHS** $= \dfrac{\sin x.\cos y + (-\sin y)(-\cos x)}{\cos x\cos y + (-\sin x)\sin y}$

$= \dfrac{\sin x\cos y + \sin y\cos x}{\cos x\cos y - \sin x\sin y}$

$= \dfrac{\sin(x+y)}{\cos(x+y)}$

$= \tan(x+y)$

$=$ **RHS** ◄

6.2 $f(x) = \sqrt{6\sin^2 x - 11(-\sin x) + 7} = 2$

$\therefore\ 6\sin^2 x + 11\sin x + 7 = 4$

$\therefore\ 6\sin^2 x + 11\sin x + 3 = 0$

$\therefore\ (2\sin x + 3)(3\sin x + 1) = 0$

$\therefore\ \sin x \neq -\frac{3}{2}$ . . . not possible $\because\ -1 \leq \sin\theta \leq 1$ for all θ

or $\sin x = -\frac{1}{3}$

$\therefore\ x = 180^\circ + 19{,}47^\circ$ or $360^\circ - 19{,}47$

$= \mathbf{199{,}47^\circ}$ or $\mathbf{340{,}53^\circ}$ ◄

6.3.1 Maximum of $g(x) = \dfrac{4 - 8\mathbf{(0)}}{3} = \mathbf{\dfrac{4}{3}}$ ◄

. . . *when $\sin^2 x$ is as small as possible*

6.3.2 $\sin^2 x = 0$ . . . *see 6.3.1*

$\therefore\ \sin x = 0$

$\therefore\ x = 0^\circ + n(180^\circ),\ n \in \mathbb{Z}$

$\therefore$ **Smallest value of x = 180°** ◄ . . . $x \in (0^\circ;\ 360^\circ]$

OR: 6.3.1 $g(x) = \dfrac{4(1 - 2\sin^2 x)}{3} = \dfrac{4\cos 2x}{3}$

Maximum of $g(x) = \dfrac{\mathbf{4(1)}}{3} = \mathbf{\dfrac{4}{3}}$ ◄

. . . *when $\cos 2x$ is as big as possible*

6.3.2 $\cos 2x = 1$

$\therefore\ 2x = 0^\circ + n(360^\circ),\ n \in \mathbb{Z}$

$\therefore\ x = 0^\circ + n(180^\circ),\ n \in \mathbb{Z}$

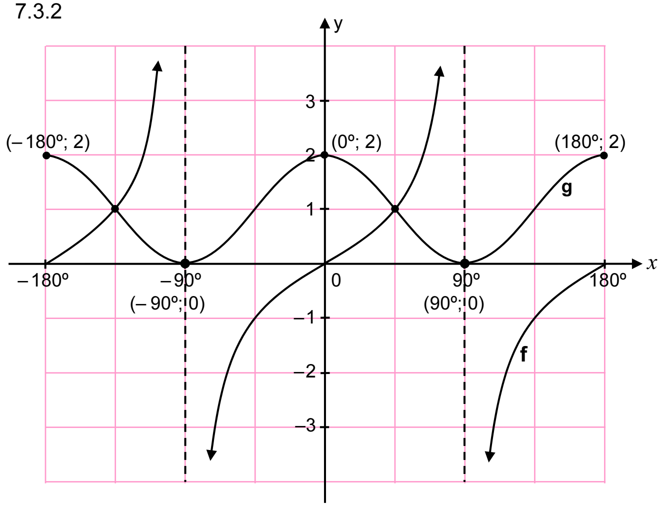
$\therefore$ **Smallest value of x = 180°** ◄ . . . $x \in (0^\circ;\ 360^\circ]$

7.1 $\mathbf{x = 90^\circ}$ ◄

7.2 $\mathbf{x = -180^\circ}$ **or** $\mathbf{-90^\circ < x \leq 0^\circ}$ ◄

7.3.1 **180°** ◄

7.3.2



7.4 $2\cos^3 x - \sin x = 0$

$(\div \cos x)$ $\therefore\ 2\cos^2 x - \tan x = 0$

$\therefore\ 2\cos^2 x = \tan x$

$\therefore\ 2\cos^2 x - 1 + 1 = \tan x$

$\therefore\ \cos 2x + 1 = \tan x$

$\therefore\ g(x) = f(x)$

$\tan x = 1$ ➡ $\cos 2x + 1 = 1$ . . . *f & g intersect*

$\therefore\ \cos 2x = 0$

$\therefore\ 2x = 90^\circ + n(360^\circ)$

$\therefore\ x = 45^\circ + n(180^\circ);\ n \in \mathbb{Z}$ ◄

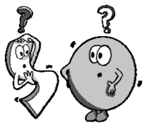
OR: By inspection, from the graphs

8.1 In ΔDCA: $\dfrac{16}{AC} = \tan 46{,}85^\circ$

$\therefore\ AC = \dfrac{16}{\tan 46{,}85^\circ}$

$= 14{,}998\ldots$

$\simeq$ **15,00 m** ◄



8.2 In ΔABC: $\dfrac{\sin C\hat{B}A}{15} = \dfrac{\sin 105{,}61^\circ}{19}$

$\therefore \sin C\hat{B}A = \dfrac{15 \sin 105{,}61^\circ}{19}$

$= 0{,}76\ldots.$

$\therefore C\hat{B}A = 49{,}50^\circ$

Then: $C\hat{A}B = 180^\circ - (49{,}50 + 105{,}61^\circ)$ . . . *∠ˢ of Δ*

$= 24{,}89^\circ$

$\therefore BC^2 = 15^2 + 19^2 - 2(15)(19)\cos 24{,}89^\circ$

$= 68{,}94$

$\therefore BC \simeq 8{,}30$ m

In ΔEBC: $B\hat{E}C = 180^\circ - 122^\circ = 58^\circ$

$\therefore E\hat{B}C = 32^\circ$ . . . *∠ˢ on a str line*

$\therefore \dfrac{EC}{BC} = \tan 32^\circ$

$\therefore EC = BC \tan 32^\circ = 8{,}30 \times \tan 32^\circ$

$\simeq 5{,}19$ m

$\therefore$ **DE ≃ 16 – 5,19 ≃ 10,81 m** ➤

▸ EUCLIDEAN GEOMETRY [41]

9.1 $\hat{C}_1 = \hat{A}_1 + \hat{A}_2$ . . . *ext ∠ of c.q.*

$\therefore 86^\circ = \hat{A}_1 + 46^\circ$

$\therefore$ **$\hat{A}_1 = 40^\circ$** ➤

9.2 $A\hat{C}E = \hat{A}_2 + \hat{B}$ . . . *ext ∠ of Δ*

$\therefore \hat{C}_2 + 86^\circ = 46^\circ + 2\hat{A}_1$

$\therefore \hat{C}_2 = -40^\circ + 2(40^\circ)$

$\therefore \hat{C}_2 = 40^\circ$

$\therefore \hat{C}_2 = \hat{A}_1$ . . . *both = 40°*

$\therefore$ **AD = DC** ➤ . . . *sides opp = ∠ˢ (in Δ)*

ⓞⓡ *equal chords; equal ∠ˢ*

10.1 Theorem

10.2.1 In ΔPWS: O midpoint PW

& R midpt PS . . . *line from centre ⊥ to chord*

$\therefore OR = \frac{1}{2} WS$ . . . *Midpt Thm*

i.e. **OR : WS = 1 : 2** ➤

Always remember the Midpt Thm!

OR: $P\hat{S}W = 90^\circ$ . . . *∠ in semi-⊙*

$\therefore P\hat{S}W = P\hat{R}O$ (= 90°)

$\therefore$ In ΔPOR & ΔPWS

(1) $\hat{P}$ is common

(2) $P\hat{R}O = P\hat{S}W$ . . . *both = 90°*

$\therefore \Delta POR \;|||\; \Delta PWS$ . . . ∠∠∠

$\therefore OR : WS = PR : PS$ . . . *||| Δˢ*

But R midpt PS . . . *OR ⊥ chord PS*

$\therefore PR : PS = 1 : 2$

$\therefore$ **OR : WS = 1 : 2** ➤

10.2.2 $P\hat{S}W = 90^\circ$ . . . *∠ in semi ⊙*

$\therefore P\hat{R}O = P\hat{S}W$

$\therefore$ (W)VS || OR . . . *corresp ∠ˢ equal*

$\therefore$ In ΔROT: $\dfrac{RS}{RT} = \dfrac{OV}{OT}$ . . . *prop thm*

$= 1 : 4$

$\therefore \dfrac{RS}{RT} = \dfrac{1}{3}$

& ST = 15

$\therefore RS = \dfrac{15}{3} = 5$ units

$\therefore PT = 2RS + ST = 2(5) + 15 =$ **25 units** ➤

11.1 [$E\hat{A}G = x$ . . . *given*]

$A\hat{F}G = x$ ➤ . . . *tan chord thm (larger ⊙)*

& **$\hat{D}_1 = x$** ➤ . . . *tan chord thm (small ⊙)*

$\hat{C}_1$ ➤ **$= \hat{D}_1$ (= x)** ➤ . . . *tan chord thm*

$\hat{C}_4$ ➤ **= vert opp $\hat{C}_1$ (= x)** ➤

11.2 $\therefore \hat{D}_1 = A\hat{F}G$ . . . *both = x in 11.1*

$\therefore$ CD || GF . . . *corresp ∠ˢ =*

$\therefore$ In ΔAGF: $\dfrac{AG}{AC} = \dfrac{AF}{AD}$ . . . *prop thm; CD || GF*

$\therefore$ **AG . AD = AC . AF** ➤

11.3 In Δˢ AGF & ABC

(1) $\hat{G} = \hat{B}_2$ (= y) . . . *∠ˢ in the same segment*

(2) $A\hat{F}G = \hat{C}_1$ . . . *both = x in 11.1*

$\therefore$ **ΔAGF ||| ΔABC** ➤ . . . ∠∠∠

11.4 $\therefore \dfrac{GF✓}{BC✓} = \dfrac{AF✓}{AC} \left(= \dfrac{AG}{AB}\right)$

$\therefore GF = \dfrac{BC✓ . AF✓}{AC}$ . . . ❶

Note
Mark the sides you **still** need: GF . FC, AD and, also AC (to cancel)

But, in Δˢ GFC and CAD

1. $\hat{G} = \hat{C}_2$ (= y, say) . . . *corresp ∠ˢ ; CD || GF in 11.2*
2. $\hat{D}_1 = \hat{C}_4$ (= x, in 11.1)

$\therefore \Delta GFC \;|||\; \Delta CAD$ . . . ∠∠∠

$\therefore \dfrac{GF}{AC} = \dfrac{FC}{AD}$

$\therefore GF = \dfrac{FC . AC}{AD}$ . . . ❷

❶ × ❷: $\therefore GF^2 = \dfrac{BC . AF}{\cancel{AC}} \times \dfrac{FC . \cancel{AC}}{AD} = \mathbf{\dfrac{BC . FC . AF}{AD}}$ ➤