KZN 2024 TAS Maths Workshop

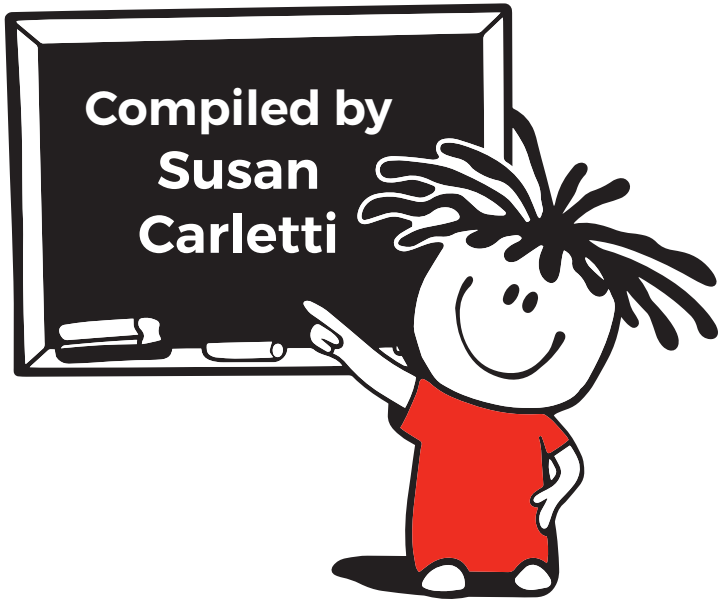
SOLUTIONS TO
PROBABILITY & COUNTING PRINCIPLES

Building confidence one question at a time

#BREAK THE 70% CEILING

LEARN HOW – Remember for a moment

LEARN WHY – Remember for a life time



THE ANSWER SERIES *Your Key to Exam Success*

CONTENTS

THE ANSWER SERIES *Your Key to Exam Success*

Question 1

1.1 $\frac{4}{20} = \frac{1}{5}$

1.2 $\frac{5}{20} = \frac{1}{4}$

1.3 $\frac{10}{20} = \frac{1}{2}$

1.4 $\frac{6}{20} = \frac{3}{10}$

1.5 $\frac{2}{20} = \frac{1}{10}$

1.6 $\frac{8}{20} = \frac{2}{5}$

1.7 $\frac{5}{20} = \frac{1}{4}$

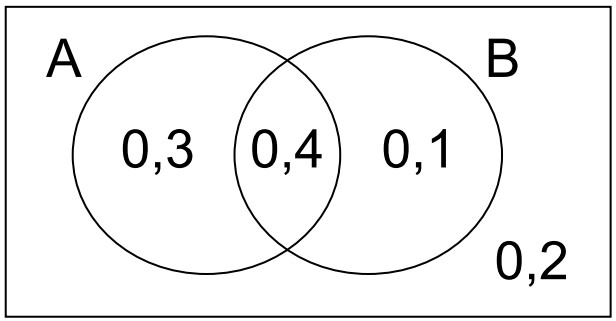
Question 2

P(not A) = 0,3
P(A or B) = 0,8
P(A and B) = 0,4

2.1 $P(A) = 1 - P(\text{not } A)$
$= 1 - 0,3$
$= 0,7$

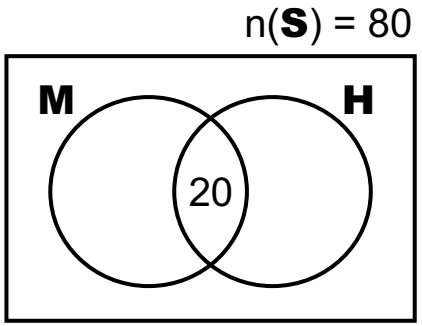
2.2 $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
$\therefore 0,8 = 0,7 + P(B) - 0,4$
$\therefore P(B) = 0,8 - 0,7 + 0,4$
$= 0,5$

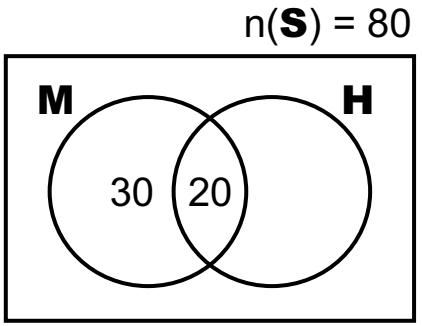
2.3

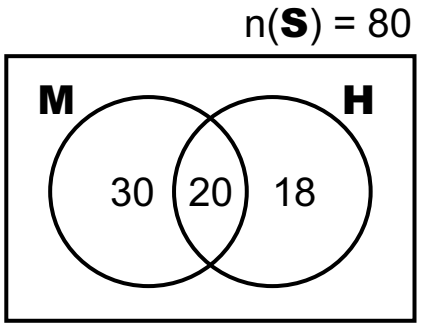


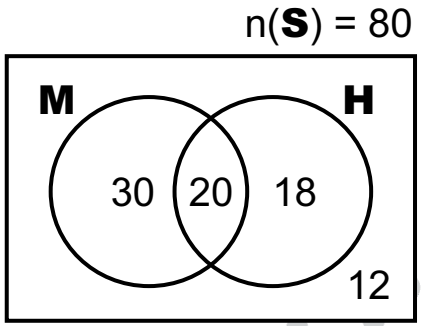
$P((\text{not } A) \text{ or } B) = P(\text{not } A) + P(B) - P((\text{not } A) \text{ and } B)$
$= 0,3 + (0,4 + 0,1) - 0,1$
$= 0,7$

Question 3





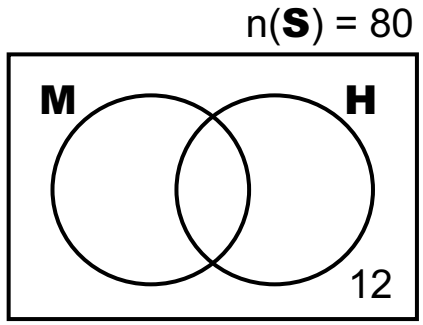


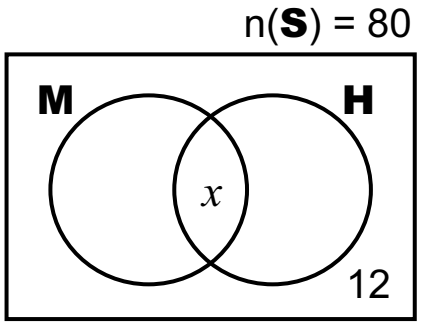


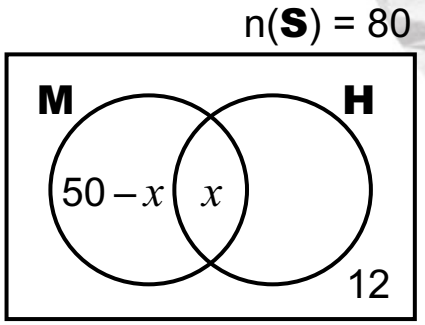
3.1 12 learners

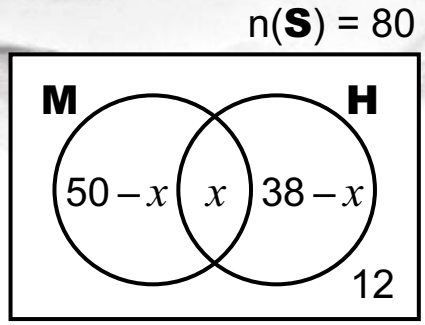
3.2 $\frac{30}{80} = \frac{3}{8}$

Question 4









$$50 - x + x + 38 - x + 12 = 80$$

$$\therefore \quad 100 - x = 80$$

$$\therefore \quad x = 20$$

There are 20 learners taking both Mathematics and History.

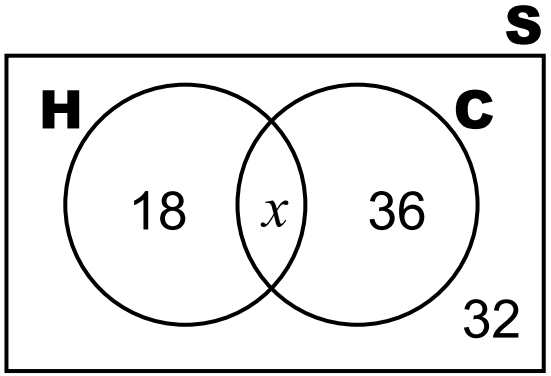
Question 5

$$x(x-3)+5+x+15 = 100$$
$$\therefore\ x^2-3x+5+x+15 = 100$$
$$\therefore\ x^2-2x-80 = 0$$
$$\therefore\ (x-10)(x+8) = 0$$
$$\therefore\ x = 10 \quad \text{or} \quad x = -8$$
$$\therefore\ x = 10$$
$$\therefore\ \text{P(D)} = \frac{10(10-3)}{100} = \frac{7}{10}$$

Question 6

$$18 \times 40 + 36 \times 15 + x \times (40+15) = 2\,855$$
$$\therefore\ 720 + 540 + 55x = 2\,855$$
$$\therefore\ 55x = 1\,595$$
$$\therefore\ x = 29$$

$\therefore$ 29 people bought both a hotdog and a cooldrink



Question 7

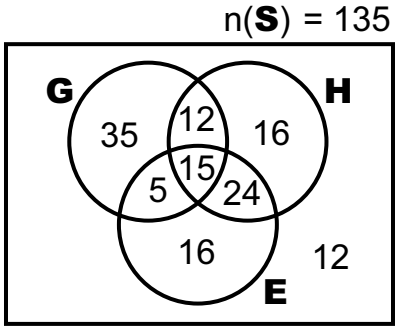
$5 + x + y + 24 = 60$

$\therefore\ x + y = 31$

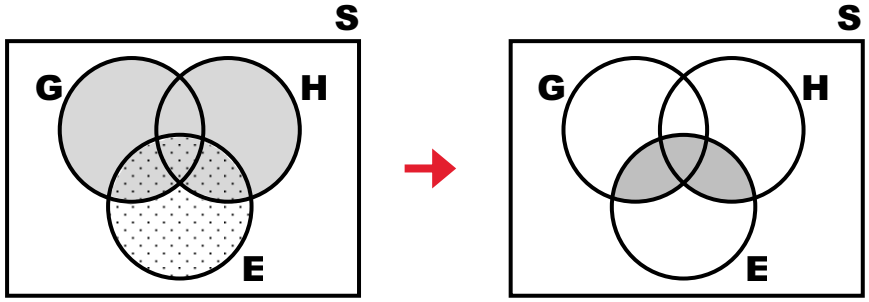
$2y + 3 + 12 + x + 5 = y + 12 + x + 24$

$\therefore\ y = 16$

$\therefore\ x = 15$



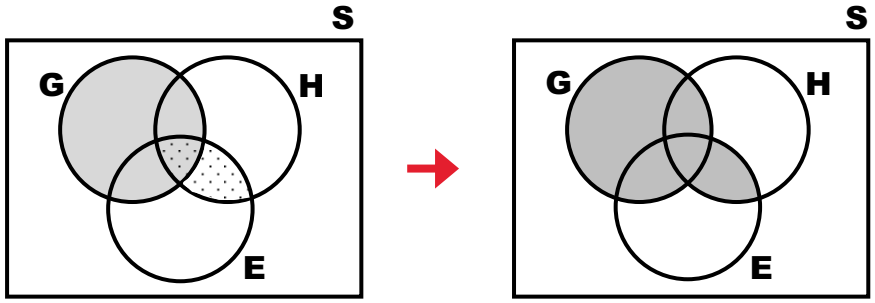
7.1



$\therefore\ P((G \text{ or } H) \text{ and } E) = \frac{5 + 15 + 24}{135}$

$= \frac{44}{135}$

7.2



$P(G \text{ or } (H \text{ and } E))$

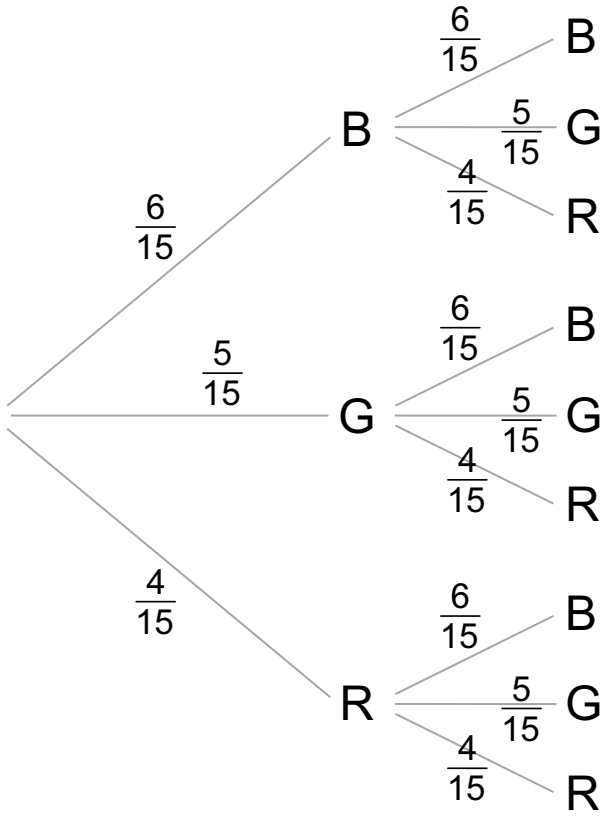
$= \frac{35 + 12 + 5 + 15 + 24}{135}$

$= \frac{91}{135}$

Question 8

8.1 P(RR)

$$= \frac{4}{15} \times \frac{4}{15} = \frac{16}{225}$$



8.2 P(BG)

$$= \frac{6}{15} \times \frac{5}{15} = \frac{2}{15}$$

8.3 P(BG or GB)

$$= \frac{6}{15} \times \frac{5}{15} + \frac{5}{15} \times \frac{6}{15} = \frac{4}{15}$$

Question 9

9.1 P(RR)

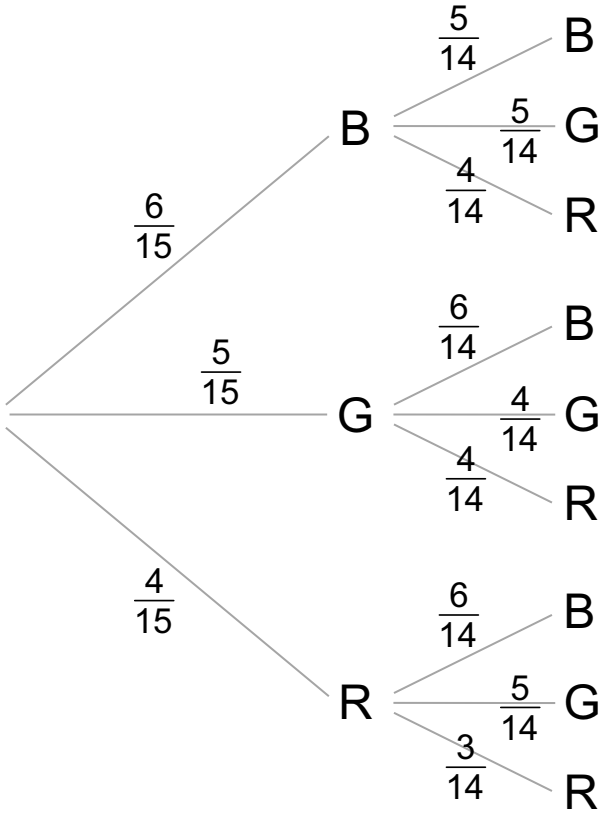
$$= \frac{4}{15} \times \frac{3}{14} = \frac{2}{35}$$

9.2 P(BG)

$$= \frac{6}{15} \times \frac{5}{14} = \frac{1}{7}$$

9.3 P(BG or GB)

$$= \frac{6}{15} \times \frac{5}{14} + \frac{5}{15} \times \frac{6}{14} = \frac{2}{7}$$



Question 10

Let the number of cheese sandwiches be x

$$\frac{120 - x}{120} \times \frac{x}{119} = \frac{18}{85}$$

$$\therefore \frac{120x - x^2}{14\,280} = \frac{18}{85}$$

$$\therefore 10\,200x - 85x^2 = 257\,040$$

$$\therefore 85x^2 - 10\,200x + 257\,040 = 0$$

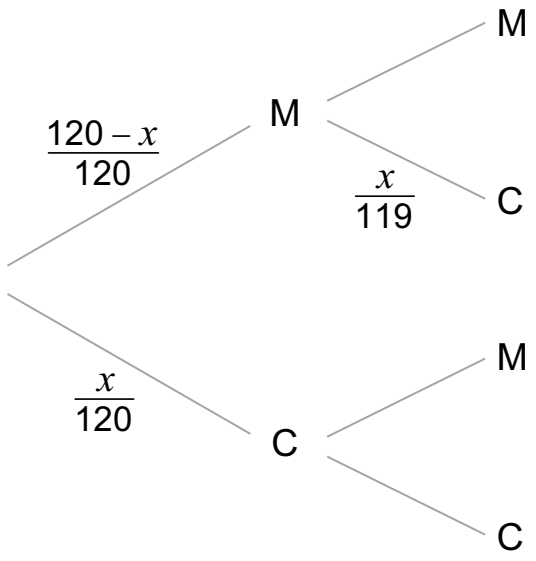
$$\therefore x^2 - 120x + 3\,024 = 0$$

$$\therefore x = \frac{120 \pm \sqrt{120^2 - 4(3\,024)}}{2}$$

$\therefore x = 84$ or $x = 36$

$\therefore$ there are 36 cheese sandwiches

$\therefore$ P(C) $= \frac{36}{120} = \frac{3}{10}$



Question 11

$$P(S \text{ or } T) = P(S) + P(T) - P(S) \times P(T)$$

$$\therefore \ 0{,}84 = y + y - y \times y$$

$$\therefore \ y^2 - 2y + 0{,}84 = 0$$

$$\therefore \ y = \frac{2 \pm \sqrt{2^2 - 4 \times 1 \times 0{,}84}}{2 \times 1}$$

$$\therefore \ y = \frac{7}{5} \text{ or } y = \frac{3}{5}$$

$$\therefore \ y = \frac{3}{5}$$

or

$$y^2 - 2y + 0{,}84 = 0$$

$$\therefore \ 100y^2 - 200y + 84 = 0$$

$$\therefore \ 25y^2 - 50y + 21 = 0$$

$$\therefore \ (5y - 7)(5y - 3) = 0$$

$$\therefore \ y = \frac{7}{5} \text{ or } y = \frac{3}{5}$$

$$\therefore \ y = \frac{3}{5}$$

Question 12

$$\frac{x}{250} = 0{,}48$$

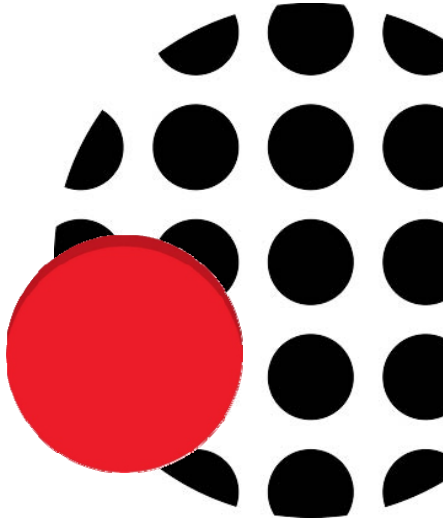
$$\therefore \ x = 120$$

$$P(J) = \frac{120 + 30}{250} = \frac{3}{5}$$

	Sandwich (S)	Pie (P)	TOTAL
Juice (J)	120	30	
Water (W)			
TOTAL	200		250

$$P(S) = \frac{200}{250} = \frac{4}{5}$$

	Sandwich (S)	Pie (P)	TOTAL
Juice (J)	120	30	
Water (W)			
TOTAL	200		250



$$P(J) \times P(S) = \frac{3}{5} \times \frac{4}{5} = \frac{12}{25}$$

$$P(J \text{ and } S) = \frac{120}{250} = \frac{12}{25}$$

	Sandwich (S)	Pie (P)	TOTAL
Juice (J)	120	30	
Water (W)			
TOTAL	200		250

$\therefore$ choosing a juice and a sandwich are independent since

$P(J \text{ and } S) = P(J) \times P(S)$

Question 13

$$\text{P(no six)} = \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = \frac{625}{1\,296}$$

$$\therefore \text{ P(at least one six)} = 1 - \frac{625}{1\,296} = \frac{671}{1\,296}$$

Question 14

14.1 $$\text{P(A)} + \text{P(B)} = \frac{2}{5} + \frac{1}{4}$$

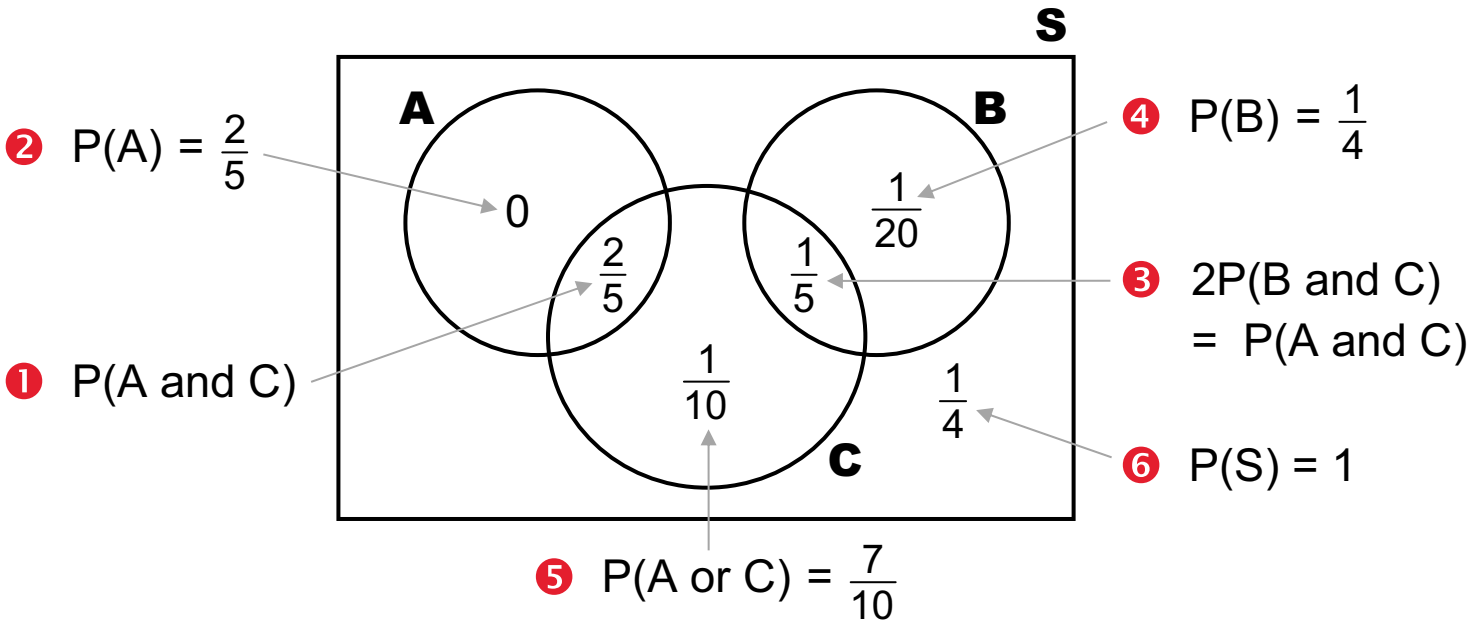
$$= \frac{8 + 5}{20}$$

$$= \frac{13}{20}$$

$$= \text{P(A or B)}$$

$\therefore$ A and B are mutually exclusive

14.2



$\therefore$ P(only C) = $\frac{1}{10}$

14.3 $\frac{1}{4}$

Question 15

Let the number of girls be x

$$\frac{10}{x+10} \times \frac{9}{x+9} = 0{,}15$$

$$\therefore \frac{90}{x^2+19x+90} = \frac{3}{20}$$

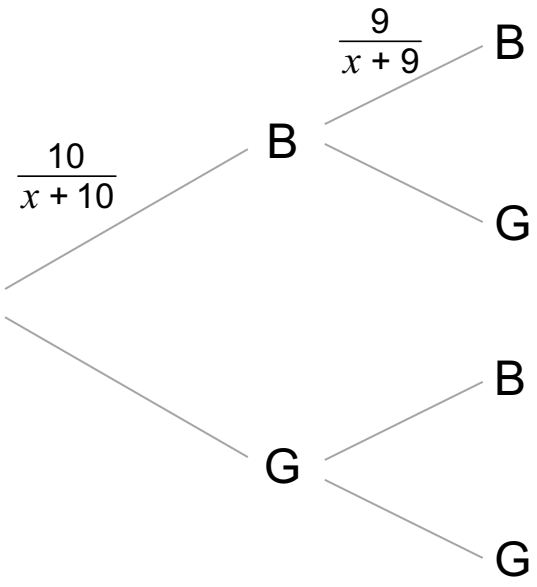
$$\therefore 3x^2 + 57x + 270 = 1\,800$$

$$\therefore 3x^2 + 57x - 1\,530 = 0$$

$$\therefore x^2 + 19x - 510 = 0$$

$$\therefore (x-15)(x+34) = 0$$

$$\therefore x = 15 \quad \text{or} \quad x \neq -34$$



There are 15 girls in the class.

OR

Let the number in the class be x

$$\frac{10}{x} \times \frac{9}{x-1} = 0{,}15$$

$$\therefore \frac{90}{x^2 - x} = \frac{3}{20}$$

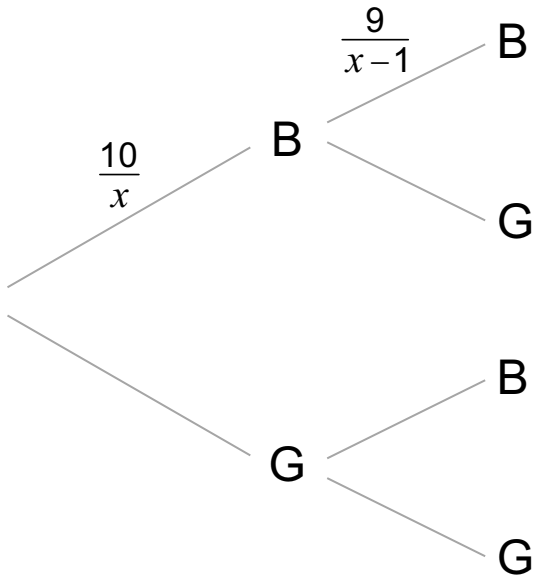
$$\therefore 3x^2 - 3x = 1\,800$$

$$\therefore 3x^2 - 3x - 1\,800 = 0$$

$$\therefore x^2 - x - 600 = 0$$

$$\therefore (x - 25)(x + 24) = 0$$

$$\therefore x = 25 \quad \text{or} \quad x \neq -24$$



There are $25 - 10 = 15$ girls in the class.

Question 16

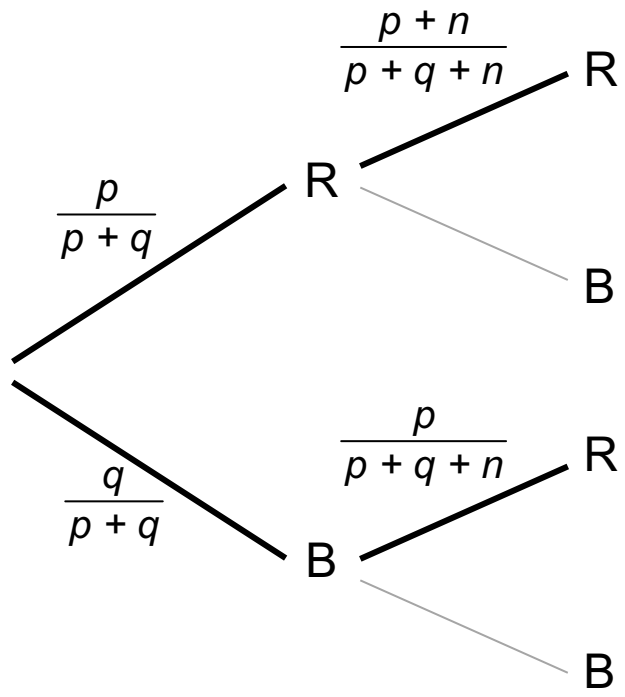
$$\frac{p(p+n)}{(p+q)(p+q+n)} + \frac{pq}{(p+q)(p+q+n)}$$

$$= \frac{p^2 + pn + pq}{(p+q)(p+q+n)}$$

$$= \frac{p(p+n+q)}{(p+q)(p+q+n)}$$

$$= \frac{p}{p+q}$$

which is independent of n



Question 1

A1	A2	A3	A4	
B1	B2	B3	B4	
C1	C2	C3	C4	∴ 12 different ways

Question 2

Starting with the mushroom dish:

MCI MCT MBI MBT MFI MFT MVI MVT

Then the soup:

SCI SCT SBI SBT SFI SFT SVI SVT

Then the pate:

PCI PCT PBI PBT PFI PFT PVI PVT

∴ 24 different combinations

Note: there is no need to list everything once you start seeing a pattern forming!

Question 3

3.1 Number of ways $= 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$

$= 10^{10}$

$= 10\ 000\ 000\ 000$

3.2 Number of ways $= 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 10!$

$= 3\ 628\ 800$

Question 4

4.1 Number of ways $= 10 \times 10 \times 10 \times 10 \times 10 \times 10$

$= 10^{6}$

$= 10\ 000\ 000$

4.2 Number of ways $= 10 \times 9 \times 8 \times 7 \times 6 \times 5$

$= 151\ 200$

Question 5

Number of ways $= 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 9!$

$= 362\,880$

Question 6

Number of codes $= 21 \times 20 \times 19 \times 7 \times 7 \times 7$

$= 2\,737\,140$

Question 7

Number of codes $= 5 \times 26 \times 26 \times 10 \times 10 \times 5$

$= 1\,690\,000$

Question 8

8.1 $8! = 40\,320$

8.2 Number of arrangements $= 1 \times 6! \times 1$

$= 720$

8.3 $P = \frac{720}{40\,320} = \frac{1}{56}$

OR

P(start with C) $= \frac{1}{8}$

P(end with M) $= \frac{1}{7}$

$\therefore$ P(start with C and end with M) $= \frac{1}{8} \times \frac{1}{7} = \frac{1}{56}$

Question 9

9.1 Number of ways $= 10 \times 9 \times 8 \times 7 \times 6$

$= 30\ 240$

9.2 Number of ways

$= (3\times7\times6\times5\times4)+(7\times3\times6\times5\times4)+(7\times6\times3\times5\times4)+(7\times6\times5\times3\times4)+(7\times6\times5\times4\times3)$

$= 3 \times 7 \times 6 \times 5 \times 4 \times 5$

$= 12\ 600$

Question 10

10.1 $8 \times 8 = 64$

10.2 $8 \times 7 \times 6 \times 5 = 1\ 680$

10.3 $1 \times 8 \times 8 \times 8 = 512$

Question 11

11.1 $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

11.2 $2 \times 4 \times 3 \times 2 \times 1 \times 1 = 48$

11.3 $4 \times 3 \times 2 \times 1 \times 2 \times 1 = 48$

11.4 $5! \times 2! = 240$

Question 12

12.1 $8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 8! = 40\,320$

12.2 $4! \times 5! = 2\,880$

12.3 $2! \times 5! \times 3! = 1\,440$

Question 13

$$\frac{9!}{3! \times 2!} = 30\ 240$$

Question 14

$$1 \times 5 \times 4 \times 3 = 60$$

Question 15

15.1 $2! \times 7! \times 4! = 241\ 920$

15.2 Total number of ways $= 11!$

Number of successful ways $= 7 \times 9! \times 4$

$$\therefore P = \frac{7 \times 9! \times 4}{11!} = \frac{14}{55}$$

OR $P(\text{shirt}) = \frac{7}{11}$

$P(\text{trousers}) = \frac{4}{10}$

$$\therefore P = \frac{7}{11} \times \frac{4}{10} = \frac{14}{55}$$

Question 16

16.1 $\frac{7!}{2! \times 2!} = 1\ 260$

16.2 Number of ways if vowels are together $\frac{5! \times 3!}{2! \times 2!} = 180$

$\therefore P = \frac{180}{1\ 260} = \frac{1}{7}$

Question 17

17.1 $\frac{10!}{2! \times 2! \times 2!} = 453\ 600$

17.2 Number of ways to start and end with C $= \frac{2 \times 8! \times 1}{2! \times 2! \times 2!} = 10\ 080$

$\therefore P = \frac{10\ 080}{453\ 600} = \frac{1}{45}$

OR $P = \frac{2}{10} \times \frac{1}{9} = \frac{1}{45}$

Question 18

Number of ways if they sit together $= 5! \times 2! = 240$

Total number of ways $= 6! = 720$

Number of ways they don't sit together $= 720 - 240 = 480$

Question 19

Total number of PIN's $= 10^5$

Number of PIN's containing no 9's $= 9^5$

Number of PIN's containing at least one 9 $= 10^5 - 9^5$

$$\therefore P = \frac{10^5 - 9^5}{10^5} = 0{,}41$$

Question 20

20.1 23 letters available (can't use E, G and O)

Remaining vowels are A, I and U, which can't be used at the beginning.

$$20 \times 22 \times 21 \times 10 \times 10 \times 10 = 9\ 240\ 000$$

20.2 You can't have a vowel in the first position.

Vowel in second position: $20 \times 3 \times 19 \times 10 \times 10 \times 5 = 570\ 000$

Vowel in third position: $20 \times 19 \times 3 \times 10 \times 10 \times 5 = 570\ 000$

$$\therefore P = \frac{570\ 000 + 570\ 000}{9\ 240\ 000} = \frac{19}{154}$$

Question 21

21.1 $9 \times 9 \times 8 = 648$

21.2 If start with 6: $1 \times 8 \times 4 = 32$

If start with 7: $1 \times 8 \times 5 = 40$

If start with 8: $1 \times 8 \times 4 = 32$

If start with 9: $1 \times 8 \times 5 = 40$

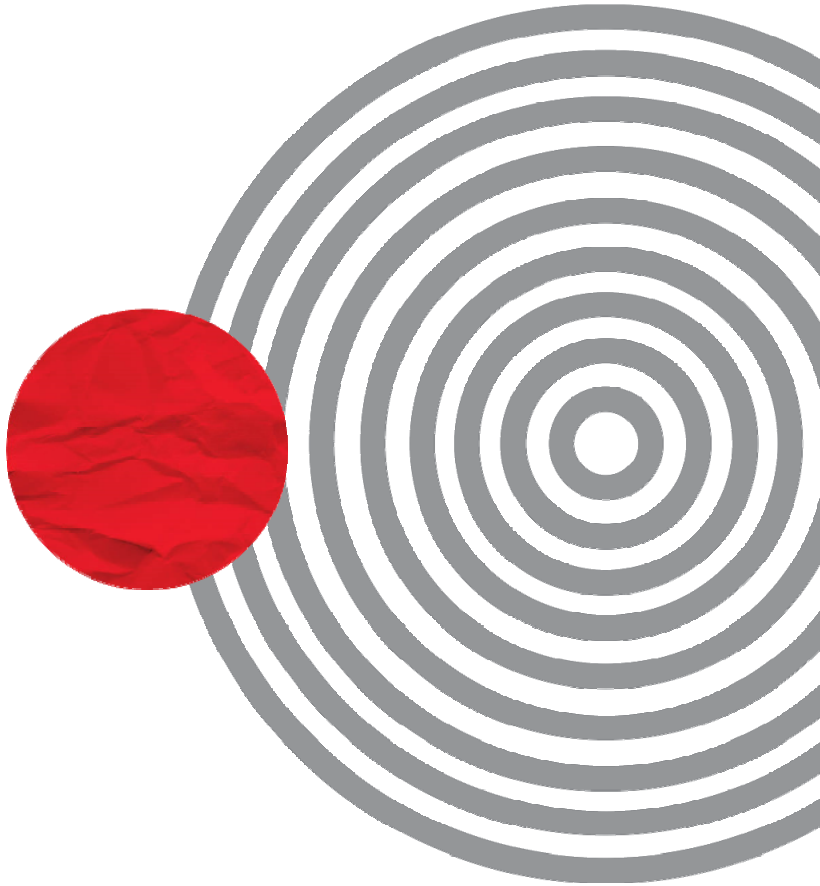
Total number of ways = 32 + 40 + 32 + 40 = 144

Question 22

22.1 7! = 5 040

22.2

ABCD	BCDE	CDEF	DEFG
ABCE	BCDF	CDEG	
ABCF	BCDG	CDFG	
ABCG	BCEF	CEFG	
ABDE	BCEG		
ABDF	BCFG		
ABDG	BDEF		
ABEF	BDEG		
ABEG	BDFG		
ABFG	BEFG		
ACDE			
ACDF			
ACDG			
ACEF			
ACEG			
ACFG			
ADEF			
ADEG			
ADFG			
AEFG			



$$\therefore P = \frac{35}{7 \times 6 \times 5 \times 4}$$

$$= \frac{1}{24}$$

22.3 F F M F M F M

$4 \times 3 \times 3 \times 2 \times 2 \times 1 \times 1 = 144$

F M F F M F M

$4 \times 3 \times 3 \times 2 \times 2 \times 1 \times 1 = 144$

F M F M F F M = 144
F M F M F M F = 144
M F F M F M F = 144
M F F M F F M = 144
M F M F F M F = 144
M F M F F F M = 144
M F M F M F F = 144
M F F F M F M = 144

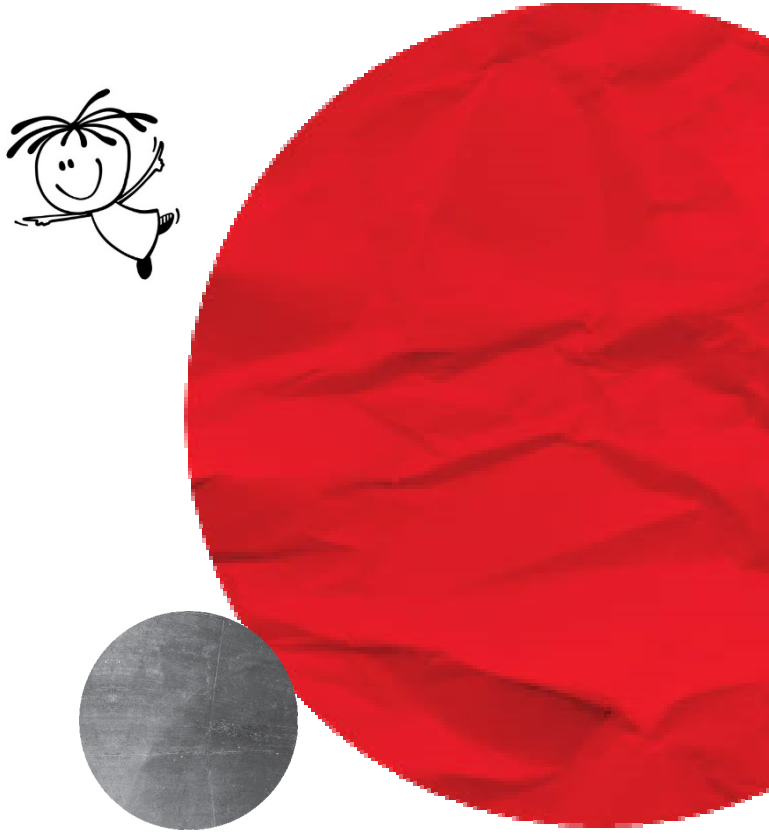
$\therefore$ $144 \times 10 = 1\,440$ ways

OR

	F		F		F		F	

Ways for males to be arranged $= 5 \times 4 \times 3$
Four females can arrange in 4! ways

$\therefore$ number of ways $= 5 \times 4 \times 3 \times 4! = 1\,440$



Question 23

Words beginning with A: $1 \times 4 \times 3 \times 2 \times 1 = 24$

Words beginning with H: $1 \times 4 \times 3 \times 2 \times 1 = 24$

Words beginning with M: MAHST

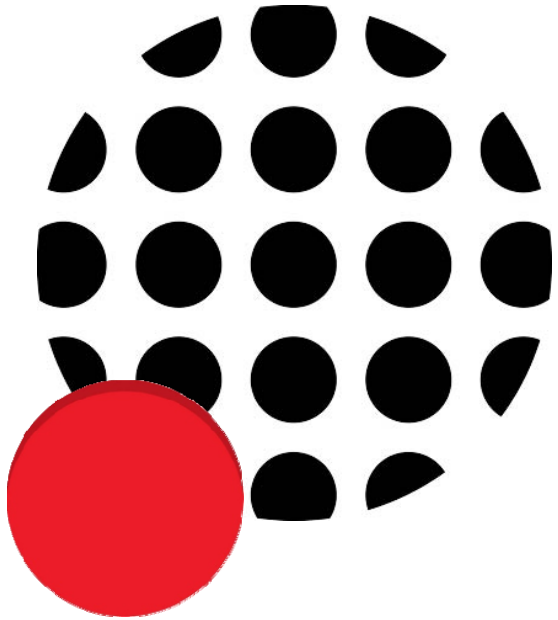
MAHTS

MASHT

MASTH

MATHS

MATHS is in the 53rd position.



Question 24

$$\frac{2023! + 2022!}{2024!} = \frac{2022!(2023 + 1)}{2024!} = \frac{2022!\ (2024)}{2024 \times 2023 \times 2022!} = \frac{1}{2023}$$

Question 25

$$\frac{((3!)!)!}{3!} = \frac{(6!)!}{6}$$

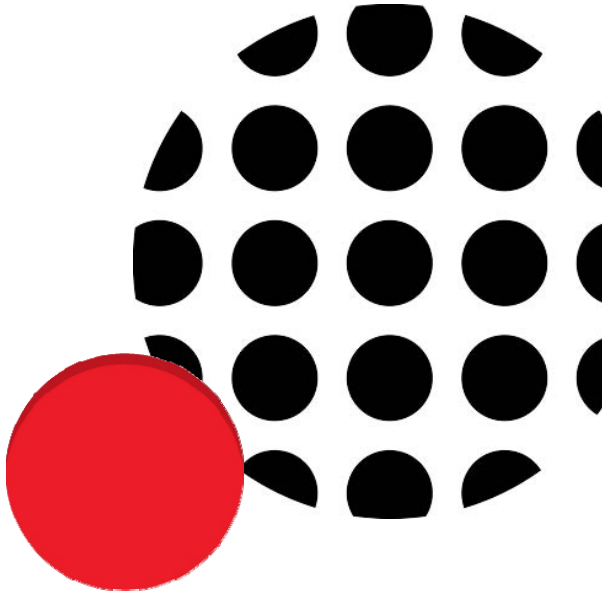
$$= \frac{720!}{6}$$

$$= \frac{720 \times 719!}{6}$$

$$= 120 \times 719!$$

$$\therefore\ a + b = 120 + 719$$

$$= 839$$



Question 26

$$\sum_{i=1}^{2023} (i!)^2 = (1!)^2 + (2!)^2 + (3!)^2 + (4!)^2 + (5!)^2 + \ldots + (2023!)^2$$

$$= 1^2 + 2^2 + 6^2 + 24^2 + 120^2 + \ldots + (2023!)^2$$

$$= 1 + 4 + 36 + 576 + 14\,400 + \ldots + (2023!)^2$$

To determine the units digit, add the units digits.

$1 + 4 + 6 + 6 = 17$

$\therefore$ the units digit is 7.

Probability

References to TAS Maths books

Gr 10 Maths 3-in-1 (Module 12)

	12.1	
Definitions and Terminology	12.1	→ 12.2
Calculating Probability	12.2	→ 12.3
Visual Representations of Probability	12.3	→ 12.11

Also see: Exemplar P1 (Q5) — E2

Gr 11 Maths 3-in-1 (Module 12)

	12.1	
Introduction to Probability	12.2	→ 12.5
Venn Diagrams	12.6	→ 12.10
Independent Events	12.11	→ 12.14
2-Way Contingency Tables	12.15	→ 12.16
Exercises	12.17	→ 12.19

Also see: Exemplar P1 (Q11 & Q12) — Q3

Gr 12 Maths 2-in-1 (Module 12)

	50	
The Probability Rules	50	
Venn Diagrams	50	→ 51
Tree Diagrams	51	→ 52
2-Way Contingency Tables	52	→ 53
Fundamental Counting Principle	53	

See Challenging Questions booklet: pages 16 → 19

See the Topic Guide on p. 147 for further exam practice.

Gr 12 Maths Past Papers Toolkit

See the Topic Guide on p. 1 for relevant questions.

THE ANSWER SERIES *Your Key to Exam Success*

THANK YOU!

CONTACT US

Maths queries:
jenny@theanswerseries.co.za

General queries:
info@theanswerseries.co.za

(021) 671 0837

064 371 8715

Please scan here to fill in the feedback form