Mathematical Literacy

CLASS TEXT & STUDY GUIDE

S. Nicol & L. van Rensburg

GRADE

10

CAPS

3-in-1

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Grade 10 Maths Literacy 3-in-1 CAPS

CLASS TEXT & STUDY GUIDE

This Grade 10 Maths Literacy 3-in-1 study guide offers both learners and teachers a comprehensive and innovative approach to Maths Literacy.

The stimulating easy-to-follow material and clear illustrations guide you effortlessly through each topic. Graded exercises placed strategically throughout this study guide, allow you to systematically bolster your knowledge of new concepts and exam techniques.

Key features:

- Easy-to-understand, step-by-step approach
- Comprehensive notes and worked examples for all 7 topics
- Exercises and 'Test your Understandings' for each topic
- Detailed answers with explanations and handy hints

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THIS CLASS TEXT & STUDY GUIDE INCLUDES

1 Notes and Worked Examples

2 Questions per Topic

3 Detailed Answers

E-book available

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2014 publication | 2020 revised edition | ISBN: 978-1-920297-81-7

080421 | NOVUS

CONTENTS

Simplifying ratios

- Ratios must always be written in their **simplest form**. This is done by **dividing both** values in the ratio by the **highest common factor**.

  Example: (a)

  25 : 625 ÷ 25 → 1 : 25

  (b)

  16 : 28 ÷ 4 → 4 : 7

- The values in a final ratio must always be given as whole numbers in their simplest form. When ratios are given in decimal form, multiply through by whichever multiple of 10 gets rid of the decimals and simplify.

  Example:

  Express the ratio 5,4 : 9,66 in its simplest form.

  5,4 : 9,66

  = 540 : 966 → make both decimals whole numbers by multiplying both sides by 100

  = 90 : 161 → simplify by dividing both sides by 6

  5,4 : 9,66 × 100 → *540 : 966* ÷ 6 → *90 : 161*

- When fractions are given as ratios, find a common denominator and convert both fractions. Since the denominators are now the same, the numerators are equivalent.

  Example:

  Express $\frac{3}{4} : \frac{2}{3}$ as a ratio in its simplest form.

  $\frac{3}{4} : \frac{2}{3} = \frac{\mathbf{9}}{12} : \frac{\mathbf{8}}{12}$ → find a common denominator and convert

  = 9 : 8 → because the denominators are the same we can now write the numerators as a ratio (Note: 9 : 8 is in its simplest form.)

Worked Examples

1. Write down any equivalent ratio for the following:

   1.1 4 : 3 ∴ *8 : 6* — *Multiply both values by the same number.*

   1.2 60 : 128 ∴ *15 : 32* — *Divide both values by the same number.*

2. Express the following ratios in their simplest form:

   2.1 Sipho's mass of 65 kg to Bongani's mass of 80 kg.

   65 : 80
   = *13 : 16*

   ÷ 5: 65 : 80 → 13 : 16 (divide by the HCF) … $\frac{65}{80} = \frac{13}{16}$

   2.2 Lindiwe's age of 38 to her mother's age of 57.

   38 : 57
   = *2 : 3*

   ÷ 19: 38 : 57 → 2 : 3 (divide by the HCF) … $\frac{38}{57} = \frac{2}{3}$

   2.3 Thandi's height of 175 cm to Ben's height of 125 cm.

   175 : 125
   = *7 : 5*

   ÷ 25: 175 : 125 → 7 : 5 (divide by the HCF) … $\frac{175}{125} = \frac{7}{5}$

   2.4 128 : 108
   = *32 : 27*

   ÷ 4: 128 : 108 → 32 : 27 (divide by the HCF) … $\frac{128}{108} = \frac{32}{27}$

3. Express the following ratios in their simplest form.

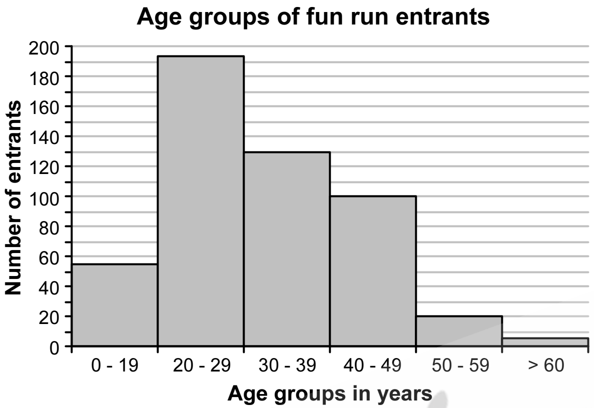
   3.1 0,72 : 0,36 = *72 : 36* *. . . × 100 to make both numbers whole numbers*
   = *2 : 1* *. . . simplify*

   × 100: 0,72 : 0,36 → 72 : 36; ÷ 36: 72 : 36 → 2 : 1

   3.2 2,4 : 48 = *24 : 480* *. . . × 10 to make both numbers whole numbers*
   = *1 : 20* *. . . simplify*

   × 10: 2,4 : 48 → 24 : 480; ÷ 24: 24 : 480 → 1 : 20

4. Study the following histogram and answer the questions that follow.



4.1 Explain, in your own words what this graph is about.

4.2 Which age group had the most number of entrants? How many entrants?

4.3 Which age group had the least number of entrants? Explain why there are so few.

4.4 How many people entered the fun run?

UNIT 2
PATTERNS AND RELATIONSHIPS

In this unit formal relationships between quantities are addressed.

The 3 formal relationships between two sets of values that need to be understood are:

- when two sets of quantities have a **constant** (fixed) **relationship**.
- when two sets of quantities have a **linear relationship** (including direct proportion).
- when two sets of quantities have an **indirect/inverse proportion relationship**.

In order to understand and be able to use, create and identify the above relationships, knowledge of the following features is essential:

Features of patterns

Independent and dependent variables

A table is often used to display patterns. The two sets of values in each row are related to each other and are called **variables**, namely the independent and dependent variable.

The **independent variable** (usually represented on the x-axis) is the one you **choose** and the **dependent variable** (usually represented on the y-axis) is the one that **depends on** the chosen (independent) variable.

Example:
The following table illustrates the cost of hiring a cottage per day:

Number of days (n)	1	2	5
Total cost (c) in Rands	500	1 000	2 500

The 'Number of days' is the **independent variable** and the 'Total cost' is the **dependent variable**. This is because the **cost of hiring** a cottage will depend on the **number of days** you hire it. Also, note that the 'Number of days' will be plotted on the x-axis, while the 'Total cost' will be plotted on the y-axis.

Note: *The independent variable must always be written in the top row of the table and the dependent variable in the second row.*

Discrete or continuous variables

Variables are classified as **discrete** when the values can only be **whole numbers**, e.g. number of people, the number of cars, etc. while **continuous** variables have values that are able to be **any type of number**, whole numbers, fractions or decimals, e.g. height, weight, temperature (any unit of measurement), etc.

Data is classified as discrete or continuous according to the **independent** variable.

❼ Reading off values from a graph

If the question is asking for the corresponding dependent variable, locate the given independent variable on the x-axis and travel vertically upwards until you hit the graph. Now at that point, travel horizontally across until you reach the y-axis. Now read off the dependent variable value.

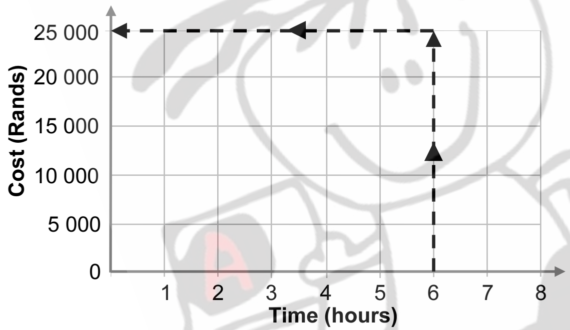
If the question is asking for the corresponding independent variable, locate the given dependent variable on the y-axis and travel horizontally across until you hit the graph. Now at that point, travel vertically downwards until you reach the x-axis. Now read off the independent variable value.

Consider the graph showing the cost of hiring the wedding venue on the previous page and answer the questions that follow:

Q: What would it cost to hire the wedding venue for 6 hours?

A: R25 000

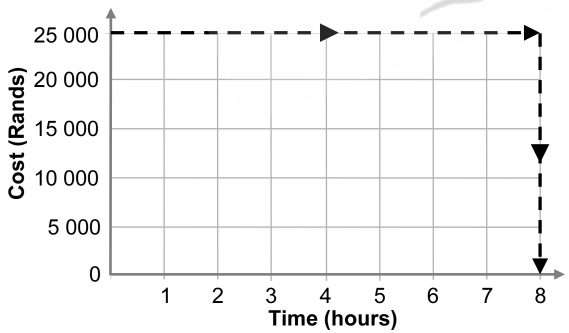
Cost of wedding venue vs time



Q: What is the maximum number of hours that the wedding venue could be hired for?

A: 8 hours

Cost of wedding venue vs time



Now try this! *Exercise 2*

Answers on page A7

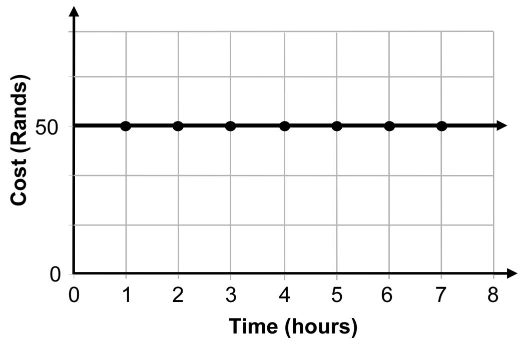
1. Jim hires a car for the day. The car rental company charges him R1 200 for the day (08:00 - 17:00) for an unlimited number of kilometres.
   - 1.1 Construct a table to represent the number of hours Jim can hire the car for versus the cost of the car.
   - 1.2 Use the table that you constructed in Question 1.1, to determine a formula for the cost of hiring the car for the day.
   - 1.3 Use the table that you constructed in Question 1.1, to draw the graph of the 'Cost of hiring the car' versus the 'Number of hours'.

2. Easy Talking cellphone company is launching a special that you only pay R50 per day, for an unlimited number of phone calls. This relationship can be represented in the table below:

Number of phone calls	0	1	5	12	20	**b**
Cost of phone calls (in Rands)	50	50	50	50	**a**	50

   - 2.1 What type of relationship is represented in the table?
   - 2.2 Determine a formula to represent the 'Cost of phone calls' using Easy Talking cellphone company.
   - 2.3 Determine the missing values **a** and **b** in the table.

3. Joan goes on holiday to a hot springs resort. They charge R50 per day for the unlimited use of their hot springs facilities. This can be illustrated in the graph below:



   - 3.1 How much would it cost Joan to use the hot springs for 4 hours?
   - 3.2 If Joan had R250, for how many days could she use the hot springs?

When investigating the most appropriate way to package cans and/or boxes for optimal use of space and the most cost effective way to package a number of cans and/or boxes, the following important things need to be remembered:

- **Practicality**
  E.g. a box that contains 48 cans that is 48 cans long and 1 can wide, would be very difficult to carry compared to a box that is 8 cans long and 6 cans wide.
- **Optimal use of space**
  The cans must be packed properly so as to ensure that the largest number of cans fit in the box, but the box must still be able to close. If the box doesn't close properly, it will take up more room than it should, reducing the amount of available space. This will increase transport costs as less boxes can fit in the truck or shipping container.
- **Cost-effectiveness**
  Packaging materials, such as cardboard, can be expensive. Therefore you need to find the smallest container that you can pack the cans in.

Example: Pattern 1 and 2 shown below waste a lot of space and the lids cannot close properly.

Pattern 1

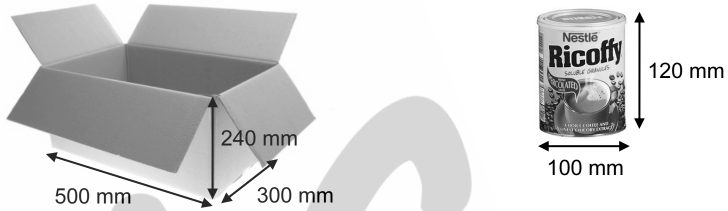
Pattern 2

In comparison, Pattern 3 maximises the available space and is therefore more cost-effective and practical to transport.

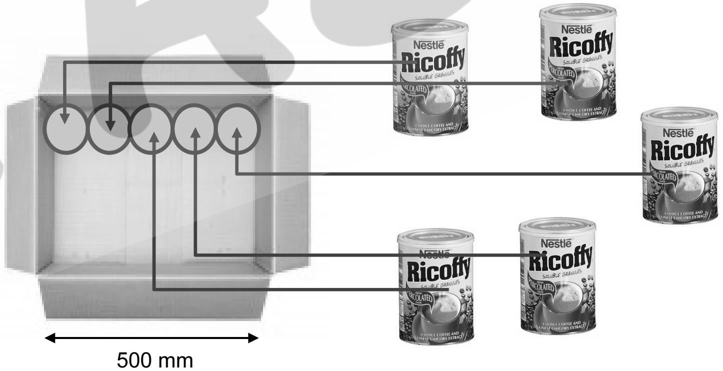
Pattern 3

Worked Examples

1. **Practical method:**
   How many tins of coffee could fit into this box if the diameter of each tin is 100 mm and its height is 120 mm?

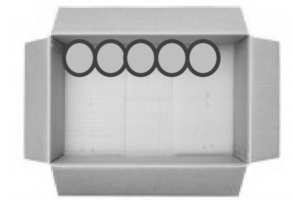


- *Place the first tin in one of the corners and keep on adding tins next to one another, until no more tins can fit. This will give you the maximum number of tins that you can fit across the length of the box, as shown below:*



∴ 5 tins fit across the length of the box.

> ***Note:*** *You will not always be able to fit the tins exactly in the box as can be seen in the picture below.*



Even though there is still space left, we would only be able to fit 5 tins across the length of the box.

CLASSIFYING AND ORGANISING DATA

categorical data	data that is generally descriptive in nature and can be classified into categories; data that cannot be measured e.g. colour; gender; shoe size
numerical data	data that has numerical values and can be quantified e.g. time; age; area
discrete data	numerical data that can be counted and only includes whole numbers e.g. number of puzzles pieces; number of building blocks
continuous data	numerical data that can be measured and includes any whole numbers, fractions or decimals e.g. length of a puzzle; weight of the building blocks
class intervals	the subdivision of data into intervals / groups; with each class interval having the same class width in order to be comparable e.g. class intervals of 10 data points: 10 - 19; 20 - 29; 30 - 39; etc.
class width	the number of data points (size) in the class interval i.e. class width = upper class boundary - lower class boundary
tally	a vertical stripe which represents the occurrence of a particular data value in a data set; and is a way of keeping count
tally table	a table whereby each data value is represented by a tally; in order to keep track of how often a piece of data appears in the raw data
frequency	the number of times something happens or appears
frequency table	a tally table which also includes a frequency column, whereby the frequencies of each data value are written in numerical format, together with the summed total of all the frequencies

SUMMARISING DATA

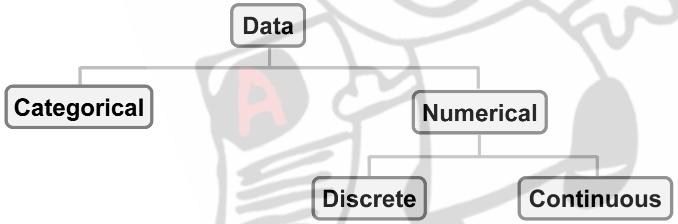
measure of central tendency	a value that provides an indication of the middle or centre of the data; and is a benchmark value again which to measure and compare the other values in the data set, as it is representative of the majority of the values in the data set *Mean, median and mode are measures of central tendency.*
mean	the 'average' of a numerical data set; and is a measure of central tendency i.e. mean = $\frac{\text{sum of all values in data set}}{\text{total number of values in data set}}$
median	the middle value of an ordered, numerical data set; and is a measure of central tendency i.e. for an even-numbered data set: median = $\frac{\text{sum of two 'middle' data values}}{2}$ *Data must first be arranged in ascending order.*
mode	the data value(s) that occur(s) most frequently in a data set; and is a measure of central tendency e.g. 1; 2; 2; 2; 3; 5 … 2 is the mode
bimodal	when two data values occur most frequently in a data set e.g. 12; 3; 6; 3; 9; 6; 11 …. bimodal data as 3 and 6 occur most frequently
measure of spread	a value that provides an indication of how 'spread out' the data is (i.e. whether the data values are very close together/clustered or whether they are very far apart/dispersed) *Range is a measure of spread.*
range	the difference between the highest and lowest values in a data set; and is a measure of spread i.e. range = highest value - lowest value

- **Numerical** data refers to data consisting of quantities or numerical values.
  - **Examples:** measurements e.g. length, height, area, volume, mass, speed, time, temperature, rainfall, humidity, sound levels, cost, members, ages, etc.
  - Numerical data can be further classified into **discrete** data or **continuous** data.

    Discrete data is a set of values that can be **counted**, e.g.
    - the number of children in a family
    - the number of cars in a parking lot
    - the number of people standing in a queue

    Continuous data is data that you **measure**, e.g.
    - the height of a learner
    - the mass of a learner
    - the time taken to run a race

- Therefore, data can be classified as follows:



In each of the above examples, the accuracy of the value recorded is limited by the measuring instrument being used, e.g.

- *It is possible to record an individual's height as 1,8 metres or 1,79 metres or 1,785 metres.*
- *It is possible to record an individual's mass as 75 kg or 74,53 kg or 74,538 kg.*
- *It is possible to record a lap time as 2 minutes or 1 minute 59 seconds or 1 minute 59,4 seconds or 1 minute 59,432 seconds*

Organising data

To organise collected data we use tallies and frequency tables.

Assume you have the following set of data:

1st questionnaire:	yes	yes	yes	yes	no	no	no	yes	no
2nd questionnaire:	yes	yes	no	no	yes	yes	yes	yes	yes

Tally tables

- As we go through each questionnaire, we put a vertical line (a tally) next to the appropriate answer (Yes/No).
- The responses of the questionnaires would be organised in a tally table as follows:

1st questionnaire

Answer	Tally
YES	~~IIII~~
NO	IIII

2nd questionnaire

Answer	Tally
YES	~~IIII~~ II
NO	II

The tallies are grouped into fives - each count is represented by a vertical line. IIII represents 4 and the fifth line is drawn horizontally through the previous 4… ~~IIII~~ to represent 5 → this makes the responses easier to count.

Frequency tables

- Another column is added to the tally table, whereby the frequency of the tallies is written in numerical form.
- The responses of the questionnaires combined would be organised in a frequency table as follows:

Answer	Tally	Frequency
YES	~~IIII~~ ~~IIII~~ II	12
NO	~~IIII~~ I	6
	Total	18

~~IIII~~ ~~IIII~~ II
5 + 5 + 2 = 12

- There are 360º in a circle, so to work out the angle of the sector, multiply the fraction by 360º:

$$\text{angle} = \frac{1}{3} \times 360^\circ = 120^\circ$$

- In one step:

$$\text{degrees of angle} = \frac{\text{money spent on shirt}}{\text{total amount spent}} \times 360^\circ = \frac{R200}{R600} \times 360^\circ = 120^\circ$$

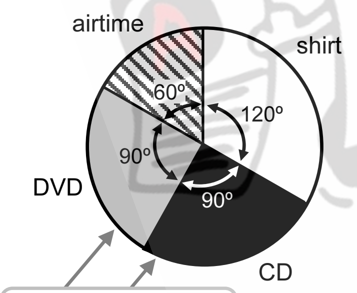
Repeat the process to find the angle of the sectors for each of the other items:

$$\text{DVD} = \frac{150}{600} \times 360^\circ = 90^\circ \qquad \text{CD} = \frac{150}{600} \times 360^\circ = 90^\circ \qquad \text{airtime} = \frac{100}{600} \times 360^\circ = 60^\circ$$

- Always check that your sector angles add up to 360º!

$$120^\circ + 90^\circ + 90^\circ + 60^\circ = 360^\circ$$

- **Pie chart showing how Siya spent his money**



Each slice of the pie is called a sector.

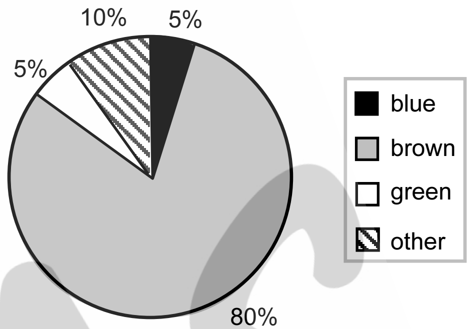
So remember: To calculate the size of a sector angle in degrees:

$$\frac{\text{category value}}{\text{total value}} \times 360^\circ$$

Worked Examples

The following pie chart was constructed from collecting data from 240 learners:

Learners' eye colour



1. Calculate the angle of each sector of the pie chart:

blue: 5% of 360º $= \frac{5}{100} \times 360^\circ = 18^\circ$

brown: 80% of 360º $= \frac{80}{100} \times 360^\circ = 288^\circ$

green: 5% of 360º $= \frac{5}{100} \times 360^\circ = 18^\circ$

other: 10% of 360º $= \frac{10}{100} \times 360^\circ = 36^\circ$

Check your answer to see if it adds up to 360º:
$18^\circ + 288^\circ + 18^\circ + 36^\circ = 360^\circ$

2. Calculate the number of learners in each segment:

blue: 5% of 240 $= \frac{5}{100} \times 240 = 12$

brown: 80% of 240 $= \frac{80}{100} \times 240 = 192$

green: 5% of 240 $= \frac{5}{100} \times 240 = 12$

other: 10% of 240 $= \frac{10}{100} \times 240 = 24$

Check your answer to see if it adds up to 240:
$12 + 192 + 12 + 24 = 240$