DBO NOV 2023 VRAESTEL 2 MEMO'S

STATISTIEK [20]

1.1 Die vergelyking: y = A + Bx

A = –23,8461…. ≃ –23,85

B = 0,2270…. ≃ 0,23

∴ **y = –23,85 + 0,23x** ➤

1.2 Stel x = 550: y = –23,85 + 0,23(550)

= **102,65 minute** ➤

1.3 Die korrelasiekoëffisiënt, r = 0,9828….

≃ **0,98** ➤

1.4 Daar is 'n baie sterk positiewe korrelasie tussen die afstand gereis en die hoeveelheid rustyd. ➤

1.5.1 Die gemiddelde, $\bar{x} = \frac{1\,200}{8}$ = **R150** ➤

1.5.2 Die standaardafwyking, σ = 50,4975….

≃ **50,50** ➤

1.5.3 Bedrag < $\bar{x}$ – 1σ

∴ Bedrag < 150 – 50,50

∴ Bedrag < 99,50

∴ **Slegs by 1 stopplek (waar hy R50 spandeer het)** ➤

2.1

AANTAL GLASE WATER PER DAG GEDRINK	AANTAL PERSONEEL-LEDE	KUMULATIEWE FREKWENSIE
$0 \le x < 2$	5	5
$2 \le x < 4$	15	20
$4 \le x < 6$	13	33
$6 \le x < 8$	5	38
$8 \le x < 10$	2	40

2.2 Daar is met 40 personeellede onderhoude gevoer ➤

2.3 33 personeellede het minder as 6 glase water gedrink ➤

2.4

AANTAL GLASE WATER PER DAG GEDRINK	MIDDELPUNT	AANTAL PERSONEEL-LEDE
$0 \le x < 2$	1	$5 + \frac{1}{2}k$
$2 \le x < 4$	3	15
$4 \le x < 6$	5	$13 + \frac{1}{2}k$
$6 \le x < 8$	7	5
$8 \le x < 10$	9	2
		40 + k

Benaderde gemiddelde = 4 wanneer k personeellede ingereken word

Totale aantal glase water

$$= \left[\left(5 + \frac{1}{2}k\right) \times 1\right] + (15 \times 3) + \left[\left(13 + \frac{1}{2}k\right) \times 5\right] + (5 \times 7) + (2 \times 9)$$

$$\therefore \text{Benaderde gemiddelde} = \frac{5 + \frac{1}{2}k + 45 + 65 + \frac{1}{2}k + 35 + 18}{40 + k} = 4$$

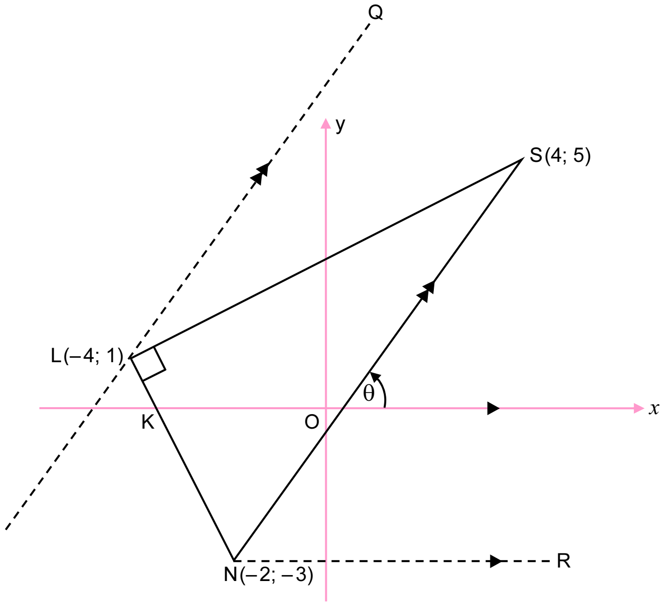
$$\therefore 168 + 3k = 160 + 4k$$

$$\therefore 8 = k$$

∴ **8 personeellede was afwesig** ➤

NOTAS

ANALITIESE MEETKUNDE [40]



3.1 $SL^2 = (4+4)^2 + (5-1)^2$
$= 64 + 16$
$= 80$
$\therefore$ SL $= \sqrt{80} =$ **$4\sqrt{5}$ eenhede** ✓

3.2 $m_{SN} = \frac{5-(-3)}{4-(-2)} = \frac{8}{6} =$ **$\frac{4}{3}$** ✓

3.3 $\tan\theta = \frac{4}{3}$
$\therefore$ **$\theta \simeq 53{,}13^\circ$** ✓

3.4 $S\hat{N}R = \theta = 53{,}13^\circ$ . . . *ooreenk. ∠e ; || lyne*

$m_{LN} = \frac{1-(-3)}{-4-(-2)} = \frac{4}{-2} = -2$

$\therefore \tan L\hat{N}R = -2$

$\therefore L\hat{N}R = 180^\circ - 63{,}43....$
$\simeq 116{,}57^\circ$

$\therefore L\hat{N}S = 116{,}57^\circ - 53{,}13^\circ$
$=$ **$63{,}44^\circ$** ✓

OF:

$LN^2 = (-4+2)^2 + (1+3)^2$
$= 4 + 16$
$= 20$
$\therefore LN = \sqrt{20} = 2\sqrt{5}$

$\therefore \tan L\hat{N}S = \frac{LS}{LN}$
$= \frac{4\sqrt{5}}{2\sqrt{5}}$
$= 2$

$\therefore$ **$L\hat{N}S = 63{,}43^\circ$** ✓

Let wel: verskil as gevolg van afronding

3.5 $m_{QL} = m_{SN} = \frac{4}{3}$ . . . *|| lyne*

Stel $m = \frac{4}{3}$ & $(-4; 1)$ in

$y - y_1 = m(x - x_1)$

$\therefore y - 1 = \frac{4}{3}(x+4)$

$\therefore$ **$y = \frac{4}{3}x + \frac{19}{3}$** ✓

3.6 Oppervlakte van $\Delta LSN = \frac{1}{2}LN.SL$

$LN^2 = (-4+2)^2 + (1+3)^2$
$= 4 + 16$
$= 20$
$\therefore LN = \sqrt{20}$

$\therefore$ Oppv. van $\Delta LSN = \frac{1}{2}\sqrt{20}\sqrt{80}$
$= \frac{1}{2}\sqrt{1\,600}$
$= \frac{1}{2}(40)$
$=$ **20 eenhede^2** ✓

3.7 P ewe ver van L, S en N

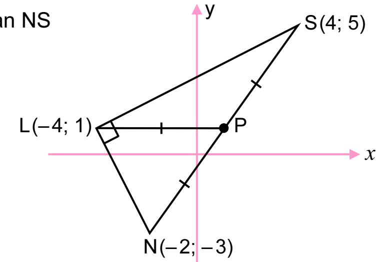
$N\hat{L}S = 90^\circ$

$\therefore$ NS is die middellyn van ⊙LSN . . . *omgekeerde ∠ in semi-⊙*

$\therefore$ P is die middelpunt van NS

$\therefore P\left(\frac{-2+4}{2}; \frac{-3+5}{2}\right)$

$\therefore$ **P(1; 1)** ✓



3.8 $L\hat{P}S = 2L\hat{N}S$ . . . *middelpunts-∠ = 2 × omtreks-∠*
$= 2(63{,}44^\circ)$
$=$ **$126{,}88^\circ$** ✓

OF: PL || *x*-as . . . $y_P = y_L$
$L\hat{P}N = 53{,}13^\circ$ . . . *verw. ∠e ; || lyne*
$\therefore L\hat{P}S = 180^\circ - 53{,}13^\circ$ . . . *∠e op 'n reguitlyn*
$=$ **$126{,}87^\circ$** ✓

Let wel: verskil as gevolg van afronding

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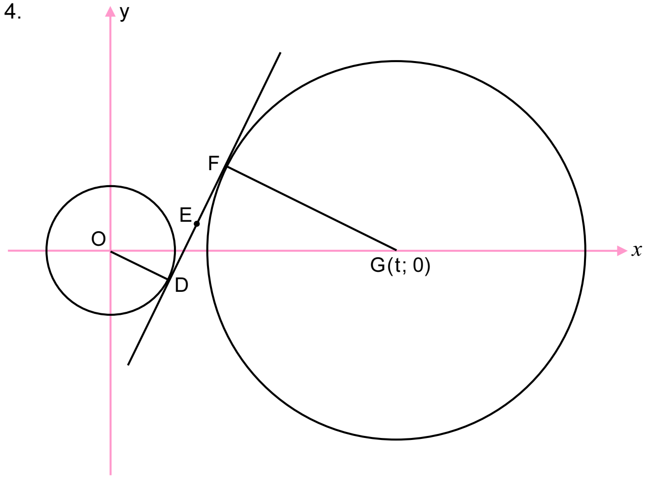
4.



4.1 D(p; –2) op ⊙O ➡ $p^2 + (-2)^2 = 20$

$\therefore\ p^2 = 16$

$\therefore\ \mathbf{p = 4}$ ◀ . . . *p > 0 in 4^de Kwadrant*

4.2 **F(8; 6)** ◀ . . . *deur inspeksie*

4.3 $m_{\text{radius OD}} = -\frac{2}{4} = -\frac{1}{2}$

$\therefore\ m_{DF} = 2$

OF: $m_{DE} = \frac{2+2}{6-4} = 2$

Stel m = 2 & (4; –2) in

$y - y_1 = m(x - x_1)$

$\therefore\ y + 2 = 2(x - 4)$

$\therefore\ \mathbf{y = 2x - 10}$ ◀

OF: $y = mx + c$

$\therefore\ -2 = (2)(4) + c$

$\therefore\ c = -10$, ens.

4.4 $m_{FG} = -\frac{1}{2}$ . . . *FG ⊥ DF*

$\therefore\ \frac{6-0}{8-t} = -\frac{1}{2}$

$\times 2(8-t)$ $\therefore\ 12 = -(8-t)$

$\therefore\ 12 = -8 + t$

$\therefore\ \mathbf{t = 20}$ ◀

OF:

gebruik $m = -\frac{1}{2}$ en (8; 6):

Vergelyking van DF:

$y - 6 = -\frac{1}{2}(x - 8)$,

& stel y = 0 in

4.5 Middelpunt G is (20; 0)

& $r^2 = FG^2 = (20-8)^2 + (0-6)^2$

$= 144 + 36$

$= 180$

$\therefore$ Vergelyking van ⊙G: $(x - 20)^2 + (y - 0)^2 = 180$

$\therefore\ x^2 - 40x + 400 + y^2 - 180 = 0$

$\therefore\ \mathbf{x^2 + y^2 - 40x + 220 = 0}$ ◀

4.6 Punt A waar die klein sirkel die x-as sny, moet beweeg na punt B waar die groot sirkel die x-as sny, of, C na H.

Klein ⊙: $r = \sqrt{20} = 2\sqrt{5}$ . . . $x^2 + y^2 = 20$

$\therefore\ x_A = -2\sqrt{5}$ en $x_C = 2\sqrt{5}$

Groot ⊙: $R = \sqrt{180} = 6\sqrt{5}$

$\therefore\ x_B = 20 - 6\sqrt{5}$ en $x_H = 20 + 6\sqrt{5}$

$\therefore$ A ➡ B: $k = 20 - 6\sqrt{5} - (-2\sqrt{5})$

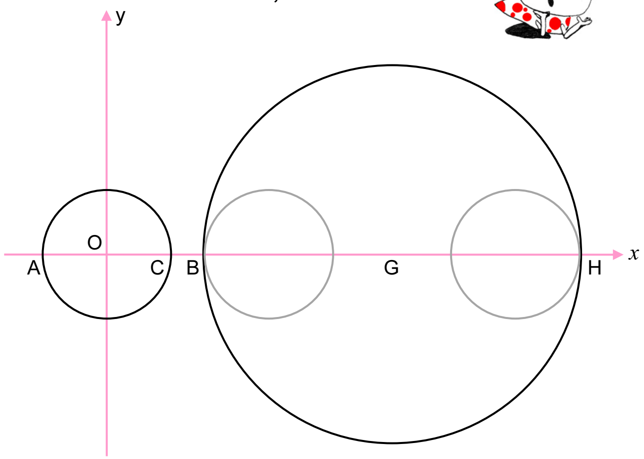
$= 20 - 4\sqrt{5}$

$\simeq$ **11,06** ◀

$\therefore$ C ➡ H: $k = 20 + 6\sqrt{5} - 2\sqrt{5}$

$= 20 + 4\sqrt{5}$

$\simeq$ **28,94** ◀



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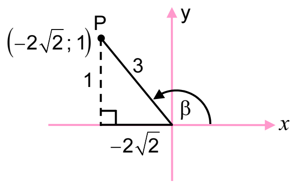
TRIGONOMETRIE [50]

5.1.1 $x_P = -\sqrt{9-1}$

$= -\sqrt{8}$

$= -2\sqrt{2}$

$\therefore \cos\beta = \frac{x}{r} = -\frac{\mathbf{2\sqrt{2}}}{\mathbf{3}}$ ◂



5.1.2 $\sin 2\beta = 2\sin\beta\cos\beta = 2\left(\frac{1}{3}\right)\left(-\frac{2\sqrt{2}}{3}\right)$

$= -\frac{\mathbf{4\sqrt{2}}}{\mathbf{9}}$ ◂

5.1.3 **Metode 1**

$\cos(450° - \beta) = \cos(90° - \beta)$

$= \sin\beta$

$= \frac{\mathbf{1}}{\mathbf{3}}$ ◂

Metode 2

$\cos(450° - \beta) = \cos 450° \cos\beta + \sin 450° \sin\beta$

$= \cos 90° \cos\beta + \sin 90° \sin\beta$

$= (0)(\cos\beta) + (1)\sin\beta$

$= \sin\beta$

$= \frac{\mathbf{1}}{\mathbf{3}}$ ◂

5.2.1 $\frac{(\cos^2 x)^2 + \sin^2 x \cos^2 x}{1 + \sin x}$

$= \frac{\cos^2 x(\cos^2 x + \sin^2 x)}{1 + \sin x}$

$= \frac{(1 - \sin^2 x)(1)}{1 + \sin x}$

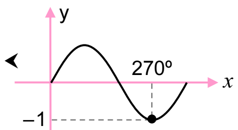
$= \frac{(1 + \sin x)(1 - \sin x)}{1 + \sin x}$

$= \mathbf{1 - \sin x}$ ◂

5.2.2 Ongedef. wanneer $1 + \sin x = 0$

$\therefore \sin x = -1$

$\therefore x = \mathbf{270°}$ ◂



5.2.3 $-1 \leq \sin x \leq 1$

Die minimum waarde van $1 - \sin x$

$= 1 - 1$

$= \mathbf{0}$ ◂

[Die minimum kom voor wanneer $\sin x = 1$]

5.3.1 $\sin(A - B) = \cos[90° - (A - B)]$

$= \cos[(90° - A) - (-B)]$

$= \cos(90° - A)\cos(-B) + \sin(90° - A)\sin(-B)$

$= \sin A \cos B + \cos A(-\sin B)$

$= \mathbf{\sin A \cos B - \cos A \sin B}$ ◂

5.3.2 Vanaf 5.3.1 $\sin A \cos B - \cos A \sin B = \sin(A - B)$

$\therefore \sin 48° \cos x - \cos 48° \sin x = \sin(48° - x)$

$\therefore \sin(48° - x) = \cos 2x$

$= \sin(90° - 2x)$

$\therefore 48° - x = 90° - 2x + n360°$

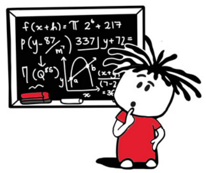
$\therefore \mathbf{x = 42° + n360°;\ n \in \mathbb{Z}}$ ◂

OF: $48° - x = 180° - (90° - 2x) + n360°$

$\therefore 48° - x = 90° + 2x + n360°$

$\therefore -3x = 42° + n360°$

$\div(-3)\ \therefore \mathbf{x = -14° + n120°;\ n \in \mathbb{Z}}$ ◂



5.4 $\frac{\sin 3x + \sin x}{\cos 2x + 1}$

$= \frac{\sin(2x + x) + \sin x}{2\cos^2 x - 1 + 1}$

$= \frac{\sin 2x \cos x + \cos 2x \sin x + \sin x}{2\cos^2 x}$

$= \frac{2\sin x \cos^2 x + (2\cos^2 x - 1)\sin x + \sin x}{2\cos^2 x}$

$= \frac{2\sin x \cos^2 x + 2\sin x \cos^2 x - \sin x + \sin x}{2\cos^2 x}$

$= \frac{4\sin x \cos^2 x}{2\cos^2 x}$

$= \mathbf{2\sin x}$ ◂

OF:

$\frac{\sin 3x + \sin x}{\cos 2x + 1}$

$= \frac{\sin(2x + x) + \sin(2x - x)}{2\cos^2 x - 1 + 1}$

$= \frac{\sin 2x \cos x + \cos 2x \sin x + \sin 2x \cos x - \cos 2x \sin x}{2\cos^2 x}$

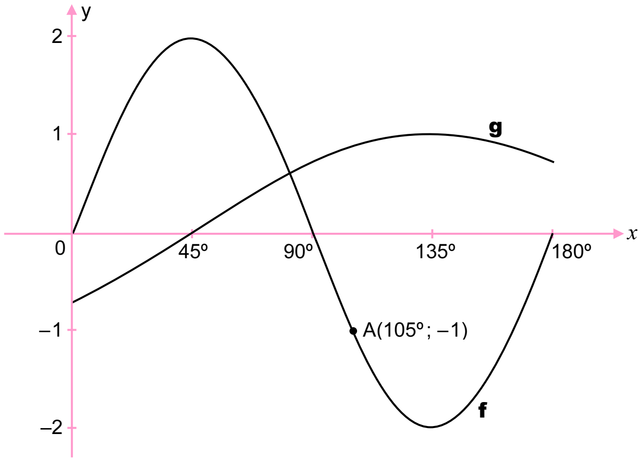
$= \frac{2\sin 2x \cos x}{2\cos^2 x}$

$= \frac{2(2\sin x \cos x)\cos x}{2\cos^2 x}$

$= \frac{4\sin x \cos^2 x}{2\cos^2 x}$

$= \mathbf{2\sin x}$ ◂

6.



6.1 **180º** ➤ . . . $\frac{1}{2} \times 360^o$

6.2 Y-afsnit : Stel $x = 0$ in, in $y = -\cos(x + 45^o)$

$\therefore\ y = -\cos 45^o$

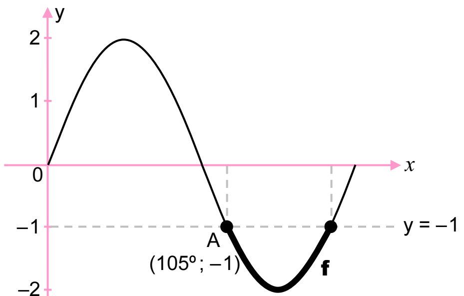
$\therefore\ y = -\frac{1}{\sqrt{2}}$

$\therefore$ Die waardeversameling van g : $\mathbf{-\frac{1}{\sqrt{2}} \leq y \leq 1}$ ➤

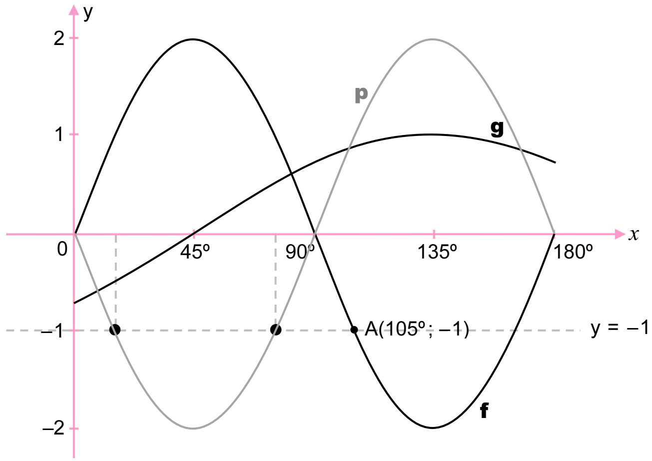
6.3.1 **$45^o < x < 90^o$** ➤ . . . *albei grafieke het dieselfde teken*

6.3.2 $f(x) \leq -1$

$\therefore$ **$105^o \leq x \leq 165^o$** ➤ . . . *deur simmetrie*



6.4 Dit kan deur inspeksie gedoen word :



k = 15º of 75º ➤

Vergelyking van p : $y = -2\sin 2x$

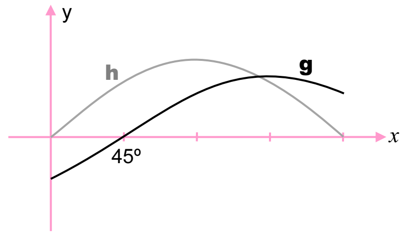
D(k; –1) op p ➨ $-2\sin 2k = -1$

$\therefore\ \sin 2k = \frac{1}{2}$

$\therefore\ 2k = 30^o$ of 150^o

$\therefore$ **k = 15º of 75º** ➤

6.5 **h(x) = sin x** ➤



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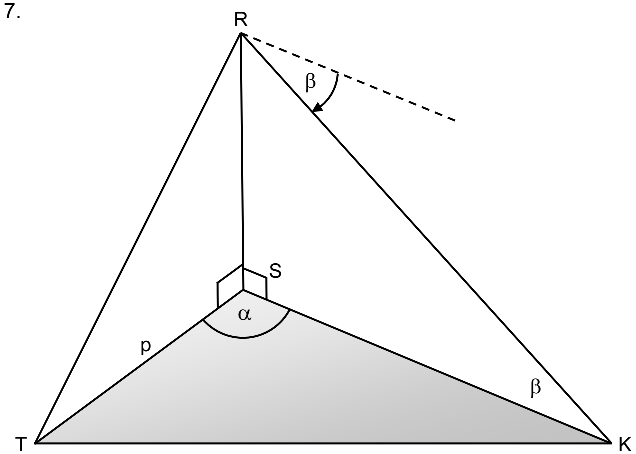
7.



7.1 $\frac{1}{2}TS.SK\sin\alpha$ = Oppervlakte van ΔSTK

$\therefore \frac{1}{2}p.SK\sin\alpha = q$

$\therefore p.SK.\sin\alpha = 2q$

$\therefore$ **SK** $= \frac{\mathbf{2q}}{\mathbf{p.\sin\alpha}}$ ➤

7.2 $R\hat{K}S = \beta$ ... *verw.* $\angle^e$; || *lyne*

In ΔRSK: $\frac{RS}{SK} = \tan\beta$

$\therefore RS = SK.\tan\beta$

Stel in vanaf V7.1 . . .

$\therefore RS = \frac{\mathbf{2q.\tan\beta}}{\mathbf{p.\sin\alpha}}$ ➤

7.3 $\therefore p.\sin\alpha.RS = 2q.\tan\beta$

$\therefore \sin\alpha = \frac{2q.\tan\beta}{p.RS}$

$= \frac{2 \times 2\,500 \times \tan 42^\circ}{80 \times 70}$

$= 0{,}803...$

$\therefore \alpha =$ **53,51°** ➤

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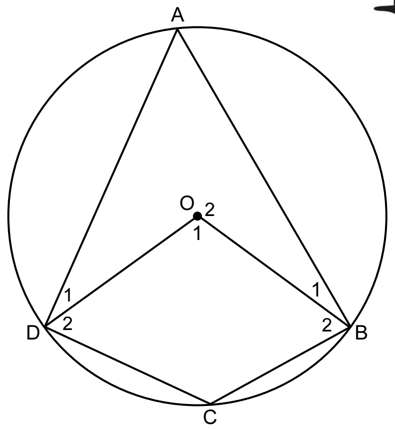
EUKLIDIESE MEETKUNDE [40]

8.1 Bewys van stelling ◄

8.2



$\hat{A} = \frac{1}{2}(4x + 100°)$ . . . *middelpunts-∠ = 2 × omtreks-∠*

$= 2x + 50°$

$\hat{A} + \hat{C} = 180°$ . . . *teenoorst. ∠^e^ van koordevierhoek*

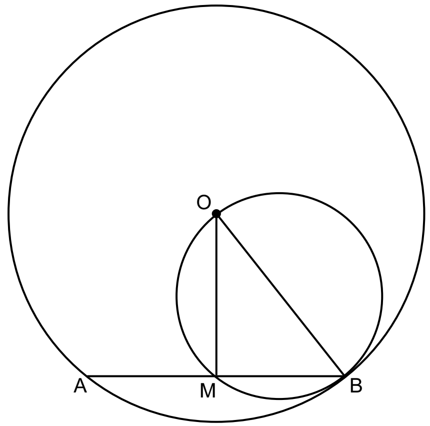
$\therefore 2x + 50° + x + 34° = 180°$

$\therefore 3x + 84° = 180°$

$\therefore 3x = 96°$

$\therefore \boldsymbol{x = 32°}$ ◄

8.3



8.3.1 $O\hat{M}B = 90°$ . . . *∠ in semi-⊙*

8.3.2 $OB^2 = OM^2 + MB^2$ . . . *Pythag*

Maar $MB = \frac{1}{2}AB$ . . . *lyn vanuit middelpt. ⊥ op koord*

$= \frac{1}{2}\sqrt{300}$

$\therefore OB^2 = 5^2 + \left(\frac{1}{2}\sqrt{300}\right)^2$

$= 25 + \left(\frac{1}{4} \times 300\right)$

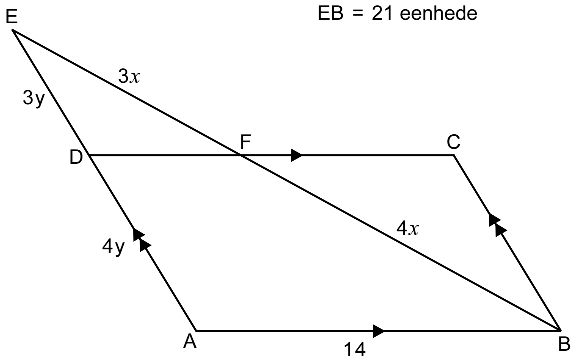
$= 25 + 75$

$= 100$

$\therefore$ **OB = 10 eenhede** ◄

NOTAS

9.



9.1 $\frac{FB}{FE} = \frac{AD}{DE} \left(= \frac{4}{3}\right)$ ... *eweredigheidstelling; DF || AB*

$\therefore$ Laat FB = $4x$ & FE = $3x$

$\therefore$ $7x$ = 21 eenhede

$\therefore$ x = 3 eenhede

$\therefore$ **FB = 12 eenhede** ‹

9.2 In Δ^e EDF & EAB

(1) $\hat{E}$ is gemeen

(2) $E\hat{F}D = E\hat{B}A$ ... *ooreenk.* $\angle^e$; *DF || AB*

$\therefore$ **ΔEDF ||| ΔEAB** ‹ ... $\angle\angle\angle$

9.3 $\frac{DF}{AB} = \frac{EF}{EB}$ ... *gelykvormige* Δ^e

$\therefore \frac{DF}{14} = \frac{9}{21}$

$\therefore DF = \frac{{}^{3}\cancel{9} \times \cancel{14}^{2}}{\cancel{21}_{\cancel{3}}}$

= 6 eenhede

Maar DC = AB = 14 eenhede ... *teenoorst. sye van* $||^m$

$\therefore$ **FC = 14 – 6 = 8 eenhede** ‹

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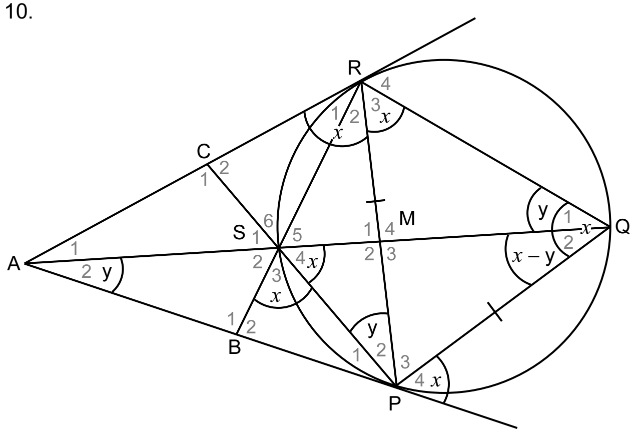
10.



10.1 Laat $\hat{S}_3 = x$

$\therefore R\hat{Q}P = x$ . . . *buite∠ van koordevierhoek*

$\therefore \hat{R}_3 = x$ . . . *∠^e^ teenoor = sye*

$\therefore \hat{S}_4 = x$ . . . *∠^e^ in dieselfde seg*

$\therefore \boldsymbol{\hat{S}_3 = \hat{S}_4}$ ◄

10.2 $R\hat{Q}P = x$

$\therefore A\hat{R}P = x$ . . . *raaklyn-koord stelling*

Maar $\hat{S}_4 = x$

$\therefore \hat{S}_4 = A\hat{R}P$

∴ **SMRC is 'n koordevierhoek** ◄ . . . *omgekeerde buite∠ van kvh.*

10.3 $\hat{P}_4 = x$ . . . *raaklyn-koord stelling*

Laat $\hat{P}_2 = y$

$\therefore \hat{Q}_1 = y$ . . . *∠^e^ in dieselfde seg*

$\therefore \hat{Q}_2 = x - y$

$\therefore \hat{A}_2 = y$ . . . *buite∠ van ΔQAP*

$\therefore \hat{P}_2 = \hat{A}_2$

∴ RS is 'n raaklyn aan die sirkel deur P, S en A ◄
. . . *omgekeerde raaklyn-koord stelling*

OF:

$\hat{P}_4 = x$ . . . *raaklyn-koord stelling*

$\therefore \hat{P}_4 = R\hat{Q}P$

∴ RQ || AP . . . *verw. ∠^e^ =*

Laat $\hat{A}_2 = y$

$\therefore \hat{Q}_1 = y$ . . . *verw. ∠^e^; RQ || AP*

$\therefore \hat{P}_2 = y$ . . . *∠^e^ in dieselfde seg*

$\therefore \hat{P}_2 = \hat{A}_2$

∴ RS is 'n raaklyn aan die sirkel deur P, S en A ◄
. . . *omgekeerde raaklyn-koord stelling*

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