

Assigning Cognitive Levels to Exam Questions

Stephen Sproule, St John's College
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Overview

- The four levels and how I see them
- Documenting the levels and examples
- Thoughts on classifying questions
- Let's try some classifying ...



The four cognitive levels

- Knowing (Kn)
- Performing routine procedures (RP)
- Performing complex procedures (CP)
- Solving problems (PS)

(TIMSS, 1999)

Why use *these* levels?

- Why do **you** want to use these levels?
- Setting better and more balanced exams.
- It's easier to use than Bloom's taxonomy.
- Does it work in lower grades?
- Enables differentiation
- These are not "levels 1 to 4"!

Mathematical demand and your learners

My rule of thumb:

Classifying the level of demand using:

- **your** teaching
- **your** learners
- the grade you are teaching
- what the rest of South Africa is doing

Knowing

Keywords:

- recall
- identify
- state
- estimate

- Recall or identify
- Use the correct formula from the information sheet
- To think mathematically, learners must first remember and use :
 - mathematical facts,
 - appropriate mathematical vocabulary and symbols

... but is the question difficult?

Knowing

Grade 10: Solve for x , $(x-1)(x+3) = 0$

Grade 11: Describe the concavity of the graph of
$$y = -2x^2 + 4$$

Grade 11: State the domain of $f(x) = 2^x - 3$

Grade 12: Determine the 25th term of the
sequence 7, 11, 15, ...

How many steps does it take?

Routine procedures

Keywords:

- solve
- simplify
- calculate
- apply
- determine
- show that

- Perform well-known procedures
(Questions practised in class and for homework.)
- Simple applications and calculations which involve a number of steps.
- Identify and use correct formula.
- Derivation from given information
- Learned proofs
- (Estimation and rounding)

Performing routine procedures

Grade 10: Solve for x , if $3^x + 3^{x+1} = 108$

Grade 10: Solve for x , $7\cos(x+30^\circ) = 1$

Grade 11: Draw a sketch graph of

$$f(x) = 3(x - 2)^2 + 1$$

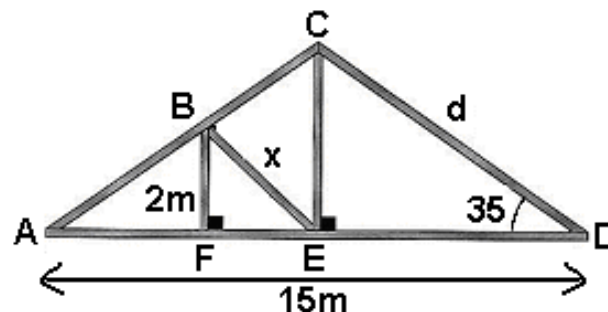
Grade 12: Show that the points P(-1; 1), Q(0; 3) and R(1; 5) are collinear.

Complex procedures

- Complex calculations and/or higher order reasoning
- Often **no obvious route** to the solution
 - Try something before you can see a route
 - Break down the question to find a starting point
- Requires deeper conceptual understanding
- **Connect** different representations or topics

Complex procedures

Grade 10: $\triangle ACD$ is a symmetrical roof truss. Solve x



Grade 11: Determine the minimum value of $\frac{1}{2^{2x-x^2}}$

Grade 12: Differentiate $\frac{d}{dx} \left(\frac{x^3 - 27}{x - 3} \right)$

Grade 12: Prove the identity $\frac{\cos 2\theta + \cos \theta + 1}{\sin 2\theta + \sin \theta} = \frac{1}{\tan \theta}$

Problem solving

Keywords:

- conjecture
- analyse
- evaluate
- generalise
- justify or prove

- **Non-routine** problems – likely to be unfamiliar to learners.
- Higher order reasoning and processes are often involved.
- Might require the ability to break a problem down into its constituent parts.
- (Transfer knowledge and thinking to a new situation.)

Problem solving

Grade 10: Solve for x , if $3^x = 7$

Grade 11: If $i^2 = -1$, determine the value of i^{2016}

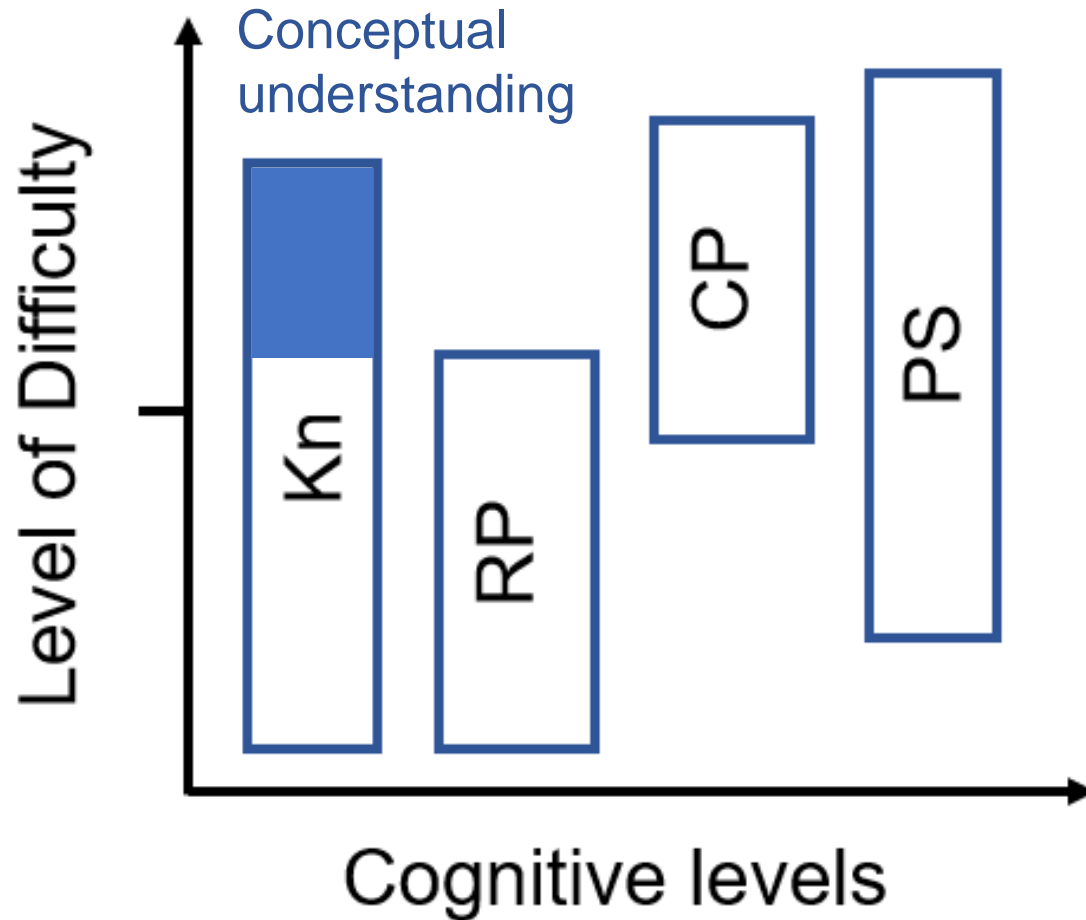
Grade 11: The average of 12 different positive integers is 12. What is the biggest possible value of any one of these numbers?

Grade 12: Find an equation for the route walked by an ant, if the ant is always that same distance from the points $(2; -1)$ and $(-1; 2)$.

Thoughts on classifying questions

- Different levels in the same question
- Prototypical examples to remember
 - CP: $3^{\sin x} + 3^{\sin x - 1} = 3^{-1}$
- Lowering the demand
- It's about your class and your school
- Putting together an exam: 55% – 30% – 15%

Also ... levels of difficulty



Let's try to classify some questions:

- Knowledge (K)
- Routine procedure (RP)
- Complex procedure (CP)
- Solving problems (PS)

1

How many real solutions does the following equation have?

$$(x+1)(2^x - 1) = 1$$

(7)

2

A photocopier valued at R24 000 depreciates at a rate of 18% p.a. on the reducing-balance method. After how many years will its value be R15 000? (4)

3

$g(x) = -8x + 20$ is a tangent to
 $f(x) = x^3 + ax^2 + bx + 18$ at $x = 1$

Calculate the values of a and b .

(5)

4

The first three terms of a geometric sequence are

6; x ; 54

Determine the value(s) of x (3)

5

The digits 1 to 7 are used to create a four-digit code to enter a locked room. How many different codes are possible if the digits may not be repeated and the code must be an even number bigger than 5 000? (5)

6

Determine the value of

$$9\,785\,627^2 - 9\,785\,630 \times 9\,785\,624$$

without using a calculator (5)

7

Given: $f(x) = 2x^2 - x$

Determine $f'(x)$ from first principles

8

The function defined as $y = \frac{a}{x+p} + q$ has the

following properties:

- The domain is $x \in \mathbb{R}, x \neq -2$
- $y = x + 6$ is an axis of symmetry
- The function is increasing for all $x \in \mathbb{R}, x \neq -2$

Draw a neat sketch graph of this function. Your sketch must include the asymptotes if they exist.

