

Algebra

Ononderhandelbaar

1.1 Los op vir x , korrek tot twee desimale plekke: $-2x^2 + 7x - 2 = 0$ (3)

$$x = \frac{-7 \pm \sqrt{7^2 - 4(-2)(-2)}}{2(-2)} \checkmark$$

$$\therefore x = 0,31 \text{ of } x = 3,19 \checkmark \checkmark$$

1.2 Los op vir x : $\sqrt{5-x} - x = 1$ (5)

$$\sqrt{5-x} = x + 1 \checkmark$$

$$\therefore 5-x = x^2 + 2x + 1 \checkmark$$

$$\therefore x^2 + 3x - 4 = 0 \checkmark$$

$$\therefore (x+4)(x-1) = 0 \text{ p}$$

$$\therefore x = -4 \text{ of } x = 1 \checkmark$$

$$\text{As } x = -4: \text{LK} = \sqrt{5 - (-4)} = 3 \quad \text{RK} = -4 + 1 = -3 \quad \therefore x \neq -4$$

$$\text{As } x = 1: \text{LK} = \sqrt{5 - 1} = 2 \quad \text{RK} = 1 + 1 = 2 \quad \therefore x = 1$$

$$\therefore x = 1 \checkmark$$

1.3 Vereenvoudig: $\frac{3^{2x+1} \cdot 15^{2x-3}}{27^{x-1} \cdot 3^x \cdot 5^{2x-4}}$ (4)

$$\frac{3^{2x+1} \cdot 15^{2x-3}}{27^{x-1} \cdot 3^x \cdot 5^{2x-4}}$$

$$= \frac{3^{2x+1} \cdot (3 \cdot 5)^{2x-3}}{(3^3)^{x-1} \cdot 3^x \cdot 5^{2x-4}}$$

$$= \frac{3^{2x+1} \cdot 3^{2x-3} \cdot 5^{2x-3}}{3^{3x-3} \cdot 3^x \cdot 5^{2x-4}} \checkmark$$

$$= 3^{2x+1+2x-3-(3x-3)-x} \cdot 5^{2x-3-(2x-4)} \checkmark$$

$$= 3^{2x+1+2x-3-3x+3-x} \cdot 5^{2x-3-2x+4}$$

$$= 3^1 \cdot 5^1 \checkmark$$

$$= 15 \checkmark$$



1.4 Los vir x en y gelyktydig op:

$$x+4=2y \text{ en } y^2 - xy + 21 = 0$$

(6)

$$x = 2y - 4 \checkmark$$

$$\therefore y^2 - (2y - 4)y + 21 = 0 \checkmark$$

$$\therefore y^2 - 2y^2 + 4y + 21 = 0 \checkmark$$

$$\therefore y^2 - 4y - 21 = 0 \checkmark$$

$$\therefore (y - 7)(y + 3) = 0$$

$$\therefore y = 7 \text{ of } y = -3 \checkmark$$

$$\therefore x = 2(7) - 4 = 10 \text{ of } x = 2(-3) - 4 = -10 \checkmark$$



Neem dit 'n stappie verder

2.1 Bepaal, sonder die gebruik van 'n sakrekenaar, die waarde van a , b en c sodat:

$$(1 - \sqrt{3})(a + b\sqrt{c}) = -10 + 2\sqrt{3}$$

(5)

$$a + b\sqrt{c} = \frac{-10 + 2\sqrt{3}}{1 - \sqrt{3}} \checkmark$$

$$\therefore a + b\sqrt{c} = \frac{-10 + 2\sqrt{3}}{1 - \sqrt{3}} \times \frac{1 + \sqrt{3}}{1 + \sqrt{3}} \checkmark$$

$$\therefore a + b\sqrt{c} = \frac{-10 - 10\sqrt{3} + 2\sqrt{3} + 6}{1 - 3} \checkmark$$

$$\therefore a + b\sqrt{c} = \frac{-4 - 8\sqrt{3}}{-2} \checkmark$$

$$\therefore a + b\sqrt{c} = 2 + 4\sqrt{3}$$

$$\therefore a = 2; b = 4; c = 3 \checkmark$$

OF

$$(1 - \sqrt{3})(a + b\sqrt{c}) = -10 + 2\sqrt{3}$$

$c = 3$... die antwoord bevat slegs $\sqrt{3}$

$$\therefore (1 - \sqrt{3})(a + b\sqrt{3}) = -10 + 2\sqrt{3} \checkmark$$

$$\therefore a + b\sqrt{3} - a\sqrt{3} - 3b = -10 + 2\sqrt{3} \checkmark$$

$\therefore a - 3b = -10$ ① ... stel rasionale dele gelyk

en $b - a = 2$ ② ... stel irrasionale dele gelyk \checkmark

$$\text{①} + \text{②} \quad \therefore -2b = -8 \checkmark$$

$$\therefore b = 4$$

$$\therefore a = 2; b = 4; c = 3 \checkmark$$



- 2.2 Twee waterpype kan 'n swembad vol maak in 20 uur. As slegs een pyp op 'n slag gebruik word, sal die pyp wat die stadigste loop nege uur langer neem om die swembad vol te maak as die pyp wat vinniger loop. Bepaal die tyd wat dit vir die pyp wat die vinnigste loop, sal neem om die swembad vol te maak. (6)

Laat die tyd wat die vinnige pyp neem x uur wees.

\therefore die tyd wat die stadiger pyp neem, is $x+9$ uur.

$$\therefore \frac{1}{x} + \frac{1}{x+9} = \frac{1}{20} \quad \checkmark \quad \dots \text{ breuk van swembad wat in 'n uur gevul word}$$

$$\therefore 20(x+9) + 20x = x(x+9) \quad \checkmark$$

$$\therefore 20x + 180 + 20x = x^2 + 9x \quad \checkmark$$

$$\therefore x^2 - 31x - 180 = 0 \quad \checkmark$$

$$\therefore (x-36)(x+5) = 0$$

$$\therefore x = 36 \text{ of } x \neq -5 \quad \checkmark$$

\therefore die tyd wat die vinnige pyp neem is 36 uur. \checkmark



Reik na die sterre

<https://www.theanswer.co.za/maths-grade-11-revision-algebra-2022/>



3. Bepaal twee nie-nul getalle sodat hul som, hul produk, en hul kwosiënt almal gelyk is. (4)

Laat die getalle x en y wees.

$$\therefore x + y = xy = \frac{x}{y} \quad \checkmark$$

$$xy = \frac{x}{y} \quad \checkmark$$

$$\therefore y^2 = 1 \quad \dots \text{ omdat } x \neq 0 \text{ kan ons deur } x \text{ deel}$$

$$\therefore y = \pm 1 \quad \checkmark$$

As $y=1$ dan $x+1=x$ wat geen oplossing het nie

$$\text{As } y=-1 \text{ dan } x-1=-x \therefore 2x=1 \therefore x=\frac{1}{2}$$

Die getalle is $\frac{1}{2}$ en -1 \checkmark



Patrone

Ononderhandelbaar

1. Beskou die aantal eenheid vierkante in die figure hieronder.

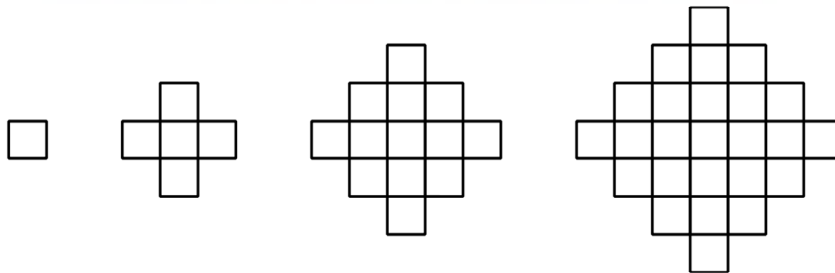


Fig. 1

Fig. 2

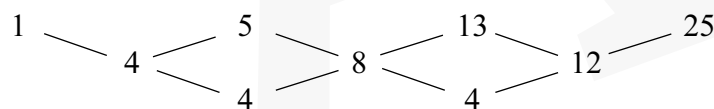
Fig. 3

Fig. 4

Figuurnommer	1	2	3	4
Aantal vierkante	1	5	13	25

Bepaal die aantal eenheid vierkante in die 100^{ste} figuur.

(6)



$$2a=4 \therefore a=2 \checkmark \checkmark$$

$$3a+b=4 \therefore b=-2 \checkmark$$

$$a+b+c=1 \therefore c=1 \checkmark$$

$$\therefore T_n = 2n^2 - 2n + 1$$

$$\therefore T_{100} = 2(100)^2 - 2(100) + 1 \checkmark$$

$$\therefore T_{100} = 19801 \checkmark$$



Neem dit 'n stappie verder

2. Die eerste vier terme van 'n kwadratiese patroon is 2 ; x ; y ; -37 .
Die eerste drie terme in die ry van eerste verskille van dieselfde getalpatroon is $2p-3$; p^2-22 ; $5p-2$. Bepaal die waarde van x en y as $p \in \mathbb{Z}$. (6)

$$\begin{array}{ccccccc}
 2 & & x & & y & & -37 \\
 & \diagdown & / & \diagdown & / & \diagdown & / \\
 & 2p-3 & & p^2-22 & & 5p-2 & \\
 & & \diagdown & / & \diagdown & / & \\
 & & p^2-2p-19 & & -p^2+5p+20 & &
 \end{array}$$

$$p^2 - 2p - 19 = -p^2 + 5p + 20 \quad \checkmark \checkmark$$

$$\therefore 2p^2 - 7p - 39 = 0 \quad \checkmark$$

$$\therefore (p+3)(2p-13) = 0$$

$$\therefore p = -3 \text{ of } p = \frac{13}{2} \quad \checkmark$$

$$\therefore p = -3 \quad \checkmark$$

... omdat $p \in \mathbb{Z}$

$$\begin{array}{ccccccc}
 2 & & x & & y & & -37 \\
 & \diagdown & / & \diagdown & / & \diagdown & / \\
 & -9 & & -13 & & -17 & \\
 & & \diagdown & / & \diagdown & / & \\
 & & & & & &
 \end{array}$$

$$\therefore x = -7 \text{ en } y = -20 \quad \checkmark$$

Reik na die sterre

<https://www.theanswer.co.za/maths-grade-11-revision-patterns-2022/>



3. In 'n kwadratiese ry is die sesde term gelyk aan 19, die negende term gelyk aan 55 en die elfde term gelyk aan 89. Bepaal die formule vir die algemene term. (7)

$$T_6 = 19 \therefore 36a + 6b + c = 19 \quad \textcircled{1} \quad \checkmark$$

$$T_9 = 55 \therefore 81a + 9b + c = 55 \quad \textcircled{2} \quad \checkmark$$

$$T_{11} = 89 \therefore 121a + 11b + c = 89 \quad \textcircled{3} \quad \checkmark$$

$$\textcircled{2} - \textcircled{1} \quad 45a + 3b = 36 \quad \textcircled{4}$$

$$\textcircled{3} - \textcircled{2} \quad 40a + 2b = 34 \quad \textcircled{5} \quad \checkmark \text{ albei}$$

$$2 \times \textcircled{4} - 3 \times \textcircled{5} \quad -30a = -30$$

$$\therefore a = 1 \quad \checkmark$$

$$45(1) + 3b = 36 \therefore b = -3 \quad \dots \text{vervang in } \textcircled{4} \quad \checkmark$$

$$36(1) + 6(-3) + c = 19 \therefore c = 1 \quad \dots \text{vervang in } \textcircled{1}$$

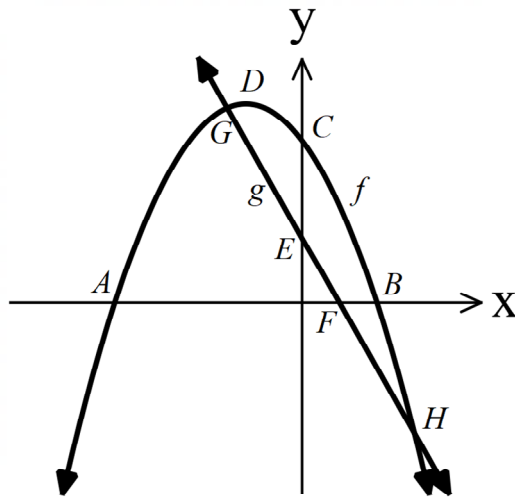
$$\therefore T_n = n^2 - 3n + 1 \quad \checkmark$$



Funksies

Ononderhandelbaar

1. Gegee $f(x) = -3x^2 - 9x + 30$, met A en B die x -afsnitte, D die draaipunt, en $g(x) = -12x + 12$.



- 1.1 Bepaal die koördinate van A en B. (3)

$$\begin{aligned} -3x^2 - 9x + 30 &= 0 \\ \therefore x^2 + 3x - 10 &= 0 \\ \therefore (x + 5)(x - 2) &= 0 \quad \checkmark \\ \therefore x &= -5 \text{ or } x = 2 \\ \therefore A(-5; 0) \text{ en } B(2; 0) &\quad \checkmark\checkmark \end{aligned}$$



- 1.2 Bepaal die koördinate van D. (2)

$$\begin{aligned} x &= -\frac{-9}{2(-3)} = -\frac{3}{2} \\ y &= -3\left(-\frac{3}{2}\right)^2 - 9\left(-\frac{3}{2}\right) + 30 = \frac{147}{4} \\ \therefore D\left(-\frac{3}{2}; \frac{147}{4}\right) \text{ of } D(-1,5; 36,75) &\quad \checkmark\checkmark \end{aligned}$$



1.3 Bepaal die koördinate van G en H, die sny punte van f en g . (4)

$$\begin{aligned} -3x^2 - 9x + 30 &= -12x + 12 \quad \checkmark \\ \therefore 3x^2 - 3x - 18 &= 0 \quad \checkmark \\ \therefore x^2 - x - 6 &= 0 \\ \therefore (x-3)(x+2) &= 0 \\ \therefore x = 3 \text{ of } x = -2 \\ \therefore G(-2; 36) \text{ en } H(3; -24) &\checkmark\checkmark \end{aligned}$$

1.4 Vir watter waardes van x is $f(x) > 0$? (2)

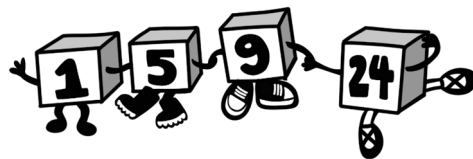
$$-5 < x < 2 \quad \checkmark\checkmark$$

1.5 Vir watter waardes van x is $f(x) \leq g(x)$? (2)

$$x \leq -2 \text{ of } x \geq 3 \quad \checkmark\checkmark$$

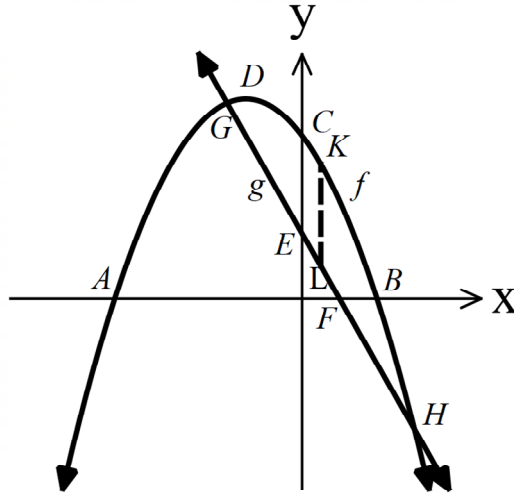
1.6 Skryf die waardeversameling van $f(x)$ neer. (1)

$$y \leq \frac{147}{4} \text{ of } y \leq 36,75 \quad \checkmark$$



Neem dit 'n stappie verder

2. Gebruik $f(x) = -3x^2 - 9x + 30$ en $g(x) = -12x + 12$ vanaf vraag 1.
KL is 'n vertikale lyn met K op f en L op g tussen punte G en H.



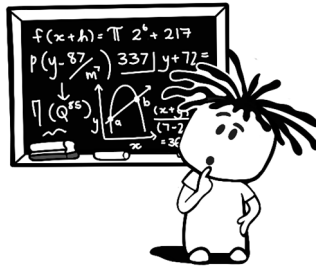
2.1 Vir watter waardes van x is $\frac{f(x)}{g(x)} \geq 0$? (3)

$$F(1;0)$$

$$\therefore -5 \leq x < 1 \text{ of } x \geq 2 \checkmark \checkmark \checkmark$$

2.2 Vir watter waardes van p sal $f(x) = p$ twee ongelyke, negatiewe wortels hê? (2)

$$30 < p < 36,75 \checkmark \checkmark$$



2.3 Bepaal die maksimum lengte van KL.

(5)

$$KL = (-3x^2 - 9x + 30) - (-12x + 12) \checkmark$$

$$\therefore KL = -3x^2 - 9x + 30 + 12x - 12$$

$$\therefore KL = -3x^2 + 3x + 18 \checkmark$$

$$\therefore KL = -3(x^2 - x - 6)$$

$$\text{of } x = -\frac{b}{2a} = -\frac{3}{2(-3)} = \frac{1}{2} \checkmark$$

$$\therefore KL = -3 \left[\left(x - \frac{1}{2} \right)^2 - \frac{1}{4} - 6 \right] \checkmark$$

$$\therefore KL = -3 \left(\frac{1}{2} \right)^2 + 3 \left(\frac{1}{2} \right) + 18 \checkmark$$

$$\therefore KL = -3 \left[\left(x - \frac{1}{2} \right)^2 - 6 \frac{1}{4} \right]$$

$$\therefore KL = \frac{75}{4}$$

$$\therefore KL = -3 \left(x - \frac{1}{2} \right)^2 + \frac{75}{4} \checkmark$$

$$\therefore \text{maksimum lengte} = \frac{75}{4} \text{ of } 18,75 \checkmark$$

2.4 Gegee $h(x) = -3x + k$. Bepaal die waarde van k as $h(x)$ 'n raaklyn aan $f(x)$ is.

(5)

$$-3x^2 - 9x + 30 = -3x + k \checkmark$$

$$\therefore 3x^2 + 6x + k - 30 = 0 \checkmark$$

$$\Delta = 6^2 - 4(3)(k - 30) \checkmark$$

$$\Delta = 0 \checkmark \quad \dots \text{ 'n raaklyn raak op een plek, dus gelyke wortels}$$

$$\therefore 36 - 12k + 360 = 0$$

$$\therefore -12k = -396$$

$$\therefore k = 33 \checkmark$$

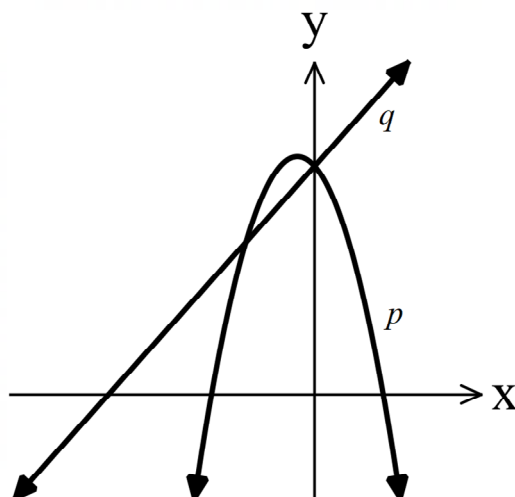


Reik na die sterre

<https://www.theanswer.co.za/maths-grade-11-revision-functions-2022/>



3. Gegee $p(x) = ax^2 + bx + c$ en $q(x) = mx + c$.



Jy word gegee dat $\frac{q(x)}{p(x)} < 0$ vir alle waardes van x as $-6 < x < -3$ of $x > 2$.

Bepaal, wys alle bewerkings, die waarde van m in terme van a .

(5)

$$p(x) = a(x+3)(x-2) \checkmark$$

$$\therefore p(x) = ax^2 + ax - 6a \checkmark$$

$$\therefore c = -6a$$

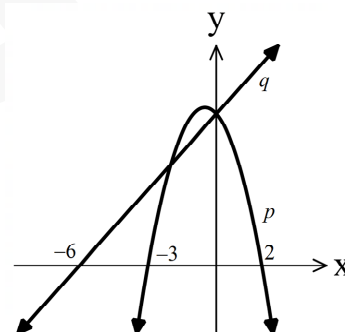
$$\therefore 6m = -6a \checkmark$$

$$\therefore m = -a \checkmark$$

$$q(x) = mx + c$$

$$\therefore 0 = -6m + c \checkmark$$

$$\therefore c = 6m$$



Finansies

Ononderhandelbaar

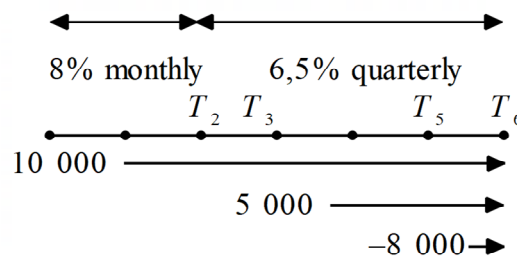
1. Bereken die oorspronklike prys van 'n iPad as die verminderde waarde na 3 jaar R7 045,32 is. Die koers waarteen die waarde verminder, is 13% p.a. gebaseer op die verminderde-saldo-metode. (3)

$$7045,32 = P(1-0,13)^3 \checkmark\checkmark$$

$$\therefore P = R10\,698,99 \checkmark$$

Neem dit 'n stappie verder

2. 'n Vrou maak 'n aanvanklike belegging van R10 000 in 'n rekening. Drie jaar later deponeer sy R5 000 in dieselfde rekening. Sy onttrek R8 000 vyf jaar na die aanvanklike belegging. Die rentekoers was 8% p.a., maandeliks saamgestel vir die eerste twee jaar, en dan verander dit daarna na 6,5% p.a., kwartaalliks saamgestel. Bepaal die finale bedrag in haar rekening na ses jaar. (5)



$$A = 10000 \left(1 + \frac{0,08}{12}\right)^{24} \left(1 + \frac{0,065}{4}\right)^{16} + 5000 \left(1 + \frac{0,065}{4}\right)^{12} - 8000 \left(1 + \frac{0,065}{4}\right)^4 \checkmark\checkmark\checkmark$$

$$\therefore A = R12\,714,00 \checkmark\checkmark$$



Reik na die sterre

<https://www.theanswer.co.za/maths-grade-11-revision-finance-2022/>



3. Die inkomstebelasting in Suid-Afrika word gehef teen 'n koers van $a\%$ vir die eerste R488 700.

Vir enige bedrag bo R488 700 is die koers $(a+5)\%$. 'n Vrou neem kennis dat haar effektiewe belasting $(a+0,34)\%$ van haar jaarlikse inkomste was. Bepaal haar jaarlikse inkomste. (5)

Laat haar jaarlikse inkomste x wees

$$488\,700 \times a\% + (x - 488\,700) \times (a+5)\% = x \times (a+0,34)\% \quad \checkmark\checkmark\checkmark$$

$$\therefore 488\,700 \times a + (x - 488\,700) \times (a+5) = x \times (a+0,34)$$

$$\therefore 488\,700a + xa + 5x - 488\,700a - 2\,443\,500 = xa + 0,34x \quad \checkmark$$

$$\therefore 4,66x = 2\,443\,500$$

$$\therefore x = R524\,356,22 \quad \checkmark$$

Of

Laat die inkomste bo R488 700, x wees

$$488\,700 \times a\% + x \times (a+5)\% = (488\,700 + x) \times (a+0,34)\% \quad \checkmark\checkmark\checkmark$$

$$\therefore 488\,700a + x(a+5) = (488\,700 + x)(a+0,34)$$

$$\therefore 488\,700a + xa + 5x = 488\,700a + 166\,158 + xa + 0,34x \quad \checkmark$$

$$\therefore 4,66x = 166\,158$$

$$\therefore x = 35\,656,22$$

$$\therefore \text{Jaarlikse inkomste} = 488\,700 + 35\,656,22 = R524\,356,22 \quad \checkmark$$



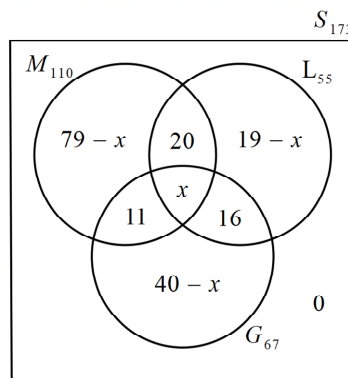
Waarskynlikheid

Ononderhandelbaar

1. 173 graad 12-leerders by 'n skool word gevra wie die volgende vakke neem: Wiskunde (M), Lewenswetenskappe (L) en Geografie (G).
- Elke graad 12-leerder neem ten minste een van die drie vakke
 - 110 neem Wiskunde
 - 55 neem Lewenswetenskappe
 - 67 neem Geografie
 - 20 neem Wiskunde en Lewenswetenskappe, maar nie Geografie nie
 - 11 neem Wiskunde en Geografie, maar nie Lewenswetenskappe nie
 - 16 neem Lewenswetenskappe en Geografie, maar nie Wiskunde nie
 - x neem al drie vakke

- 1.1 Teken 'n Venn-diagram om bostaande inligting voor te stel. (4)

✓✓✓✓



- 1.2 Bepaal die waarde van x . (2)

$$\begin{aligned}79 - x + 19 - x + 40 - x + 20 + 11 + 16 + x &= 173 \quad \checkmark \\ \therefore -2x &= -12 \\ \therefore x &= 6 \quad \checkmark\end{aligned}$$

- 1.3 Bepaal die waarskynlikheid dat 'n leerder presies een van hierdie drie vakke neem. (2)

$$\begin{aligned}P &= \frac{79 - 6 + 19 - 6 + 40 - 6}{173} \quad \checkmark \\ \therefore P &= \frac{120}{173} \quad \checkmark\end{aligned}$$

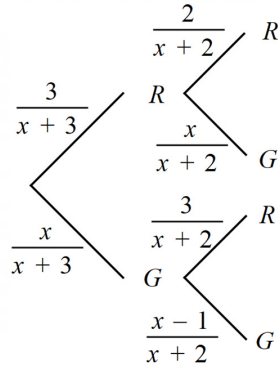


Neem dit 'n stappie verder

2. 'n Sak bevat drie rooi albasters en x groen albasters. Twee albasters word uit die sak gehaal, sonder om terug te plaas. Die waarskynlikheid om een albaster van elke kleur te kry, is $\frac{4}{7}$.

Bepaal die waarde van x .

(5)



$$\frac{3}{x+3} \times \frac{x}{x+2} + \frac{x}{x+3} \times \frac{3}{x+2} = \frac{4}{7} \checkmark \checkmark$$

$$\therefore \frac{6x}{(x+3)(x+2)} = \frac{4}{7}$$

$$\therefore 42x = 4(x+3)(x+2) \checkmark$$

$$\therefore 42x = 4x^2 + 20x + 24$$

$$\therefore 4x^2 - 22x + 24 = 0 \checkmark$$

$$\therefore x^2 - 11x + 12 = 0$$

$$\therefore (x-4)(2x-3) = 0$$

$$\therefore x = 4 \text{ of } x = \frac{3}{2} \checkmark$$

$$\therefore x = 4 \checkmark$$

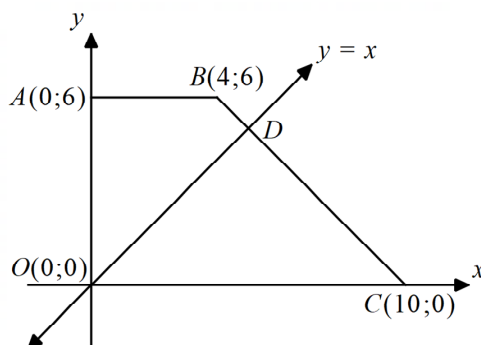


Reik na die sterre

<https://www.theanswer.co.za/maths-grade-11-revision-probability-2022/>



3. 'n Punt $(x; y)$ word lukraak binne 'n vierhoek met hoekpunte $O(0;0)$, $A(0;6)$, $B(4;6)$ en $C(10;0)$ gekies. Wat is die waarskynlikheid dat $y \geq x$? (7)



$$m_{BC} = \frac{6-0}{4-10} = -1 \quad \checkmark$$

$$BC: y - 0 = -1(x - 10)$$

$$\therefore y = -x + 10 \quad \checkmark$$

$$D: -x + 10 = x \quad \checkmark$$

$$\therefore -2x = -10$$

$$\therefore x = 5$$

$$\therefore D(5;5) \quad \checkmark$$

$$Area_{OACB} = \frac{1}{2}(10+4)(6) = 42 \quad \checkmark$$

$$Area_{\triangle ODC} = \frac{1}{2}(10)(5) = 25 \quad \checkmark$$

$$\therefore Area_{\triangle ABD} = 42 - 25 = 17$$

$$\therefore P = \frac{17}{42} \quad \checkmark$$



Datahantering

Ononderhandelbaar

Die tabel hieronder wys die punte behaal deur 'n graad 11-klas vir 'n Wiskunde-toets uit 50.

Punte	Frekwensie	Kumulatiewe Frekwensie
$0 < x \leq 10$	3	
$10 < x \leq 20$		11
$20 < x \leq 30$	15	
$30 < x \leq 40$	10	
$40 < x \leq 50$		40

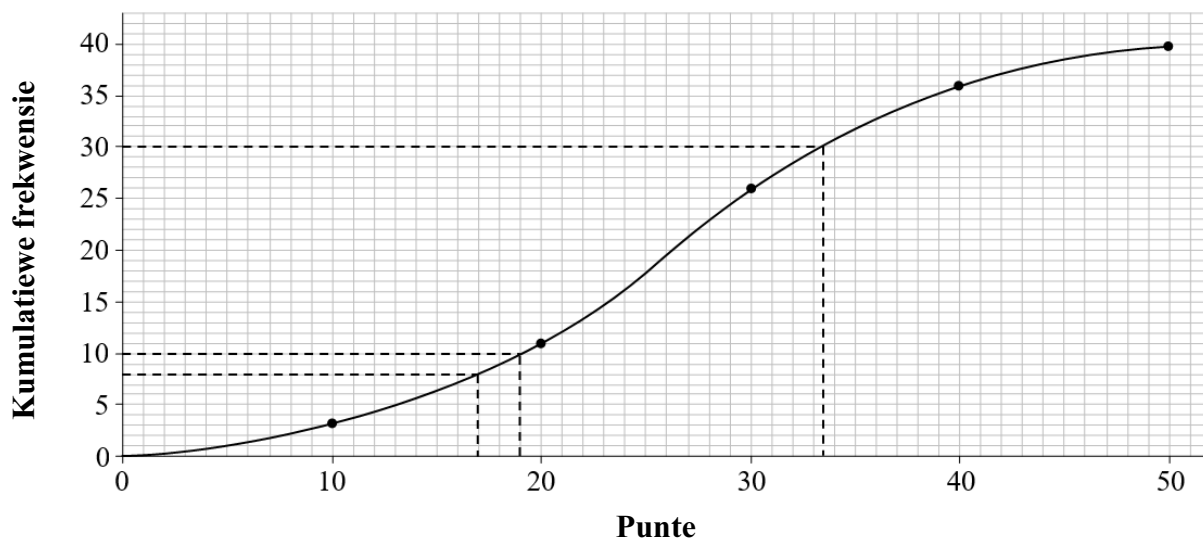
1.1 Voltooi die ontbrekende inligting in die tabel. (2)

✓✓ -1 per fout

Punte	Frekwensie	Kumulatiewe Frekwensie
$0 < x \leq 10$	3	3
$10 < x \leq 20$	8	11
$20 < x \leq 30$	15	26
$30 < x \leq 40$	10	36
$40 < x \leq 50$	4	40

1.2 Teken 'n ogief (kumulatiewe frekwensiekurve) om die bostaande inligting voor te stel. (3)

- ✓ plot die boonste grens
- ✓ plot die korrekte kumulatiewe frekwensie
- ✓ gladde kurwe



- 1.3 Bepaal, deur van die ogief gebruik te maak, die interkwartielomvang. (3)

$$LQ=19 \checkmark$$

$$UQ=33,5 \checkmark$$

$$\therefore IQR=33,5-19=14,5 \checkmark$$

- 1.4 As 20% van die klas die toets drui, gebruik die ogief om die slaagsyfer te bepaal. (2)

$$20\% = 8 \text{ leeders } \checkmark$$

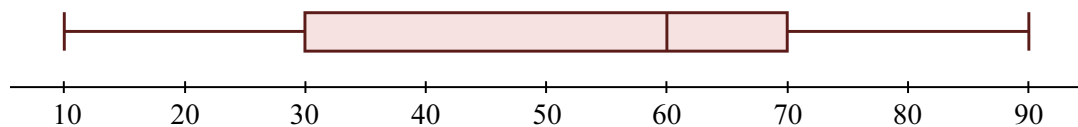
$$\therefore \text{ slaagsyfer is bo 17 uit 50. } \checkmark$$

Neem dit 'n stappie verder

2. Die punte behaal deur 80 graad 11-leeders word hieronder gewys.

Punte	Frekwensie
$10 < x \leq 20$	7
$20 < x \leq 30$	A
$30 < x \leq 40$	B
$40 < x \leq 50$	4
$50 < x \leq 60$	10
$60 < x \leq 70$	C
$70 < x \leq 80$	12
$80 < x \leq 90$	D

'n Mond-en-snor-diagram van die data is geteken. Geen leerder behaal presies 30, 60, of 70 punte.



Bepaal die waardes van A, B, C en D. (4)

Elke kwartiel het 20 leeders.

$$10 < x \leq 30 \quad A = 20 - 7 = 13 \checkmark$$

$$30 < x \leq 60 \quad B = 20 - 14 = 6 \checkmark$$

$$60 < x \leq 70 \quad C = 20 \checkmark$$

$$70 < x \leq 90 \quad D = 20 - 12 = 8 \checkmark$$



Reik na die sterre

<https://www.theanswer.co.za/maths-grade-11-revision-data-2022/>



3. Ag getalle word in stygende orde geskryf.

5; x ; 13; 17; 21; 21; y ; 31

Die gemiddeld van die getalle is 18 en die interkwartielomvang is 11. Bepaal die waarde van x en y .

$$\frac{5+x+13+17+21+21+y+31}{8} = 18 \checkmark$$

$$\therefore 108+x+y=144$$

$$\therefore x+y=36 \quad \textcircled{1} \checkmark$$

$$Q_1 = \frac{x+13}{2} \text{ en } Q_3 = \frac{21+y}{2} \checkmark$$

$$\therefore \frac{21+y}{2} - \frac{x+13}{2} = 11 \checkmark$$

$$\therefore 21+y-x-13=22$$

$$\therefore y-x=14 \quad \textcircled{2} \checkmark$$

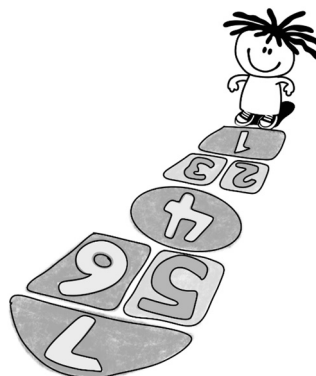
$$\textcircled{1} + \textcircled{2} \quad 2y=50$$

$$\therefore y=25 \checkmark$$

$$\therefore x+25=36$$

$$\therefore x=11 \checkmark$$

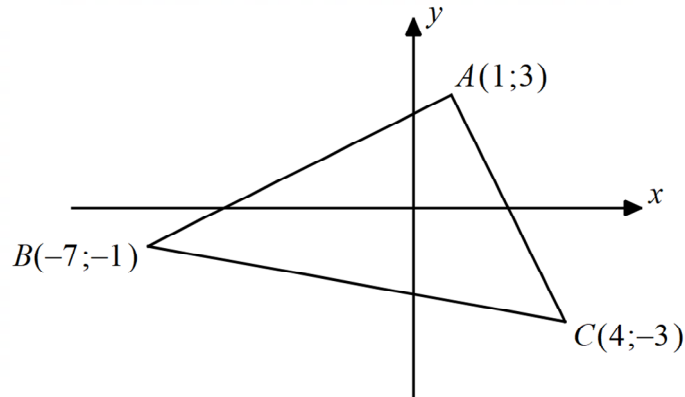
$$\therefore x=11 \text{ en } y=25$$



Analitiese Meetkunde

Ononderhandelbaar

1. In die diagram word $A(1;3)$, $B(-7;-1)$ en $C(4;-3)$ gegee.



- 1.1 Bepaal die lengte van AB, in eenvoudigste wortelvorm. (2)

$$AB = \sqrt{(1 - (-7))^2 + (3 - (-1))^2} \checkmark$$
$$\therefore AB = \sqrt{80}$$
$$\therefore AB = 4\sqrt{5} \checkmark$$

- 1.2 Bepaal die koördinate van Q, die middelpunt van BC. (2)

$$Q\left(\frac{-7+4}{2}; \frac{-1-3}{2}\right) = Q\left(-\frac{3}{2}; -2\right) \checkmark\checkmark$$

- 1.3 Bepaal die gradiënt van AB. (2)

$$m_{AB} = \frac{3 - (-1)}{1 - (-7)} = \frac{1}{2} \checkmark\checkmark$$

- 1.4 Bepaal die vergelyking van die lyn parallel aan AB, wat deur Q gaan. (3)

$$y = \frac{1}{2}x + c \checkmark$$
$$\therefore -2 = \frac{1}{2}\left(-\frac{3}{2}\right) + c \checkmark$$
$$\therefore c = -\frac{5}{4}$$
$$\therefore y = \frac{1}{2}x - \frac{5}{4} \checkmark$$



1.5 Bewys dat $AB \perp AC$. (2)

$$m_{AC} = \frac{3 - (-3)}{1 - 4} = -2 \checkmark$$

$$\therefore AB \perp AC \quad (m_{AB} \times m_{AC} = -1) \checkmark$$

1.6 Bepaal die koördinate van D as ABCD 'n parallelogram is. (2)

$$x_B \rightarrow x_A + 8$$

$$x_C \rightarrow x_D + 8$$

$$\therefore x_D = 12$$

$$y_B \rightarrow y_A + 4$$

$$y_C \rightarrow y_D + 4$$

$$\therefore y_D = 1$$

$$\therefore D(12;1) \checkmark \checkmark$$



Neem dit 'n stappie verder

2. Gebruik die diagram in vraag 1.

2.1 Bepaal die grootte van \hat{ABC} , korrek tot twee desimale plekke. (4)

$$m_{AB} = \frac{1}{2}$$

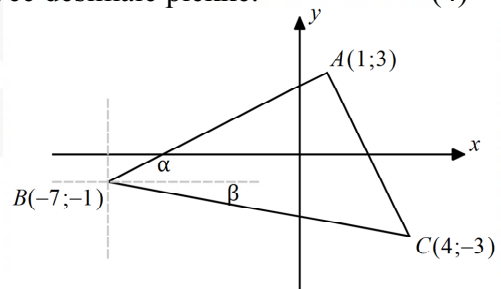
$$\therefore \tan \alpha = \frac{1}{2} \therefore \alpha = 26,57^\circ \checkmark$$

$$m_{BC} = \frac{-1 - (-3)}{-7 - 4} = -\frac{2}{11} \checkmark$$

$$\therefore \tan \beta = \frac{2}{11} \therefore \beta = 10,30^\circ \checkmark \dots \text{gebruik positiewe gradiënt omdat } \beta \text{ 'n}$$

skerphoek is

$$\therefore \hat{ABC} = 36,87^\circ \checkmark$$



2.2 P is 'n punt op BC sodat die oppervlakte van ΔABC vier keer die oppervlakte van ΔABP is. Bepaal die koördinate van P. (2)

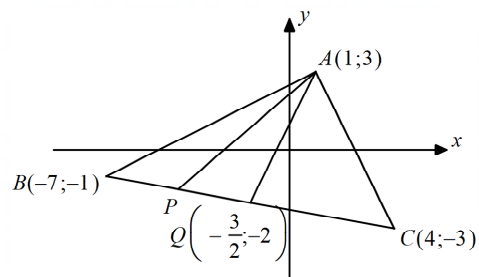
ΔABC en ΔABP het dieselfde hoogte

$$\therefore BP = \frac{1}{4} BC$$

$Q\left(-\frac{3}{2}; -2\right)$ is die middelpunt van BC

$\therefore P$ sal die middelpunt van BQ wees

$$\therefore P\left(\frac{-7 - \frac{3}{2}}{2}; \frac{-1 - 2}{2}\right) = P\left(-\frac{17}{4}; -\frac{3}{2}\right)$$

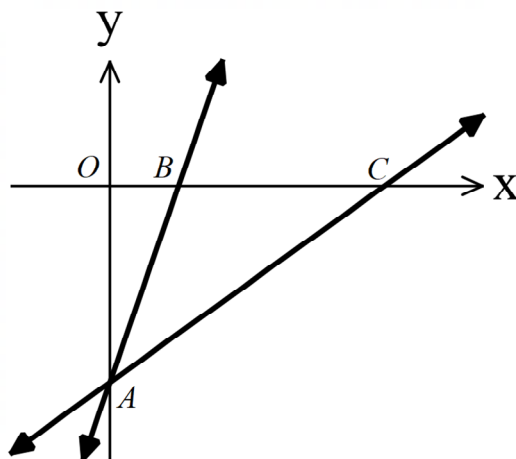


Reik na die sterre



<https://www.theanswer.co.za/maths-grade-11-revision-analytical-geometry-2022/>

3. In die diagram is die vergelykings van die twee reguit lyne $2y - x + 2q = 0$ en $y - 2x + q = 0$.



Bepaal die waarde van q as die oppervlakte van $\triangle ABC$ 48 eenhede² is. (6)

$$AB: y - 2q + q = 0 \therefore y = 2x - q$$

$$AC: 2y - x + 2q = 0 \therefore y = \frac{1}{2}x - q$$

$$\therefore A(0; -q); B\left(\frac{q}{2}; 0\right); C(2q; 0) \checkmark\checkmark\checkmark$$

$$Area = \frac{1}{2} \times BC \times OA$$

$$\therefore \frac{1}{2} \times \frac{3q}{2} \times q = 48 \checkmark$$

$$\therefore 3q^2 = 192 \checkmark$$

$$\therefore q^2 = 64$$

$$\therefore q = \pm 8$$

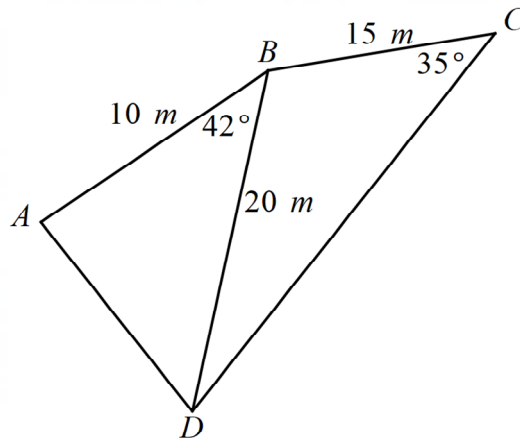
$$\therefore q = 8 \checkmark$$



Trigonometrie

Ononderhandelbaar

1. In die diagram, wat nie volgens skaal geteken is nie, is $AB = 10$ m, $BD = 20$ m, $BC = 15$ m, $\widehat{ABD} = 42^\circ$ en $\widehat{BCD} = 35^\circ$.



Bepaal, korrek tot twee desimale plekke:

- 1.1 die oppervlakte van $\triangle ABD$. (2)

$$A = \frac{1}{2} \times 10 \times 20 \times \sin 42^\circ \checkmark$$
$$\therefore A = 66,91 \text{ m}^2 \checkmark$$

- 1.2 die lengte van AD. (2)

$$AD^2 = 10^2 + 20^2 - 2 \times 10 \times 20 \times \cos 42^\circ \checkmark$$
$$\therefore AD = 14,24 \text{ m} \checkmark$$

- 1.3 die grootte van \widehat{BDC} . (3)

$$\frac{\sin \widehat{BDC}}{15} = \frac{\sin 35^\circ}{20} \checkmark$$
$$\therefore \sin \widehat{BDC} = 0,4301... \checkmark$$
$$\therefore \widehat{BDC} = 25,48^\circ \checkmark$$



Neem dit 'n stappie verder

2. Los op vir x , korrek tot twee desimale syfers:

2.1 $27^{\tan x} = 9; x \in [-180^\circ; 360^\circ]$ (4)

$$\therefore 3^{3 \tan x} = 3^2 \checkmark$$

$$\therefore 3 \tan x = 2$$

$$\therefore \tan x = \frac{2}{3} \checkmark$$

$$\therefore x = 33,69 + n180^\circ \checkmark$$

$$\therefore x = 33,69^\circ; 213,69^\circ; -146,31^\circ \checkmark$$

2.2 $2\sin^2 x - 6\sin x \cos x = 3\cos x - \sin x$. Gee die algemene oplossing. (6)

$$\therefore 2\sin^2 x - 6\sin x \cos x - 3\cos x + \sin x = 0$$

$$\therefore 2\sin x(\sin x - 3\cos x) - (3\cos x - \sin x) = 0 \checkmark$$

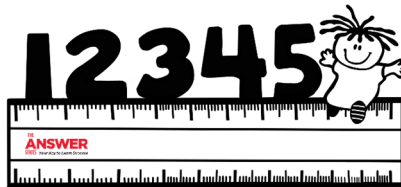
$$\therefore 2\sin x(\sin x - 3\cos x) + (\sin x - 3\cos x) = 0$$

$$\therefore (\sin x - 3\cos x)(2\sin x + 1) = 0 \checkmark$$

$$\therefore \sin x = 3\cos x \text{ of } \sin x = -\frac{1}{2} \checkmark$$

$$\therefore \tan x = 3 \text{ of } \sin x = -\frac{1}{2} \checkmark$$

$$\therefore x = 71,57^\circ + n180^\circ \text{ of } x = 210^\circ + n360^\circ \text{ of } x = 330^\circ + n360^\circ; n \in \mathbb{Z} \checkmark \checkmark$$

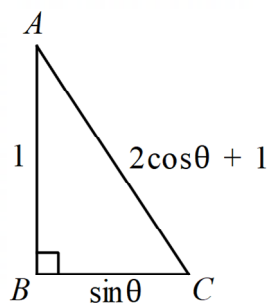


Reik na die sterre

<https://www.theanswer.co.za/maths-grade-11-revision-trigonometry-2022/>



3. In die diagram is $BC = \sin \theta$, $AB = 1$ en $AC = 2 \cos \theta + 1$.



Bepaal $\tan A$ sonder die gebruik van 'n sakrekenaar.

(5)

$$(2 \cos \theta + 1)^2 = 1^2 + \sin^2 \theta \text{ (Pythag)} \checkmark$$

$$\therefore 4 \cos^2 \theta + 4 \cos \theta + 1 = 1 + \sin^2 \theta \checkmark$$

$$\therefore 4 \cos^2 \theta + 4 \cos \theta + 1 = 1 + 1 - \cos^2 \theta$$

$$\therefore 5 \cos^2 \theta + 4 \cos \theta - 1 = 0 \checkmark$$

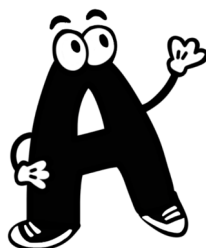
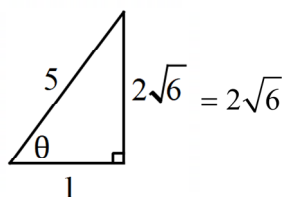
$$\therefore (5 \cos \theta - 1)(\cos \theta + 1) = 0$$

$$\therefore \cos \theta = \frac{1}{5} \text{ of } \cos \theta = -1$$

$$\therefore \cos \theta = \frac{1}{5} \checkmark \quad \dots \text{ as } \cos \theta = -1, \text{ dan } AC = -1 \text{ wat onmoontlik is.}$$

$$\tan A = \frac{\sin \theta}{1}$$

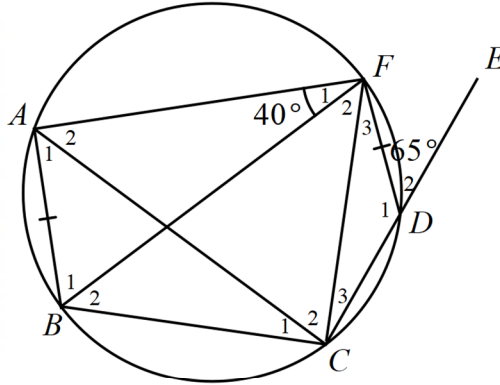
$$\therefore \tan A = \frac{2\sqrt{6}}{5} \checkmark$$



Euklidiese Meetkunde

Ononderhandelbaar

1. In die diagram het sirkel ABCDF 'n middellyn BF. $\hat{D}_2 = 65^\circ$, $\hat{F}_1 = 40^\circ$ en $AB = DF$.



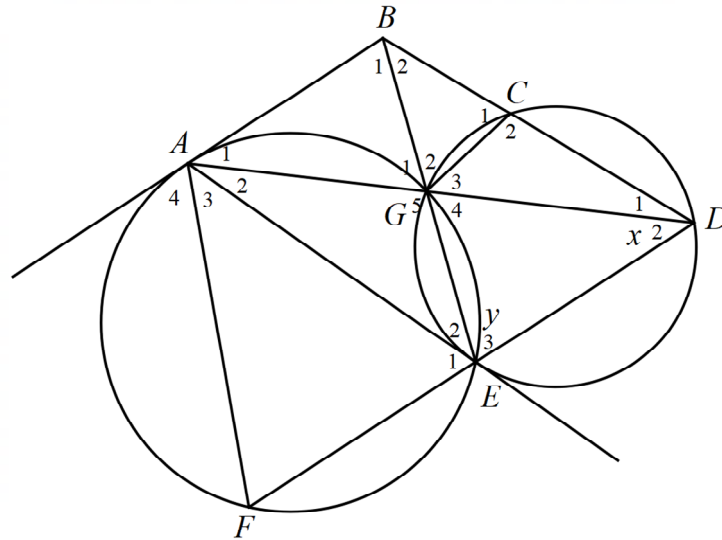
Bepaal, met redes, die grootte van die hoeke.

- 1.1 $\hat{B}AF$ (2)
 $\hat{B}AF = 90^\circ$ (\angle in semi-sirkel) ✓✓
- 1.2 \hat{B}_1 (2)
 $\hat{B}_1 = 50^\circ$ (\angle som van $\triangle BAF$) ✓✓
- 1.3 \hat{C}_1 (2)
 $\hat{C}_1 = 40^\circ$ (\angle e in dieselfde seg) ✓✓
- 1.4 \hat{C}_3 (2)
 $\hat{C}_3 = 40^\circ$ (gelyke koorde, gelyke hoeke) ✓✓
- 1.5 \hat{B}_2 (2)
 $\hat{B}_2 = 65^\circ$ (buite \angle van kvh) ✓✓
- 1.6 \hat{F}_2 (2)
 $\hat{F}_2 = 180^\circ - (50^\circ + 65^\circ + 40^\circ)$
 $\therefore \hat{F}_2 = 25^\circ$ (teenoorst \angle e van kvh) ✓✓



Neem dit 'n stappie verder

2. In die diagram is AB 'n raaklyn aan sirkel $AFEG$ by A . AE is 'n raaklyn aan sirkel $EDCG$ by E . BE en AD sny by G . Die twee sirkels sny mekaar by E en F .
 $\hat{D}_2 = x$ en $\hat{E}_3 = y$.



Bewys dat:

2.1 $DF \parallel BA$

$$\begin{aligned} \hat{E}_2 &= x \text{ (rlyn koord st)} \checkmark \\ \therefore \hat{A}_1 &= x \text{ (rlyn koord st)} \checkmark \\ \therefore \hat{D}_2 &= \hat{A}_1 \checkmark \\ \therefore DF &\parallel BA \text{ (verw } \angle\text{e gelyk)} \checkmark \end{aligned}$$



(4)

2.2 AB 'n raaklyn aan sirkel BCG is.

$$\begin{aligned} \hat{B}_1 &= y \text{ (verw } \angle\text{e; } DF \parallel BA) \checkmark \\ \hat{C}_1 &= y \text{ (buite } \angle \text{ van kvh)} \checkmark \\ \therefore \hat{B}_1 &= \hat{C}_1 \checkmark \\ \therefore AB &\text{ is 'n raaklyn aan sirkel } BCG \text{ (omgekeerde rlyn koord st)} \checkmark \end{aligned}$$

(4)

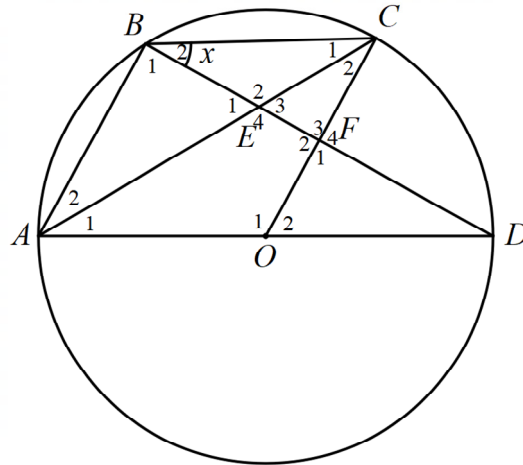


Reik na die sterre

<https://www.theanswer.co.za/maths-grade-11-revision-euclidean-geometry-2022/>



3. In die diagram is A, B, C en D punte op 'n sirkel met middelpunt O. OC sny BD by F, die middelpunt van koord BD. $\hat{B}_2 = x$.



- 3.1 Bewys dat BC 'n raaklyn is aan die sirkel wat deur A, B en E gaan. (8)

$\hat{B}_1 = 90^\circ$ (\angle in semi-sirkel) ✓
 $\hat{F}_1 = 90^\circ$ (lyn van midpt na midpt van koord) ✓
 $\therefore \hat{B}_1 = \hat{F}_1$
 $\therefore AB \parallel OC$ (ooreenk \angle e gelyk) ✓
 $\hat{A}_1 = x$ (\angle e in dieselfde seg) ✓
 $\therefore \hat{O}_2 = 2x$ (\angle by midpt = $2 \times \angle$ by omtrek) ✓
 $\therefore \hat{A}_2 = x$ (ooreenk \angle e; $AB \parallel OC$) ✓
 $\therefore \hat{B}_2 = \hat{A}_2$ ✓
 $\therefore BC$ is 'n raaklyn aan sirkel ABE (omgekeerde rlyn koord st) ✓

- 3.2 Bewys dat $AB^2 = 4AO^2 - 4BC^2 + 4CF^2$ (4)

$AB^2 = AD^2 - BD^2$ (Pythag in $\triangle ABD$) ✓
 $\therefore AB^2 = (2AO)^2 - (2BF)^2$ ✓
 $\therefore AB^2 = 4AO^2 - 4BF^2$ ✓
 $\therefore AB^2 = 4AO^2 - 4(BC^2 - CF^2)$ (Pythag in $\triangle BCF$) ✓
 $\therefore AB^2 = 4AO^2 - 4BC^2 + 4CF^2$

