Algebra Solutions

Non-negotiable

1. Solve for x: $\sqrt{5-x}-1=x$ (5)

$\sqrt{5-x}=x+1$ ✓

$\therefore 5-x=x^2+2x+1$ ✓

$\therefore x^2+3x-4=0$ ✓

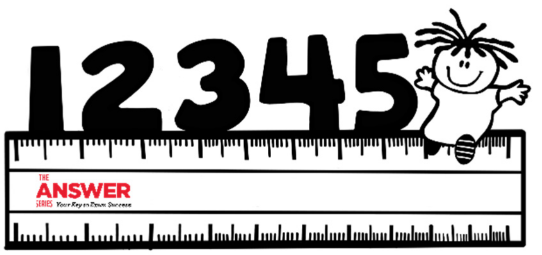
$\therefore (x+4)(x-1)=0$

$\therefore x=-4$ or $x=1$ ✓

If $x=-4$: $LHS=\sqrt{5-(-4)}=3$ and $RHS=-4+1=-3$ $\therefore LHS \neq RHS$

If $x=1$: $LHS=\sqrt{5-1}=2$ and $RHS=1+1=2$ $\therefore LHS = RHS$

$\therefore x=1$ ✓



Take it up a notch

2. Given $A = \dfrac{\sqrt{x+4}.(x-3)}{(x-1)^2}$

For what value(s) of x is:

2.1 $A = 0$? (2)

$x+4=0$ or $x-3=0$

$\therefore x=-4$ ✓ or $x=3$ ✓

2.2 A undefined? (1)

$x-1=0$

$\therefore x=1$ ✓

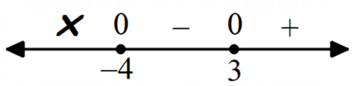
2.3 A non-real? (2)

$x+4<0$ ✓

$\therefore x<-4$ ✓

2.4 $A \leq 0$? (3)

$\sqrt{x+4}.(x-3) \leq 0$ … since $(x-1)^2 \geq 0$ for all x



$\therefore -4 \leq x \leq 3$ ✓✓; $x \neq 1$ ✓

Reach for the stars

https://www.theanswer.co.za/maths-grade-12-revision-algebra-2022/

3. A closed box has the shape of a rectangular prism with a square base. The sides of the base are x cm long. The height is y cm. The surface area of the box is 288 cm^2. The lengths of the edges are such that $2x + y = 21$. Determine the values of x and y. (7)

$SA = 2x^2 + 4xy = 288$ ✓

$\therefore x^2 + 2xy = 144$

$y = 21 - 2x$ ✓

$\therefore x^2 + 2x(21 - 2x) = 144$ ✓

$\therefore x^2 + 42x - 4x^2 = 144$ ✓

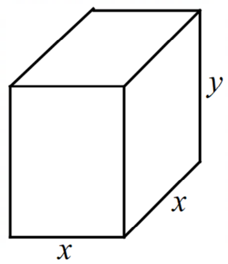
$\therefore 3x^2 - 42x + 144 = 0$ ✓

$\therefore x^2 - 14x + 48 = 0$

$\therefore (x - 6)(x - 8) = 0$

$\therefore x = 6$ or $x = 8$ ✓

$\therefore y = 9$ or $y = 5$ ✓



Sequences and Series Solutions

Non-negotiable

1. The first three terms of a convergent geometric sequence are $7x+1;\ 2x+2;\ x-1$.
Determine the value of x. (7)

$$\frac{2x+2}{7x+1}=\frac{x-1}{2x+2}\ \checkmark$$

$$\therefore (2x+2)^2=(7x+1)(x-1)\ \checkmark$$

$$\therefore 4x^2+8x+4=7x^2-6x-1\ \checkmark$$

$$\therefore 3x^2-14x-5=0\ \checkmark$$

$$\therefore (3x+1)(x-5)=0$$

$$\therefore x=-\frac{1}{3}\text{ or } x=5\ \checkmark$$

If $x=-\frac{1}{3}$: series is $-\frac{4}{3};\ \frac{4}{3};\ -\frac{4}{3}$ which has $r=-1$, $\therefore$ not convergent

If $x=5$: series is 36; 12; 4 which has $r=\frac{1}{3}$, $\therefore$ convergent since $-1<r<1$

$$\therefore x=5\ \checkmark\checkmark$$

Take it up a notch

2. The first four terms of a quadratic pattern are $-124;\ x;\ x^2;\ x^3$. Determine T_n of the sequence. (8)

$$\begin{array}{ccccccc} -124 & & x & & x^2 & & x^3 \\ & x+124 & & x^2-x & & x^3-x^2 & \\ & & x^2-2x-124\ \checkmark & & x^3-2x^2+x\ \checkmark & & \end{array}$$

$x^2-2x-124=x^3-2x^2+x$

$\therefore x^3-3x^2+3x+124=0$ ✓

$\therefore (x+4)(x^2-7x+31)=0$ ✓

$\therefore x=-4$ only ✓ ...If $x^2-7x+31=0$ $\therefore \Delta=(-7)^2-4(1)(31)=-75$ $\therefore$ imag roots

$$\begin{array}{ccccccc} -124 & & -4 & & 16 & & -64 \\ & 120 & & 20 & & -80 & \\ & & -100 & & -100 & & \end{array}$$

$2a=-100$	$\therefore a=-50$ ✓
$3a+b=120$	$\therefore b=270$ ✓
$a+b+c=-124$	$\therefore c=-344$ ✓

$\therefore T_n=-50n^2+270n-344$

Reach for the stars

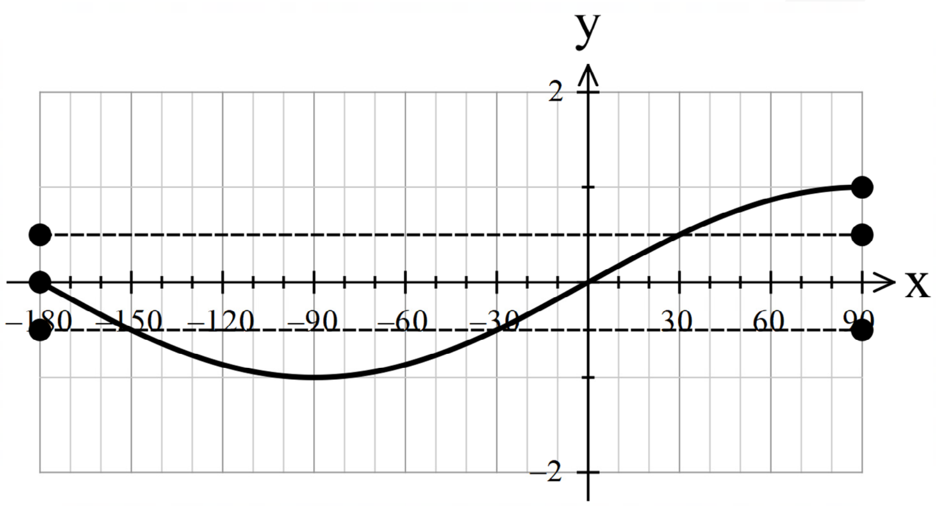
https://www.theanswer.co.za/maths-grade-12-revision-patterns-and-sequences-2022/

3. Given $\sin x - 2\sin^2 x + 4\sin^3 x - 8\sin^4 x + \ldots$
   For what values of x, with $x \in [-180°; 90°]$ will the series converge? (6)

$r = -2\sin x$ ✓

$\therefore -1 < -2\sin x < 1$ ✓

$\therefore \frac{1}{2} > \sin x > -\frac{1}{2}$ ✓

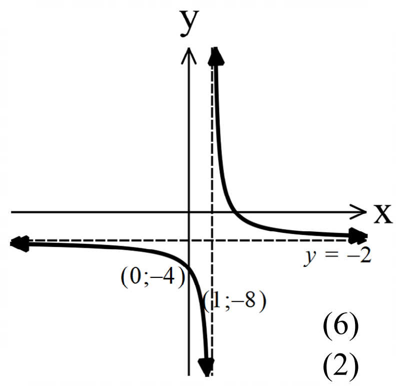


$\therefore -180° < x < -150°$ ✓ or $-30° < x < 30°$ ✓ with $x \neq 0°$ ✓

Functions Solutions

Non-negotiable

1. The graph of $f(x)=\dfrac{k}{x+r}+d$ is sketched below.



The graph passes through the points $(0;-4)$ and $(1;-8)$, and has a horizontal asymptote of $y=-2$.

1.1 Determine the values of k, r and d. (6)

1.2 Write down the range of f. (2)

1.3 Write down an equation for the axis of symmetry of f that has a negative gradient. (3)

1.1 $y=\dfrac{k}{x+r}-2$ ✓ … horizontal asymptote at $-2 \therefore d=-2$

$-4=\dfrac{k}{0+r}-2$ ✓ subs. $(0;-4)$

$\therefore -2=\dfrac{k}{r}$

$\therefore k=-2r$

$-8=\dfrac{k}{1+r}-2$ ✓ subs. $(1;-8)$

$\therefore -6=\dfrac{k}{1+r}$

$\therefore k=-6-6r$

$\therefore -2r=-6-6r$ ✓

$\therefore 4r=-6$

$\therefore r=-\dfrac{3}{2}$ ✓

$\therefore k=-2r=-2\left(-\dfrac{3}{2}\right)=3$ ✓

$\therefore k=3;\ r=-\dfrac{3}{2};\ d=-2$

1.2 $y\in\mathbb{R}$✓; $y\neq -2$ ✓

1.3 Asymptotes cut at $\left(\dfrac{3}{2};-2\right)$

$\therefore y+2=-\left(x-\dfrac{3}{2}\right)$ ✓✓

$\therefore y=-x-\dfrac{1}{2}$ ✓

Take it up a notch

2. Given: $f(x) = \frac{8}{x} + 2$. Determine the value of $f(4) + f'(4) + f^{-1}(4)$. (7)

$x = \frac{8}{y} + 2$ ✓

$\therefore xy = 8 + 2y$

$\therefore y(x-2) = 8$

$\therefore f^{-1}(x) = \frac{8}{x-2}$ ✓

$f(x) = 8x^{-1} + 2$

$\therefore f'(x) = -8x^{-2}$

$\therefore f'(x) = -\frac{8}{x^2}$ ✓

$\therefore f(4) + f'(4) + f^{-1}(4)$

$= \left[\frac{8}{4} + 2\right]$ ✓ $+ \left[-\frac{8}{4^2}\right]$ ✓ $+ \left[\frac{8}{4-2}\right]$ ✓

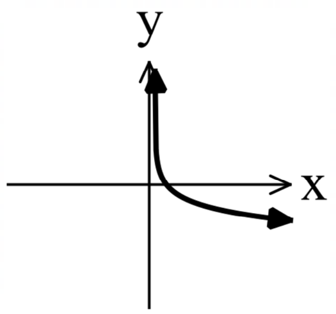
$= 4 + \left(-\frac{1}{2}\right) + 4$

$= 7\frac{1}{2}$ ✓

3. Given: $f(x) = a^x$ with $0 < a < 1$, and $h(x) = \dfrac{k}{x}$ with $k > 0$.

Determine the values of x which are common to the domains of $f^{-1}(x)$ and $h(x)$. (4)

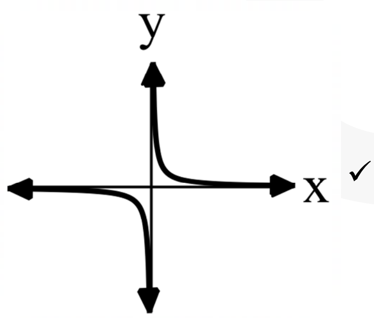
$f^{-1}(x) = \log_a x$ ✓ with $0 < a < 1$



Domain: $x > 0$

$\therefore x > 0$ ✓

$h(x) = \dfrac{k}{x}$ with $k > 0$



Domain: $x \in \mathbb{R};\ x \neq 0$

Finance Solutions

Non-negotiable

1. On 30 June 2013 and at the end of each month thereafter, Asif deposited R2 500 into a bank account that pays interest at 6% per annum, compounded monthly. He wants to continue to deposit this amount until 31 May 2018. Calculate how much money Asif will have in this account immediately after depositing R2 500 on 31 May 2018. (5)

$$F = \frac{2500\left[\left(1+\frac{0,06}{12}\right)^{60}-1\right]}{\frac{0,06}{12}} \checkmark\checkmark\checkmark$$

$$\therefore F = R174\ 425,08 \checkmark\checkmark$$

Take it up a notch

2. A woman invests R108 706,86 in a bank account and begins withdrawing R6 000 per quarter after three months. How many withdrawals can she make before all the money is withdrawn, if interest is calculated at 15% p.a. compounded quarterly? (5)

$$108\ 706,86 = \frac{6000\left[1-\left(1+\frac{0,15}{4}\right)^{-n}\right]}{\frac{0,15}{4}} \checkmark$$

$$\therefore \left(\frac{83}{80}\right)^{-n} = 0,3205... \checkmark$$

$$\therefore -n = \log_{\frac{83}{80}} 0,3205... \checkmark$$

$$\therefore -n = -30,9017... \checkmark$$

$\therefore$ she can make 31 withdrawals where the last one will be less than R6 000 $\checkmark$

Reach for the stars

https://www.theanswer.co.za/maths-grade-12-revision-finance-2022/

3. The Bradford's were granted a bond of R2 million at a rate of 13,5% p.a., compounded monthly, to be amortised in 20 years. Consider the implications of the interest rate changing to 14% p.a., compounded monthly, two years after the bond was granted.

   3.1 If the bond is still to be amortised in 20 years, determine the increased monthly payments. (7)

   3.2 If the monthly payments remain the same, determine how much longer it will take to pay off the bond. (4)

3.1 $$2\ 000\ 000 = \frac{x\left[1-\left(1+\frac{0{,}135}{12}\right)^{-240}\right]}{\frac{0{,}135}{12}}\ ✓$$

$$\therefore x = 24\ 147{,}49\ ✓$$

$$BO = 2\ 000\ 000\left(1+\frac{0{,}135}{12}\right)^{24} - \frac{24\ 147{,}49\left[\left(1+\frac{0{,}135}{12}\right)^{24}-1\right]}{\frac{0{,}135}{12}}\ ✓✓$$

$$\therefore BO = 1\ 954\ 896{,}67\ ✓$$

$$\therefore 1\ 954\ 896{,}67 = \frac{x\left[1-\left(1+\frac{0{,}14}{12}\right)^{-216}\right]}{\frac{0{,}14}{12}}\ ✓$$

$$\therefore x = R24\ 834{,}68\ ✓$$

3.2 $$1\ 954\ 896{,}67 = \frac{24\ 147{,}49\left[1-\left(1+\frac{0{,}14}{12}\right)^{-n}\right]}{\frac{0{,}14}{12}}\ ✓$$

$$\therefore \left(\frac{607}{600}\right)^{-n} = 0{,}0555...\ ✓$$

$$\therefore -n = \log_{\frac{607}{600}} 0{,}0555...$$

$$\therefore n = 249{,}26\ ✓$$

$$\therefore n = 250$$

$\therefore$ it will take 10 more months to pay off the bond ✓

Calculus Solutions

Non-negotiable

1. Given $f(x) = -3x^2$. Determine $f'(x)$ using first principles. (5)

$$f'(x) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$$

$$\therefore f'(x) = \lim_{h\to 0}\frac{-3(x+h)^2 - (-3x^2)}{h} \checkmark$$

$$\therefore f'(x) = \lim_{h\to 0}\frac{-3x^2 - 6xh - 3h^2 + 3x^2}{h} \checkmark$$

$$\therefore f'(x) = \lim_{h\to 0}\frac{-6xh - 3h^2}{h} \checkmark$$

$$\therefore f'(x) = \lim_{h\to 0}(-6x - 3h) \checkmark$$

$$\therefore f'(x) = -6x \checkmark$$

Take it up a notch

2. The line $g(x) = 5x + 1$ is a tangent to the curve of a function f at the point where $x = 2$. Calculate the value of $f(2) + f'(2)$. (4)

Since f and g meet at the point where $x = 2$:

$\therefore f(2) = g(2) = 5(2) + 1 = 11$ ✓✓

The gradient of f and g when $x = 2$ is the same:

$\therefore f'(2) = g'(2) = 5$ ✓

$\therefore f(2) + f'(2) = 11 + 5 = 16$ ✓

Reach for the stars

https://www.theanswer.co.za/maths-grade-12-revision-calculus-2022/

3. Given:

- $f(x)=x^3+bx^2+cx+d$
- $f'(x)>0$ for $x<-1$ or $x>2$
- $f'(x)<0$ for $-1<x<2$

Determine the values of b and c. (5)

$f'(x)=3x^2+2bx+c$ ✓

$f'(-1)=f'(2)=0$

$3(-1)^2+2b(-1)+c=0$ ✓

$\therefore -2b+c=-3$ ①

$3(2)^2+2b(2)+c=0$ ✓

$\therefore 4b+c=-12$ ②

$\therefore 6b=-9$ Eqn ② – ①

$\therefore b=-\frac{3}{2}$ ✓ and $c=-6$ ✓

Probability Solutions

Non-negotiable

1. On a randomly chosen day, the probability that James travels to school by train, by bus or on foot is $\frac{1}{2}$, $\frac{1}{6}$ and $\frac{1}{3}$ respectively. The probability of being late when using these methods of travel is $\frac{1}{5}$, $\frac{2}{5}$ and $\frac{1}{10}$ respectively.

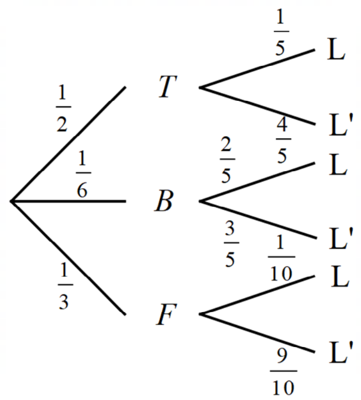
1.1 Draw a tree diagram to represent this information. (3)

1.2 Find the probability that on a randomly chosen day:

1.2.1 James travels by foot and is late. (2)

1.2.2 James is not late. (4)

1.1



✓✓✓

1.2.1 $P(\text{foot and late}) = \frac{1}{3} \times \frac{1}{10} ✓ = \frac{1}{30}$ ✓

1.2.2 $P(\text{not late}) = \frac{1}{2} \times \frac{4}{5} ✓ + \frac{1}{6} \times \frac{3}{5} ✓ + \frac{1}{3} \times \frac{9}{10} ✓ = \frac{4}{5}$ ✓

Take it up a notch

2. Given that $P(A) = 0,35$, $P(B) = 0,45$ and $P(A \text{ and } B) = 0,1$.

2.1 Find $P(A \text{ or } B)$. (2)

2.2 It is further given that $P(C) = 0,2$. The events A and C are mutually exclusive and events B and C are independent.

2.2.1 Determine $P(B \text{ and } C)$. (2)

2.2.2 Draw a Venn diagram showing A, B and C, and all probabilities. (4)

2.2.3 Determine $P[(B \text{ and } C)']$. (2)

2.1 $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\therefore P(A \text{ or } B) = 0,35 + 0,45 - 0,1$ ✓

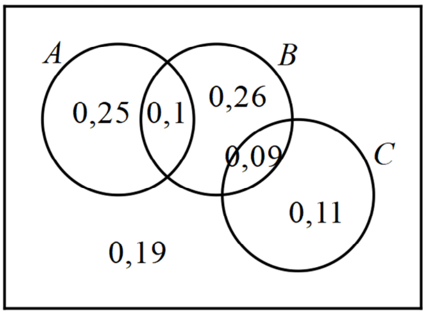
$\therefore P(A \text{ or } B) = 0,7$ ✓

2.2.1 $P(B \text{ and } C) = P(B) \times P(C)$

$\therefore P(B \text{ and } C) = 0,45 \times 0,2$ ✓

$\therefore P(B \text{ and } C) = 0,09$ ✓

2.2.2



✓✓✓✓

2.2.3 $P[(B \text{ and } C)'] = 1 - 0,09 = 0,91$ ✓✓

https://www.theanswer.co.za/maths-grade-12-revision-probability-2022/

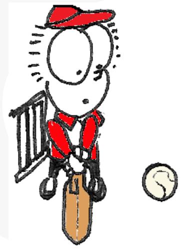
3. There are 11 players in a cricket team. They are asked to stand in a straight line for a photo. Three of the players, Andrew, Bobby, and Cuan refuse to stand next to each other. The other team members do not mind where they stand. Determine the number of ways in which the 11 players can be positioned for the photo. (5)

Total number of ways the players can stand together
$= 11! = 39\ 916\ 800$ ✓

Number of ways all three of A, B and C can stand together
$= 9! \times 3! = 2\ 177\ 280$ ✓

Number of ways exactly two of A, B and C can stand together
$= 8! \times 3 \times 2! \times 9 \times 8 = 17\ 418\ 240$ ✓✓

Number of ways they can be positioned for the photo
$= 39\ 916\ 800 - 2\ 177\ 280 - 17\ 418\ 240$
$= 20\ 321\ 280$ ✓



Data Handling Solutions

Non-negotiable

1. Various Grade 3 classes were tested for their reading ability. The following information was gathered.

Class size (x)	Mean reading score (y)
35	70
30	80
35	60
35	72
40	58
33	71
38	68
30	75
29	72
39	62

1.1 Determine the equation of the least squares regression line. (3)

1.2 Write down the correlation coefficient. (1)

1.3 Comment on the correlation coefficient, and explain how class size affects the mean reading score. (2)

1.1 $a = 117,78$ ✓

$b = -1,42$ ✓

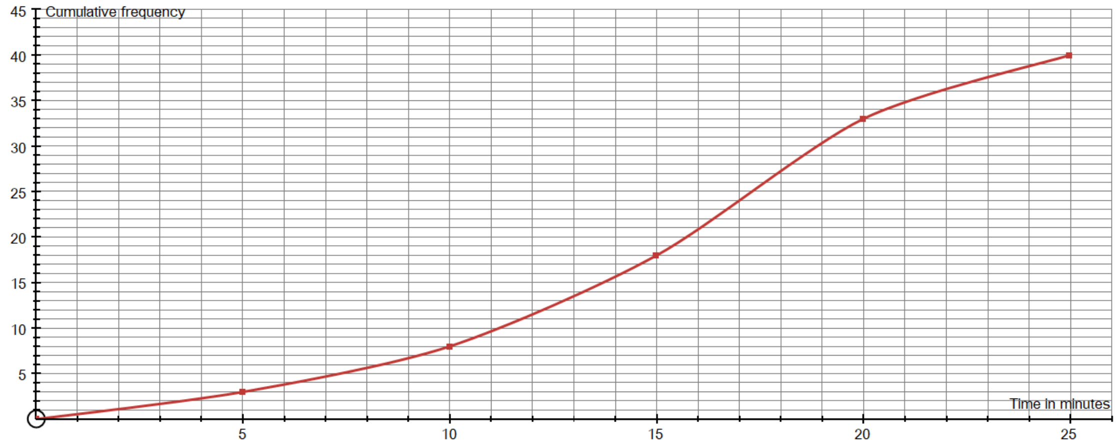
$\therefore y = 117,78 - 1,42x$ ✓

1.2 $r = -0,80$ ✓

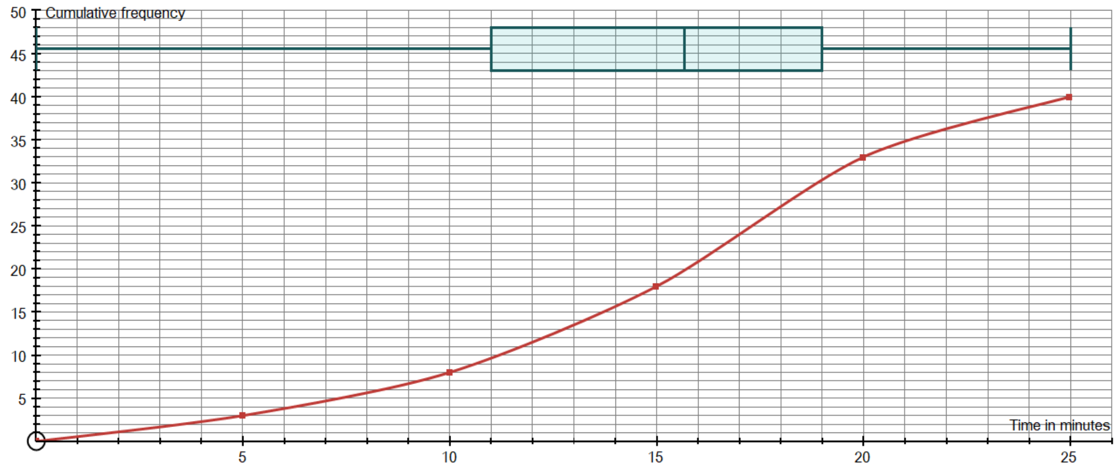
1.3 There is a strong, negative correlation coefficient. ✓ This means the bigger the class, the lower the reading score. ✓

Take it up a notch

2. The length of time, in minutes, of a certain number of cell phone calls was recorded. No call lasted longer than 25 minutes. A cumulative frequency diagram of this data is shown below.



Draw a box and whisker plot from the cumulative frequency curve. (5)



✓ minimum and maximum ✓ lower quartile ✓ median ✓ upper quartile ✓ shape

Reach for the stars

https://www.theanswer.co.za/maths-grade-12-revision-data-2022/

3. Five numbers, a, b, c, d and e, are written in ascending order. Determine the upper quartile if the largest number is 42, the range is 17, the lower quartile is 27, c and d are equal, and the mean of the five numbers is 34. (4)

$e = 42$

$a = 42 - 17 = 25$ ✓

$\frac{a+b}{2} = 27 \quad \therefore b = 29$ ✓ … the lower quartile is halfway between a and b

$\frac{a+b+c+d+e}{5} = 34 \quad \therefore c = d = 37$ ✓

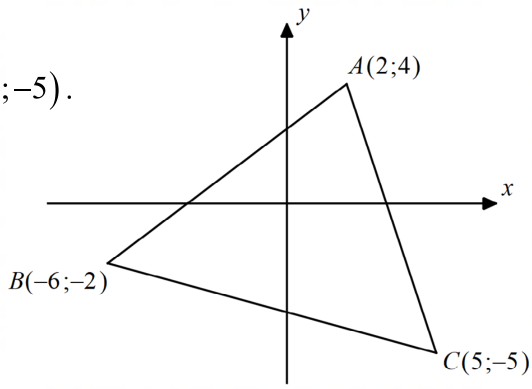
The five numbers are 25; 29; 37; 37; 42

$\therefore$ upper quartile $= \frac{37+42}{2} = 39\frac{1}{2}$ ✓

Analytical Geometry Solutions

Non-negotiable

1. ΔABC has $A(2;4)$, $B(-6;-2)$ and $C(5;-5)$.



1.1	Determine the co-ordinates of M, the midpoint of AB.	(2)
1.2	Determine the gradient of AC.	(2)
1.3	Determine the equation of the line passing through M parallel to AC.	(3)
1.4	Determine the length of AC.	(2)
1.5	Write down, with a reason and correct to two decimal places, the length of MN if $MN \parallel AC$ with N on BC.	(2)

1.1 $M\left(\dfrac{-6+2}{2};\dfrac{-2+4}{2}\right)$

$\therefore M(-2✓;1✓)$

1.2 $m_{AC}=\dfrac{4-(-5)}{2-5}✓=-3$ ✓

1.3 $y=-3x+c$ ✓

$\therefore 1=-3(-2)+c$ ✓

$\therefore c=-5$

$\therefore y=-3x-5$ ✓

1.4 $AC=\sqrt{(2-5)^2+(4-(-5))^2}$ ✓

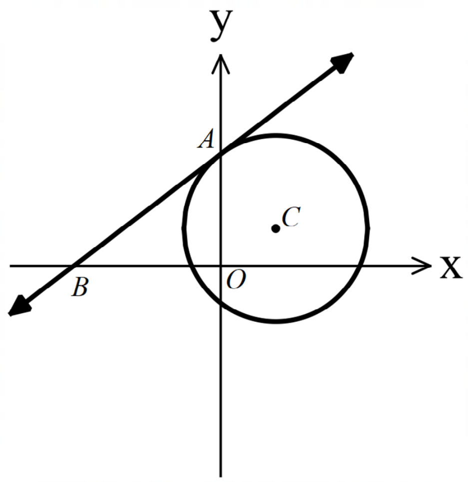
$\therefore AC=3\sqrt{10}$ $\quad$ (or $9,49$) ✓

1.5 $MN=\dfrac{1}{2}AC$ $\quad$ (conv midpt thm) ✓

$\therefore MN=4,74$ ✓

Take it up a notch

2. In the diagram, the circle with centre C and with equation $x^2 - 6x + y^2 - 4y = 12$ cuts the y-axis at A. BA is a tangent to the circle with B on the x-axis.



2.1	Determine the co-ordinates of C.	(4)
2.2	Determine the equation of BA.	(6)

2.1 $x^2 - 6x + y^2 - 4y = 12$

$\therefore (x-3)^2 \checkmark + (y-2)^2 \checkmark = 12 + 9 + 4 \checkmark$

$\therefore (x-3)^2 + (y-2)^2 = 25$

$\therefore C(3;2) \checkmark$

2.2 $(0)^2 - 6(0) + y^2 - 4y = 12 \checkmark$

$\therefore y^2 - 4y - 12 = 0$

$\therefore (y-6)(y+2) = 0$

$\therefore y = 6$ or $y = -2 \checkmark$

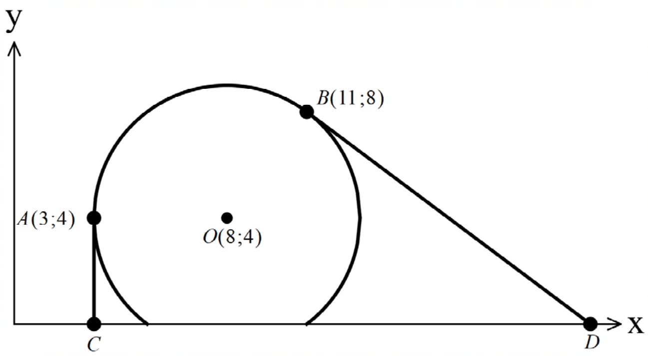
$\therefore A(0;6) \checkmark$

$m_{AC} = \dfrac{6-2}{0-3} = -\dfrac{4}{3} \checkmark$

$\therefore m_{\tan} = \dfrac{3}{4} \checkmark$

$\therefore BA: y = \dfrac{3}{4}x + 6 \checkmark$

3. A circle centred at $O(8;4)$ passes through $A(3;4)$ and $B(11;8)$. AC is a vertical line and BD is a tangent to the circle at B. A piece of wire is stretched from C to A, round the circle to B, then to D on the x axis.



Determine the length of the wire. (10)

AC: $AC = 4$ ✓

AB: $m_{OB} = \frac{8-4}{11-8} = \frac{4}{3}$ ✓

$\therefore \angle$ of incl $= \tan^{-1}\frac{4}{3} = 53,13°$

$\therefore A\hat{O}B = 180° - 53,13° = 126,87°$ ✓

$\therefore$ arc $AB = \frac{126,87°}{360°} \times 2\pi(5)$✓$= 11,07$ ✓

BD: $m_{BD} = -\frac{3}{4}$ ✓

$\therefore 8 = -\frac{3}{4}(11) + c$

$\therefore c = \frac{65}{4}$

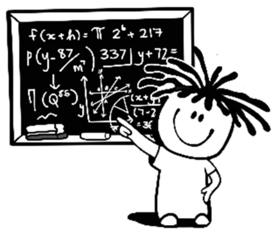
$\therefore y = -\frac{3}{4}x + \frac{65}{4}$ ✓

At D: $y = 0$

$\therefore D\left(\frac{65}{3};0\right)$ ✓

$\therefore BD = \sqrt{\left(11 - \frac{65}{3}\right)^2 + (8-0)^2} = \frac{40}{3}$ ✓

$\therefore$ length of wire $= 4 + 11,07 + \frac{40}{3} = 28,40$ units ✓



Trigonometry Solutions

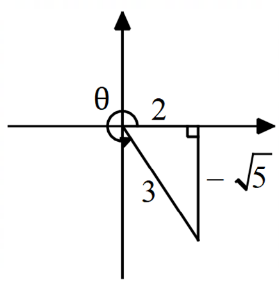
Non-negotiable

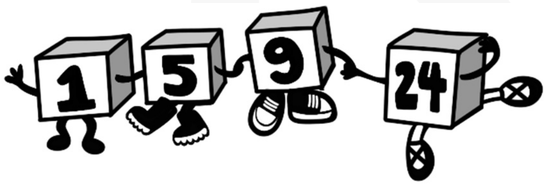
1. If $\cos\theta = \frac{2}{3}$ and $\theta > 90^\circ$, determine the value of $\cos\left(\theta + 45^\circ\right)$ without the use of a calculator. Leave your answer in surd form. (4)

$$\cos\left(\theta + 45^\circ\right) = \cos\theta\cos 45^\circ - \sin\theta\sin 45^\circ \ \checkmark$$

$$\therefore \cos\left(\theta + 45^\circ\right) = \frac{2}{3} \times \frac{1}{\sqrt{2}} \checkmark - \left(-\frac{\sqrt{5}\checkmark}{3}\right) \times \frac{1}{\sqrt{2}}$$

$$\therefore \cos\left(\theta + 45^\circ\right) = \frac{2+\sqrt{5}}{3\sqrt{2}} \ \checkmark$$





Take it up a notch

2. Prove the identity:

$$2\cos\theta.\cos 2\theta + \frac{\sin^2 2\theta}{\cos\theta} = 2\cos\theta \qquad (4)$$

$$\begin{aligned} LHS &= 2\cos\theta\left(1-2\sin^2\theta\checkmark\right)+\frac{\left(2\sin\theta\cos\theta\checkmark\right)^2}{\cos\theta} \\ &= 2\cos\theta - 4\sin^2\theta\cos\theta + \frac{4\sin^2\theta\cos^2\theta}{\cos\theta}\ \checkmark \\ &= 2\cos\theta - 4\sin^2\theta\cos\theta + 4\sin^2\theta\cos\theta\ \checkmark \\ &= 2\cos\theta \\ &= RHS \end{aligned}$$

OR

$$\begin{aligned} LHS &= 2\cos\theta\left(\cos^2\theta-\sin^2\theta\checkmark\right)+\frac{\left(2\sin\theta\cos\theta\checkmark\right)^2}{\cos\theta} \\ &= 2\cos^3\theta - 2\cos\theta\sin^2\theta + \frac{4\sin^2\theta\cos^2\theta}{\cos\theta}\ \checkmark \\ &= \frac{2\cos^4\theta - 2\cos^2\theta\sin^2\theta + 4\sin^2\theta\cos^2\theta}{\cos\theta} \\ &= \frac{2\cos^4\theta + 2\cos^2\theta\sin^2\theta}{\cos\theta} \\ &= \frac{2\cos^2\theta\left(\cos^2\theta+\sin^2\theta\right)}{\cos\theta}\ \checkmark \\ &= \frac{2\cos^2\theta(1)}{\cos\theta} \\ &= 2\cos\theta \\ &= RHS \end{aligned}$$

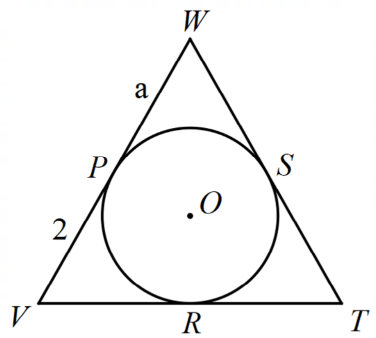
OR

$$\begin{aligned} LHS &= 2\cos\theta\left(2\cos^2\theta-1\checkmark\right)+\frac{\left(2\sin\theta\cos\theta\checkmark\right)^2}{\cos\theta} \\ &= 4\cos^3\theta - 2\cos\theta + \frac{4\sin^2\theta\cos^2\theta}{\cos\theta}\ \checkmark \\ &= 4\cos^3\theta - 2\cos\theta + 4\sin^2\theta\cos\theta \\ &= 4\cos\theta\left(\cos^2\theta+\sin^2\theta\right) - 2\cos\theta\ \checkmark \\ &= 4\cos\theta(1) - 2\cos\theta \\ &= 4\cos\theta - 2\cos\theta \\ &= 2\cos\theta \\ &= RHS \end{aligned}$$

Reach for the stars

https://www.theanswer.co.za/maths-grade-12-revision-trig-2022/

3. In the diagram, circle PSR with centre O is drawn inside ΔWVT, as shown.
WV = WT
WP = a units
PV = 2 units



Determine $\cos V$ in terms of a. (7)

$WS = a$ (tans from same pt) ✓
$ST = 2$ ($WV = WT$) ✓
$VR = 2$ (tans from same pt) ✓
$TR = 2$ (tans from same pt) ✓

$$\therefore \cos V = \frac{(a+2)^2 + 4^2 - (a+2)^2}{2(a+2)(4)} \checkmark$$

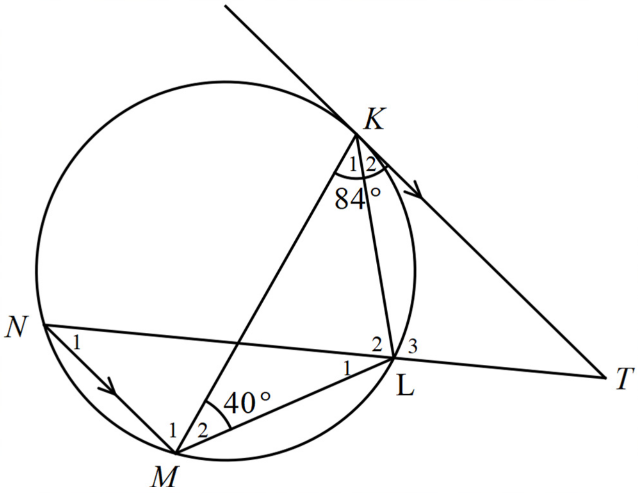
$$\therefore \cos V = \frac{16}{8(a+2)} \checkmark$$

$$\therefore \cos V = \frac{2}{a+2} \checkmark$$

Euclidean Geometry Solutions

Non-negotiable

1. In the diagram, tangent KT to the circle at K is parallel to the chord NM. NT cuts the circle at L. ΔKML is drawn. $\widehat{M}_2 = 40°$ and $M\widehat{K}T = 84°$.



Determine, giving reasons, the size of:

1.1	$\widehat{K}_2$	(2)
1.2	$\widehat{N}_1$	(3)
1.3	$\widehat{T}$	(2)
1.4	$\widehat{L}_2$	(2)
1.5	$\widehat{L}_1$	(1)

1.1 $\widehat{K}_2 = 40°$ ✓ (tan chord thm) ✓

1.2 $\widehat{K}_1 = 44°$ ✓

$\therefore \widehat{N}_1 = 44°$ ✓ (∠'s in same seg) ✓

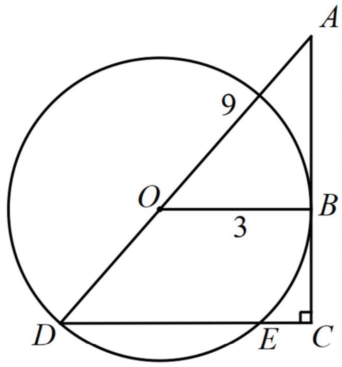
1.3 $\widehat{T} = 44°$ ✓ (alt ∠'s; KT || NM) ✓

1.4 $\widehat{L}_2 = 84°$ ✓ (ext ∠ of ΔKLT) ✓

1.5 $\widehat{L}_1 = 12°$ (∠ sum of ΔKLM) ✓

Take it up a notch

2. O is the centre of the circle. Tangent ABC meets chord DE produced to C such that $\hat{C} = 90°$. DO produced meets the tangent at A. $AO = 9$ units and $OB = 3$ units.



Calculate, giving reasons:

2.1 AB (4)

2.2 BC (5)

2.1 $O\hat{B}A = 90°$ ✓ (tan ⊥ rad) ✓

$\therefore AB = \sqrt{9^2 - 3^2} = 6\sqrt{2}$ ✓ units (Pythag) ✓

2.2 OB || DC (corres ∠s =) ✓

$OD = 3$ (radii) ✓

$\frac{AO}{OD} = \frac{AB}{BC}$ (prop thm; OB || DC) ✓

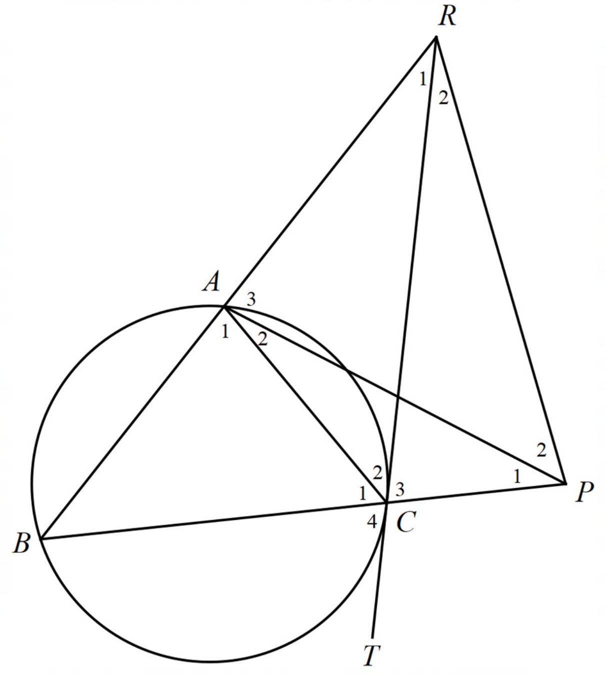
$\therefore \frac{9}{3} = \frac{6\sqrt{2}}{BC}$ ✓

$\therefore BC = 2\sqrt{2}$ units ✓

Reach for the stars

https://www.theanswer.co.za/maths-grade-12-revision-euclidean-2022/

3. In the diagram, chord BA and tangent TC of circle ABC are produced to meet at R. BC is produced to P with RC = RP. AP is not a tangent.



3.1 Prove that:

3.1.1 ACPR is a cyclic quadrilateral. (5)

3.1.2 $\Delta CBA \parallel\!| \Delta RPA$ (4)

3.1.3 $RC = \dfrac{CB.RA}{AC}$ (2)

3.1.4 $RB.AC = RC.CB$ (4)

3.2 Hence, prove that $RC^2 = RA.RB$ (3)

3.1

3.1.1 Let $\hat{C}_3 = x$

$\therefore \hat{C}_4 = x$ (vert opp ∠'s) ✓

$\therefore \hat{A}_1 = x$ (tan chord thm) ✓

$R\hat{P}C = x$ (∠'s opp equal sides) ✓

$\therefore \hat{A}_1 = R\hat{P}C$ ✓

$\therefore ACPR$ is a cyclic quadrilateral (ext ∠ = int opp ∠) ✓

3.1.2 In ΔCBA and ΔRPA

1. $\hat{C}_1 = A\hat{R}P$ (ext ∠ of cyc quad) ✓
2. $\hat{A}_3 = x$ (∠'s in same seg) ✓

   $\therefore \hat{A}_1 = \hat{A}_3$ ✓

$\therefore \Delta CBA \parallel\mkern-9mu\mid \Delta RPA$ (AAA) ✓

3.1.3 $\frac{CB}{RP} = \frac{BA}{PA} = \frac{CA}{RA}$ $(\Delta CBA \parallel\mkern-9mu\mid \Delta RPA)$ ✓

$\therefore RP = \frac{CB.RA}{CA}$ ✓

$\therefore RC = \frac{CB.RA}{CA}$ $(RP = RC)$

3.1.4 In ΔRBP and ΔCBA

1. $\hat{B}$ is common ✓
2. $B\hat{P}R = \hat{A}_1$ (ext ∠ of cyc quad) ✓

$\therefore \Delta RBP \parallel\mkern-9mu\mid \Delta CBA$ (AAA) ✓

$\therefore \frac{RB}{CB} = \frac{BP}{BA} = \frac{RP}{CA}$ $(\Delta RBP \parallel\mkern-9mu\mid \Delta CBA)$

$\therefore RB.CA = RP.CB$ ✓

$\therefore RB.CA = RC.CB$ $(RP = RC)$

3.2 $RC = \frac{RB.AC}{CB}$ ✓ (from 3.1.4)

$\therefore RC^2 = \frac{\cancel{CB}.RA}{\cancel{AC}} \times \frac{RB.\cancel{AC}}{\cancel{CB}}$ ✓✓

$\therefore RC^2 = RA.RB$