ALGEBRA (65%): DBE NOVEMBER 2022

QUESTION 1 65%

1.1 Solve for x:

76% 1.1.1 $(3x - 6)(x + 2) = 0$ (2)

1.1.2 $2x^2 - 6x + 1 = 0$ *(correct to TWO decimal places)* (3)

1.1.3 $x^2 - 90 > x$ (4)

Memo

1.1.1 $(3x - 6)(x + 2) = 0$

$\therefore\ 3(x - 2)(x + 2) = 0$

$\therefore\ x - 2 = 0$ or $x + 2 = 0$

$\therefore\ \boldsymbol{x = \pm 2}$ ◄

1.1.2 $2x^2 - 6x + 1 = 0$

$\therefore\ x = \dfrac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(1)}}{2(2)}$ . . . $\boxed{x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$

$= \dfrac{6 \pm \sqrt{36 - 8}}{4}$

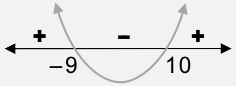
$= \dfrac{6 \pm \sqrt{28}}{4}$

$\approx$ **2,82 or 0,18** ◄

1.1.3 $x^2 - 90 > x$

$\therefore\ x^2 - x - 90 > 0$

$\therefore\ (x + 9)(x - 10) > 0$



$\therefore\ \boldsymbol{x < -9}$ **or** $\boldsymbol{x > 10}$ ◄

Common Errors and Misconceptions

(a) Some candidates **multiplied the brackets out** but **then factorised incorrectly** in **Q1.1.1**.

(b) Rounding off the answers to two decimal places is still a problem for some candidates. Some candidates still lack basic calculator skills. A few candidates took the value of b in **Q1.1.2** to be 6 instead of –6.

(c) In answering **Q1.1.3** many candidates **treated the inequality as an equation**. Their answer read: $(x + 9)(x - 10) > 0$ followed by $x > -9$ or $x > 10$.

Many candidates struggled to interpret the correct answer from the inequality.

$x^2 - x - 90 > 0$

$(x + 9)(x - 10) > 0$

$\therefore\ x = -9$ or $x = 10$

$\therefore\ -9 < x < 10$

Some candidates drew a **sketch** but were unable to use it to write down the required answer. Another common error was the incorrect notation in the answer. Candidates wrote the answer as $-9 < x > 10$ instead of $x < -9$ or $x > 10$.

1.1.4 $x - 7\sqrt{x} = -12$ (4)

1.1.4 $x - 7\sqrt{x} = -12$

$\therefore \left(\sqrt{x}\right)^2 - 7\sqrt{x} + 12 = 0$

$\therefore \left(\sqrt{x} - 3\right)\left(\sqrt{x} - 4\right) = 0$

$\therefore \sqrt{x} = 3$ or 4

$\therefore$ **$x = 9$ or 16** ➤ *. . . both valid*

OR: $+7\sqrt{x} = +x + 12$

$49x = x^2 + 24x + 144$

$0 = x^2 - 25x + 144$

$0 = (x - 9)(x - 16)$

$\therefore$ **$x = 9$ or $x = 16$** ➤

1.2 Solve for x and y simultaneously:

85%

$2x - y = 2$

$xy = 4$ (5)

1.2 $2x - y = 2$

$\therefore 2x - 2 = y$ . . . ❶

$xy = 4$ . . . ❷

❶ in ❷: $\therefore x(2x - 2) = 4$

$\therefore 2x^2 - 2x - 4 = 0$

$(\div 2)$ $\therefore x^2 - x - 2 = 0$

$\therefore (x - 2)(x + 1) = 0$

$\therefore x = 2$ or -1

For $x = 2$: $y = 2(2) - 2 = 2$

& For $x = -1$: $y = 2(-1) - 2 = -4$

$\therefore$ Solution: **(2; 2)** **or** **(–1; –4)** ➤

Common Errors and Misconceptions

(d) Most candidates had some idea that they had to square both sides of the equation in **Q1.1.4**. However, few candidates were able to **isolate the *surd* before** squaring both sides of the equation.

The following errors were noted:

$(x - 7\sqrt{x})^2 = (-12)^2$

$\therefore x^2 - 49x = 144$

or

$x - 7\sqrt{x} = -12$

$\therefore \sqrt{x} = \dfrac{-12}{x - 7}$

No idea!

(e) While many candidates answered **Q1.2** correctly, a few made errors when they wrote the linear equation in terms of one variable. Some candidates disregarded a valid solution. They confused simultaneous equations with surd equations.

1.3 Show that $2.5^n - 5^{n+1} + 5^{n+2}$ is even for all positive integer values of n. (3)

41%

Memo

1.3

$$\begin{aligned} & 2.5^n - 5^{n+1} + 5^{n+2} \\ &= 2.5^n - 5^n.5 + 5^n.5^2 \\ &= 5^n(2 - 5 + 25) \\ &= 5^n(22) \\ &= 2(5^n \times 11),\ \text{an even number for all } n \in \mathbb{N} \end{aligned}$$

1.4 Rephrased question.

22%

Determine a pair of integral values of x and y for which:

$$\frac{3^{y+1}}{32} = \sqrt{96^x}.$$

(4) **[25]**

Memo

1.4

$$\frac{3^{y+1}}{32} = \sqrt{96^x}$$

$$\therefore \frac{3^{y+1}}{2^5} = \sqrt{(2^5.3)^x}$$

$$\therefore 2^{-5}.3^{y+1} = 2^{\frac{5x}{2}}.3^{\frac{x}{2}}$$

2	96
2	48
2	24
2	12
2	6
3	3
	1

$96 = 2^5.3$

If we equate the exponents of the 2s and the 3s

$\therefore \frac{5x}{2} = -5$ and $\frac{x}{2} = y + 1$

$\therefore 5x = -10$ $\therefore \frac{-2}{2} = y + 1$

$\therefore x = -2$ $\therefore y = -2$

$\therefore$ A solution is: **(–2; –2)** ➤

Common Errors and Misconceptions

(f) In **Q1.3** many candidates were able to write the exponents with separate bases, i.e. $2.5^n - 5^{n+1} + 5^{n+2} = 2.5^n - 5^n.5 + 5^n.5^2$.

However, some failed to factorise the resulting expression correctly, i.e. $2.5^n - 5^n.5 + 5^n.5^2 = 5^n(2.1^n - 1^n.5 + 1^n.5^2)$.

The majority of the candidates could not explain why $5^n(22)$ will be even numbers for all positive integral values of n.

(g) Many candidates did not know how to answer **Q1.4**. They were unable to write both sides of the equation as exponents which had the same bases.

Note:

One needs **2** equations in x and y to find 0,1 or 2 solutions. There are actually infinite solutions when only 1 equation is given. However, because integral solutions are asked for, there is only one possible solution.

QUESTION 1: Suggestions for Improvement

(a) Much of the **work** in this question is **covered in Grade 11**. It is therefore important for teachers to set **revision tasks** on these sections **throughout the Grade 12 year**.

(b) Teachers should not take for granted that learners know how to round off a number to the required number of places. Where necessary, this skill should be retaught in Grades 11 and 12. Teachers should penalise learners in SBA tasks when they do not round off to the correct number of places.

(c) Teachers should take some time, preferably in **Grade 10**, to focus on teaching learners how to represent **inequalities** (e.g. $-3 < x < 5$; $x < -3$ or $x > 5$) **on a number line and** also how **to write an inequality** from the illustration on a number line. This will benefit learners as they are required to write inequality solutions for a number of questions in both examination papers. Emphasis on the correct notation is essential when writing down the solutions to inequalities.

Inequalities

(d) Teachers should explain the difference between ***and*** and ***or*** in the context of **inequalities**. Learners cannot use these words interchangeably as they have different meanings.

(e) When dealing with ***surd* equations**, learners should be reminded that they need to isolate the *radical* before they can square both sides of the equation. Teachers must emphasise that **implicit restrictions** are placed on *surd* equations and that learners should continue to **test** whether their **answers** satisfy the original equation.

(f) Learners need to be exposed to complex questions involving *surds* and *exponents*.

PATTERNS & SEQUENCES (56,4%): DBE NOV. 2022

QUESTION 2 50%

2.1 The first term of a geometric series is 14 and the 6th term is 448.

55%

2.1.1 Calculate the value of the constant ratio, r. (2)

2.1.2 Determine the number of consecutive terms that must be added to the first 6 terms of the series in order to obtain a sum of 114 674. (4)

2.1.3 If the first term of another series is 448 and the 6th term is 14, calculate the sum to infinity of the new series. (3)

Memo

2. G.S.: $a = 14$ and $T_6 = 448$; r?

2.1.1 $T_6 = ar^5 = 448$ . . . $\boxed{\mathbf{T_n = ar^{n-1}}}$

$\therefore 14r^5 = 448$

$\therefore r^5 = 32$

$\therefore \mathbf{r = 2}$ ✓

2.1.2 $114\,674 = \dfrac{14(2^n - 1)}{2 - 1}$ . . . $\boxed{\mathbf{S_n = \dfrac{a(r^n - 1)}{r - 1}}}$

$\therefore \dfrac{114\,674}{14} = 2^n - 1$

$\therefore \dfrac{114\,674}{14} + 1 = 2^n$

$\therefore 8\,192 = 2^n$

$\therefore 2^{13} = 2^n$

$\therefore n = 13$

$\therefore$ No. of consecutive terms added $= 13 - 6 = \mathbf{7}$ ✓

2.1.3 $T_1 = 448$ & $T_6 = 14$

$\therefore a = 448$ & $r = \frac{1}{2}$ . . . *The given series is reversed!*

$\therefore \boxed{\mathbf{S_\infty = \dfrac{a}{1 - r}}} = \dfrac{448}{1 - \frac{1}{2}} = \mathbf{896}$ ✓

Common Errors and Misconceptions

(a) Many candidates did not understand what was required in **Q2.1.2**.

They correctly calculated that the sum of the first 13 terms was 114 674. However, they did not subtract 6 from 13 in order to answer the required question.

(b) While many candidates answered **Q2.1.3** correctly, some candidates failed to realise that for a **convergent geometric series** the value of r must lie in the interval: $\mathbf{-1 < r < 1}$.

2.2 If $\sum_{p=0}^{k}\left(\frac{1}{3}p+\frac{1}{6}\right) = 20\frac{1}{6}$, determine the value of k. (5)

42%

[14]

Memo

2.2 $\sum_{p=0}^{k}\left(\frac{1}{3}p+\frac{1}{6}\right) = \left[\frac{1}{3}(0)+\frac{1}{6}\right]+\left[\frac{1}{3}(1)+\frac{1}{6}\right]+\left[\frac{1}{3}(2)+\frac{1}{6}\right]+\dots$

$= \frac{1}{6}+\frac{1}{2}+\frac{5}{6}+\dots$

$\left[T_3 - T_2 = \frac{5}{6}-\frac{1}{2} = \frac{1}{3} \ \& \ T_2 - T_1 = \frac{1}{2}-\frac{1}{6} = \frac{1}{3}\right]$

AP $a = \frac{1}{6}$ and $d = \frac{1}{3}$

$\boxed{\mathbf{S_n = \frac{n}{2}[2a+(n-1)d]}}$

$\therefore\ 20\frac{1}{6} = \frac{n}{2}\left[2\left(\frac{1}{6}\right)+(n-1)\left(\frac{1}{3}\right)\right]$

$\therefore\ \frac{121}{6} = \frac{n}{2}\left[\frac{1}{3}+\frac{1}{3}n-\frac{1}{3}\right]$

$\therefore\ 121 = 3n\left[\frac{1}{3}n\right]$

$= n^2$

$\therefore\ n = 11$

$\therefore\ \mathbf{k = 10}$ ✓ *(because p starts at 0)*

Common Errors and Misconceptions

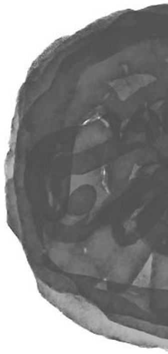
(c) In **Q2.2** many candidates could not interpret the ***sigma notation*** correctly. Some of them mistook the general term given in the *sigma notation* to represent a geometric series. A fair number of candidates were unable to calculate the first three terms of the series and consequently were unable to calculate the common difference.

A number of candidates failed to make the connection that $n = k + 1$. Some candidates incorrectly assumed that $T_n = 20\frac{1}{6}$ instead of $S_n = 20\frac{1}{6}$.

QUESTION 2: Suggestions for Improvement

(a) Learners must be taught how to **identify** the **type of sequence** they are working with and **which formulae** are applicable to it.

(b) Make learners aware of which formulae on the information sheet apply to which type of sequence. It is good practice for learners to use the information sheet in class so that they become familiar with it.



(c) Teach learners how to **identify whether** the question requires them to calculate the value of the n^{th} term **or** the sum of the first n terms.

(d) Learners must read the questions carefully so that they know what is required of them.

(e) Remind learners that **n** cannot be a negative number, zero or a fraction. When solving for n, learners should arrive at **a natural number** solution. If this is not the case, then they should know that they have made a mistake in their working. Learners must be told that if they round off their answers to a natural number, then they will not be awarded any marks.

(f) It is important to demonstrate, by way of example, the concept of a convergent geometric series, first by taking a value of $r > 1$ and then taking a value of $-1 < r < 1$. This should alert learners to the condition for which a geometric series will converge.

(g) More attention should be given to expanding a series that is given in ***sigma notation*** form.

QUESTION 3 64%

It is given that the general term of a quadratic number pattern is

$T_n = n^2 + bn + 9$ and the first term of the first differences is 7.

3.1 Show that $b = 4$. (2)

77%

3.2 Determine the value of the 60th term of this number pattern. (2)

88%

Memo

3.1 Quadratic number pattern

$$\mathbf{T_n = n^2 + bn + 9}$$

$$T_1 = 1 + b + 9 = b + 10$$

$$T_2 = 2^2 + 2b + 9 = 2b + 13$$

$$\therefore (2b + 13) - (b + 10) = 7 \quad \ldots$$ *The 1st term of the 1st differences*

$$\therefore b + 3 = 7$$

$$\therefore \mathbf{b = 4} \blacktriangleleft$$

3.2 $T_{60} = 60^2 + (4)(60) + 9$

$= \mathbf{3\ 849} \blacktriangleleft$

Common Errors and Misconceptions

(a) When answering **Q3.1**, many candidates failed to identify from the question that $a = 1$ and that **$3a + b$** $= 7$.

Instead, they used $b = 4$ in their calculations to prove that **the first of the first differences** is 7.

3.3 **58%** Determine the general term for the sequence of first differences of the quadratic number pattern.

Write your answer in the form $T_p = mp + q$. (3)

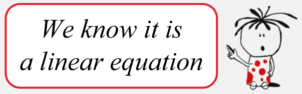
3.4 **46%** Which TWO consecutive terms in the quadratic number pattern have a first difference of 157? (3)

[10]

Memo

3.3 $\quad 14 \quad 21 \quad 30 \quad 41$

$\qquad\quad 7 \quad\ 9 \quad 11$



$\mathbf{T_p = 2p + 5}$ ✓ . . . by inspection

OR: $T_p = mp + q$

$\therefore T_1 = m + q = 7$ . . . ❶

& $T_2 = 2m + q = 9$ . . . ❷

❷ – ❶: $\therefore m = 2$

❶: & $q = 5$

$\therefore$ The general term is: $\mathbf{T_p = 2p + 5}$ ✓

3.4 The general term of the 1st differences:

$2n + 5 = 157$

$\therefore 2n = 152$

$\therefore n = 76$

$\therefore$ The 2 consecutive terms of the quadratic number pattern are the **76th and 77th** ✓

OR: Let the consecutive terms be $\mathbf{T_n}$ and $\mathbf{T_{n+1}}$; then:

$$T_{n+1} - T_n = 157$$

$$\therefore [(n+1)^2 + 4(n+1) + 9] - [n^2 + 4n + 9] = 157$$

$$\therefore \cancel{n^2} + 2n + 1 + 4n + 4 + 9 - \cancel{n^2} - 4n - 9 = 157$$

$$\therefore 2n + 5 = 157$$

$$\therefore 2n = 152$$

$$\therefore n = 76\text{, etc.}$$

Common Errors and Misconceptions

(b) Many candidates were able to answer **Q3.3** correctly. However, they failed to make the connection between Q3.3 and **Q3.4**. In answering Q3.4, many candidates incorrectly stated that $n^2 + 4n + 9 = 157$.

QUESTION 3: Suggestions for Improvement

(a) When teaching *quadratic number patterns*, it is essential to show learners how the formulae: $\mathbf{T_1 = a + b + c}$, the first term of the first differences = $\mathbf{3a + b}$ and the second difference = $\mathbf{2a}$, are deduced.

(b) The sequence of first differences of a ***quadratic number pattern*** forms an ***arithmetic pattern***. This implies that an arithmetic sequence is embedded within a quadratic number pattern. Learners must read the question very carefully in order to establish to which pattern the question is referring. Glossing over words in the question leads to learners making incorrect statements.

(c) Remind learners that n cannot be a negative number, zero or a fraction. When solving for n, learners should arrive at a natural number solution. If this is not the case, then they have made a mistake in their working.

(d) Teachers should integrate quadratic number patterns with arithmetic patterns and quadratic functions. This will provide learners with the skills to deal with higher-order questions.

GRAPHS & FUNCTIONS (54%): DBE NOVEMBER 2022

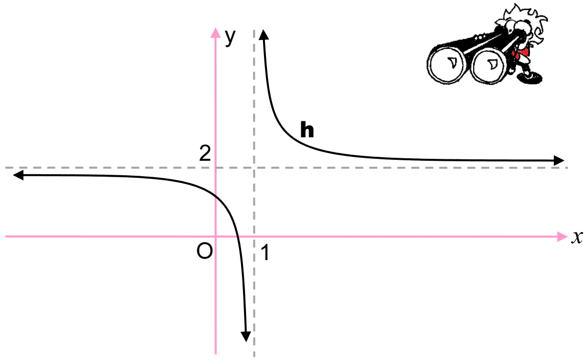
QUESTION 4 54%

Standard form!

4.1 Sketched below is the graph of $h(x) = \dfrac{1}{x + p} + q$.

53%

The asymptotes of h intersect at the point (1; 2).



4.1.1 Write down the values of p and q. (2)

4.1.2 Calculate the coordinates of the x-intercept of h. (2)

4.1.3 Write down the x-coordinate of the x-intercept of g if $g(x) = h(x + 3)$. (2)

4.1.4 The equation of an axis of symmetry of h is $y = x + t$.

Determine the value of t. (2)

4.1.5 Determine the values of x for which $-2 \le \dfrac{1}{x - 1}$. (3)

Memo

4.1.1 **p = –1** ◄ **q = 2** ◄

4.1.2 ***x*-int.**: Subst. y = 0 in $y = \dfrac{1}{x-1} + 2$

$\therefore\ -2 = \dfrac{1}{x-1}$

$\therefore\ x - 1 = -\dfrac{1}{2}$

$\therefore\ x = \dfrac{1}{2}$

$\therefore\ \left(\dfrac{1}{2}; 0\right)$ ◄

4.1.3 The x-coordinate: $x = -2\dfrac{1}{2}$ ◄ . . . $\dfrac{1}{2} - 3$

[Note: g is h moved 3 units to the left.]

4.1.4 [Note: This axis of symmetry has a positive gradient.]

The axis of symmetry passes through the point (1; 2)

$\therefore$ Substitute in $y = x + t$

$2 = 1 + t$

$\therefore\ \mathbf{t = 1}$ ◄

4.1.5 $h(x) = \dfrac{1}{x-1} + 2$

$h(x) \ge 0$ ➡ $\dfrac{1}{x-1} + 2 \ge 0$

$\therefore\ \dfrac{1}{x-1} \ge -2$

$\mathbf{h(x) \ge 0}$ **for** $\mathbf{x \le \frac{1}{2}}$ **or** $\mathbf{x > 1}$ ◄

Common Errors and Misconceptions

(a) A number of candidates incorrectly stated that p = 1 instead of **p = –1** when answering **Q4.1.1**.

Right shift, then p is neg.

(b) Many candidates answered **Q4.1.2** correctly. However, they did not realise that they had to translate the x-intercept 3 units to the left to obtain the answer for **Q4.1.3**. The challenge in this question was that candidates were unable to interpret the notation correctly.

Left shift, then p is pos.

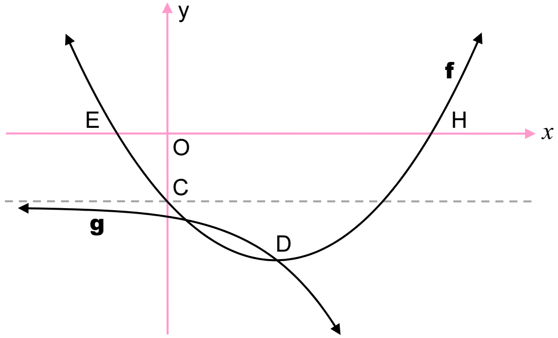
(c) The most common error candidates made when answering **Q4.1.5** was to solve the equation algebraically. Candidates failed to relate the given equation to the given hyperbola.

USE THE GRAPH to solve the inequality!

4.2 The graphs of $f(x) = x^2 - 4x - 5$ and $g(x) = a.2^x + q$ are sketched below.

55%

- E and H are the x-intercepts of f.
- C is the y-intercept of f and lies on the asymptote of g.
- The two graphs intersect at D, the turning point of f.



4.2.1 Write down the y-coordinate of C. (1)

4.2.2 Determine the coordinates of D. (2)

4.2.3 Determine the values of a and q. (3)

4.2.4 Write down the range of g. (1)

4.2.5 Determine the values of k for which the value of $f(x) - k$ will always be positive. (2)

[20]

Memo

4.2.1 $\mathbf{y_C = -5}$ ✓

4.2.2 At D:

$$x = -\frac{b}{2a} = -\frac{-4}{2(1)} = 2$$

OR: $f'(x) = 0$ $\therefore 2x - 4 = 0$ $\therefore x = 2$ etc.

& $y = 2^2 - 4(2) - 5 = -9$

$\therefore$ **Pt D is (2; –9)** ✓

OR: $y = x^2 - 4x - 5$ $\therefore y = (x+1)(x-5)$ $\therefore$ The roots are -1 & 5 $\therefore x_D = \frac{-1+5}{2} = 2$ etc.

4.2.3 $g(x) = a.2^x + q$

$\mathbf{q = y_C = -5}$ ✓ . . . *$y = q$ is the asymptote of g*

D(2; –9) on g: $\therefore -9 = a.2^2 - 5$ $\therefore -4 = 4a$ $\therefore \mathbf{a = -1}$ ✓

4.2.4 $\mathbf{y \in \mathbb{R};\ y < -5}$ ✓

4.2.5 $y = f(x) + 9$ would be ≥ 0 (i.e. touch the x-axis)

$\therefore -k > 9$

$\therefore \mathbf{k < -9}$ ✓

OR: for $f(x) = x^2 - 4x - 5 - k$, $\Delta < 0$, etc.

(d) In **Q4.2.2** many candidates were able to calculate the axis of symmetry correctly **but** were **unable** to calculate the y-coordinate of the turning point correctly.

(e) Despite having obtained the value of q correctly, many candidates were unable to calculate the value of a correctly in **Q4.2.3**.

(f) The common error in **Q4.2.4** was **notation**. Candidates wrote $y \in (-5\,;-\infty)$ or $y \in (-\infty\,;-5]$ instead of $y \in (-\infty\,;-5)$.

RANGE

(g) Very few candidates attempted **Q4.2.5** because they **failed to interpret** the notation used in the question.

USE GRAPHS to find k

QUESTION 4: Suggestions for Improvement

(a) Teachers need to pay more attention to using the correct **notation** when writing **intervals**. Pay attention to the meaning of the notation and the difference between the **various notations**. Teachers must not condone the use of incorrect notation in SBA tasks.

INTERVALS

(b) A hyperbola can be defined in two ways, viz. $y = \frac{a}{x+p} + q$ and $\cancel{y = \frac{a}{x-p} + q}$.

Teachers are advised to use $\mathbf{y = \frac{a}{x+p} + q}$ as it is referred to **in the *CAPS***. It would seem that learners do not pay attention to the difference in signs between the two equations. However, they are penalised when they write down the value of *p* incorrectly.

Transformations are in the Senior Phase CAPS curriculum but excluded in the current ATPs!

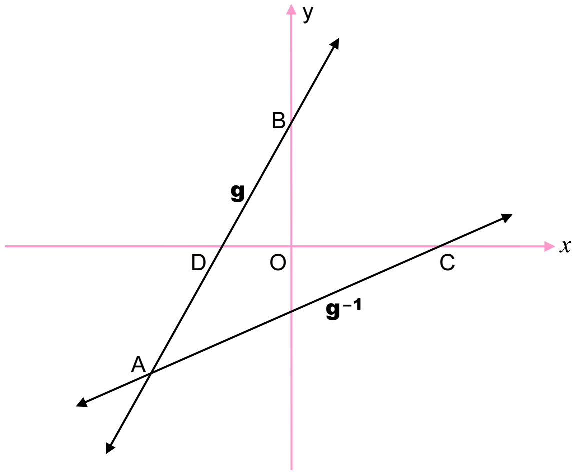
(c) ***Transformation*** is no longer a topic in the curriculum. **However**, in **the section of functions, the basic graph is transformed to obtain all other graphs**. The effect of the transformation on the equation of the basic graph must be discussed. In doing so, it is suggested that teachers also pay attention to the effect that the transformation has on the basic equation, or vice versa.

(d) Learners need to be made aware that an inequality containing fractions cannot be solved in the same way as an equation. When solving an equation, we multiply both sides by the LCD to eliminate the fractions. When solving an inequality, we multiply both sides by the $(LCD)^2$.

QUESTION 5 54%

The graphs of $g(x) = 2x + 6$ and g^{-1}, the inverse of g, are shown in the diagram below.

- D and B are the x- and y-intercepts respectively of g.
- C is the x-intercept of g^{-1}.
- The graphs of g and g^{-1} intersect at A.



5.1 Write down the y-coordinate of B. (1)

91%

5.2 Determine the equation of g^{-1} in the form $g^{-1}(x) = mx + n$. (2)

62%

Memo

5.1 $\mathbf{y_B = 6}$ ✓

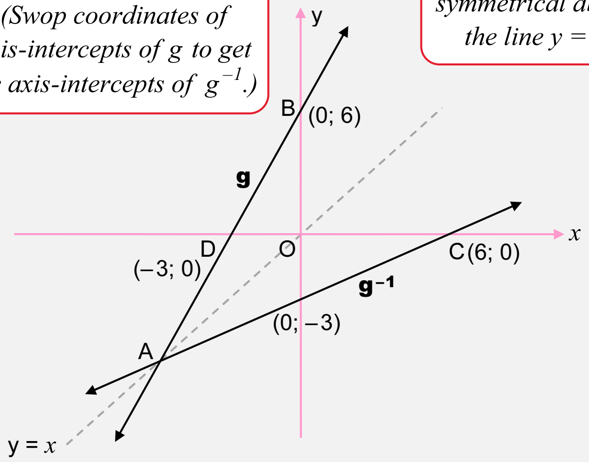
5.2 Equation of g^{-1}: $\mathbf{y = \frac{1}{2}x - 3}$ ✓

By inspection!

(Swop coordinates of axis-intercepts of g to get the axis-intercepts of g^{-1}.)

Note:

g and g^{-1} are symmetrical about the line $y = x$



OR: Equation of g: $y = 2x + 6$

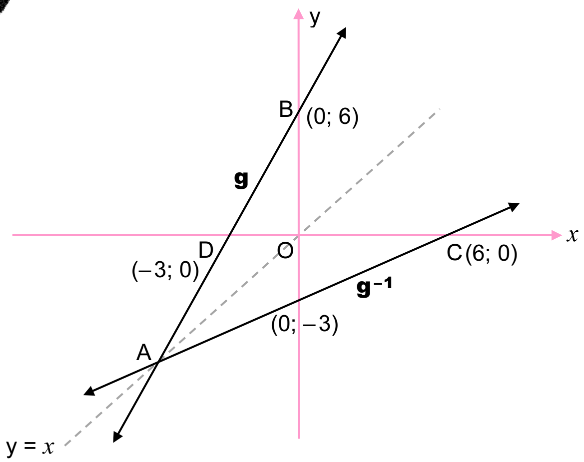
$\therefore$ Equation of g^{-1}: $x = 2y + 6$

$\therefore -2y = -x + 6$

$\therefore \mathbf{y = \frac{1}{2}x - 3}$ ✓

Common Errors and Misconceptions

(a) In **Q5.2** most of the candidates were able to correctly swop x and y in the equation. However, some did not make y the subject of the formula while others made errors when transposing terms. Many candidates did not understand the **difference** in the notations: $\mathbf{g'(x)}$ **and** $\mathbf{g^{-1}(x)}$. Some candidates simply wrote down *log* when they saw $g^{-1}(x)$. They believe that the log function is the inverse of all functions. This is incorrect.



5.3 Determine the coordinates of A. (3)

63%

5.4 Calculate the length of AB. (2)

69%

Memo

5.3 At A: $y = \frac{1}{2}x - 3$ and $y = 2x + 6$

$\therefore \frac{1}{2}x - 3 = 2x + 6$

$\therefore -\frac{3}{2}x = 9$

$\times\left(-\frac{2}{3}\right) \quad \therefore x = -6$

& $y = 2(-6) + 6 = -6$

$\therefore$ **A(−6; −6)** ◄

5.4 $AB^2 = (6+6)^2 + (0+6)^2$

$= 144 + 36$

$= 180$

$\therefore AB = \sqrt{180} = 6\sqrt{5} \approx$ **13,42 units** ◄

Common Errors and Misconceptions

(b) When answering **Q5.3**, many candidates calculated the **intercepts** with the axes **instead of** calculating the **point of intersection** of the two lines. Many candidates did not show any working but simply wrote down incorrect coordinates of A as if they had read them off the graph.

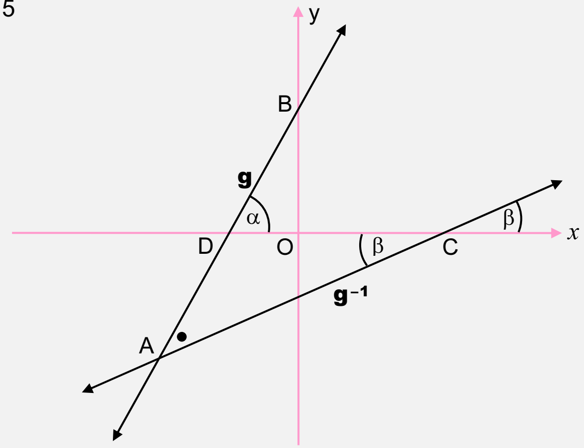
5.5 Calculate the area of ΔABC. (5)

[13]

32%

Memo

5.5



$m_{AB} = 2$ ➡ $\alpha = 63{,}434...^\circ$

$m_{AC} = \frac{1}{2}$ ➡ $\beta = 26{,}565...^\circ$

$\therefore\ B\hat{A}C = \alpha - \beta = 36{,}869...$

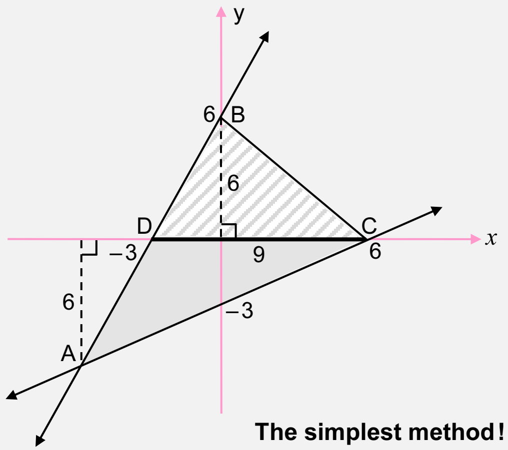
The area of $\Delta ABC = \frac{1}{2} AB.AC \sin B\hat{A}C$

where $AC = AB = \sqrt{180}$

$\therefore$ Area of $\Delta ABC = \frac{1}{2}\sqrt{180}\sqrt{180}\ \sin 36{,}869...^\circ$

$= $ **54 units²** ◄

OR:



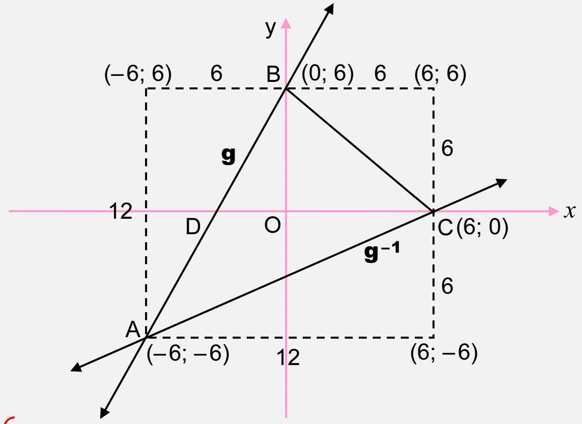
The simplest method!

Area of ΔABC

$= \Delta ADC + \Delta BDC$

$= \frac{1}{2}(9)(6) + \frac{1}{2}(9)(6)$

$= $ **54 units²** ◄



OR: Area of ΔABC

$= (12 \times 12) - \frac{1}{2}(12 \times 6) - \frac{1}{2}(6 \times 6) - \frac{1}{2}(12 \times 6)$

Area of a square & Area of Δ^s

There are several methods.

Common Errors and Misconceptions

(c) Many candidates made incorrect assumptions when answering **Q5.5**. The most common of these was to assume that AB was perpendicular to BC. Some candidates used the incorrect angle when using the area formula: $A = \frac{1}{2} ab \sin C$.

They used an angle of inclination instead of the angle between the two sides of the triangle.

QUESTION 5: Suggestions for Improvement

(a) Teachers should spend some time discussing that **all points on the x-axis** have a **y-coordinate of 0** and **all points on the y-axis** have a **x-coordinate of 0**. The domain is always a set of x-values and the range is always a set of y-values.

(b) Learners must be taught to respond to the question. They should write down the **coordinates** of a point when the question requires them to do so.

(c) The section of inverses opens itself to an investigative approach to teaching. Teachers should take the opportunity to allow learners to discover that **a function and its inverse are symmetrical about the line $y = x$** and that the point of intersection, where it exists, of a function and its inverse will also lie on the line $y = x$.

(d) Teachers need to emphasise that the height of a triangle is perpendicular to its base.

FINANCE (47%): DBE NOVEMBER 2022

QUESTION 6 57%

6.1 **49%** R12 000 was invested in a fund that paid interest at m% p.a., compounded quarterly. After 24 months, the value of the investment was R13 459.

Determine the value of m. (4)

6.2 **66%** On 31 January 2022, Tino deposited R1 000 in an account that paid interest at 7,5% p.a., compounded monthly. He continued depositing R1 000 on the last day of every month. He will make the last deposit on 31 December 2022.

Will Tino have sufficient funds in the account on 1 January 2023 to buy a computer that costs R13 000?

Justify your answer by means of an appropriate calculation. (4)

Memo

6.1

$$13\,459 = 12\,000\left(1 + \frac{m\%}{4}\right)^{2\times 4} \qquad \boxed{\mathbf{A = P(1+i)^n}}$$

$$\therefore \frac{13\,459}{12\,000} = \left(1 + \frac{m\%}{4}\right)^{8}$$

$$\therefore 4\left(\sqrt[8]{\frac{13\,459}{12\,000}} - 1\right) = m\%$$

$$\therefore m\% = \mathbf{5{,}78\%} \blacktriangleleft$$

6.2

$$\boxed{\mathbf{F_v = \frac{x\left[(1+i)^n - 1\right]}{i}}}$$

$$= \frac{1\,000\left[\left(1 + \frac{7{,}5\%}{12}\right)^{12} - 1\right]}{\frac{7{,}5\%}{12}}$$

$$= \text{R}12\,421{,}22$$

$\therefore$ **Tino will not have sufficient funds.** ◄

Common Errors and Misconceptions

(a) In **Q6.1** some candidates used the simple interest formula instead of the compound interest formula.

Many candidates used the incorrect value of n in the compound interest formula. They used 4, 12 or 24 as the value of n instead of 8.

6.3 Thabo plans to buy a car that costs R250 000. He will pay a deposit of 15% and take out a loan for the balance. The interest on the loan is 13% p.a., compounded monthly.

33%

6.3.1 Calculate the value of the loan. (1)

6.3.2 The first repayment will be made 6 months after the loan has been granted. The loan will be repaid over a period of 6 years after it has been granted. Calculate the MONTHLY instalment. (5)

[14]

Memo

6.3.1 15% of 250 000 = R37 500

$\therefore$ 250 000 – 37 500 = **R212 500** ✓

6.3.2 13% p.a. monthly

$$\boxed{P_V = \frac{x\left[1-(1+i)^{-n}\right]}{i}}$$

$$212\,500 = \frac{x\left[1-\left(1+\frac{13\%}{12}\right)^{-(6\times 12-5)}\right]}{\frac{13\%}{12}} \cdot \left(1+\frac{13\%}{12}\right)^{-5}$$

x = **R4 724,96** ✓

$$\text{OR: } 212\,500\left(1+\frac{0,13}{12}\right)^{5} = \frac{x\left[1-\left(1+\frac{0,13}{12}\right)^{-67}\right]}{\frac{0,13}{12}}$$

x = **R4 724,96** ✓

$$\text{OR: } 212\,500\left(1+\frac{0,13}{12}\right)^{72} = \frac{x\left[\left(1+\frac{0,13}{12}\right)^{67}-1\right]}{\frac{0,13}{12}}$$

x = **R4 724,96** ✓

Common Errors and Misconceptions

(b) Many candidates failed to take into account that **Q6.3.2** was based on a **deferred payment**. Consequently, they did not **add interest to the initial loan amount**. Common errors were:

$$212\,500 = \frac{x\left[1-\left(1+\frac{0,13}{12}\right)^{-72}\right]}{\frac{0,13}{12}} \quad \textbf{or} \quad 212\,500 = \frac{x\left[1-\left(1+\frac{0,13}{12}\right)^{-67}\right]}{\frac{0,13}{12}}$$

(See memo)

QUESTION 6: Suggestions for Improvement

(a) Learners need deeper insight into the relevance of each of the formulae and under which circumstances each can be used. The **variables** in each formula must be explained. More practice in Financial Mathematics is necessary so that learners can identify when to use the different formulae.

(b) Teachers should demonstrate all the steps required when using a **calculator**. Learners should be penalised for rounding off early in formal assessment tasks at school.

(c) The difference between ***compound interest*** and ***future value annuities*** must be explained.

(d) The correct Financial Mathematics language should be used in class and learners should read the question with understanding.

(e) Teachers should expose learners to the various scenarios for loan repayments, viz. deferred payments, missed payments, outstanding balance immediately after a certain number of payments, final payment, etc.

CALCULUS (41,5%): DBE NOVEMBER 2022

QUESTION 7 61%

7.1 Determine $f'(x)$ from first principles if $f(x) = x^2 + x$. (5)

76%

7.2 Determine $f'(x)$ if $f(x) = 2x^5 - 3x^4 + 8x$. (3)

88%

Memo

7.1
$$\begin{aligned} f(x) &= x^2 + x \\ \therefore\ f(x+h) &= (x+h)^2 + x + h \\ &= x^2 + 2xh + h^2 + x + h \\ \therefore\ f(x+h) - f(x) &= 2xh + h^2 + h \\ \therefore\ \frac{f(x+h)-f(x)}{h} &= \frac{2xh + h^2 + h}{h} \\ &= 2x + h + 1 \end{aligned}$$

$$\therefore\ \boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}} = \lim_{h \to 0} (2x + h + 1)$$
$$= \mathbf{2x + 1} \ ◄$$

7.2
$$f(x) = 2x^5 - 3x^4 + 8x$$
$$\therefore\ \mathbf{f'(x) = 10x^4 - 12x^3 + 8} \ ◄$$

Common Errors and Misconceptions

(a) In **Q7.1** some candidates made the following **notational errors**:

$$f(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad \text{or} \quad \lim_{h \to 0} = \frac{f(x+h) - f(x)}{h}.$$

Serious notational errors

They lost a mark for these errors.

Other candidates made **errors in substitution**, i.e.:

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 + (x+h) - x^2 + x}{h} \quad \textbf{or} \quad f'(x) = \lim_{h \to 0} \frac{(x+h)^2 + x - (x^2 + x)}{h}$$

Another common error was **incorrect factorisation**, i.e.:

$$\begin{aligned} f'(x) &= \lim_{h \to 0} \frac{2xh + h^2 + h}{h} \\ &= \lim_{h \to 0} \frac{h(2x + h)}{h} \\ &= \lim_{h \to 0} (2x + h) \\ &= 2x \end{aligned}$$

ALGEBRA

7.3 The tangent to $g(x) = ax^3 + 3x^2 + bx + c$ has a minimum gradient at the point (–1; –7).

22%

For which values of x will g be concave up? (4)

[12]

Memo

7.3 $g(x) = ax^3 + 3x^2 + bx + c$

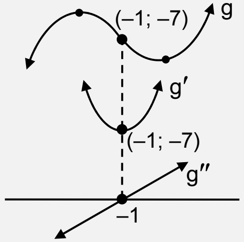
The gradient of the tangent to g

$= g'(x) = 3ax^2 + 6x + b$

& the minimum of this gradient occurs when $x = -1$

$\therefore$ g wil be concave up for $\boldsymbol{x > -1}$ ◂

Note: This is where $g''(x) > 0$



OR:

$$
\begin{aligned}
g(x) &= ax^3 + 3x^2 + bx + c\\
\therefore\ g'(x) &= 3ax^2 + 6x + b\\
\therefore\ g''(x) &= 6ax + 6\\
\therefore\ 0 &= 6a(-1) + 6\\
\therefore\ 6a &= 6\\
\therefore\ a &= 1
\end{aligned}
$$

$g''(x) = 6x + 6$ ➡ $g''(x) > 0$

$\therefore\ 6x + 6 > 0$

$\therefore\ 6x > -6$

$\therefore\ \boldsymbol{x > -1}$ ◂

Common Errors and Misconceptions

(b) When answering **Q7.3**, many candidates calculated the first and second derivatives correctly. However, they substituted incorrectly into the second derivative, i.e. $6a(-1)^2 + 6 = -7$. They confused the value of the second derivative at the point of inflection with the minimum gradient of the tangent.

QUESTION 7: Suggestions for Improvement

(a) Emphasis should be placed on the use of the correct notation when determining the derivative, either when using first principles or the rules.

(b) Teachers should explain the need for **brackets** when determining the derivative from **first principles**.

This prevents the incorrect simplification that follows.

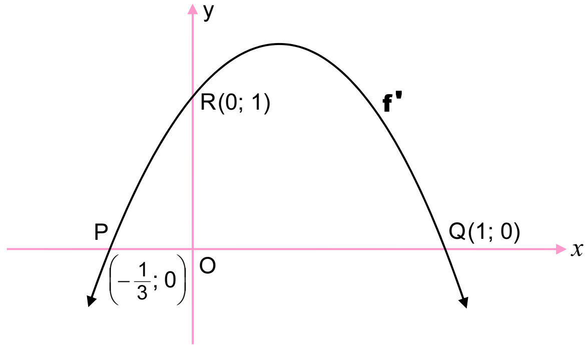
(c) The **concepts** of **gradient of tangent** and **point of inflection** must be understood. Teachers should explain the difference between the two concepts.

(d) The focus when teaching *cubic functions* should not only be on calculating the critical points, but also on **interpreting** the critical points on the graph.

QUESTION 8 43%

The graph of $y = f'(x) = mx^2 + nx + k$ is drawn below.

The graph passes the points $P\left(-\frac{1}{3};\ 0\right)$, $Q(1;\ 0)$ and $R(0;\ 1)$.



8.1 Determine the values of m, n and k. (6)

49%

Memo

8.1 $k = 1$ . . . *y-intercept*

$\&\ y = m\left(x + \frac{1}{3}\right)(x-1)$ . . . *roots* $-\frac{1}{3}$ *& 1*

$\therefore\ y = m\left(x^2 - \frac{2}{3}x - \frac{1}{3}\right)$

Substitute $(0;\ 1)$

$\therefore\ 1 = m\left(-\frac{1}{3}\right)$

$\therefore\ \mathbf{m = -3}$ ✓

$\therefore$ The equation is: $y = -3\left(x^2 - \frac{2}{3}x - \frac{1}{3}\right)$

$\therefore\ y = -3x^2 + 2x + 1$

$\therefore\ \mathbf{n = 2}$ ✓

$\&\ \mathbf{k = 1}$ ✓

Common Errors and Misconceptions

(a) Many candidates failed to realise that **Q8.1** required them to determine the equation of a parabola given two x-intercepts and a point on the parabola. Some candidates assumed that the value of m was -1 because the parabola had a maximum turning point. A few candidates used the function given in Q8.2 to calculate the values of m, n and k. They were not awarded any marks for this.

8.2 If it is further given that $f(x) = -x^3 + x^2 + x + 2$:

50% 8.2.1 Determine the coordinates of the turning points of f. (3)

8.2.2 Draw the graph of f. Indicate on your graph the coordinates of the turning points and the intercepts with the axes. (5)

Memo

8.2.1

TPs of f are at $x = -\frac{1}{3}$ and $x = 1$

. . . where the derivative, $f'(x)$, is zero.

$$f(x) = -x^3 + x^2 + x + 2$$

$$\therefore f\left(-\frac{1}{3}\right) = -\left(-\frac{1}{3}\right)^3 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right) + 2$$

$$= \frac{49}{27}$$

$$= 1{,}81 \text{ or } 1\frac{22}{27}$$

$$\& \quad f(1) = -(1)^3 + (1)^2 + (1) + 2$$

$$= 3$$

$\therefore$ The turning points: $\left(-\frac{1}{3};\ 1{,}81\right)$ **&** **(1; 3)** ◄

8.2.2 **The Y-int.** ($x = 0$): $f(0) = 2$ $\therefore$ **(0; 2)** ◄

The X-int. (y = 0): $-x^3 + x^2 + x + 2 = 0$

$$\therefore x^3 - x^2 - x - 2 = 0$$

$$\therefore (x-2)(x^2 + x + 1) = 0 \quad \ldots f(2) = 0$$

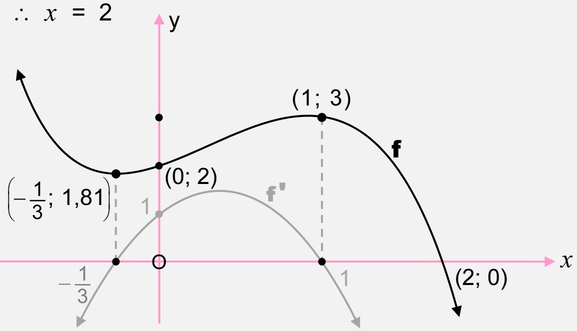
$$\therefore x = 2$$

$\therefore$ **(2; 0)** ◄

For $x^2 + x + 1$

$\Delta = 1^2 - 4(1)(1) = -3$

$\Delta < 0$ ➡ *'no roots'*



Common Errors and Misconceptions

(b) In **Q8.2.1** many candidates calculated the x-coordinates of the turning points by differentiating the given cubic function instead of reading them off the given graph. These candidates could not **relate** the given parabola to the first derivative of the cubic function.

(c) Many candidates failed to calculate the x-intercepts of the cubic function and hence were unable to draw the cubic graph correctly in **Q8.2.2**.

8.3 Points E and W are two variable points on f ′ and are on the same horizontal line.

12%

- h is a tangent to f ′ at E.
- g is a tangent to f ′ at W.
- h and g intersect at D(a; b).

8.3.1 Write down the value of a. (1)

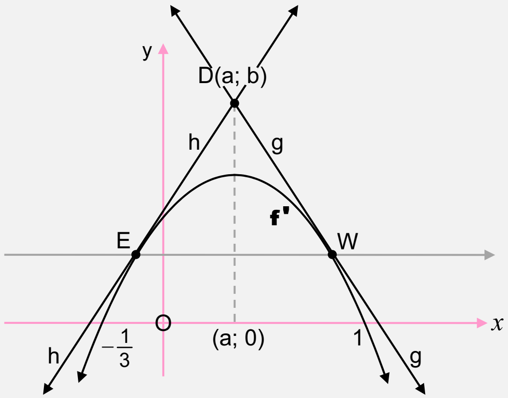
8.3.2 Rephrased question.

For which value(s) of b is it impossible to draw two distinct tangents from D(a; b) to f ′? (2)

[17]

Memo

8.3



8.3.1 $a = \dfrac{-\frac{1}{3} + 1}{2}$ . . . *midpoint*

$\therefore$ **$a = \frac{1}{3}$** ✓

8.3.2 Turning point of f ′ is $\left(\frac{1}{3}; \frac{4}{3}\right)$

$b \leq \frac{4}{3}$ ✓ . . . *Read the rephrased question carefully!*

Common Errors and Misconceptions

(d) **Q8.3.1** and **Q8.3.2** posed **a reading challenge** to many candidates. They did not realise that these questions were based on the turning point of the parabola. In addition, these questions required candidates to make a graphical interpretation. This was beyond many candidates’ capabilities.

QUESTION 8: Suggestions for Improvement

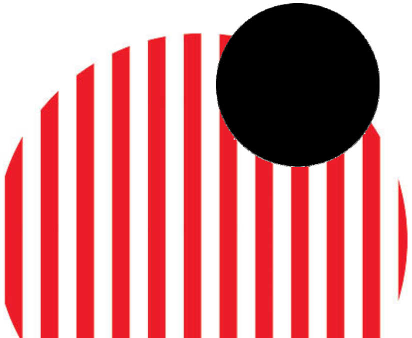
(a) Learners should be taught to determine the equation of a graph from Grade 10 to 12 in a progressive manner.

(b) When teaching *factorisation of third-degree polynomials*, teachers should include examples where there is only one real root.

(c) Teachers should demonstrate the **relationship** between a ***cubic graph*** and ***the graphs of its first and second derivatives***.

CONCAVITY

(d) The application of Calculus lends itself to many applications. Teachers need to expose learners to a wide variety of questions.



QUESTION 9 9%

Given f(x) = x^2.

Determine the minimum distance between the point (10; 2) and a point on f. **[8]**

Memo

9. Points on f are $(x; x^2)$

The distance between (10; 2) & $(x; x^2)$

$= \sqrt{(x-10)^2 + (x^2-2)^2}$

$= \sqrt{x^2 - 20x + 100 + x^4 - 4x^2 + 4}$

$= \sqrt{x^4 - 3x^2 - 20x + 104}$

To find the minimum of $\sqrt{A}$, we can find the minimum of A, since the smaller A is, the smaller $\sqrt{A}$ is. So the minimum occurs when the derivative of what is under the root sign equals 0.

$\therefore\ 4x^3 - 6x - 20 = 0$

$\therefore\ 2x^3 - 3x - 10 = 0$

This is true when $x = 2$ . . . *trial and error*

$\therefore\ 2x^3 - 3x - 10$

$= (x-2)(2x^2 \ldots + 5)$

$= (x-2)(2x^2 + 4x + 5)$

For $2x^2 + 4x + 5$:
$\Delta = 4^2 - 4(2)(5) = -24$
& $\Delta < 0$ ➡ *'no roots'*

$\therefore\ x = 2$ is the only real root of this equation

$\therefore$ The point is (2; 4)

$\therefore$ The minimum distance $= \sqrt{(2-10)^2 + (4-2)^2}$

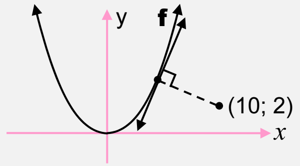
$= \sqrt{64 + 4}$

$= \sqrt{68}$

$= 2\sqrt{17}$

$\approx$ **8,25 units** ◄

OR: The shortest line drawn from (10; 2) to f is the perpendicular to the tangent at that point.



The gradient of the tangent $= f'(x) = 2x$

& The gradient of the line is $\dfrac{x^2-2}{x-10}$

$\therefore\ 2x \times \dfrac{x^2-2}{x-10} = -1$ . . . *product of the gradients = –1*

$\therefore\ 2x(x^2-2) = -x + 10$

$\therefore\ 2x^3 - 4x + x - 10 = 0$

$\therefore\ 2x^3 - 3x - 10 = 0$, etc. (as above)

Common Errors and Misconceptions

Any point on f is $(x\,;\,x^2)$

The vast majority of the candidates did not attempt this question because they were unable to determine **the distance function** from the given information. Some candidates used trial and error to obtain the minimum distance between the points. They were not awarded any marks.

QUESTION 9: Suggestions for Improvement

(a) Learners appear to be dependent on the formulae being given when solving *optimisation* problems. It is advisable that learners interrogate the optimum function even when it is given in a question. This should help their conceptual development.

(b) Teachers should ensure that there is enough time for learners to understand the application of Calculus fully.

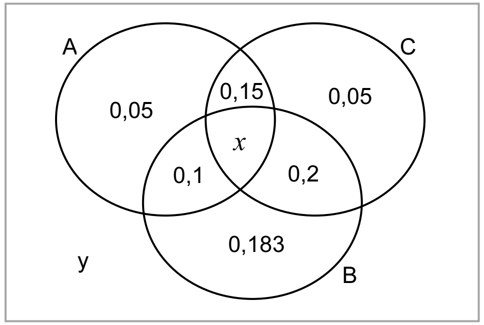
(c) Reading for understanding should be ongoing if learners are to improve their responses to word problems.

PROBABILITY (21%): DBE NOVEMBER 2022

QUESTION 10 21%

10.1 A, B and C are three events. The probabilities of these events (or any combination of them) occurring is given in the Venn-diagram below.

25%



10.1.1 If it is given that the probability that at least one of the events will occur is 0,893, calculate the value of:

(a) y, the probability that none of the events will occur. (1)

(b) x, the probability that all three events will occur. (1)

Memo

10.1.1 (a) y, the probability that none of the events will occur.

= **1 –** the probability that at least one of the events will occur. . . . *complementary events*

= 1 – 0,893

= **0,107** ◄

(b) The **sum** of all the probabilities is **1**

$\therefore\ x + 0{,}05 + 0{,}1 + 0{,}15 + 0{,}2 + 0{,}05 + 0{,}183 + 0{,}107 = 1$

$\therefore\ x = 1 - 0{,}84$

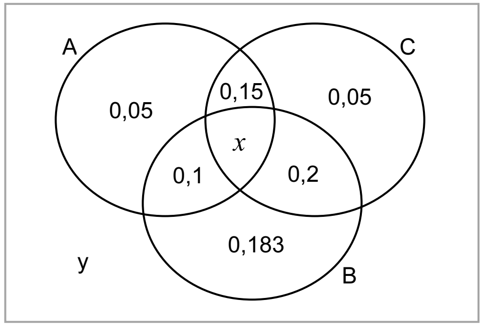
$= \mathbf{0{,}16}$ ◄

Tick them off as you enter each one!

Common Errors and Misconceptions

(a) Many candidates did not realise that *the probability of at least one of the three events will occur is 0,893* represented the sum of the probabilities of all the regions enclosed by the three circles.

Thus they were unable to answer **Q10.1.1(a)** and **Q10.1.1(b)** correctly.



10.1.2 Determine the probability that at least two of the events will take place. (2)

10.1.3 Are events B and C independent? Justify your answer. (5)

Memo

10.1.2 P(at least 2 events will take place) = 0,1 + 0,16 + 0,15 + 0,2
= **0,61** ✓

10.1.3 P(B and C) = x + 0,2 = 0,16 + 0,2 = 0,36

& P(B) × P(C) = (0,1 + 0,16 + 0,183 + 0,2) × (0,16 + 0,15 + 0,2 + 0,05)
= 0,643 × 0,56
= 0,360 08

∴ P(B and C) ≠ P(B) × P(C)

∴ **Events B and C are not independent** ✓

Note:

Rounding off of decimals is only done **if necessary** (which isn't the case here).

However, it could be argued that, when rounding which is 'customary',
P(B and C) **=** P(B) × P(C) . . . = *0,36* & then events B and C are independent ✓

Common Errors and Misconceptions

(b) In answering **Q10.1.2** many candidates were unable to correctly identify the regions in the diagram that satisfied the condition of *at least two events*.

(c) In **Q10.1.3** a number of candidates read off P(A) from the sketch instead of P(C). They went on to test the independence of events A and B instead of events B and C. Some candidates confused P(B) with P(B only) and incorrectly took the value of P(B) as 0,183.

10.2 A four-digit code is required to open a combination lock.

17% The code must be even-numbered and may not contain the digits 0 or 1. Digits may not be repeated.

10.2.1 How many possible 4-digit combinations are there to open the lock? (3)

10.2.2 Calculate the probability that you will open the lock at the first attempt if it is given that the code is greater than 5 000 and the third digit is 2. (5)

[17]

TOTAL: 150

Memo

10.2 $\underline{7}\ \underline{6}\ \underline{5}\ \underline{4}$ 2, 3, 4, 5, 6, 7, 8, 9

(8)

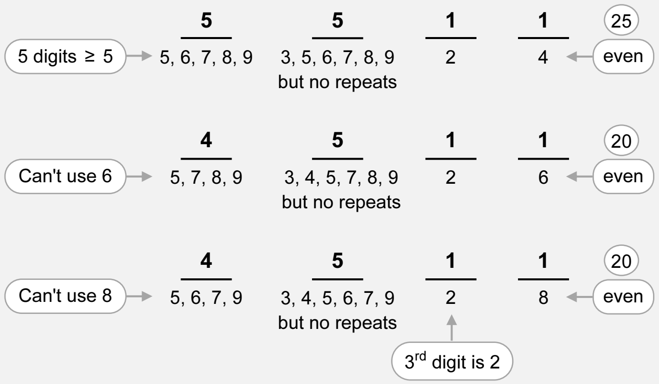
↑ 2, 4, 6, 8

10.2.1 The number of possible 4-digit combinations

$= 7 \times 6 \times 5 \times 4$

$= \mathbf{840}$ ➤

10.2.2



∴ The number of different ways

$= (5 \times 5 \times 1 \times 1) + (4 \times 5 \times 1 \times 1) + (4 \times 5 \times 1 \times 1)$

$= 25 + 20 + 20$

$= 65$

∴ The probability for this combination

$= \frac{n(E)}{n(S)} = \mathbf{\frac{1}{65}}$ ➤

Common Errors and Misconceptions

(d) Many candidates were unable to answer **Q10.2.1** and **Q10.2.2** correctly because they were unable to take the **restrictions** placed on the digits into account. The problem was compounded by the fact that the digits may **not be repeated.**

QUESTION 10: Suggestions for Improvement

(a) Teaching **basic concepts** cannot be overlooked. When learners understand the basic concepts well enough, then the more complex concepts are easier to grasp.

(b) It must be stressed that the **probability of an event A** lies in the interval $0 \leq P(A) \leq 1$.

(c) Reading for understanding must be a regular practice in the classroom. This should equip learners with the skills to deal with word problems in assessment tasks.

(d) **Use Venn diagrams to teach *probability***. It helps with the understanding of the different areas that make up the events, e.g. only A, only B, A and B, A or B, not A, not B, not A and not B and not A or not B.

(e) Teach learners the ***Fundamental Counting Principle*** in such a way that they will be able to base their answers on their **reasoning**, rather than on the rule.

2023 DIAGNOSTIESE VERSLAG (DBO NOV 2022 V1)

VRAAG 1 65%

1.1 Los op vir x:

76% 1.1.1 $(3x - 6)(x + 2) = 0$ (2)

1.1.2 $2x^2 - 6x + 1 = 0$ *(korrek tot TWEE desimale plekke)* (3)

1.1.3 $x^2 - 90 > x$ (4)

Memo

1.1.1 $(3x - 6)(x + 2) = 0$

$\therefore\ 3(x - 2)(x + 2) = 0$

$\therefore\ x - 2 = 0$ of $x + 2 = 0$

$\therefore\ \boldsymbol{x = \pm 2}$ ◄

1.1.2 $2x^2 - 6x + 1 = 0$

$$\therefore\ x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(1)}}{2(2)} \quad \ldots \quad \boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

$$= \frac{6 \pm \sqrt{36 - 8}}{4}$$

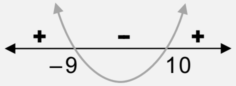
$$= \frac{6 \pm \sqrt{28}}{4}$$

$\approx$ **2,82 of 0,18** ◄

1.1.3 $x^2 - 90 > x$

$\therefore\ x^2 - x - 90 > 0$

$\therefore\ (x + 9)(x - 10) > 0$



$\therefore\ \boldsymbol{x < -9}$ **of** $\boldsymbol{x > 10}$ ◄

Algemene Foute en Wanopvattings

(a) In **V1.1.1** het sommige kandidate **die hakies eerste vermenigvuldig**, maar **het dan verkeerd gefaktoriseer**.

(b) Dit is steeds vir sommige kandidate moeilik om die antwoorde tot twee desimale plekke af te rond. Sommige kandidate beskik steeds nie oor basiese sakrekenaarvaardighede nie. 'n Paar kandidate het die waarde van b in **V1.1.2** as 6 in plaas van –6 geneem.

(c) In die beantwoording van **V1.1.3** het baie kandidate **die ongelykheid as 'n vergelyking** hanteer. Hulle antwoord het gelees: $(x + 9)(x - 10) > 0$ gevolg deur $x > -9$ of $x > 10$.

Baie kandidate het gesukkel om die korrekte antwoord vanaf die ongelykheid te interpreteer.

$x^2 - x - 90 > 0$

$(x + 9)(x - 10) > 0$

$\therefore\ x = -9$ of $x = 10$

$\therefore\ -9 < x < 10$

Sommige kandidate het 'n **skets** gemaak, maar kon dit nie gebruik om die antwoord wat vereis is, neer te skryf nie. Nog 'n algemene fout was die verkeerde notasie in die antwoord. Kandidate het die antwoord as $-9 < x > 10$ geskryf in plaas van $x < -9$ of $x > 10$.

1.1.4 $x - 7\sqrt{x} = -12$ (4)

1.1.4 $x - 7\sqrt{x} = -12$

$\therefore \left(\sqrt{x}\right)^2 - 7\sqrt{x} + 12 = 0$

$\therefore \left(\sqrt{x} - 3\right)\left(\sqrt{x} - 4\right) = 0$

$\therefore \sqrt{x} = 3$ of 4

$\therefore$ **$x = 9$ of 16** ◂ . . . *beide geldig*

OF: $+7\sqrt{x} = +x + 12$

$49x = x^2 + 24x + 144$

$0 = x^2 - 25x + 144$

$0 = (x - 9)(x - 16)$

$\therefore$ **$x = 9$ of $x = 16$** ◂

1.2 Los gelyktydig vir x en y op:

85%

$2x - y = 2$

$xy = 4$ (5)

1.2 $2x - y = 2$

$\therefore 2x - 2 = y$ . . . ❶

$xy = 4$ . . . ❷

❶ in ❷: $\therefore x(2x - 2) = 4$

$\therefore 2x^2 - 2x - 4 = 0$

$(\div 2)$ $\therefore x^2 - x - 2 = 0$

$\therefore (x - 2)(x + 1) = 0$

$\therefore x = 2$ of -1

Vir $x = 2$: $y = 2(2) - 2 = 2$

& Vir $x = -1$: $y = 2(-1) - 2 = -4$

$\therefore$ Oplossing: **(2; 2) of (–1; –4)** ◂

Algemene Foute en Wanopvattings

(d) Die meeste kandidate het een of ander idee gehad dat hulle albei kante van die vergelyking in **V1.1.4** moes kwadreer. 'n Paar kandidate kon egter **die *wortelvorm* isoleer voordat** hulle albei kante van die vergelyking kwadreer het.

Die volgende foute is opgemerk:

$$(x - 7\sqrt{x})^2 = (-12)^2 \qquad \therefore x^2 - 49x = 144$$

of

$$x - 7\sqrt{x} = -12 \qquad \therefore \sqrt{x} = \frac{-12}{x - 7}$$

Geen idee nie!

(e) Alhoewel baie kandidate **V1.2** korrek beantwoord het, het 'n paar foute gemaak toe hulle die lineêre vergelyking in terme van een veranderlike geskryf het. Sommige kandidate het 'n geldige oplossing geïgnoreer. Hulle het gelyktydige vergelykings met wortelvergelykings verwar.

1.3 Toon dat $2.5^n - 5^{n+1} + 5^{n+2}$ ewe is vir alle positiewe heelgetalwaardes van n. (3)

41%

Memo

1.3 $2.5^n - 5^{n+1} + 5^{n+2}$

$= 2.5^n - 5^n.5 + 5^n.5^2$

$= 5^n(2 - 5 + 25)$

$= 5^n(22)$

$= 2(5^n \times 11)$, 'n ewe getal vir alle $n \in \mathbb{N}$

1.4 Geherformuleerde vraag.

22%

Bepaal een paar heelgetalwaardes van x en y waarvoor:

$$\frac{3^{y+1}}{32} = \sqrt{96^x}.$$

(4) **[25]**

Memo

1.4 $\dfrac{3^{y+1}}{32} = \sqrt{96^x}$

$\therefore \dfrac{3^{y+1}}{2^5} = \sqrt{(2^5.3)^x}$

$\therefore 2^{-5}.3^{y+1} = 2^{\frac{5x}{2}}.3^{\frac{x}{2}}$

2	96
2	48
2	24
2	12
2	6
3	3
	1

$96 = 2^5.3$

As ons die eksponente van die 2's en die 3's gelykstel

$\therefore \dfrac{5x}{2} = -5$ en $\dfrac{x}{2} = y + 1$

$\therefore 5x = -10$ $\therefore \dfrac{-2}{2} = y + 1$

$\therefore x = -2$ $\therefore y = -2$

$\therefore$ 'n Oplossing is: **(–2; –2)** ➤

(f) In **V1.3** kon baie kandidate die eksponente met afsonderlike grondtalle skryf, d.w.s. $2.5^n - 5^{n+1} + 5^{n+2} = 2.5^n - 5^n.5 + 5^n.5^2$.

Sommige kon egter nie die daaropvolgende uitdrukking korrek faktoriseer nie, d.w.s. $2.5^n - 5^n.5 + 5^n.5^2 = 5^n(2.1^n - 1^n.5 + 1^n.5^2)$.

Die meerderheid van die kandidate kon nie verduidelik waarom $5^n(22)$ ewe getalle vir alle positiewe heelgetalwaardes van n sal wees nie.

(g) Baie kandidate het nie geweet hoe om **V1.4** te beantwoord nie. Hulle kon nie albei kante van die vergelyking as eksponente wat dieselfde grondtalle het, skryf nie.

Let Wel:

Daar is **2** vergelykings in x en y nodig om 0,1 of 2 oplossings te vind.
Daar is eintlik 'n oneindige aantal oplossings indien slegs 1 vergelyking gegee word.
Daar word egter heeltallige oplossings gevra, daarom is daar slegs een moontlike oplossing.

VRAAG 1: Voorstelle vir Verbetering

(a) Baie van die **werk** in hierdie vraag is **in Graad 11 behandel**. Dit is dus belangrik dat onderwysers **regdeur die Graad 12-jaar hersieningstake** oor hierdie afdelings opstel.

(b) Onderwysers moenie sommer net aanneem dat leerders weet hoe om 'n getal tot 'n vereiste aantal plekke af te rond nie. Hierdie vaardigheid moet, waar nodig, in Graad 11 en 12 weer aangeleer word. Onderwysers moet leerders penaliseer indien hulle in SBA-take nie tot die korrekte aantal plekke afrond nie.

(c) Onderwysers moet, verkieslik in **Graad 10**, tyd afstaan om daarop te fokus om vir leerders te leer hoe om **ongelykhede** (bv. $-3 < x < 5$; $x < -3$ of $x > 5$) **op 'n getallelyn voor te stel en** ook hoe **om 'n ongelykheid** vanaf die illustrasie op 'n getallelyn neer te skryf. Leerders sal hierby baat vind aangesien hulle vir 'n hele aantal vrae in albei eksamenvraestelle ongelykheidsoplossings moet kan neerskryf. Die klem op die korrekte notasie is noodsaaklik wanneer oplossings vir ongelykhede neergeskryf word.

Ongelykhede

(d) Onderwysers moet die verskil tussen ***en*** en ***of*** in die konteks van **ongelykhede** verduidelik. Leerders kan nie sommer net die een óf die ander gebruik nie, aangesien hulle verskillende betekenisse het.

(e) Wanneer met ***wortel*vergelykings** gewerk word, moet leerders herinner word dat hulle die wortelteken moet isoleer voordat hulle albei kante van die vergelyking kan kwadreer. Onderwysers moet klem daarop lê dat daar **vanselfsprekende beperkings** op *wortel*vergelykings geplaas word en dat leerders moet aanhou **toets** of hul **antwoorde** die oorspronklike vergelyking bevredig.

(f) Leerders moet aan ingewikkelde vrae wat *wortelvorme* en *eksponente* insluit, blootgestel word.

VRAAG 2 50%

2.1 Die eerste term van 'n meetkundige reeks is 14 en die 6^{de} term is 448.

55%

2.1.1 Bereken die waarde van die konstante verhouding, r. (2)

2.1.2 Bepaal die aantal opeenvolgende terme wat by die eerste 6 terme van die reeks getel moet word om 'n som van 114 674 te kry. (4)

2.1.3 Indien die eerste term van 'n ander reeks 448 en die 6^{de} term 14 is, bereken die som tot oneindigheid van die nuwe reeks. (3)

Memo

2. M.R.: $a = 14$ en $T_6 = 448$; r?

2.1.1 $T_6 = ar^5 = 448$ . . . $\boxed{\mathbf{T_n = ar^{n-1}}}$

$\therefore 14r^5 = 448$

$\therefore r^5 = 32$

$\therefore \mathbf{r = 2}$ ≺

2.1.2 $114\,674 = \dfrac{14(2^n - 1)}{2 - 1}$ . . . $\boxed{\mathbf{S_n = \dfrac{a(r^n - 1)}{r - 1}}}$

$\therefore \dfrac{114\,674}{14} = 2^n - 1$

$\therefore \dfrac{114\,674}{14} + 1 = 2^n$

$\therefore 8\,192 = 2^n$

$\therefore 2^{13} = 2^n$

$\therefore n = 13$

$\therefore$ Aantal opeenvolgende terme bygevoeg $= 13 - 6$
$= \mathbf{7}$ ≺

2.1.3 $T_1 = 448$ & $T_6 = 14$

$\therefore a = 448$ & $r = \frac{1}{2}$ . . . *Die gegewe reeks is omgekeer!*

$\therefore \boxed{\mathbf{S_\infty = \dfrac{a}{1 - r}}} = \dfrac{448}{1 - \frac{1}{2}} = \mathbf{896}$ ≺

Algemene Foute en Wanopvattings

(a) Baie kandidate het nie verstaan wat in **V2.1.2** van hulle verwag is nie.

Hulle het korrek bereken dat die som van die eerste 13 terme 114 674 was. Hulle het egter nie 6 van 13 afgetrek om die vereiste vraag te beantwoord nie.

(b) Terwyl baie kandidate **V2.1.3** korrek beantwoord het, het sommige kandidate nie besef dat vir 'n **konvergerende meetkundige reeks,** die waarde van r in die interval **$-1 < r < 1$** moet lê nie.

2.2 Indien $\sum_{p=0}^{k}\left(\frac{1}{3}p+\frac{1}{6}\right) = 20\frac{1}{6}$, bepaal die waarde van k.

42%

(5)

[14]

Memo

2.2 $$\sum_{p=0}^{k}\left(\frac{1}{3}p+\frac{1}{6}\right) = \left[\frac{1}{3}(0)+\frac{1}{6}\right]+\left[\frac{1}{3}(1)+\frac{1}{6}\right]+\left[\frac{1}{3}(2)+\frac{1}{6}\right]+\dots$$

$$= \frac{1}{6}+\frac{1}{2}+\frac{5}{6}+\dots$$

$$\left[T_3 - T_2 = \frac{5}{6}-\frac{1}{2} = \frac{1}{3} \quad \& \quad T_2 - T_1 = \frac{1}{2}-\frac{1}{6} = \frac{1}{3}\right]$$

RR $a = \frac{1}{6}$ en $d = \frac{1}{3}$

$$\boxed{S_n = \frac{n}{2}[2a+(n-1)d]}$$

$$\therefore\ 20\frac{1}{6} = \frac{n}{2}\left[2\left(\frac{1}{6}\right)+(n-1)\left(\frac{1}{3}\right)\right]$$

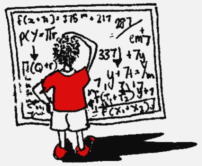
$$\therefore\ \frac{121}{6} = \frac{n}{2}\left[\frac{1}{3}+\frac{1}{3}n-\frac{1}{3}\right]$$

$$\therefore\ 121 = 3n\left[\frac{1}{3}n\right]$$

$$= n^2$$

$$\therefore\ n = 11$$

$\therefore$ **k = 10** ◄ *(want p begin by 0)*



(c) In **V2.2** kon baie kandidate nie die ***sigma-notasie*** korrek interpreteer nie. Party van hulle het verkeerdelik gedink die algemene term wat in die *sigma-notasie* gegee is, stel 'n meetkundige reeks voor. 'n Redelike aantal kandidate kon nie die eerste drie terme van die reeks bereken nie en was gevolglik nie in staat om die gemene verskil te bereken nie.

'n Aantal kandidate kon nie die afleiding maak dat $n = k + 1$ is nie.

Sommige kandidate het verkeerdelik aangeneem dat $T_n = 20\frac{1}{6}$ in plaas van $S_n = 20\frac{1}{6}$.

VRAAG 2: Voorstelle vir Verbetering

(a) Leerders moet geleer word hoe om die **tipe reeks** waarmee hul werk, en **watter formules** daarop van toepassing is, te **identifiseer**.

(b) Maak leerders bewus van watter formules op die inligtingsblad op watter tipe reeks van toepassing is. Dit is goeie oefening vir leerders om die inligtingsblad in die klas te gebruik sodat hulle dit kan gewoond raak.

(c) Leer vir leerders hoe om te **identifiseer of** die vraag van hulle verwag om die waarde van die n^{de} term **of** die som van die eerste n terme te bereken.

(d) Leerders moet die vrae noukeurig lees sodat hulle weet wat van hulle verwag word.

(e) Herinner leerders dat **n** nie 'n negatiewe getal, nul of 'n breuk kan wees nie. Wanneer n opgelos word, moet leerders by **'n natuurlike getal**-oplossing uitkom. As dit nie die geval is nie, moet hulle weet hulle het êrens in hul berekeninge 'n fout gemaak. Daar moet vir leerders gesê word dat hulle geen punte sal kry indien hulle hul antwoorde tot 'n natuurlike getal afrond nie.

(f) Dit is belangrik om, met behulp van 'n voorbeeld, die konsep van 'n konvergerende meetkundige reeks te demonstreer, deur eers 'n waarde van $r > 1$ en dan 'n waarde van $-1 < r < 1$ te neem. Dit sal leerders bedag maak op die voorwaarde waarvoor 'n meetkundige reeks sal konvergeer.

(g) Daar moet meer aandag gegee word aan die uitbreiding van 'n reeks wat in ***sigma-notasie*** vorm gegee word.

VRAAG 3 64%

Daar word gegee dat die algemene term van 'n kwadratiese getalpatroon $T_n = n^2 + bn + 9$ is en dat die eerste term van die eerste verskille 7 is.

3.1 Toon dat $b = 4$. (2)

77%

3.2 Bepaal die waarde van die 60^{ste} term van hierdie getalpatroon. (2)

88%

Memo

3.1 Kwadratiese getalpatroon

$$\mathbf{T_n = n^2 + bn + 9}$$

$$T_1 = 1 + b + 9 = b + 10$$

$$T_2 = 2^2 + 2b + 9 = 2b + 13$$

$$\therefore\ (2b + 13) - (b + 10) = 7 \quad \ldots \textit{ Die } 1^{ste} \textit{ term van die } 1^{ste} \textit{ verskille}$$

$$\therefore\ b + 3 = 7$$

$$\therefore\ \mathbf{b = 4} \ \blacktriangleleft$$

3.2 $T_{60} = 60^2 + (4)(60) + 9$

$= \mathbf{3\ 849} \ \blacktriangleleft$

Algemene Foute en Wanopvattings

(a) Met die beantwoording van **V3.1**, kon baie kandidate nie daarin slaag om uit die vraag te identifiseer dat $a = 1$ en dat **$3a + b$** $= 7$ nie. In plaas daarvan het hulle in hul berekeninge $b = 4$ gebruik om te bewys dat die **eerste van die eerste verskille**, 7 is.

58% 3.3 Bepaal die algemene term vir die ry van eerste verskille van die kwadratiese getalpatroon.
Skryf jou antwoord in die vorm $T_p = mp + q$. (3)

46% 3.4 Watter TWEE opeenvolgende terme in die kwadratiese getalpatroon het 'n eerste verskil van 157? (3)
[10]

Memo

3.3 14 21 30 41
7 9 11

Ons weet dit is 'n lineêre vergelyking

$\mathbf{T_p = 2p + 5}$ ✓ . . . deur inspeksie

OF: $T_p = mp + q$

$\therefore\ T_1 = m + q = 7$ . . . ❶

& $T_2 = 2m + q = 9$ . . . ❷

❷ – ❶: $\therefore\ m = 2$

❶: & $q = 5$

$\therefore$ Die algemene term is: $\mathbf{T_p = 2p + 5}$ ✓

3.4 Die algemene term van die 1ste verskille:

$2n + 5 = 157$

$\therefore\ 2n = 152$

$\therefore\ n = 76$

$\therefore$ Die 2 opeenvolgende terme van die kwadratiese getalpatroon is die **76ste en 77ste** ✓

OF: Laat die opeenvolgende terme wees $\mathbf{T_n}$ en $\mathbf{T_{n+1}}$; dan:

$$T_{n+1} - T_n = 157$$

$$\therefore\ [(n+1)^2 + 4(n+1) + 9] - [n^2 + 4n + 9] = 157$$

$$\therefore\ \cancel{n^2} + 2n + 1 + 4n + 4 + 9 - \cancel{n^2} - 4n - 9 = 157$$

$$\therefore\ 2n + 5 = 157$$

$$\therefore\ 2n = 152$$

$$\therefore\ n = 76, \text{ ens.}$$

(b) Baie kandidate kon **V3.3** korrek beantwoord. Hulle kon egter nie die verband tussen V3.3 en **V3.4** sien nie. In die beantwoording van V3.4, het baie kandidate verkeerdelik gestel dat $n^2 + 4n + 9 = 157$.

VRAAG 3: Voorstelle vir Verbetering

(a) Wanneer kwadratiese getalpatrone aangeleer word, is dit noodsaaklik om leerders te wys hoe die formules:

$T_1 = a + b + c$, die eerste term van die eerste verskille = **$3a + b$** en die tweede verskil = **$2a$**, afgelei word.

(b) Die reeks eerste verskille van 'n ***kwadratiese getalpatroon*** vorm 'n ***rekenkundige patroon***. Dit impliseer dat 'n rekenkundige reeks in 'n kwadratiese getalpatroon ingesluit word. Leerders moet die vraag baie noukeurig lees om vas te stel na watter patroon die vraag verwys. Deur die woorde net vinnig met die oog te vang, lei daartoe dat leerders verkeerde stellings maak.

(c) Herinner leerders dat n nie 'n negatiewe getal, nul of 'n breuk kan wees nie. Wanneer n opgelos word, moet leerders by 'n natuurlike getal-oplossing uitkom. As dit nie die geval is nie, moet hulle weet hulle het êrens in hul berekeninge 'n fout gemaak.

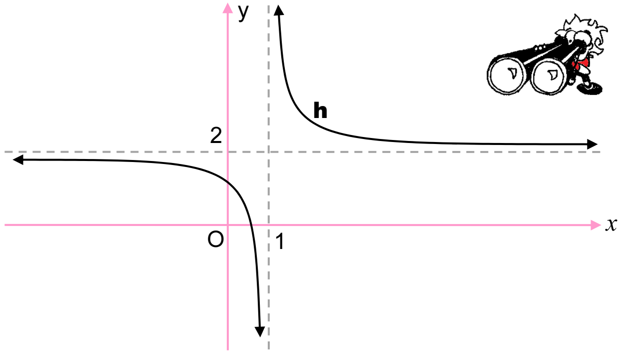
(d) Onderwysers moet kwadratiese getalpatrone met rekenkundige patrone en kwadratiese funksies integreer. Dit sal aan leerders die vaardighede gee om hoërorde vrae te kan hanteer.

VRAAG 4 54%

Standaardvorm!

4.1 Die grafiek van $\boxed{h(x) = \frac{1}{x + p} + q}$ is hieronder geskets.

53% Die asimptoot van h sny by die punt (1; 2).



4.1.1 Skryf die waardes van p en q neer. (2)

4.1.2 Bereken die koördinate van die x-afsnit van h. (2)

4.1.3 Skryf die x-koördinaat van die x-afsnit van g neer, indien $g(x) = h(x + 3)$. (2)

Memo

4.1.1 **p = –1** ◄ **q = 2** ◄

4.1.2 **X-afsnit**.: Stel y = 0 in, in $y = \frac{1}{x - 1} + 2$

$$\therefore\ -2 = \frac{1}{x - 1}$$

$$\therefore\ x - 1 = -\frac{1}{2}$$

$$\therefore\ x = \frac{1}{2}$$

$\therefore \left(\frac{1}{2}; 0\right)$ ◄

4.1.3 Die x- koördinaat: $x = -2\frac{1}{2}$ ◄ . . . $\frac{1}{2} - 3$

[LW: g is h 3 eenhede na links geskuif.]

Algemene Foute en Wanopvattings

(a) Met die beantwoording van **V4.1.1** het 'n aantal kandidate verkeerdelik gestel dat p = 1 in plaas van **p = –1**.

Skuif regs dan is p neg.

(b) Baie kandidate het **V4.1.2** korrek beantwoord. Hulle het egter nie besef dat hulle die x-afsnit 3 eenhede na links moes transleer om by die antwoord vir **V4.1.3** uit te kom nie. Die uitdaging in hierdie vraag was dat kandidate nie die notasie korrek kon interpreteer nie.

Skuif links dan is p pos.

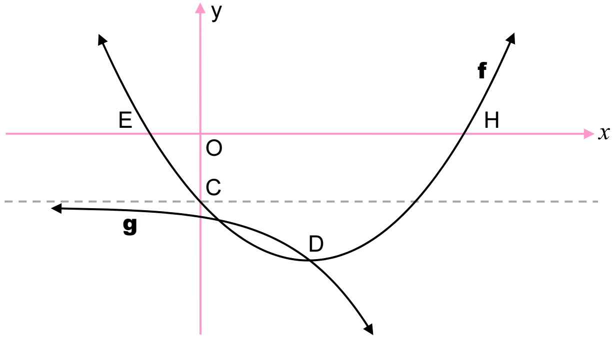
4.1.4 Die vergelyking van 'n simmetrie-as van h is $y = x + t$. Bepaal die waarde van t. (2)

4.1.5 Bepaal die waardes van x waarvoor $-2 \leq \frac{1}{x-1}$. (3)

55%

4.2 Die grafieke van $f(x) = x^2 - 4x - 5$ en $g(x) = a.2^x + q$ is hieronder geskets.

- E en H is die x-afsnitte van f.
- C is die y-afsnit van f en lê op die asimptoot van g.
- Die twee grafieke sny by D, die draaipunt van f.



4.2.1 Skryf die y-koördinaat van C neer. (1)

Memo

4.1.4 [LW: Hierdie simmetrie-as het 'n positiewe gradiënt.]

Die simmetrie-as gaan deur die punt (1; 2)

$\therefore$ Stel in, in $y = x + t$

$2 = 1 + t$

$\therefore$ **$t = 1$** <

4.1.5 $h(x) = \frac{1}{x-1} + 2$

$h(x) \geq 0$ ➡ $\frac{1}{x-1} + 2 \geq 0$

$\therefore \frac{1}{x-1} \geq -2$

$h(x) \geq 0$ vir $x \leq \frac{1}{2}$ of $x > 1$ <

4.2.1 **$y_C = -5$** <

(c) Die algemeenste fout wat kandidate in die beantwoording van **V4.1.5** gemaak het, was om die vergelyking algebraïes op te los. Kandidate kon nie daarin slaag om die gegewe ongelykheid met die gegewe hiperbool in verband te bring nie.

GEBRUIK DIE GRAFIEK om die ongelykheid op te los!

4.2.2 Bepaal die koördinate van D. (2)

4.2.3 Bepaal die waardes van a en q. (3)

4.2.4 Skryf die waardeversameling van g neer. (1)

4.2.5 Bepaal die waardes van k waarvoor die waarde van $f(x) - k$ altyd positief sal wees. (2)

[20]

Memo

4.2.2 By D: $x = -\frac{b}{2a}$

$= -\frac{-4}{2(1)}$

$= 2$

OF: $f'(x) = 0$

$\therefore 2x - 4 = 0$

$\therefore x = 2$ ens.

& $y = 2^2 - 4(2) - 5$

$= -9$

OF: $y = x^2 - 4x - 5$

$\therefore y = (x + 1)(x - 5)$

∴ Die wortels is -1 & 5

$\therefore x_D = \frac{-1 + 5}{2} = 2$ ens.

∴ Pt D is (2; –9) ◄

4.2.3 $g(x) = a.2^x + q$

$\mathbf{q = y_C = -5}$ ◄ . . . *$y = q$ is die asimptoot van g*

D(2; –9) op g: $\therefore -9 = a.2^2 - 5$

$\therefore -4 = 4a$

$\therefore \mathbf{a = -1}$ ◄

4.2.4 $\mathbf{y \in \mathbb{R};\ y < -5}$ ◄

4.2.5 $y = f(x) + 9$ sal ≥ 0 wees (d.w.s. raak die x-as)

$\therefore -k > 9$

$\therefore \mathbf{k < -9}$ ◄

OF: vir $f(x) = x^2 - 4x - 5 - k$, $\Delta < 0$, ens.

(d) In **V4.2.2** kon baie kandidate die simmetrie-as korrek bereken, **maar kon nie** die y-koördinaat van die draaipunt korrek bereken nie.

(e) Alhoewel baie kandidate in **V4.2.3** die waarde van q korrek gehad het, kon hulle nie die waarde van a korrek bereken nie.

(f) Die algemeenste fout in **V4.2.4** was **notasie**. Kandidate het $y \in (-5\ ;\ -\infty)$ of $y \in (-\infty\ ;\ -5)$ in plaas van slegs $y \in (-\infty\ ;\ -5)$ geskryf.

WAARDEVERSAMELING

(g) Baie min kandidate het probeer om **V4.2.5** te beantwoord omdat hulle **nie** die notasie wat in die vraag gebruik is, kon **interpreteer nie**.

GEBRUIK GRAFIEKE om k te bepaal

VRAAG 4: Voorstelle vir Verbetering

(a) Onderwysers moet meer aandag daaraan gee om die korrekte **notasie** te gebruik wanneer hulle **intervalle** skryf. Gee aandag aan die betekenis van die notasie en die verskil tussen die **verskillende notasies**. Onderwysers moenie die gebruik van die verkeerde notasie in SBA-take toelaat nie.

INTERVALLE

(b) 'n Hiperbool kan op twee maniere gedefinieer word, nl. $y = \frac{a}{x+p} + q$ en $\cancel{y = \frac{a}{x-p} + q}$.

Onderwysers word aangeraai om $\mathbf{y = \frac{a}{x+p} + q}$ te gebruik aangesien **in die *KABV*** daarna verwys word. Dit lyk asof die leerders nie aandag gee aan die verskil in tekens tussen die twee vergelykings nie. Hulle word egter gepenaliseer indien hulle die waarde van *p* verkeerd neerskyf.

Transformasies is in die Senior Fase KABV kurrikulum, maar is in die huidige ATP's (Aangepaste Onderrigprogramme) uitgesluit.

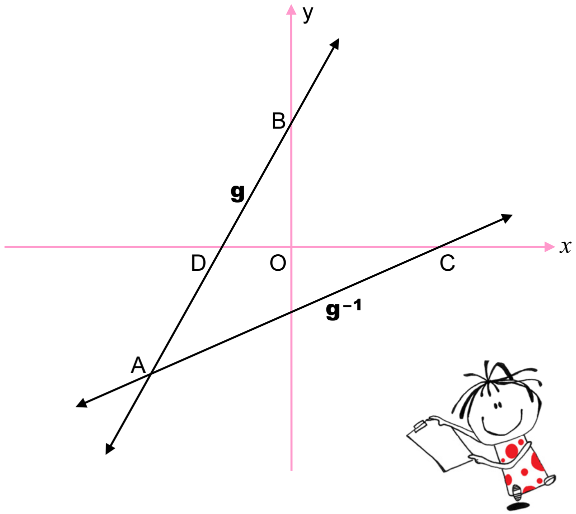
(c) ***Transformasie*** is nie meer 'n onderwerp in die kurrikulum nie. In **die afdeling oor funksies word die basiese grafiek egter getransformeer om alle ander grafieke te verkry.** Die effek van die transformasie op die vergelyking van die basiese grafiek moet bespreek word. Wanneer hulle dit doen, word ook voorgestel dat onderwysers sal aandag gee aan die effek wat die transformasie op die basiese vergelyking het, of andersom.

(d) Leerders moet daarvan bewus gemaak word dat 'n ongelykheid wat breuke bevat, nie op dieselfde manier as 'n vergelyking opgelos kan word nie. Wanneer 'n vergelyking opgelos word, vermenigvuldig ons beide kante met die KGD om die breuke te elimineer. Wanneer 'n ongelykheid opgelos word, vermenigvuldig ons albei kante met die $(\text{KGD})^2$.

VRAAG 5 54%

Die grafieke van g$(x) = 2x + 6$ en g^{-1}, die inverse van g, word in die diagram hieronder getoon.

- D en B is onderskeidelik die x- en y-afsnitte van g.
- C is die x-afsnit van g^{-1}.
- Die grafieke van g en g^{-1} sny by A.



5.1 Skryf die y-koördinaat van B neer. (1)

91%

5.2 Bepaal die vergelyking van g^{-1} in die vorm g$^{-1}(x) = $ mx + n. (2)

62%

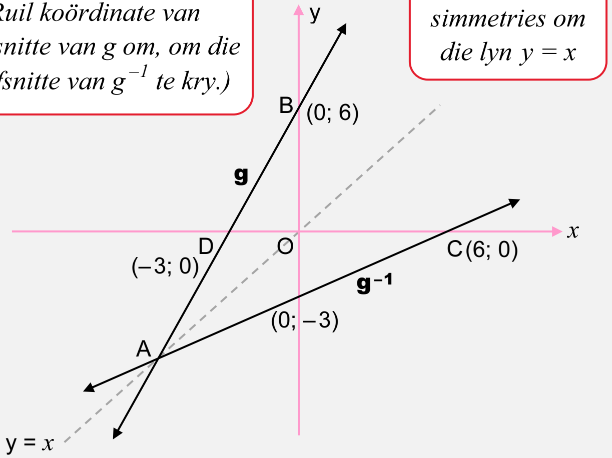
Memo

5.1 **$y_B = 6$** ✓

5.2 Vergelyking van g^{-1}: **$y = \frac{1}{2}x - 3$** ✓

Deur inspeksie!
(Ruil koördinate van as-afsnitte van g om, om die as-afsnitte van g^{-1} te kry.)

LW:
g en g^{-1} is simmetries om die lyn $y = x$



OF: Vergelyking van g: $y = 2x + 6$

$\therefore$ Vergelyking van g^{-1}: $x = 2y + 6$

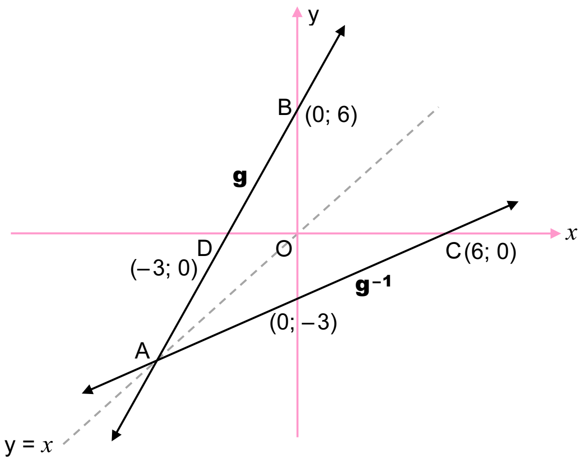
$\therefore -2y = -x + 6$

$\therefore$ **$y = \frac{1}{2}x - 3$** ✓

Algemene Foute en Wanopvattings

(a) In **V5.2** kon die meeste kandidate x en y in die vergelyking korrek omruil. Sommige het egter nie vir y die onderwerp van die formule gemaak nie, terwyl ander foute gemaak het met die oorbring van die terme. Baie kandidate het nie die **verskil** in die notasies **g$'(x)$ en g$^{-1}(x)$** verstaan nie.

Sommige kandidate het net eenvoudig log neergeskryf toe hulle g$^{-1}(x)$ sien. Hulle dink die logfunksie is die inverse/omgekeerde van alle funksies. Dit is verkeerd.



5.3 Bepaal die koördinate van A. (3)

63%

5.4 Bereken die lengte van AB. (2)

69%

Memo

5.3 By A: $y = \frac{1}{2}x - 3$ en $y = 2x + 6$

$$\therefore \frac{1}{2}x - 3 = 2x + 6$$

$$\therefore -\frac{3}{2}x = 9$$

$$\times\left(-\frac{2}{3}\right) \quad \therefore x = -6$$

$$\& \quad y = 2(-6) + 6 = -6$$

$\therefore$ **A(−6; −6)** ◂

5.4 $AB^2 = (6+6)^2 + (0+6)^2$

$= 144 + 36$

$= 180$

$\therefore AB = \sqrt{180} = 6\sqrt{5} \approx$ **13,42 eenhede** ◂

(b) Met die beantwoording van **V5.3** het baie kandidate die **afsnitte** met die asse bereken **in plaas daarvan** om die **snypunt** van die twee lyne te bereken. Baie kandidate het nie enige berekeninge getoon nie, maar net eenvoudig verkeerde koördinate van A neergeskryf, asof hulle dit van die grafiek afgelees het.

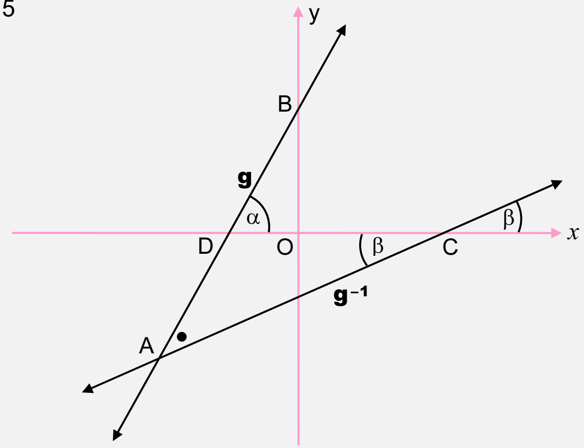
5.5 Bereken die oppervlakte van ΔABC. (5)

[13]

32%

Memo

5.5



$m_{AB} = 2$ ➡ $\alpha = 63,434...^\circ$

$m_{AC} = \frac{1}{2}$ ➡ $\beta = 26,565...^\circ$

∴ $B\hat{A}C = \alpha - \beta = 36,869...$

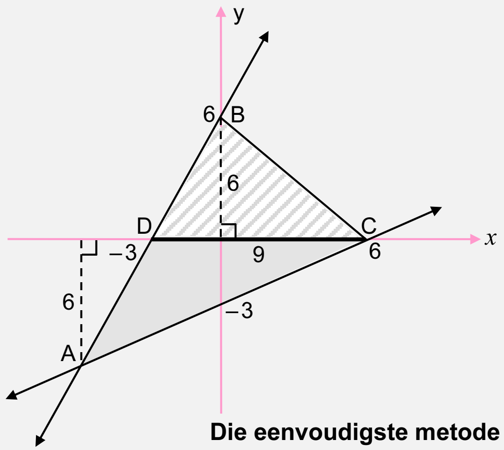
Die oppervlakte van ΔABC = $\frac{1}{2} AB . AC \sin B\hat{A}C$

waar $AC = AB = \sqrt{180}$

∴ Oppv. van ΔABC = $\frac{1}{2}\sqrt{180}\,\sqrt{180}\,\sin 36,869...^\circ$

= **54 eenhede²** ◄

OF:



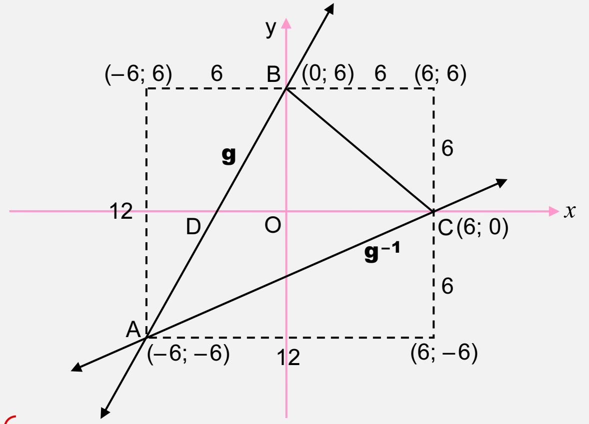
Die eenvoudigste metode!

Oppv. van ΔABC

= ΔADC + ΔBDC

= $\frac{1}{2}(9)(6) + \frac{1}{2}(9)(6)$

= **54 eenhede²** ◄



OF: Oppervlakte van ΔABC

= $(12 \times 12) - \frac{1}{2}(12 \times 6) - \frac{1}{2}(6 \times 6) - \frac{1}{2}(12 \times 6)$

Oppervlakte van 'n vierkant & Oppervlakte van Δ^e

Daar is verskeie metodes.

(c) Baie kandidate het verkeerde aannames gemaak met die beantwoording van **V5.5**.

Die algemeenste hiervan was om aan te neem dat AB loodreg op BC is. Sommige kandidate het die verkeerde hoek in die oppervlakformule $A = \frac{1}{2} ab \sin C$ gebruik. Hulle het 'n inklinasiehoek in plaas van die hoek tussen die twee sye van die driehoek, gebruik.

VRAAG 5: Voorstelle vir Verbetering

(a) Onderwysers moet tyd spandeer om te bespreek dat **alle punte op die x-as** 'n **y-koördinaat van 0** het en dat **alle punte op die y-as** 'n **x-koördinaat van 0** het. Die definisieversameling is altyd 'n stel x-waardes en die waardeversameling is altyd 'n stel y-waardes.

(b) Leerders moet geleer word om die vraag te beantwoord. Hulle moet die **koördinate** van 'n punt neerskryf indien die vraag dit van hulle verwag.

(c) Die afdeling oor inverses lei op sigself tot 'n ondersoekende benadering in onderrig. Onderwysers moet die geleentheid gebruik om leerders toe te laat om te ontdek dat **'n funksie en sy inverse simmetries om die lyn $y = x$** is en dat die snypunt, waar dit bestaan, van 'n funksie en sy inverse ook op die lyn $y = x$ sal lê.

(d) Onderwysers moet dit beklemtoon dat die hoogte van 'n driehoek loodreg op sy basis is.

VRAAG 6 57%

6.1 R12 000 is in 'n fonds belê wat rente teen m% p.j., kwartaalliks saamgestel, betaal het. Na 24 maande was die waarde van die belegging R13 459.

49%

Bepaal die waarde van m. (4)

6.2 Op 31 Januarie 2022 het Tino R1 000 in 'n rekening gedeponeer wat rente teen 7,5% p.j., maandeliks saamgestel het. Hy het aangehou om R1 000 op die laaste dag van elke maand te deponeer. Hy sal die laaste deposito op 31 Desember 2022 maak.

66%

Sal Tino op 1 Januarie 2023 genoeg geld in die rekening hê om 'n rekenaar wat R13 000 kos, te kan koop? Motiveer jou antwoord deur 'n toepaslike berekening te gebruik. (4)

Memo

6.1

$$13\,459 = 12\,000\left(1+\frac{m\%}{4}\right)^{2\times 4} \qquad \boxed{\mathbf{A = P(1+i)^n}}$$

$$\therefore \frac{13\,459}{12\,000} = \left(1+\frac{m\%}{4}\right)^{8}$$

$$\therefore 4\left(\sqrt[8]{\frac{13\,459}{12\,000}}-1\right) = m\%$$

$$\therefore m\% = \mathbf{5{,}78\%} \blacktriangleleft$$

6.2

$$\boxed{\mathbf{F_v = \frac{\mathit{x}\left[(1+i)^n-1\right]}{i}}}$$

$$= \frac{1\,000\left[\left(1+\frac{7{,}5\%}{12}\right)^{12}-1\right]}{\frac{7{,}5\%}{12}}$$

$= $ R12 421,22

$\therefore$ **Tino sal nie genoeg fondse hê nie.** ◄

Algemene Foute en Wanopvattings

(a) In **V6.1** het sommige kandidate die formule vir enkelvoudige rente in plaas van die formule vir saamgestelde rente gebruik. Baie kandidate het die verkeerde waarde van n in die formule vir saamgestelde rente gebruik. Hulle het 4, 12 of 24 as die waarde van n gebruik, in plaas van 8.

33%

6.3 Thabo beplan om 'n kar wat R250 000 kos, te koop. Hy sal 'n deposito van 15% betaal en 'n lening vir die balans uitneem. Die rente op die lening is 13% p.j., maandeliks saamgestel.

6.3.1 Bereken die waarde van die lening. (1)

6.3.2 Die eerste terugbetaling sal gemaak word 6 maande nadat die lening toegestaan is. Die lening sal oor 'n tydperk van 6 jaar nadat dit toegestaan is, afbetaal word. Bereken die MAANDELIKSE paaiement. (5)

[14]

Memo

6.3.1 15% of 250 000 = R37 500

$\therefore$ 250 000 – 37 500 = **R212 500** ◄

6.3.2 13% p.j. maandeliks

$$\boxed{P_v = \frac{x\left[1-(1+i)^{-n}\right]}{i}}$$

$$212\,500 = \frac{x\left[1-\left(1+\frac{13\%}{12}\right)^{-(6\times 12-5)}\right]}{\frac{13\%}{12}} \cdot \left(1+\frac{13\%}{12}\right)^{-5}$$

x = **R4 724,96** ◄

OF: $$212\,500\left(1+\frac{0,13}{12}\right)^{5} = \frac{x\left[1-\left(1+\frac{0,13}{12}\right)^{-67}\right]}{\frac{0,13}{12}}$$

x = **R4 724,96** ◄

OF: $$212\,500\left(1+\frac{0,13}{12}\right)^{72} = \frac{x\left[\left(1+\frac{0,13}{12}\right)^{67}-1\right]}{\frac{0,13}{12}}$$

x = **R4 724,96** ◄

(b) Baie kandidate het nie in ag geneem dat **V6.3.2** op 'n **uitgestelde betaling** gebaseer is nie. Gevolglik het hulle nie **rente by die aanvanklike leningsbedrag bygetel nie**. Algemene foute was:

$$212\,500 = \frac{x\left[1-\left(1+\frac{0,13}{12}\right)^{-72}\right]}{\frac{0,13}{12}}$$

of (Sien memo)

$$212\,500 = \frac{x\left[1-\left(1+\frac{0,13}{12}\right)^{-67}\right]}{\frac{0,13}{12}}$$

VRAAG 6: Voorstelle vir Verbetering

(a) Leerders het dieper insig nodig in die toepaslikheid van elk van die formules en onder watter omstandighede elkeen gebruik kan word. Die **veranderlikes** in elke formule moet verduidelik word. Meer oefening oor Finansiële Wiskunde is nodig sodat leerders kan identifiseer wanneer om die verskillende formules te gebruik.

(b) Onderwysers moet al die stappe wat nodig is wanneer met 'n **sakrekenaar** gewerk word, demonstreer. Leerders moet in formele assesseringstake by die skool gepenaliseer word indien hulle vroegtydig afrond.

(c) Die verskil tussen **saamgestelde rente** en **toekomstige waarde-annuïteite** moet verduidelik word.

(d) Die korrekte Finansiële Wiskunde-taal moet in die klaskamer gebruik word en leerders moet die vraag met begrip lees.

(e) Onderwysers moet leerders aan die verskillende scenario's vir terugbetalings van lenings, nl. uitgestelde betalings, oorgeslaande betalings, uitstaande saldo onmiddellik na 'n sekere aantal betalings, finale betaling, ens., blootstel.

VRAAG 7 61%

7.1 Bepaal $f'(x)$ vanuit eerste beginsels indien

76% $f(x) = x^2 + x$. (5)

7.2 Bepaal $f'(x)$ indien $f(x) = 2x^5 - 3x^4 + 8x$. (3)

88%

Memo

7.1

$$\begin{aligned} f(x) &= x^2 + x \\ \therefore\ f(x+h) &= (x+h)^2 + x + h \\ &= x^2 + 2xh + h^2 + x + h \\ \therefore\ f(x+h) - f(x) &= 2xh + h^2 + h \\ \therefore\ \frac{f(x+h) - f(x)}{h} &= \frac{2xh + h^2 + h}{h} \\ &= 2x + h + 1 \end{aligned}$$

$$\therefore\ \boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}} = \lim_{h \to 0}(2x + h + 1)$$

$$= \mathbf{2x + 1}\ ◄$$

7.2

$$f(x) = 2x^5 - 3x^4 + 8x$$

$$\therefore\ \mathbf{f'(x) = 10x^4 - 12x^3 + 8}\ ◄$$

Algemene Foute en Wanopvattings

(a) In **V7.1** het sommige kandidate die volgende **notasiefoute** gemaak:

$$f(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad \text{of} \quad \lim_{h \to 0} = \frac{f(x+h) - f(x)}{h}.$$

Ernstige notasie-foute

Hulle het 'n punt verloor vir hierdie foute.

Ander kandidate het **foute gemaak met substitusie**, d.w.s.:

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 + (x+h) - x^2 + x}{h} \quad \textbf{of} \quad f'(x) = \lim_{h \to 0} \frac{(x+h)^2 + x - (x^2 + x)}{h}$$

Nog 'n algemene fout was **verkeerde faktorisering**, d.w.s.:

$$\begin{aligned} f'(x) &= \lim_{h \to 0} \frac{2xh + h^2 + h}{h} \\ &= \lim_{h \to 0} \frac{h(2x + h)}{h} \\ &= \lim_{h \to 0} (2x + h) \\ &= 2x \end{aligned}$$

ALGEBRA

7.3 Die raaklyn aan $g(x) = ax^3 + 3x^2 + bx + c$ het 'n minimum helling (gradiënt) by die punt $(-1; -7)$.

22%

Vir watter waardes van x sal g konkaaf op wees? (4)

[12]

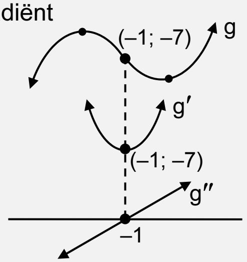
Memo

7.3 $g(x) = ax^3 + 3x^2 + bx + c$

Die gradiënt van die raaklyn aan g

$= g'(x) = 3ax^2 + 6x + b$

& die minimum van hierdie gradiënt kom voor wanneer $x = -1$



$\therefore$ g sal konkaaf op wees vir $\boldsymbol{x > -1}$ ◄

LW: Dit is waar $g''(x) > 0$

OF:

$$g(x) = ax^3 + 3x^2 + bx + c$$
$$\therefore\ g'(x) = 3ax^2 + 6x + b$$
$$\therefore\ g''(x) = 6ax + 6$$
$$\therefore\ 0 = 6a(-1) + 6$$
$$\therefore\ 6a = 6$$
$$\therefore\ a = 1$$

$g''(x) = 6x + 6$ ➡ $g''(x) > 0$

$\therefore\ 6x + 6 > 0$

$\therefore\ 6x > -6$

$\therefore\ \boldsymbol{x > -1}$ ◄

(b) Met die beantwoording van **V7.3**, het baie kandidate die eerste en tweede afgeleides korrek bereken. Hulle het egter die substitusie/instel in die tweede afgeleide verkeerd gedoen, d.w.s. $6a(-1)^2 + 6 = -7$. Hulle het die waarde van die tweede afgeleide by die buig-/infleksiepunt met die minimum gradiënt van die raaklyn verwar.

VRAAG 7: Voorstelle vir Verbetering

(a) Daar moet klem geplaas word op die gebruik van die korrekte notasie wanneer die afgeleide bepaal word, óf deur die gebruik van eerste beginsels óf die reëls.

(b) Onderwysers moet die feit dat **hakies** nodig is wanneer die afgeleide vanuit **eerste beginsels** bepaal word, verduidelik. Dit verhoed die verkeerde vereenvoudiging wat volg.

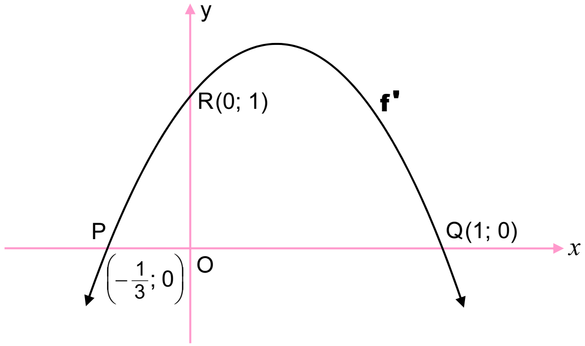
(c) Die **konsepte** van **gradiënt van raaklyn** en **buigpunt** moet verstaan word. Onderwysers moet die verskil tussen die twee konsepte verduidelik.

(d) Met die onderrig van derdegraadse funksies moet die fokus nie net op die berekening van kritieke punte wees nie, maar ook op die **interpretering** van die kritieke punte op die grafiek.

VRAAG 8 43%

Die grafiek van $y = f'(x) = mx^2 + nx + k$ is hieronder geteken.

Die grafiek gaan deur die punte $P\left(-\frac{1}{3};\ 0\right)$, Q(1; 0) en R(0; 1).



8.1 Bepaal die waardes van m, n en k. (6)

49%

Memo

8.1 $k = 1$ . . . *y-afsnit*

$\&\ y = m\left(x + \frac{1}{3}\right)(x - 1)$ . . . *wortels* $-\frac{1}{3}$ *& 1*

$\therefore\ y = m\left(x^2 - \frac{2}{3}x - \frac{1}{3}\right)$

Stel (0; 1) in

$\therefore\ 1 = m\left(-\frac{1}{3}\right)$

$\therefore$ **m = –3** ➤

$\therefore$ Die vergelyking is: $y = -3\left(x^2 - \frac{2}{3}x - \frac{1}{3}\right)$

$\therefore\ y = -3x^2 + 2x + 1$

$\therefore$ **n = 2** ➤

& **k = 1** ➤

Algemene Foute en Wanopvattings

(a) Baie kandidate het nie besef dat **V8.1** van hulle verwag om die vergelyking van 'n parabool, gegewe twee x-afsnitte en 'n punt op die parabool, te bepaal nie. Sommige kandidate het aangeneem dat m, –1 was, omdat die parabool 'n maksimum draaipunt gehad het. 'n Paar kandidate het die funksie wat in V8.2 gegee is, gebruik om die waardes van m, n en k te bereken. Hulle het geen punte hiervoor gekry nie.

8.2 Indien dit verder gegee word dat f(x) = $-x^3 + x^2 + x + 2$:

50%

8.2.1 Bepaal die koördinate van die draaipunte van f. (3)

Memo

8.2.1

DP'e van f is by $x = -\frac{1}{3}$ *en* $x = 1$

. . . waar die afgeleide, $f'(x)$*, nul is.*

$$\text{f}(x) = -x^3 + x^2 + x + 2$$

$$\therefore\ \text{f}\left(-\frac{1}{3}\right) = -\left(-\frac{1}{3}\right)^3 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right) + 2$$

$$= \frac{49}{27}$$

$$= 1{,}81 \quad \text{of} \quad 1\frac{22}{27}$$

$$\&\ \ \text{f}(1) = -(1)^3 + (1)^2 + (1) + 2$$

$$= 3$$

$\therefore$ Die draaipunte: $\left(-\frac{\mathbf{1}}{\mathbf{3}};\ \mathbf{1{,}81}\right)$ **& (1; 3)** ◄

(b) In **V8.2.1** het baie kandidate die x-koördinate van die draaipunte bereken deur differensiasie van die gegewe derdegraadse funksie in plaas daarvan om dit van die gegewe grafiek af te lees. Hierdie kandidate kon nie die gegewe parabool en die eerste afgeleide van die derdegraadse funksie **met mekaar in verband bring** nie.

8.2.2 Skets die grafiek van f. Dui die koördinate van die draaipunte en die afsnitte met die asse op jou grafiek aan. (5)

Memo

8.2.2 **Die Y-afsnit** ($x = 0$): $f(0) = 2$ $\therefore$ **(0; 2)** ◂

Die X-afsnit (y = 0):

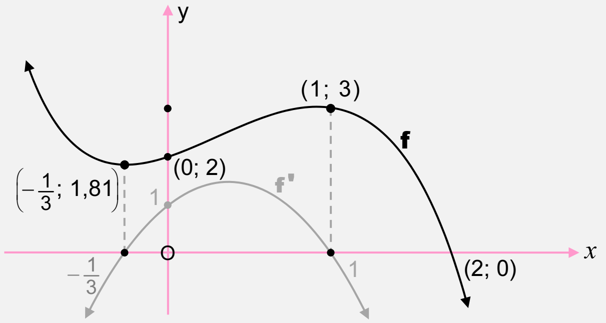
$$\begin{aligned} -x^3 + x^2 + x + 2 &= 0 \\ \therefore\ x^3 - x^2 - x - 2 &= 0 \\ \therefore\ (x-2)(x^2+x+1) &= 0 \quad \ldots f(2) = 0 \\ \therefore\ x &= 2 \end{aligned}$$

Vir $x^2 + x + 1$

$\Delta = 1^2 - 4(1)(1) = -3$

$\Delta < 0$ ➡ 'geen wortels'

$\therefore$ **(2; 0)** ◂



(c) In **V8.2.2** kon baie kandidate nie daarin slaag om die x-afsnitte van die derdegraadse funksie te bereken nie en was gevolglik nie in staat om die derdegraadse grafiek korrek te teken nie.

12%

8.3 Punte E en W is twee veranderlike punte op f' en is op dieselfde horisontale lyn.

- h is 'n raaklyn aan f' by E.
- g is 'n raaklyn aan f' by W.
- h en g sny by D(a; b).

8.3.1 Skryf die waarde van a neer. (1)

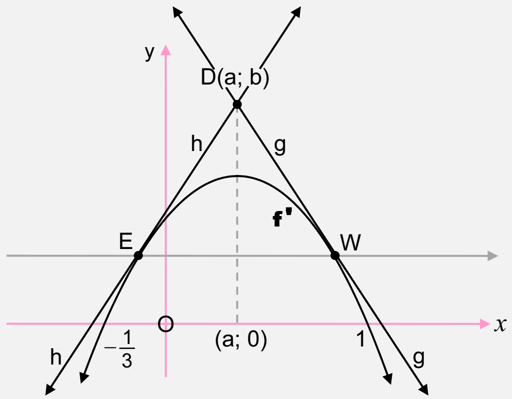
8.3.2 Geherformuleerde vraag.

Vir watter waarde(s) van b is dit onmoontlik om twee verskillende raaklyne vanaf D(a; b) tot f' te trek? (2)

[17]

Memo

8.3



8.3.1 $a = \dfrac{-\frac{1}{3} + 1}{2}$ . . . *middelpunt*

$\therefore$ **$a = \frac{1}{3}$** ✓

8.3.2 Draaipunt van f' is $\left(\frac{1}{3}; \frac{4}{3}\right)$

$b \leq \frac{4}{3}$ ✓ . . . *Lees die geherformuleerde vraag aandagtig deur!*

(d) **V8.3.1** en **V8.3.2** het vir baie kandidate **'n leesuitdaging** ingehou.

Hulle het nie besef dat hierdie vrae op die draaipunt van die parabool gebaseer was nie. Hierdie vrae het dan ook nog van die kandidate verwag om 'n grafiese interpretasie te maak. Dit was bo baie kandidate se vuurmaakplek/vermoëns.

VRAAG 8: Voorstelle vir Verbetering

(a) Leerders moet vanaf Graad 10 tot 12 op 'n progressiewe manier geleer word om die vergelyking van 'n grafiek te bepaal.

(b) Wanneer *faktorisering van derdegraadse polinome* onderrig word, moet onderwysers voorbeelde waar daar slegs een reële wortel is, insluit.

(c) Onderwysers moet die **verband** tussen 'n **derdegraadse grafiek** en **die grafieke van sy eerste en tweede afgeleides** demonstreer.

KONKAWITEIT

(d) Die toepassing van Differensiaalrekene lei op sigself tot baie toepassings. Onderwysers moet leerders aan 'n wye verskeidenheid vrae blootstel.

VRAAG 9 9%

Gegee f$(x) = x^2$.

Bepaal die minimum afstand tussen die punt (10; 2) en 'n punt op f. **[8]**

Memo

9. Punte op f is $(x;\ x^2)$

Die afstand tussen (10; 2) & $(x;\ x^2)$

$$= \sqrt{(x-10)^2 + (x^2-2)^2}$$

$$= \sqrt{x^2 - 20x + 100 + x^4 - 4x^2 + 4}$$

$$= \sqrt{x^4 - 3x^2 - 20x + 104}$$

Om die minimum van $\sqrt{A}$ te vind, kan ons die minimum van A vind, want hoe kleiner A is, hoe kleiner is $\sqrt{A}$. Die minimum kom dus voor wanneer die afgeleide van wat onder die wortelteken is gelyk aan 0 is.

$\therefore\ 4x^3 - 6x - 20 = 0$

$\therefore\ 2x^3 - 3x - 10 = 0$

Dit is waar wanneer $x = 2$ . . . *probeer en tref*

$\therefore\ 2x^3 - 3x - 10$

$= (x-2)(\ 2x^2 \ldots + 5)$

$= (x-2)(\ 2x^2 + 4x + 5)$

> *Vir* $2x^2 + 4x + 5$:
> $\Delta = 4^2 - 4(2)(5) = -24$
> & $\Delta < 0$ ➡ *'geen wortels'*

$\therefore\ x = 2$ is die enigste reële wortel van hierdie vergelyking

$\therefore$ Die punt is (2; 4)

$\therefore$ Die minimum afstand $= \sqrt{(2-10)^2 + (4-2)^2}$

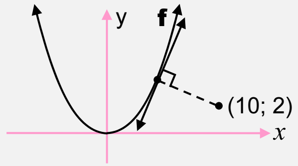
$= \sqrt{64 + 4}$

$= \sqrt{68}$

$= 2\sqrt{17}$

$\approx$ **8,25 eenhede** ◄

OF: Die kortste lyn getrek Van (10; 2) tot f is die loodlyn op die raaklyn by daardie punt.



Die gradiënt van die raaklyn $= \text{f}'(x) = 2x$

& Die gradiënt van die lyn is $\dfrac{x^2-2}{x-10}$

$\therefore\ 2x \times \dfrac{x^2-2}{x-10} = -1$ . . . *produk van die gradiënte* $= -1$

$\therefore\ 2x(x^2-2) = -x + 10$

$\therefore\ 2x^3 - 4x + x - 10 = 0$

$\therefore\ 2x^3 - 3x - 10 = 0$, ens. (soos hierbo)

Algemene Foute en Wanopvattings

Enige punt op f is
$(x\ ;\ x^2)$

Die oorgrote meerderheid van die kandidate het nie 'n poging aangewend om hierdie vraag te beantwoord nie omdat hulle nie **die afstandsfunksie** uit die gegewe inligting kon bepaal nie. Sommige kandidate het 'n probeer-en-trefmetode gebruik om die minimum afstand tussen die punte te verkry. Hulle het geen punte daarvoor gekry nie.

VRAAG 9: Voorstelle vir Verbetering

(a) Dit lyk asof leerders van formules wat gegee word afhanklik is wanneer dit by die oplos van optimeringsomme kom.

Dit is raadsaam dat leerders die optimumfunksie bevraagteken al word dit in 'n vraag gegee. Dit sal help met leerders se begripsontwikkeling.

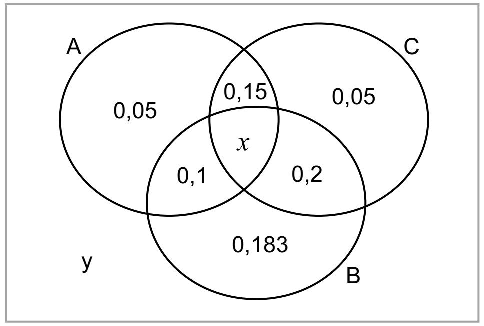
(b) Onderwysers moet seker maak dat daar genoeg tyd is vir leerders om die toepassing van Differensiaalrekene ten volle te verstaan.

(c) Leerders moet deurlopend met begrip lees indien hulle hul reaksies op woordsomme wil verbeter.

VRAAG 10 21%

10.1 A, B en C is drie gebeurtenisse. Die waarskynlikheid dat hierdie gebeurtenisse (of enige kombinasie daarvan) sal plaasvind, word in die Venn-diagram hieronder gegee.

25%



10.1.1 Indien daar gegee word dat die waarskynlikheid dat ten minste een van die gebeurtenisse sal plaasvind, 0,893 is, bereken die waarde van:

(a) y, die waarskynlikheid dat nie een van hierdie gebeurtenisse sal plaasvind nie. (1)

(b) x, die waarskynlikheid dat al drie gebeurtenisse sal plaasvind. (1)

Memo

10.1.1 (a) y, die waarskynlikheid dat nie een van die gebeurtenisse sal plaasvind nie.

= **1 –** die waarskynlikheid dat ten minste een van die gebeurtenisse sal plaasvind.

. . . *komplementêre gebeurtenisse*

= 1 – 0,893

= **0,107** ➤

(b) Die **som** van al die waarskynlikhede is **1**

$\therefore\ x + 0{,}05 + 0{,}1 + 0{,}15 + 0{,}2 + 0{,}05 + 0{,}183 + 0{,}107 = 1$

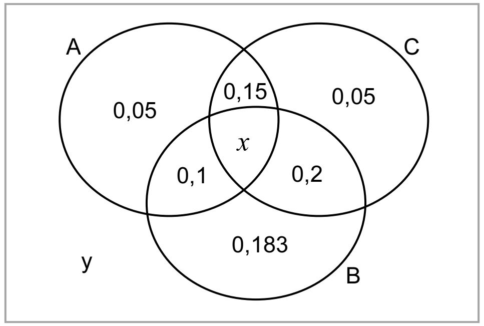
$\therefore\ x = 1 - 0{,}84$

$= \mathbf{0{,}16}$ ➤

Merk elkeen af soos jy dit inskryf!

Algemene Foute en Wanopvattings

(a) Baie kandidate het nie besef dat *die waarskynlikheid dat ten minste een van die drie gebeurtenisse sal plaasvind 0,893 is,* die som van die waarskynlikhede van al die areas wat deur die drie sirkels omsluit word, voorgestel het nie. Hulle kon dus nie **V10.1.1(a)** en **V10.1.1(b)** korrek beantwoord nie.



10.1.2 Bepaal die waarskynlikheid dat ten minste twee van die gebeurtenisse sal plaasvind. (2)

10.1.3 Is gebeurtenisse B en C onafhanklik? Motiveer jou antwoord. (5)

Memo

10.1.2 P(ten minste 2 gebeurtenisse sal plaasvind)

$= 0{,}1 + 0{,}16 + 0{,}15 + 0{,}2$

$= \mathbf{0{,}61}$ ◂

10.1.3 $\text{P(B en C)} = x + 0{,}2 = 0{,}16 + 0{,}2 = 0{,}36$

& $\text{P(B)} \times \text{P(C)} = (0{,}1 + 0{,}16 + 0{,}183 + 0{,}2) \times (0{,}16 + 0{,}15 + 0{,}2 + 0{,}05)$

$= 0{,}643 \times 0{,}56$

$= 0{,}360\ 08$

$\therefore$ $\text{P(B en C)} \neq \text{P(B)} \times \text{P(C)}$

$\therefore$ **Gebeurtenisse B en C is nie onafhanklik nie** ◂

Let wel:

Afronding van desimale word slegs gedoen **indien nodig** (wat nie hier die geval is nie).

Daar kan egter geargumenteer word dat, wanneer soos gewoonlik afgerond word, is $\text{P(B en C)} = \text{P(B)} \times \text{P(C)}$
$\ldots = 0{,}36$

& dan is gebeurtenisse B en C onafhanklik ◂

(b) In die beantwoording van **V10.1.2** was baie kandidate nie in staat om die areas in die diagram, wat die voorwaarde van *ten minste twee gebeurtenisse* bevredig, korrek te identifiseer nie.

(c) In **V10.1.3** het 'n aantal kandidate P(A), in plaas van P(C), van die skets afgelees. Hulle het verder gegaan om die onafhanklikheid van gebeurtenisse A en B, in plaas van gebeurtenisse B en C, te toets. Sommige kandidate het P(B) met P(slegs B) verwar en die waarde van P(B) verkeerdelik as 0,183 geneem.

10.2 'n Viersyferkode word benodig om 'n kombinasieslot oop te maak. Die kode moet 'n ewe getal wees en mag nie die syfers 0 of 1 bevat nie. Syfers mag nie herhaal word nie.

17%

10.2.1 Hoeveel moontlike 4-syfer-kombinasies is daar om die slot oop te maak? (3)

10.2.2 Bereken die waarskynlikheid dat jy die slot met die eerste poging sal oopmaak as daar gegee word dat die kode groter as 5 000 is en dat die derde syfer 2 is. (5)

[17]

TOTAAL : 150

Memo

10.2 $\underline{7}\ \ \underline{6}\ \ \underline{5}\ \ \underline{4}$ 2, 3, 4, 5, 6, 7, 8, 9

(8)

↑ 2, 4, 6, 8

10.2.1 Die aantal moontlike 4-syfer kombinasies

$= 7 \times 6 \times 5 \times 4$

$= \mathbf{840}$ ◄

10.2.2

5 syfers ≥ 5 →	**5** 5, 6, 7, 8, 9	**5** 3, 5, 6, 7, 8, 9 geen herhalings nie	**1** 2	**1** 4	㉕ ← ewe
Kan nie 6 gebruik nie →	**4** 5, 7, 8, 9	**5** 3, 4, 5, 7, 8, 9 geen herhalings nie	**1** 2	**1** 6	⑳ ← ewe
Kan nie 8 gebruik nie →	**4** 5, 6, 7, 9	**5** 3, 4, 5, 6, 7, 9 geen herhalings nie	**1** 2 ↑ 3^{de} syfer is 2	**1** 8	⑳ ← ewe

∴ Die aantal verskillende maniere

$= (5 \times 5 \times 1 \times 1) + (4 \times 5 \times 1 \times 1) + (4 \times 5 \times 1 \times 1)$

$= 25 + 20 + 20$

$= 65$

∴ Die waarskynlikheid vir hierdie kombinasie

$= \frac{n(E)}{n(S)} = \mathbf{\frac{1}{65}}$ ◄

(d) Baie kandidate was nie in staat om **V10.2.1** en **V10.2.2** korrek te beantwoord nie omdat hulle nie in staat was om die **beperkings** wat op die syfers geplaas is, in berekening te bring nie. Die probleem is vererger deur die feit dat die syfers **nie herhaal mag word nie**.

VRAAG 10: Voorstelle vir Verbetering

(a) Om **basiese konsepte** te onderrig, kan nie oor die hoof gesien word nie. Wanneer leerders die basiese konsepte goed genoeg verstaan, is dit makliker om die meer ingewikkelde konsepte te begryp.

(b) Dit moet benadruk word dat die **waarskynlikheid van 'n gebeurtenis A** in die interval $\mathbf{0 \leq P(A) \leq 1}$ lê.

(c) Om met begrip te lees moet gereeld in die klaskamer geoefen word. Dit behoort leerders met die vaardighede om woordsomme in assesseringstake te kan hanteer, toe te rus.

(d) **Gebruik Venn-diagramme om *waarskynlikheid* te onderrig.** Dit help met die begrip van die verskillende areas wat die gebeurtenisse uitmaak, bv. slegs A, slegs B, A en B, A of B, nie A, nie B, nie A en nie B en nie A of nie B.

(e) Leer leerders die ***Basiese Telbeginsel*** op so 'n manier dat hulle in staat sal wees om hul antwoorde op hul **beredenering**, eerder as op die reël, te grond.