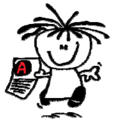


DBE NOV 2024 PAPER 1

ALGEBRA & EQUATIONS & INEQUALITIES [27]

1.1.1 $x(x-3) = 0$

$\therefore$ $\boldsymbol{x = 0}$ ◄ or $x - 3 = 0$

$\therefore$ $\boldsymbol{x = 3}$ ◄

1.1.2 $2x^2 + 1 = 4x$

$2x^2 - 4x + 1 = 0$

$\therefore x = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(1)}}{2(2)}$ . . . $\boxed{x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$

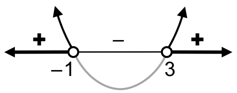
$= \dfrac{4 \pm \sqrt{8}}{4}$

$\approx$ **1,71 or 0,29** ◄

1.1.3 $x^2 - 2x - 3 > 0$

$\therefore (x+1)(x-3) > 0$

Critical values: -1 & 3



$\therefore$ $\boldsymbol{x < -1}$ **or** $\boldsymbol{x > 3}$ ◄

1.1.4 $2^{2x} - 2^{x+2} - 32 = 0$

$\therefore (2^x)^2 - 2^x.2^2 - 32 = 0$

$\therefore (2^x)^2 - 4(2^x) - 32 = 0$

$\therefore (2^x + 4)(2^x - 8) = 0$

OR: Let $2^x = k$,
$k > 0 \quad \because 2^x > 0$
$\therefore k^2 - 4k - 32 = 0$
$\therefore (k-8)(k+4) = 0$
$\therefore k = 8$, etc.

$2^x \neq -4$ . . . *$2^x > 0$ for all $x \in \mathbb{R}$*

$\therefore 2^x = 8$

$\therefore$ $\boldsymbol{x = 3}$ **only** ◄

1.1.5 $\sqrt{-2x+4} - x = 2$

$\therefore \sqrt{-2x+4} = x + 2$ . . . ❶

$\therefore \left(\sqrt{-2x+4}\right)^2 = (x+2)^2$

$\therefore -2x + \cancel{4} = x^2 + 4x + \cancel{4}$

$\therefore 0 = x^2 + 6x$

$\therefore x(x+6) = 0$

$\therefore$ $\boldsymbol{x = 0}$ ◄ $x \neq -6 \quad \because \sqrt{\ } \geq 0$ in ❶

Remember to check the answer(s)

1.2 $2x + y = 3$

$\therefore y = 3 - 2x$ . . . ❶

❷: $y^2 + xy = 2$

❶ in ❷: $\therefore (3-2x)^2 + x(3-2x) - 2 = 0$

$\therefore 9 - 12x + 4x^2 + 3x - 2x^2 - 2 = 0$

$\therefore 2x^2 - 9x + 7 = 0$

$\therefore (2x-7)(x-1) = 0$

$\therefore x = \dfrac{7}{2}$ or 1

❶: For $x = \dfrac{7}{2}$: $y = 3 - 2\left(\dfrac{7}{2}\right) = -4$

& For $x = 1$: $y = 3 - 2(1) = 1$

$\therefore$ Solution: $\left(\dfrac{7}{2}; -4\right)$ **or** **(1; 1)** ◄

1.3 $\left(1 + \dfrac{1}{2}\right)\left(1 + \dfrac{1}{3}\right)\left(1 + \dfrac{1}{4}\right)\left(1 + \dfrac{1}{5}\right) \ldots \left(1 + \dfrac{1}{n}\right)$

$= \left(\dfrac{\cancel{3}}{2}\right)\left(\dfrac{\cancel{4}}{\cancel{3}}\right)\left(\dfrac{\cancel{5}}{\cancel{4}}\right)\left(\dfrac{\cancel{6}}{\cancel{5}}\right) \ldots \left(\dfrac{n+1}{\cancel{n}}\right)$ . . .

Note the cancelling

$= \dfrac{n+1}{2}$

So, n + 1 must be any even number ≥ 4

$\therefore$ **n must be any odd number greater than 2** ◄

i.e. n = 3 ; 5 ; 7 ; 9 ; . . . ◄

PATTERNS & SEQUENCES [24]

2.1 **A.S:** a = 7 ; d = 5 ; n = 20

2.1.1 $\mathbf{S_n = \dfrac{n}{2}[2a + (n-1)d]}$

$\therefore \mathbf{S_{20}} = \dfrac{20}{2}[2(7) + (20-1)(5)]$

$= 10(14 + 95)$

$= \mathbf{1\ 090}$ ◄

2.1.2 **n** = 75 ; $\mathbf{S_{75}}$ = 14 400 (& **a** = 7)

$\mathbf{T_n = a + (n-1)d}$

$= 7 + (n-1)(5)$

$= 7 + 5n - 5$

$= 5n + 2$

$\therefore$ Sum of the terms added, T_{21} to T_{75},

$\therefore \displaystyle\sum_{n=21}^{75} (5n+2) = S_{75} - S_{20} = 14\ 400 - 1\ 090 = \mathbf{13\ 310}$ ◄

2.2.1 $T_1 \quad T_2 \quad T_3 \quad T_4 \quad \ldots \quad T_{98} \quad T_{99}$

$1 \quad 3 \quad 5 \quad \ldots \quad *$

1^{st} differences terms: $2n - 1$ . . . *odd numbers*

$\therefore T_{99} - T_{98} = 2(98) - 1 = 195$ . . . $*$

$\therefore \mathbf{T_{98}} = T_{99} - 195$

$= 9\ 632 - 195$

$= \mathbf{9\ 437}$ ◄

OR: $T_1 \quad T_2 \quad T_3 \quad T_4 \quad T_5 \quad \ldots \quad T_{98} \quad 9\ 632$

$1 \quad 3 \quad 5 \quad \ldots \quad 195$

$\therefore T_{99} = 9\ 632 - 195 = 9\ 437$

Explanation:

The 1^{st} difference lies between T_1 and T_2.

The 2^{nd} difference lies between T_2 and T_3.

The 98^{th} difference lies between T_{98} and T_{99}.

The general term for 1; 3; 5; . . . is given by $T_k = 2k - 1$

$\therefore T_{98} = 2(98) - 1 = 195$ (98^{th} first difference)

2.2.2 If $T_3 = 32$, then $T_2 = 32 - 3 = 29$

& $T_1 = 29 - 1 = 28$

	28	29	32
1st differences:	1	3	5
2nd differences:	2	2	

The general term, $\mathbf{T_n = an^2 + bn + c}$

$\mathbf{2a = 2}$ ∴ $a = 1$ *(common diff.)*

$\mathbf{3a + b = 1}$ ∴ $3 + b = 1$ ∴ $b = -2$ *(1st 1st difference)*

(T_1 =) $\mathbf{a + b + c = 28}$ ∴ $1 - 2 + c = 28$ ∴ $c = 29$ *(1st term)*

$\therefore \mathbf{T_n = n^2 - 2n + 29}$ ◄

Check: $T_{98} = 98^2 - 2(98) + 29 = 9\,437$

3.1 **G.S.:** The radii: $6\ ;\ 3\ ;\ \frac{3}{2}\ ;\ \ldots$ The radii halve each time

∴ **The radius of the 3rd circle is $\frac{3}{2}$ cm** ◄

3.2 $\mathbf{S_n = \dfrac{a(1 - r^n)}{1 - r}}$ The areas of the circles are: $36\pi\ ;\ 9\pi\ ;\ 2{,}25\pi\ ;$ etc.

$\mathbf{S_{10}?}$; $\mathbf{n} = 10$; $\mathbf{a} = 36\pi$; $\mathbf{r} = \frac{1}{4}$

$$\therefore S_{10} = \frac{36\pi\left[1 - \left(\frac{1}{4}\right)^{10}\right]}{1 - \frac{1}{4}} \simeq \mathbf{150{,}80\ cm^2}\ ◄$$

3.3 Use $\mathbf{T_n = ar^{n-1}}$ for the radii:

$\mathbf{n?}$; $\mathbf{a} = 6$; $\mathbf{r} = \frac{1}{2}$; $\mathbf{T_n} = \frac{1}{2}$ **of** $\frac{3}{128} = \frac{3}{256}$

$$\therefore \frac{3}{256} = 6.\left(\frac{1}{2}\right)^{n-1}$$

$$\therefore \left(\frac{1}{2}\right)^{n-1} = \frac{\cancel{3}}{256} \times \frac{1}{\cancel{6}_2} \quad \ldots$$

$$\therefore \left(\frac{1}{2}\right)^{n-1} = \frac{1}{2^8 \times 2}$$

$$\therefore \left(\frac{1}{2}\right)^{n-1} = \left(\frac{1}{2}\right)^9$$

∴ $n - 1 = 9$

∴ $n = 10$ ∴ **The 10th circle** ◄

OR: Use logs

$n - 1 = \log_{\frac{1}{2}}\left(\frac{1}{512}\right)$

∴ $n - 1 = 9$

∴ $\mathbf{n = 10}$ ◄

FUNCTIONS & GRAPHS [34]

4.1 Subst. B$\left(2;\ \frac{-5}{9}\right)$ in $y = a^x - 1$

$$\therefore \frac{-5}{9} = a^2 - 1$$

$$\therefore \frac{4}{9} = a^2$$

$\therefore \mathbf{a = \frac{2}{3}}$ ◄ ... *a > 0*

4.2 $f(x) = \left(\frac{2}{3}\right)^x - 1$

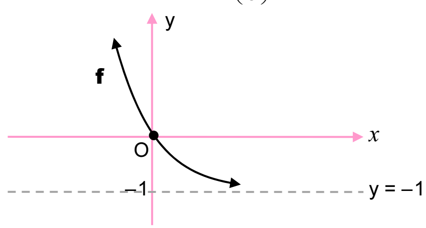
Range of f: $\mathbf{y > -1,\ y \in \mathbb{R}}$ ◄

4.3 **X-intercept** (y = 0): $\left(\frac{2}{3}\right)^x - 1 = 0$

$$\left(\frac{2}{3}\right)^x = 1$$

$\therefore \mathbf{x = 0}$ ◄

Y-intercept (x = 0): $y = \left(\frac{2}{3}\right)^0 - 1 = 1 - 1 = 0$



4.4 Subst. C$\left(x;\ \frac{19}{8}\right)$ in $y = \left(\frac{2}{3}\right)^x - 1$

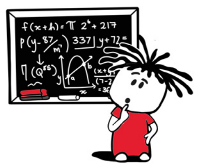
$$\therefore \frac{19}{8} = \left(\frac{2}{3}\right)^x - 1$$

$$\therefore \frac{27}{8} = \left(\frac{2}{3}\right)^x$$

$$\therefore \left(\frac{3}{2}\right)^3 = \left(\frac{3}{2}\right)^{-x}$$

∴ $x = -3$

∴ Pt C$\left(-3;\ \frac{19}{8}\right)$



∴ **Pt C′$\left(\frac{19}{8};\ -3\right)$** ◄ ... *refl about y = x*; *(x; y)* ➡ *(y; x)*

5.1 $\mathbf{p = -1}$ ◄ ... *a shift 1 unit to the right*

5.2 Subst. $x = 1$ in $y = x - 3$ ... *(1; y) on g*

∴ $y = 1 - 3$
$= -2$

∴ Eqn of horizontal asymptote: $\mathbf{y = -2}$ ◄

5.3 Subst. (0; 1), p = –1 & q = –2:

$f(x) = \frac{a}{x + p} + q$: $1 = \frac{a}{0 - 1} - 2$

∴ $3 = -a$

∴ $\mathbf{a = -3}$ ◄

5.4 **X-int.** (y = 0): $0 = \frac{-3}{x - 1} - 2$

$$\therefore 2 = \frac{-3}{x - 1}$$

∴ $2(x - 1) = -3$

∴ $x - 1 = -\frac{3}{2}$

∴ $x = -\frac{1}{2}$

∴ $f(x) \geq 0$ for: $\mathbf{-\frac{1}{2} \leq x < 1}$ ◄

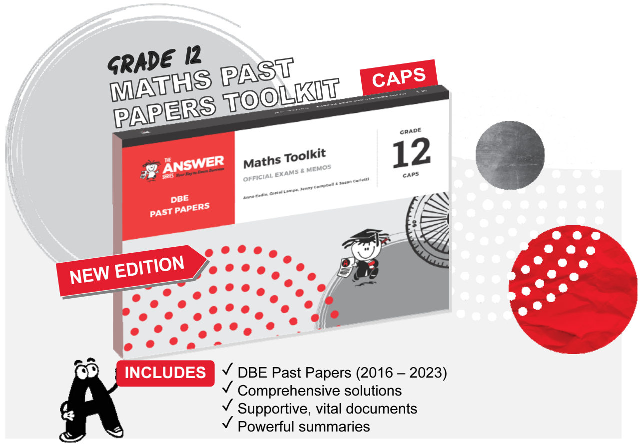
5.5 **A reflection in the line x = 1**

or A reflection in the line y = –2 ◄

Reflections in the lines of symmetry

OR: Rotation 90º anticlockwise about (1; –2)

OR: Rotation 90º clockwise about (1; –2)



6.1 $y = -x^2 + 4x + 5$

Axis of Sym.: $x = -\frac{b}{2a} = -\frac{4}{2(-1)} = 2$

$\therefore$ Max $y = -2^2 + 4(2) + 5$
$= -4 + 8 + 5$
$= 9$

$\therefore$ **B(2; 9)** ➤

OR: $f'(x) = -2x + 4 = 0$ at the turning point
$\therefore -2x = -4$
$\therefore x = 2$, etc.

6.2 Eqn of f: $y = -(x^2 - 4x - 5)$
$\therefore y = -(x + 1)(x - 5)$
$\therefore x = -1$ at A (& $x = 5$ at D)

$\therefore m_{AC} = \frac{8 - 0}{3 - (-1)} = \frac{8}{4} = 2$

Subst. (3; 8) & m = 2 in

$\mathbf{y - y_1 = m(x - x_1)}$
$y - 8 = 2(x - 3)$
$\therefore y = 2x - 6 + 8$

OR: $y = mx + c$
$\therefore 8 = (2)(3) + c$
$\therefore c = 2$, etc.

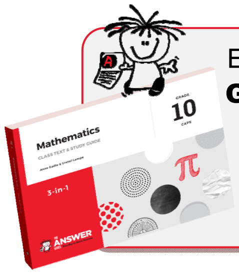
$\therefore$ $\mathbf{g(x) = 2x + 2}$ ➤

6.3 $EH = f(x) - g(x)$ . . . *vertical length*
$= (-x^2 + 4x + 5) - (2x + 2)$
$= -x^2 + 2x + 3$

Max when $x = -\frac{2}{2(-1)} = 1$

$\therefore$ Max length of EH
$= -1^2 + 2(1) + 3$
$=$ **4 units** ➤

OR: $D_x[-x^2 + 2x + 3] = 0$
$\therefore -2x + 2 = 0$
$\therefore -2x = -2$
$\therefore x = 1$, etc.



6.4 g a tangent to k
➡ The equation $k(x) = g(x)$ has equal roots. So, $\Delta = 0$

$-x^2 - 2mx - m^2 + 4x + 4m + 5 = 2x + 2$
$\therefore -x^2 - 2mx + 2x - m^2 + 4m + 3 = 0$
$\therefore -x^2 - (2m - 2)x - (m^2 - 4m - 3) = 0$
$\therefore x^2 + (2m - 2)x + (m^2 - 4m - 3) = 0$

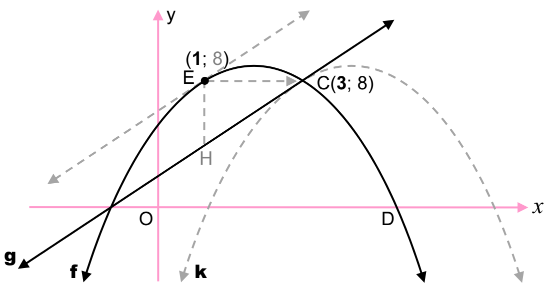
$\Delta = b^2 - 4ac$
$= (2m - 2)^2 - 4(1)(m^2 - 4m - 3)$
$= 4m^2 - 8m + 4 - 4m^2 + 16m + 12$
$= 8m + 16$

$\Delta = 0$ ➡ $8m + 16 = 0$
$\therefore 8m = -16$
$\therefore$ $\mathbf{m = -2}$ ➤

OR: $k(x) = f(x + m)$ means that k is f shifted horizontally by m units. (f would have to shift to the right)

A tangent to f (with gradient, 2) would touch f at E, when EH is at maximum length, when $x = 1$ (See 6.3).

The gradient of g is 2
& The gradient of f $= f'(x) = -2x + 4$
$\therefore$ At E: $-2x + 4 = 2$
$\therefore -2x = -2$
$\therefore x_E = 1$



E must shift horizontally to C . . . ($y_E = 8$)
$\therefore$ by 2 units
$\therefore$ $\mathbf{m = -2}$ ➤

FINANCE, GROWTH & DECAY [14]

7.1 **P** = R5 000 ; $\boldsymbol{i}$ = 6,8% ÷ 4 ; **n** = 16 × 4 ; **A?**

$\mathbf{A = P(1 + \mathit{i})^n}$ ➡ $\mathbf{A} = 5\,000(1 + 0{,}017)^{64}$
$\simeq$ **R14 706,56** ➤

7.2 **n** = 4 ; **P** = original value ; **A** = $\frac{1}{2}$**P** ; $\boldsymbol{i}$?

$\mathbf{A = P(1 - \mathit{i}n)}$

$\therefore \frac{1}{2}P = P(1 - i.4)$

(÷P) $\therefore \frac{1}{2} = 1 - 4i$
$\therefore 4i = \frac{1}{2}$
$\therefore i = \frac{1}{8}$ (= 0,125)

$\therefore$ The rate of depreciation was **12,5% per annum** ➤

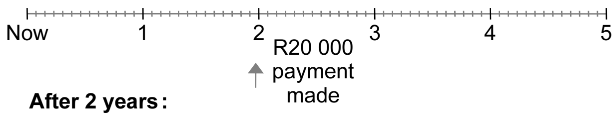
7.3.1 The value of the payments
$= 5 \times 12 \times$ R2 300,98
$=$ R138 058,80

The value of the loan = R100 000

$\therefore$ The interest = R138 058,80 – R100 000 . . . *the difference!*
$=$ **R38 058,80** ➤

7.3.2 Up to 1 March 2024, before the R20 000 payment:
$\boldsymbol{x}$ = R2 300,98 ; **n** = 2 × 12 = 24 ; $\boldsymbol{i} = \frac{13{,}5\%}{12} = \frac{0{,}135}{12}$

12 monthly payments for each year



After 2 years:
The balance of the loan (before the payment)
$=$ **A – Fv** **A:** the accrued value of the loan
Fv: the accrued value of the payments

$= \mathbf{P(1 + \mathit{i})^n} - \frac{x\left[(1 + i)^n - 1\right]}{i}$

$= 100\,000\left(1 + \frac{0{,}135}{12}\right)^{24} - \dfrac{2\,300{,}98\left[\left(1 + \frac{0{,}135}{12}\right)^{24} - 1\right]}{\frac{0{,}135}{12}}$

$\simeq$ R67 805,20 . . . *Now subtract the payment*

Subtract the payment of R20 000, then the balance of the loan

= R47 805,20

= the Present value (P_V) of the remaining **n** payments

$$P_v = \frac{x\left[1-(1+i)^{-n}\right]}{i}$$

$P_V = 47\ 805,20$; $x = 2\ 300,98$; **n?** ; $i = \frac{0,135}{12}$

$$\therefore \frac{2\ 300,98\left[1-\left(1+\frac{0,135}{12}\right)^{-n}\right]}{\frac{0,135}{12}} = 47\ 805,20$$

$$\therefore \left[1-\left(1+\frac{0,135}{12}\right)^{-n}\right] = 0,2337\ldots$$

$$\therefore 0,76626\ldots = \left(1+\frac{0,135}{12}\right)^{-n}$$

$$\therefore -n = \log_{\left(1+\frac{0,135}{12}\right)} 0,76626\ldots$$

$$= -23,796\ldots$$

$\therefore$ **n** $\simeq$ 24 months . . . *The 24th payment would be a lesser payment*

$\therefore$ **12 months earlier** ➤ . . . *It would have been another 3 years, i.e. 36 months*

OR:

Using Present values: $P_V = A(1+i)^{-n}$ and $P_V = \frac{x\left[1-(1+i)^{-n}\right]}{i}$

Let n be the total number of payments made.

$$100\ 000 = 20\ 000\left(1+\frac{0,135}{12}\right)^{-24} + \frac{2\ 300,98\left[1-\left(1+\frac{0,135}{12}\right)^{-n}\right]}{\frac{0,135}{12}}$$

$$\therefore 1-\left(1+\frac{0,135}{12}\right)^{-n} = 0,41416\ldots$$

$$\therefore \left(1+\frac{0,135}{12}\right)^{-n} = 0,585\ldots$$

$$\therefore -n = \log_{\left(1+\frac{0,135}{12}\right)} 0,585\ldots = -47,796\ldots$$

$$\therefore n = 47,796\ldots$$

n = the total number of payments made on the original loan

He needs to make 48 payments to pay off the loan.

$\therefore$ he will pay off the loan 12 months earlier than originally planned.

Finance Formulae

$$\mathbf{A = P(1 \pm in)} \qquad \mathbf{A = P(1 \pm i)^n}$$

$$\mathbf{F_v = \frac{x\left[(1+i)^n - 1\right]}{i}} \qquad \mathbf{P_v = \frac{x\left[1-(1+i)^{-n}\right]}{i}}$$

$$\mathbf{1 + i_{eff} = \left(1 + \frac{i_{nom}}{m}\right)^m}$$

DIFFERENTIAL CALCULUS [34]

8.1.1 $\frac{d}{dx}\left[3x - 5x^2\right] = \mathbf{3 - 10x}$ ➤

8.1.2 $g(x) = 2x^{-2} - x^{\frac{7}{3}}$

$\therefore g'(x) = \mathbf{-4x^{-3}} - \frac{7}{3}x^{\frac{4}{3}}$ ➤

$\left(= -\frac{4}{x^3} - \frac{7}{3}\sqrt[3]{x^4}\right)$ ➤

8.2 $f(x) = x^3 - 4x^2 + 2x + 3$

$\therefore f'(x) = 3x^2 - 8x + 2$ . . . *the gradient of the tangent to f*

$\therefore f'(2) = 3(2)^2 - 8(2) + 2$ . . . *the gradient of the tangent to f at x = 2*

$= 12 - 16 + 2$

$= -2$

& $f(2) = 2^3 - 4(2)^2 + 2(2) + 3$

$= 8 - 16 + 4 + 3$

$= -1$

$\therefore$ Point of contact is (2; –1)

Subst. m = –2 & pt (2; –1) in

$\mathbf{y - y_1 = m(x - x_1)}$

$\therefore y + 1 = -2(x - 2)$

$\therefore \mathbf{y = -2x + 3}$ ➤

OR: $\mathbf{y = mx + c}$

$\therefore -1 = (-2)(2) + c$

$\therefore c = 3$, etc.

8.3.1 $f(x) = -6x^2$

$\therefore f(x + h) = -6(x + h)^2$

$= -6(x^2 + 2xh + h^2)$

$= -6x^2 - 12xh - 6h^2$

$\therefore f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

def of a derivative

$= \lim_{h \to 0} \frac{-6x^2 - 12xh - 6h^2 - (-6x^2)}{h}$

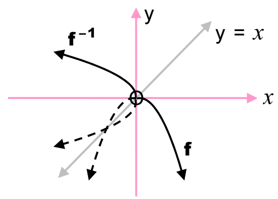
$= \lim_{h \to 0} \frac{12xh - 6h^2}{h}$

$= \lim_{h \to 0} (-12x - 6h)$

$= \mathbf{-12x}$ ➤

8.3.2 $\mathbf{x \geq 0 ;\ x \in \mathbb{R}}$ ➤

or $\mathbf{x \leq 0 ;\ x \in \mathbb{R}}$ ➤



8.3.3 Equation of f: $y = -6x^2$

$\therefore$ Equation of f^{-1}: $x = -6y^2$

$\therefore 6y^2 = -x$

$\therefore y^2 = -\frac{x}{6}$

$\therefore \mathbf{y = -\sqrt{-\frac{x}{6}}}$ where $x \leq 0$ ➤ . . . $f^{-1}(x) \leq 0$

9.1 $\mathbf{1 < x < \frac{5}{2}}$ ➤ . . . *$1 \leq x \leq \frac{5}{2}$ will also be accepted*

9.2 x-intercepts of f':

(1; 0) & $\left(\frac{5}{2}; 0\right)$ ➤

9.3 At the point of inflection:

$x = \frac{1 + \frac{5}{2}}{2} = 1\frac{3}{4}$

$\therefore$ f is concave up for $\mathbf{x > 1\frac{3}{4}}$ ➤

9.4 **–9 < k < –8** ➤

10.1 The speed $= s'(t) = -3t^2 + 18t$

$\therefore$ The maximum speed occurs when $s''(t) = 0$

$\therefore -6t + 18 = 0$

$\therefore -6t = -18$

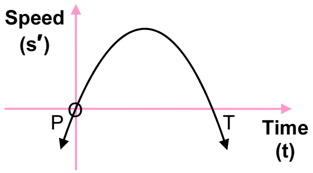
$\therefore t = 3$

$\therefore$ The maximum speed $= -3(3)^2 + 18(3)$

$= -27 + 54$

$=$ **27 km/h** ✓

10.2 At the start and the end i.e. at town P and town T, the speed $= 0$



$\therefore -3t^2 + 18t = 0$

$\therefore -3t(t-6) = 0$

$\therefore t = 0$ (at P) or $t = 6$ (at T)

$\therefore$ The time taken is 6 hours

The distance:

$s(t) = -t^3 + 9t^2$ $\quad \ldots$ *by inspection*, Note: s′(0) = 0*

$\therefore s(6) = -(6)^3 + 9(6)^2$

$=$ **108 km** ✓

*OR:

$s(t) = at^3 + bt^2 + ct + d$

$s'(t) = 3at^2 + 2bt + c$

We have: $s'(t) = -3t^2 + 18t$ $\quad \ldots s'(0) = c = 0$

$\therefore 3a = -3$ $\qquad 2b = 18$ $\qquad c = 0$ $\qquad d = 0$

$\therefore a = -1$ $\qquad \therefore b = 9$ $\qquad \because s(0) = 0$

$\therefore s(t) = -t^3 + 9t^2$, etc.

PROBABILITY [17]

11. All sit for at least one examination.

$n(M) = 22$; $n(T) = 16$; $n(G) = 18$

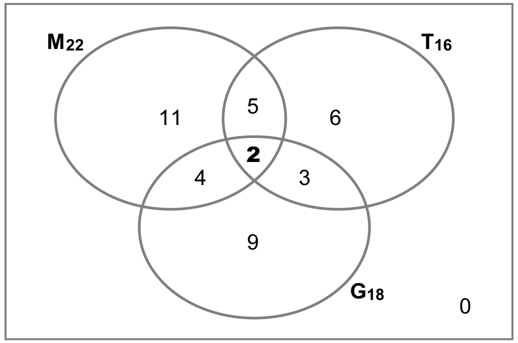
$n(M \cap T \cap G') = 5$

$n(M \cap G \cap T') = 4$

$n(T \cap G \cap M') = 3$

$n(T \text{ only}) = 6$

11.1



$n(T) = 16 \Rightarrow n(M \cap T \cap G) = 2$

11.2 The total number of learners $= 40$

The number taking at least 2 subjects

$= 4 + 2 + 3 + 5 = 14$

$\therefore$ P(at least 2 subjects) $= \frac{14}{40} = \frac{\mathbf{7}}{\mathbf{20}}$ $\quad \ldots$ **(= 0,35 = 35%)** ✓

11.3 $P(M) = \frac{22}{40}$ & $P(T) = \frac{16}{40}$

$\therefore P(M) \times P(T) = \frac{22}{40} \times \frac{16}{40} = \frac{11}{50}$

whereas $P(M \text{ and } T) = \frac{7}{40}$

$\therefore$ **P(M) × P(T) ≠ P(M and T)**

$\therefore$ **The events are not independent** ✓

12.1 <u>26</u> <u>10</u> <u>26</u> <u>10</u>

$\therefore$ No of different codes $= 26 \times 10 \times 26 \times 10$

$=$ **67 600** ✓

12.2 20 letters are used

Choice of 18 for 1st spot

No repeats of letters or digits

Last slot must be odd

<u>18</u> <u>9</u> <u>19</u> <u>5</u>

$\therefore$ No of different codes $= 18 \times 9 \times 19 \times 5$

$=$ **15 390** ✓

12.3 <u>24</u> <u>9</u> <u>25</u> <u>5</u>

$\therefore$ No of different codes $= 27\,000$

% increase $= \frac{27\,000 - 15\,390}{15\,390}$ %

$=$ **75,44%** ✓