CURRICULUM: Maths FET

CAPS: Extracts & Errata

See highlighted text from CAPS errata document (December 2016)

Latest changes

Curriculum: CAPS Documents > CAPS for the FET > Nonlanguages in English > Mathematics

Errata: CAPS Documents > CAPS Errata: Grade 12 and SP

CONTENTS

The essence of the CAPS curriculum lies far deeper than pure content. The emphasis on substantial algebra, logic and interpretation is quite clear. Geometry encourages the development of spacial concepts, investigation, presentation of findings and reasoning. At the heart of this curriculum lies the need for learners to observe, analyse, explain, generalise, predict – indeed, the true art of doing maths!

EXAMINATIONS: Mark distribution

Grades 10 – 12 Mathematics end of year papers

Grade 10: 2 hours for each paper **Grade 11 & 12:** 3 hours for each paper

PAPER 1: Grade 12 : bookwork: maximum 6 marks			
Description	**GRADE 10**	**GRADE 11**	**GRADE 12**
Algebra and Equations (and inequalities); *Exponents*	30 ± 3	45 ± 3	25 ± 3
Patterns & Sequences	15 ± 3	25 ± 3	25 ± 3
Finance and growth *(Financial Maths)*	10 ± 3	/	/
Finance, growth and decay	/	15 ± 3	15 ± 3
Functions & Graphs	30 ± 3	45 ± 3	35 ± 3
Differential Calculus	/	/	35 ± 3
Probability	15 ± 3	20 ± 3	15 ± 3
TOTAL	**100**	**150**	**150**

PAPER 2: Grades 11 and 12 : theorems and/or trigonometric proofs: maximum 12 marks			
Description	**GRADE 10**	**GRADE 11**	**GRADE 12**
Statistics	15 ± 3	20 ± 3	20 ± 3
Analytical Geometry	15 ± 3	30 ± 3	40 ± 3
Trigonometry	40 ± 3	50 ± 3	50 ± 3
Euclidean Geometry and Measurement	30 ± 3	50 ± 3	40 ± 3
TOTAL	**100**	**150**	**150**

NOTE:

- Modelling as a process should be included in all papers, thus contextual questions can be set on any topic.
  Trigonometric functions will be examined in paper 2.
- Questions will not necessarily be compartmentalised in sections, as this table indicates. Various topics can be integrated in the same question.
- A formula sheet must not be provided for tests, but only for final examinations in Grades 10 and 11.

PTO for the weighting of the cognitive levels used in all assessments.

COGNITIVE LEVELS: Percentages & Descriptors

The four cognitive levels used to guide all assessment tasks is based on those suggested in the TIMSS study of 1999. Descriptors for each level and the approximate percentage of tasks, tests and examinations which should be at each level are given below:

Cognitive levels	Description of skills to be demonstrated	Examples
1: Knowledge **20%**	• Straight recall • Identification and direct use of correct formula on the information sheet (no changing of the subject) • Use of mathematical facts • Appropriate use of mathematical vocabulary	1. Write down the domain of the function $y = f(x) = \frac{3}{x} + 2$ (Grade 10) 2. The angle $A\hat{O}B$ subtended by arc AB at the centre O of a circle . . . (Grade 11)
2: Routine procedures **35%**	• Estimation and appropriate rounding of numbers • Proofs of prescribed theorems and derivation of formulae • Perform well known procedures • Simple applications and calculations which might involve a few steps • Derivation from given information may be involved • Identification and use (after changing the subject) of correct formula • Generally similar to those encountered in class.	1. Solve for x : $x^2 - 5x = 14$ (Grade 10) 2. Determine the general solution of the equation $2\sin(x - 30^\circ) + 1 = 0$ (Grade 11) 3. Prove that the angle $A\hat{O}B$ subtended by arc AB at the centre O of a circle is double the size of the angle $A\hat{C}B$ which the same arc subtends at the circle. (Grade 11)
3: Complex procedures **30%**	• Problems involve complex calculations and/or higher order reasoning • There is often not an obvious route to the solution • Problems need not be based on a real world context • Could involve making significant connections between different representations • Require conceptual understanding	1. What is the average speed covered on a round trip to and from a destination if the average speed going to the destination is100 km / h and the average speed for the return journey is 80km/ h? (Grade 11) 2. Differentiate $\frac{(x+2)^2}{\sqrt{x}}$ with respect to x. (Grade 12)
4: Problem solving **15%**	• Non-routine problems (which are not necessarily difficult) • Higher order reasoning and processes are involved • Might require the ability to break the problem down into its constituent parts	Suppose a piece of wire could be tied tightly around the earth at the equator. Imagine that this wire is then lengthened by exactly one metre and held so that it is still around the earth at the equator. Would a mouse be able to crawl between the wire and the earth? Why or why not? (Any grade)

The Programme of Assessment is designed to set formal assessment tasks in all subjects in a school throughout the year.

THE CAPS CURRICULUM: OVERVIEW OF TOPICS

Paper 1

1. FUNCTIONS

Grade 10	Grade 11	Grade 12
Work with relationships between variables in terms of numerical, graphical, verbal and symbolic representations of functions and convert flexibly between these representations (tables, graphs, words and formulae). Include linear and some quadratic polynomial functions, exponential functions, some rational functions and trigonometric functions.	Extend Grade 10 work on the relationships between variables in terms of numerical, graphical, verbal and symbolic representations of functions and convert flexibly between these representations (tables, graphs, words and formulae). Include linear and quadratic polynomial functions, exponential functions, some rational functions and trigonometric functions.	Introduce a more formal definition of a function and extend Grade 11 work on the relationships between variables in terms of numerical, graphical, verbal and symbolic representations of functions and convert flexibly between these representations (tables, graphs, words and formulae). Include linear, quadratic and some cubic polynomial functions, exponential and logarithmic functions, and some rational functions.
Generate as many graphs as necessary, initially by means of point-by-point plotting, supported by available technology, to make and test conjectures and hence generalise the effect of the parameter which results in a vertical shift and that which results in a vertical stretch and /or a reflection about the x-axis.	Generate as many graphs as necessary, initially by means of point-by-point plotting, supported by available technology, to make and test conjectures and hence generalise the effects of the parameter which results in a horizontal shift and that which results in a horizontal stretch and/or reflection about the y-axis.	The inverses of prescribed functions and be aware of the fact that, in the case of many-to-one functions, the domain has to be restricted if the inverse is to be a function.
Problem solving and graph work involving the prescribed functions.	Problem solving and graph work involving the prescribed functions. Ave. gradient between 2 points.	Problem solving & graph work involving the prescribed functions (incl. the logarithmic function).

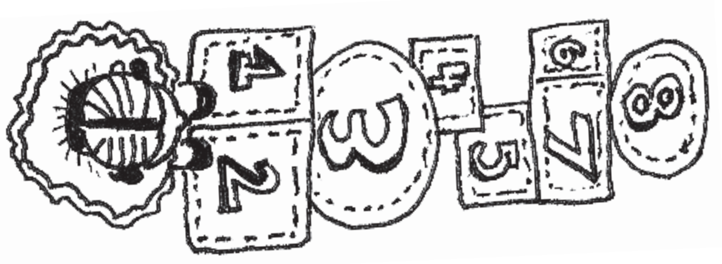
2. NUMBER PATTERNS, SEQUENCES AND SERIES

Grade 10	Grade 11	Grade 12
Investigate number patterns leading to those where there is a constant difference between consecutive terms, and the general term is therefore linear.	Investigate number patterns leading to those where there is a constant second difference between consecutive terms, and the general term is therefore quadratic.	Identify and solve problems involving number patterns that lead to arithmetic and geometric sequences and series, including infinite geometric series.

3. FINANCE, GROWTH AND DECAY

Grade 10	Grade 11	Grade 12
Use simple and compound growth formulae $A = P(1 + in)$ and $A = P(1 + i)^n$ to solve problems (including interest, hire purchase, inflation, population growth and other real life problems).	Use simple and compound decay formulae $A = P(1 - in)$ and $A = P(1 - i)^n$ to solve problems (including straight line depreciation and depreciation on a reducing balance). Link to work on functions.	(a) Calculate the value of n in the formulae $A = P(1 + i)^n$ and $A = P(1 - i)^n$ (b) Apply knowledge of geometric series to solve annuity and bond repayment problems.
The implications of fluctuating foreign exchange rates.	The effect of different periods of compounding growth & decay (incl. effective and nominal interest rates).	Critically analyse different loan options.

4. ALGEBRA					
	(a) Understand that real numbers can be irrational or rational.		Take note that there exist numbers other than those on the real number line, the so-called non-real numbers. It is possible to square certain non-real numbers and obtain negative real numbers as answers.		
	(a) Simplify expressions using the laws of exponents for rational exponents. (b) Establish between which two integers a given simple surd lies. (c) Round real numbers to an appropriate degree of accuracy (to a given number of decimal digits).		(a) Apply the laws of exponents to expressions involving rational exponents. (b) Add, subtract, multiply and divide simple surds.		Demonstrate an understanding of the definition of a logarithm and any laws needed to solve real life problems.
	Manipulate algebraic expressions by: • multiplying a binomial by a trinomial; • factorising trinomials; • factorising the difference and sums of two cubes; • factorising by grouping in pairs; and • simplifying, adding and subtracting algebraic fractions with denominators of cubes (limited to sum and difference of cubes).		Revise factorisation.		• Take note, and understand, the Remainder and Factor Theorems for polynomials up to the third degree. • Factorise third-degree polynomials (including examples which require the Factor Theorem).
	Solve: • linear equations; • quadratic equations; • literal equations (changing the subject of a formula); • exponential equations; • linear inequalities; • system of linear equations; and • word problems.		Solve: • quadratic equations (by factorisation, completing the square and by using the quadratic formula) • quadratic inequalities in one variable and interpret the solution graphically; and • equations in two unknowns, one of which is linear the other quadratic, algebraically or graphically. Determine: • the nature of the roots and the conditions for which the roots are real, non-real, equal, unequal, rational and irrational.		



5. DIFFERENTIAL CALCULUS					
					(a) An intuitive understanding of the concept of a limit. (b) Differentiation of specified functions from first principles. (c) Use of the specified rules of differentiation. (d) The equations of tangents to graphs. (e) The ability to sketch graphs of cubic functions. (f) Practical problems involving optimization and rates of change (including the calculus of motion).
6. PROBABILITY					
	(a) Compare the relative frequency of an experimental outcome with the theoretical probability of the outcome. (b) Venn diagrams as an aid to solving probability problems. (c) Mutually exclusive events and complementary events. (d) The identity for any two events A & B: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$		(a) Dependent and independent events. (b) Venn diagrams or contingency tables and tree diagrams as aids to solving probability problems (where events are not necessarily independent).		(a) Generalisation of the fundamental counting principle. (b) Probability problems using the fundamental counting principle.

Paper 2

7. EUCLIDEAN GEOMETRY AND MEASUREMENT					
	(a) Revise basic results established in earlier grades. (b) Investigate line segments joining the mid-points of two sides of a triangle. (c) Properties of special quadrilaterals.		(a) Investigate and prove theorems of the geometry of circles assuming results from earlier grades, together with one other result concerning tangents and radii of circles. (b) Solve circle geometry problems, providing reasons for statements when required. (c) Prove riders.		(a) Revise earlier (Grade 9) work on the necessary and sufficient conditions for polygons to be similar. (b) Prove (accepting results established in earlier grades): • that a line drawn parallel to one side of a triangle divides the other two sides proportionally (and the Mid-point Theorem as a special case of this theorem); • that equiangular triangles are similar; • that triangles with sides in proportion are similar; • the Pythagorean Theorem by similar triangles; and • riders
	Solve problems involving volume and surface area of solids studied in earlier grades as well as spheres, pyramids and cones and combinations of those objects.		Revise Grade 10 work.		

8. TRIGONOMETRY					
	(a) Definitions of the trigonometric ratios $sin\ \theta$, $cos\ \theta$ and $tan\ \theta$ in a right-angled triangle. (b) Extend the definitions of $sin\ \theta$, $cos\ \theta$ and $tan\ \theta$ to $0^o \leq \theta \leq 360^o$. (c) Derive and use values of the trigonometric ratios (without using a calculator) for the special angles $\theta \in \{0^o; 30^o; 45^o; 60^o; 90^o\}$. (d) Define the reciprocals of trigonometric ratios.		(a) Derive and use the identities: $tan\ \theta = \frac{sin\ \theta}{cos\ \theta}$ and $sin^2\theta + cos^2\theta = 1$ (b) Derive the reduction formulae. (c) Determine the general solution and / or specific solutions of trigonometric equations. (d) Prove and apply the sine, cosine and area rules.		Prove the identity $cos\ (\alpha - \beta) = cos\ \alpha\ cos\ \beta + sin\ \alpha\ sin\ \beta$ and derive other compound angle identities.
	Solve problems in two dimensions.		Solve problems in 2-dimensions.		Solve problems in two and three dimensions.
9. ANALYTICAL GEOMETRY					
	Represent geometric figures in a Cartesian co-ordinate system, and derive and apply, for any two points $(x_1\ ;\ y_1)$ and $(x_2\ ;\ y_2)$, a formula for calculating: • the distance between the two points; • the gradient of the line segment joining the points; • conditions for parallel and perpendicular lines; and • the co-ordinates of the mid-point of the line segment joining the points.		Use a Cartesian co-ordinate system to derive and apply: • the equation of a line through two given points; • the equation of a line through one point and parallel or perpendicular to a given line; and • the inclination of a line.		Use a two-dimensional Cartesian co-ordinate system to derive and apply: • the equation of a circle (any centre); and • the equation of a tangent to a circle at a given point on the circle.
10. STATISTICS					
	(a) Collect, organise and interpret univariate numerical data in order to determine: • measures of central tendency; • five number summary; • box and whisker diagrams; and • measures of dispersion.		(a) Represent measures of central tendency and dispersion in univariate numerical data by: • using ogives; and • calculating the variance and standard deviation of sets of data manually (for small sets of data) and using calculators (for larger sets of data) and representing results graphically. (b) Represent skewed data in box and whisker diagrams, and frequency polygons. Identify outliers in the context of box and whisker diagrams.		(a) Represent bivariate numerical data as a scatter plot and suggest intuitively and by simple investigation whether a linear, quadratic or exponential function would best fit the data. (b) Use a calculator to calculate the linear regression line which best fits a given set of bivariate numerical data. (c) Use a calculator to calculate the correlation co-efficient of a set of bivariate numerical data and make relevant deductions.

CONTENT

TERM BY TERM
FOR GRADE 10

3.2.3 Topic allocation per term

GRADE 10: TERM 1			
No of Weeks	**Topic**	**Curriculum statement**	**Clarification**
			Where an example is given, the cognitive demand is suggested: knowledge (K), routine procedure (R), complex procedure (C) or problem-solving (P)
3	**Algebraic expressions**	1. Understand that real numbers can be rational or irrational. 2. Establish between which two integers a given simple surd lies. 3. Round real numbers to an appropriate degree of accuracy. 4. Multiplication of a binomial by a trinomial. 5. Factorisation to include types taught in grade 9 and: • trinomials • grouping in pairs • sum and difference of two cubes 6. Simplification of algebraic fractions using factorization with denominators of cubes (limited to sum and difference of cubes).	**Examples to illustrate the different cognitive levels involved in factorisation:** 1. Factorise fully: 1.1. $m^2 - 2m + 1$ (revision) Learners must be able to recognise the simplest perfect squares. (R) 1.2. $2x^2 - x - 3$ This type is routine and appears in all texts. (R) 1.3. $\frac{y^2}{2} - \frac{13y}{2} + 18$ Learners are required to work with fractions and identify when an expression has been “fully factorised”. (R) 2. Simplify $\frac{1-2x}{4x^2-1} - \frac{x+4}{2x^2-3x+1} + \frac{1}{1-x}$ (C)
2	**Exponents**	1. Revise laws of exponents learnt in Grade 9 where $x, y > 0$ and $m, n \in Z$: • $x^m \times x^n = x^{m+n}$ • $x^m \div x^n = x^{m-n}$ • $(x^m)^n = x^{mn}$ • $x^m \times y^m = (xy)^m$ Also by definition: • $x^{-n} = \frac{1}{x^n}$, $x \neq 0$, and • $x^0 = 1$, $x \neq 0$ 2. Use the laws of exponents to simplify expressions and solve equations, accepting that the rules also hold for $m, n \in Q$.	**Examples:** 1. Simplify: $(3 \times 5^2)^3 - 75$ A simple two-step procedure is involved. (R) 2. Simplify $\frac{9^x - 1}{3^x + 1}$ Assuming this type of question has not been taught, spotting that the numerator can be factorised as a difference of squares requires insight. (P) 3. Solve for x: 3.1 $2^x = 0{,}125$ (R) 3.2 $2x^{\frac{3}{2}} = 54$ (R) 3.3 $3^{x+1} + 3^{x-1} = \frac{10}{9}$ (C) 3.4 $x^{\frac{1}{2}} + 3x^{\frac{1}{4}} - 18 = 0$ (C)

GRADE 10: TERM 1			
No of Weeks	**Topic**	**Curriculum statement**	**Clarification**
1	**Number patterns**	Patterns: Investigate number patterns leading to those where there is a constant difference between consecutive terms, and the general term (without using a formula) is therefore linear.	**Comment:** • Arithmetic sequence is done in Grade 12, hence $T_n = a + (n-1)d$ is not used in Grade 10. **Examples:** 1. Determine the 5th and the nth terms of the number pattern 10; 7; 4; 1; …. There is an algorithmic approach to answering such questions. (R) 2. If the pattern MATHSMATHSMATHS… is continued in this way, what will the 267th letter be? It is not immediately obvious how one should proceed, unless similar questions have been tackled. (P)
2	**Equations and Inequalities**	1. Revise the solution of linear equations. 2. Solve quadratic equations (by factorisation). 3. Solve simultaneous linear equations in two unknowns. 4. Solve word problems involving linear, quadratic or simultaneous linear equations. 5. Solve literal equations (changing the subject of a formula). 6. Solve linear inequalities (and show solution graphically). Interval notation must be known.	**Examples**: 1. Solve for x: $\frac{2x-3}{3} - 3x = \frac{2x}{6}$ (R) 2. Solve for m: $2m^2 - m = 1$ (R) 3. Solve for x and y: $x + 2y = 1$; $\frac{x}{3} + \frac{y}{2} = 1$ (C) 4. Solve for r in terms of V, Π and h: $V = \Pi r^2 h$ (R) 5. Solve for x: $-1 \le 2 - 3x \le 8$ (C)

GRADE 10: TERM 1			
No of Weeks	**Topic**	**Curriculum statement**	**Clarification**
3	**Trigonometry**	1. Define the trigonometric ratios sin θ, cos θ and tan θ, using right-angled triangles. 2. Extend the definitions of sin θ, cos θ and tan θ for $0^0 \leq \theta \leq 360^0$. 3. Define the reciprocals of the trigonometric ratios cosec θ, sec θ and cot θ, using right-angled triangles (these three reciprocals should be examined in grade 10 only). 4. Derive values of the trigonometric ratios for the special cases (without using a calculator) $\theta \in \{0^0; 30^0; 45^0; 60^0; 90^0\}$. 5. Solve two-dimensional problems involving right-angled triangles. 6. Solve simple trigonometric equations for angles between 0^0 and 90^0. 7. Use diagrams to determine the numerical values of ratios for angles from 0^0 to 360^0.	**Comment**: It is important to stress that: • similarity of triangles is fundamental to the trigonometric ratios sin θ, cos θ and tan θ; • trigonometric ratios are independent of the lengths of the sides of a similar right-angled triangle and depend (uniquely) only on the angles, hence we consider them as functions of the angles; • doubling a ratio has a different effect from doubling an angle, for example, generally $2\sin\theta \neq \sin 2\theta$; and • Solve equation of the form $\sin x = c$, or $2\cos x = c$, or $\tan 2x = c$, where c is a constant. **Examples**: 1. If $5\sin\theta + 4 = 0$ and $0^0 \leq \theta \leq 270^0$, calculate the value of $\sin^2\theta + \cos^2\theta$ without using a calculator. (R) 2. Let *ABCD* be a rectangle, with AB = 2*cm*. Let *E* be on *AD* such that $A\hat{B}E = 45^0$ and $B\hat{E}C = 75^0$. Determine the area of the rectangle. (P) 3. Determine the length of the hypotenuse of a right-angled triangle *ABC,* where $\hat{B} = 90^0$, $\hat{A} = 30^0$ and AB = 10cm (K) 4. Solve for *x*: $4\sin x - 1 = 3$ for $x \in [0°;90°]$ (C)

Assessment Term 1:

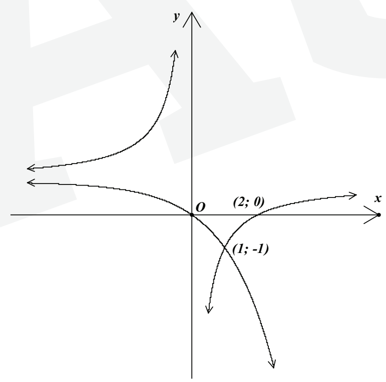
1. Investigation <u>or project</u> (only one project per year) (at least 50 marks)

   Example of an investigation:

   Imagine a cube of white wood which is dipped into red paint so that the surface is red, but the inside still white. If one cut is made, parallel to each face of the cube (and through the centre of the cube), then there will be 8 smaller cubes. Each of the smaller cubes will have 3 red faces and 3 white faces. Investigate the number of smaller cubes which will have 3, 2, 1 and 0 red faces if 2/3/4/…/*n* equally spaced cuts are made parallel to each face. This task provides the opportunity to investigate, tabulate results, make conjectures and justify or prove them.

2. <u>Test</u> (at least 50 marks and 1 hour). Make sure all topics are tested.

   Care needs to be taken to set questions on all four cognitive levels: approximately 20% knowledge, approximately 35% routine procedures, 30% complex procedures and 15% problem-solving.

GRADE 10: TERM 2			
Weeks	**Topic**	**Curriculum statement**	**Clarification**
(4 + 1) **5**	**Functions**	1. The concept of a function, where a certain quantity (output value) uniquely depends on another quantity (input value) should be emphasized. Work with relationships between variables using tables, graphs, words and formulae. Convert flexibly between these representations. Note: that the graph defined by $y = x$ should be known from Grade 9. 2. Point by point plotting of basic graphs defined by $y = x^2$, $y = \frac{1}{x}$ and $y = b^x$; $b > 0$ and $b \neq 1$ to discover shape, domain (input values), range (output values), asymptotes, axes of symmetry, turning points and intercepts on the axes (where applicable). 3. Investigate the effect of a and q on the graphs defined by $y = a.f(x) + q$, where $f(x) = x$, $f(x) = x^2$, $f(x) = \frac{1}{x}$ and $f(x) = b^x$, $b > 0, b \neq 1$. 4. Point by point plotting of basic graphs defined by $y = \sin\theta$, $y = \cos\theta$ and $y = \tan\theta$ for $\theta \in [0°; 360°]$ 5. Study the effect of a and q on the graphs defined by: $y = a\sin\theta + q$; $y = a\cos\theta + q$; and $y = a\tan\theta + q$ where $a, q \in \mathbb{Q}$ for $\theta \in [0°;360°]$. 6. Sketch graphs, find the equations of given graphs and interpret graphs. **Note**: Sketching of the graphs must be based on the observation of number 3 and 5.	**Comments**: • A more formal definition of a function follows in Grade 12. At this level it is enough to investigate the way (unique) output values depend on how input values vary. The terms independent (input) and dependent (output) variables might be useful. • After summaries have been compiled about basic features of prescribed graphs and the effects of parameters a and q have been investigated: a: a vertical stretch (and/or a reflection about the x-axis) and q a vertical shift. The following examples might be appropriate: • Remember that graphs in some practical applications may be either discrete or continuous. **Examples:** 1. Sketched below are graphs of $f(x) = \frac{a}{x} + q$ and $g(x) = nb^x + t$. The horizontal asymptote of both graphs is the line y = 1. Determine the values of a , b , n , q and t . (C)  2. Sketch the graph defined by $y = -\sin x + \frac{1}{2}$ for $x \in [0°;360°]$ (R)

GRADE 10: TERM 2

Weeks	Topic	Curriculum statement	Clarification
3	**Euclidean Geometry**	1. Revise basic results established in earlier grades regarding lines, angles and triangles, especially the similarity and congruence of triangles. 2. Investigate line segments joining the mid-points of two sides of a triangle. 3. Define the following special quadrilaterals: the kite, parallelogram, rectangle, rhombus, square and trapezium. Investigate and make conjectures about the properties of the sides, angles, diagonals and areas of these quadrilaterals. Prove these conjectures.	**Comments:** • Triangles are similar if their corresponding angles are equal, or if the ratios of their sides are equal: Triangles ABC and DEF are similar if $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$. They are also similar if $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$. • We could define a parallelogram as a quadrilateral with two pairs of opposite sides parallel. Then we investigate and prove that the opposite sides of the parallelogram are equal, opposite angles of a parallelogram are equal, and diagonals of a parallelogram bisect each other. • It must be explained that a single counter example can disprove a Conjecture, but numerous specific examples supporting a conjecture do not constitute a general proof. **Example**: In quadrilateral KITE, KI = KE and IT = ET. The diagonals intersect at M. Prove that: 1. IM = ME and (R) 2. KT is perpendicular to IE. (P) As it is not obvious, first prove that $\Delta KIT \equiv \Delta KET$.
3	**Mid-year examinations**		

Assessment term 2:

1. <u>Assignment / test</u> (at least 50 marks)
2. <u>Mid-year examination</u> (at least 100 marks)

   One paper of 2 hours (100 marks) **or** Two papers - one, 1 hour (50 marks) and the other, 1 hour (50 marks)

GRADE 10: TERM 3			
Weeks	**Topic**	**Curriculum statement**	**Clarification**
2	**Analytical Geometry**	Represent geometric figures on a Cartesian co-ordinate system. Derive and apply for any two points $(x_1;y_1)$ and $(x_2;y_2)$ the formulae for calculating the: 1. distance between the two points; 2. gradient of the line segment connecting the two points (and from that identify parallel and perpendicular lines); and 3. coordinates of the mid-point of the line segment joining the two points.	**Example**: Consider the points $P(2;5)$ and $Q(-3;1)$ in the Cartesian plane. 1.1. Calculate the distance PQ. (K) 1.2 Find the coordinates of R if M $(-1;0)$ is the mid-point of PR. (R) 1.3 Determine the coordinates of S if PQRS is a parallelogram. (C) 1.4 Is PQRS a rectangle? Why or why not? (R)
2	**Finance and growth**	Use the simple and compound growth formulae $A = P(1+in)$ and $A = P(1+i)^n$ to solve problems, including annual interest, hire purchase, inflation, population growth and other real-life problems. Understand the implication of fluctuating foreign exchange rates (e.g. on the petrol price, imports, exports, overseas travel).	

<table>
<tr><th colspan="4">GRADE 10: TERM 3</th></tr>
<tr><th>Weeks</th><th>Topic</th><th>Curriculum statement</th><th>Clarification</th></tr>
<tr>
<td>2</td>
<td>Statistics</td>
<td>
1. Revise measures of central tendency in ungrouped data.
2. Measures of central tendency in grouped data:
calculation of mean estimate of grouped and ungrouped data and identification of modal interval and interval in which the median lies.
3. Revision of range as a measure of dispersion and extension to include percentiles, quartiles, interquartile and semi interquartile range.
4. Five number summary (maximum, minimum and quartiles) and box and whisker diagram.
5. Use the statistical summaries (measures of central tendency and dispersion), and graphs to analyse and make meaningful comments on the context associated with the given data.
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Comment:
In grade 10, the intervals of grouped data should be given using inequalities, that is, in the form $0 \le x < 20$ rather than in the form 0-19, 20-29, …
Example:
The mathematics marks of 200 grade 10 learners at a school can be summarised as follows:
<table>
<tr><th>Percentage obtained</th><th>Number of candidates</th></tr>
<tr><td>$0 \le x < 20$</td><td>4</td></tr>
<tr><td>$20 \le x < 30$</td><td>10</td></tr>
<tr><td>$30 \le x < 40$</td><td>37</td></tr>
<tr><td>$40 \le x < 50$</td><td>43</td></tr>
<tr><td>$50 \le x < 60$</td><td>36</td></tr>
<tr><td>$60 \le x < 70$</td><td>26</td></tr>
<tr><td>$70 \le x < 80$</td><td>24</td></tr>
<tr><td>$80 \le x < 100$</td><td>20</td></tr>
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1. Calculate the approximate mean mark for the examination. (R)
2. Identify the interval in which each of the following data items lies:
2.1. the median; (R)
2.2. the lower quartile; (R)
2.3. the upper quartile; and (R)
2.4. the thirtieth percentile. (R)
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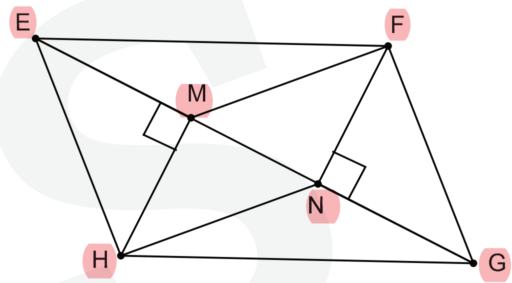
GRADE 10: TERM 3			
Weeks	**Topic**	**Curriculum statement**	**Clarification**
2	**Trigonometry**	Problems in two dimensions.	**Example:** Two flagpoles are 30 m apart. The one has height 10 m, while the other has height 15 m. Two tight ropes connect the top of each pole to the foot of the other. At what height above the ground do the two ropes intersect? What if the poles were at different distance apart? (P)
1	**Euclidean Geometry**	Solve problems and prove riders using the properties of parallel lines, triangles and quadrilaterals.	**Comment:** Use congruency and properties of quads, esp. parallelograms. **Example:** EFGH is a parallelogram. Prove that MFNH is a parallelogram.  (C)
1	**Measurement**	1. Revise the volume and surface areas of right-prisms and cylinders. 2. Study the effect on volume and surface area when multiplying any dimension by a constant factor *k*. 3. Calculate the volume and surface areas of spheres, right pyramids and right cones.	**Example:** The height of a cylinder is 10 cm, and the radius of the circular base is 2 cm. A hemisphere is attached to one end of the cylinder and a cone of height 2 cm to the other end. Calculate the volume and surface area of the solid, correct to the nearest cm^3 and cm^2, respectively. (R) **Comments:** • In case of pyramids, bases must either be an equilateral triangle or a square. • Problem types must include composite figure.

Assessment term 3: Two (2) Tests (at least 50 marks and 1 hour) covering all topics in approximately the ratio of the allocated teaching time.

GRADE 10: TERM 4			
No of Weeks	**Topic**	**Curriculum statement**	**Clarification**
2	**Probability**	1. The use of probability models to compare the relative frequency of events with the theoretical probability. 2. The use of Venn diagrams to solve probability problems, deriving and applying the following for any two events A and B in a sample space S: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ A and B are mutually exclusive if $P(A \text{ and } B) = 0$; A and B are complementary if they are mutually exclusive ; and $P(A) + P(B) = 1$. Then $P(B) = P(\text{ not } (A)) = 1 - P(A)$.	**Comment:** • It generally takes a very large number of trials before the relative frequency of a coin falling heads up when tossed approaches 0,5. **Example**: In a survey 80 people were questioned to find out how many read newspaper S or D or both. The survey revealed that 45 read D, 30 read S and 10 read neither. Use a Venn diagram to find how many read 1. S only; (C) 2. D only; and (C) 3. both D and S. (C)
4	**Revision**		**Comment:** The value of working through past papers cannot be over emphasised.
3	**Examinations**		

Assessment term 4

1. Test (at least 50 marks)
2. Examination

Paper 1: 2 hours (100 marks made up as follows: 15±3 on number patterns, 30±3 on algebraic expressions, equations and inequalities , 30±3 on functions, 10±3 on finance and growth and 15±3 on probability.

Paper 2: 2 hours (100 marks made up as follows: 40±3 on trigonometry, 15±3 on Analytical Geometry, 30±3 on Euclidean Geometry and Measurement, and 15±3 on Statistics

CONTENT

TERM BY TERM
FOR GRADE 11

GRADE 11: TERM 1			
No fWeeks	**Topic**	**Curriculum statement**	**Clarification**
			Where an example is given, the cognitive demand is suggested: knowledge (K), routine procedure (R), complex procedure (C) or problem solving (P)
3	**Exponents and surds**	1. Simplify expressions and solve equations using the laws of exponents for rational exponents where $x^{\frac{p}{q}} = \sqrt[q]{x^p};\ x > 0; q > 0$. 2. Add, subtract, multiply and divide simple surds. 3. Solve simple equations involving surds.	**Example**: 1. Determine the value of $9^{\frac{3}{2}}$. (R) 2. Simplify: $(3+\sqrt{2})(3-\sqrt{2})$. (R) 3. Solve for x: $\sqrt{x-2} = 3$. (P)
3	**Equations and Inequalities**	1. Solve: 1.1 Quadratic equations (by factorisation, completing the square and using the quadratic formula). 1.2 Quadratic inequalities in one unknown (Interpret solutions graphically). 1.3 Equations in two unknowns, one of which is linear and the other quadratic. NB: It is recommended that the solving of equations in two unknowns is important to be used in other equations like hyperbola-straight line as this is normal in the case of graphs. 2. Determine the nature of the roots and the conditions for which the roots are real, non-real, equal, unequal, rational and irrational.	**Example**: 1. I have 12 metres of fencing. What are the dimensions of the largest rectangular area I can enclose with this fencing by using an existing wall as one side?Hint: let the length of the equal sides of the rectangle be x metres and formulate an expression for the area of the rectangle.(C) (Without the hint this would probably be problem solving.) 2.1. Show that the roots of $x^2 - 2x - 7 = 0$ are irrational. (R) 2.2. Show that $x^2 + x + 1 = 0$ has no real roots. (R) 3. Solve for x: $x^2 \leq 4$. (R) 4. Solve for x: $(x+1)(2x-3) \leq 3$. (R) 5. Two machines, working together, take 2 hours 24 minutes to complete a job. Working on its own, one machine takes 2 hours longer than the other to complete the job. How long does the slower machine take on its own? (P)
2	**Number patterns**	Patterns: Investigate number patterns leading to those where there is a constant second difference between consecutive terms, and the general term is therefore quadratic.	**Example**: In the first stage of the World Cup Soccer Finals there are teams from four different countries in each group. Each country in a group plays every other country in the group once. How many matches are there for each group in the first stage of the finals? How many games would there be if there were five teams in each group? Six teams? *n* teams? (P)

GRADE 11: TERM 1

No fWeeks	Topic	Curriculum statement	Clarification
3	**Analytical Geometry**	Derive and apply: 1. the equation of a line through two given points; 2. the equation of a line through one point and parallel or perpendicular to a given line; and 3. the inclination (θ) of a line, where $m = \tan\theta$ is the gradient of the line $(0° \leq \theta < 180°)$.	**Example**: Given the points $A(2;5)$; $B(-3;-4)$ and $C(4;-2)$ determine: 1. the equation of the line AB; and (R) 2. the size of $B\hat{A}C$. (C)

Assessment Term 1:

1. An Investigation or a project (a maximum of one project in a year) (at least 50 marks)

Notice that an assignment is generally an extended piece of work undertaken at home. ***Linear programming questions can be used as projects.***

Example of an assignment: Ratios and equations in two variables.

(This assignment brings in an element of history which could be extended to include one or two ancient paintings and examples of architecture which are in the shape of a rectangle with the ratio of sides equal to the golden ratio.)

Task 1

If $2x^2 - 3xy + y^2 = 0$ then $(2x - y)(x-y) = 0$ so $x = \frac{y}{2}$ or $x = y$. Hence the ratio $\frac{x}{y} = \frac{1}{2}$ or $\frac{x}{y} = \frac{1}{1}$. In the same way find the possible values of

the ratio $\frac{x}{y}$ if it is given that $2x^2 - 5xy + y^2 = 0$

Task

Most paper is cut to internationally agreed sizes: A0, A1, A2, …, A7 with the property that the A1 sheet is half the size of the A0 sheet and similar to the A0

sheet, the A2 sheet is half the size of the A1 sheet and similar to the A1 sheet, etc. Find the ratio of the length x to the breadth y of A0, A1, A2, …, A7 paper (in simplest surd form).

Task 3

The golden rectangle has been recognised through the ages as being aesthetically pleasing. It can be seen in the architecture of the Greeks, in sculptures and in Renaissance paintings. Any golden rectangle with length x and breadth y has the property that when a square the length of the shorter side (y) is cut from it, another rectangle similar to it is left. The process can be continued indefinitely, producing smaller and smaller rectangles. Using this information, calculate the ratio $x: y$ in surd form.

Example of project: Collect the heights of at least 50 sixteen-year-old girls and at least 50 sixteen-year-old boys. Group your data appropriately and use these two sets of grouped data to draw frequency polygons of the relative heights of boys and of girls, in different colours, on the same sheet of graph paper. Identify the modal intervals, the intervals in which the medians lie and the approximate means as calculated from the frequencies of the grouped data. By how much does the approximate mean height of your sample of sixteen-year-old girls differ from the actual mean? Comment on the symmetry of the two frequency polygons and any other aspects of the data which are illustrated by the frequency polygons.

2. Test (at least 50 marks and 1 hour). Make sure all topics are tested.

Care needs to be taken to ask questions on all four cognitive levels: approximately 20% knowledge, approximately 35% routine procedures, 30% complex procedures and 15% problem-solving.

GRADE 11: TERM 2			
No of Weeks	**Topic**	**Curriculum statement**	**Clarification**
4	**Functions**	1. Revise the effect of the parameters *a* and *q* and investigate the effect of *p* on the graphs of the functions defined by: 1.1. $y = f(x) = a(x+p)^2 + q$ 1.2. $y = f(x) = \frac{a}{x+p} + q$ 1.3. $y = f(x) = ab^{x+p} + q$ where *b>0, b≠1* Investigate numerically the average gradient between two points on a curve and develop an intuitive understanding of the concept of the gradient of a curve at a point. 3. Point by point plotting of basic graphs defined by $y = \sin\theta$, $y = \cos\theta$ and $y = \tan\theta$ for $\theta \in [-360°; 360°]$ 4. Investigate the effect of the parameter *k* on the graphs of the functions defined by $y = \sin(kx)$, $y = \cos(kx)$ and $y = \tan(kx)$. 5. Investigate the effect of the parameter *p* on the graphs of the functions defined by $y = \sin(x+p)$, $y = \cos(x+p)$ and $y = \tan(x+p)$. 6. Draw sketch graphs defined by: $y = a \sin k(x+p)$, $y = a \cos k(x+p)$ and $y = a \tan k(x+p)$ at most two parameters at a time.	**Comment:** • Once the effects of the parameters have been established, various problems need to be set: drawing sketch graphs, determining the defining equations of functions from sufficient data, making deductions from graphs. Real life applications of the prescribed functions should be studied. • Two parameters at a time can be varied in tests or examinations in trigonometric graphs only. **Example**: Sketch the graphs defined by $y = -\frac{1}{2}\sin(x+30°)$ and $f(x) = \cos(2x - 120°)$ on the same set of axes, where $-360° \le x \le 360°$. (C)

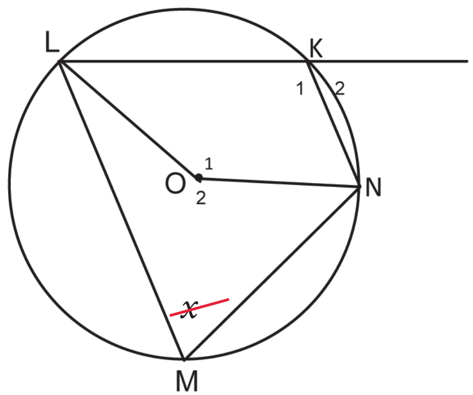
GRADE 11: TERM 2			
No of Weeks	**Topic**	**Curriculum statement**	**Clarification**
4	**Trigonometry**	1. Derive and use the identities $\tan\theta = \frac{\sin\theta}{\cos\theta}$, $\theta \neq k.90°, k$ an odd integer; and $\sin^2\theta + \cos^2\theta = 1$. 2. Derive and use reduction formulae to simplify the following expressions: 2.1. $\sin(90° \pm \theta); \cos(90° \pm \theta)$; 2.2. $\sin(180° \pm \theta); \cos(180° \pm \theta)$; $\tan(180° \pm \theta)$; 2.3. $\sin(360° \pm \theta); \cos(360° \pm \theta)$; $\tan(360° \pm \theta)$; and 2.4. $\sin(-\theta); \cos(-\theta); \tan(-\theta)$. 3. Determine for which values of a variable an identity holds. 4. Determine the general solutions of trigonometric equations. Also, determine solutions in specific intervals.	**Comment:** • Teachers should explain where reduction formulae come from. **Examples**: 1. Prove that $\frac{1}{\tan\theta} + \tan\theta = \frac{\tan\theta}{\sin^2\theta}$. (R) 2. For which values of θ is $\frac{1}{\tan\theta} + \tan\theta = \frac{\tan\theta}{\sin^2\theta}$ undefined? (R) 3. Simplify $\frac{\cos(180° - x)\sin(x - 90°) - 1}{\tan^2(540° + x)\sin(90° + x)\cos(-x)}$ (R) 4. Determine the general solutions of $\cos^2\theta + 3\sin\theta = -3$. (C)
3	**Mid-year examinations**		

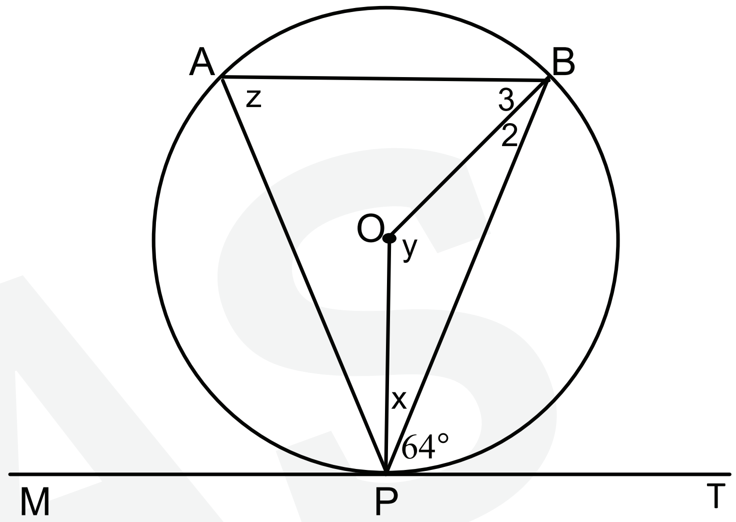
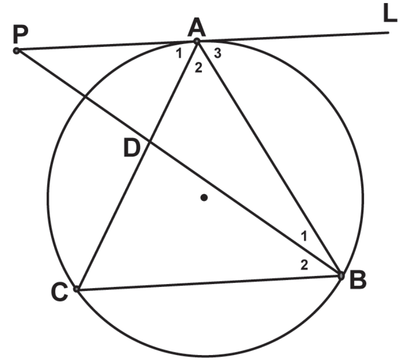
Assessment term 2:

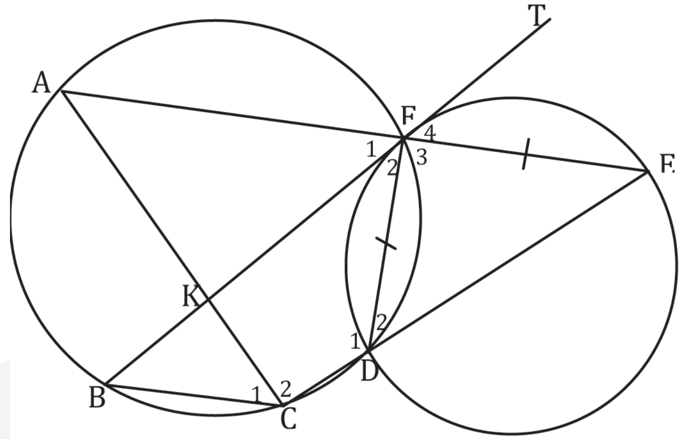
1. Assignment (at least 50 marks)
2. Mid-year examination:

Paper 1: 2 hours (100 marks made up as follows: general algebra (25±3) equations and inequalities (35±3); number patterns (15±3); functions (25±3)

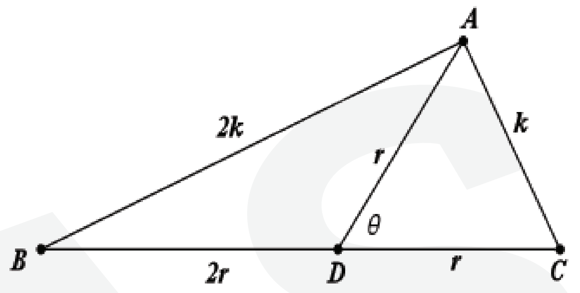
Paper 2: 2 hours (100 marks made up as follows: analytical geometry (30±3) and trigonometry (70±3)).

GRADE 11: TERM 3			
No. of weeks	**Topic**	**Curriculum Statement**	**Clarification**
1	**Measurement**	1. Revise the Grade 10 work.	
3	**Euclidean Geometry**	Accept results established in earlier grades as axioms and also that a tangent to a circle is perpendicular to the radius, drawn to the point of contact. Then investigate and prove the theorems of the geometry of circles: • The line drawn from the centre of a circle perpendicular to a chord bisects the chord; • The perpendicular bisector of a chord passes through the centre of the circle; • The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre); • Angles subtended by a chord of the circle, on the same side of the chord, are equal; • The opposite angles of a cyclic quadrilateral are supplementary; • Two tangents drawn to a circle from the same point outside the circle are equal in length; • The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment. Use the above theorems and their converses, where they exist, to solve riders.	**Comments:** Proofs of theorems can be asked in examinations, but their converses (wherever they hold) cannot be asked. **Example:** *1.* *AB* and *CD* are two chords of a circle with centre O. *M* is on *AB* and *N* is on *CD* such that *OM* ⊥ *AB* and *ON* ⊥ *CD*. Also, *AB* = 50mm, *OM* = 40mm and *ON* = 20mm. Determine the radius of the circle and the length of *CD*. (C) 2. *O* is the centre of the circle below and $\hat{O}_1 = 2x$.  2.1. Determine $\hat{O}_2$ and $\hat{M}$ in terms of x. (R) 2.2. Determine $\hat{K}_1$ and $\hat{K}_2$ in terms of x. (R) 2.3. Determine $\hat{K}_1 + \hat{M}$. What do you notice? (R) 2.4. Write down your observation regarding the measures of $\hat{K}_2$ and $\hat{M}$. (R)

GRADE 11: TERM 3			
No. of weeks	**Topic**	**Curriculum Statement**	**Clarification**
			3. *O* is the centre of the circle above and MPT is a tangent. Also, $OP \perp MT$. Determine, with reasons, x, y and z . (C)  4. Given: $AB = AC$, $AP \parallel BC$ and $\hat{A}_2 = \hat{B}_2$ . .  Prove that: 4.1 *PAL* is a tangent to circle *ABC*; (P) 4.2 *AB* is a tangent to circle *ADP*. (P)

<table>
<tr><th colspan="4">GRADE 11: TERM 3</th></tr>
<tr><th>No. of weeks</th><th>Topic</th><th>Curriculum Statement</th><th>Clarification</th></tr>
<tr><td></td><td></td><td></td><td>5. In the accompanying figure, two circles intersect at F and D.


BFT is a tangent to the smaller circle at F. Straight line AFE is drawn such that FD = FE. CDE is a straight line and chord AC and BF cut at K.
Prove that:
5.1 BT // CE (C)
5.2 BCEF is a parallelogram (P)
5.3 AC = BF (P)</td></tr>
</table>

GRADE 11: TERM 3			
No. of weeks	**Topic**	**Curriculum Statement**	**Clarification**
2	**Trigonometry**	1. Prove and apply the sine, cosine and area rules. 2. Solve problems in two dimensions using the sine, cosine and area rules.	**Comment:** • The proofs of the sine, cosine and area rules are examinable. **Example:** In ΔADC, D is on BC, $A\hat{D}C = \theta$, $DA = DC = r$, $BD = 2r$, $AC = k$ and $BA = 2k$.  Show that $\cos\theta = \frac{1}{4}$. (P)
2	**Finance, growth and decay**	1. Use simple and compound decay formulae: $A = P(1 - in)$ and $A = P(1 - i)^n$ to solve problems (including straight line depreciation and depreciation on a reducing balance). 2. The effect of different periods of compound growth and decay, including nominal and effective interest rates.	**Examples**: 1. The value of a piece of equipment depreciates from R10 000 to R5 000 in four years. What is the rate of depreciation if calculated on the: 1.1 straight line method; and (R) 1.2 reducing balance? (C) 2. Which is the better investment over a year or longer: 10,5% p.a. compounded daily or 10,55% p.a. compounded monthly? (R) 3. R50 000 is invested in an account which offers 8% p.a. interest compounded quarterly for the first 18 months. The interest then changes to 6% p.a. compounded monthly. Two years after the money is invested, R10 000 is withdrawn. How much will be in the account after 4 years? (C) **Comment**: • The use of a timeline to solve problems is a useful technique. • Stress the importance of not working with rounded answers, but of using the maximum accuracy afforded by the calculator right to the final answer when rounding might be appropriate.

GRADE 11: TERM 3			
No. of weeks	**Topic**	**Curriculum Statement**	**Clarification**
2	**Probability**	1. Revise the addition rule for mutually exclusive events: $P(A \text{ or } B) = P(A) + P(B)$, the complementary rule: $P(\text{not } A) = 1 - P(A)$ and the identity P(A or B) = P(A) + P(B) - P(A and B) 2. Identify dependent and independent events and the product rule for independent events: $P(A \text{ and } B) = P(A) \times P(B)$. 3. The use of Venn diagrams to solve probability problems, deriving and applying formulae for any three events A, B and C in a sample space S. 4. Use tree diagrams for the probability of consecutive or simultaneous events which are not necessarily independent. 5. The use of contingency tables to solve probability problems for three events in a sample space.	**Comment**: • Venn Diagrams or Contingency tables can be used to study dependent and independent events. **Examples**: 1. P(A) = 0,45, P(B) = 0,3 and P(A or B) = 0,615. Are the events A and B mutually exclusive, independent or neither mutually exclusive nor independent? (R) 2. What is the probability of throwing at least one six in four rolls of a regular six sided die? (C) 3. In a group of 50 learners, 35 take Mathematics and 30 take History, while 12 take neither of the two. If a learner is chosen at random from this group, what is the probability that he/she takes both Mathematics and History? (C) 4. A study was done to test how effective three different drugs, A, B and C were in relieving headaches. Over the period covered by the study, 80 patients were given the opportunity to use all two drugs. The following results were obtained:from at least one of the drugs? (R) 40 reported relief from drug A 35 reported relief from drug B 40 reported relief from drug C 21 reported relief from both drugs A and C 18 reported relief from drugs B and C 68 reported relief from at least one of the drugs 7 reported relief from all three drugs. 4.1 Record this information in a Venn diagram. (C) 4.2 How many subjects got no relief from any of the drugs? (K) 4.3 How many subjects got relief from drugs A and B, but not C? (R) 4.4 What is the probability that a randomly chosen subject got relief from at least one of the drugs? (R)

GRADE 11: TERM 4			
No. of weeks	**Topic**	**Curriculum Statement**	**Clarification**
3	**Statistics**	1. Histograms 2. Frequency polygons 3. Ogives (cumulative frequency curves) 4. Variance and standard deviation of ungrouped data 5. Symmetric and skewed data 6. Identification of outliers	**Comments:** • Variance and standard deviation may be calculated using calculators. • Problems should cover topics related to health, social, economic, cultural, political and environmental issues. • Identification of outliers should be done in the context of box and whisker diagrams.
3	**Revision**		
3	**Examinations**		

Assessment term 4:

1. Test (at least 50 marks)
2. Examination (300 marks)

   Paper 1: 3 hours (150 marks made up as follows: (25±3) on number patterns, on (45±3) exponents and surds, equations and inequalities, (45±3) on functions, (15±3) on finance growth and decay, (20±3) on probability).

   Paper 2: 3 hours (150 marks made up as follows: (50±3) on trigonometry, (30±3) on Analytical Geometry, (50±3) on Euclidean Geometry and Measurement, (20±3) on Statistics).

CONTENT

TERM BY TERM
FOR GRADE 12

GRADE 12: TERM 1			
No. of Weeks	**Topic**	**Curriculum statement**	**Clarification**
3	**Patterns, sequences, series**	1. Number patterns, including arithmetic and geometric sequences and series 2. Sigma notation 3. Derivation and application of the formulae for the sum of arithmetic and geometric series: 3.1 $S_n = \frac{n}{2}[2a + (n-1)d]$; $S_n = \frac{n}{2}(a + l)$; 3.2 $S_n = \frac{a(r^n - 1)}{r - 1}; (r \neq 1)$; and 3.3 $S_\infty = \frac{a}{1-r}; (-1 < r < 1)$, $\cancel{(r \neq 1)}$	**Comment:** Derivation of the formulae is examinable. **Examples:** 1. Write down the first five terms of the sequence with general term $T_k = \frac{1}{3k-1}$. (K) 2. Calculate $\sum_{k=0}^{3}(3k-1)$. (R) 3. Determine the 5th term of the geometric sequence of which the 8th term is 6 and the 12th term is 14. (C) 4. Determine the largest value of n such that $\sum_{i=1}^{n}(3i-2) < 2000$. (R) 5. Show that $0,99\dot{9} = 1$. (P)
3	**Functions**	1. Definition of a *function.* 2. General concept of the *inverse of a function* and how the domain of the function may need to be restricted (in order to obtain a one-to-one function) to ensure that the inverse is a function. 3. Determine and sketch graphs of the inverses of the functions defined by $y = ax + q$; $y = ax^2$ $y = b^x; (b > 0, b \neq 1)$ Focus on the following characteristics: domain and range, intercepts with the axes, turning points, minima, maxima, asymptotes (horizontal and vertical), shape and symmetry, average gradient (average rate of change), intervals on which the function increases /decreases.	**Examples:** 1. Consider the function f where $f(x) = 3x - 1$. 1.1 Write down the domain and range of f. (K) 1.2 Show that f is a one-to-one relation. (R) 1.3 Determine the inverse function f^{-1}. (R) 1.4 Sketch the graphs of the functions f, f^{-1} and $y = x$ line on the same set of axes. What do you notice? (R) 2. Repeat Question 1 for the function $f(x) = -x^2$ and $x \leq 0$. (C) **Caution:** 1. Do not confuse the inverse function f^{-1} with the reciprocal $\frac{1}{f(x)}$. For example, for the function where $f(x) = \sqrt{x}$, the reciprocal is $\frac{1}{\sqrt{x}}$, while $f^{-1}(x) = x^2$ for $x \geq 0$. 2. Note that the notation $f^{-1}(x) = ...$ is used only for one-to-one relation and must not be used for inverses of many-to-one relations, since in these cases the inverses are not functions.

GRADE 12: TERM 1			
No. of Weeks	**Topic**	**Curriculum statement**	**Clarification**
1	**Functions: exponential and logarithmic**	1. Revision of the exponential function and the exponential laws and graph of the function defined by $y = b^x$ where $b > 0$ and $b \neq 1$ 2. Understand the definition of a logarithm: $y = \log_b x \Leftrightarrow x = b^y$, where $b > 0$ and $b \neq 1$. 3. The graph of the function define $y = \log_b x$ for both the cases $0 < b < 1$ and $b > 1$.	**Comment:** The four logarithmic laws that will be applied, only in the context of real-life problems related to finance, growth and decay, are: $\log_b(AB) = \log_b A + \log_b B$; $\log_b(\frac{A}{B}) = \log_b A - \log_b B$; $\log A^n = n\log A$; and $\log_B A = \frac{\log A}{\log B}$. They follow from the basic exponential laws (term 1 of grade 10). • Manipulation involving the logarithmic laws will not be examined. **Caution:** 1. Make sure learners know the difference between the two functions defined by $y = b^x$ and $y = x^b$ where b is a positive (constant) real number. 2. Manipulation involving the logarithmic laws will not be examined. **Examples:** 1. Solve for x : $75(1{,}025)^{x-1} = 300$ (R) 2. Let $f(x) = a^x$, $a > 0$. 2.1 Determine a if the graph of f goes through the point $(2;\frac{25}{16})$. (R) 2.2 Determine the function f^{-1}. (R) 2.3 For which values of x is $f^{-1}(x) > -1$? (C) 2.4 Determine the function h if the graph of h is the reflection of the graph of f about the y-axis. (C) 2.5 Determine the function k if the graph of k is the reflection of the graph of f about the x-axis. (C) 2.6 Determine the function p if the graph of p is obtained by shifting the graph of f two units to the left. (C) 2.7 Write down the domain and range for each of the functions f, f^{-1}, h, k and p. (R) 2.8 Represent all these functions graphically. (R)

GRADE 12: TERM 1			
No. of Weeks	**Topic**	**Curriculum statement**	**Clarification**
2	**Finance, growth and decay**	1. Solve problems involving present value and future value annuities. 2. Make use of logarithms to calculate the value of n, the time period, in the equations $A = P(1+i)^n$ or $A = P(1-i)^n$. 3. Critically analyse investment and loan options and make informed decisions as to best option(s) (including pyramid).	**Comment:** Derivation of the formulae for present and future values using the geometric series formula, $S_n = \frac{a(r^n - 1)}{r-1}; r \neq 1$, should be part of the teaching process to ensure that the learners understand where the formulae come from. The two annuity formulae: $F = \frac{x[(1+i)^n - 1]}{i}$ and $P = \frac{x[1-(1+i)^{-n}]}{i}$ hold only when payment commences one period from the present and ends after n periods. **Comment**: The use of a timeline to analyse problems is a useful technique. **Examples:** 1. Given that a population increased from 120 000 to 214 000 in 10 years, at what annual (compound) rate did the population grow? (R) 2. In order to buy a car, John takes out a loan of R25 000 from the bank. The bank charges an annual interest rate of 11%, compounded monthly. The instalments start a month after he has received the money from the bank. 2.1 Calculate his monthly instalments if he has to pay back the loan over a period of 5 years. (R) 2.2 Calculate the outstanding balance of his loan after two years (immediately after the 24th instalment). (C)
2	**Trigonometry**	Compound angle identities: $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$; $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$; $\sin 2\alpha = 2\sin\alpha\cos\alpha$; $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$; $\cos 2\alpha = 2\cos^2\alpha - 1$; and $\cos 2\alpha = 1 - 2\sin^2\alpha$.	2. Prove $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$ and then derive the other compound angle identities. (C) 3. Determine the general solution of $\sin 2x + \cos x = 0$. (R) 4. Prove that $\frac{1+\sin 2x}{\cos 2x} = \frac{\cos x + \sin x}{\cos x - \sin x}$. (C)

GRADE 12: TERM 1			
No. of Weeks	Topic	Curriculum statement	Clarification

Assessment Term 1:

1. Investigation or project. (at least 50 marks)

   Only one investigation or project per year is required.

1.1 Example of an investigation which revises the sine, cosine and area rules:

1.2 Investigation: Polygons with 12 Matches

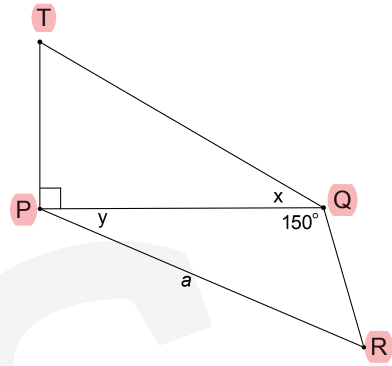
- How many different triangles can be made with a perimeter of 12 matches?
- Which of these triangles has the greatest area?
- What regular polygons can be made using all 12 matches?
- Investigate the areas of polygons with a perimeter of 12 matches in an effort to establish the maximum area that can be enclosed by the matches.

  Any extensions or generalisations that can be made, based on this task, will enhance your investigation. But you need to strive for quality, rather than simply producing a large number of trivial observations.

Note:

The focus of this task is on mathematical processes. Some of these processes are: specialising, classifying, comparing, inferring, estimating, generalising, making conjectures, validating, proving and communicating mathematical ideas.

2. Assignment or Test. (at least 50 marks)
3. Test. (at least 50 marks)

GRADE 12: TERM 2			
No. of Weeks	**Topic**	**Curriculum statement**	**Clarification**
2	**Trigonometry continued**	1. Solve problems in two and three dimensions.	**Examples:**  1. TP is a tower. Its foot, P, and the points Q and R are on the same horizontal plane. From Q the angle of elevation to the top of the building is x. Furthermore, $P\hat{Q}R = 150°$, $Q\hat{P}R =$ y and the distance between P and R is a metres. Prove that $TP = a\tan x(\cos y - \sqrt{3}\sin y)$ (C) 2. In ΔABC, $AD \perp BC$. Prove that: 2.1 $a = b\cos C + c\cos B$ where $a = BC$; $b = AC$ and $c = AB$. (R) 2.2 $\dfrac{\cos B}{\cos C} = \dfrac{c - b\cos A}{b - c\cos A}$ (on condition that $\hat{C} \neq 90°$). (P) 2.3 $\tan A = \dfrac{a\sin C}{b - a\cos C}$ (on condition that $\hat{A} \neq 90°$). (P) 2.4 $a + b + c = (b + c)\cos A + (c + a)\cos B + (a + b)\cos C$. (P)
1	**Functions: polynomials**	Factorise third-degree polynomials. Apply the Remainder and Factor Theorems to polynomials of degree at most 3 (no proofs required).	Any method may be used to factorise third degree polynomials but it should include examples which require the Factor Theorem. **Examples:** 1. Solve for x: $x^3 + 8x^2 + 17x + 10 = 0$ (R) 2. If $a(x) = -2x^3 + px - 1$ is divided by $x - 1$, the remainder is $-\frac{1}{2}$. Determine the value of p. (P)

GRADE 12: TERM 2			
No. of Weeks	**Topic**	**Curriculum statement**	**Clarification**
3	**Differential Calculus**	1. An intuitive understanding of the limit concept, in the context of approximating the rate of change or gradient of a function at a point. 2. Use limits to define the derivative of a function f at any x : $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$. Generalise to find the derivative of f at any point x in the domain of f , i.e., define the derivative function $f'(x)$ of the function $f(x)$. Understand intuitively that $f'(a)$ is the gradient of the tangent to the graph of f at the point with x -coordinate a . 3. Using the definition (first principle), find the derivative, $f'(x)$ for *a*, *b* and c constants: 3.1 $f(x) = ax^2 + bx + c$; 3.2 $f(x) = ax^3$; 3.3 $f(x) = \frac{a}{x}$ for $x \neq 0$; 3.4 $f(x) = c$.	**Comment:** Differentiation from first principles will be examined on any of the types described in 3.1, 3.2 and 3.3. **Examples:** 1. In each of the following cases, find the derivative of $f(x)$ at the point where $x = -1$, using the definition of the derivative: 1.1 $f(x) = x^2 + 2$ (R) 1.2 $f(x) = \frac{1}{2}x^2 + x - 2$ (R) 1.3 $f(x) = -x^3$ (R) 1.4 $f(x) = -\frac{1}{x}$ for $x \neq 0$; (C) for x ≠ 0 **Caution**: Care should be taken not to apply the sum rule for differentiation 4.1 in a similar way to products. • Determine $\frac{d}{dx}((x+1)(x-1))$. • Determine $\frac{d}{dx}(x+1) \times \frac{d}{dx}(x-1)$. • Write down your observation. 2. Use differentiation rules to do the following: 2.1 Determine $f'(x)$ if $f(x) = (x+2)^2$ (R) 2.2 Determine $f'(x)$ if $f(x) = \frac{(x+2)^3}{\sqrt{x}}$ (C) (P)

GRADE 12: TERM 2

No. of Weeks	Topic	Curriculum statement	Clarification
		4. Use the formula $\frac{d}{dx}(ax^n) = anx^{n-1}$, (for any real number n) together with the rules 4.1 $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$ and 4.2 $\frac{d}{dx}[kf(x)] = k\frac{d}{dx}[f(x)]$ (k a constant) 5. Find equations of tangents to graphs of functions. 6. Introduce the second derivative of $f(x)$ $f''(x) = \frac{d}{dx}(f'(x))$ $\cancel{\text{of } f(x)}$ and how it determines the concavity of a function. 7. Sketch graphs of cubic polynomial functions using differentiation to determine the co-ordinate of stationary points, and points of inflection (where concavity changes). Also, determine the x-intercepts of the graph using the factor theorem and other techniques. 8. Solve practical problems concerning optimisation and rate of change, including calculus of motion.	2.3 Determine $\frac{dy}{dt}$ if $y = \frac{t^2-1}{2t+2}$ (R) 2.4 Determine $f'(\theta)$ if $f(\theta) = (\theta^{3/2} - 3\theta^{-1/2})^2$ (C) 3. Determine the equation of the tangent to the graph defined by $y = (2x+1)^2(x+2)$ where $x = \frac{3}{4}$. (C) 4. Sketch the graph defined by $y = -x^3 + 4x^2 - x$ by: 4.1 finding the intercepts with the axes; (C) 4.2 finding maxima, minima and the co-ordinate of the point of inflection; (R) 4.3 looking at the behaviour of y as $x \to \infty$ and as $x \to -\infty$. (P) (Remember: To understand points of inflection, an understanding of concavity is necessary. This is where the second derivative plays a role.) 5. The radius of the base of a circular cylindrical can is x cm, and its volume is 430 cm^3. 5.1 Determine the height of the can in terms of x . (R) 5.2 Determine the area of the material needed to manufacture the can (that is, determine the total surface area of the can) in terms of x . (C) 5.3 Determine the value of x for which the least amount of material is needed to manufacture such a can. (C) 5.4 If the cost of the material is R500 per m^2 , what is the cost of the cheapest can (labour excluded)? (P)

GRADE 12: TERM 2			
No. of Weeks	**Topic**	**Curriculum statement**	**Clarification**
2	**Analytical Geometry**	1. The equation $(x-a)^2+(y-b)^2=r^2$ defines a circle with radius r and centre $(a;b)$. 2. Determination of the equation of a tangent to a given circle.	**Examples:** 1. Determine the equation of the circle with centre $(-1;2)$ and radius $\sqrt{6}$ (K) 2. Determine the equation of the circle which has the line segment with endpoints $(5;3)$ and $(-3;6)$ as diameter. (R) 3. Determine the equation of a circle with a radius of 6 units, which intersects the x-axis at $(-2;0)$ and the y-axis at $(0;3)$. How many such circles are there? (P) 4. Determine the equation of the tangent that touches the circle defined by $x^2-2x+y^2+4y=5$ at the point $(-2;-1)$. (C) 5. The line with the equation $y=x+2$ intersects the circle defined by $x^2+y^2=20$ at A and B. 5.1 Determine the co-ordinates of A and B. (R) 5.2 Determine the length of chord AB. (K) 5.3 Determine the co-ordinates of M, the midpoint of AB. (K) 5.4 Show that $OM \perp AB$, where O is the origin. (C) 5.5 Determine the equations of the tangents to the circle at the points A and B. (C) 5.6 Determine the co-ordinates of the point C where the two tangents in 5.5 intersect. (C) 5.7 Verify that $CA=CB$. (R) 5.8 Determine the equations of the two tangents to the circle, both parallel to the line with the equation $y=-2x+4$. (P)
3	**Mid-Year Examinations**		

Assessment term 2:

1. Assignment (at least 50 marks)
2. Examination (300 marks)

GRADE 12 TERM 3			
No. of Weeks	**Topic**	**Curriculum statement**	**Clarification**
2	**Euclidean Geometry**	1. Revise earlier work on the necessary and sufficient conditions for polygons to be similar. 2. Prove (accepting results established in earlier grades): • that a line drawn parallel to one side of a triangle divides the other two sides proportionally (and the Mid-point Theorem as a special case of this theorem) ; • that equiangular triangles are similar; • that triangles with sides in proportion are similar; and • the Pythagorean Theorem by similar triangles.	**Example:** Consider a right triangle ABC with $\hat{B} = 90°$. Let $BC = a$ and $AB = c$. Let D be on AC such that $BD \perp AC$. Determine the length of BD in terms of a and c. (P)
2	**Statistics (regression and correlation)**	1. Revise symmetric and skewed data. 2. Use statistical summaries, scatterplots, regression (in particular the least squares regression line) and correlation to analyse and make meaningful comments on the context associated with given bivariate data, including interpolation, extrapolation and discussions on skewness.	**Example:** The following table summarises the number of revolutions x (per minute) and the corresponding power output y (horse power) of a Diesel engine: <table><tr><td>x</td><td>400</td><td>500</td><td>600</td><td>700</td><td>750</td></tr><tr><td>y</td><td>580</td><td>1030</td><td>1420</td><td>1880</td><td>2100</td></tr></table>1. Find the least squares regression line $y = a + bx$ (K) 2. Use this line to estimate the power output when the engine runs at 800m. (R) 3. Roughly how fast is the engine running when it has an output of 1200 horse power? (R)

GRADE 12 TERM 3			
No. of Weeks	**Topic**	**Curriculum statement**	**Clarification**
2	**Counting and probability**	1. Revise: • dependent and independent events; • the product rule for independent events: P(A and B) = P(A) × P(B). • the sum rule for mutually exclusive events A and B: $P(A \text{ or } B) = P(A) + P(B)$ • the identity: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ • the complementary rule: $P(\text{ not } A) = 1 - P(A)$ 2. Probability problems using Venn diagrams, tree diagrams, two-way contingency tables and other techniques to solve probability problems (where events are not necessarily independent). 3. Apply the fundamental counting principle to solve probability problems.	**Examples:** 1. How many three-character codes can be formed if the first character must be a letter and the second two characters must be digits? (K) 2. What is the probability that a random arrangement of the letters BAFANA starts and ends with an 'A'? (R) 3. A drawer contains twenty envelopes. Eight of the envelopes each contain five blue and three red sheets of paper. The other twelve envelopes each contain six blue and two red sheets of paper. One envelope is chosen at random. A sheet of paper is chosen at random from it. What is the probability that this sheet of paper is red? (C) 4. Assuming that it is equally likely to be born in any of the 12 months of the year, what is the probability that in a group of six, at least two people have birthdays in the same month? (P)
3	**Examinations / Revision**		

Assessment Term 3:

1. Test (at least 50 marks)
2. Preliminary examinations (300 marks)

Important:

Take note that at least one of the examinations in terms 2 and 3 must consist of two three-hour papers with the same or very similar structure to the final NSC papers. The other examination can be replaced by tests on relevant sections.

IMPORTANT ADVICE FOR MASTERING MATHS

Don't focus on what you *haven't* done in the past.
Put that behind you and start today! Give it your all – it is well worth it!

❶ TIMETABLE / PLANNING

Draw up a timetable of study times. Revise your schedule from time to time to ensure optimum focus and awareness of time. Motivation will not be a problem once you've done this, because you will see that you need to use every minute!

❷ ROUTINE

Routine is really important. Start early in the morning, at the same time every day, and don't work beyond 11 at night.

Arrange some 1 hour and some 2 hour sessions on particular subjects.

Schedule more difficult pieces of work for early in the day and easier bits for later when you're tired.

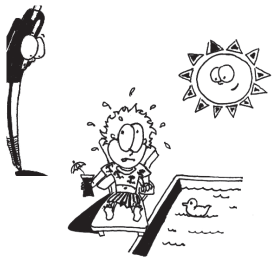
Reward yourself with an early night now and again!

Allow some time for physical exercise – at least ½ hour a day. Any sport or walking, jogging (or skipping when it rains) will improve your concentration.

❸ 'NO-NO'S'

Limit the time you spend

- on your phone
- at the television
- on Facebook and any other social networks
- in the sun



All these activities break down your commitment, focus and energy.

❹ WORK FOCUS

Don't worry about marks. Just focus on the work and the marks will take care of themselves. Worrying is tiring and time-wasting and gets in the way of your progress! Your marks will gradually improve if you work consistently.

❺ YOUR APPROACH

The most important thing of all is to remain positive. Some times will be tough, some exams WILL BE TOUGH, but in the end, your results *will* reflect all the effort that you have put in.

- **Despair can destroy your Mathematics**. Mathematics should be taken on as a continual challenge (or not at all!). Teach your ego to suffer the 'knocks' which it may receive – like a poor test result. Instead of being negative about your mistakes (e.g. 'I'll never be able to do these sums'), **learn** from them. As you address each one, they will **help** you to understand the work and do better next time!

- **Work with a friend occasionally**. Discussing Mathematics makes it alive and enjoyable.

❻ A GREAT GENERAL STUDY TIP

Don't just read through work! Study a section and then, on your own, write down all you can remember. Knowing that you're going to do this makes you study in a logical, alert way. You're then only left to learn the few things which you left out. This applies to all subjects. In this text, read the explanations very carefully and actively, trying all worked examples yourself as you master each topic. A subject like Maths also requires you to practise and apply the concepts regularly.

❼ ABOUT THE MATHS

- **Try each problem on your own first – no matter how inadequately – before consulting the answer**. It is only by **encountering** the difficulties which you personally have that you will be able, firstly, to **pin-point them**, and then, secondly, to understand and **rectify them** and then make sure you don't make them again!!

- **Learn to keep asking yourself 'why'?**
  It is when you learn to REASON that you really start enjoying Maths and, quite coincidentally, start doing well at it!!
  Answers are by no means the most important thing in Mathematics.
  When you've done a problem, don't be satisfied only to check the answer.

  Check also on your **layout** and reasoning (logic). Systematic, to-the-point, logical and neat presentation is **very** important.

- **Revise earlier work as often as possible**. Set up a revision plan with at least one session a week for this purpose. Familiarity is the key to success in Maths!

❽ EXAM PREPARATION

The best way to prepare for your exam is to start early – in fact, on the first day of the year!!! Working past papers is excellent preparation for any exam – and The Answer Series provides these – but, only when you're ready. Focus on WORKING ON ONE TOPIC AT A TIME first. It is the most effective way to improve, particularly as you will build up your confidence this way. The Answer Series provides thorough topic treatment for all subjects, enabling you to cover all aspects of each topic, from the basics to the top level questions. Thereafter, working past papers is a worthwhile and rewarding exercise.

❾ THE EXAMS

Finally, for the exams themselves, make sure you have all you need (e.g. your calculator, ruler, etc.) and don't allow yourself to be upset by panicking friends. Plan your time in the exam well – allowing some time to check at the end. Whatever you do, don't allow yourself to get stuck on any difficult issues in the exam. Move on, and rather come back to problem questions if you have time left. If you're finding an exam difficult, just continue to do your absolute best right until the end! Partial answers can earn marks.

We wish you the best of luck in your studies and hope that this book will be the key to your success – enjoy it!!

Anne Eadie and Gretel Lampe

WEIGHTINGS FOR 2023 & S024 ACCORDING TO 2023/24 ATPS

a) Number of Assessment Tasks and Weighting:

Learners are expected to have seven (7) formal assessment tasks for their school-based assessment. The number of tasks and their weighting are listed below:

		GRADE 10		GRADE 11		GRADE 12	
		TASKS	**WEIGHT (%)**	**TASKS**	**WEIGHT (%)**	**TASKS**	**WEIGHT (%)**
School-based Assessment	**Term 1**	**Project /Investigation** **Test**	**15** **14**	**Project /Investigation** **Test**	**15** **14**	**Project /Investigation** **Test** **Assignment**	**15** **15** **10**
	Term 2	**Assignment** **Mid-Year Examination**	**15** **14**	**Assignment** **Mid-Year Examination**	**15** **14**	**Assignment** **Mid-Year Examination**	**15** **15**
	Term 3	**Test** **Test**	**14** **14**	**Test** **Test**	**14** **14**	**Test** **Trial Examination**	**15** **25**
	Term 4	**Test**	**14**	**Test**	**14**		
School-based Assessment mark			**100**		**100**		**100**
School-based Assessment mark (as % of promotion mark)			**25%**		**25%**		**25%**
End-of-year examinations			**75%**		**75%**		
Promotion mark			**100%**		**100%**		

Note:

- Although the project/investigation is indicated in the first term, it could be scheduled in term 2. Only **ONE** project/investigation should be set per year.
- Tests should be at least ONE hour long and count at least 50 marks.
- Project or investigation must contribute 25% of term 1 marks while the test marks contribute 75% of the term 1 marks.
  The combination (25% and 75%) of the marks must appear in the learner's report.
- Graphic and programmable calculators are not allowed (for example, to factorise $a^2 - b^2 = (a-b)(a+b)$, or to find roots of equations) will be allowed. Calculators should only be used to perform standard numerical computations and to verify calculations by hand.
- **Formula sheet must not be provided for tests, but only for final examinations in Grades 10 and 11.**