INDEPENDENT EXAMINATIONS BOARD

INTERNATIONAL SECONDARY CERTIFICATE
(IEB)

CURRICULUM AND ASSESSMENT MANUAL

FURTHER STUDIES MATHEMATICS

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LIST OF ABBREVIATIONS

BLO broad learning outcomes
CAM curriculum and assessment manual
CLL competence for lifelong learning
ICT information communication technology
IKS indigenous knowledge systems
QSO qualification-specific outcomes
SSO subject-specific outcomes

1. INTRODUCTION

The IEB supports democratic principles, contributing to the building of a fair and equitable educational landscape across the globe. Furthermore, the IEB is committed to building the reputation of educational offerings by demonstrating that the continent of Africa produces exceptional students through a quality-assured assessment process that is well recognised and respected. As an organ of civil society, the IEB believes it has an important role in providing commentary on, actively participating in, and contributing constructively to educational debate.

The IEB is clear that education is as much a philosophical undertaking, underpinned by social and educational values, as it is a technical exercise carried out by a professional body of experts. The IEB's desire to make the philosophical underpinnings of its work explicit led to the formulation of *The intentional educational beliefs of the IEB.*

This vision explains what the IEB sees as its educational purpose and outlines the learning that it values. Our world is characterised by rapid change, increasing complexity, and uncertainty. The complexity of societies in Africa and the world requires citizens who are courageous, able to determine what is needed to create a better world and, more importantly, how to bring that better world into being. In its assessments and related activities, the IEB encourages learning institutions and teachers to provide students with opportunities to explore topics, participate in activities and be exposed to discussions and debates that develop the kinds of skills, attitudes, and attributes it believes every student entering society needs to master. The IEB makes every effort to ensure that its assessments are globally competitive, giving students access to assessments that compete at the highest level.

It is this belief in the quality of students from nations in Africa and further afield that inspires the IEB to try to influence the educational experience beyond assessment.

With this as the backdrop, the IEB presents the following vision of what it stands for as an educational institution:

1.1 The Intentional Educational Beliefs

The **vision** of the IEB is to advance quality teaching and learning through an assessment process of integrity, innovation, ethical practice, and international comparability.

The IEB is cognisant that assessment is essential in developing students who are responsible citizens within their own communities and who, at the same time, can negotiate the challenges of a global, interconnected world.

Through its assessments, the IEB is motivated to develop students who can successfully navigate the complexities of constant change and mushrooming uncertainty. They are equipped with rigorous content knowledge that is supported by outcomes that articulate the skills, competencies, attributes, and attitudes required to be an active citizen. All education starts with learning, understanding, and knowing, which leads to developing values that inform action in relevant and authentic communities locally, regionally, and globally.
In summary, students completing the ISC will be:

- critical users of information
- ethical reasoners
- problem solvers
- creative and reflective thinkers
- lifelong learners
- society members respectful of diversity, particularly in the African context
- active citizens who are committed to upholding the principles of democracy and respect and the wellbeing of all people

The IEB supports the position that actively promoting quality education for every student is fundamental in establishing just, open societies based on democratic values, social justice, and fundamental human rights, in which cultural diversity is appreciated and embraced.

The IEB's **mission** is to make a significant, on-going contribution to human resource development through the design, delivery, and promotion of a wide range of high-quality, affordable assessment products and services to all sectors within independent education.

The work of the IEB is underpinned by a commitment to the following **values**: integrity, respect, commitment and service, professionalism, communication, teamwork, and quality.

1.2 Qualification-Specific Outcomes of the International Secondary Certificate (ISC)

The *intentional educational beliefs* provide a philosophical underpinning of the ISC. The overarching intention in the teaching, learning and assessment that underpins the ISC is to use the intentional educational beliefs to define qualification-specific outcomes (QSOs).

These QSOs are cross-curricular outcomes that articulate specific skills and competencies that characterise the ISC. They may also be considered transversal skills or competencies. These outcomes are the basis for equipping students with the knowledge, skills, values, attributes, and attitudes to navigate their personal journey in learning, living, and working. They are not independently assessed in the qualification. Instead, students develop and use these outcomes in the context of a specific subject and when they encounter unfamiliar or challenging situations during learning.

Students learn better when connections are established in the brain. QSOs encourage the development of skills in an integrated way. They are further articulated in the subject-specific outcomes (SSOs) of the various subjects.

Students have multiple learning opportunities to develop, apply and demonstrate these QSOs in each subject through the content topics, themes, or strands.

Teachers are not required to formally evaluate and report on these QSOs separately. Instead, teachers must give students feedback on the development of these competencies as they relate to SSOs and activities. The QSOs are defined below:

1. CRITICAL THINKING AND ETHICAL REASONING

Students must be able to think critically by analysing and evaluating multiple sources of information, using the framework of a particular discipline and the related concepts. They should then be able to draw reasonable inferences within that discipline that show intellectual integrity, ethical engagement, and cultural sensitivity.

2. PROBLEM SOLVING

Students must be able to solve problems in a variety of contexts and disciplines using a series of steps that involve defining a problem; determining the cause of the problem; identifying, prioritising, and selecting alternatives for a solution; evaluating the effectiveness of the decided course of action that is ethical, environmentally conscious, and culturally aware; and implementing the solution or course of action. A key aspect of problem solving is the ability to apply known concepts in unfamiliar contexts.

3. CREATIVITY AND INNOVATION

Students must be able to generate and apply ideas to create something new, recognise opportunities to apply ideas in new ways by considering aesthetics and interpretive options and provide a reasoned explanation for their choices. They must be able to take risks, show initiative and a broad awareness of social and cultural frameworks.

4. TAKE RESPONSIBILITY FOR LEARNING

Students must be able to identify their learning goals and believe that they can achieve these goals through a thoughtful growing awareness of their values and priorities so that they can manage their learning purposely and effectively.

5. UNDERSTAND HOW LEARNING HAPPENS

Students must be able to think about their learning as it is happening so that they can develop a thoughtful growing awareness of it, using strategies that organise it in such a way that they are able to identify the success or failure of these strategies and revise them appropriately.

6. LIFELONG LEARNING

Students must be able to show understanding that learning goes beyond the classroom and that to integrate and adapt effectively in the world of work, they will be learning, unlearning, and relearning as their social, environmental, economic, and cultural context changes.

7. DIGITAL CITIZENSHIP

Students must be able to show knowledge and skills that use digital technology to enhance their learning in different ways and in different contexts so that they can use it confidently, safely, healthily, responsibly, and ethically in school, at home and in their communities.

8. COMMUNICATION

Students must be able to share ideas through oral, written, or non-verbal media; engage in formal and informal exchanges with others; demonstrate respect, empathy, and responsibility; and consider how culture, context, and experience impact messaging. This includes visual literacy, media literacy, and technological literacy.

9. COLLABORATION AND CO-OPERATION

Students must be able to co-operate with a range of others in a group to successfully work together to achieve a common task or goal, by:

- participating,
- exchanging ideas,
- sharing responsibilities,
- being aware of the impact of their behaviour on the group,
- eliciting views of others,
- respecting competing views,
- adapting,
- being willing to compromise and use strategies that help to resolve conflict,
- nurturing positive relationships, and
- helping the group come to valid conclusions.

10. ENVIRONMENTAL AWARENESS

Students must be able to show understanding of the finite nature of the earth's resources, and the impact of climate change and environmental degradation on the planet so that they can appreciate our common humanity and engage in strategies to promote sustainable living for all members of society both locally and globally.

11. CONNECTING, CREATING AND EXPANDING KNOWLEDGE TO INCLUDE INDIGENOUS KNOWLEDGE SYSTEMS (IKS)

Students must be able to actively engage with cultural, environmental, political, or economic knowledge systems by:

- understanding the limitations of seeing the world in one way;
- realising the importance and value of expanding the boundaries of their knowledge to include their local community's traditional technology; and
- social, economic, and philosophical learning grounded in skills, practices, and ways of being in nature.

Identifying factors that influence their own opinions and beliefs about the world reveal significant community issues, and articulate new global perspectives so they can make informed decisions.

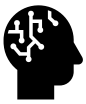
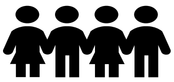
12. ACTIVE, CONFIDENT CITIZENSHIP

Students must show commitment to upholding democratic principles, constitutional values of equality, freedom, and dignity to ensure the rights and responsibilities including the wellbeing of all people in their local communities, their country and even the world.

1.3 Broad Learning Outcomes and their Relationship to Qualification-Specific Outcomes

Qualification-specific outcomes (QSOs) can be organised into four broad learning outcomes (BLOs). The value of this is that it makes it easier to map them with subject-specific outcomes (SSOs).

Table 1: The four broad learning outcomes and associated qualification-specific outcomes

ICON	Broad Learning Outcome	Qualification-Specific Outcomes
	**Broad Learning Outcome 1: Thinking, reasoning, and innovation skills**	1. Critical thinking and ethical reasoning 2. Problem solving 3. Creativity and innovation
	**Broad Learning Outcome 2: Understanding learning**	4. Lifelong learning 5. Taking responsibility for learning (Agency) 6. Understanding how learning happens (Metacognition)
	**Broad Learning Outcome 3: Life skills**	7. Digital citizenship 8. Communication 9. Collaboration and co-operation
	**Broad Learning Outcome 4: Social responsibility and cultural, geopolitical and environmental awareness**	10. Environmental awareness 11. Active citizenship 12. Connecting, creating, and expanding knowledge to include Indigenous Knowledge Systems

1.4 Linking the levels of the curriculum through the *Curriculum Framework*

The ***Curriculum Framework*** describes links between the different levels within this qualification. It outlines the intended learning outcomes at the different levels namely the BLOs and QSOs or transversal competencies that are underpinned by the intentional educational beliefs and further articulated in the SSOs of each subject.

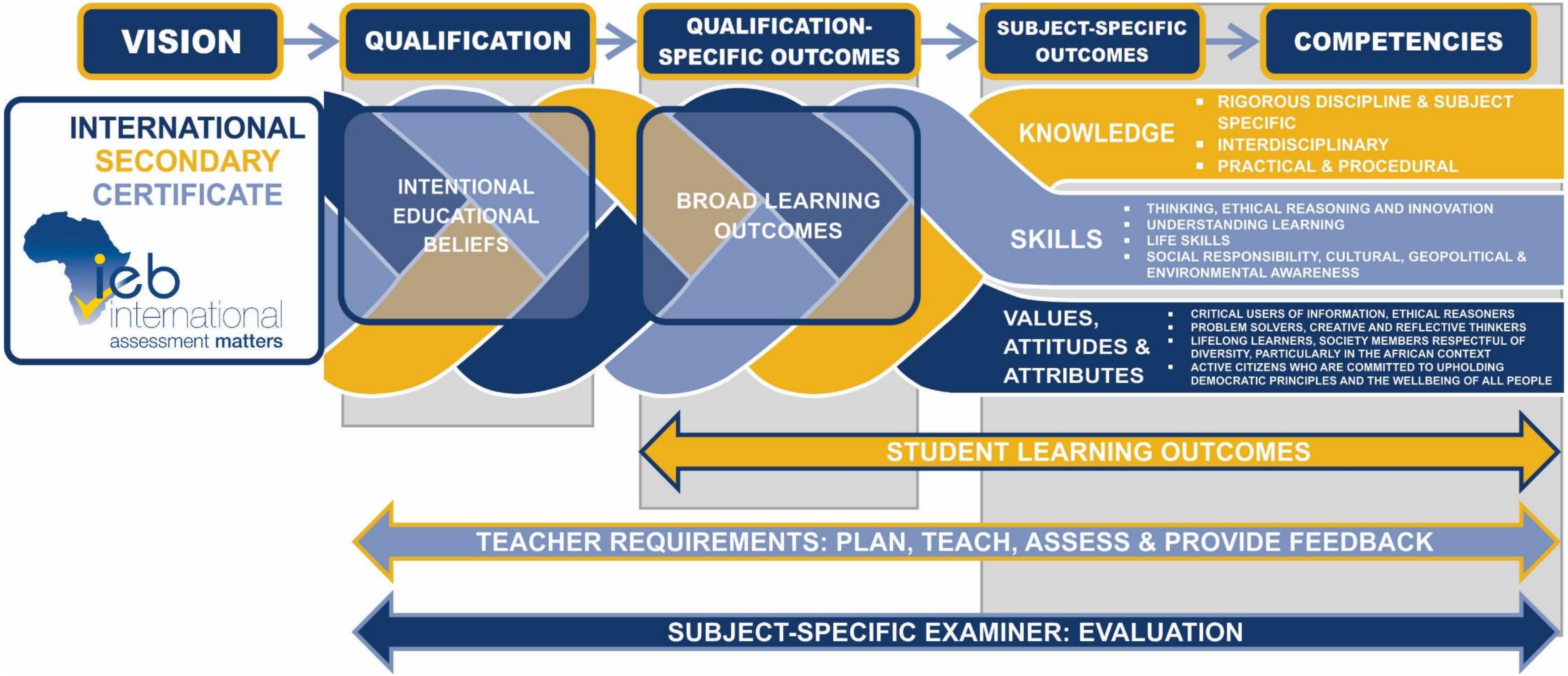


Figure 1: Representation of the embedded outcomes at each level of the *Curriculum Framework* for the ISC

1.5 The relationship between the *lived environment* and the *learning environment*

The *Curriculum Framework* also maps the relationship between a student's lived environment and experiences and their learning environment. Learning begins at birth. Children are born with innate curiosity and learn through constantly questioning the world in which they find themselves. Opportunities to find answers to their questions are provided by various aspects of the lived and learning environments of the child.

The informal learning environment is where the students are exposed to learning opportunities in a less structured way among people who are close to them e.g., the family, extended family, close friends, and community. The broader local community lived environment is where the child experiences learning outside the home, e.g., sports clubs, hobbies, community events, societies and extra-mural activities at the school, home, or place of religious observance.

The home country and the world lived environment is where the student is exposed to national and global issues outside of their local community e.g., universal human commonalities and differences, global and national problems and features of agreement and disagreement on a global level.

The formal learning environment is where students formally experience teaching and learning in a purposeful, structured way.

The responsibility for the development of BLOs is shared between the formal learning environment and the community within which the student lives. It is in the formal learning environment where learning opportunities are provided to raise awareness and find strategies to develop these outcomes. The broader local community provides the environment within which the BLOs are practised. The specific subjects contribute to raising awareness and developing strategies to develop these competencies and teachers are obliged to create overt learning opportunities to actively emphasise and provide feedback on the progress in developing these transversal skills.

The curriculum is the vehicle through which a student's lived environment is connected to the learning environment, the formal learning pathway through which a student's learning achievements are developed and finally recognised by the award of a qualification.

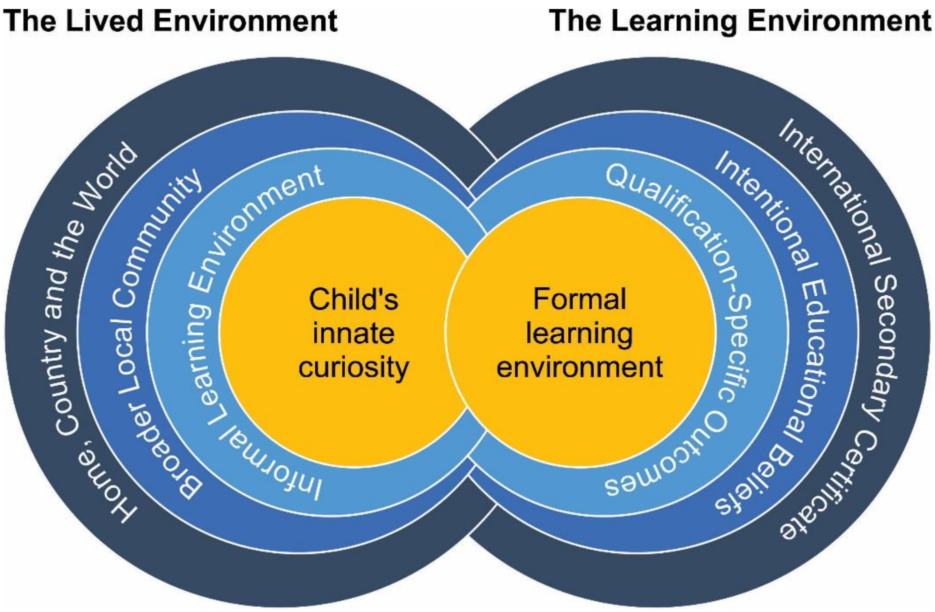


Figure 2: Representation of the intersection of the ISC *Curriculum Framework* and the student's *lived environment*.

2. APPROACH TO ASSESSMENT

2.1 Integrating teaching, learning and assessment

The IEB believes that the value of the curriculum as a vehicle for education is experienced through the way in which it is assessed. The assessment of learning for understanding is much more than the recognition or recall of facts. Assessment should enable students to show the complexity of their learning, understanding, and thinking. They need to be able to apply key concepts, knowledge, skills, and attitudes in ways that are authentic and encourage them to engage in higher-order thinking.

The IEB through the ISC aims to harness the positive impact of appropriate assessment methodologies on learning and teaching and, through its instruments, to offer teachers and students the opportunity to develop the necessary knowledge, skills, values, and attitudes required in our complex, constantly changing world. In-depth study and rigorous assessment prepare students for the demands of tertiary study, including the development of the affective skills such as time management, prioritisation, and perseverance.

Assessment in all its forms is a fundamental aspect of learning and teaching and hence should:

- allow teachers and students to identify gaps in learning prior to the introduction of new learning;
- inform students of the degree to which they have mastered the subject-specific concepts and skills that have been taught;
- inform students of the degree to which they have mastered transversal skills as articulated in the QSOs;
- allow both teachers and students to monitor the process of learning and make the necessary interventions to ensure further learning;
- inform teachers of the strengths and gaps of both individuals and groups of students, thereby providing the teacher with information about where teaching of a skill or concept may need a different approach or a targeted intervention; and
- allow students to participate in the assessment and learning process through peer, group, and self-assessment, developing valuable metacognitive, social, and affective skills and leading to a consolidation of their learning.

In the assessment of learning, the methodologies used should address the intended outcomes of the curriculum as well as inform the progression of learning to reach those outcomes. They should provide valid and reliable information to support credible and defensible results so that they can be used for access to the world of work or further study. Assessment methods, including tests and examinations, can serve these purposes. However, the term *assessment* encompasses a range of other assessment methodologies that serve to provide opportunities for students to demonstrate:

- the skills and knowledge they have acquired over a period in a rich variety of products of learning, such as portfolios, exhibitions, performances, presentations, simulations, case studies, multimedia projects, and other written, oral, and visual tasks. This rich variety of assessments provides students with a range of learning experiences and ways of communicating their competence.
- skills and, as appropriate, knowledge that cannot be assessed in a written test or formal examination but is, however, critical to the specific subject. These methodologies include oral assessment, particularly in the study of languages, which is also a useful assessment method to build confidence in speaking in a more formal environment in other subjects; practical competence in the arts, sciences, and semi-vocational subjects; digital competence; and opportunities to provide time to inspire creativity e.g., in creative writing.

While assessment of these skills may well form part of the final assessment, their development is the result of ongoing learning experiences, opportunities, and exposure to challenges, which are a necessary part of preparation for the final assessment.

2.2 Feedback

Best practice requires that teachers and students are fully aware of the criteria against which their performance will be assessed. Hence the IEB provides teachers with information that details the cognitive demand, levels of difficulty, and curriculum coverage of each question in the examination paper, as well as the marking guidelines. The IEB also provides detailed reports per institution on how their students performed in each question, thus encouraging the development of focussed interventions at institution level. The IEB advocates that the criteria associated with all site-based assessments be shared with students, not only encouraging a transparent learning environment, but also providing valuable feedback for students when they reflect on their efforts.

2.3 The setting and marking of assessment tasks and examination papers

The assessment requirements in the *Curriculum and Assessment Manuals* (CAMs) for each subject detail the expectations for the external examination. Depending on the subject and the curriculum requirements, students will need to develop oral competence, performance competence and/or practical competence that will be assessed and contribute towards their final achievement of the qualification. Guidelines for each subject provide the weighting of the cognitive skills as they are reflected in the final assessments as well as the weighting of the various topics, which broadly aligns with the amount of time allocated to the teaching and learning processes. As is the IEB's practice, exemplars of the kinds of assessment are made available as part of its commitment to a publicly recognisable standard.

The assessment instruments developed by the examining panels in all subjects explicitly align with the SSOs and educational opportunities stipulated in the CAM for each subject. This naturally leads to a range of assessment tasks and activities that may look rather different from more conventional forms of assessment since their focus is to develop, instil and assess a more searching range of thinking skills, processes, and values.

The IEB follows an authentic approach to assessment, setting, as far as possible, contextualised problems that require the use of skills and knowledge that encourage the application of the 'known' in an unfamiliar context. Examination papers are developed by experienced examining panels through an engaging and reflective process. The papers are moderated internally by very experienced assessors who have had previous examining experience, and externally by the quality assurance body.

The marking is completed by qualified teachers in the IEB teaching community. Appropriate structures and protocols are in place to ensure moderation of the marking process, oversee the standardisation of results, and deal with any irregularities that may have occurred. Appeal processes are available.

3. THE ROLE OF TECHNOLOGY

The ability to use technology inside and outside the classroom effectively and safely provides students with access to information, accelerates their learning, allows them to explore new subjects, and deepens their understanding of difficult concepts. However, access to connectivity and devices does not automatically provide engaging educational opportunities or a quality educational experience.

Today's students are digital natives. They know what information communication technology (ICT) is available and inevitably know how to use it in all its permutations and platforms. However, technical proficiency or regular use of technology does not necessarily mean digital literacy. Digital literacy is defined as 'the ability to use technology, information and communication technologies safely and effectively, to find, evaluate, create, and communicate information responsibly. This requires both cognitive and technical skills.' This ability to think critically about how to use technology to create, build, explore and collaborate, rather than simply consume information passively, is an important outcome of any curriculum.

Clearly students of today need to understand the tools and skills necessary to process the enormous quantity of information they encounter daily. This is a qualification-specific outcome or transversal skill. However, the relevance of ICT and digital literacy varies from subject area to subject area. Hence, embedded in the content of each subject is an awareness of how ICT impacts on the subject area and how it can be used to advance a positive and effective learning culture.

The learning environment should provide students with access to computers and the internet. Equally important is that it allows for the development of key skills associated with digital literacy so that our students become responsible digital citizens.

4. RESOURCES

4.1 What kind of teacher does the ISC need?

The teachers who facilitate the learning required for the ISC must:

- be appropriately qualified to teach the subject they are teaching and, as professionals, keep up to date with developments in education generally and in their discipline and subject areas specifically;
- be mindful of the Intentional Educational Beliefs, the QSOs of the qualification and provide opportunities for students to develop them within the context of the SSOs;
- clearly articulate the SSOs in their planning, teaching, and assessment so that they can provide the necessary feedback to each student on their progress as they engage with the process of learning;
- have competence in managing the learning process, including planning, assessment, and monitoring progress;
- have competence in developing assessment tasks that are appropriate for the outcomes being assessed and appropriately judging the evidence of learning to positively impact on further learning;
- be mediators of learning, mindful of the range of theories of learning and provide opportunities in the classroom that harness the positive aspects of each theory;
- be aware of the impact of different religious, cultural, and social practices on the opinions and beliefs of the students;
- be aware of any unconscious bias that they may hold that could impact on the students;
- be mindful of the contention that may arise during discussion in the classroom and hence be sensitive and caring in managing different points of view and ideas;
- be mindful of the power they hold and the impact they have on influencing the learning experience of each student; and
- be a lifelong learner and model the process of learning and metacognition for their students.

Obviously, the teacher should operate in a functional, well-managed school where they are supported. Teachers must provide clear teaching and assessment plans that overtly address the curriculum and its specific requirements, thereby providing a bridge between the intended outcomes of the curriculum and the implemented curriculum.

4.2 Textbooks

The IEB encourages teachers to make use of a variety of textbooks as well as other learning resources that support specific learning activities. These include a range of sources e.g., the media; the internet; video and other digital material. The intention of every teacher should be to develop a bank of resources that make learning interesting and sufficiently varied to cater for individual learning approaches in the context within which the learning takes place.

4.3 Teacher Support

The IEB hosts subject-specific conferences for teachers annually and has well-run Professional Learning Networks that provide ample opportunity for engagement among teachers. These vehicles promote the professional development of teachers where they can, inter alia, support one another towards best practice, share resources, and discuss and debate issues that pertain to their subject.

In addition, the IEB assessment specialists provide guidance. The IEB website has resources, including exemplars, past examination papers, and marking guidelines, for teachers to use.

4.4 Time allocation for the ISC

The time allocations below are *indicative* of the amount of time required per subject. Educational institutions, however, have some latitude regarding how the timetabling of the subjects is achieved, given the consideration of the stipulated length of the school day and the number of teaching weeks available within a country. Independent institutions normally have leeway to extend beyond the minimum requirements. Students registered directly with the IEB also have significant leeway regarding the time allocated per subject and the number of subjects that they take at a time.

(a) The recommended instructional time for each level of the ISC is as follows:

Subject	**Minimum time allocation (hours) per week (5-day cycle)**
Primary Language	4,5
Additional Language	4,5
CLL*, e.g., Active Citizenry	2
A minimum of four subjects selected from Annexures A5–A8 of the *Qualification Framework*, subject to the provisions stipulated in section 2.6 of the *Qualification Framework*.	16 (4 × 4 h)
Total	27

*Competence for Lifelong Learning

(b) The recommended instructional time for the Further Studies Programmes in the ISC for each level is as follows:

Subject	Minimum time allocation (hours) per week (5-day cycle)
Further Studies Mathematics: (Standard level) (Extended level)	 5 6
Further Studies Language together with Primary Language	6 (1,5 + 4,5)
Further Studies Physics Further Studies Chemistry	6 6
The offering of Further Studies Programmes is subject to the provisos stipulated in section 2.6 of the *Qualification Framework*.	

5. OVERALL STRUCTURE OF THE FURTHER STUDIES MATHEMATICS CURRICULUM AND ASSESSMENT MANUAL

5.1 DEFINITION AND PURPOSE

Further Studies Mathematics is an extension of Mathematics and is similarly based on the following view of the nature of the discipline.

Further Studies Mathematics is a course designed for students who intend to follow tertiary studies in mathematics, engineering, and the sciences including actuarial science.

Standard Level focuses on Number, Algebra, Functions, Trigonometry and Differential & Integral Calculus. ,.

The intrigue and richness of mathematics is accentuated at the Extended level, where students broaden their perspective of the diverse journeys available in mathematics and develop a passion for the continued learning of mathematics. In the Extended Level, different elective branches of mathematics including Statistics and Probability; Finance and Mathematical Modelling; Matrices and Graph Theory are offered in addition to the content of the Standard Level.

5.1.1 OVERALL OUTCOMES

ISC students are able to:

- demonstrate the value of mathematics and its role in the development of our contemporary society.
- use their mathematical knowledge to solve unfamiliar problems in the world around them and to grow in confidence in this ability.
- use mathematical processes to solve and/or pose problems creatively and critically.
- demonstrate the patience and perseverance to work both independently and cooperatively on problems that require more time to solve.
- contribute to quantitative arguments relating to local, national, and global issues.
- focus on the processes of mathematics, rather than only on the correct answers.
- demonstrate self-reliance in using mathematics and validating their own answers.
- communicate mathematical problems, ideas, and scenarios through reading, writing and mathematical language.
- critically engage with multiple knowledge systems including Indigenous Knowledge Systems.
- use appropriate technology effectively to solve problems.

5.1.2 MEANS OF ASSESSMENT

A. Further Studies Standard: one 2-hour examination (Paper 1).

Further Studies Extended: two examinations, one 2-hour examination (Paper 1) and one 1 hour-examination (Paper 2):

Paper 1	2 hours	200 marks
Paper 2	1 hour	100 marks

B. Examination paper content requirements

Paper 1
Calculus
Algebra

Paper 2
Statistics & Probability

OR

Finance & Mathematical Modelling

OR

Matrices & Graph Theory

C. Structure of Further Studies Mathematics: Standard & Extended

Module	Further Studies Standard Level	Further Studies Extended Level
1A Calculus	✓	✓
1B Algebra	✓	✓
ELECTIVES		
2 Statistics & Probability **OR**		✓
3 Finance & Mathematical Modelling **OR**		✓
4 Matrices & Graph Theory		✓

- Included with the papers is a formulae sheet.
- The **Assessment Addendum for Further Studies Mathematics** includes the formulae sheet and additional information that specifically relates to the assessment.

5.2 Content Outline

There are three levels and represent the progression of teaching and learning in each content topic.

Topic: Calculus (Module 1A)

The student is able to establish, define, manipulate, determine, and represent the derivative and integral, both as an anti-derivative and as the area under a curve, of various algebraic and trigonometric functions, and solve related problems.

	Sub-Topics		
	Level 1	**Level 2**	**Level 3**
	We know this when the student is able to:	We know this when the student is able to:	We know this when the student is able to:
1		(a) Sketch the graphs of mathematical functions including split domain and composite functions (comprised of linear, quadratic, hyperbolic, absolute value, and exponential functions). (b) Manipulate, analyse split domain, and composite functions using the definition of a function and the graph of the function.	
2		(a) Define and use trigonometric and reciprocal trigonometric functions to: • manipulate trigonometric statements. • solve trigonometric problems in realistic and mathematical contexts. • find the general solution of trigonometric equations. (b) convert between angles measured in degrees and radians. (c) use trigonometric functions defined in terms of a real variable (angle in radians) and the *x*, *y,* and r-definition to: • calculate the lengths of arcs of circles, • calculate the area of sectors and segments of circles.	

	Level 1	Level 2	Level 3
	We know this when the student is able to:	We know this when the student is able to:	We know this when the student is able to:
3		(a) Compare the graphical, numerical, and symbolic representations of the limit of a function. (b) Determine the limit of an algebraic function at a point, including from the right and left, and to infinity algebraically. (Note: The limit at a point is defined as the limit from the left and from the right.) (c) Illustrate the continuity of a function graphically and apply the definition of continuity at a point to simple algebraic functions, including split domain functions.	(a) Use first principles and graphs to determine the continuity and differentiability at a given point of algebraic functions, including split domain functions. (b) Without proof, apply the theorem and its converse, "A function that is differentiable at a point is continuous at that point", and deal with examples to indicate that the converse is not valid. (c) Demonstrate the derivative of a function at a point as the rate of change, by graphical, numerical, and symbolic representations.
4		(a) Illustrate the differentiability of a function graphically and determine the derivative as the gradient of a function at a point using limits. (b) Establish the derivatives of functions of the form $ax^2 + bx + c$ & $mx + c$, from first principles.	(a) Establish the derivatives of functions of the form $\sqrt{x}$, $\sqrt{ax+b}$, $\frac{1}{ax+b}$, $\frac{1}{\sqrt{ax+b}}$ from first principles. (b) Use the following rules of differentiation: $\frac{d}{dx}\left[f(x).(gx)\right] = g(x).\frac{d}{dx}\left[f(x)\right] + f(x).\frac{d}{dx}\left[g(x)\right]$ $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x).\frac{d}{dx}\left[f(x)\right] - f(x).\frac{d}{dx}\left[g(x)\right]}{\left[g(x)\right]^2}$ $\frac{d}{dx}\left[f(g(x))\right] = f'(g(x)).g'(x)$

	Level 1	Level 2	Level 3
	We know this when the student is able to:	We know this when the student is able to:	We know this when the student is able to:
			applied to: • polynomial, rational, radical, natural logarithm, ef(x) and trigonometric functions, (Students may assume without proof that $\frac{d}{dx}\sin x = \cos x$), • higher order derivatives, but the n-th derivative is not examinable, and • functions in two variables using implicit differentiation. (c) Newton's method. (d) Calculate maximum and minimum values of a function using calculus methods. Determine both absolute and relative maximum and minimum values of a function over a given interval. (e) Use calculus methods to sketch the curves of polynomial and rational functions determining: • intervals over which a function is increasing or decreasing. • *y*-intercepts and *x*-intercepts, using Newton's Method if necessary. • the coordinates and nature of stationary points. • any vertical, horizontal, or oblique asymptotes. (f) Use methods learned above to solve practical problems involving optimisation and rates of change in real, realistic, and abstract mathematical contexts. Verify the results of the calculus modelling by referring to the practical context.

	Level 1	Level 2	Level 3
	We know this when the student is able to:	We know this when the student is able to:	We know this when the student is able to:
5	(a) Approximate the area between familiar curves, such as straight lines, parabolas, hyperbolae and exponential graphs, and the *x*-axis using the Rectangle Method on an interval of the *x*-axis. (b) Estimate the margin of error of the approximate area determined by the upper and lower sums method. (c) Experiment with the accuracy of the approximation by varying the width and number of rectangles. (Not examinable in grade 12)	(a) Investigate and develop a formula for the upper and lower sums method of approximating area under the curve of $y = x^n$, for $n \in$ N and $x \geq 0$ on the interval [a; b] $\int_a^b x^n \, dx = \left[\frac{1}{n+1} x^{n+1} \right|_a^b$ (b) Use available technology to manipulate the width of sub-intervals and the accuracy of the approximate area under a polynomial function. (c) Investigate and intuitively develop the Riemann (definite) integral as the approximating rectangles are made narrower and the number of rectangles, *n*, increases. (Generating and simplifying the formula $Area = \lim_{n \to \infty} \left(\frac{b-a}{n} \right) \sum_{i=1}^{n} f\left(x_i\right)$ is not examinable. Students may be required to interpret the formula.)	(a) Appreciate the Fundamental Theorem of Calculus and its significance in recognising anti-differentiation as the reverse of differentiation. (b) Manipulate and then integrate algebraic, natural logarithm, $e^{f(x)}$ and trigonometric functions of the form: • $\int ax^n dx$, *a* is a constant and $n \in$ Q, • $\int p(x)\,dx$ and $\int \frac{p(x)}{q(x)}dx$, *p*(*x*) and *q*(*x*) are polynomials or radicals, • $\int \frac{f(x)}{g(x)}dx$, leading to partial fractions • $\int f\left(g(x)\right).g'(x)dx$ • where the anti-derivative of a trigonometric function can be directly determined from the derivative (e.g. $\int \cos x dx$, $\int \sec^2 x dx$, $\int \sec x \tan x dx$) • $\int \sin^n x dx$ and $\int \cos^n x dx$ where n $\in$ N and n $\leq$ 3 • $\int \sin mx\, dx$; $\int \sin mx \cos nx dx$ and similar functions

	Level 1	Level 2	Level 3
	We know this when the student is able to:	We know this when the student is able to:	We know this when the student is able to:
			using only the following methods: • direct anti-derivatives • simplification of trigonometric functions using appropriate squares, compound angle and given product-sum formulae • integration by u-substitution • integration with a given substitution • integration by parts (c) Find the definite integral of any function using a calculator. (d) Apply the definite integral and techniques of integration to solve area and volume problems by: • Calculating the area under or between curves using the manipulation of intervals. • Calculating the volume generated by rotating a function about the x-axis in mathematical and real-world contexts.

Topic: Algebra (Module 1B)

The student is able to represent, investigate, analyse, manipulate, and prove conjectures about numerical and algebraic relationships and functions, and solve related problems.

	Sub-Topics		
	Level 1	**Level 2**	**Level 3**
	We know this when the student is able to:	We know this when the student is able to:	We know this when the student is able to:
6	(a) Characterise and discuss the nature and relevance of the roots of $x^2+1=0$. (b) Classify numbers using sub-fields of the Complex numbers. (c) Determine the roots of equations of the form $ax^2+bx+c=0$ and classify the roots as real or imaginary. Determine the real and complex roots of quadratic and cubic equations using: • factorisation, • the quadratic formula, and • the factor theorem to find the first real root of cubic equations. (d) Perform the four basic operations (+, –, /, x) on Complex numbers and their conjugates without the use of a calculator. (e) Illustrate a Complex number on an Argand diagram.	Solve: (a) equations containing multi-term algebraic fractions using algebraic methods. (b) polynomial and rational inequalities. (c) absolute value equations.	Solve exponential equations using the laws of exponents, algebraic manipulation, and logs.

	Level 1	Level 2	Level 3
	We know this when the student is able to:	We know this when the student is able to:	We know this when the student is able to:
7	Simplify compound fractions.	Decompose algebraic fractions into partial fractions when the denominator is of the form: • $(a_1x+b_1)(a_2x+b_2)\ \dots\ (a_nx+b_n)$, using the 'cover up' method. • $(ax+b)^2(cx+d)^2$, by comparing coefficients.	(a) Simplify and manipulate algebraic expressions using the laws of exponents and logarithms. (b) Demonstrate an understanding of *e* and its role in exponents and logarithms by using *e* freely in problem solving.
8	(a) Demonstrate an understanding of the absolute value of an algebraic expression as a distance from the origin. $\lvert x\rvert = \begin{cases} x & if\ x \geq 0 \\ -x & if\ x < 0 \end{cases}$ (b) Solve linear absolute value equations. (c) Draw graphs of the form $y = a\lvert x\rvert + q$.	(a) Draw graphs of $y = a\lvert x-p\rvert + q$, $y = \lvert f(x)\rvert$ and $y = f(\lvert x\rvert)$ by inference where $f(x)$ is a simple function (e.g. see 11.1.1). (b) Find the equation of the absolute value function, in the form $y = a\lvert x-p\rvert + q$, given the graph and necessary points on the graph. (c) Interpret the graphs of absolute value functions to determine the: • domain and range; • intercepts with the axes; • turning points, minima and maxima; • shape and symmetry; • transformations. (d) Solve absolute value inequalities using graphical representations of the associated functions.	(a) Draw exponential graphs, including $y = e^x$. (b) Draw logarithmic graphs, including y = ln x. (c) Find the equation for the reflections of the exponential or logarithmic functions in the lines x = 0, y = 0 and y = x, the inverse of the function. (d) transformations of ln(x) / e^x graphs.
9			Use mathematical induction to prove: (a) statements of summation of series. (b) statements about factors and factoring with Natural numbers.

Topic: Statistics and Probability (Module 2)

The student is able to organise, summarise, analyse, and interpret data to identify, formulate and test statistical and probability models, and solve related problems.

<table>
<tr><th></th><th colspan="3">Sub-Topics</th></tr>
<tr><th></th><th>Level 1</th><th>Level 2</th><th>Level 3</th></tr>
<tr><td></td><td>We know this when the student is able to:</td><td>We know this when the student is able to:</td><td>We know this when the student is able to:</td></tr>
<tr><td>1</td><td>(a) Use Venn diagrams as an aid to solve probability problems of random events.

(b) Use Tree diagrams as an aid to solve probability problems of random events.

(c) Use contingency tables to solve probability problems.

(d) Recognise and then determine the probability of conditional events using diagrams and the formula: $P(A|B) = \frac{P(A \cap B)}{P(B)}$

(e) Identify and determine the probability of mutually exclusive and independent events.

(f) Use the Laws of Probability to evaluate simple random events.</td><td>(a) Identify, apply, and calculate the probability of the following distributions of discrete random events:

• Hypergeometric distribution model
• Binomial distribution model

The mean and variance of the binomial distribution should be known and applied.

(b) Formulate and/or apply a probability mass function (including simple continuous models) and find the expected value and variance.

(c) Identify and apply the Normal distribution model to the probability of continuous random events, using statistical tables and calculations, as necessary.</td><td>(a) Understand how the Central Limit Theorem is applied when a large (n > 30) sample is taken from any distribution.

(b) Apply the Normal distribution model to a sample to estimate a confidence interval for the population mean or proportion, using statistical tables to deal with various confidence levels.

(c) Apply the normal approximation to the binomial distribution, utilising continuity corrections, as appropriate.

(d) Formulate a probability mass or density function for a:

• Hypergeometric distribution
• Binomial distribution
• Simple continuous probability models</td></tr>
<tr><td>2</td><td>Count arrangements using permutations (including those where repetition occurs).</td><td>Count arrangements and choices using permutations (including those where repetition occurs) and combinations.</td><td>–</td></tr>
</table>

	Level 1	Level 2	Level 3
	We know this when the student is able to:	We know this when the student is able to:	We know this when the student is able to:
3	(a) Calculate the standard deviation. (b) Understand the mean and standard deviation of a population as applied to the normal distribution. The effect of these parameters on the shape of the curve should be understood. (c) The percentage of values that lie within 1, 2 and 3 standard deviations of the mean should be known.		Perform a one-tail and/or two-tail hypothesis test on a mean or difference of means. This includes being able to: • Distinguish between one-tail and two-tail events. • Establish a null hypothesis based on the prevalent condition.

Topic: Finance and Mathematics Modelling (Module 3)

The student is able to investigate, represent and model growth and decay problems using formulae, difference equations and series.

	Sub-Topics		
	Level 1	**Level 2**	**Level 3**
	We know this when the student is able to:	We know this when the student is able to:	We know this when the student is able to:
1	(a) Generalise number patterns using first order linear difference equations of the form $u_n = k.u_{n-1} + c$. (b) Use appropriate technology to solve higher terms in first order linear difference equations. (c) Convert first order linear difference equations into a general solution in explicit form. (d) With the aid of appropriate technology, use first order linear difference equations to solve future and present value annuities.	(a) Generalise or produce number patterns using second order homogenous linear difference equations $(u_n = p.u_{n-1} + q.u_{n-2} + c)$. (b) Use appropriate technology to solve higher terms in second order homogenous linear difference equations. (c) Model simple population growth and decay problems using a discrete Malthusian population model of the form $P_{n+1} = (1+r).P_n + c$.	(a) Model simple population growth and decay problems using • a discrete Logistic population model of the form $P_{n+1} = P_n + a(1 - b.P_n).P_n$ • a discrete two species Lotka-Volterra predator-prey population model written in difference equation form $R_{n+1} = R_n + a.R_n(1 - R_n / k) - b.R_n.F_n$ $F_{n+1} = F_n + e.b.R_n.F_n - c.F_n$ and interpret characteristics of the parameters and attributes of the models (e.g., carrying capacity, equilibrium populations) (b) Evaluate a realistic population scenario and apply the most suitable model for a given scenario.

	Level 1	**Level 2**	**Level 3**
	We know this when the student is able to:	We know this when the student is able to:	We know this when the student is able to:
2	(a) Use simple and compound growth formulae to solve problems in various contexts including but not limited to: • simple interest and straight-line depreciation, • compound interest and reducing balance depreciation, • compound growth and decay problems (b) Investigate and derive the future value annuity formula using first order linear difference equations in explicit form.	Formulate timelines and apply future and present value annuity formulae to: (a) Calculate the present value or future value of an annuity, or the termly payment. (b) Calculate the balance outstanding on a loan at a specified point in the amortisation period. (c) Establish a sinking fund in a given context. (d) Calculate the value of a deferred annuity. (e) Convert effective and nominal interest rates to solve problems with different accumulation periods.	Formulate timelines and apply future and present value annuity formulae to: (a) Determine the number of repayment periods using logarithms. (b) Calculate the number of payments and the final payment when a loan is repaid by fixed instalments. (c) Solve annuity problems involving changing circumstances such as changes to time periods, repayments (including missed payments), withdrawals and interest rates.

Topic: Matrices and Graph Theory (Module 4)

The student is able to identify, represent and manipulate discrete variables using graphs and matrices, applying algorithms in modelling finite systems.

	Sub-Topics		
	Level 1	**Level 2**	**Level 3**
	We know this when the student is able to:	We know this when the student is able to:	We know this when the student is able to:
1	(a) Arrange numbers in a suitable rectangular array or matrix to facilitate problem solving. (b) Knowing when a matrix operation is possible, perform the following operations on a matrix or matrices: • addition, • multiplication of a matrices, and • multiplication by a scalar. (c) Solve systems of two variable linear equations using the method of diagonalisation. (d) Determine the inverse of 2 × 2 matrices by a sequence of row transformation $[A : I_{nxn}] = [I_{nxn} : A^{-1}]$. (e) Solve systems of linear equations using the inverse matrix.	Use 2 × 2 matrices to transform points and figures in the Cartesian Plane by: (a) A translation, given in the form $\begin{pmatrix} a \\ b \end{pmatrix}$. (b) Rotation through any given angle about the origin. (c) Reflection in any given line through the origin. (d) Enlargement, using construction, with positive or negative scale factors and the centre of enlargement at the origin. (e) Shear and stretch with the x or y axis as the invariant line using negative or positive shear/stretch factors.	Use matrices to: (a) Solve systems of three variable linear equations using the method of diagonalisation (i.e., Gaussian Reduction). (b) Determine the inverse matrix by a sequence of row transformations using $[A : I_{nxn}] = [I_{nxn} : A^{-1}]$. (c) Calculate the determinant of the matrix. (d) Solve systems of linear equations using the inverse matrix up to a 4 by 4 matrix.

	Level 1	Level 2	Level 3
	We know this when the student is able to:	We know this when the student is able to:	We know this when the student is able to:
2	(a) Define simple, regular, and connected graphs, their vertices, edges, and the degree of the graph. (b) Identify complete, complementary, and isomorphic graphs. (c) Define walks, paths, trails, cycles and circuits. (d) Evaluate and determine Eulerian paths within a graph. (e) Evaluate and classify graphs, intuitively and algorithmically, as Eulerian Circuits or Hamiltonian Circuits. (f) Use Euler's, Fleury's, and Dirac's algorithms to test the nature of the paths and circuits in a graph.	(a) Determine the number of different graphs that can be drawn on $n \leq 6$ vertices. (b) Represent simple network problems using a weighted graph. (c) Solve simple optimisation problems using weighted graphs. (d) Determine the shortest path of a network or weighted graph using Dijkstra's shortest path algorithm. (e) Optimise route inspection (Chinese postman) problems using Eulerian circuits, paths and the shortest path algorithm.	(a) Solve minimum connector problems using graphs, matrices and the Kruskal and Prim algorithms. (b) Solve by finding an upper bound for simple travelling salesman problems using graphs and matrices and the nearest-neighbour algorithm. (c) Solve simple travelling salesman problems using simple algorithms researched in the literature. (d) Use matrices to represent graphs and to solve optimisation problems.