

Algebra

Non-negotiable

1.1 Solve for x , correct to two decimal places: $-2x^2 + 7x - 2 = 0$ (3)

$$x = \frac{-7 \pm \sqrt{7^2 - 4(-2)(-2)}}{2(-2)} \checkmark$$

$$\therefore x = 0,31 \text{ or } x = 3,19 \checkmark \checkmark$$

1.2 Solve for x : $\sqrt{5-x} - x = 1$ (5)

$$\sqrt{5-x} = x+1 \checkmark$$

$$\therefore 5-x = x^2 + 2x + 1 \checkmark$$

$$\therefore x^2 + 3x - 4 = 0 \checkmark$$

$$\therefore (x+4)(x-1) = 0 \text{ p}$$

$$\therefore x = -4 \text{ or } x = 1 \checkmark$$

$$\text{If } x = -4: LHS = \sqrt{5 - (-4)} = 3 \quad RHS = -4 + 1 = -3 \quad \therefore x \neq -4$$

$$\text{If } x = 1: LHS = \sqrt{5 - 1} = 2 \quad RHS = 1 + 1 = 2 \quad \therefore x = 1$$

$$\therefore x = 1 \checkmark$$

1.3 Simplify: $\frac{3^{2x+1} \cdot 15^{2x-3}}{27^{x-1} \cdot 3^x \cdot 5^{2x-4}}$ (4)

$$\frac{3^{2x+1} \cdot 15^{2x-3}}{27^{x-1} \cdot 3^x \cdot 5^{2x-4}}$$

$$= \frac{3^{2x+1} \cdot (3 \cdot 5)^{2x-3}}{(3^3)^{x-1} \cdot 3^x \cdot 5^{2x-4}}$$

$$= \frac{3^{2x+1} \cdot 3^{2x-3} \cdot 5^{2x-3}}{3^{3x-3} \cdot 3^x \cdot 5^{2x-4}} \checkmark$$

$$= 3^{2x+1+2x-3-(3x-3)-x} \cdot 5^{2x-3-(2x-4)} \checkmark$$

$$= 3^{2x+1+2x-3-3x+3-x} \cdot 5^{2x-3-2x+4}$$

$$= 3^1 \cdot 5^1 \checkmark$$

$$= 15 \checkmark$$



1.4 Solve for x and y simultaneously if:

$$x+4=2y \text{ and } y^2-xy+21=0$$

(6)

$$x=2y-4 \quad \checkmark$$

$$\therefore y^2-(2y-4)y+21=0 \quad \checkmark$$

$$\therefore y^2-2y^2+4y+21=0 \quad \checkmark$$

$$\therefore y^2-4y-21=0 \quad \checkmark$$

$$\therefore (y-7)(y+3)=0$$

$$\therefore y=7 \text{ or } y=-3 \quad \checkmark$$

$$\therefore x=2(7)-4=10 \text{ or } x=2(-3)-4=-10 \quad \checkmark$$



Take it up a notch

2.1 Determine, without the use of a calculator, the value of a , b and c such that:

$$(1-\sqrt{3})(a+b\sqrt{c})=-10+2\sqrt{3}$$

(5)

$$a+b\sqrt{c}=\frac{-10+2\sqrt{3}}{1-\sqrt{3}} \quad \checkmark$$

$$\therefore a+b\sqrt{c}=\frac{-10+2\sqrt{3}}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}} \quad \checkmark$$

$$\therefore a+b\sqrt{c}=\frac{-10-10\sqrt{3}+2\sqrt{3}+6}{1-3} \quad \checkmark$$

$$\therefore a+b\sqrt{c}=\frac{-4-8\sqrt{3}}{-2} \quad \checkmark$$

$$\therefore a+b\sqrt{c}=2+4\sqrt{3}$$

$$\therefore a=2; b=4; c=3 \quad \checkmark$$

Or

$$(1-\sqrt{3})(a+b\sqrt{c})=-10+2\sqrt{3}$$

$c=3$... the answer only has $\sqrt{3}$ in it

$$\therefore (1-\sqrt{3})(a+b\sqrt{3})=-10+2\sqrt{3} \quad \checkmark$$

$$\therefore a+b\sqrt{3}-a\sqrt{3}-3b=-10+2\sqrt{3} \quad \checkmark$$

$$\therefore a-3b=-10 \quad \textcircled{1} \quad \dots \text{equate rational parts}$$

$$\text{and } b-a=2 \quad \textcircled{2} \quad \dots \text{equate irrational parts } \checkmark$$

$$\textcircled{1} + \textcircled{2} \quad \therefore -2b = -8 \quad \checkmark$$

$$\therefore b=4$$

$$\therefore a=2; b=4; c=3 \quad \checkmark$$



- 2.2 Two hoses can fill a pool in 20 hours. If only one hose had been used at a time, the slower hose would have taken nine hours more than the faster hose to fill the pool. Determine the time taken by the faster hose to fill the pool. (6)

Let the time taken by the faster hose be x hours.

\therefore the time taken by the slower hose is $x + 9$ hours.

$$\therefore \frac{1}{x} + \frac{1}{x+9} = \frac{1}{20} \quad \checkmark \quad \dots \text{fraction of pool filled in one hour}$$

$$\therefore 20(x+9) + 20x = x(x+9) \quad \checkmark$$

$$\therefore 20x + 180 + 20x = x^2 + 9x \quad \checkmark$$

$$\therefore x^2 - 31x - 180 = 0 \quad \checkmark$$

$$\therefore (x - 36)(x + 5) = 0$$

$$\therefore x = 36 \text{ or } x = -5 \quad \checkmark$$

\therefore the time taken by the faster hose is 36 hours. \checkmark



Reach for the stars

<https://www.theanswer.co.za/maths-grade-11-revision-algebra-2022/>



3. Determine two non-zero numbers such that their sum, their product, and their quotient are all equal. (4)

Let the numbers be x and y

$$\therefore x + y = xy = \frac{x}{y} \quad \checkmark$$

$$xy = \frac{x}{y} \quad \checkmark$$

$$\therefore y^2 = 1 \quad \dots \text{since } x \neq 0 \text{ we can divide by } x$$

$$\therefore y = \pm 1 \quad \checkmark$$

If $y = 1$ then $x + 1 = x$ which has no solution

$$\text{If } y = -1 \text{ then } x - 1 = -x \therefore 2x = 1 \therefore x = \frac{1}{2}$$

The numbers are $\frac{1}{2}$ and -1 \checkmark



Patterns

Non-negotiable

1. Consider the number of unit squares in the figures below.

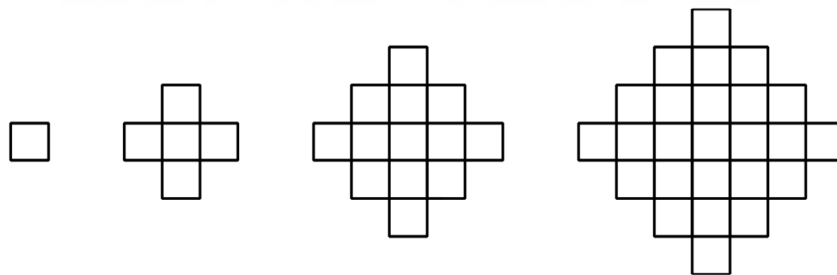


Fig. 1

Fig. 2

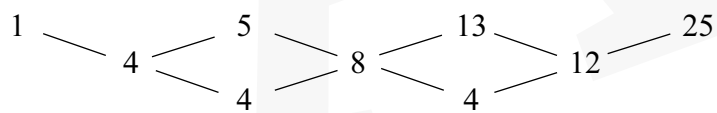
Fig. 3

Fig. 4

Figure number	1	2	3	4
Number of squares	1	5	13	25

Determine the number of unit squares in the 100th figure.

(6)



$$2a = 4 \therefore a = 2 \checkmark \checkmark$$

$$3a + b = 4 \therefore b = -2 \checkmark$$

$$a + b + c = 1 \therefore c = 1 \checkmark$$

$$\therefore T_n = 2n^2 - 2n + 1$$

$$\therefore T_{100} = 2(100)^2 - 2(100) + 1 \checkmark$$

$$\therefore T_{100} = 19801 \checkmark$$



Take it up a notch

2. The first four terms of a quadratic number pattern are 2; x ; y ; -37 .
The first three terms in the row of first differences of the same number pattern are $2p-3$; p^2-22 ; $5p-2$. Determine the value of x and y if $p \in \mathbb{Z}$. (6)

$$\begin{array}{ccccccc}
 & & & x & & y & & \\
 2 & \diagdown & & \diagup & & \diagdown & & \diagup & & -37 \\
 & & 2p-3 & & p^2-22 & & 5p-2 & & \\
 & & & \diagdown & & \diagup & & & \\
 & & & p^2-2p-19 & & -p^2+5p+20 & & &
 \end{array}$$

$$p^2 - 2p - 19 = -p^2 + 5p + 20 \quad \checkmark \checkmark$$

$$\therefore 2p^2 - 7p - 39 = 0 \quad \checkmark$$

$$\therefore (p+3)(2p-13) = 0$$

$$\therefore p = -3 \text{ or } p = \frac{13}{2} \quad \checkmark$$

$$\therefore p = -3 \quad \checkmark \quad \dots \text{ since } p \in \mathbb{Z}$$

$$\begin{array}{ccccccc}
 & & & x & & y & & \\
 2 & \diagdown & & \diagup & & \diagdown & & \diagup & & -37 \\
 & & -9 & & -13 & & -17 & & \\
 & & & & & & & &
 \end{array}$$

$$\therefore x = -7 \text{ and } y = -20 \quad \checkmark$$

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<https://www.theanswer.co.za/maths-grade-11-revision-patterns-2022/>



3. A quadratic sequence has the sixth term equal to 19, the ninth term equal to 55 and the eleventh term is 89. Determine the formula for the general term. (7)

$$T_6 = 19 \therefore 36a + 6b + c = 19 \quad \textcircled{1} \quad \checkmark$$

$$T_9 = 55 \therefore 81a + 9b + c = 55 \quad \textcircled{2} \quad \checkmark$$

$$T_{11} = 89 \therefore 121a + 11b + c = 89 \quad \textcircled{3} \quad \checkmark$$

$$\textcircled{2} - \textcircled{1} \quad 45a + 3b = 36 \quad \textcircled{4}$$

$$\textcircled{3} - \textcircled{2} \quad 40a + 2b = 34 \quad \textcircled{5} \quad \checkmark \text{ both}$$

$$2 \times \textcircled{4} - 3 \times \textcircled{5} \quad -30a = -30$$

$$\therefore a = 1 \quad \checkmark$$

$$45(1) + 3b = 36 \therefore b = -3 \quad \dots \text{ subs into } \textcircled{4} \quad \checkmark$$

$$36(1) + 6(-3) + c = 19 \therefore c = 1 \quad \dots \text{ subs into } \textcircled{1}$$

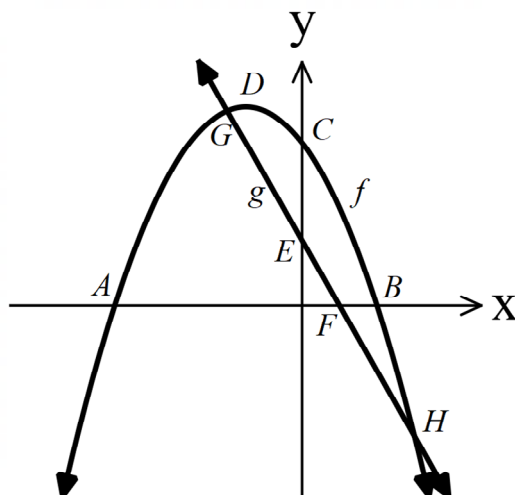
$$\therefore T_n = n^2 - 3n + 1 \quad \checkmark$$



Functions

Non-negotiable

1. Given $f(x) = -3x^2 - 9x + 30$, with A and B the x -intercepts and D the turning point, and $g(x) = -12x + 12$.



- 1.1 Determine the co-ordinates of A and B. (3)

$$\begin{aligned} -3x^2 - 9x + 30 &= 0 \\ \therefore x^2 + 3x - 10 &= 0 \\ \therefore (x + 5)(x - 2) &= 0 \quad \checkmark \\ \therefore x &= -5 \text{ or } x = 2 \\ \therefore A(-5; 0) \text{ and } B(2; 0) &\quad \checkmark \checkmark \end{aligned}$$



- 1.2 Determine the co-ordinates of D. (2)

$$\begin{aligned} x &= -\frac{-9}{2(-3)} = -\frac{3}{2} \\ y &= -3\left(-\frac{3}{2}\right)^2 - 9\left(-\frac{3}{2}\right) + 30 = \frac{147}{4} \\ \therefore D\left(-\frac{3}{2}; \frac{147}{4}\right) \text{ or } D(-1,5; 36,75) &\quad \checkmark \checkmark \end{aligned}$$



- 1.3 Determine the co-ordinates of G and H, the intersection of f and g .

(4)

$$\begin{aligned} -3x^2 - 9x + 30 &= -12x + 12 \quad \checkmark \\ \therefore 3x^2 - 3x - 18 &= 0 \quad \checkmark \\ \therefore x^2 - x - 6 &= 0 \\ \therefore (x-3)(x+2) &= 0 \\ \therefore x &= 3 \text{ or } x = -2 \\ \therefore G(-2; 36) \text{ and } H(3; -24) &\quad \checkmark \checkmark \end{aligned}$$

- 1.4 For what values of x is $f(x) > 0$?

(2)

$$-5 < x < 2 \quad \checkmark \checkmark$$

- 1.5 For what values of x is $f(x) \leq g(x)$?

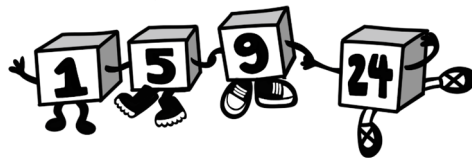
(2)

$$x \leq -2 \text{ or } x \geq 3 \quad \checkmark \checkmark$$

- 1.6 Write down the range of $f(x)$.

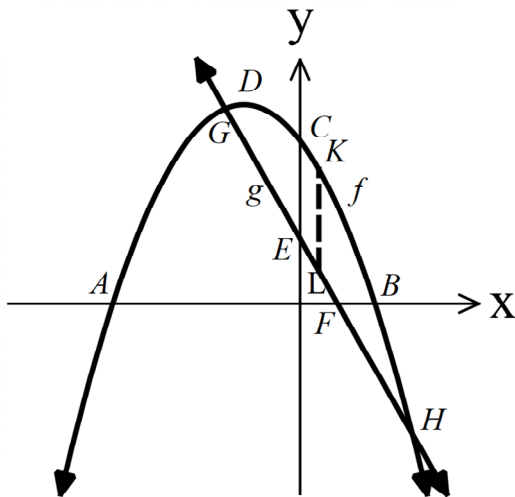
(1)

$$y \leq \frac{147}{4} \text{ or } y \leq 36,75 \quad \checkmark$$



Take it up a notch

2. Continue with $f(x) = -3x^2 - 9x + 30$ and $g(x) = -12x + 12$ from question 1.
 KL is a vertical line with K on f and L on g between points G and H.



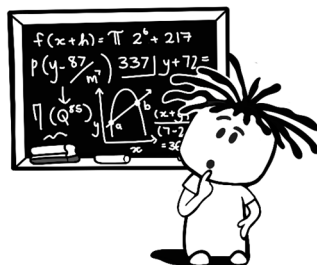
2.1 For what values of x is $\frac{f(x)}{g(x)} \geq 0$? (3)

$$F(1;0)$$

$$\therefore -5 \leq x < 1 \text{ or } x \geq 2 \checkmark \checkmark \checkmark$$

2.2 For what value of p will $f(x) = p$ have two unequal, negative roots? (2)

$$30 < p < 36,75 \checkmark \checkmark$$



2.3 Determine the maximum length of KL.

(5)

$$KL = (-3x^2 - 9x + 30) - (-12x + 12) \checkmark$$

$$\therefore KL = -3x^2 - 9x + 30 + 12x - 12$$

$$\therefore KL = -3x^2 + 3x + 18 \checkmark$$

$$\therefore KL = -3(x^2 - x - 6)$$

$$\text{or } x = -\frac{b}{2a} = -\frac{3}{2(-3)} = \frac{1}{2} \checkmark$$

$$\therefore KL = -3 \left[\left(x - \frac{1}{2} \right)^2 - \frac{1}{4} - 6 \right] \checkmark$$

$$\therefore KL = -3 \left(\frac{1}{2} \right)^2 + 3 \left(\frac{1}{2} \right) + 18 \checkmark$$

$$\therefore KL = -3 \left[\left(x - \frac{1}{2} \right)^2 - 6 \frac{1}{4} \right]$$

$$\therefore KL = \frac{75}{4}$$

$$\therefore KL = -3 \left(x - \frac{1}{2} \right)^2 + \frac{75}{4} \checkmark$$

$$\therefore \text{maximum length} = \frac{75}{4} \text{ or } 18,75 \checkmark$$

2.4 Given $h(x) = -3x + k$. Determine the value of k if $h(x)$ is a tangent to $f(x)$.

(5)

$$-3x^2 - 9x + 30 = -3x + k \checkmark$$

$$\therefore 3x^2 + 6x + k - 30 = 0 \checkmark$$

$$\Delta = 6^2 - 4(3)(k - 30) \checkmark$$

$$\Delta = 0 \checkmark \quad \dots \text{ a tangent touches once, so equal roots}$$

$$\therefore 36 - 12k + 360 = 0$$

$$\therefore -12k = -396$$

$$\therefore k = 33 \checkmark$$

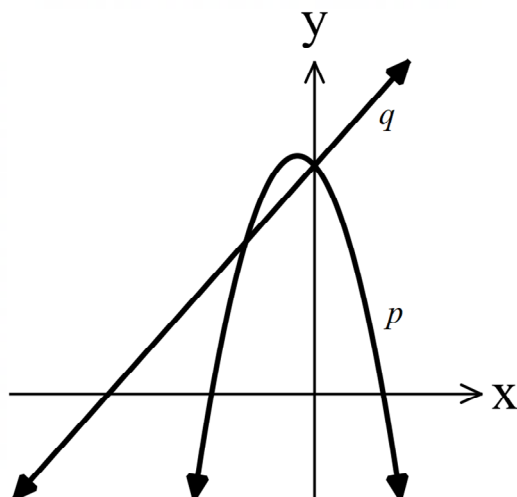


Reach for the stars

<https://www.theanswer.co.za/maths-grade-11-revision-functions-2022/>



3. Given $p(x) = ax^2 + bx + c$ and $q(x) = mx + c$.



You are given that $\frac{q(x)}{p(x)} < 0$ for all values of x when $-6 < x < -3$ or $x > 2$.

Determine, showing working, the value of m in terms of a .

(5)

$$p(x) = a(x+3)(x-2) \quad \checkmark$$

$$\therefore p(x) = ax^2 + ax - 6a \quad \checkmark$$

$$\therefore c = -6a$$

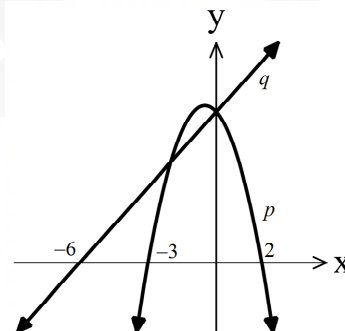
$$\therefore 6m = -6a \quad \checkmark$$

$$\therefore m = -a \quad \checkmark$$

$$q(x) = mx + c$$

$$\therefore 0 = -6m + c \quad \checkmark$$

$$\therefore c = 6m$$



Finance

Non-negotiable

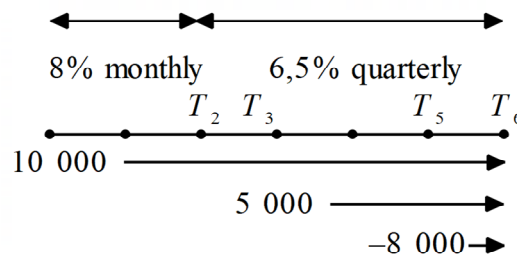
1. Calculate the original price of an iPad if the depreciated value after 3 years is R7 045,32. The rate of depreciation is 13% p.a. based on the reducing balance method. (3)

$$7045,32 = P(1 - 0,13)^3 \checkmark\checkmark$$

$$\therefore P = R10\,698,99 \checkmark$$

Take it up a notch

2. A woman made an initial investment of R10 000 into an account. Three years later she deposited R5 000 into the same account. She withdrew R8 000 five years after the initial investment. The interest rate was 8% p.a. compounded monthly for the first two years, and then it changed to 6,5% p.a. compounded quarterly after that. Determine the final amount in her account after six years. (5)



$$A = 10000 \left(1 + \frac{0,08}{12}\right)^{24} \left(1 + \frac{0,065}{4}\right)^{16} + 5000 \left(1 + \frac{0,065}{4}\right)^{12} - 8000 \left(1 + \frac{0,065}{4}\right)^4 \checkmark\checkmark\checkmark$$

$$\therefore A = R12\,714,00 \checkmark\checkmark$$



Reach for the stars

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3. The income tax in South Africa is levied at a rate of $a\%$ for the first R488 700. For any amount above R488 700 the rate is $(a + 5)\%$. A woman noticed her effective tax was $(a + 0,34)\%$ of her annual income. Determine her annual income. (5)

Let her annual income be x

$$488\,700 \times a\% + (x - 488\,700) \times (a + 5)\% = x \times (a + 0,34)\% \quad \checkmark\checkmark\checkmark$$

$$\therefore 488\,700 \times a + (x - 488\,700) \times (a + 5) = x \times (a + 0,34)$$

$$\therefore 488\,700a + xa + 5x - 488\,700a - 2\,443\,500 = xa + 0,34x \quad \checkmark$$

$$\therefore 4,66x = 2\,443\,500$$

$$\therefore x = R524\,356,22 \quad \checkmark$$

Or

Let the income above R488 700 be x

$$488\,700 \times a\% + x \times (a + 5)\% = (488\,700 + x) \times (a + 0,34)\% \quad \checkmark\checkmark\checkmark$$

$$\therefore 488\,700a + x(a + 5) = (488\,700 + x)(a + 0,34)$$

$$\therefore 488\,700a + xa + 5x = 488\,700a + 166\,158 + xa + 0,34x \quad \checkmark$$

$$\therefore 4,66x = 166\,158$$

$$\therefore x = 35\,656,22$$

$$\therefore \text{annual income} = 488\,700 + 35\,656,22 = R524\,356,22 \quad \checkmark$$



Probability

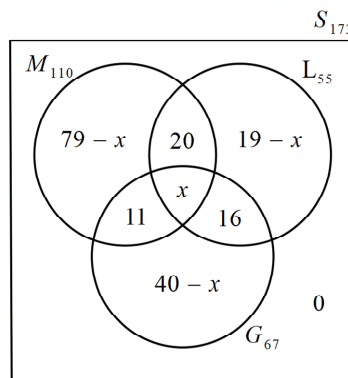
Non-negotiable

1. 173 Grade 12's at a school were surveyed to see who took Mathematics (M), Life Sciences (L) and Geography (G).
- Every Grade 12 took at least one of these three subjects
 - 110 take Mathematics
 - 55 take Life Sciences
 - 67 take Geography
 - 20 take Mathematics and Life Sciences, but not Geography
 - 11 take Mathematics and Geography, but not Life Sciences
 - 16 take Life Sciences and Geography, but not Mathematics
 - x take all three subjects

1.1 Draw a Venn diagram to illustrate the above information.

(4)

✓✓✓✓



1.2 Determine the value of x .

(2)

$$79 - x + 19 - x + 40 - x + 20 + 11 + 16 + x = 173 \quad \checkmark$$
$$\therefore -2x = -12$$
$$\therefore x = 6 \quad \checkmark$$

1.3 Determine the probability that a student takes exactly one of these three subjects.

(2)

$$P = \frac{79 - 6 + 19 - 6 + 40 - 6}{173} \quad \checkmark$$
$$\therefore P = \frac{120}{173} \quad \checkmark$$

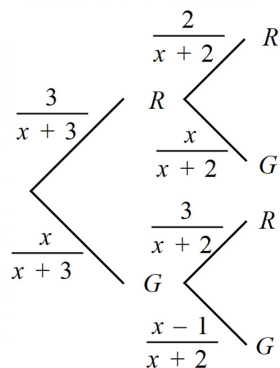


Take it up a notch

2. A bag contains three red marbles and x green marbles. Two marbles are pulled from the bag without replacement. The probability of getting one marble of each colour is $\frac{4}{7}$.

Determine the value of x .

(5)



$$\frac{3}{x+3} \times \frac{x}{x+2} + \frac{x}{x+3} \times \frac{3}{x+2} = \frac{4}{7} \quad \checkmark \checkmark$$

$$\therefore \frac{6x}{(x+3)(x+2)} = \frac{4}{7}$$

$$\therefore 42x = 4(x+3)(x+2) \quad \checkmark$$

$$\therefore 42x = 4x^2 + 20x + 24$$

$$\therefore 4x^2 - 22x + 24 = 0 \quad \checkmark$$

$$\therefore x^2 - 11x + 12 = 0$$

$$\therefore (x-4)(2x-3) = 0$$

$$\therefore x = 4 \text{ or } x = \frac{3}{2} \quad \checkmark$$

$$\therefore x = 4 \quad \checkmark$$

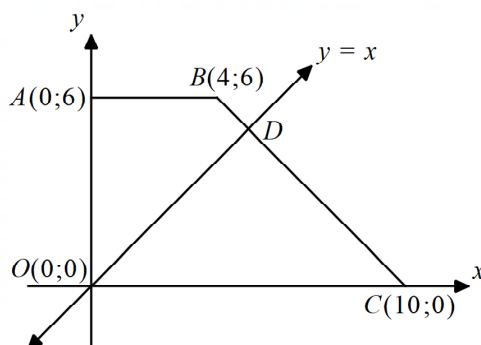


Reach for the stars

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3. A point $(x; y)$ is randomly picked inside a quadrilateral with vertices $O(0;0)$, $A(0;6)$, $B(4;6)$ and $C(10;0)$. What is the probability that $y \geq x$? (7)



$$m_{BC} = \frac{6-0}{4-10} = -1 \quad \checkmark$$

$$BC: y - 0 = -1(x - 10)$$

$$\therefore y = -x + 10 \quad \checkmark$$

$$D: -x + 10 = x \quad \checkmark$$

$$\therefore -2x = -10$$

$$\therefore x = 5$$

$$\therefore D(5;5) \quad \checkmark$$

$$Area_{OACB} = \frac{1}{2}(10+4)(6) = 42 \quad \checkmark$$

$$Area_{\triangle ODC} = \frac{1}{2}(10)(5) = 25 \quad \checkmark$$

$$\therefore Area_{\triangle OADB} = 42 - 25 = 17$$

$$\therefore P = \frac{17}{42} \quad \checkmark$$



Data Handling

Non-negotiable

1. The table below shows the marks obtained by a Grade 11 class for a Maths test out of 50.

Marks	Frequency	Cumulative Frequency
$0 < x \leq 10$	3	
$10 < x \leq 20$		11
$20 < x \leq 30$	15	
$30 < x \leq 40$	10	
$40 < x \leq 50$		40

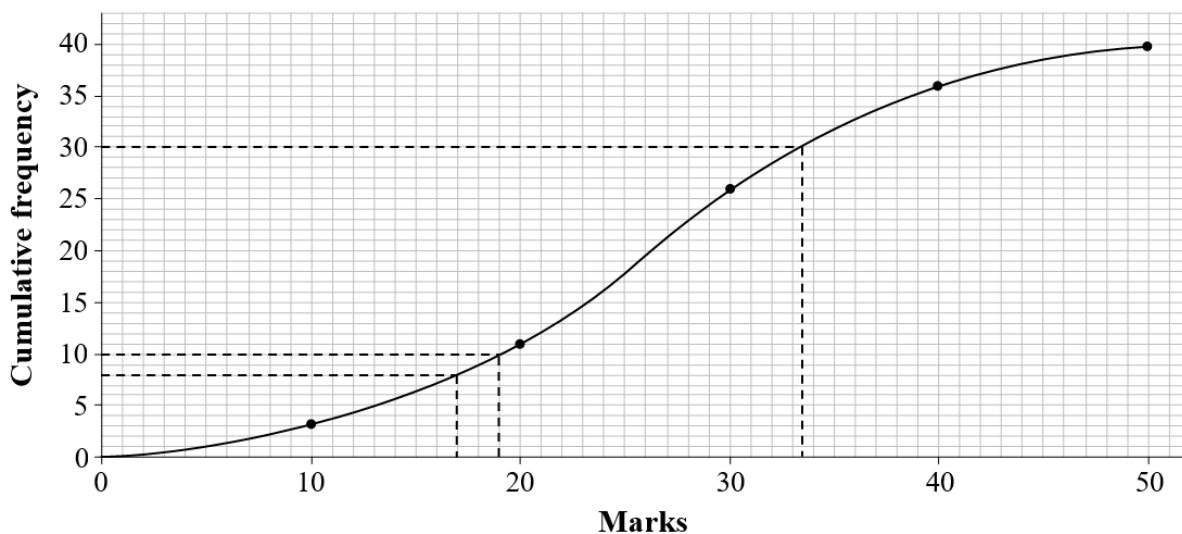
- 1.1 Complete the missing information in the table. (2)

✓✓ -1 per mistake

Marks	Frequency	Cumulative Frequency
$0 < x \leq 10$	3	3
$10 < x \leq 20$	8	11
$20 < x \leq 30$	15	26
$30 < x \leq 40$	10	36
$40 < x \leq 50$	4	40

- 1.2 Draw an ogive (cumulative frequency curve) representing the above information. (3)

✓ plotted at upper bound
✓ plotted correct cumulative frequency
✓ smooth curve



1.3 Determine, using your ogive, the interquartile range.

(3)

$$LQ = 19 \checkmark$$

$$UQ = 33,5 \checkmark$$

$$\therefore IQR = 33,5 - 19 = 14,5 \checkmark$$

1.4 If 20% of the class failed the test, use the ogive to determine the pass mark.

(2)

$$20\% = 8 \text{ people } \checkmark$$

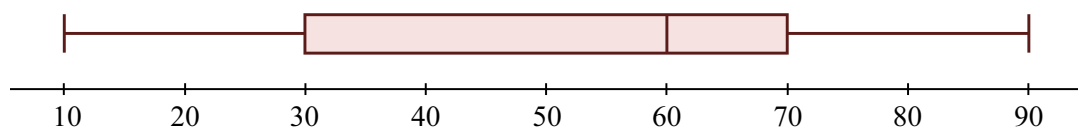
$$\therefore \text{pass mark is above 17 out of 50. } \checkmark$$

Take it up a notch

2. The marks obtained by 80 Grade 11 learners are shown below.

Marks	Frequency
$10 < x \leq 20$	7
$20 < x \leq 30$	A
$30 < x \leq 40$	B
$40 < x \leq 50$	4
$50 < x \leq 60$	10
$60 < x \leq 70$	C
$70 < x \leq 80$	12
$80 < x \leq 90$	D

A box and whisker plot is drawn of the data. No learner got exactly 30, 60, or 70 marks.



Determine the values of A, B, C, and D.

(4)

Each quartile has 20 learners.

$$10 < x \leq 30 \quad A = 20 - 7 = 13 \checkmark$$

$$30 < x \leq 60 \quad B = 20 - 14 = 6 \checkmark$$

$$60 < x \leq 70 \quad C = 20 \checkmark$$

$$70 < x \leq 90 \quad D = 20 - 12 = 8 \checkmark$$



Reach for the stars

<https://www.theanswer.co.za/maths-grade-11-revision-data-2022/>



3. Eight numbers are written in ascending order.

5; x ; 13; 17; 21; 21; y ; 31

The mean of the numbers is 18 and the interquartile range is 11. Determine the value of x and y .

(7)

$$\frac{5+x+13+17+21+21+y+31}{8} = 18 \checkmark$$

$$\therefore 108+x+y=144$$

$$\therefore x+y=36 \quad \textcircled{1} \checkmark$$

$$LQ = \frac{x+13}{2} \text{ and } UQ = \frac{21+y}{2} \checkmark$$

$$\therefore \frac{21+y}{2} - \frac{x+13}{2} = 11 \checkmark$$

$$\therefore 21+y-x-13=22$$

$$\therefore y-x=14 \quad \textcircled{2} \checkmark$$

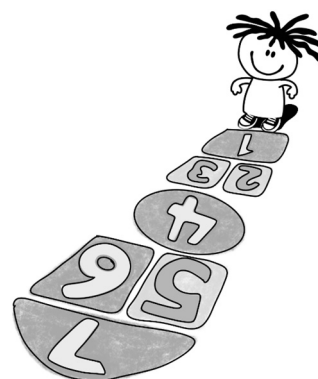
$$\textcircled{1} + \textcircled{2} \quad 2y = 50$$

$$\therefore y = 25 \checkmark$$

$$\therefore x + 25 = 36$$

$$\therefore x = 11 \checkmark$$

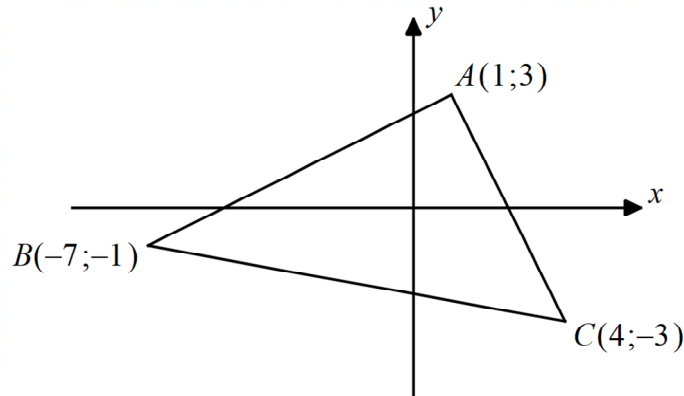
$$\therefore x = 11 \text{ and } y = 25$$



Analytical Geometry

Non-negotiable

1. In the diagram, $A(1;3)$, $B(-7;-1)$ and $C(4;-3)$ are given.



- 1.1 Determine the length of AB, in simplified surd form. (2)

$$AB = \sqrt{(1 - (-7))^2 + (3 - (-1))^2} \checkmark$$
$$\therefore AB = \sqrt{80}$$
$$\therefore AB = 4\sqrt{5} \checkmark$$

- 1.2 Determine the co-ordinates of Q, the midpoint of BC. (2)

$$Q\left(\frac{-7+4}{2}; \frac{-1-3}{2}\right) = Q\left(-\frac{3}{2}; -2\right) \checkmark\checkmark$$

- 1.3 Determine the gradient of AB. (2)

$$m_{AB} = \frac{3 - (-1)}{1 - (-7)} = \frac{1}{2} \checkmark\checkmark$$

- 1.4 Determine the equation of the line parallel to AB, passing through Q. (3)

$$y = \frac{1}{2}x + c \checkmark$$
$$\therefore -2 = \frac{1}{2}\left(-\frac{3}{2}\right) + c \checkmark$$
$$\therefore c = -\frac{5}{4}$$
$$\therefore y = \frac{1}{2}x - \frac{5}{4} \checkmark$$



1.5 Prove that $AB \perp AC$. (2)

$$m_{AC} = \frac{3 - (-3)}{1 - 4} = -2 \quad \checkmark$$

$$\therefore AB \perp AC \quad (m_{AB} \times m_{AC} = -1) \quad \checkmark$$

1.6 Determine the co-ordinates of D if ABCD is a parallelogram. (2)

$$x_B \rightarrow x_A + 8$$

$$x_C \rightarrow x_D + 8$$

$$\therefore x_D = 12$$

$$y_B \rightarrow y_A + 4$$

$$y_C \rightarrow y_D + 4$$

$$\therefore y_D = 1$$

$$\therefore D(12;1) \quad \checkmark \checkmark$$



Take it up a notch

2. Use the diagram in question 1.

2.1 Determine the size of \widehat{ABC} , correct to two decimal places. (4)

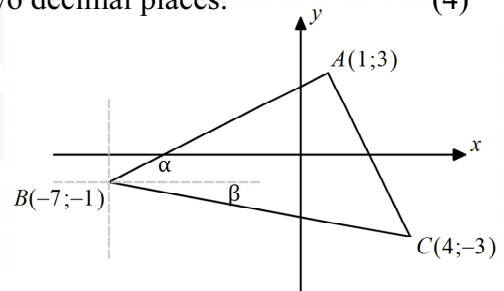
$$m_{AB} = \frac{1}{2}$$

$$\therefore \tan \alpha = \frac{1}{2} \quad \therefore \alpha = 26,57^\circ \quad \checkmark$$

$$m_{BC} = \frac{-1 - (-3)}{-7 - 4} = -\frac{2}{11} \quad \checkmark$$

$$\therefore \tan \beta = \frac{2}{11} \quad \therefore \beta = 10,30^\circ \quad \checkmark \dots \text{ use positive gradient since } \beta \text{ is an acute angle}$$

$$\therefore \widehat{ABC} = 36,87^\circ \quad \checkmark$$



2.2 P is a point on BC such that the area of ΔABC is four times the area of ΔABP . Determine the co-ordinates of P. (2)

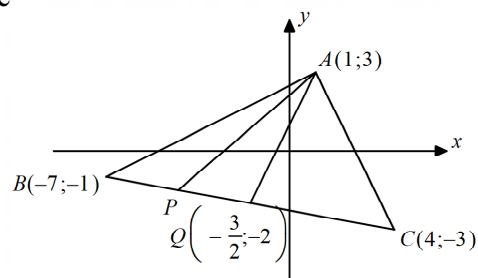
ΔABC and ΔABP have the same altitude

$$\therefore BP = \frac{1}{4} BC$$

$Q\left(-\frac{3}{2}; -2\right)$ is the midpoint of BC

$\therefore P$ will be the midpoint of BQ

$$\therefore P\left(\frac{-7 - \frac{3}{2}}{2}; \frac{-1 - 2}{2}\right) = P\left(-\frac{17}{4}; -\frac{3}{2}\right)$$

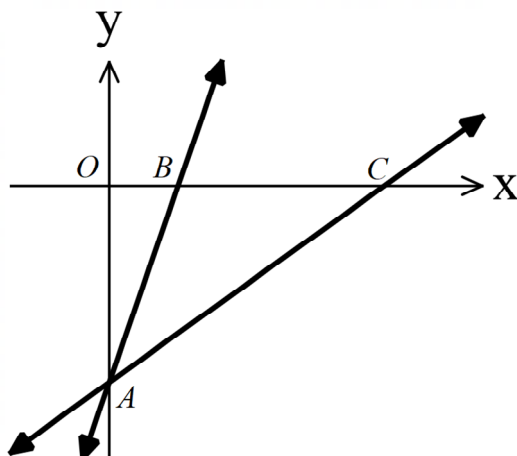


Reach for the stars



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3. In the diagram, the equations of the two straight lines are $2y - x + 2q = 0$ and $y - 2x + q = 0$.



Determine the value of q if the area of $\triangle ABC$ is 48 units².

(6)

$$AB: y - 2q + q = 0 \therefore y = 2x - q$$

$$AC: 2y - x + 2q = 0 \therefore y = \frac{1}{2}x - q$$

$$\therefore A(0; -q); B\left(\frac{q}{2}; 0\right); C(2q; 0) \checkmark\checkmark\checkmark$$

$$\text{Area} = \frac{1}{2} \times BC \times OA$$

$$\therefore \frac{1}{2} \times \frac{3q}{2} \times q = 48 \checkmark$$

$$\therefore 3q^2 = 192 \checkmark$$

$$\therefore q^2 = 64$$

$$\therefore q = \pm 8$$

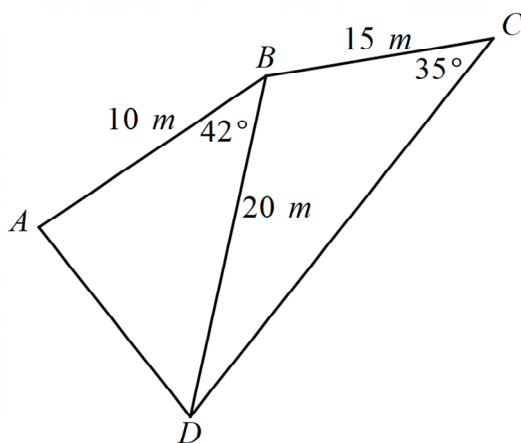
$$\therefore q = 8 \checkmark$$



Trigonometry

Non-negotiable

1. In the diagram, which is not drawn to scale, $AB = 10 \text{ m}$, $BD = 20 \text{ m}$, $BC = 15 \text{ m}$, $\widehat{ABD} = 42^\circ$ and $\widehat{BCD} = 35^\circ$.



Determine, correct to two decimal places:

- 1.1 the area of $\triangle ABD$. (2)

$$A = \frac{1}{2} \times 10 \times 20 \times \sin 42^\circ \checkmark$$
$$\therefore A = 66,91 \text{ m}^2 \checkmark$$

- 1.2 the length of AD. (2)

$$AD^2 = 10^2 + 20^2 - 2 \times 10 \times 20 \times \cos 42^\circ \checkmark$$
$$\therefore AD = 14,24 \text{ m} \checkmark$$

- 1.3 the size of \widehat{BDC} . (3)

$$\frac{\sin \widehat{BDC}}{15} = \frac{\sin 35^\circ}{20} \checkmark$$
$$\therefore \sin \widehat{BDC} = 0,4301... \checkmark$$
$$\therefore \widehat{BDC} = 25,48^\circ \checkmark$$



Take it up a notch

2. Solve for x , correct to two decimal places:

2.1 $27^{\tan x} = 9; x \in [-180^\circ; 360^\circ]$ (4)

$$\therefore 3^{3 \tan x} = 3^2 \checkmark$$

$$\therefore 3 \tan x = 2$$

$$\therefore \tan x = \frac{2}{3} \checkmark$$

$$\therefore x = 33,69 + n180^\circ \checkmark$$

$$\therefore x = 33,69^\circ; 213,69^\circ; -146,31^\circ \checkmark$$

2.2 $2\sin^2 x - 6\sin x \cos x = 3\cos x - \sin x$. Give the general solution. (6)

$$\therefore 2\sin^2 x - 6\sin x \cos x - 3\cos x + \sin x = 0$$

$$\therefore 2\sin x(\sin x - 3\cos x) - (3\cos x - \sin x) = 0 \checkmark$$

$$\therefore 2\sin x(\sin x - 3\cos x) + (\sin x - 3\cos x) = 0$$

$$\therefore (\sin x - 3\cos x)(2\sin x + 1) = 0 \checkmark$$

$$\therefore \sin x = 3\cos x \text{ or } \sin x = -\frac{1}{2} \checkmark$$

$$\therefore \tan x = 3 \text{ or } \sin x = -\frac{1}{2} \checkmark$$

$$\therefore x = 71,57^\circ + n180^\circ \text{ or } x = 210^\circ + n360^\circ \text{ or } x = 330^\circ + n360^\circ; n \in \mathbb{Z} \checkmark \checkmark$$

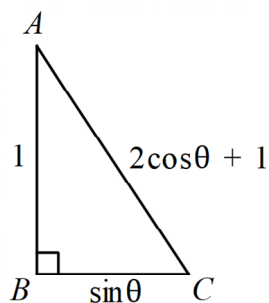


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3. In the diagram, $BC = \sin \theta$, $AB = 1$ and $AC = 2 \cos \theta + 1$.



Determine $\tan A$ without the use of a calculator.

(5)

$$(2 \cos \theta + 1)^2 = 1^2 + \sin^2 \theta \text{ (Pythag)} \checkmark$$

$$\therefore 4 \cos^2 \theta + 4 \cos \theta + 1 = 1 + \sin^2 \theta \checkmark$$

$$\therefore 4 \cos^2 \theta + 4 \cos \theta + 1 = 1 + 1 - \cos^2 \theta$$

$$\therefore 5 \cos^2 \theta + 4 \cos \theta - 1 = 0 \checkmark$$

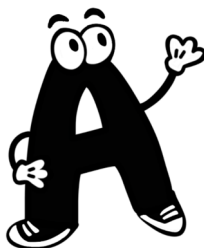
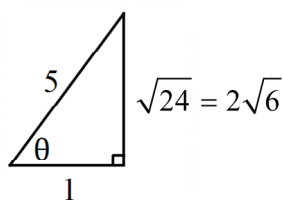
$$\therefore (5 \cos \theta - 1)(\cos \theta + 1) = 0$$

$$\therefore \cos \theta = \frac{1}{5} \text{ or } \cos \theta = -1$$

$$\therefore \cos \theta = \frac{1}{5} \checkmark \quad \dots \text{ if } \cos \theta = -1, \text{ then } AC = -1 \text{ which is not possible.}$$

$$\tan A = \frac{\sin \theta}{1}$$

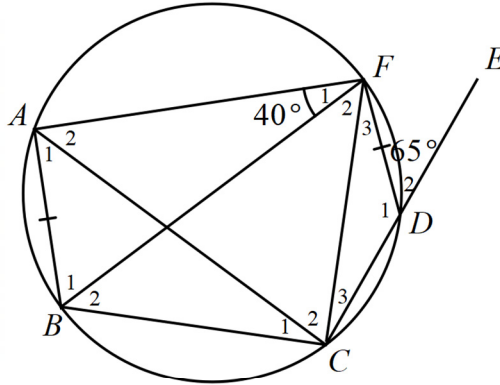
$$\therefore \tan A = \frac{2\sqrt{6}}{5} \checkmark$$



Euclidean Geometry

Non-negotiable

1. In the diagram, circle ABCDF has BF as a diameter. $\widehat{D}_2 = 65^\circ$, $\widehat{F}_1 = 40^\circ$ and $AB = DF$.



Determine the size of the following angles, giving reasons.

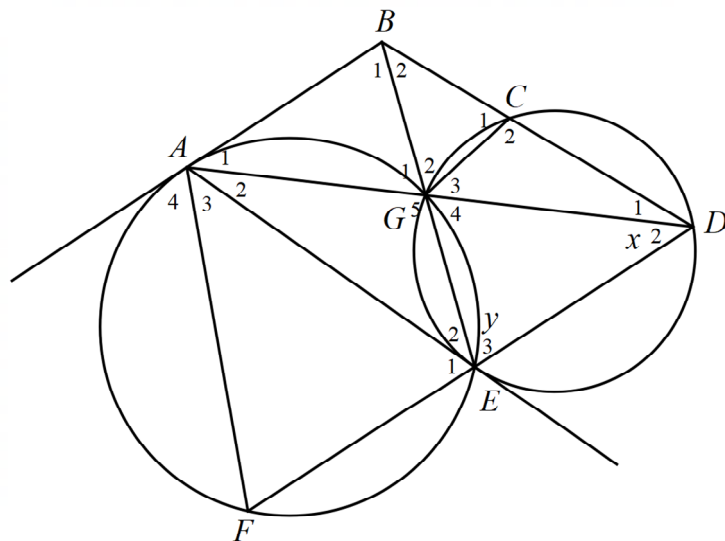
- 1.1 \widehat{BAF} (2)
 $\widehat{BAF} = 90^\circ$ (\angle in semicircle) ✓✓
- 1.2 \widehat{B}_1 (2)
 $\widehat{B}_1 = 50^\circ$ (\angle sum of $\triangle BAF$) ✓✓
- 1.3 \widehat{C}_1 (2)
 $\widehat{C}_1 = 40^\circ$ (\angle s in same seg) ✓✓
- 1.4 \widehat{C}_3 (2)
 $\widehat{C}_3 = 40^\circ$ (equal chords; equal angles) ✓✓
- 1.5 \widehat{B}_2 (2)
 $\widehat{B}_2 = 65^\circ$ (ext \angle of cyclic quad) ✓✓
- 1.6 \widehat{F}_2 (2)
 $\widehat{F}_2 = 180^\circ - (50^\circ + 65^\circ + 40^\circ)$
 $\therefore \widehat{F}_2 = 25^\circ$ (opp \angle s of cyclic quad) ✓✓



Take it up a notch

2. In the diagram, AB is a tangent to circle $AFEG$ at A . AE is a tangent to circle $EDCG$ at E . BE and AD intersect at G . The two circles intersect each other at E and G .

$$\widehat{D}_2 = x \text{ and } \widehat{E}_3 = y.$$



Prove that:

2.1 $DF \parallel BA$ (4)

$$\begin{aligned} \widehat{E}_2 &= x \text{ (tan chord thm)} \checkmark \\ \therefore \widehat{A}_1 &= x \text{ (tan chord thm)} \checkmark \\ \therefore \widehat{D}_2 &= \widehat{A}_1 \checkmark \\ \therefore DF &\parallel BA \text{ (alt } \angle\text{s equal)} \checkmark \end{aligned}$$



2.2 AB is a tangent to circle BCG . (4)

$$\begin{aligned} \widehat{B}_1 &= y \text{ (alt } \angle\text{s; } DF \parallel BA) \checkmark \\ \widehat{C}_1 &= y \text{ (ext } \angle \text{ of cyclic quad)} \checkmark \\ \therefore \widehat{B}_1 &= \widehat{C}_1 \checkmark \\ \therefore AB &\text{ is a tangent to circle } BCG \text{ (converse tan chord thm)} \checkmark \end{aligned}$$

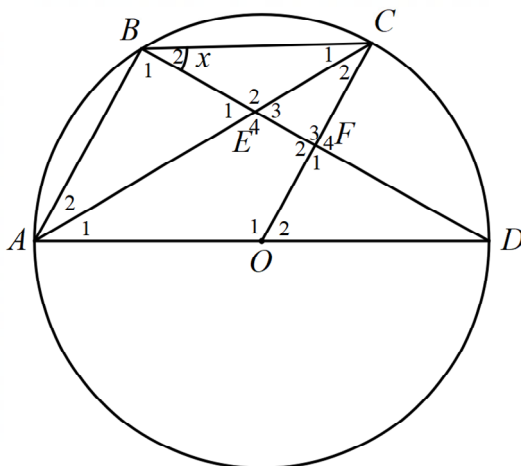


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3. In the diagram, A, B, C and D are points on a circle with centre O. OC intersects BD at F, the midpoint of chord BD. $\widehat{B}_2 = x$.



- 3.1 Prove that BC is a tangent to the circle that passes through A, B and E. (8)

$$\widehat{B}_1 = 90^\circ \text{ (}\angle \text{ in semicircle) } \checkmark$$

$$\widehat{F}_1 = 90^\circ \text{ (line from centre to midpt of chord) } \checkmark$$

$$\therefore \widehat{B}_1 = \widehat{F}_1$$

$$\therefore AB \parallel OC \text{ (corresp } \angle \text{s equal) } \checkmark$$

$$\widehat{A}_1 = x \text{ (}\angle \text{s in same seg) } \checkmark$$

$$\therefore \widehat{O}_2 = 2x \text{ (}\angle \text{ at centre} = 2 \times \angle \text{ at circumference) } \checkmark$$

$$\therefore \widehat{A}_2 = x \text{ (corresp } \angle \text{s; } AB \parallel OC) \checkmark$$

$$\therefore \widehat{B}_2 = \widehat{A}_2 \checkmark$$

$$\therefore BC \text{ is a tangent to circle ABE (converse tan chord thm) } \checkmark$$

- 3.2 Prove that $AB^2 = 4AO^2 - 4BC^2 + 4CF^2$ (4)

$$AB^2 = AD^2 - BD^2 \text{ (Pythag in } \triangle ABD) \checkmark$$

$$\therefore AB^2 = (2AO)^2 - (2BF)^2 \checkmark$$

$$\therefore AB^2 = 4AO^2 - 4BF^2 \checkmark$$

$$\therefore AB^2 = 4AO^2 - 4(BC^2 - CF^2) \text{ (Pythag in } \triangle BCF) \checkmark$$

$$\therefore AB^2 = 4AO^2 - 4BC^2 + 4CF^2$$

