# **GeoGebra …** Links to sketches

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**Topics:**

- Analytical Geometry
- Calculus
- Functions
- Geometry (angles and lines, triangles, circles)
- Trigonometry

# Analytical Geometry

https://simi.education/online-resources/links-to-sketches-for-analytical-geometry/

Explore the concepts with this digital scratchpad

Use this applet to explore circles.

Grow your understanding of angle of inclination with this applet.

# Calculus

https://simi.education/online-resources/links-to-sketches-for-calculus/

Use this sketch to better understand the first and second derivatives and what they tell us about the function, its gradient and its concavity.

# Functions

https://simi.education/online-resources/links-to-sketches-for-functions/

Loads of online sketches to help you explore the FUNctions

The linear function

The quadratic function in vanilla form

The quadratic function in standard form

The hyperbolic function

The exponential function in vanilla form

The exponential function in standard form

Exponential and Log graphs

Reflections

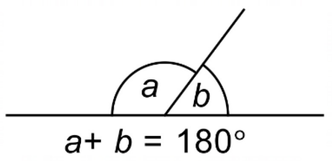
# Geometry

https://simi.education/online-resources/links-to-sketches-for-geometry/

## Interactive Geometry

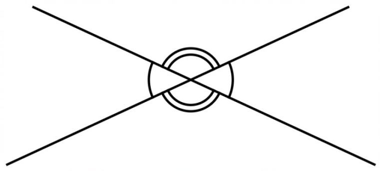
**Complementary** means *"add to 90°"* while **supplementary** means *"add to 180°"*

Angles on straight line add to 180°
($\angle s$ on str. line)



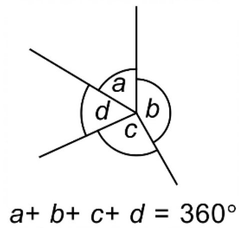

sketch

Vertically opposite angles are equal
(vert. opp. $\angle s$)



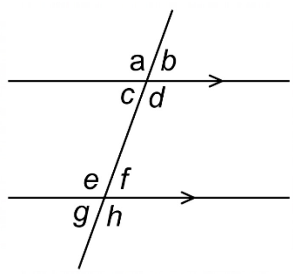
sketch

Angles round a point add to 360°
($\angle s$ round a point)




sketch

**Parallel lines**




corresponding angles (a&e ; b&f ; c&g ; d&h) are equal (corr. $\angle s$ on || lines)

sketch

alternate angles (c&f ; d&e) are equal (alt. $\angle s$ on || lines)

co-interior angles (c&e ; d&f) are supplementary (co-int. $\angle s$ on || lines)

sketch

**Proving lines parallel**

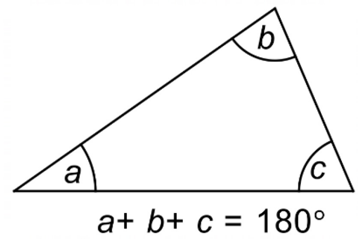
If corresponding angles are equal, then the lines are parallel (corr. $\angle s$ equal)

If alternate angles are equal, then the lines are parallel ($\alpha t.\ \angle s$ equal)

If co-interior angles are supplementary, then the lines are parallel (co-int. $\angle s$ equal)

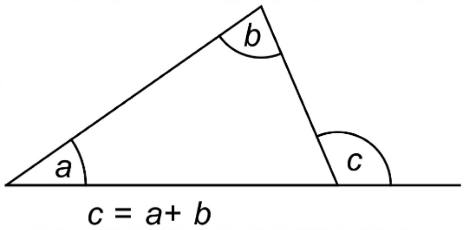
Sketch

**Triangles**



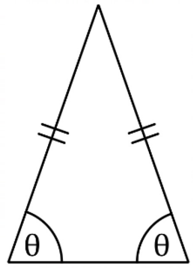

sketch

The **angles of a triangle** add to 180º ($\angle s$ of $\Delta$)



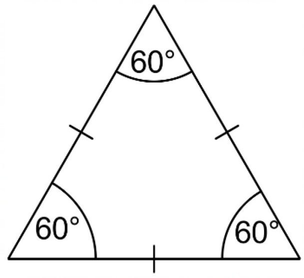

sketch

The **exterior** angle of a triangle is equal to the sum of the opposite interior angles
(ext. $\angle$ of $\Delta$)




sketch

An **isosceles** triangle has two equal sides and the angles opposite them equal.
(isos. $\Delta$)




sketch

An **equilateral** triangle has all three sides equal and each angle equal to 60º
(equilateral $\Delta$)

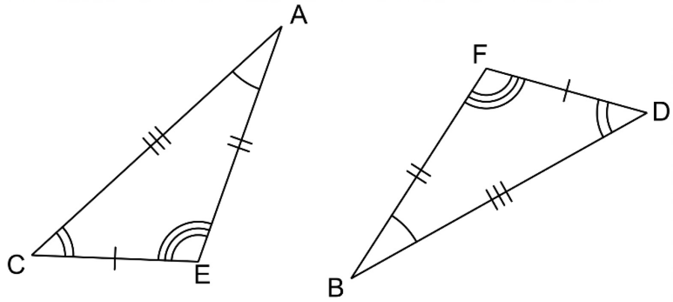
**Congruency**

One can prove **congruence** (‘identicalness’) of triangles in one of four ways:

1. Three sides equal (SSS)
2. Two angles and a side in the same corresponding position (AAS)
3. Two sides and an **included** angle (SAS)
4. Right angle, hypotenuse and a side (RHS)

The symbol for congruency is ≡. We always list congruent triangles in corresponding order.

If congruent triangles are properly listed, then one can extract information as per the example below:




$\Delta ACE \equiv \Delta BDF$

$AE = BF$, $C\hat{E}A = D\hat{F}B$ etc.

sketch

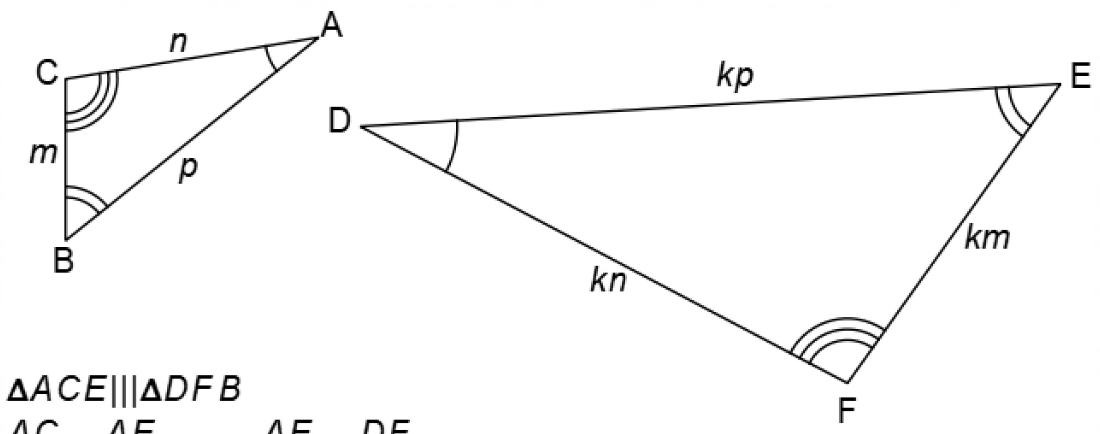
**Similarity**

One can prove **similarity** of triangles in one of two ways:

1. Prove them equiangular (AAA)
2. Prove that the corresponding sides are in proportion (corr. sides in proportion)

In fact, both are necessary to prove shapes similar, but with triangles the one guarantees the other.

The symbol for similarity is |||. We always list similar triangles in corresponding order, following which we can pull out equal ratios as per the example below:




$\Delta ACE|||\Delta DFB$

$\frac{AC}{DF} = \frac{AE}{DB}$ and $\frac{AE}{CE} = \frac{DF}{FB}$ etc.

sketch

**Areas of similar shapes**

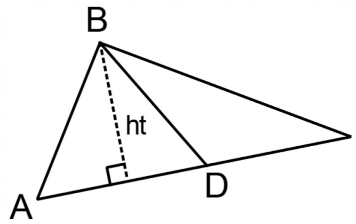
Shapes that are similar with sides in the ratio $a : b$ will have **areas in the ratio** $a^2 : b^2$. In the event of 3-dimensional solids their **volumes will be in the ratio** $a^3 : b^3$

In the above example the sides are in the ratio $1 : 2$ so the areas will be in the ratio $1^2 : 2^2 = 1 : 4$

$\Delta ABC = \frac{1}{4}\Delta \mathrm{DEF}$ or $\Delta \mathrm{DEF} = 4\Delta ABC$

sketch

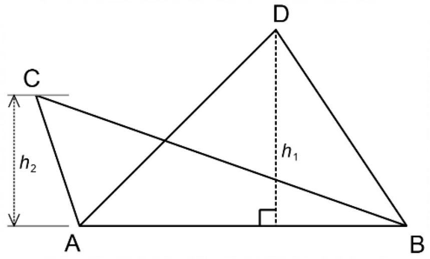
## Areas of triangles sharing bases or heights




$$\frac{\Delta ABD}{\Delta ABC} = \frac{\frac{1}{2} \times AD \times ht}{\frac{1}{2} \times AC \times ht} = \frac{AD}{AC}$$

sketch

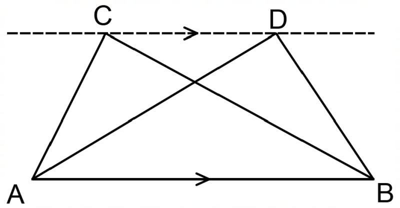
The areas of triangles with a shared / common height are in the same ratio as their bases.
(shared heights)




$$\frac{\Delta ABC}{\Delta ABD} = \frac{\frac{1}{2} \times AB \times h_2}{\frac{1}{2} \times AB \times h_1} = \frac{h_2}{h_1}$$

sketch

The areas of triangles with a shared base are in the same ratio as their heights.
(shared bases)



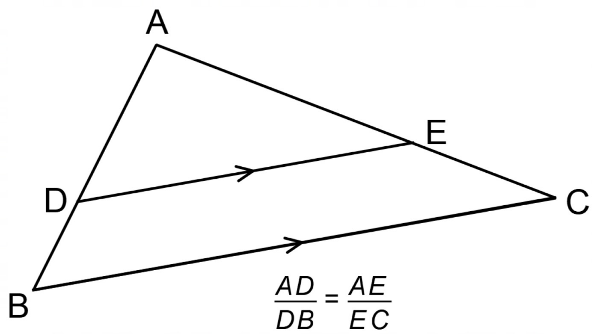

$$\Delta ABC = \Delta ABD$$

sketch

Triangles on the same base between parallel lines have equal areas.
(shared base between parallels)

## The proportional intercept theorem

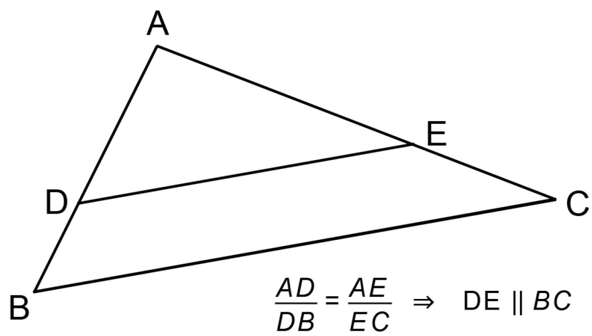
A line drawn parallel to one side of a triangle cuts the other two sides in the same proportion (prop. int)




$$\frac{AD}{DB} = \frac{AE}{EC}$$

sketch

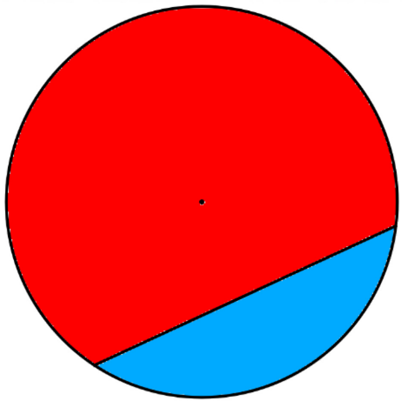
The converse is true. In other words, if a line cuts two sides of a triangle in the same proportion, then it will be parallel to the third side (conv. prop. int)



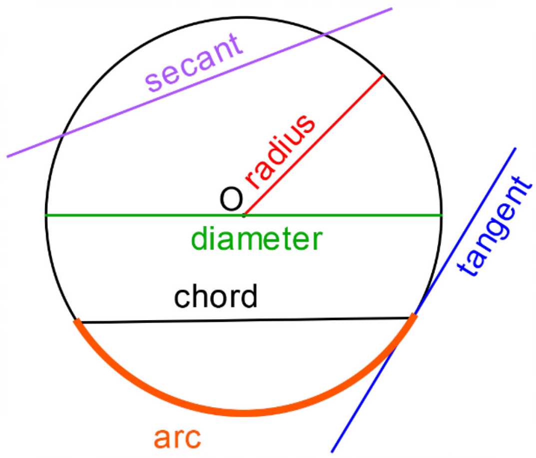

$$\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \text{DE} \parallel BC$$

sketch

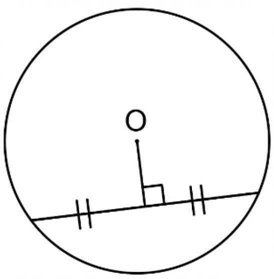
**The geometry of the circle**



A chord divides a circle into two segments, major and minor



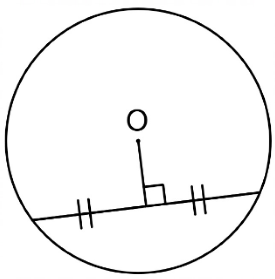

In each case the centre of the circle is marked with O if given.




If a line is drawn from the centre of a circle perpendicular to a chord, then it bisects the chord.

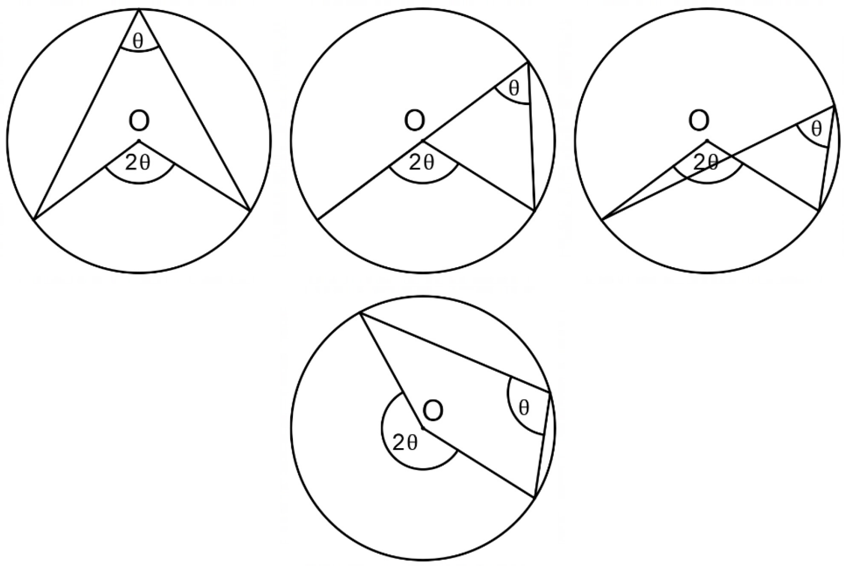
(⊥ from centre to chord)

sketch




If a line is drawn from the centre of a circle to the mid-point of a chord, then it will be perpendicular to the chord.

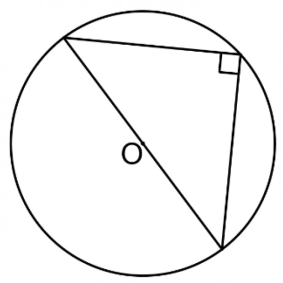
(bisector from centre to chord)




The angle subtended at the centre of a circle is double the angle subtended at the circumference.

(∠ at centre)

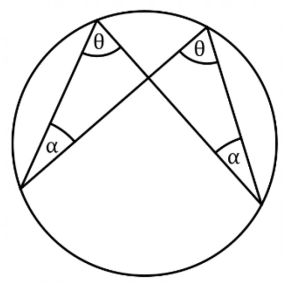
sketch




A diameter subtends an angle of 90° at the circumference.

(∠ in semi-circle)

sketch




Angles subtended in the same segment of a circle are equal.

(∠*s* in same segment)

sketch 1

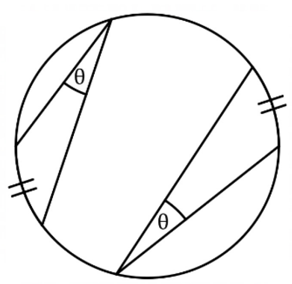
sketch 2

The converse is true.

If a line joining two points subtends equal angles at two other points on the same side of it then all 4 points are concyclic – i.e. they are the corners of a cyclic quad.

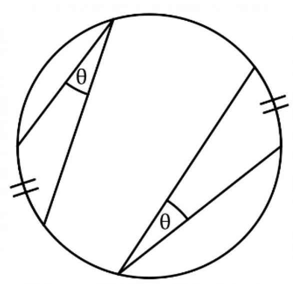
(conv. ∠s in same segment)

sketch




Arcs of equal length subtend equal angles.

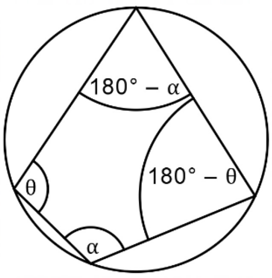
( = arcs subtend = ∠s)




Equal angles are subtended by equal arcs

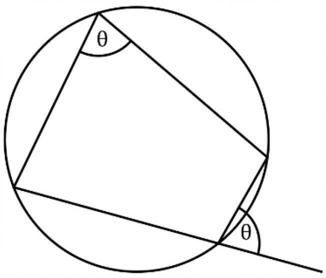
(= ∠*s* subtended by = arcs)

sketch




The opposite angles of a cyclic quadrilateral are supplementary.

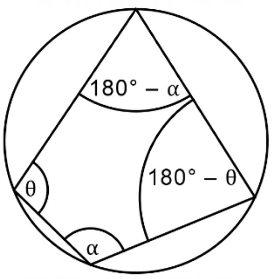
(opp. ∠s cyclic quad)




The exterior angle of a cyclic quad is equal to the opposite interior angle.

(ext. ∠ cyclic quad)

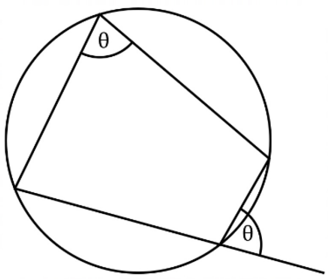
sketch




The converse is true.

If opp. angles supplementary, then it is a cyclic quadrilateral

(conv. opp. ∠s cyclic quad)

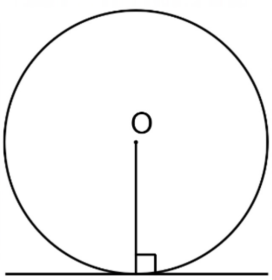



The converse is true.

If ext. angle equals opp. interior angle, then it is a cyclic quadrilateral

(conv. ext. ∠ cyclic quad)

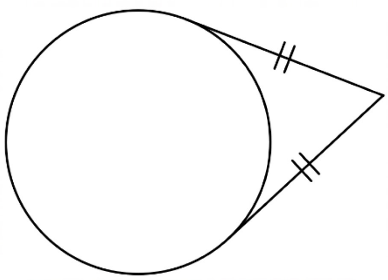
sketch




Radius is perpendicular to tangent.

(rad. ⊥ tan )

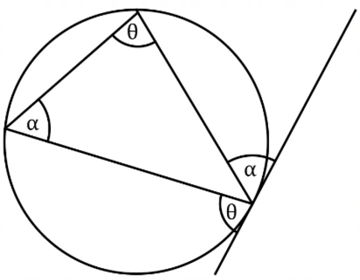
sketch



Tangents from a common point are equal in length

(tangents from common pt)

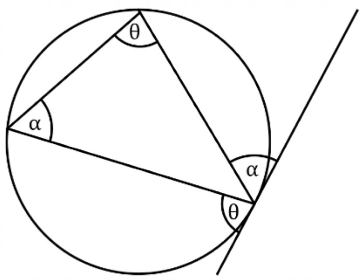
sketch




The angle between a tangent and a chord is equal to the angle in the alternate segment

(tan chord)

sketch




Converse is true. If angles equal then a tangent.

(conv. tan chord)

sketch

One can prove a line is a diameter by using the converse of the angle in semi-circle theorem.

One can prove a quadrilateral is cyclic by proving:

- Opposite angles supplementary (opp. $\angle s$ supplementary)
- Exterior angle equal to the opposite interior angle (ext. $\angle$ = opp. int. $\angle$)
- Converse angles in same segment. (conv. $\angle s$ in same segment)

One can prove a tangent using the converse of the tan-chord theorem.

# Trigonometry

https://simi.education/online-resources/links-to-sketches-for-trigonometry/

**A digital Trig. Board**
This interactive online applet will help you understand how and why the ratios work. It will also help you understand reductions.

**Trig. Graphs**
This interactive online applet will allow you to explore the parameter associated with trig. graphs. You can also modify the domain and range.

**The solution of triangles**
Sine rule, cos rule and area rule applets to help you understand how they work.