STATISTICS [20]

1.1 The equation: y = A + Bx

A = −23,8461…. ≃ −23,85

B = 0,2270…. ≃ 0,23

∴ **y = −23,85 + 0,23x** ◄

1.2 Subst. x = 550: y = −23,85 + 0,23(550)

= **102,65 minutes** ◄

1.3 The correlation coefficient, r = 0,9828….

≃ **0,98** ◄

1.4 There is a very strong positive correlation between the distance travelled and the amount of rest time. ◄

1.5.1 The mean, $\bar{x} = \frac{1\,200}{8}$ = **R150** ◄

1.5.2 The standard deviation, σ = 50,4975….

≃ **50,50** ◄

1.5.3 Amount < $\bar{x}$ − 1σ

∴ Amount < 150 − 50,50

∴ Amount < 99,50

∴ **Only 1 stop was less (when he spent R50)** ◄

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2.1

NUMBER OF GLASSES OF WATER DRANK PER DAY	NUMBER OF STAFF MEMBERS	CUMULATIVE FREQUENCY
$0 \le x < 2$	5	5
$2 \le x < 4$	15	20
$4 \le x < 6$	13	33
$6 \le x < 8$	5	38
$8 \le x < 10$	2	40

2.2 40 staff members were interviewed ◄

2.3 33 staff members drank fewer than 6 glasses of water ◄

2.4

NUMBER OF GLASSES OF WATER DRANK PER DAY	MIDPOINT	NUMBER OF STAFF MEMBERS
$0 \le x < 2$	1	$5 + \frac{1}{2}k$
$2 \le x < 4$	3	15
$4 \le x < 6$	5	$13 + \frac{1}{2}k$
$6 \le x < 8$	7	5
$8 \le x < 10$	9	2
		40 + k

Estimated mean = 4 when k staff members are added

Total no. of glasses of water

$$= \left[\left(5 + \tfrac{1}{2}k\right) \times 1\right] + (15 \times 3) + \left[\left(13 + \tfrac{1}{2}k\right) \times 5\right] + (5 \times 7) + (2 \times 9)$$

$$\therefore \text{ Estimated mean } = \frac{5 + \frac{1}{2}k + 45 + 65 + \frac{1}{2}k + 35 + 18}{40 + k} = 4$$

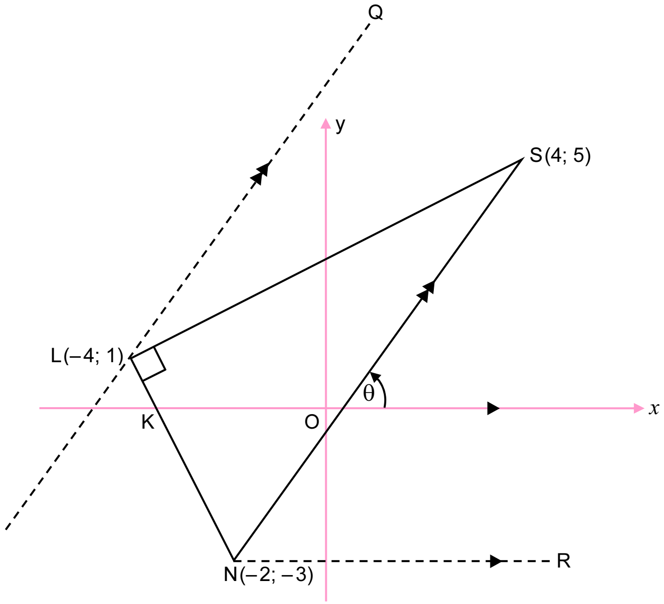
∴ 168 + 3k = 160 + 4k

∴ 8 = k

∴ **8 staff members were absent** ◄

NOTES

ANALYTICAL GEOMETRY [40]



3.1 $SL^2 = (4+4)^2 + (5-1)^2$
$= 64 + 16$
$= 80$
$\therefore$ SL $= \sqrt{80} = \mathbf{4\sqrt{5}}$ **units** ◂

3.2 $m_{SN} = \frac{5-(-3)}{4-(-2)} = \frac{8}{6} = \mathbf{\frac{4}{3}}$ ◂

3.3 $\tan\theta = \frac{4}{3}$
$\therefore\ \theta \simeq \mathbf{53{,}13^\circ}$ ◂

3.4 $S\hat{N}R = \theta = 53{,}13^\circ$ ... *corresp* $\angle^s$; || *lines*

$m_{LN} = \frac{1-(-3)}{-4-(-2)} = \frac{4}{-2} = -2$

$\therefore \tan L\hat{N}R = -2$

$\therefore L\hat{N}R = 180^\circ - 63{,}43....$
$\simeq 116{,}57^\circ$

$\therefore L\hat{N}S = 116{,}57^\circ - 53{,}13^\circ$
$= \mathbf{63{,}44^\circ}$ ◂

OR:

$LN^2 = (-4+2)^2 + (1+3)^2$
$= 4 + 16$
$= 20$
$\therefore LN = \sqrt{20} = 2\sqrt{5}$

$\therefore \tan L\hat{N}S = \frac{LS}{LN}$
$= \frac{4\sqrt{5}}{2\sqrt{5}}$
$= 2$
$\therefore \mathbf{L\hat{N}S = 63{,}43^\circ}$ ◂

Note: difference due to rounding

3.5 $m_{QL} = m_{SN} = \frac{4}{3}$ ... || *lines*

Subst. $m = \frac{4}{3}$ & $(-4; 1)$ in

$y - y_1 = m(x - x_1)$

$\therefore y - 1 = \frac{4}{3}(x+4)$

$\therefore \mathbf{y = \frac{4}{3}x + \frac{19}{3}}$ ◂

3.6 Area of $\Delta LSN = \frac{1}{2}LN.SL$

$LN^2 = (-4+2)^2 + (1+3)^2$
$= 4 + 16$
$= 20$
$\therefore LN = \sqrt{20}$

$\therefore$ Area of $\Delta LSN = \frac{1}{2}\sqrt{20}\sqrt{80}$
$= \frac{1}{2}\sqrt{1\,600}$
$= \frac{1}{2}(40)$
$= \mathbf{20\ units^2}$ ◂

3.7 P equidistant from L, S and N

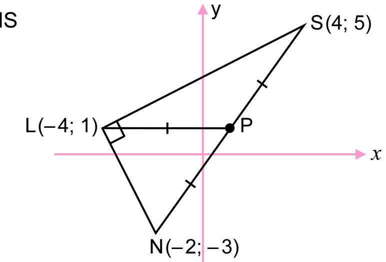
$N\hat{L}S = 90^\circ$

$\therefore$ NS is the diameter of ⊙LSN ... *conv.* $\angle$ *in semi-*⊙

$\therefore$ P is the midpoint of NS

$\therefore P\left(\frac{-2+4}{2}; \frac{-3+5}{2}\right)$

$\therefore$ **P(1; 1)** ◂



3.8 $L\hat{P}S = 2L\hat{N}S$ ... $\angle$ *at centre* $= 2 \times \angle$ *at circumference*
$= 2(63{,}44^\circ)$
$= \mathbf{126{,}88^\circ}$ ◂

OR: PL || x-axis ... $y_P = y_L$

$L\hat{P}N = 53{,}13^\circ$ ... *alt* $\angle^s$; || *lines*

$\therefore L\hat{P}S = 180^\circ - 53{,}13^\circ$ ... $\angle^s$ *on a str line*
$= \mathbf{126{,}87^\circ}$ ◂

Note: difference due to rounding

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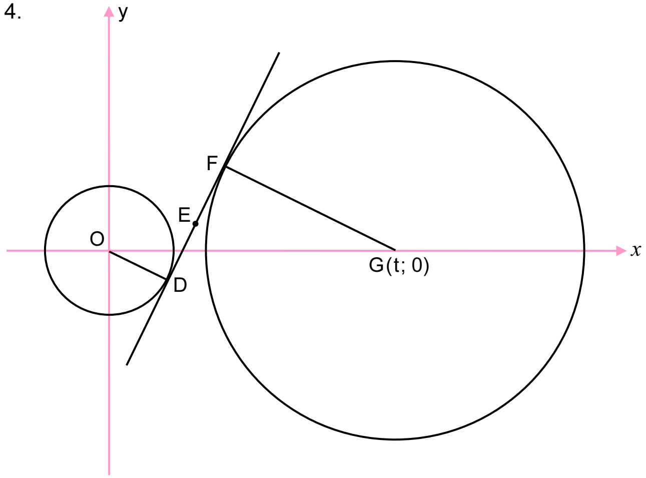
4.



4.1 D(p; –2) on ⊙O ➡ $p^2 + (-2)^2 = 20$

$\therefore\ p^2 = 16$

$\therefore$ **p = 4** ◄ . . . *p > 0 in 4th Quadrant*

4.2 **F(8; 6)** ◄ . . . *by inspection*

4.3 $m_{\text{radius OD}} = -\frac{2}{4} = -\frac{1}{2}$

$\therefore\ m_{DF} = 2$

OR: $m_{DE} = \frac{2+2}{6-4} = 2$

Subst. m = 2 & (4; –2) in

$y - y_1 = m(x - x_1)$

$\therefore\ y + 2 = 2(x - 4)$

$\therefore$ $\mathbf{y = 2x - 10}$ ◄

OR: $y = mx + c$

$\therefore\ -2 = (2)(4) + c$

$\therefore\ c = -10$, etc.

4.4 $m_{FG} = -\frac{1}{2}$ . . . *FG ⊥ DF*

$\therefore\ \frac{6-0}{8-t} = -\frac{1}{2}$

×2(8 – t) $\therefore\ 12 = -(8 - t)$

$\therefore\ 12 = -8 + t$

$\therefore$ **t = 20** ◄

OR:

use $m = -\frac{1}{2}$ and (8; 6):

Eqn of DF:

$y - 6 = -\frac{1}{2}(x - 8)$,

& subst. y = 0

4.5 Centre G is (20; 0)

& $r^2 = FG^2 = (20 - 8)^2 + (0 - 6)^2$

$= 144 + 36$

$= 180$

$\therefore$ Eqn of ⊙G: $(x - 20)^2 + (y - 0)^2 = 180$

$\therefore\ x^2 - 40x + 400 + y^2 - 180 = 0$

$\therefore$ $\mathbf{x^2 + y^2 - 40x + 220 = 0}$ ◄

4.6 Point A where the small circle cuts the x-axis must move to point B where the large circle cuts the x-axis, or, C to H.

Small ⊙: $r = \sqrt{20} = 2\sqrt{5}$ . . . $x^2 + y^2 = 20$

$\therefore\ x_A = -2\sqrt{5}$ and $x_C = 2\sqrt{5}$

Large ⊙: $R = \sqrt{180} = 6\sqrt{5}$

$\therefore\ x_B = 20 - 6\sqrt{5}$ and $x_H = 20 + 6\sqrt{5}$

$\therefore$ A ➡ B: $k = 20 - 6\sqrt{5} - (-2\sqrt{5})$

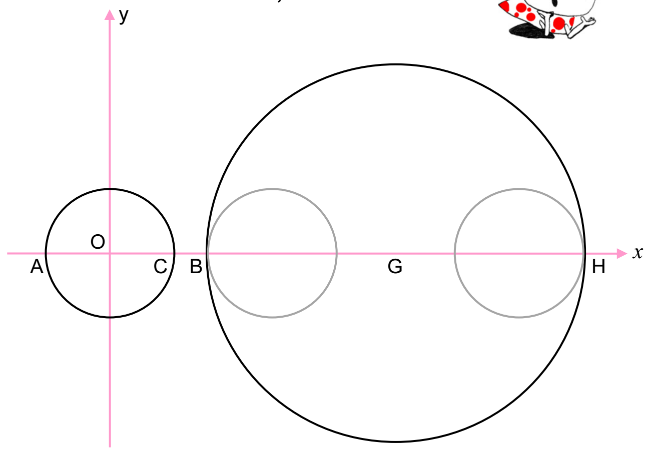
$= 20 - 4\sqrt{5}$

$\simeq$ **11,06** ◄

$\therefore$ C ➡ H: $k = 20 + 6\sqrt{5} - 2\sqrt{5}$

$= 20 + 4\sqrt{5}$

$\simeq$ **28,94** ◄

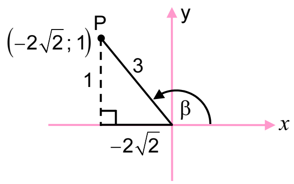


NOTES

TRIGONOMETRY [50]

5.1.1 $x_P = -\sqrt{9-1}$
$= -\sqrt{8}$
$= -2\sqrt{2}$

$\therefore \cos\beta = \dfrac{x}{r} = -\dfrac{\mathbf{2\sqrt{2}}}{\mathbf{3}}$ ✓



5.1.2 $\sin 2\beta = 2\sin\beta\cos\beta = 2\left(\dfrac{1}{3}\right)\left(-\dfrac{2\sqrt{2}}{3}\right)$
$= -\dfrac{\mathbf{4\sqrt{2}}}{\mathbf{9}}$ ✓

5.1.3 **Method 1**

$\cos(450^\circ - \beta) = \cos(90^\circ - \beta)$
$= \sin\beta$
$= \dfrac{\mathbf{1}}{\mathbf{3}}$ ✓

Method 2

$\cos(450^\circ - \beta) = \cos 450^\circ \cos\beta + \sin 450^\circ \sin\beta$
$= \cos 90^\circ \cos\beta + \sin 90^\circ \sin\beta$
$= (0)(\cos\beta) + (1)\sin\beta$
$= \sin\beta$
$= \dfrac{\mathbf{1}}{\mathbf{3}}$ ✓

5.2.1 $\dfrac{(\cos^2 x)^2 + \sin^2 x\cos^2 x}{1+\sin x}$

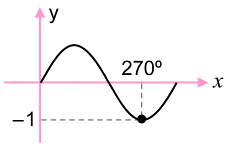
$= \dfrac{\cos^2 x(\cos^2 x + \sin^2 x)}{1+\sin x}$

$= \dfrac{(1-\sin^2 x)(1)}{1+\sin x}$

$= \dfrac{(1+\sin x)(1-\sin x)}{1+\sin x}$

$= \mathbf{1 - \sin x}$ ✓

5.2.2 Undefined when $1 + \sin x = 0$
$\therefore \sin x = -1$
$\therefore \boldsymbol{x = 270^\circ}$ ✓



5.2.3 $-1 \le \sin x \le 1$

The minimum value of $1 - \sin x$
$= 1 - 1$
$= \mathbf{0}$ ✓

[The minimum occurs when $\sin x = 1$]

5.3.1 $\sin(A-B) = \cos[90^\circ - (A-B)]$
$= \cos[(90^\circ - A) - (-B)]$
$= \cos(90^\circ - A)\cos(-B) + \sin(90^\circ - A)\sin(-B)$
$= \sin A\cos B + \cos A(-\sin B)$
$= \mathbf{\sin A\cos B - \cos A\sin B}$ ✓

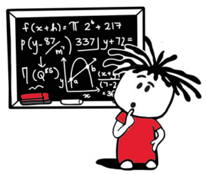
5.3.2 From 5.3.1 $\sin A\cos B - \cos A\sin B = \sin(A-B)$

$\therefore \sin 48^\circ\cos x - \cos 48^\circ\sin x = \sin(48^\circ - x)$

$\therefore \sin(48^\circ - x) = \cos 2x$
$= \sin(90^\circ - 2x)$

$\therefore 48^\circ - x = 90^\circ - 2x + n360^\circ$
$\therefore \boldsymbol{x = 42^\circ + n360^\circ;\ n \in \mathbb{Z}}$ ✓

OR: $48^\circ - x = 180^\circ - (90^\circ - 2x) + n360^\circ$
$\therefore 48^\circ - x = 90^\circ + 2x + n360^\circ$
$\therefore -3x = 42^\circ + n360^\circ$
$\div(-3)\ \therefore \boldsymbol{x = -14^\circ + n120^\circ;\ n \in \mathbb{Z}}$ ✓



5.4 $\dfrac{\sin 3x + \sin x}{\cos 2x + 1}$

$= \dfrac{\sin(2x + x) + \sin x}{2\cos^2 x - 1 + 1}$

$= \dfrac{\sin 2x\cos x + \cos 2x\sin x + \sin x}{2\cos^2 x}$

$= \dfrac{2\sin x\cos^2 x + (2\cos^2 x - 1)\sin x + \sin x}{2\cos^2 x}$

$= \dfrac{2\sin x\cos^2 x + 2\sin x\cos^2 x - \sin x + \sin x}{2\cos^2 x}$

$= \dfrac{4\sin x\cos^2 x}{2\cos^2 x}$

$= \mathbf{2\sin \boldsymbol{x}}$ ✓

OR:

$\dfrac{\sin 3x + \sin x}{\cos 2x + 1}$

$= \dfrac{\sin(2x + x) + \sin(2x - x)}{2\cos^2 x - 1 + 1}$

$= \dfrac{\sin 2x\cos x + \cos 2x\sin x + \sin 2x\cos x - \cos 2x\sin x}{2\cos^2 x}$

$= \dfrac{2\sin 2x\cos x}{2\cos^2 x}$

$= \dfrac{2(2\sin x\cos x)\cos x}{2\cos^2 x}$

$= \dfrac{4\sin x\cos^2 x}{2\cos^2 x}$

$= \mathbf{2\sin \boldsymbol{x}}$ ✓

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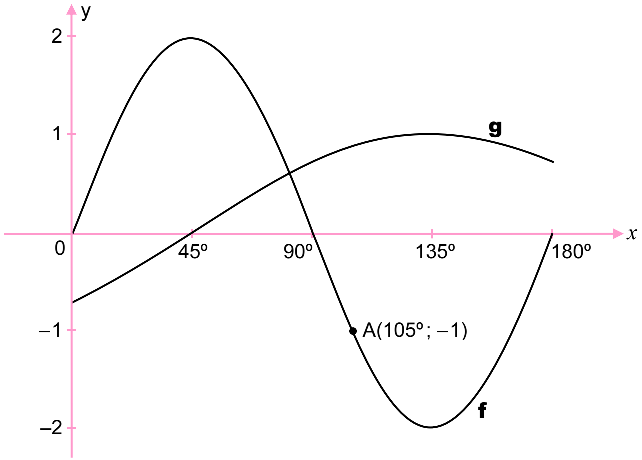
6.



6.1 **180º** ◄ . . . $\frac{1}{2} \times 360^o$

6.2 Y-int: Subt. $x = 0$ in $y = -\cos(x + 45^o)$

$\therefore\ y = -\cos 45^o$

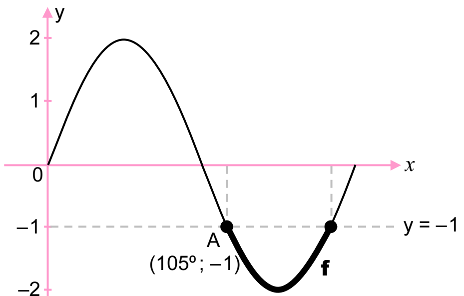
$\therefore\ y = -\frac{1}{\sqrt{2}}$

$\therefore$ The range of g: $\mathbf{-\frac{1}{\sqrt{2}} \leq y \leq 1}$ ◄

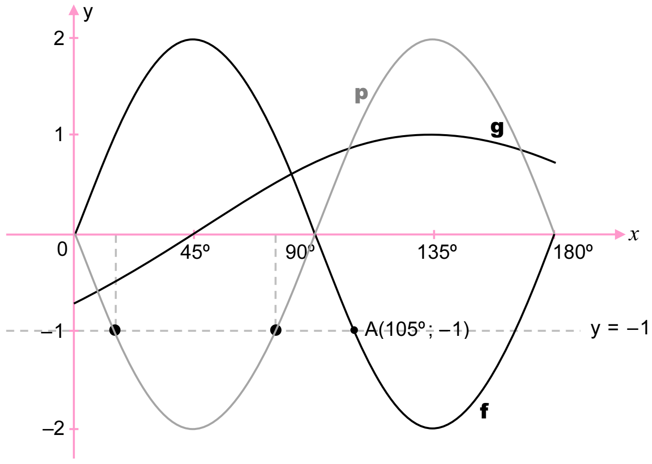
6.3.1 **45º < x < 90º** ◄ . . . *both graphs have the same sign*

6.3.2 $f(x) \leq -1$

$\therefore$ **105º ≤ x ≤ 165º** ◄ . . . *by symmetry*



6.4 This can be done by inspection:



k = 15º or 75º ◄

Eqn of p: $y = -2 \sin 2x$

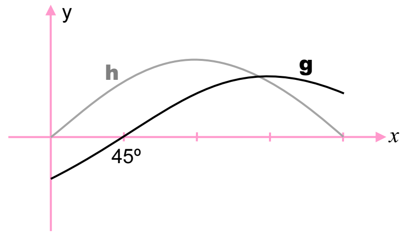
D(k; –1) on p ➡ $-2 \sin 2k = -1$

$\therefore\ \sin 2k = \frac{1}{2}$

$\therefore\ 2k = 30^o$ or 150^o

$\therefore$ **k = 15º or 75º** ◄

6.5 **h(x) = sin x** ◄



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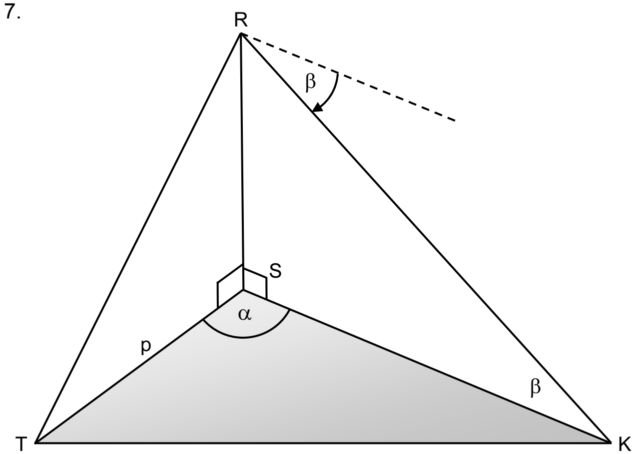
7.



7.1 $\frac{1}{2}$TS.SK sin α = Area of ΔSTK

∴ $\frac{1}{2}$p.SK sin α = q

∴ p.SK.sin α = 2q

∴ **SK = $\frac{2q}{p.\sin\alpha}$** ◂

7.2 $R\hat{K}S = \beta$ . . . *alt ∠s; || lines*

In ΔRSK: $\frac{RS}{SK}$ = tan β

∴ RS = SK.tan β

Substitute from Q7.1 . . .

∴ RS = $\frac{\mathbf{2q.\tan\beta}}{\mathbf{p.\sin\alpha}}$ ◂

7.3 ∴ p.sin α.RS = 2q.tan β

∴ sin α = $\frac{2q.\tan\beta}{p.RS}$

= $\frac{2 \times 2\,500 \times \tan 42^\circ}{80 \times 70}$

= 0,803...

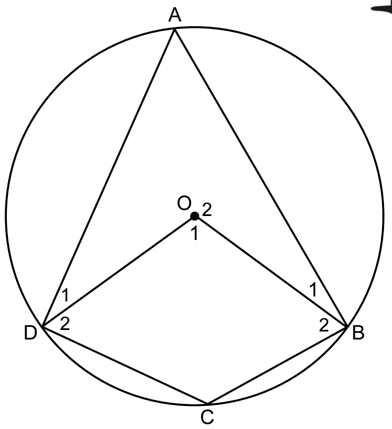
∴ α = **53,51°** ◂

NOTES

EUCLIDEAN GEOMETRY [40]

8.1 Theorem proof ➤

8.2



$\hat{A} = \frac{1}{2}(4x + 100°)$ . . . *∠ at centre = 2 × ∠ at circum*

$= 2x + 50°$

$\hat{A} + \hat{C} = 180°$ . . . *opp ∠s of cyclic quad*

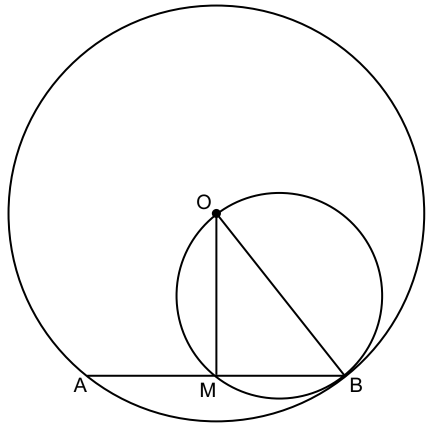
$\therefore\ 2x + 50° + x + 34° = 180°$

$\therefore\ 3x + 84° = 180°$

$\therefore\ 3x = 96°$

$\therefore\ \boldsymbol{x = 32°}$ ➤

8.3



8.3.1 $O\hat{M}B = 90°$ . . . *∠ in semi-⊙*

8.3.2 $OB^2 = OM^2 + MB^2$ . . . *Pythag*

But $MB = \frac{1}{2}AB$ . . . *line from centre ⊥ to chord*

$= \frac{1}{2}\sqrt{300}$

$\therefore\ OB^2 = 5^2 + \left(\frac{1}{2}\sqrt{300}\right)^2$

$= 25 + \left(\frac{1}{4} \times 300\right)$

$= 25 + 75$

$= 100$

$\therefore$ **OB = 10 units** ➤

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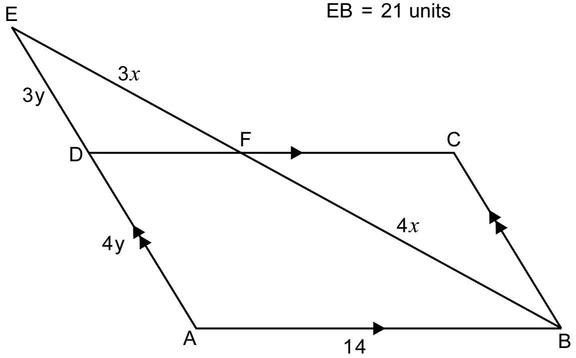
NOTES

9.

EB = 21 units



9.1 $\frac{FB}{FE} = \frac{AD}{DE} \left(= \frac{4}{3} \right)$ . . . *prop thm ; DF || AB*

$\therefore$ Let FB = $4x$ & FE = $3x$

$\therefore$ $7x$ = 21 units

$\therefore$ x = 3 units

$\therefore$ **FB = 12 units** ◂

9.2 In Δ^s EDF & EAB

(1) $\hat{E}$ is common

(2) $E\hat{F}D = E\hat{B}A$ . . . *corresp $\angle^s$; DF || AB*

$\therefore$ **ΔEDF ||| ΔEAB** ◂ . . . $\angle\angle\angle$

9.3 $\frac{DF}{AB} = \frac{EF}{EB}$ . . . *similar Δ^s*

$\therefore$ $\frac{DF}{14} = \frac{9}{21}$

$\therefore$ DF $= \frac{{}^{3}\cancel{9} \times \cancel{14}^{2}}{\cancel{21}_{\cancel{3}}}$

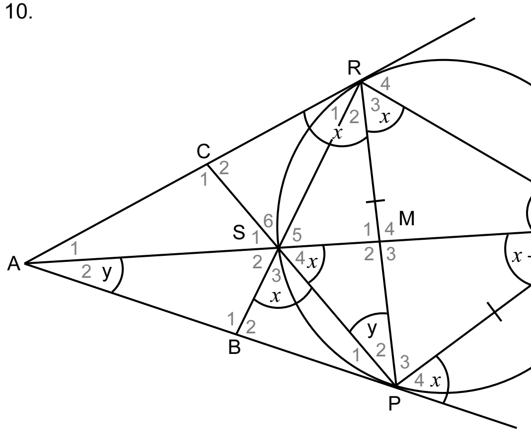
= 6 units

But DC = AB = 14 units . . . *opp sides of $||^m$*

$\therefore$ **FC = 14 – 6 = 8 units** ◂

NOTES

10.



10.1 Let $\hat{S}_3 = x$

$\therefore\ R\hat{Q}P = x$ . . . *ext ∠ of cyclic quad*

$\therefore\ \hat{R}_3 = x$ . . . *∠^s opp = sides*

$\therefore\ \hat{S}_4 = x$ . . . *∠^s in the same seg*

$\therefore\ \mathbf{\hat{S}_3 = \hat{S}_4}$ ◂

10.2 $R\hat{Q}P = x$

$\therefore\ A\hat{R}P = x$ . . . *tan chord theorem*

But $\hat{S}_4 = x$

$\therefore\ \hat{S}_4 = A\hat{R}P$

∴ **SMRC is a cyclic quad** ◂ . . . *converse ext ∠ of c.q.*

10.3 $\hat{P}_4 = x$ . . . *tan chord theorem*

Let $\hat{P}_2 = y$

$\therefore\ \hat{Q}_1 = y$ . . . *∠^s in the same seg*

$\therefore\ \hat{Q}_2 = x - y$

$\therefore\ \hat{A}_2 = y$ . . . *ext ∠ of ΔQAP*

$\therefore\ \hat{P}_2 = \hat{A}_2$

∴ RS is a tangent to the circle through P, S and A ◂

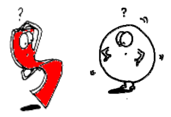
. . . *conv tan chord thm*

OR:

$\hat{P}_4 = x$ . . . *tan chord thm*

$\therefore\ \hat{P}_4 = R\hat{Q}P$

∴ RQ || AP . . . *alt ∠^s =*



Let $\hat{A}_2 = y$

$\therefore\ \hat{Q}_1 = y$ . . . *alt ∠^s; RQ || AP*

$\therefore\ \hat{P}_2 = y$ . . . *∠^s in the same seg*

$\therefore\ \hat{P}_2 = \hat{A}_2$

∴ RS is a tangent to the circle through P, S and A ◂

. . . *conv tan chord thm*

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