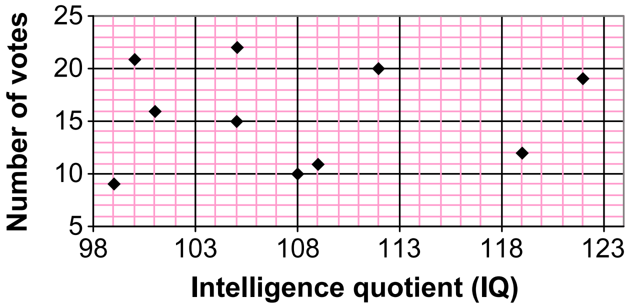
STATISTICS (62,8%): DBE NOVEMBER 2022

QUESTION 1 72%

The matric class of a certain high school had to vote for the chairperson of the RCL (representative council of learners). The scatter plot below shows the IQ (intelligence quotient) of the 10 learners who received the most votes and the number of votes that they received.

SCATTER PLOT



Before the election, the popularity of each of these ten learners was established and a popularity score (out of 100) was assigned to each. The popularity scores and the number of votes of the same 10 learners who received the most votes are shown in the table below.

Popularity score (x)	32	89	35	82	50	59	81	40	79	65
Number of votes (y)	9	22	10	21	11	15	20	12	19	16

1.1 Calculate the:

84%

1.1.1 Mean number of votes that these 10 learners received. (2)

1.1.2 Standard deviation of the number of votes that these 10 learners received. (1)

Common Errors and Misconceptions

(a) In **Q1.1.1** many candidates did not read the question correctly and calculated the mean of the popularity score instead of the mean number of votes. This may be as a result of them being used to calculating $\bar{x}$ and not $\bar{y}$.

(b) In **Q1.1.2** many candidates calculated the standard deviation for the popularity score instead of the number of votes.

MEMOS

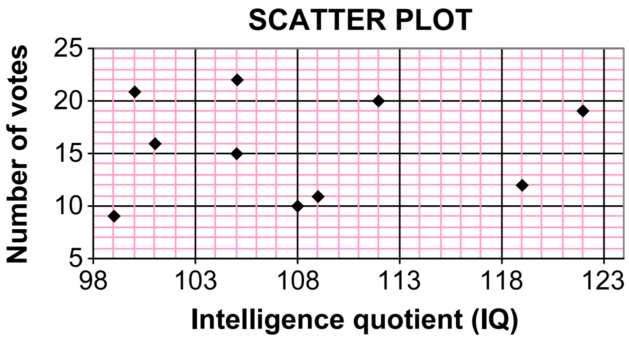
1.1.1 mean $= \frac{155}{10}$

$= \mathbf{15,5}$ ✓

1.1.2 standard deviation = **4,59** ✓

THE ANSWER SERIES Your Key to Exam Success

QUESTION 1 (cont.)



Popularity score (x)	32	89	35	82	50	59	81	40	79	65
Number of votes (y)	9	22	10	21	11	15	20	12	19	16

1.2 The learners who received fewer votes than one standard deviation below the mean were not invited for an interview. How many learners were invited? (2)
55%

1.3 Determine the equation of the least squares regression line for the data given in the table. (3)
88%

1.4 Predict the number of votes that a learner with a popularity score of 72 will receive. (2)
87%

THE ANSWER SERIES *Your Key to Exam Success*

Common Errors and Misconceptions

(c) It is evident that a number of candidates did not **understand Q1.2**. They calculated the one standard deviation interval but did not conclude whether this was for *invited or uninvited learner*. Some candidates calculated the limit for $\bar{y} + \sigma_y$ instead of $\bar{y} - \sigma_y$.

(d) When writing the equation in **Q1.3**, a few candidates interchanged the values of a and b. Some calculated the values of a and b correctly, but did not write the equation.

(e) In **Q1.4** some candidates used other values besides 72 that was given in the question. A few candidates substituted y = 72 and then solved for x.

MEMOS

1.1.1 mean = **15,5** ✓

1.1.2 standard deviation = **4,59** ✓

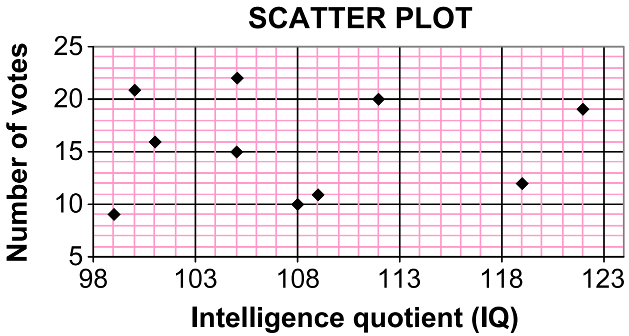
1.2 mean – std. dev. = 15,5 – 4,59 = 10,91
∴ **8 learners were invited** ✓

1.3 **A** = 1,77
B = 0,22
∴ **y = 1,77 + 0,22x** ✓

1.4 y = 1,77 + 0,22(72)
= 17,61
∴ **18 votes or 17 votes?** ✓

If you use **A** *and* **B** *from the calculator, you get 17,92*

QUESTION 1 (cont.)



Popularity score (x)	32	89	35	82	50	59	81	40	79	65
Number of votes (y)	9	22	10	21	11	15	20	12	19	16

Rephrased question.

30%

1.5.1 **Using the scatter plot** above, provide a reason why IQ not a good indicator of the number of votes that a learner could receive. (1)

1.5.2 **Using the table** above, provide a reason why the prediction in QUESTION 1.4 is reliable. (1)

[12]

Common Errors and Misconceptions

(f) Candidates were confused when answering **Q1.5.1**. They provided general responses instead of making use of the *scatterplot* to justify their answer.

(g) In **Q1.5.2** most candidates stated that the prediction would be reliable but could not justify this statement with a comment about the correlation coefficient.

MEMOS

1.5.1 The data is randomly spread. ✓

1.5.2 r = 0,98 ∴ because the correlation coefficient is very high, it is reliable ✓

QUESTION 1: Suggestions for Improvement

(a) When teaching *Statistics*, the focus should not be on the calculations only. Teachers should also pay attention to the **meaning** of the different concepts, e.g. *mean, standard deviation, correlation coefficient*, etc. The values obtained in the calculations should then become more meaningful for learners.

(b) Understanding of statistical terminology is developed by using these terms frequently in the class. **Using diagrams** when explaining the concepts of *standard deviation* and *deviation intervals* from the *mean* should help learners to understand these concepts.

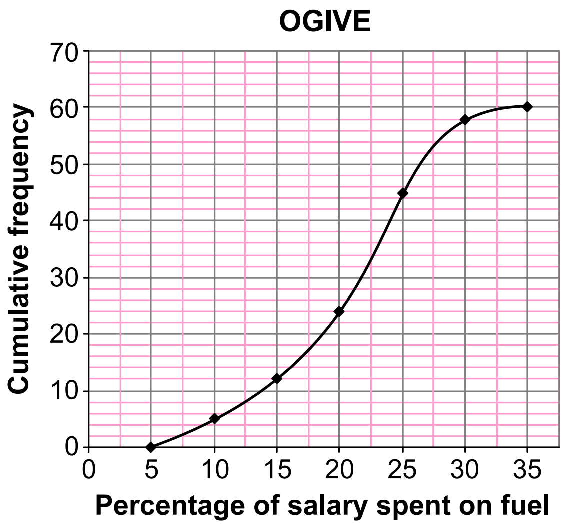
(c) **Practise calculator skills** with learners. When calculating the standard deviation, the population standard deviation (σ_x) should be used and **not** the sample standard deviation (s_x). Learners are advised to become familiar with and use the same brand of calculator in the examinations.

(d) When determining the equation of the least squares regression line, it is advisable that learners write down the values of **A** and **B** and then write down **the equation** of the regression line. In this way, they can get the CA mark for the equation.

(e) Learners should be able to use the values of their calculations to make predictions and comments about the data. Time should be devoted to **interpretation** questions.

QUESTION 2   49%

A company conducted research among all its employees on what percentage of their monthly salary was spent on fuel in a particular month. The data is represented in the ogive (cumulative frequency graph) below.



2.1 How many people are employed at this company? (1)

79%

2.2 Write down the modal class of the data. (1)

48%

2.3 How many employees spent **more than 22,5%** of their monthly salary on fuel? (2)

47%

Common Errors and Misconceptions

(a) A few candidates gave the answer to **Q2.1** as 70, the highest y-value on the cumulative frequency axis, instead of the highest value that **the graph** attained on the cumulative frequency axis.

(b) In **Q2.2** many candidates were unable to state the modal class. They provided a single value instead of **an interval**.

(c) In **Q2.3** many candidates gave the answer as 34, the cumulative frequency of employees who spent 22,5% or less on fuel. This was incorrect as the question required candidates to subtract this value from 60 in order to get the number of employees who spent more than 22,5% on fuel.

MEMOS

2.1 **60** ✓

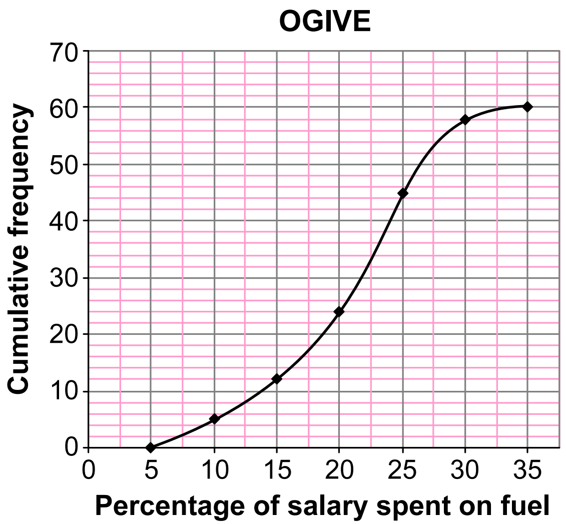
2.2 $\mathbf{20 \leq x < 25}$ ✓

2.3 60 – 34 = **26** ✓

THE ANSWER SERIES *Your Key to Exam Success*

QUESTION 2 (cont.)

A company conducted research among all its employees on what percentage of their monthly salary was spent on fuel in a particular month. The data is represented in the ogive (cumulative frequency graph) below.



2.4 An employee spent R2 400 of his salary on fuel in that particular month. Determine the monthly salary of this employee if he spends 7% of his salary on fuel. (2)

57%

Common Errors and Misconceptions

(d) In **Q2.4**, some candidates worked out the monthly salary as 7% of R2 400 while others worked out the monthly salary as R2 400 × 7. Both of these were **incorrect**.

MEMOS

2.4 7% is R2 400

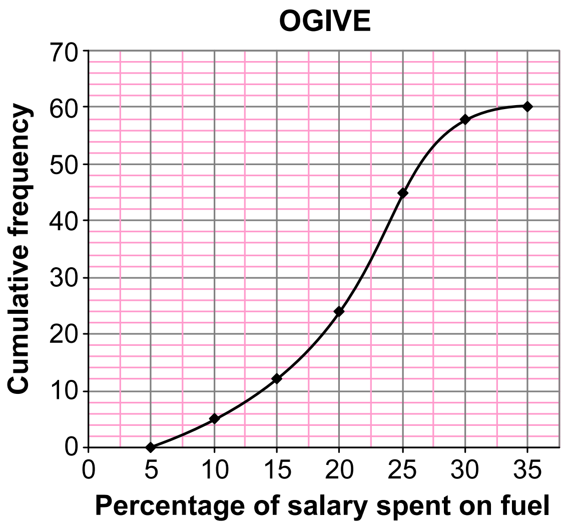
$\therefore$ 1% is $\dfrac{2\ 400}{7}$

$\therefore$ 100% is $\dfrac{2\ 400}{7} \times 100 = \text{R}34\ 285{,}71$

$\therefore$ monthly salary = **R34 285,71** ✓

QUESTION 2 (cont.)

A company conducted research among all its employees on what percentage of their monthly salary was spent on fuel in a particular month. The data is represented in the ogive (cumulative frequency graph) below.



2.5 The monthly salaries of these employees remains constant and the number of litres of fuel used in each month also remains constant. If the **fuel price increases** from R21,43 per litre to R22,79 per litre at the beginning of the next month, how will the above ogive change? (2)

27%

[8]

Common Errors and Misconceptions

(e) Many candidates failed to interpret **Q2.5** and consequently did not attempt this question. They were unable to relate the increase in fuel price to the *ogive*. Some provided an unrelated response.

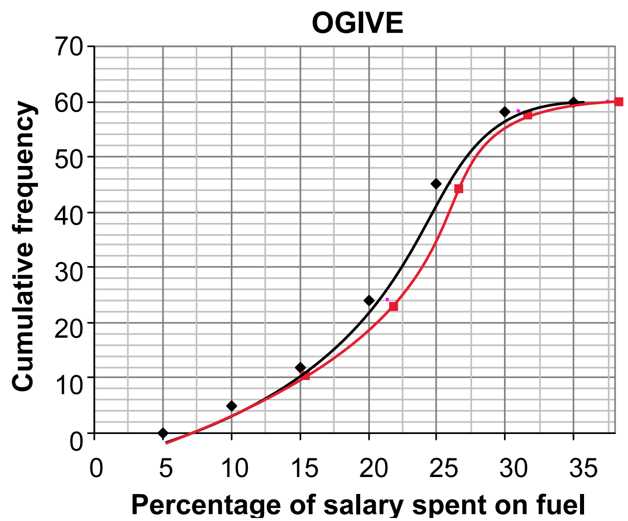
MEMOS

2.5 The **graph** will be **stretched to the right** with a cumulative frequency still at 60.

OR

The **graph will lie below the original graph** with a cumulative frequency still at 60.

Note that the lower bound might change to 10 and there might need to be a new interval from 35 to 40.



QUESTION 2: Suggestions for Improvement

(a) The **frequency table** should be viewed alongside **the ogive**. In this way, learners should be able to identify the *modal* class as that part on the *ogive* where the gradient is the steepest.

(b) Greater care needs to be taken when reading off a graph. While the values for the major gridlines are given, learners must take care when establishing the units of the minor gridlines. The minor gridlines are not always 1 unit apart.

(c) Learners should be exposed to **interpretation-type** questions as part of their classwork. This should build confidence in them when they are faced with similar questions in tests and examinations.

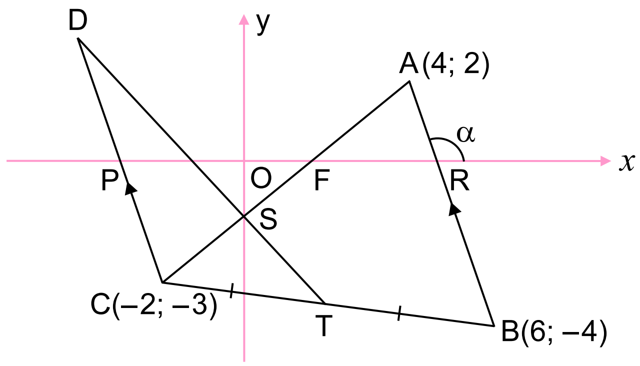
(d) **Reading for understanding** is a fundamental requirement in the *Data Handling* section. This skill needs to be developed in classroom activities.

THE ANSWER SERIES *Your Key to Exam Success*

ANALYTICAL GEOMETRY (57%): DBE NOVEMBER 2022

QUESTION 3 66%

In the diagram, A(4; 2), B(6; –4) and C(–2; –3) are vertices of $\triangle$ABC. T is the midpoint of CB. The equation of line AC is $5x - 6y = 8$. The angle of inclination of AB is α. $\triangle$DCT is drawn such that CD || BA. The lines AC and DT intersect at S, the y-intercept of AC. P, F and R are the x-intercepts of DC, AC and AB respectively.



3.1 Calculate the:

88%

3.1.1	Gradient of AB	(2)
3.1.2	Size of α	(2)
3.1.3	Coordinates of T	(2)
3.1.4	Coordinates of S	(2)

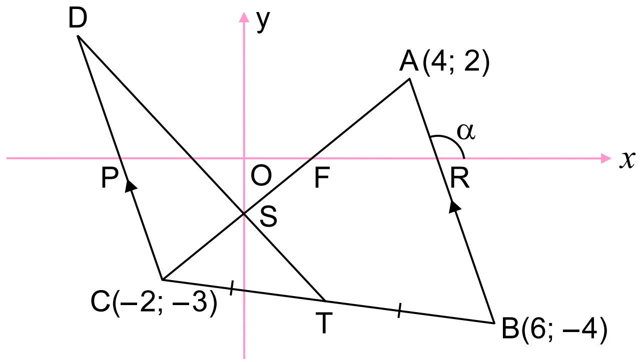
Common Errors and Misconceptions

(a) In **Q3.1.1** some candidates used the incorrect gradient formula despite it being given on the information sheet. Others substituted incorrectly, i.e. $m_{AB} = \frac{2-(-4)}{6-4}$ instead of $m_{AB} = \frac{2-(-4)}{4-6}$. Some substituted correctly into the formula but arrived at the incorrect answer of $m_{AB} = -\frac{3}{2}$.

(b) In **Q3.1.2** many candidates ignored the fact that α was an *obtuse angle* in the diagram. They correctly wrote down that $\tan\alpha = -3$ but then calculated $\alpha = 71{,}57^\circ$ instead of $\alpha = 108{,}43^\circ$.

(c) Some candidates **assumed** that S was the midpoint of AC when answering **Q3.1.4**. They failed to realise that S was a y-intercept. Others substituted $x = 0$ into the equation of AC but were unable to work correctly with the fractions to arrive at the correct answer for y.

QUESTION 3



3.1 Calculate the:

3.1.1 Gradient of AB (2)

3.1.2 Size of α (2)

3.1.3 Coordinates of T (2)

3.1.4 Coordinates of S (2)

MEMOS

3.1.1 $m_{AB} = \frac{2-(-4)}{4-6} = \frac{6}{-2} = \mathbf{-3}$ ➤

3.1.2 $\tan\alpha = -3$

$\therefore\ \alpha = 180° - 71{,}57° = \mathbf{108{,}43°}$ ➤

3.1.3 Pt T is $\left(\frac{-2+6}{2}; \frac{-3+(-4)}{2}\right)$

$\therefore\ \mathbf{T\left(2; -3\frac{1}{2}\right)}$ ➤

T midpoint of BC - by inspection!

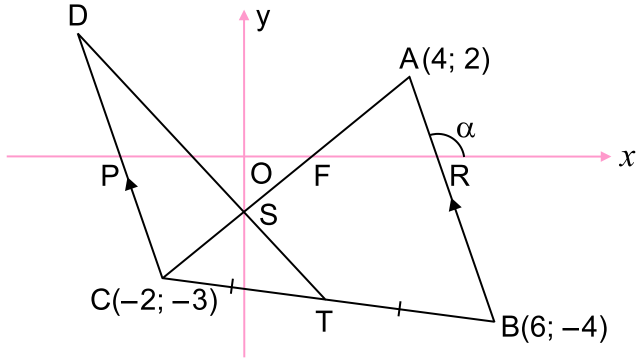
3.1.4 At S, $x = 0$

$\therefore\ 5(0) - 6y = 8$

$\therefore\ y = -\frac{4}{3}$

$\therefore\ \mathbf{S\left(0; -\frac{4}{3}\right)}$ ➤

QUESTION 3 (cont.)



3.2 Determine the equation of CD in the form $y = mx + c$. (3)

87%

3.3 Calculate the:

39% 3.3.1 Size of $D\hat{C}A$ (4)

3.3.2 Area of POSC (5)

[20]

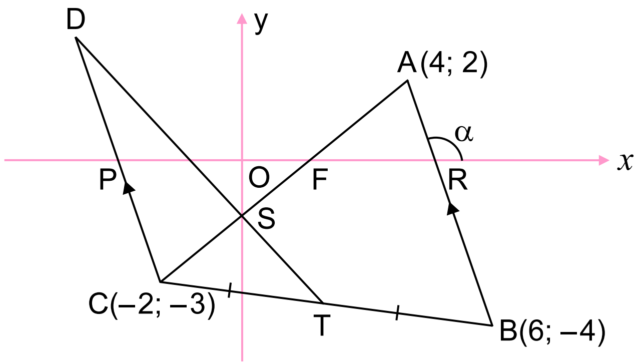
Common Errors and Misconceptions

(d) In **Q3.2** some candidates used the incorrect gradient for the line. Many calculated the gradient as if AB was perpendicular to CD instead of AB being parallel to CD. Some candidates substituted the coordinates of C instead of the coordinates of A.

(e) In answering **Q3.3.1** many candidates correctly calculated the gradient of AC as $\frac{5}{6}$. However, they were unable to link the angle of inclination of AC with $A\hat{C}D$. Some candidates **incorrectly assumed** that DC was perpendicular to AB.

(f) In **Q3.3.2** many candidates made the incorrect assumption that POSC was a *trapezium*. Some calculated random lengths of sides and incorrectly used these in the *area formula*.

QUESTION 3 (cont.)



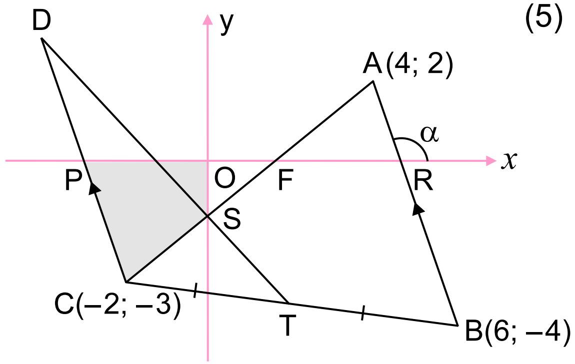
From Q3.1.2: $\alpha = 108{,}43^\circ$

3.2 Determine the equation of CD in the form $y = mx + c$. (3)

3.3 Calculate the:

3.3.1 Size of $D\hat{C}A$ (4)

3.3.2 Area of POSC (5)



[20]

MEMOS

3.2 $m_{CD} = m_{AB} = -3$ . . . *CD || AB*

$\therefore$ Subst. $m = -3$ & $C(-2; -3)$ in

$$\mathbf{y - y_1 = m(x - x_1)}$$
$$\therefore\ y + 3 = -3(x + 2)$$
$$\therefore\ \mathbf{y = -3x - 9}$$

OR: $\mathbf{y = mx + c}$
$\therefore\ -3 = (-3)(-2) + c$
$\therefore\ -9 = c$, etc.

3.3.1 $m_{CA} = \dfrac{2-(-3)}{4-(-2)} = \dfrac{5}{6}$

$\therefore\ m_{AC} = \dfrac{5}{6}$

OR: Eqn of AC: $5x - 6y = 8$
$\therefore\ -6y = -5x + 8$

$\therefore\ A\hat{F}R = 39{,}8055\ldots$

$\therefore\ F\hat{A}R = 108{,}4349\ldots - 39{,}8055\ldots$ . . . *ext ∠ of ΔAFR*
$= 68{,}63^\circ$

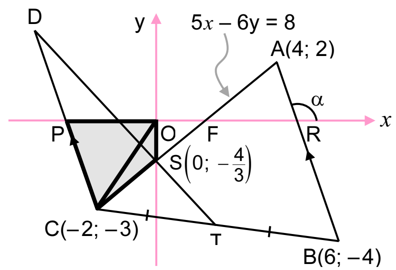
$\therefore\ D\hat{C}A = \mathbf{68{,}63^\circ}$ . . . *alt ∠s; CD || BA*

3.3.2 Area of POSC

$= \Delta OPC + \Delta OSC$

$= \dfrac{1}{2}(3 \times 3) + \dfrac{1}{2}\left(\dfrac{4}{3} \times 2\right)$

$= \mathbf{\dfrac{35}{6}\ units^2}$



QUESTION 3: Suggestions for Improvement

(a) If learners are not sure, they should consult the information sheet for the correct formula.

(b) Teachers should request learners to label the coordinates as $(x_1; y_1)$ and $(x_2; y_2)$ on the diagram. This should prevent learners from making mistakes when substituting the coordinates into a formula. The order of substitution must be consistent, especially when using the gradient formula.

(c) Teachers need to emphasise the relationship between the sign of the gradient and the size of the angle of inclination of the line.

(d) Emphasise to learners that it is **not acceptable to make any assumptions**, e.g. that a certain point is the midpoint of a line. Even if it looks as if the point is the midpoint, it may not just be assumed and used. These need to be proven first before the results can be used in an answer.

(e) Teachers should encourage learners to write down the values that they have already calculated (lengths, angles and gradients) on the diagram. This will assist learners when they answer follow-up questions.

(f) To answer questions in analytical geometry well, learners should master the properties of **quadrilaterals** and **triangles**. **Constant revision of Analytical Geometry concepts taught in Grades 10 and 11** is essential, as **much of the Grade 12 work revolves around these concepts**.

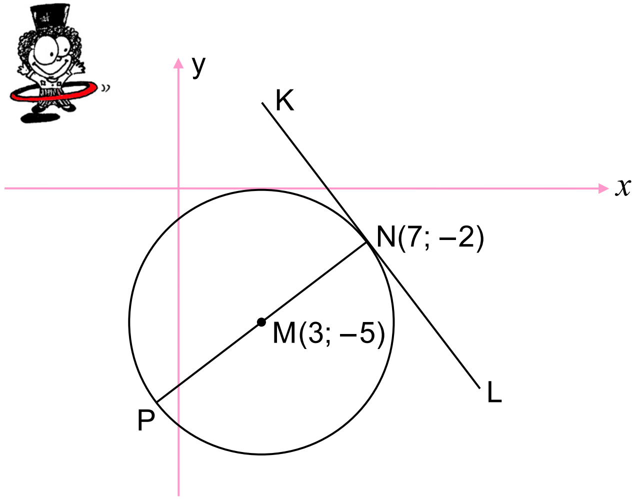
(g) Teachers should develop in learners the skill of dividing a quadrilateral into two triangles.

(h) Teachers should show learners different orientations of the base and the perpendicular height of a triangle. This should give learners more options when calculating the area of a triangle.

(i) **The different topics in Mathematics should be integrated**. Learners must be able to establish the **connection** between **Euclidean Geometry** and **Analytical Geometry**.

QUESTION 4 48%

In the diagram, M(3; –5) is the centre of the circle having PN as its diameter.
KL is a tangent to the circle at N(7; –2).



4.1 Calculate the coordinates of P. (2)

77%

4.2 Determine the equation of:

78%

4.2.1 The circle in the form $(x - a)^2 + (y - b)^2 = r^2$ (3)

4.2.2 KL in the form $y = mx + c$ (5)

Common Errors and Misconceptions

(a) In **Q4.1** some candidates substituted incorrectly into the midpoint formula. They calculated the midpoint of the given points. An endpoint and the midpoint were given. The question required candidates to calculate the other endpoint.

(b) In **Q4.2.1** some candidates incorrectly used the coordinates of N as the centre of the circle. Other candidates wrote the equation as either $(x - 3)^2 + (y + 5)^2 = 5$ or $(x + 3)^2 + (y - 5)^2 = 25$ instead of $(x - 3)^2 + (y + 5)^2 = 25$

(c) Many candidates correctly calculated the gradient of the radius in **Q4.2.2**. However, they used this as the gradient of the tangent, which was incorrect. Other candidates incorrectly used $-\frac{3}{4}$ as the gradient of the tangent.

MEMOS

4.1 **P(–1; –8)** ✓ . . . *M midpt PN;* **By Inspection**

4.2.2 $m_{MN} = \frac{-2+5}{7-3} = \frac{3}{4}$

$\therefore\ m_{KL} = -\frac{4}{3}$

Subst. $m = -\frac{4}{3}$ & N(7; –2) in

$$\mathbf{y - y_1 = m(x - x_1)}$$

$\therefore\ y + 2 = -\frac{4}{3}(x - 7)$

$\therefore\ y = -\frac{4}{3}x + \frac{28}{3} - 2$

$\therefore\ \mathbf{y = -\frac{4}{3}x + \frac{22}{3}}$ ✓

OR: $\mathbf{y = mx + c}$

$\therefore\ -2 = \left(-\frac{4}{3}\right)(7) + c$

$\therefore\ \frac{22}{3} = c$, etc.

4.2.1 $MN^2 = (7-3)^2 + (-2+5)^2$

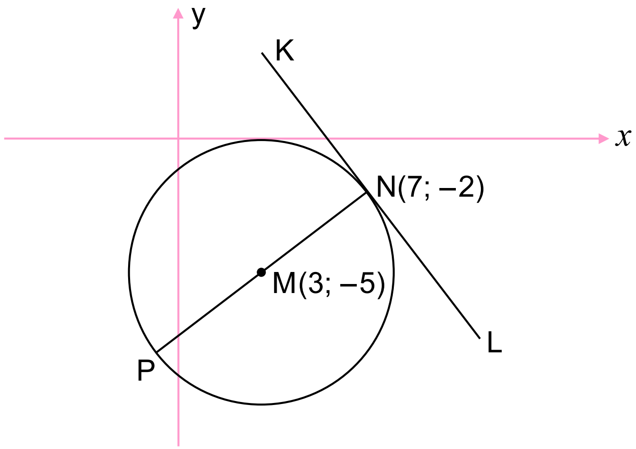
$= 16 + 9$

$= 25$

$\therefore$ MN = 5 units

$\therefore$ Eqn of ⊙M: $(x - 3)^2 + (y + 5)^2 = \mathbf{25}$ ✓

QUESTION 4 (cont.)



4.3 For which values of k will $y = -\frac{3}{4}x + k$ be a secant to the circle? (4)

20%

Common Errors and Misconceptions

(d) The vast majority of candidates were unable to answer **Q4.3** correctly. They were not familiar with the term **secant** used in the question.

MEMOS

4.3 From Q4.1: P is the point (–1; –8)

From Q4.2.2: The equation of tangent KL is $y = -\frac{4}{3}x - \frac{22}{3}$

A secant is a line that cuts the circle twice

Eqn of tangent at P:

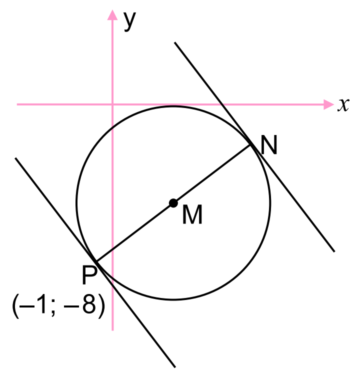
Subst. $m = -\frac{4}{3}$ & P(–1; –8)

$\therefore\ y + 8 = -\frac{4}{3}(x + 1)$

$\therefore\ y = -\frac{4}{3}x - \frac{4}{3} - 8$

$\therefore\ y = -\frac{4}{3}x - \frac{28}{3}$

$\therefore\ -\frac{28}{3} < k < \frac{22}{3}$ ◄



THE ANSWER SERIES *Your Key to Exam Success*

QUESTION 4 (cont.)

4.4 Points A (t; t) and B are not shown on the diagram.

18%

From point A, another tangent is drawn to touch the circle with centre M at B.

4.4.1 Show that the **length of the tangent** AB is given by

$\sqrt{2t^2 + 4t + 9}$ (2)

Common Errors and Misconceptions

(e) Many candidates did not attempt **Q4.4.1** because they could not place points A and B correctly on the diagram.

MEMOS

4.4.1 From Q4.2.1: The equation of the circle is $(x - 3)^2 + (y + 5)^2 = 25$

$M\hat{B}A = 90^\circ$ . . . *tan ⊥ radius*

In $\triangle ABM$:

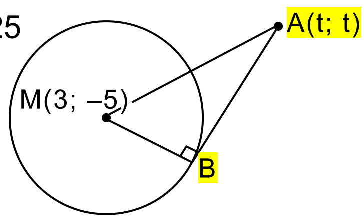
$AB^2 = AM^2 - BM^2$ . . . **(Pythagoras)**

$= (t - 3)^2 + (t + 5)^2 - 5^2$

$= t^2 - 6t + 9 + t^2 + 10t + 25 - 25$

$= 2t^2 + 4t + 9$

$\therefore$ **AB** $= \sqrt{2t^2 + 4t + 9}$ ➤



4.4.2 Determine the **minimum** length of AB. (4)

[20]

Common Errors and Misconceptions

(f) In **Q4.4.2**, while a fair number of candidates had some knowledge of how to calculate the minimum value of AB, the fact that AB was given as a square root posed a challenge to many. Some candidates incorrectly took the square root of each of the terms, i.e. $AB = \sqrt{2t^2 + 4t + 9}$ was then incorrectly simplified to $AB = \sqrt{2t^2} + \sqrt{4t} + \sqrt{9}$. They were then able to calculate the derivative of AB from this simplified expression.

Algebra!

MEMOS

4.4.2 Minimum occurs when

$4t + 4 = 0$ . . . *deriv = 0*

$\therefore 4t = -4$

$\therefore t = -1$

OR: $t = -\frac{4}{2(2)}$

$= -1$

. . . *axis of symmetry*

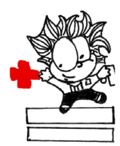
$\therefore$ Minimum length of AB

$= \sqrt{2(-1)^2 + 4(-1) + 9}$

$= \sqrt{2 - 4 + 9}$

$= \sqrt{7}$

$\approx$ **2,65 units** ➤



QUESTION 4: Suggestions for Improvement

(a) Teachers should encourage learners to **analyse the diagram** before attempting any questions. They must first write down any given information on the diagram and then make deductions from this information.

(b) Teachers need to revise the concept of perpendicular lines and gradients, particularly that the **tangent** is **perpendicular to the radius** at the point of contact.

(c) Learners should be reminded to refer to the information sheet for the relevant formula.

(d) Teachers should show learners how to visualise and make rough drawings of all extra information given in *Analytical Geometry* questions.

(e) Teachers should ensure that learners are exposed to **assessments** that **integrate** various **topics** in Mathematics. Learners must also be exposed to **higher-order questions** in class and in school-based assessment tasks.

THE ANSWER SERIES *Your Key to Exam Success*

QUESTION 5 40%

No marks given.

5.1

There was an error in the question.
We have offered the intended question

Given that $\sqrt{13}\sin x - 3 = 0$, where $x \in (0°; 90°)$.

Without using a calculator, determine the value of:

5.1.1 $\sin(360° + x)$ (2)

5.1.2 $\tan x$ (3)

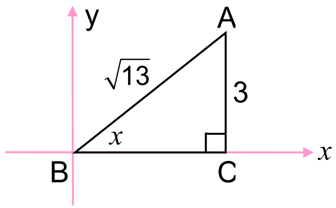
5.1.3 $\cos(180° + x)$ (2)

MEMOS

5.1 $\sqrt{13}\sin x - 3 = 0$

$\therefore \sqrt{13}\sin x = 3$

$\therefore \sin x = \frac{3}{\sqrt{13}}$



5.1.1 $\sin(360° + x) = \sin x = \mathbf{\frac{3}{\sqrt{13}}}$ ➤

5.1.2 $BC^2 = \left(\sqrt{13}\right)^2 - 3^2 = 13 - 9 = 4$

$\therefore$ BC = 2 units

$\therefore \mathbf{\tan x = \frac{3}{2}}$ ➤

5.1.3 $\cos(180° + x) = -\cos x = -\frac{2}{\sqrt{13}}$

QUESTION 5 (cont.)

5.2 Determine the value of the following expression, **without using a calculator**: $\dfrac{\cos(90^\circ + \theta)}{\sin(\theta - 180^\circ) + 3\sin(-\theta)}$ (5)

64%

5.3 Determine the general solution of the following equation:
$(\cos x + 2\sin x)(3\sin 2x - 1) = 0$ (6)

39%

Common Errors and Misconceptions

(a) It was encouraging that many candidates had some knowledge to answer **Q5.2**. Some candidates incorrectly reduced cos(90 + θ) to –cos θ or sin θ. Other candidates incorrectly simplified $\dfrac{-\sin\theta}{-4\sin\theta}$ to 4 instead of $\dfrac{1}{4}$.

A few candidates incorrectly cancelled terms,
i.e. $\dfrac{-\sin\theta}{-\sin\theta - 3\sin\theta}$ was simplified to $\dfrac{1}{1 - 3\sin\theta}$
instead of $\dfrac{-\sin\theta}{-4\sin\theta}$.

Algebra!

(b) In **Q5.3** a number of candidates multiplied out the factors on the LHS and then attempted to factorise again, but made errors when factorising. Many candidates were **unable to solve the equations**: $\cos x + 2\sin x = 0$ and $\sin 2x = \dfrac{1}{3}$.

MEMO

5.2 $\dfrac{\cos(90^\circ + \theta)}{\sin(\theta - 180^\circ) + 3\sin(-\theta)} = \dfrac{-\sin\theta}{-\sin\theta - 3\sin\theta}$

$= \dfrac{-\sin\theta}{-4\sin\theta}$

$= \dfrac{1}{4}$ ✓

5.3 $(\cos x + 2\sin x)(3\sin 2x - 1) = 0$

$\therefore \cos x + 2\sin x = 0$

$\therefore 2\sin x = -\cos x$

$(\div \cos x)\ \therefore 2\tan x = -1$

$\therefore \tan x = -\dfrac{1}{2}$

$\therefore x = 180^\circ - 26{,}57^\circ + k(180^\circ),\ k \in \mathbb{Z}$

$\mathbf{= 153{,}43^\circ + k(180^\circ)}$ ✓

or $3\sin 2x - 1 = 0$

$\therefore 3\sin 2x = 1$

$\therefore \sin 2x = \dfrac{1}{3}$

$\therefore 2x = 19{,}47\ldots^\circ + k(360^\circ),\ k \in \mathbb{Z}$

$\therefore \mathbf{x = 9{,}74^\circ + k(180^\circ)}$ ✓

or $2x = 180^\circ - 19{,}47\ldots^\circ + k(360^\circ)$

$\therefore \mathbf{x = 80{,}26^\circ + k(180^\circ)}$ ✓

QUESTION 5 (cont.)

5.4 Given the identity:

36% $\cos(x + y) . \cos(x - y) = 1 - \sin^2 x - \sin^2 y$

5.4.1 Prove the identity. (4)

5.4.2 Hence, determine the value of $1 - \sin^2 45° - \sin^2 15°$, **without using a calculator**. (3)

Common Errors and Misconceptions

(c) When answering **Q5.4.1**, many candidates left out the brackets in the expansions.

Algebra!

They wrote $\cos(x + y) . \cos(x - y)$ as $\cos x \cos y - \sin x \sin y . \cos x \cos y + \sin x \sin y$ instead of **(**$\cos x \cos y - \sin x \sin y$**)(**$\cos x \cos y + \sin x \sin y$**)**.

This created challenges when simplifying further.

Many candidates did not use the **squares' identities** when simplifying further.

Theory!

(d) Most candidates did not see **the link** between **Q5.4.1** and **Q5.4.2**. Many ignored the instruction and used a **calculator** to answer **Q5.4.2**. They were not awarded any marks for the answer.

MEMOS

5.4.1 **LHS** $= \cos(x + y) . \cos(x - y)$

$= (\cos x \cos y - \sin x \sin y)(\cos x \cos y + \sin x \sin y)$

$= \cos^2 x \cos^2 y - \sin^2 x \sin^2 y$

$= (1 - \sin^2 x)(1 - \sin^2 y) - \sin^2 x \sin^2 y$

$= 1 - \sin^2 x - \sin^2 y + \sin^2 x \sin^2 y - \sin^2 x \sin^2 y$

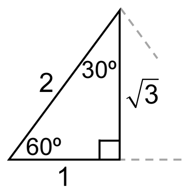
$= 1 - \sin^2 x - \sin^2 y$

= **RHS** ◀

5.4.2 Let $x = 45°$ and $y = 15°$

$\therefore 1 - \sin^2 45° - \sin^2 15° = \cos(45° + 15°) . \cos(45° - 15°)$

$= \cos 60° . \cos 30°$



$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \left(\frac{\sqrt{3}}{4}\right)$ ◀

THE ANSWER SERIES Your Key to Exam Success

QUESTION 5 (cont.)

5.5 Consider the trigonometric expression:

24%

$$16 \sin x \,.\, \cos^3 x - 8 \sin x \,.\, \cos x$$

5.5.1 Rewrite the expression as a single trigonometric ratio. (4)

5.5.2 **For which value of** x in the interval $x \in [0°; 90°]$ will $16 \sin x \,.\, \cos^3 x - 8 \sin x \,.\, \cos x$ have its minimum value? (1)

[30]

Common Errors and Misconceptions

(e) In **Q5.5.1** many candidates were unable to factorise the expression correctly into an expression involving a *sine of a double angle*. Some candidates got as far as $4 \sin 2x \cos 2x$ but **did not recognise** that this can be written as $2 \sin 4x$.

(f) The vast majority of the candidates could not answer **Q5.5.2** because they were unable to write the expression in **Q5.5.1** in terms of one trigonometric ratio. A few who managed to reduce the expression to $2 \sin 4x$ wrote -2 as the answer. They gave the minimum value of the expression **instead of the value of** x for which the minimum will occur.

MEMOS

5.5.1 $16 \sin x \,.\, \cos^3 x - 8 \sin x \,.\, \cos x$

$= 8 \sin x \cos x \,(2 \cos^2 x - 1)$

$= 4(2 \sin x \cos x)(\cos 2x)$

$= 4 \,.\, \sin 2x \,.\, \cos 2x$

$= 2(2 \sin 2x \,.\, \cos 2x)$

$= \mathbf{2 \sin 4x}$ ✓

Algebra!

5.5.2 The minimum value occurs when

$\sin 4x = -1$

$\therefore\ 4x = 270°$

$\therefore\ \mathbf{x = 67{,}5°}$ ✓

$-1 \leq \sin \theta \leq 1$ for all values of θ. Min value of $\sin \theta$ is -1.

QUESTION 5: Suggestions for Improvement

(a) Learners find it **difficult to recall** the Trigonometry taught in **Grades 10 and 11**. Revision of this work must be ongoing. It is better to revise small sections of work at a time than to give learners a comprehensive revision task.

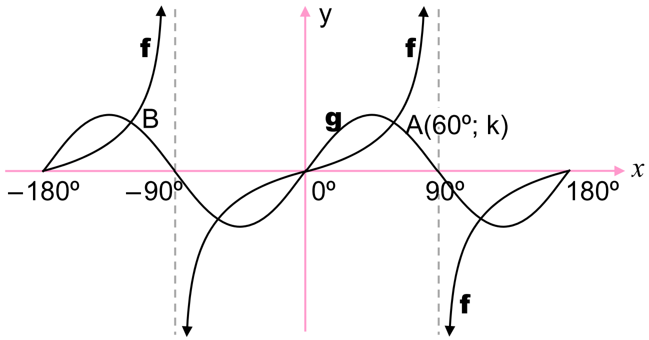
(b) Remind learners that **the same simplification skills used in Algebra** also **apply to Trigonometry**. Regular practice can remediate the poor algebraic and manipulation skills.

(c) More emphasis should be placed on **solving** simple **trigonometric equations**, particularly that a basic trigonometric equation has two solutions in the interval from 0º to 360º. The key to solving trigonometric equations lies in **understanding** in which quadrants a trigonometric function is positive or negative.

(d) Learners should be given exercises to practise solving complex trigonometric equations.

QUESTION 6 29%

In the diagram below, the graphs of $f(x) = \tan x$ and $g(x) = 2 \sin 2x$ are drawn for the interval $x \in [-180^\circ; 180^\circ]$. A(60º; k) and B are two points of intersection of f and g.



66%

6.1 Write down the **period** of g. (1)

54%

6.2 Calculate the:

6.2.1 value of k   6.2.2 coordinates of B (1)(1)

36%

6.3 Write down the range of $2g(x)$. (2)

MEMOS

6.1 **The period of g is 180º** ◄ . . . $\frac{360^o}{2}$

6.2.1 $k = \tan 60^\circ = \sqrt{3}$ ◄

(OR: $k = 2 \sin 2(60^\circ) = 2 \sin 120^\circ = 2 \sin 60^\circ = 2\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}$ ◄)

6.2.2 $\mathbf{B(-120^\circ; \sqrt{3})}$ ◄ . . . $x_B = x_A - 180^o$

6.3 The range of $g(x)$: $-2 \le y \le 2$

$\therefore$ The range of $2g(x)$: $\mathbf{-4 \le y \le 4}$ ◄

(OR: $\mathbf{y \in [-4; 4]}$ ◄)

Common Errors and Misconceptions

(a) In **Q6.1** some candidates gave the **domain** (which is an interval) instead of the **period** (which is a single value). Some candidates ignored the effect that the coefficient of x had on the period of the graph. They wrote the period as 360º, which was incorrect.

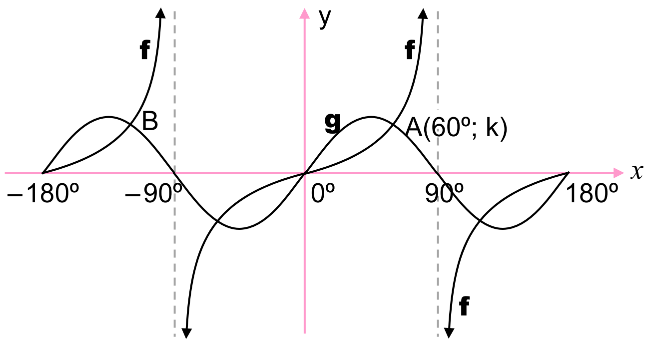
(b) In answering **Q6.2.1**, many candidates did not realise that they could have **substituted the x-coordinate** into the equation **to calculate the y-coordinate**. Some candidates rounded off the value of k from 1,73 to 2. This was not accepted as a correct answer as the amplitude of the graph was 2 and k was not the y-coordinate of a turning point.

(c) Although point B was to the left of the y-axis, some candidates wrote 120º as the answer to **Q6.2.2**. Others gave the answer as –60º. Both these answers were incorrect.

(d) When answering **Q6.3**, many candidates gave the answer as $-2 \le y \le 2$, the range of $g(x)$, instead of giving the range of **2**$g(x)$. Other candidates incorrectly wrote the range as $4 \le y \le -4$ or $y \in [4; -4]$.

Intervals!

QUESTION 6 (cont.)



In the diagram, the graphs of $f(x) = \tan x$ and $g(x) = 2 \sin 2x$ are drawn for the interval $x \in [-180^\circ; 180^\circ]$.

(60º; k) and B are two points of intersection of f and g.

6.4 For which values of x will $g(x + 5^\circ) - f(x + 5^\circ) \leq 0$ in the interval $x \in [-90^\circ; 0^\circ]$? (2)

11%

6.5 Determine the values of p for which $\sin x . \cos x = p$ will have exactly two real roots in the interval $x \in [-180^\circ; 180^\circ]$. (3)

7%

[10]

Common Errors and Misconceptions

(e) Most candidates were unable to **link** **Q6.4** to the given graphs. Some candidates gave the answer as $-60^\circ \leq x \leq 0^\circ$ instead of $-65^\circ \leq x \leq -5^\circ$. These candidates knew where g was below f but did not take into account the **translation of 5º** to the **left**.

(f) Most candidates had little idea how the graphs could be used to answer **Q6.5**. They could not make the **association** between **sin x cos x** and **sin 2x**.

MEMOS

6.4 From 6.2.2: $B\left(-120^\circ; \sqrt{3}\right)$

$g(x + 5^\circ) - f(x + 5^\circ) \leq 0 \;\Rightarrow\; g(x + 5^\circ) \leq f(x + 5^\circ)$

> $g(x) \leq f(x)$ for $-60^\circ \leq x \leq 0^\circ$;
> then both graphs shift 5º to the left

$\therefore\ \mathbf{-65^\circ \leq x \leq -5^\circ}$ ✓

OR: $\mathbf{x \in [-65^\circ; -5^\circ]}$ ✓

6.5

$$\sin x \cos x = p$$
$$\therefore\ 2 \sin x \cos x = 2p$$
$$\therefore\ \sin 2x = 2p$$
$$\therefore\ 2 \sin 2x = 4p$$
$$\text{i.e. } g(x) = 4p$$

$\therefore\ 4p = 2$ or -2 . . . *where the line $y = 4p$ touches g twice*

$\therefore\ \mathbf{p = \frac{1}{2}}$ **or** $\mathbf{-\frac{1}{2}}$ ✓

QUESTION 6: Suggestions for Improvement

(a) When teaching the drawing of trigonometric graphs, it is strongly recommended that the approach should be to ensure that learners know the basic graphs – the *mother graphs* – very well. Thereafter, teachers should explain how to draw the required graph by applying knowledge of transformations.

(b) Although these concepts are discussed in Grade 10, it is necessary for learners to be reminded constantly of the meaning of concepts like *period*, *domain*, *amplitude* and *range*.

(c) Learners should be told that the period of a trigonometric function is the length of a function's cycle. Since this value is a length, it is a single number and not an interval of values.

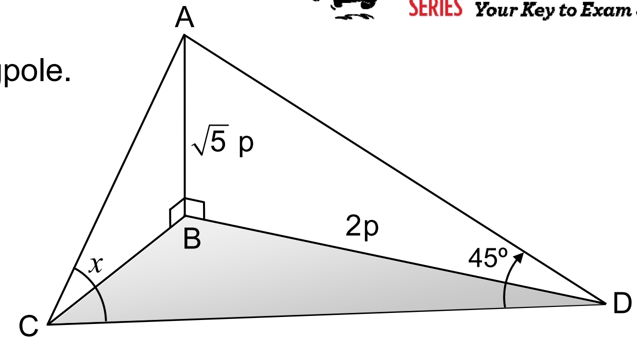
(d) When teaching trigonometric functions, teachers should emphasise the **meaning** and effect of each of the parameters: **a, k, p and q** in the equation $y = a\sin(kx + p) + q$, for example.

(e) Teachers should make learners aware of the **cyclic nature** of trigonometric graphs. This is useful in determining the coordinates of other points on the graph.

THE ANSWER SERIES *Your Key to Exam Success*

QUESTION 7 35%

AB is a vertical flagpole that is $\sqrt{5}$ p metres long. AC and AD are two cables anchoring the flagpole.
B, C and D are in the same horizontal plane. BD = 2p metres, $A\hat{C}D = x$ and $A\hat{D}C = 45°$.



59% 7.1 Determine the length of AD in terms of p. (2)

24% 7.2 Show that the length of CD = $\dfrac{3p(\sin x + \cos x)}{\sqrt{2}\sin x}$ (5)

Common Errors and Misconceptions

(a) In **Q7.1** many candidates were able to substitute correctly into the *Theorem of Pythagoras*, i.e. $AD^2 = \left(\sqrt{5}p\right)^2 + (2p)^2$. However, they made errors when calculating the length of AD. They either gave the length of AD as $\sqrt{29}$p or 7p. Some candidates attempted to calculate the length of AD by making use of trigonometric ratios. However, these candidates failed to first calculate the size of $B\hat{A}D$.

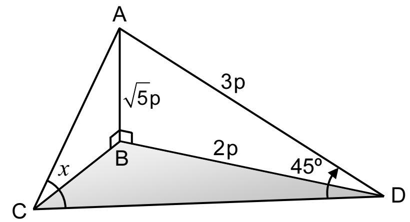
(b) In their attempt to answer **Q7.2**, some candidates used the *sine formula* with side AC and $A\hat{D}C$. However, this did not allow them to **arrive at the required expression for CD**. Some candidates were unable to calculate the size of $\mathbf{C\hat{A}D}$ correctly. They arrived at $C\hat{A}D = 180° - (45° + x) = 135°$ or $\sin C\hat{A}D = \sin[180° - (45° + x)] = \sin(45° - x)$. Both of which were incorrect. A fair number of candidates were unsure whether they should use the *sine formula or the cosine formula* to answer this question.

MEMOS

7.1 In right $\angle^d$ ΔABD:

$AD^2 = \left(\sqrt{5}p\right)^2 + (2p)^2$ . . . *Pythagoras*
$= 5p^2 + 4p^2$
$= 9p^2$

$\therefore$ **AD = 3p metres** ➤



7.2 In non-right $\angle^d$ ΔACD: $C\hat{A}D = 180° - (x + 45°)$ . . . *$\angle$ sum of ΔACD*
$= 135° - x$

$\dfrac{CD}{\sin C\hat{A}D} = \dfrac{3p}{\sin x}$

$\therefore$ CD $= \dfrac{\mathbf{3p \sin(135° - x)}}{\sin x}$

$= \dfrac{3p\left[\sin 135° \cos x - \cos 135° \sin x\right]}{\sin x}$

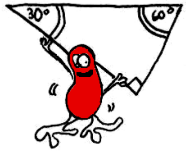
$= \dfrac{3p\left[\sin 45° \cos x - (-\cos 45°)\sin x\right]}{\sin x}$

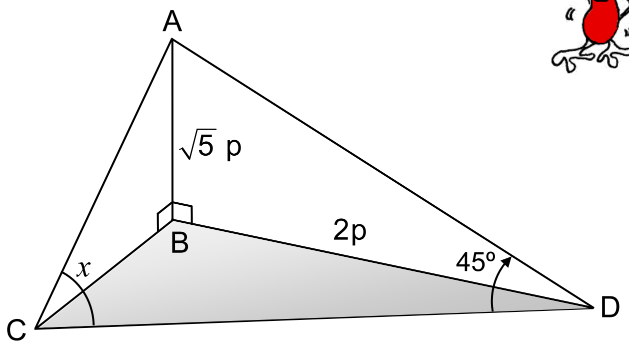
$= \dfrac{3p\left[\left(\frac{1}{\sqrt{2}}\right)\cos x - \left(-\frac{1}{\sqrt{2}}\right)\sin x\right]}{\sin x}$

$= \dfrac{3p \cdot \frac{1}{\sqrt{2}}(\cos x + \sin x)}{\sin x} \times \dfrac{\sqrt{2}}{\sqrt{2}}$

$= \dfrac{\mathbf{3p(\sin x + \cos x)}}{\mathbf{\sqrt{2}\sin x}}$ ➤

QUESTION 7 (cont.)





7.3 If it is further given that p = 10 and $x = 110°$, calculate

37% the area of ΔADC. (3)

[10]

Common Errors and Misconceptions

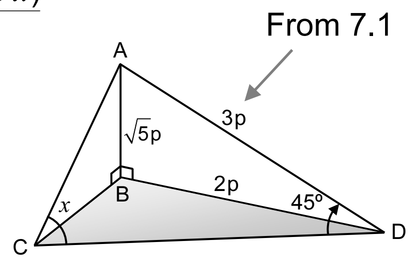
(c) Some candidates incorrectly attempted to calculate the area of ΔADC by using the formula:

area of triangle = $\frac{1}{2}$base $\times$ height.

They were under the impression that they were working in a right-angled triangle.

MEMOS

7.3 From 7.2: $CD = \frac{3p(\sin x + \cos x)}{\sqrt{2}\sin x}$



Area of ΔADC

$= \frac{1}{2} \cdot CD \cdot 3p \sin 45°$

$= \frac{1}{2} \cdot \frac{3(10)(\sin 110° + \cos 110°)}{\sqrt{2}\sin 110°} \cdot 3(10) \cdot \frac{1}{\sqrt{2}}$

$= \frac{900(\sin 110° + \cos 110°)}{4 \sin 110°}$

$\approx$ **143,11 m²** ◄

THE ANSWER SERIES *Your Key to Exam Success*

QUESTION 7: Suggestions for Improvement

(a) A careful analysis of the information provided will give learners some idea of the concepts required in solving a triangle.

(b) Teachers need to develop **strategies** to be used when solving **right-angled triangles** and **triangles that are not right-angled**. Teach learners the conditions that determine which rule should be used to solve the question.

(c) It might be a good idea to give learners an exercise in which they identify which rule is to be used to solve the question. The learners must also give a reason why they think that the rule that they have selected applies to the question.

(d) Learners should be encouraged to highlight the different triangles using different colours.

(e) Initially, expose learners to numeric questions when solving 3D problems. This makes it easier for them to develop strategies on how to solve such questions. Once learners have gained confidence with numeric type questions, they should be exposed to non-numeric and higher-order questions.

EUCLIDEAN GEOMETRY (36,7%): DBE NOVEMBER 2022

QUESTION 8 55%

8.1 In the diagram, O is the centre of the circle. MNPR is a cyclic quadrilateral and SN is a diameter of the circle.
71%

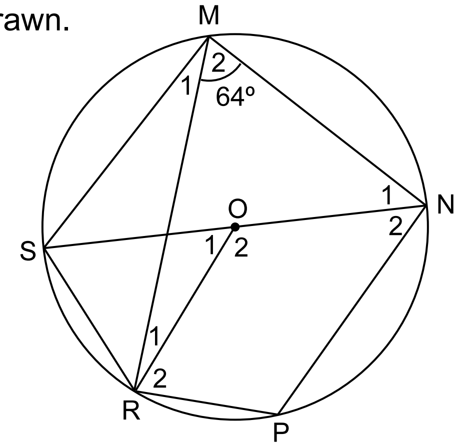
Chord MS and radius OR are drawn.
$\hat{M}_2 = 64°$.

Determine, giving reasons, the size of the following angles:

8.1.1 $\hat{P}$ (2)

8.1.2 $\hat{M}_1$ (2)

8.1.3 $\hat{O}_1$ (2)



MEMOS

8.1.1 $\hat{P} = 180° - 64°$ . . . **opp ∠s of cyclic quad**

$= \mathbf{116°}$ ✓

8.1.2 $S\hat{M}N = 90°$ . . . **∠ in semi-⊙**

$\therefore \hat{M}_1 = 90° - 64°$

$= \mathbf{26°}$ ✓

8.1.3 $\hat{O}_1 = 2\hat{M}_1$ . . . **∠ at centre = 2 × ∠ at circumference**

$= \mathbf{52°}$ ✓

Common Errors and Misconceptions

(a) In **Q8.1.1** some candidates incorrectly stated that $\hat{P} = \hat{M}_2$ with the reason that the opposite angles of a cyclic quadrilateral are equal. Other candidates did not state adequate information in **the reason**. Opposite angles and opposite angles are supplementary were not accepted as correct.

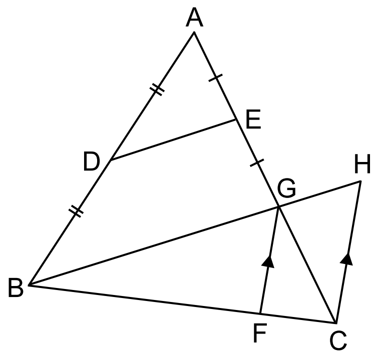
(b) When answering **Q8.1.2**, some candidates incorrectly stated that $\hat{M}_1 = \hat{O}_1$ with the reason that they were angles in the same segment. Some candidates gave the reason right-angled triangle. This was not accepted as correct.

(c) In **Q8.1.3** some candidates failed to see the relationship between $\hat{M}_1$ and $\hat{O}_1$ as being angle at centre is equal to twice the angle at the circumference.

THE ANSWER SERIES *Your Key to Exam Success*

QUESTION 8 (cont.)

8.2 In the diagram, ΔABG is drawn.
42% D and E are midpoints of AB and AG respectively.
AG and BG are produced to C and H respectively.
F is a point on BC such that FG || CH.



8.2.1 Give a reason why DE || BH. (1)

8.2.2 If it is further given that $\frac{FC}{BF} = \frac{1}{4}$, DE = $3x - 1$ and GH = $x + 1$, calculate, giving reasons, the value of x. (6)

[13]

Common Errors and Misconceptions

(d) In **Q8.2.1** many candidates were unable to give the correct reason for the lines being parallel. They confused the theorem with its converse. Answers given were the *proportionality theorem* instead of the ***converse** proportionality theorem*; or the ***converse** midpoint theorem* instead of the *midpoint theorem*.

MEMO

8.2.1 **CONVERSE** Midpoint Theorem

For those not familiar with **the Midpoint Theorem**, one could use the converse of the Proportion Theorem.

Common Errors and Misconceptions

(e) In **Q8.2.2** many candidates **assumed** that BG = DE even though this was not the case in the diagram given. Some candidates made **algebraic errors**, e.g. $4(x + 1) = 4x + 1$.

Algebra!

Other candidates incorrectly wrote down $\frac{BF}{FC} = \frac{BG}{DE}$ instead of $\frac{BF}{FC} = \frac{BG}{GH}$. Some candidates did not **mention the parallel lines in the reason**.

MEMO

8.2.2 In ΔABG:

$BG = 2(3x - 1)$ . . . *midpoint theorem*

$\therefore BG = 6x - 2$

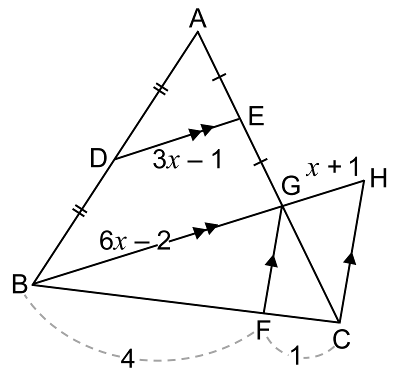
& In ΔBCH: $\frac{GH}{BG} = \frac{1}{4}$ . . . *prop thm;* **FG || CH**

$\therefore \frac{x + 1}{6x - 2} = \frac{1}{4}$

$\therefore 6x - 2 = 4x + 4$

$\therefore 2x = 6$

$\therefore \boldsymbol{x = 3}$



THE ANSWER SERIES *Your Key to Exam Success*

QUESTION 8: Suggestions for Improvement

(a) The key to answering Euclidean Geometry successfully is to be fully conversant with the **terminology** in this section. To this end, teachers should explain the meaning of *chord*, *tangent*, *cyclic quadrilateral*, etc. so that learners will be able to use them correctly.

(b) Teachers must cover the **basic work** thoroughly. An explanation of the theorem should be accompanied by showing the relationship in a diagram.

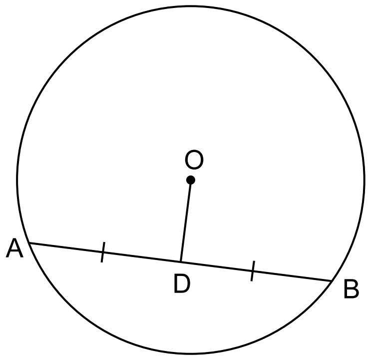
(c) Teachers are encouraged to use the **'Acceptable Reasons'** in the *Examination Guidelines* when teaching. This should start from as early as Grade 8.

(d) Learners should be encouraged to scrutinise the given information and the diagram for **clues** about **which theorems** could be used when answering the question.

(e) Learners should be taught that **all statements must be accompanied by reasons**. It is essential that the **parallel lines** be mentioned when stating that corresponding angles are equal, alternate angles are equal, the sum of the co-interior angles is 180° or when stating the proportional intercept theorem

QUESTION 9 37%

9.1 In the diagram, O is the centre of a circle. OD bisects chord AB.
56%



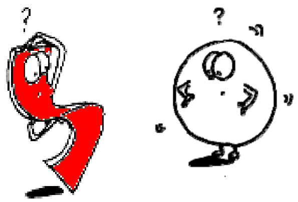
Prove the theorem that states that the line from the centre of a circle that bisects a chord is perpendicular to the chord, i.e. OD $\perp$ AB. (5)

Common Errors and Misconceptions

(a) In **Q9.1** many candidates omitted the reason that angles on a straight line were supplementary. Some candidates used ***similarity*** instead of ***congruency*** to prove this theorem. Many candidates **stated that OD was perpendicular to AB** in the proof. This resulted in a **breakdown** as these candidates were unclear about what information was given and what they had to prove.

MEMOS

9.1 Theorem proof

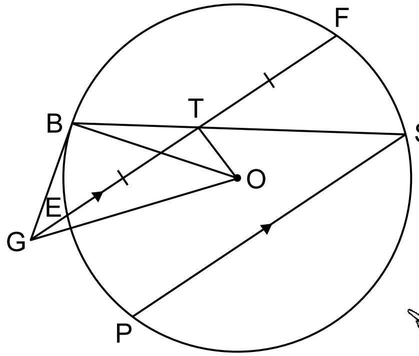


QUESTION 9 (cont.)

26%

9.2 In the diagram, E, B, F, S and P are points on the circle centred at O. GB is a tangent to the circle at B. FE is produced to meet the tangent at G. OT is drawn such that T is the midpoint of EF. GO and BO are drawn. BS is drawn through T. PS || GF.

Sketch redrawn.



Prove, giving reasons, that:

9.2.1 OTBG is a cyclic quadrilateral (5)

9.2.2 $G\hat{O}B = \hat{S}$ (4)

[14]

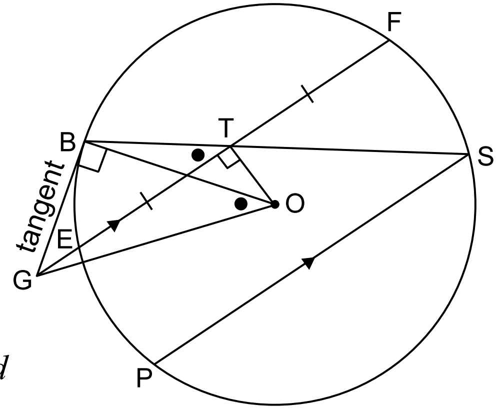
Common Errors and Misconceptions

(b) In **Q9.2.1** many candidates could not identify that the radius was drawn to the midpoint of the chord. Those who could prove that OTBG was a *cyclic quadrilateral* gave the incorrect reason that angles in the same segment instead of **converse** angles in the same segment.

(c) When answering **Q9.2.2**, some candidates stated that BG and OG were equal because they were tangents from a common point. This was incorrect because OG was not a tangent to the circle. A big challenge in this question was the poor labelling of angles. Candidates would refer to $\hat{T}$ while there are a number of angles around point T.

MEMOS

9.1 Theorem proof



9.2.1 $G\hat{B}O = 90°$ . . . *tan ⊥ radius*

$O\hat{T}G = 90°$ . . . *line from centre to midpt of chord*

∴ $G\hat{B}O = O\hat{T}G$

∴ **OTBG is a cyclic quadrilateral** ✓ . . . **converse** *∠s in the same segment*

9.2.2 $G\hat{O}B = G\hat{T}B$ . . . *∠s in the same segment*

$= \hat{S}$ ✓ . . . *corresp ∠s; PS || GF*

QUESTION 9: Suggestions for Improvement

(a) Learners should be taught that a **construction** is required in order to prove a theorem. If the construction is not shown, then the proof is regarded as **a breakdown** and they get no marks. Teachers should **reinforce theory** in short tests and assignments.

(b) Teachers should focus on developing learners' skills to analyse the question and the diagram for **clues** on **which theorems** are required to answer the questions correctly.

(c) Learners should be forced to use **acceptable reasons** in Euclidean Geometry. Teachers should explain the difference between a ***theorem* and its *converse***. They should also explain the **conditions for which theorems are applicable** and when the converse will apply.

(d) Learners need to be exposed to questions in Euclidean Geometry that include the theorems and the converses. When proving that a quadrilateral is cyclic, no circle terminology may be used when referring to the quadrilateral.

(e) Learners should be discouraged from writing correct statements that are not related to the solution. No marks are awarded for statements that do not lead to solving the problem.

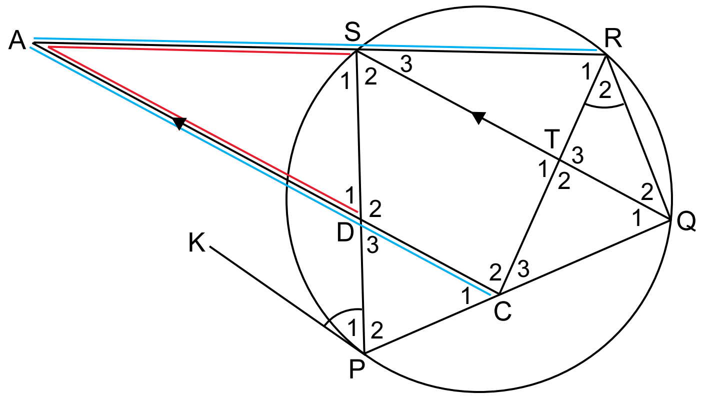
(f) Learners need to be told that success in answering Euclidean Geometry comes from regular practice, starting off with the easy and progressing to the difficult.

(g) Teachers should take some time to discuss the naming of angles. The acceptable methods are $\hat{T}$ or $\hat{T}_1$ or $O\hat{T}S$. Teachers should also clarify when it is acceptable to refer to an angle as $\hat{T}$ and when to refer to it as $\hat{T}_1$.

QUESTION 10 18%

In the diagram, PQRS is a cyclic quadrilateral. KP is a tangent to the circle at P. C and D are points on chords PQ and PS respectively and CD produced meets RS produced at A.

CA || QS. RC is drawn. $\hat{P}_1 = \hat{R}_2$.



Prove, giving reasons, that:

19%	10.1	$\hat{S}_1 = \hat{T}_2$	(4)
25%	10.2	$\frac{AD}{AR} = \frac{AS}{AC}$	(5)
7%	10.3	$AC \times SD = AR \times TC$	(4)
			[13]

TOTAL: 150

THE ANSWER SERIES *Your Key to Exam Success*

Common Errors and Misconceptions

(a) A fair number of candidates made **incorrect assumptions** when answering **Q10.1**. Among them were that: $\hat{S}_1 = 90°$ and $\hat{C}_2 = 90°$, $\hat{P}_1 = \hat{R}_1 + \hat{R}_2$ with the reason *exterior angle of cyclic quadrilateral*, $\hat{P}_1 = \hat{C}_1$ with the reason *tan-chord theorem* and $\hat{S}_2 = \hat{R}_2$ with the reason *angles in the same segment*.

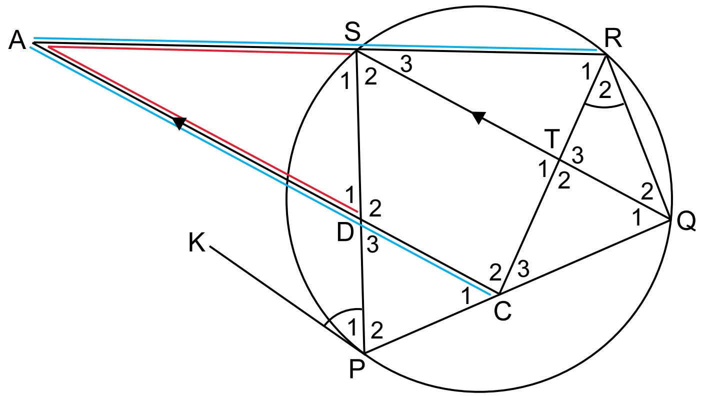
(b) In **Q10.2** some candidates attempted to prove the ratios equal by using the ***proportionality theorem*** instead of **similar triangles**. A common error made by candidates attempting to prove that $\triangle ASD$ was similar to $\triangle ACR$ was to merely state that $\hat{S}_1 = \hat{C}_2$ without any proof or reasons. This was seen as a **breakdown** in the answer.

(c) **Q10.3** required candidates to obtain a proportion from the **similar triangles** in **Q10.2**, using the ***proportional intercept theorem*** in $\triangle RAC$ to establish a second proportion and then to combine the two. Many candidates failed to establish one or the other proportion and therefore could not make the conclusion.

QUESTION 10

In the diagram, PQRS is a cyclic quadrilateral. KP is a tangent to the circle at P. C and D are points on chords PQ and PS respectively and CD produced meets RS produced at A.

CA || QS. RC is drawn. $\hat{P}_1 = \hat{R}_2$.



Prove, giving reasons, that:

10.1 $\hat{S}_1 = \hat{T}_2$ (4)

10.2 $\dfrac{AD}{AR} = \dfrac{AS}{AC}$ (5)

10.3 $AC \times SD = AR \times TC$ (4)

[13]

TOTAL: 150

MEMOS

10.1 Let $\hat{P}_1 = \hat{R}_2 = x$

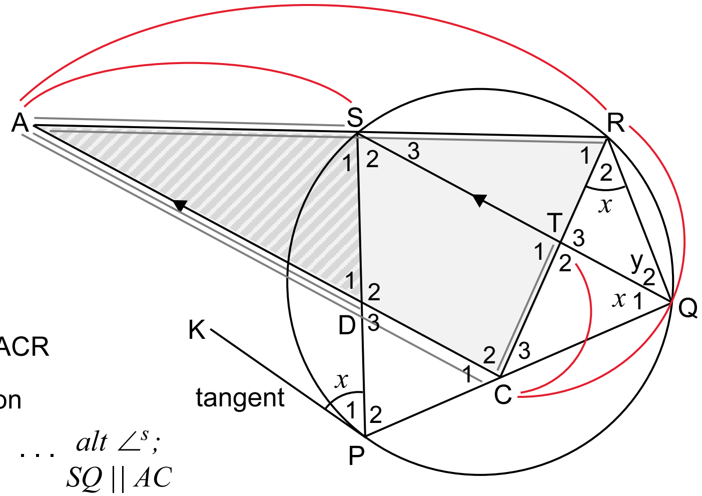
$\hat{Q}_1 = x$ . . . *tan chord theorem*

Let $\hat{Q}_2 = y$

$\therefore \hat{T}_2 = x + y$ . . . *ext ∠ of ΔRTQ*

$\therefore \hat{S}_1 = x + y$ . . . *ext ∠ of cyclic quad*

$\therefore \hat{S}_1 = \hat{T}_2$



10.2 In Δ^s ASD and ACR

(1) $\hat{A}$ is common

(2) $\hat{C}_2 = x + y$ . . . *alt ∠s; SQ || AC*

$\therefore \hat{S}_1 = \hat{C}_2$

$\therefore$ ΔASD ||| ΔACR . . . ∠∠∠

$\therefore \dfrac{AD}{AR} = \dfrac{AS}{AC}$ ◀ . . . $= \dfrac{SD}{CR}$

10.3 $\dfrac{AS}{AC} = \dfrac{SD}{CR}$ . . . *similar* Δ^s *in 10.2*

$\therefore AS.CR = AC.SD$ . . . ❶

& In ΔACR: $\dfrac{AS}{AR} = \dfrac{CT}{CR}$ . . . *prop thm; CA || TS*

$\therefore AS.CR = AR.CT$ . . . ❷

From ❶ & ❷:

$\therefore$ **AC . SD = AR . TC** ◀

QUESTION 10: Suggestions for Improvement

(a) **More time needs to be spent on the teaching of Euclidean Geometry in all grades**. More practice in Grade 11 and 12 Euclidean Geometry will help learners to understand theorems and diagram analysis. They should read the given information carefully without making any **assumptions**. This work covered in class must include different activities and all levels of the taxonomy.

(b) Teachers should require learners to make **use** of the **diagrams** in the Answer Book to **indicate angles and sides** that are equal and **record information** that has been calculated.

(c) Learners need to be made aware that writing correct, but **irrelevant statements**, **will not earn them any marks** in an examination.

EUKLIDIESE MEETKUNDE

INHOUDSRAAMWERK

- LYNE
- DRIEHOEKE
- VIERHOEKE

(Gr 8 → 10)

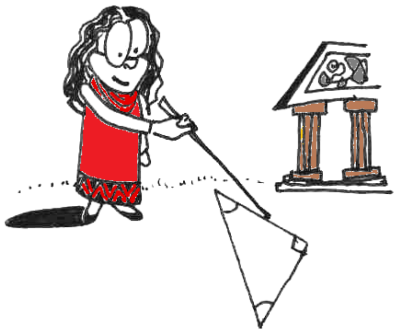
- SIRKELS (Gr 11)

Gr 12?

Gr 12: Stelling van Pythagoras (Gr 8)

Gelykvormige Δ^e (Gr 9)

Middelpuntstelling (Gr 10)



& Die Eweredigheidstelling

Verhouding Eweredigheid Oppervlakte

SIRKELMEETKUNDE

Die Taal (Woordeskat)

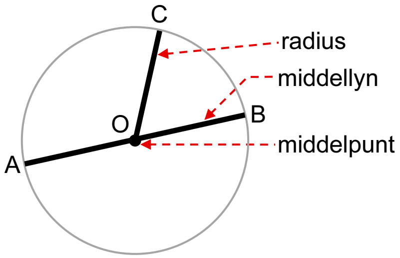
GROEP I : Sirkels met middelpunt

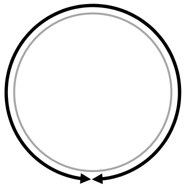
GROEP II : Sirkels met geen middelpunt

Middelpunt

Middellyn

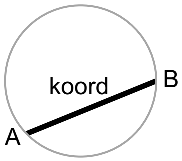
Radius



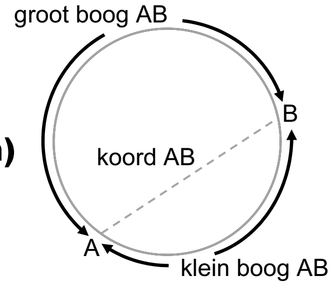


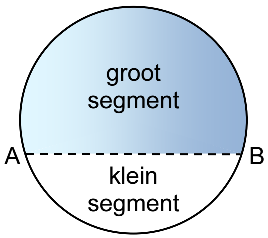
Omtrek

Koorde



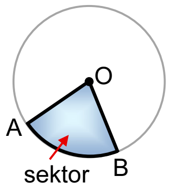
Boë (groot & klein)



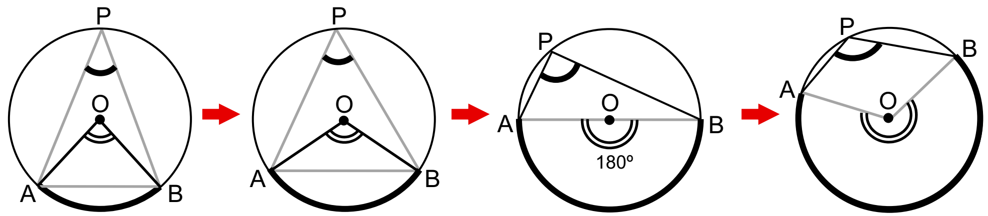


Segmente (groot & klein)

Sektore



'ONDERSPAN' . . . Verstaan die woord!



Figuur 1 Figuur 2 Figuur 3 Figuur 4

Middelpunts- en Omtrekshoeke

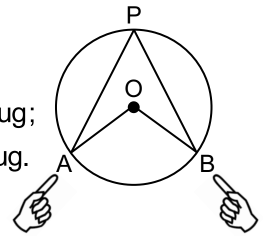
In al die figure **onderspan** boog AB $(\overset{\frown}{AB})$, of koord AB:

- 'n **middelpuntshoek**, **A$\hat{\text{O}}$B**, by die **middelpunt** van die sirkel, en
- 'n **omtrekshoek**, **A$\hat{\text{P}}$B**, op die **omtrek** van die sirkel.

Neem in ag dat **onderspan, ondersteun** beteken.

Om te verseker dat jy die betekenis van die woord 'onderspan' verstaan:

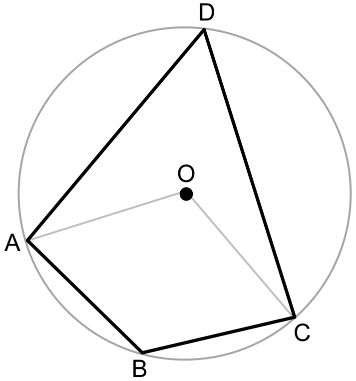
- Neem **elk** van die figure:
  - Plaas jou wysvingers op A & B;
  - beweeg langs die radiusse om by O te ontmoet en terug;
  - beweeg dan om by P op die omtrek te ontmoet en terug.



- Draai jou boek onderstebo en skuins.
  Jy moet die verskillende aansigte van hierdie situasies kan herken.
- Let op of die hoeke skerp, stomp, reghoekig, gestrek of inspringend (refleks) is.
- Teken Figure 1 tot 4 (hierbo) oor, laat die koord AB heeltemal uit en **let op die boog** wat die middelpunts- en omtrekshoeke in elke geval, onderspan.

GROEP III : Koordevierhoeke

'n **Koordevierhoek** is 'n vierhoek met al vier hoekpunte op die omtrek van 'n sirkel.



Punte A, B, C en D is **konsiklies**, d.w.s. hulle lê op dieselfde sirkel.

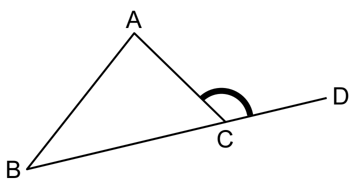
Let Wel: Vierhoek AOCB is **nie** 'n koordevierhoek nie, want punt O is **nie** op die omtrek **nie**! (A, **O**, C en B is **nie** konsiklies **nie**)

Ons benoem *vierhoeke* deur kloksgewys, of antikloksgewys te gaan en opeenvolgende hoekpunte te gebruik, d.w.s. ABCD of ADCB, **nie** ADBC **nie**.

Buitehoeke van veelhoeke

Die **buitehoek** van enige veelhoek is 'n hoek wat gevorm word tussen een sy van die veelhoek en 'n ander sy wat *verleng* is.

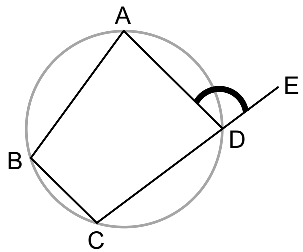
bv. 'n driehoek



$A\hat{C}D$ is 'n **buite**∠ van △ABC.

[LW: BCD is 'n reguitlyn!]

bv. 'n vierhoek/koordevierhoek



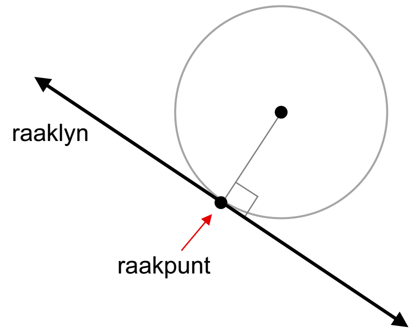
$A\hat{D}E$ is 'n **buite**∠ van kvh. ABCD.

[LW: CDE is 'n reguitlyn!]

GROEP IV : Raaklyne

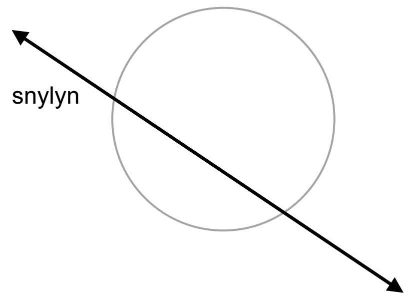
Spesiale lyne

- 'n **Raaklyn** is 'n lyn wat 'n sirkel by slegs een punt *raak*.



LW:
Daar word aangeneem dat die raaklyn aan 'n sirkel loodreg is op die radius/middellyn van die sirkel by die raakpunt.

- 'n **Snylyn/sekans** is 'n lyn wat 'n sirkel (in twee punte) *sny*.



Die 4 Groepe Sirkelmeetkunde Stellings

Ons verdeel die Sirkelmeetkunde stellings in 4 groepe, wat dit makliker maak om al die stellings sistematies te herroep. (Sien die opsomming op die volgende bladsy.)

GROEPERING VAN SIRKELMEETKUNDE STELLINGS

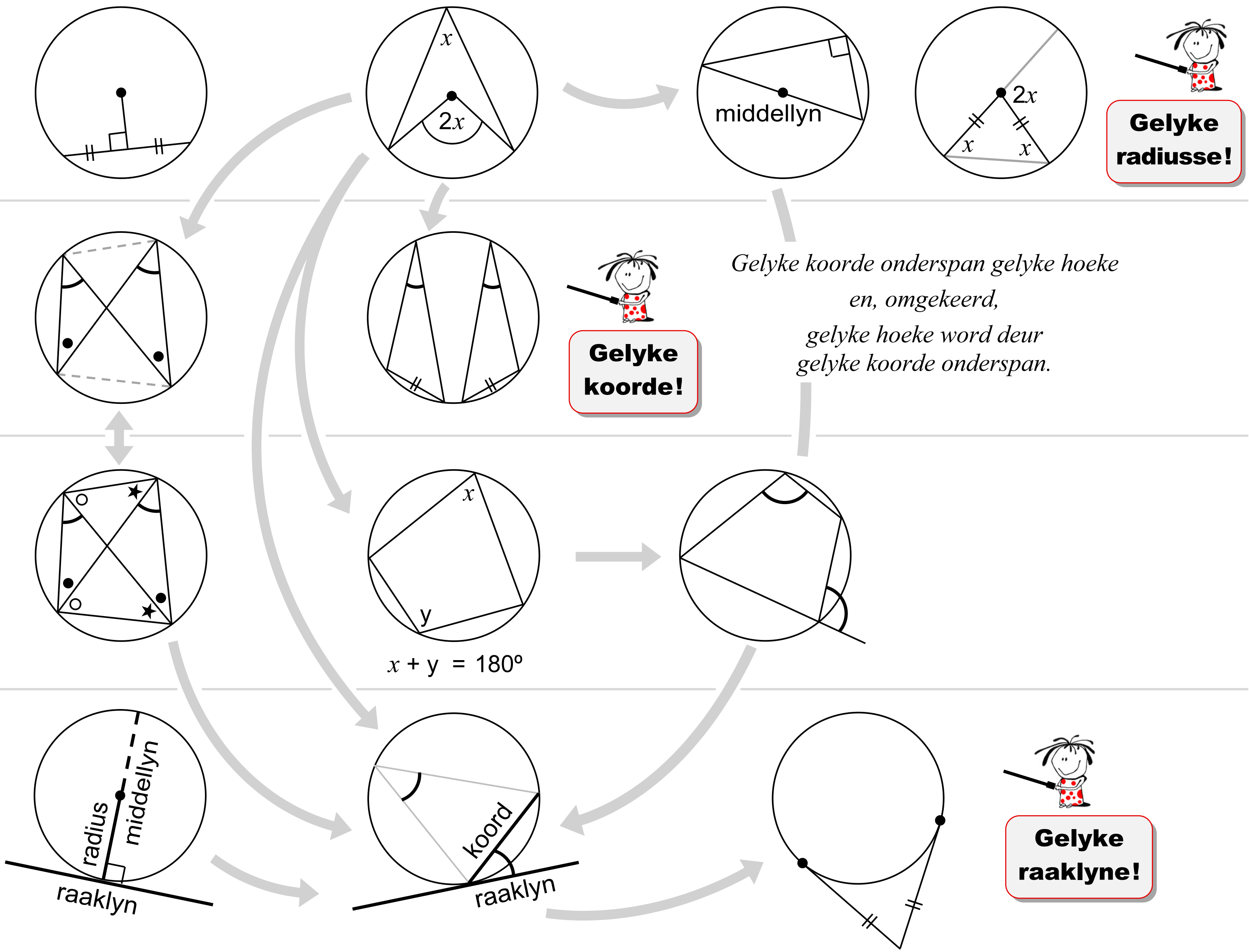
Die grys pyle dui aan hoe verskillende stellings gebruik word om daaropvolgendes te bewys

I — **Die 'Middelpunt' groep**

II — **Die 'Geen Middelpunt' groep**

III — **Die 'Koordevierhoek' groep**

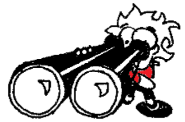
IV — **Die 'Raaklyn' groep**



VISUALISEER BEWYSE VAN STELLINGS

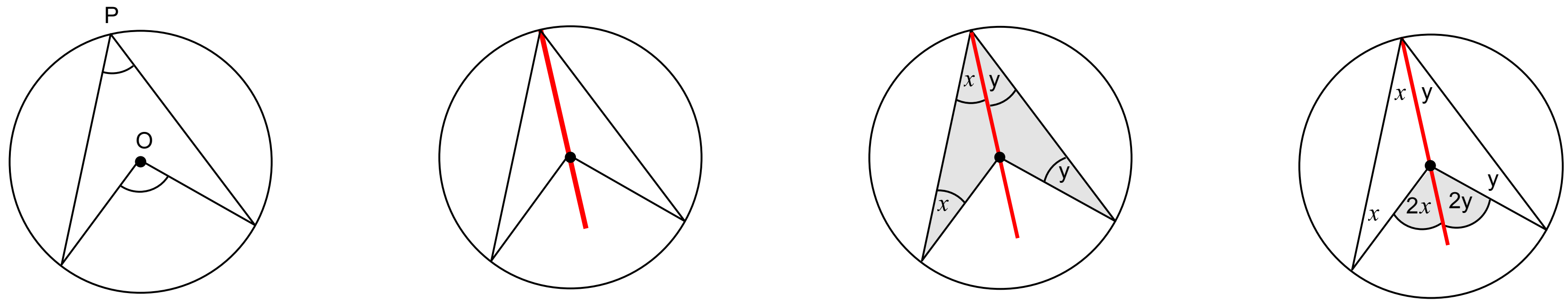
Die gebruik van VISUALISERING om bewyse van stellings te verstaan en te bemeester . . .

Om suksesvol te wees in Meetkunde, is dit baie belangrik om die bewyse van stellings te verstaan.



Woorde vs. Voorstellings?

Bewys die stelling: Middelpuntshoek is 2 keer die omtrekshoek (Stelling 3 op bl. 1.5)

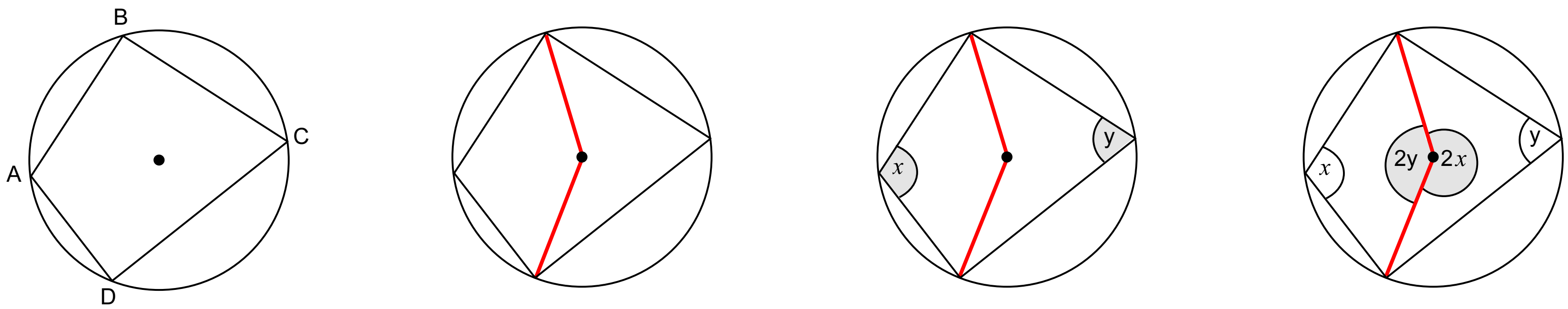


$2x + 2y$

$= 2(x + y)$

Gedoen!

Bewys die stelling: Teenoorstaande hoeke van 'n koordevierhoek is supplementêr (Stelling 4 op bl. 1.5)



$2x + 2y = 360°$

. . . *omwenteling*

$\therefore \; x + y = 180°$

Gedoen!

EUKLIDIESE MEETKUNDE (36,7%): DBO NOVEMBER 2022

VRAAG 8 55%

8.1 In die diagram is O die middelpunt van die sirkel. MNPR is
71% 'n koordevierhoek en SN is 'n middellyn van die sirkel.

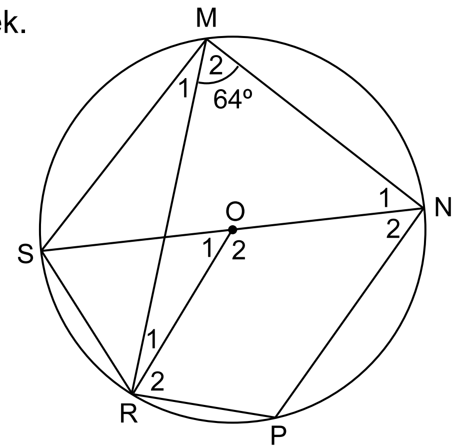
Koord MS en radius OR is getrek.
$\hat{M}_2 = 64^\circ$.

Bepaal, met redes, die grootte van die volgende hoeke:

8.1.1 $\hat{P}$ (2)

8.1.2 $\hat{M}_1$ (2)

8.1.3 $\hat{O}_1$ (2)



MEMO'S

8.1.1 $\hat{P} = 180^\circ - 64^\circ$ . . . **teenoorst. ∠ᵉ van kvh**

$= \mathbf{116^\circ}$ ➤

8.1.2 $S\hat{M}N = 90^\circ$ . . . **∠ in semi-⊙**

$\therefore \hat{M}_1 = 90^\circ - 64^\circ$

$= \mathbf{26^\circ}$ ➤

8.1.3 $\hat{O}_1 = 2\hat{M}_1$ . . . **middelpunts∠ = 2 × omtreks∠**

$= \mathbf{52^\circ}$ ➤

Algemene Foute en Wanopvattings

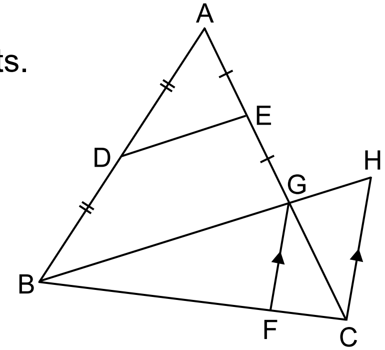
(a) In **V8.1.1** het sommige kandidate dit verkeerdelik gestel dat $\hat{P} = \hat{M}_2$ met die rede dat die teenoorstaande hoeke van 'n koordevierhoek gelyk is. Ander kandidate het nie voldoende inligting in **die rede** verskaf nie. Teenoorstaande hoeke, en teenoorstaande hoeke is supplementêr, is nie as korrek aanvaar nie.

(b) In die beantwoording van **V8.1.2**, het sommige kandidate verkeerdelik gestel dat $\hat{M}_1 = \hat{O}_1$ met die rede dat hulle hoeke in dieselfde segment was. Sommige kandidate het die rede as reghoekige driehoek gegee. Dit is nie as korrek aanvaar nie.

(c) In **V8.1.3** kon sommige kandidate nie daarin slaag om die verband tussen $\hat{M}_1$ en $\hat{O}_1$ te sien, waar die middelpuntshoek twee maal die omtrekshoek is nie.

VRAAG 8 (verv.)

8.2 In die diagram is $\triangle$ABG geskets.
42% D en E is middelpunte van AB en AG onderskeidelik. AG en BG word na C en H onderskeidelik verleng. F is 'n punt op BC sodanig dat FG || CH.



8.2.1 Gee 'n rede waarom DE || BH. (1)

8.2.2 Indien dit verder gegee word dat $\frac{FC}{BF} = \frac{1}{4}$, DE = $3x - 1$ en GH = $x + 1$, bereken, met redes, die waarde van x. (6) **[13]**

Algemene Foute en Wanopvattings

(d) In **V8.2.1** kon baie kandidate nie die korrekte rede gee vir die lyne wat ewewydig is nie. Hulle het die stelling met sy omgekeerde verwar. Antwoorde wat gegee is, was die *eweredigheidstelling*, in plaas van die ***omgekeerde*** *eweredigheidstelling*; of die ***omgekeerde*** *middelpuntstelling,* in plaas van die *middelpuntstelling*.

MEMO

8.2.1 **OMGEKEERDE** Middelpuntstelling

Vir diegene wat nie vertroud is met **die Middelpuntstelling** nie; jy kan die omgekeerde van die Eweredigheidstelling gebruik.

Algemene Foute en Wanopvattings

(e) In **V8.2.2** het baie kandidate **aangeneem** dat BG = DE alhoewel dit nie in die gegewe diagram die geval was nie. Sommige kandidate het **algebraïese foute**, bv. $4(x + 1) = 4x + 1$ gemaak.

Algebra!

Ander kandidate het verkeerdelik neergeskryf $\frac{BF}{FC} = \frac{BG}{DE}$ in plaas van $\frac{BF}{FC} = \frac{BG}{GH}$. Sommige kandidate het nie **die ewewydige lyne in die rede genoem nie.**

MEMO

8.2.2 In $\triangle$ABG:

BG = $2(3x - 1)$ . . . *middelpuntstelling*

$\therefore$ BG = $6x - 2$

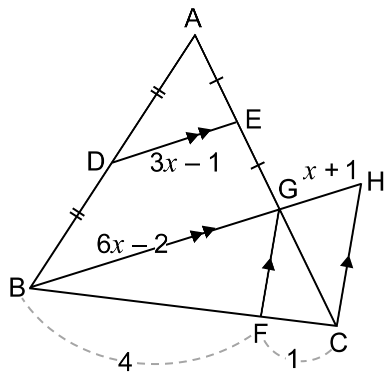
& In $\triangle$BCH: $\frac{GH}{BG} = \frac{1}{4}$ . . . *middelpuntstelling;* **FG || CH**

$\therefore \frac{x + 1}{6x - 2} = \frac{1}{4}$

$\therefore 6x - 2 = 4x + 4$

$\therefore 2x = 6$

$\therefore$ $\boldsymbol{x = 3}$ ◄



VRAAG 8: Voorstelle vir Verbetering

(a) Die geheim om Euklidiese Meetkunde suksesvol te beantwoord, is om ten volle bekend te wees met die **terminologie** van hierdie afdeling. Hiervoor moet onderwysers die betekenis van koord, raaklyn, koordevierhoek, ens. verduidelik sodat leerders in staat sal wees om dit korrek te gebruik.

(b) Onderwysers moet die **basiese werk** deeglik behandel. 'n Verduideliking van die stelling moet van 'n diagram, waar die verband getoon word, vergesel word.

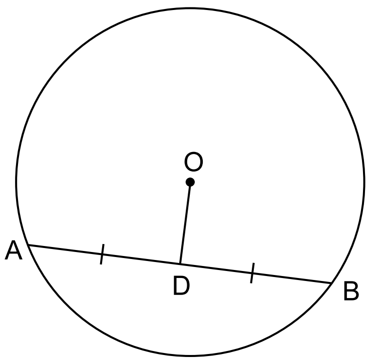
(c) Onderwysers word aangemoedig om met onderrig die **'Aanvaarbare Redes'** in die *Eksamenriglyne* te gebruik. Dit moet reeds van so vroeg soos Graad 8 gedoen word.

(d) Leerders moet aangemoedig word om die gegewe inligting en die diagram deeglik deur te werk vir **leidrade** oor **watter stellings** gebruik kan word, wanneer die vraag beantwoord word.

(e) Leerders moet geleer word dat **alle stellings van redes vergesel moet word**. Dit is noodsaaklik dat die **ewewydige lyne** genoem word wanneer gestel word dat ooreenkomstige hoeke gelyk is, verwisselende hoeke gelyk is, die som van die ko-binnehoeke 180° is of wanneer die eweredigheid-afsnitstelling gebruik word.

VRAAG 9 37%

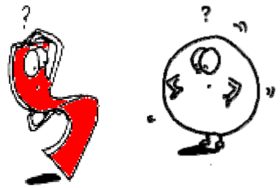
9.1 In die diagram is O die middelpunt
56% van 'n sirkel. OD halveer koord AB.



Bewys die stelling wat beweer dat die lyn wat vanaf die middelpunt van 'n sirkel getrek word en 'n koord halveer, loodreg op die koord is, met ander woorde $OD \perp AB$. (5)

Algemene Foute en Wanopvattings

(a) In **V9.1** het baie kandidate die rede dat hoeke op 'n reguitlyn supplementêr is, uitgelaat. Sommige kandidate het ***gelykvormigheid*** in plaas van ***kongruensie*** gebruik om hierdie stelling te bewys. Baie kandidate het in die bewys gestel dat **OD loodreg op AB was**. Dit het tot 'n **ineenstorting** gelei omdat hierdie kandidate onduidelik was oor watter inligting gegee was en wat hul moes bewys.



MEMO'S

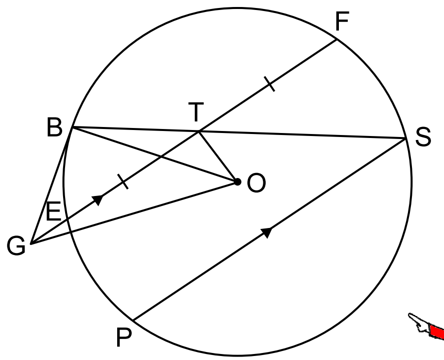
9.1 Bewys van stelling

VRAAG 9 (verv.)

26%

9.2 In die diagram is E, B, F, S en P punte op die sirkel met middelpunt O. GB is 'n raaklyn aan die sirkel by B. FE word verleng om die raaklyn by G te ontmoet. OT is getrek sodanig dat T die middelpunt van EF is. GO en BO is getrek. BS is deur T getrek. PS || GF.

Skets oorgeteken.



Bewys, met redes, dat:

9.2.1 OTBG 'n koordevierhoek is (5)

9.2.2 $G\hat{O}B = \hat{S}$ (4)

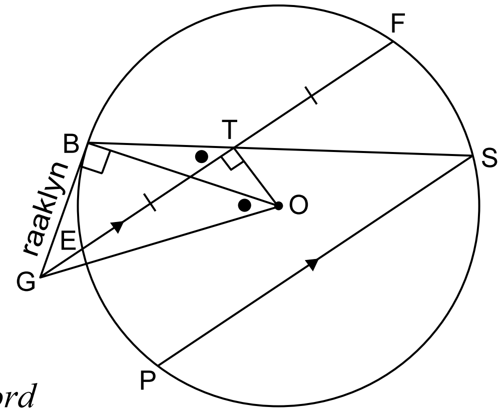
[14]

Algemene Foute en Wanopvattings

(b) In **V9.2.1** kon baie kandidate nie identifiseer dat die radius na die middelpunt van die koord getrek is nie. Dié wat kon bewys dat OTBG 'n koordevierhoek was, het die verkeerde rede, hoeke in dieselfde segment, in plaas van **omgekeerde** hoeke in dieselfde segment, gegee.

(c) Met die beantwoording van **V9.2.2**, het sommige kandidate gestel dat BG en OG gelyk was omdat hulle raaklyne vanuit 'n gemeenskaplike punt was. Dit was verkeerd omdat OG nie 'n raaklyn aan die sirkel was nie. 'n Groot uitdaging in hierdie vraag was die swak benoeming van hoeke. Kandidate sou na $\hat{T}$ verwys terwyl daar 'n hele aantal hoeke rondom punt T is.

MEMO'S



9.1 Bewys van stelling

9.2.1 $G\hat{B}O = 90°$ . . . *raaklyn ⊥ radius*

$O\hat{T}G = 90°$ . . . *lyn vanuit midpt na midpt van koord*

$\therefore G\hat{B}O = O\hat{T}G$

$\therefore$ **OTBG is 'n koordevierhoek** ✓ . . . **omgekeerde** *∠e in dieselfde segment*

9.2.2 $G\hat{O}B = G\hat{T}B$ . . . *∠e in dieselfde segment*

$= \hat{S}$ ✓ . . . *ooreenkomstige ∠e; PS || GF*

VRAAG 9: Voorstelle vir Verbetering

(a) Leerders moet geleer word dat ŉ **konstruksie** vereis word om ŉ stelling te bewys. Indien die konstruksie nie getoon word nie, word die bewys as ŉ **ineenstorting** beskou en kry hulle geen punte nie. Onderwysers moet **teorie** in kort toetse en take **vaslê**.

(b) Onderwysers moet daarop fokus om leerders se vaardighede te ontwikkel om die vraag en die diagram te analiseer vir **leidrade** oor **watter stellings** nodig is om die vrae korrek te beantwoord.

(c) Leerders moet gedwing word om **aanvaarbare redes** in Euklidiese Meetkunde te gebruik. Onderwysers moet die verskil tussen ŉ ***stelling*** **en sy** ***omgekeerde*** verduidelik. Hulle moet ook die **voorwaardes vir watter stellings van toepassing is** en wanneer die omgekeerde van toepassing sal wees, verduidelik.

(d) Leerders moet in Euklidiese Meetkunde blootgestel word aan vrae wat die stellings en die omgekeerdes insluit. Wanneer bewys word dat ŉ vierhoek ŉ koordevierhoek is, mag sirkelterminologie nie gebruik word wanneer na die vierhoek verwys word nie.

(e) Leerders moet ontmoedig word om korrekte stellings wat nie met die oplossing verband hou nie, neer te skryf. Geen punte word toegeken vir stellings wat nie tot die oplos van die probleem lei nie.

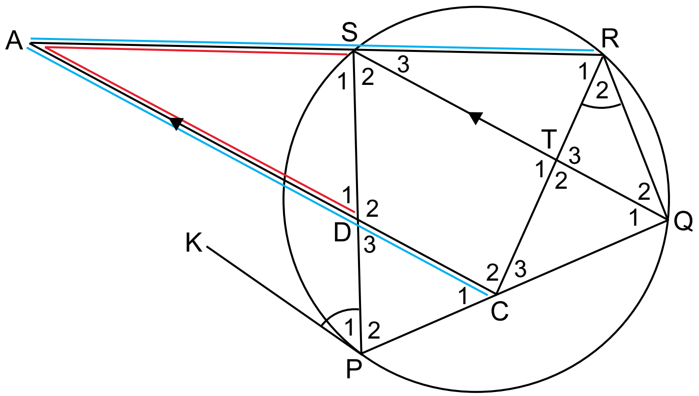
(f) Daar moet vir leerders gesê word dat die sukses in die beantwoording van Euklidiese Meetkunde in gereelde oefening lê, waar daar by dit wat maklik is begin word en mettertyd na dit wat moeilik is, gewerk word.

(g) Onderwysers moet tyd spandeer om die benoeming van hoeke te bespreek. Die aanvaarbare metodes is $\hat{T}$ of $\hat{T}_1$ of $O\hat{T}S$. Onderwysers moet dit duidelik maak wanneer dit aanvaarbaar is om na ŉ hoek as $\hat{T}$ te verwys en wanneer daarna as $\hat{T}_1$ verwys moet word.

VRAAG 10 18%

In die diagram is PQRS ’n koordevierhoek. KP is ’n raaklyn aan die sirkel by P. C en D is punte op koorde PQ en PS onderskeidelik en CD verleng, ontmoet RS verleng, by A.

CA || QS. RC is getrek. $\hat{P}_1 = \hat{R}_2$.



Bewys, met redes, dat:

19%	10.1	$\hat{S}_1 = \hat{T}_2$	(4)
25%	10.2	$\dfrac{AD}{AR} = \dfrac{AS}{AC}$	(5)
7%	10.3	$AC \times SD = AR \times TC$	(4)
			[13]

Algemene Foute en Wanopvattings

(a) ‘n Redelike aantal kandidate het **verkeerde aannames** gemaak in die beantwoording van **V10.1**.
Onder andere dat: $\hat{S}_1 = 90^\circ$ en $\hat{C}_2 = 90^\circ$, $\hat{P}_1 = \hat{R}_1 + \hat{R}_2$ met die rede *buitehoek van koordevierhoek,*
$\hat{P}_1 = \hat{C}_1$ met die rede *raaklyn-koord stelling* en
$\hat{S}_2 = \hat{R}_2$ met die rede *hoeke in dieselfde segment*.

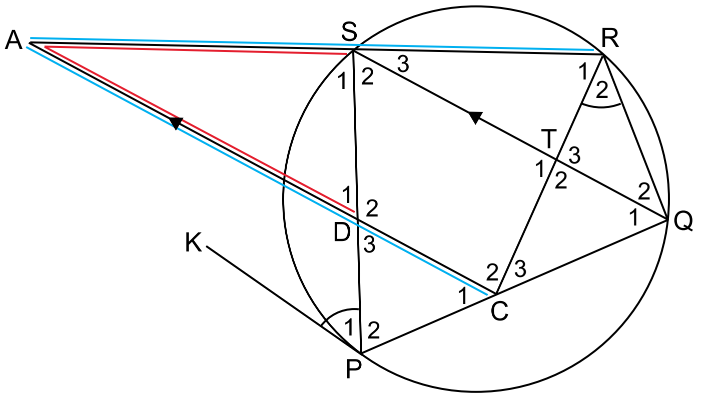
(b) In **V10.2** het sommige kandidate probeer om te bewys die verhoudings is gelyk deur die **eweredigheidstelling** te gebruik in plaas van **gelykvormige driehoeke**.
‘n Algemene fout wat kandidate wat wou bewys dat △ASD gelykvormig is aan △ACR, gemaak het, was om net te stel dat $\hat{S}_1 = \hat{C}_2$ sonder enige bewys of redes. Dit is as ‘n **ineenstorting** in die antwoord beskou.

(c) Daar is in **V10.3** van kandidate verwag om ‘n eweredigheid vanaf die **gelykvormige driehoeke** in **V10.2** te verkry, die **eweredigheid-afsnitstelling** in △RAC te gebruik om ‘n tweede eweredigheid daar te stel en dan die twee te kombineer. Baie kandidate kon nie daarin slaag om die een of die ander eweredigheid te kry nie en kon dus nie by die gevolgtrekking uitkom nie.

VRAAG 10

In die diagram is PQRS 'n koordevierhoek. KP is 'n raaklyn aan die sirkel by P. C en D is punte op koorde PQ en PS onderskeidelik en CD verleng, ontmoet RS verleng, by A.

CA || QS. RC is getrek. $\hat{P}_1 = \hat{R}_2$.



Bewys, met redes, dat:

10.1 $\hat{S}_1 = \hat{T}_2$ (4)

10.2 $\dfrac{AD}{AR} = \dfrac{AS}{AC}$ (5)

10.3 $AC \times SD = AR \times TC$ (4)

[13]

MEMO'S

10.1 Laat $\hat{P}_1 = \hat{R}_2 = x$

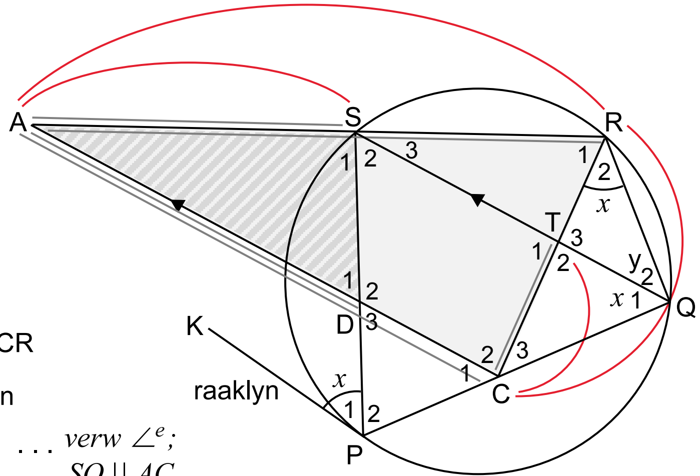
$\hat{Q}_1 = x$ . . . *raaklyn-koord stelling*

Laat $\hat{Q}_2 = y$

$\therefore \hat{T}_2 = x + y$ . . . *buite ∠ van ΔRTQ*

$\therefore \hat{S}_1 = x + y$ . . . *buite ∠ van koordevierhoek*

$\therefore \hat{S}_1 = \hat{T}_2$



10.2 In Δ^e ASD en ACR

(1) $\hat{A}$ is gemeen

(2) $\hat{C}_2 = x + y$ . . . *verw ∠^{e}; SQ || AC*

$\therefore \hat{S}_1 = \hat{C}_2$

$\therefore$ ΔASD ||| ΔACR . . . ∠∠∠

$\therefore \dfrac{AD}{AR} = \dfrac{AS}{AC}$ ◄ . . . $= \dfrac{SD}{CR}$

10.3 $\dfrac{AS}{AC} = \dfrac{SD}{CR}$ . . . *gelykvormige Δ^e in 10.2*

$\therefore$ AS . CR = AC . SD . . . ❶

& In ΔACR: $\dfrac{AS}{AR} = \dfrac{CT}{CR}$ . . . *eweredigh.stelling; CA || TS*

$\therefore$ AS . CR = AR . CT . . . ❷

Vanaf ❶ & ❷:

$\therefore$ **AC . SD = AR . TC** ◄

VRAAG 10: Voorstelle vir Verbetering

(a) **In alle grade moet meer tyd aan die onderrig van Euklidiese Meetkunde spandeer word.** Meer oefening in Graad 11 en 12 Euklidiese Meetkunde sal leerders help om die analise van stellings en diagramme te verstaan. Hulle moet die gegewe inligting noukeurig lees sonder om enige **aannames** te maak. Wanneer hierdie werk in die klas behandel word, moet dit verskillende aktiwiteite en alle vlakke van die taksonomie insluit.

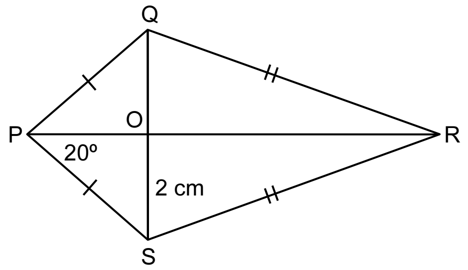
(b) Onderwysers moet van leerders verwag om die **diagramme** in die Antwoordboek te **gebruik** om **hoeke en sye** wat gelyk is, **aan te dui** en om **inligting** wat bereken is **daarop aan te dui**.

(c) Leerders moet daarop gewys word dat daar in die eksamen **geen punte toegeken word** vir die neerskryf van korrekte, maar **irrelevante stellings,** nie.

GR 10 – 12 EKSEMPLAAR MEETKUNDE

GRAAD 10: VRAE

1. PQRS is 'n vlieër sodanig dat die hoeklyne mekaar by O sny. OS = 2 cm en $\hat{OPS}$ = 20º.

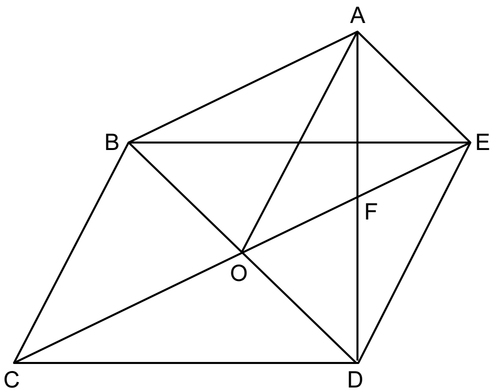


1.1 Skryf die lengte van OQ neer. (2)

1.2 Skryf die grootte van $\hat{POQ}$ neer. (2)

1.3 Skryf die grootte van $\hat{QPS}$ neer. (2) [6]

2. In die diagram is BCDE en AODE parallelogramme.



2.1 Bewys dat OF || AB. (4)

2.2 Bewys dat ABOE 'n parallelogram is. (4)

2.3 Bewys dat ΔABO ≡ ΔEOD. (5) [13]

GRAAD 10: MEMO'S

1.1 OQ = 2 cm ✓ . . . *die langer hoeklyn van'n vlieër halveer die korter hoeklyn*

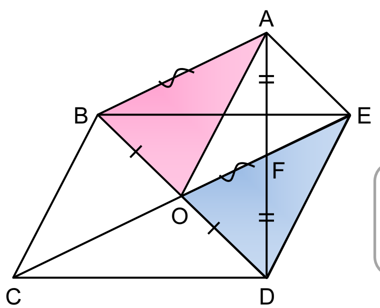
1.2 $\hat{POQ}$ = 90º ✓ . . . *die hoeklyne van 'n vlieër sny mekaar reghoekig*

1.3 $\hat{QPO}$ = 20º . . . *die langer hoeklyn van 'n vlieër halveer die (teenoorstaande) hoeke van 'n vlieër*

∴ $\hat{QPS}$ = 40º ✓

---

2. **Wenk:**
Gebruik kleurpotlode om die verskillende $||^{me}$ en Δ^{e} aan te dui.



Die ingekleurde Δ^{e} (en hul sye) verwys na Vraag 2.3.

2.1 In ΔDBA:

O is die midpt van BD . . . *hoeklyne van $||^{m}$ BCDE halveer mekaar*

& F is die midpt van AD . . . *hoeklyne van $||^{m}$ AODE halveer mekaar*

∴ OF || AB ✓ . . . *die lyn wat die middelpunte van twee sye van 'n Δ verbind is || aan die 3^{de} sy*

2.2 AE || OD . . . *teenoorst. sye van $||^{m}$ AODE*

∴ AE || BO

en OF || AB . . . *hierbo bewys*

∴ OE || AB

∴ ABOE is 'n $||^{m}$ . . . *albei pare teenoorstaande sye is ewewydig*

OF: In $||^{m}$ AODE: AE = en || OD . . . *teenoorst.. sye van $||^{m}$*

Maar OD = BO . . . *O bewys midpt van BD in 2.1*

∴ AE = en || BO

∴ ABOE is 'n $||^{m}$ ✓ . . . *1 pr teenoorst. sye = en ||*

2.3 In Δ^{e} ABO en EOD

1) AB = EO . . . *teenoorst. sye van $||^{m}$ ABOE*

2) BO = OD . . . *bewys in 2.1*

3) AO = ED . . . *teenoorst. sye van $||^{m}$ AODE*

∴ ΔABO ≡ ΔEOD ✓ . . . *SSS*

GRAAD 11: VRAE

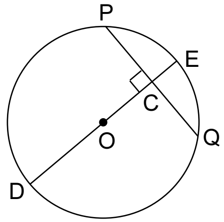
1.1 Voltooi die stelling sodat dit geldig is:

Die lyn wat van die middelpunt van 'n sirkel loodreg op 'n koord getrek word . . . (1)

1.2 In die diagram is O die middelpunt van die sirkel.

Die middellyn DE sny die koord PQ loodreg by C.

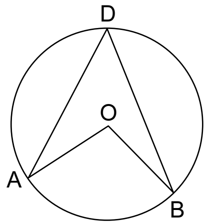
DE = 20 cm en CE = 2 cm.



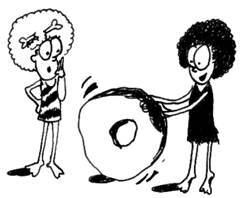
Bereken, met redes, die lengte van die volgende:

1.2.1 OC 1.2.2 PQ (2)(4) [7]

2.1 In die diagram is O die middelpunt van die sirkel en A, B en D is punte op die sirkel.



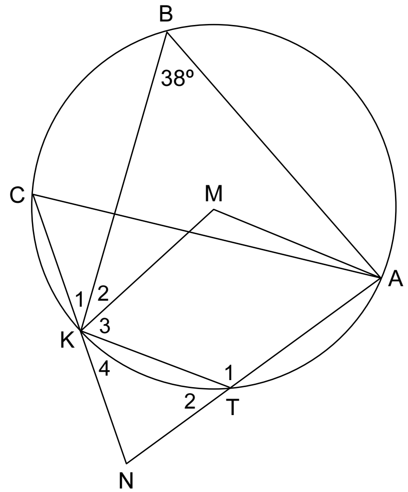
Gebruik Euklidiese meetkundemetodes om die stelling te bewys wat beweer dat $A\hat{O}B = 2A\hat{D}B$. (5)



2.2 In die diagram is M die middelpunt van die sirkel. A, B, C, K en T lê op die sirkel.

AT verleng en CK verleng ontmoet in N.

Verder is NA = NC en $\hat{B} = 38°$.



2.2.1 Bereken, met redes, die grootte van die volgende hoeke:

(a) $K\hat{M}A$ (b) $\hat{T}_2$ (2)(2)

(c) $\hat{C}$ (d) $\hat{K}_4$ (2)(2)

2.2.2 Toon aan dat NK = NT. (2)

2.2.3 Bewys dat AMKN 'n koordevierhoek is. (3)

[18]

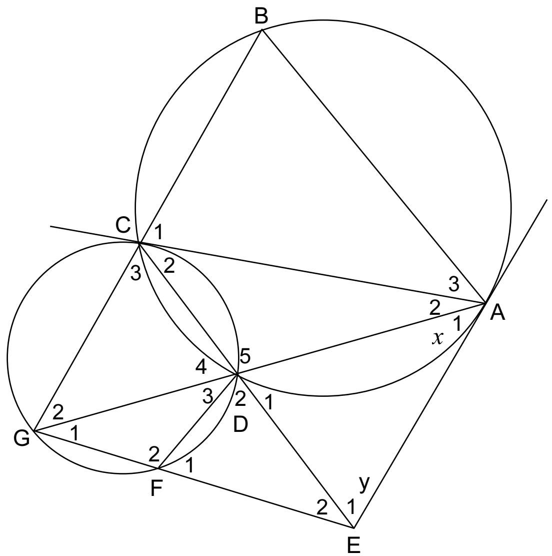
3.1 Voltooi die volgende stelling sodat dit geldig is:

Die hoek tussen 'n koord en 'n raaklyn by die raakpunt is . . . (1)

3.2 In die diagram is EA 'n raaklyn aan sirkel ABCD by A.

AC is 'n raaklyn aan sirkel CDFG by C.

CE en AG sny mekaar in D.



As $\hat{A}_1 = x$ en $\hat{E}_1 = y$, bewys die volgende met redes:

3.2.1 BCG || AE (5)

3.2.2 AE is 'n raaklyn aan sirkel FED (5)

3.2.3 AB = AC (4) [15]

GRAAD 11: MEMO'S

1.1 . . . **halveer die koord** ◄

1.2.1 OE = OD = $\frac{1}{2}$(20) = 10 cm | *radiusse* $= \frac{1}{2}$ *middellyn*

$\therefore$ **OC = 8 cm** ◄ . . . *CE = 2 cm*

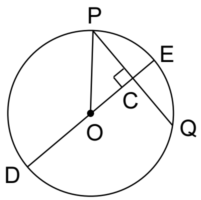
1.2.2 In ΔOPC:

PC^2 = OP^2 - OC^2 . . . *Pythagoras*

= 10^2 - 8^2

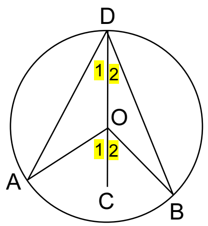
= 36

$\therefore$ PC = 6 cm



$\therefore$ **PQ = 12 cm** ◄ . . . *lyn vanuit midpt. $\perp$ koord*

2.1 Konstruksie: Verbind DO en verleng dit na C



Bewys:

Laat $\hat{D}_1 = x$

dan is $\hat{A} = x$ . . . *radiusse; $\angle^e$ teenoor = sye*

$\therefore$ $\hat{O}_1 = 2x$ . . . *buite$\angle$ van ΔDAO*

Net so: Laat $\hat{D}_2$ = y

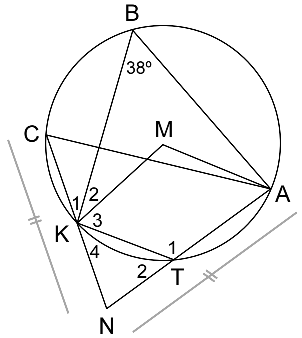
dan is $\hat{O}_2$ = 2y

$\therefore$ A$\hat{O}$B = $2x$ + 2y

= 2(x + y)

= **2 A$\hat{D}$B** ◄

2.2



2.2.1 (a) K$\hat{M}$A = 2(38°) . . . *middelpunts$\angle$ = 2 $\times$ omtreks$\angle$*

= **76°** ◄

(b) $\hat{T}_2$ = **38°** ◄ . . . *buite$\angle$ van kvh. BKTA*

(c) $\hat{C}$ = **38°** ◄ . . . *$\angle^e$ in dieselfde segment of, buite$\angle$ van kvh. CKTA*

(d) N$\hat{A}$C = 38° . . . *$\angle^e$ teenoor = sye*

$\therefore$ $\hat{K}_4$ = **38°** ◄ . . . *buite$\angle$ van kvh. CKTA*

2.2.2 In ΔNKT: $\hat{K}_4 = \hat{T}_2$ . . . *beide = 38° in 2.2.1*

$\therefore$ **NK = NT** ◄ . . . *sye teenoor gelyke $\angle^e$*

2.2.3 K$\hat{M}$A = 2(38°) . . . *sien 2.2.1(a)*

& $\hat{N}$ = 180° - 2(38°) . . . *som van $\angle^e$ in ΔNKT (sien 2.2.2)*

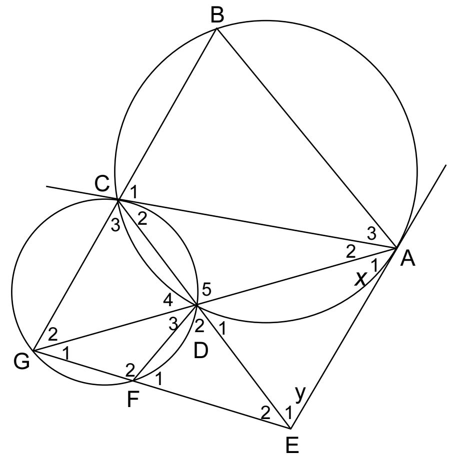
$\therefore$ K$\hat{M}$A + $\hat{N}$ = 180°

$\therefore$ **AMKN is 'n koordevierhoek** ◄

. . . *teenoorstaande $\angle^e$ supplementêr*

3.1 . . . **gelyk aan die hoek onderspan deur die koord in die teenoorstaande segment.** ◄

3.2



3.2.1 $\hat{A}_1 = x$ . . . *gegee*

$\therefore$ $\hat{C}_2 = x$ . . . *raaklyn koord stelling*

$\therefore$ $\hat{G}_2 = x$ . . . *raaklyn koord stelling*

$\therefore$ $\hat{A}_1$ = (verwisselende) $\hat{G}_2$

$\therefore$ **BCG || AE** ◄ . . . *(verwisselende $\angle^e$ gelyk)*

3.2.2 $\hat{F}_1 = \hat{C}_3$ . . . *buite$\angle$ van koordevierhoek. CGFD*

= $\hat{E}_1$ (= y) . . . *verwisselende $\angle^e$; BCG || AE*

$\therefore$ **AE is 'n raaklyn aan $\odot$FED** ◄

. . . *omgekeerde van raaklyn koord stelling*

3.2.3 $\hat{C}_1$ = C$\hat{A}$E . . . *verwisselende $\angle^e$; BCG || AE*

= $\hat{B}$ . . . *raaklyn koord stelling*

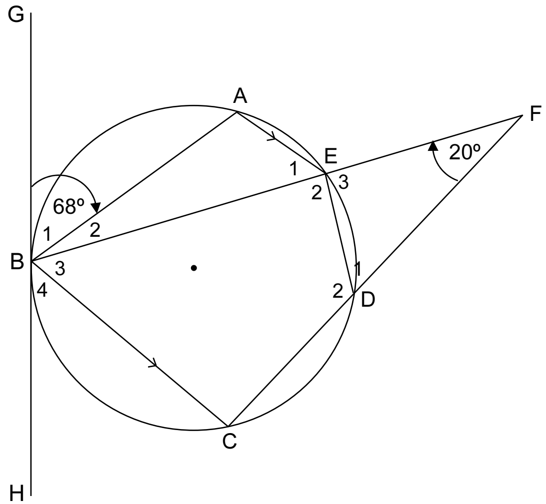
$\therefore$ **AB = AC** ◄ . . . *sye teenoor gelyke $\angle^e$*

GRAAD 12: VRAE

1.1 Voltooi die volgende bewering:

Die hoek tussen 'n raaklyn en 'n koord by die raakpunt is gelyk aan . . . (1)

1.2 In die diagram is A, B, C, D en E punte op die omtrek van die sirkel sodat AE || BC. BE en CD verleng ontmoet in F. GBH is 'n raaklyn aan die sirkel by B. $\hat{B}_1 = 68°$ en $\hat{F} = 20°$.



Bepaal die grootte van elk van die volgende:

1.2.1 $\hat{E}_1$ (2)

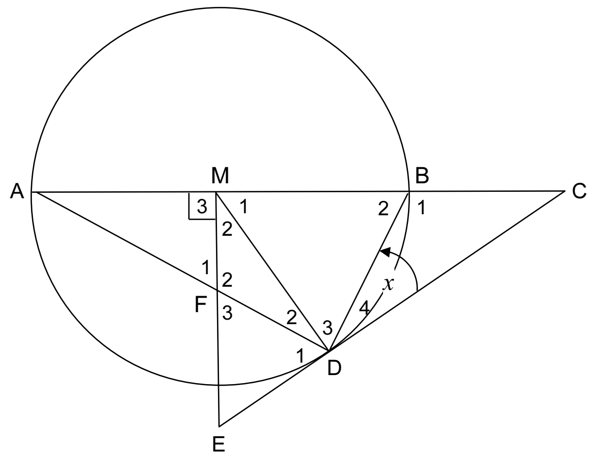
1.2.2 $\hat{B}_3$ (1)

1.2.3 $\hat{D}_1$ (2)

1.2.4 $\hat{E}_2$ (1)

1.2.5 $\hat{C}$ (2) [9]

2. In die diagram is M die middelpunt van die sirkel en middellyn AB is verleng na C. ME is loodreg op AC getrek sodat CDE 'n raaklyn aan die sirkel by D is. ME en koord AD sny in F. MB = 2BC



2.1 As $\hat{D}_4 = x$, skryf, met redes, TWEE ander hoeke neer wat gelyk is aan x. (3)

2.2 Bewys dat CM 'n raaklyn by M is aan die sirkel wat deur M, E en D gaan. (4)

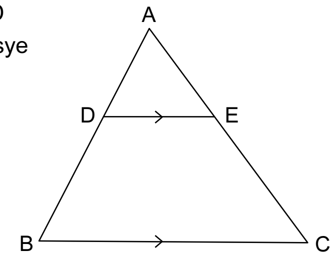
2.3 Bewys dat FMBD 'n koordevierhoek is. (3)

2.4 Bewys dat $DC^2 = 5BC^2$. (3)

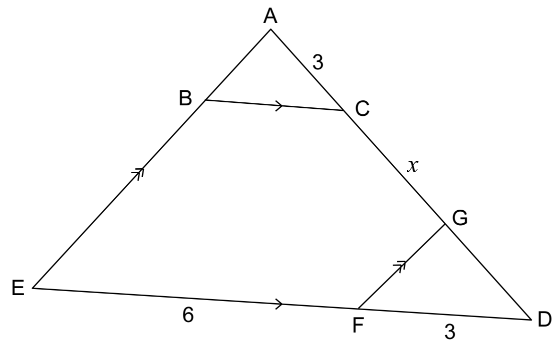
2.5 Bewys dat ΔDBC ||| ΔDFM. (4)

2.6 Bepaal vervolgens die waarde van $\frac{DM}{FM}$. (2) [19]

3.1 In die diagram lê punte D en E op onderskeidelik sye AB en AC van ΔABC sodat DE || BC. Gebruik Euklidiese meetkunde-metodes om die stelling te bewys wat beweer dat $\frac{AD}{DB} = \frac{AE}{EC}$. (6)



3.2 In die diagram is ADE 'n driehoek met BC || ED en AE || GF. Verder word ook gegee dat AB : BE = 1 : 3, AC = 3 eenhede, EF = 6 eenhede, FD = 3 eenhede en CG = x eenhede.



Bereken, met redes:

3.2.1 die lengte van CD (3)

3.2.2 die waarde van x (4)

3.2.3 die lengte van BC (5)

3.2.4 die waarde van $\frac{\text{oppervlakte } \Delta ABC}{\text{oppervlakte } \Delta GFD}$ (5) [23]

GRAAD 12: MEMO'S

1.1 . . . die hoek onderspan deur die koord in die oorstaande segment.

1.2.1 $\hat{E}_1 = \hat{B}_1$ . . . *raaklyn koord stelling*
$= \mathbf{68^\circ}$ ◀

1.2.2 $\hat{B}_3 = \hat{E}_1$ . . . *verw.* $\angle^e$ *; AE || BC*
$= \mathbf{68^\circ}$ ◀

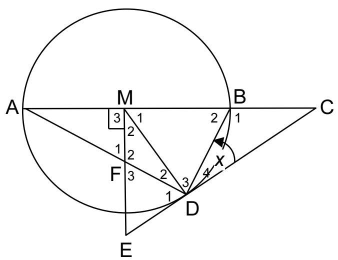
1.2.3 $\hat{D}_1 = \hat{B}_3$ . . . *buite* $\angle$ *van koordevierhoek*
$= \mathbf{68^\circ}$ ◀

1.2.4 $\hat{E}_2 = \hat{D}_1 + 20^\circ$ . . . *buite* $\angle$ *van* Δ
$= \mathbf{88^\circ}$ ◀

1.2.5 $\hat{C} = 180^\circ - \hat{E}_2$ . . . *teenoorst.* $\angle^e$ *van kvh.*
$= \mathbf{92^\circ}$ ◀

2.1 $\hat{A} = x$ . . . *raaklyn koord stelling*
$\hat{D}_2 = x$ . . . $\angle^e$ *teenoor gelyke sye*

2.2



$\hat{M}_1 = \hat{A} + \hat{D}_2$ . . . *buite* $\angle$ *van* Δ
$= 2x$
$\therefore \hat{M}_2 = 90^\circ - 2x$ . . . *ME* $\perp$ *AC*

& $M\hat{D}E = 90^\circ$ . . . *radius MD* $\perp$ *raaklyn CDE*
$\therefore \hat{E} = 2x$ . . . *som van* $\angle^e$ *in* ΔMED
$\therefore \hat{M}_1 = \hat{E}$

$\therefore$ **CM is 'n raaklyn by M aan ⊙MED** ◀ . . . *omgekeerde raaklyn koord stelling*

2.3 $A\hat{D}B = 90^\circ$ . . . $\angle$ *in semi-⊙*
& $\hat{M}_3 = 90^\circ$ . . . *ME* $\perp$ *AC*
$\therefore \hat{M}_3 = A\hat{D}B$

$\therefore$ **FMBD is 'n kvh.** ◀ . . . *omgekeerde buite* $\angle$ *van kvh.*

2.4 Laat BC = a ; dan is MB = 2a
$\therefore$ MD = 2a . . . *radiusse*

In ΔMDC: $M\hat{D}C = 90^\circ$ . . . *radius* $\perp$ *raaklyn*

$\therefore DC^2 = MC^2 - MD^2$ . . . *stelling van Pythagoras*
$= (3a)^2 - (2a)^2$
$= 9a^2 - 4a^2$
$= 5a^2$
$= \mathbf{5BC^2}$ ◀

2.5 In Δ^e DBC en DFM

(1) $\hat{B}_1 = \hat{F}_2$ . . . *buite* $\angle$ *van kvh. FMBD*

(2) $\hat{D}_4 = \hat{D}_2$ . . . *beide = x*

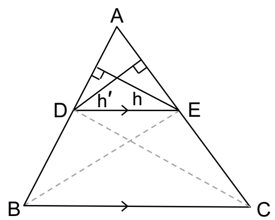
$\therefore$ **ΔDBC ||| ΔDFM** ◀ . . . *gelykhoekige* Δ^e

2.6 $\therefore \dfrac{DM}{FM} = \dfrac{DC}{BC}$ . . . $|||\ \Delta^e$

$= \dfrac{\sqrt{5}\,BC}{BC}$ . . . *sien 2.4*

$= \mathbf{\sqrt{5}}$ ◀

3.1 **Konstruksie:**
Verbind DC en EB
en hoogtes h en h′



Bewys:

$\dfrac{\text{oppv. van } \Delta ADE}{\text{oppv. van } \Delta DBE} = \dfrac{\frac{1}{2}AD.\,h}{\frac{1}{2}DB.\,h}$

$= \dfrac{AD}{DB}$ . . . *gelyke hoogtes*

& $\dfrac{\text{oppv. van } \Delta ADE}{\text{oppv. van } \Delta EDC} = \dfrac{\frac{1}{2}AE.\,h'}{\frac{1}{2}EC.\,h'} = \dfrac{AE}{EC}$ . . . *gelyke hoogtes*

Maar, oppv. van ΔDBE = oppv. van ΔEDC . . . *dies. basis DE & tussen dies. || lyne, d.w.s. dies. hoogte*

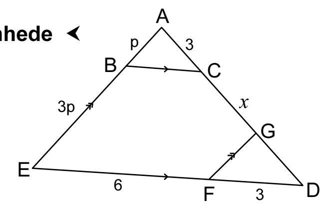
$\therefore \dfrac{\text{oppv. van } \Delta ADE}{\text{oppv. van } \Delta DBE} = \dfrac{\text{oppv. van } \Delta ADE}{\text{oppv. van } \Delta EDC}$

$\therefore \mathbf{\dfrac{AD}{DB} = \dfrac{AE}{EC}}$ ◀

3.2.1 Laat AB = p ; dan is BE = 3p

In ΔAED: $\dfrac{CD}{3} = \dfrac{3p}{p}$ . . . *ewer. stelling; BC || ED*

$\times 3)$ $\therefore$ **CD = 9 eenhede** ◀



3.2.2 CG = x ; dus is GD = $9 - x$

In ΔDAE: $\dfrac{9 - x}{x + 3} = \dfrac{3}{6}$ . . . *ewer. stelling; AE || GF*

$\therefore 54 - 6x = 3x + 9$
$\therefore -9x = -45$
$\therefore \mathbf{x = 5}$ ◀

3.2.3 In Δ^e ABC en AED

(1) $\hat{A}$ is gemeen

(2) $A\hat{B}C = \hat{E}$ . . . *ooreenk.* $\angle^e$*; BC || ED*

$\therefore \Delta ABC\ |||\ \Delta AED$ . . . *gelykhoekige* Δ^e

$\therefore \dfrac{BC}{ED} = \dfrac{AB}{AE}$ . . . $|||\ \Delta^e$

$\therefore \dfrac{BC}{9} = \dfrac{p}{4p}$

$\times 9)$ $\therefore$ **BC = $\mathbf{\dfrac{9}{4}}$ eenhede** ◀

3.2.4 $\dfrac{\text{oppv. van } \Delta ABC}{\text{oppv. van } \Delta GFD} = \dfrac{\frac{1}{2}AC.\,BC \sin A\hat{C}B}{\frac{1}{2}DG.\,DF \sin \hat{D}}$

$= \dfrac{\frac{1}{2}.\,3.\,\frac{9}{4}.\sin \hat{D}}{\frac{1}{2}.\,4.\,3.\sin \hat{D}}$ . . . *ooreenk.* $\angle^e$*; BC || ED*

$= \dfrac{\frac{9}{4}}{4}$

$= \mathbf{\dfrac{9}{16}}$ ◀

OF: $\dfrac{\text{oppv. van } \Delta ABC}{\text{oppv. van } \Delta AED} = \dfrac{\frac{1}{2}.\,p.\,3.\sin \hat{A}}{\frac{1}{2}.\,4p.\,12.\sin \hat{A}} = \dfrac{1}{16}$

$\therefore$ oppv. van ΔABC $= \dfrac{1}{16}$ oppv. van ΔAED . . . ❶

& $\dfrac{\text{oppv. van } \Delta GFD}{\text{oppv. van } \Delta AED} = \dfrac{\frac{1}{2}.\,4.\,3.\sin D}{\frac{1}{2}.\,12.\,9.\sin D} = \dfrac{1}{9}$

$\therefore$ oppv. van ΔGFD $= \dfrac{1}{9}$ oppv. van ΔAED . . . ❷

❶ ÷ ❷: $\therefore \dfrac{\text{oppv. van } \Delta ABC}{\text{oppv. van } \Delta GFD} = \dfrac{\frac{1}{16}\text{ oppv. van } \Delta AED}{\frac{1}{9}\text{ oppv. van } \Delta AED}$

$= \mathbf{\dfrac{9}{16}}$ ◀

Euklidiese Meetkunde: Stellings & Aanvaarbare Redes

LYNE

Aanliggende hoeke op 'n reguitlyn is supplementêr.	∠^{e} op reguitlyn
As aanliggende hoeke supplementêr is, lê die buitenste bene van die hoeke in 'n reguitlyn.	aanliggende ∠^{e} suppl.
Die som van die aanliggende hoeke om 'n punt is 360º.	∠^{e} om 'n pt. **OF** ∠^{e} in 'n omw.
Regoorstaande hoeke is gelyk.	regoorst. ∠^{e}
As AB || CD, dan is die verwisselende hoeke gelyk.	verw. ∠^{e}; AB || CD
As AB || CD, dan is die ooreenkomstige hoeke gelyk.	ooreenk. ∠^{e}; AB || CD
As AB || CD, dan is die ko-binnehoeke supplementêr.	ko-binne ∠^{e}; AB || CD
As die verwisselende hoeke tussen twee lyne gelyk is, dan is die lyne ewewydig.	verw. ∠^{e} =
As die ooreenkomstige hoeke tussen twee lyne gelyk is, dan is die lyne ewewydig.	ooreenk. ∠^{e} =
As die ko-binnehoeke tussen twee lyne supplementêr is, dan is die lyne ewewydig.	ko-binne ∠^{e} suppl.

DRIEHOEKE

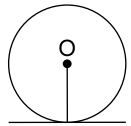
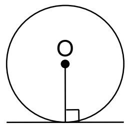
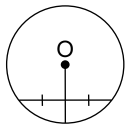
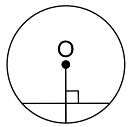
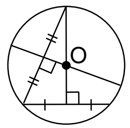
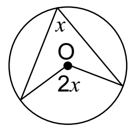
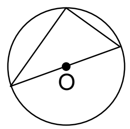
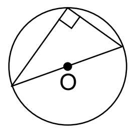
Die binnehoeke van 'n driehoek is supplementêr.	∠ som van Δ **OF** som van ∠^{e} in Δ **OF** binne ∠^{e} in Δ
Die buitehoek van 'n driehoek is gelyk aan die som van die twee teenoorstaande binnehoeke.	buite∠ van Δ
Die hoeke teenoor die gelyke sye van 'n gelykbenige driehoek, is gelyk.	∠^{e} teenoor gelyke sye
Die sye teenoor die gelyke hoeke van 'n gelykbenige driehoek, is gelyk.	sye teenoor gelyke ∠^{e}
In 'n reghoekige driehoek is die vierkant op die skuinssy gelyk aan die som van die vierkante op die ander twee sye.	Pythagoras **OF** Stelling van Pythagoras
As die vierkant op een sy van 'n driehoek gelyk is aan die som van die vierkante op die ander twee sye, dan is die driehoek reghoekig.	**Omgekeerde** Pythagoras **OF Omgekeerde** Stelling van Pythagoras
As drie sye van een driehoek onderskeidelik gelyk is aan drie sye van 'n ander driehoek, dan is die driehoeke kongruent.	SSS
As twee sye en 'n ingeslote hoek van een driehoek onderskeidelik gelyk is aan twee sye en 'n ingeslote hoek van 'n ander driehoek, dan is die twee driehoeke kongruent.	SHS **OF** S∠S
As twee hoeke en 'n sy van een driehoek onderskeidelik gelyk is aan twee hoeke en 'n ooreenstemmende sy van 'n ander driehoek, dan is die twee driehoeke kongruent.	HHS **OF** ∠∠S
As die skuinssy en 'n reghoeksy van 'n reghoekige driehoek onderskeidelik gelyk is aan die skuinssy en 'n reghoeksy van 'n ander reghoekige driehoek, dan is die twee driehoeke kongruent.	RHS **OF** 90ºHS
Die lynstuk wat die middelpunte van twee sye van 'n driehoek verbind, is ewewydig aan en gelyk aan die helfte van die derde sy.	Midpt.-stelling
Die lynstuk wat van die middelpunt van een sy van 'n driehoek ewewydig aan die tweede sy getrek word, halveer die derde sy.	lyn deur midpt || 2de sy
Die lyn ewewydig aan een sy van 'n driehoek verdeel die ander twee sye in eweredige dele.	lyn || een sy van Δ **OF** eweredigheidstelling; noem || lyne
As 'n lyn twee sye van 'n driehoek in eweredige dele verdeel, is die lyn ewewydig aan die derde sy.	lyn verdeel twee sye van Δ eweredig
As twee driehoeke gelykhoekig is, is hulle ooreenstemmende sye eweredig (en is driehoeke dus gelykvormig).	||| Δ^e **OF** gelykhoekige Δ^e
As die ooreenstemmende sye van twee driehoeke eweredig is, is die driehoeke gelykhoekig (en is driehoeke dus gelykvormig).	sye van Δ^e eweredig
Driehoeke (of parallelogramme) op dieselfde basis en tussen dieselfde ewewydige lyne is gelyk in oppervlakte.	dieselfde basis; dieselfde hoogte **OF** gelyke basis; gelyke hoogte

VIERHOEKE

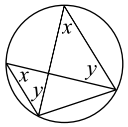
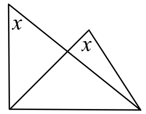
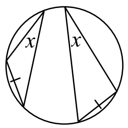
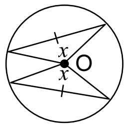
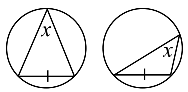
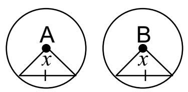
Die som van die binnehoeke van 'n vierhoek is 360º.	som van $\angle^e$ in vierhoek
Die teenoorstaande sye van 'n parallelogram is ewewydig.	tenoorst. sye van ||m
As die teenoorstaande sye van 'n vierhoek ewewydig is, dan is die vierhoek 'n parallelogram.	teenoorst sye van vierh is || **OF** **omgekeerde** teenoorst. sye van ||m
Die teenoorstaande sye van 'n parallelogram is gelyk in lengte.	teenoorst. sye van ||m
As die teenoorstaande sye van 'n vierhoek gelyk is, dan is die vierhoek 'n parallelogram.	teenoorst. sye van vierh = **OF** **omgekeerde** teenoorst sye van ||m
Die teenoorstaande hoeke van 'n parallelogram is gelyk.	teenoorst. $\angle^e$ van ||m
As die teenoorstaande hoeke van 'n vierhoek gelyk is, dan is die vierhoek 'n parallelogram.	teenoorst. $\angle^e$ van vierh = **OF** **omgekeerde** teenoorst. $\angle^e$ van ||m
Die hoeklyne van 'n parallelogram halveer mekaar.	hoeklyne van ||m
As die hoeklyne van 'n vierhoek mekaar halveer, dan is die vierhoek 'n parallelogram.	hoeklyne van vierh halveer mekaar **OF** **omgekeerde** hoeklyne van ||m
As een paar teenoorstaande sye van 'n vierhoek gelyk en ewewydig is, dan is die vierhoek 'n parallelogram.	teenoorst. sye = en ||
Die hoeklyne van 'n parallelogram halveer die oppervlakte van die parallelogram.	hoeklyn van ||m halveer opp.
Die hoeklyne van 'n ruit halveer mekaar reghoekig.	hoeklyne van ruit
Die hoeklyne van 'n ruit halveer die teenoorst. binnehoeke.	hoeklyne van ruit
Al vier sye van 'n ruit is gelyk.	sye van ruit
Al vier sye van 'n vierkant is gelyk.	sye van vierkant
Die hoeklyne van 'n reghoek is ewe lank.	hoeklyne van reghoek
Die hoeklyne van 'n vlieër sny mekaar reghoekig.	hoeklyne van vlieër
Die een hoeklyn van 'n vlieër halveer die ander hoeklyn.	hoeklyne van vlieër
Een hoeklyn van 'n vlieër halveer die teenoorstaande binnehoeke.	hoeklyne van vlieër

SIRKELS

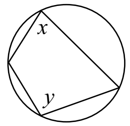
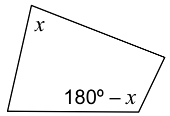
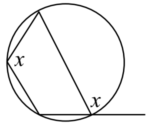
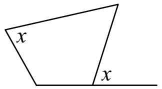
GROEP I

 O	'n Raaklyn aan 'n sirkel is loodreg op die radius/middellyn van die sirkel by die raakpunt.	raaklyn $\perp$ radius raaklyn $\perp$ middellyn
 O	As 'n lyn loodreg getrek word na die radius/middellyn by die punt waar die radius/middellyn die sirkel ontmoet, dan is die lyn 'n raaklyn aan die sirkel.	lyn $\perp$ radius **OF** **omgekeerde** raaklyn $\perp$ radius **OF** **omgekeerde** raaklyn $\perp$ middellyn
 O	Die lynstuk wat die middelpunt van 'n sirkel met die middelpunt van 'n koord verbind, is loodreg op die koord.	lyn vanuit midpt na midpt van koord
 O	Die loodlyn uit die middelpunt van 'n sirkel na 'n koord, halveer die koord.	lyn vanuit midpt $\perp$ op koord
 O	Die middelloodlyn van 'n koord gaan deur die middelpunt van die sirkel.	middelloodlyn van koord
	Die hoek wat 'n koord by die middel-punt van 'n sirkel onderspan, is dubbel die hoek wat dit by enige punt op die omtrek onderspan (aan dieselfde kant van die koord as die midpt)	midpts$\angle$ = 2 $\times$ omtreks$\angle$
 O	Die omtrekshoek wat deur die middellyn onderspan word, is 90º.	$\angle^e$ in halwe sirkel **OF** middelloodlyn onderspan regte hoek **OF** $\angle$ in ½ ⊙
 O	As 'n koord van 'n sirkel 'n regte hoek by die omtrek onderspan, dan is die koord 'n middellyn.	koord onderspan 90º **OF** **omgekeerde** $\angle^e$ in halwe sirkel

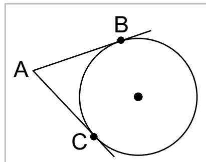
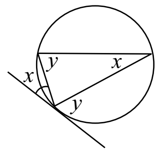
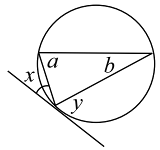
GROEP II

	Hoeke onderspan deur 'n koord van 'n sirkel, aan dieselfde kant van die koord, is gelyk.	$\angle^e$ in dieselfde segment
	As 'n lynstuk wat twee punte verbind, gelyke hoeke by twee ander punte aan dieselfde kant van die lynstuk onderspan, dan is die vier punte konsiklies. (d.w.s. hulle lê op die omtrek van 'n sirkel).	lynstuk onderspan gelyke $\angle^e$ **OF** **omgekeerde** $\angle^e$ in dieselfde segment
	Gelyke koorde onderspan gelyke omtrekshoeke.	gelyke koorde; gelyke $\angle^e$
	Gelyke koorde onderspan gelyke middelpuntshoeke.	gelyke koorde; gelyke $\angle^e$
	Gelyke koorde in gelyke sirkels onderspan gelyke omtrekshoeke.	gelyke sirkels; gelyke koorde; gelyke $\angle^e$
	Gelyke koorde in gelyke sirkels onderspan gelyke middelpuntshoeke. (A en B dui die middelpunte van die sirkels aan)	gelyke sirkels; gelyke koorde; gelyke $\angle^e$

GROEP III

	Die teenoorstaande hoeke van 'n koordvierhoek is supplementêr. (d.w.s. x en y is supplementêr)	teenoorst. $\angle^e$ van kvh
	As die teenoorstaande hoeke van 'n vierhoek supplementêr is, dan is die vierhoek 'n koordevierhoek.	teenoorst. $\angle^e$ van vierhk is supp **OF** **omgekeerde** teenoorst $\angle^e$ van koordevierhoek
	Die buitehoek van 'n koordevierhoek is gelyk aan die teenoorstaande binnehoek.	buite$\angle$ van kvh
	As die buitehoek van 'n vierhoek gelyk is aan die teenoorstaande binhoek, dan is die vierhoek 'n koordevierhoek.	buite$\angle$van vierhoek = teenoorst. binne$\angle$ **OF** **omgekeerde** buite$\angle$ van kvh

GROEP IV

	Twee raaklyne wat vanaf dieselfde punt buite 'n sirkel na 'n sirkel getrek word, is ewe lank (AB = AC).	Raaklyne vanuit gemeensk. punt **OF** Raaklyne vanaf dieselfde punt
	Die hoek wat gevorm word tussen 'n raaklyn aan 'n sirkel en 'n koord wat vanuit die raakpunt getrek word, is gelyk aan die hoek in die oorstaande segment.	raaklyn koord stelling
	As 'n lyn deur die eindpunt van 'n koord 'n hoek met die koord vorm wat gelyk is aan die hoek in die oorstaande segment, dan is die lyn 'n raaklyn aan die sirkel. (Indien $x = b$ of indien $y = a$ dan is die lyn 'n raaklyn aan die sirkel)	**omgekeerde** raaklyn koord stelling **OF** $\angle$ tussen lyn en koord

Euklidiese Meetkunde

Verwysings na TAS Wiskundeboeke

THE ANSWER SERIES *Your Key to Exam Success*

Gr 10 Wiskunde 3-in-1 (Module 7)

# 1:	Lyne, hoeke & driehoeke: hersiening • woordeskat & feite	7.1 ⟶ 7.7
# 2:	Vierhoeke: hersiening • definisies • stellings • oppervlaktes	7.8 ⟶ 7.15
# 3:	Die middelpuntstelling	7.16 ⟶ 7.17
# 4:	Veelhoeke/Poligone: definisies & tipes • binnehoeke • buitehoeke	7.18

Let wel: Die Gr 10 Eksemplaarvraestelle en Memo's is agter in die boek

Gr 11 Wiskunde 3-in-1 (Module 9)

# 1:	Hersiening van vorige grade	9.1 ⟶ 9.5
# 2:	Sirkelmeetkunde	9.6 ⟶ 9.26

Let wel: Die Gr 11 Eksemplaarvraestelle en Memo's is agter in die boek

Gr 12 Wiskunde 2-in-1 (Module 10)

# 1:	Sirkelmeetkunde	36 ⟶ 40
# 2:	Eweredigheidstelling	40 ⟶ 42
# 3:	Gelykvormige Driehoeke	42 ⟶ 43
# 4:	Gemeng	43
Agterbladsye:	Sirkelmeetkunde, Eweredigheid en Gelykvormige Driehoeke STELLINGS	i ⟶ iii
	Groepering van Sirkelmeetkunde Stellings	viii
	Omgekeerde Stellings in Sirkelmeetkunde	Ix
	Stellings & Aanvaarbare Redes	x ⟶ xii

Sien Uitdagende Vraeboekie bladsye 29 ⟶ 38

Sien die Onderwerpgids op bl. 148 vir verdere eksamenoefening

Gr 12 Wiskunde Ou Vraestelle 'Toolkit'

Agterbladsye:	Sirkelmeetkunde, Eweredigheid en Gelykvormige Driehoeke STELLINGS	i ⟶ iii
	Groepering van Sirkelmeetkunde Stellings	xiii
	Stellings & Aanvaarbare Redes	xiv ⟶ xvi

Sien die Onderwerpgidse: DBO: bl. 2 & IEB: bl. 40