INDEPENDENT EXAMINATIONS BOARD

INTERNATIONAL SECONDARY CERTIFICATE (IEB)

FURTHER STUDIES MATHEMATICS (STANDARD & EXTENDED LEVEL)

GRADES 10–12

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1. WHAT KIND OF STUDENT DOES OUR WORLD NEED?

As our history would suggest, the IEB supports democratic principles, contributing to the building of a fair and equitable education landscape across the globe. Furthermore, the IEB is committed to building the reputation of educational offerings by demonstrating that the region produces exceptional students through a quality assured assessment process that is well-recognised and respected. As an organ of civil society, the IEB believes it has an important role in providing commentary on, actively participating in and contributing constructively to educational debate.

The IEB is clear that education is as much a philosophical undertaking, underpinned by social and educational values, as it is a technical exercise carried out by a professional body of experts. The IEB's desire to make the philosophical underpinnings of the IEB's work explicit led to the formulation of *The Intentional Educational Beliefs of the IEB.*

In the statement of its intentional beliefs, the IEB explains what it sees as the educational purpose of an organisation such as the IEB and outlines the learning that it values. Our complex world and the complexity of our societies in this region demand citizens who are courageous, able to determine what we need to create a better world and more importantly, how to bring that better world into being. In its assessments and related activities, the IEB encourages learning institutions and teachers to provide students with the opportunities to explore topics, participate in activities and be exposed to discussions and debates that develop the kinds of skills and attitudes we believe every modern member of society needs to master. The IEB makes every effort to ensure that our assessments are globally competitive, giving our students access to assessments that compete at the highest level.

It is this belief in the quality of students from nations in Africa and further afield that inspires the IEB to try and influence the educational experience beyond assessment.

With this as the backdrop, the IEB presents the following statement of what it stands for as an educational institution.

1.1 The Intentional Educational Beliefs of the IEB

The **vision** of the IEB is to advance quality teaching and learning through an assessment process of integrity, innovation and international comparability.

The IEB is cognisant that assessment is essential in developing students who are responsible citizens within their own communities and at the same time, are able to negotiate the challenges of a global world. Through our assessments, we are motivated to develop students who are:

- critical users of information
- ethical reasoners
- problem solvers
- creative and reflective thinkers
- lifelong students
- society members respectful of diversity
- active citizens who are committed to upholding democratic principles and the wellbeing of all people.

The IEB supports the position that actively promoting quality education for every student is fundamental in establishing just, open societies based on democratic values, social justice and fundamental human rights, in which cultural diversity is appreciated and embraced.

The IEB's **mission** is to make a significant, on-going contribution to human resource development through the design, delivery and promotion of a wide range of high quality, affordable assessment products and services to all sectors.

The work of the IEB is underpinned by a commitment to the following **values**: Integrity, respect, commitment and service, professionalism, communication, quality, teamwork.

1.2 Broad Learning Outcomes of the curriculum

The overriding objective in the education that underpins the ISC then is to develop a citizen who has skills, knowledge and attitudes that

- enable a thorough understanding and caring for self and others;
- enable and encourage life-long learning;
- enable contribution to the well-being of society through work;
- enable and encourage responsible and constructive civic participation.

Each subject curriculum is designed to support the development of thinking capabilities, personal attributes and mindful attitudes.

Thinking capabilities include but are not limited to critical thinking, problem solving, understanding of the origins of knowledge and the impact of phenomena through research, specific disciplinary skills and knowledge, integration of knowledge and skills across disciplines and ethical reasoning. These all require active curiosity with a commitment and determination to learn.

Personal attributes and mindful attitudes include but are not limited to integrity, courage, awareness of self and others, generosity of spirit and openness to engage. Mindfulness encompasses appropriate personal attributes which include but are not limited to responsiveness, responsibility, self-belief and reflection, resourcefulness, perseverance and openness to change.

1.3 Specific Outcomes of the qualification

The Specific Outcomes of the qualification are as follows:

- **Communicate** in at least two languages: the student is able to demonstrate the ability to listen carefully, speak, read and write. This communication outcome also presupposes basic quantitative reasoning, visual literacy and computer skills as one can no longer be regarded as literate without these.
- **Co-operate** with others in situations that require a group to work successfully together towards a common task or goal: the student is able to demonstrate the ability to elicit the views of others, to resolve disagreements, to display tolerance for the views of others, and to help the group come to valid conclusions, including consideration of ethical factors. The student is able to show an awareness of his/her own behaviour in interacting with a range of other people and able to assess his/her effect on the group. This outcome also includes the ability to acknowledge conflict and to use strategies that help to resolve and neutralise conflict.
- **Think critically, analyse**, **and evaluate** multiple sources of information to reach an opinion: the student is able to establish meaning, to show an understanding of a phenomenon and the relationship and the value of those parts to each other and the whole, and draw reasonable inferences from that understanding. Furthermore, the student is able to display the ability to draw on their own experience, knowledge, reasoning and beliefs to form well-substantiated personal responses that engage critically with ideas. This outcome extends into being able to understand the framework of a particular discipline, the related concepts and the ability to draw reasonable inferences within that discipline.
- **Solve problems** in a variety of contexts and disciplines: this outcome requires that the student is able to

  - define the problem
  - devise a solution identifying the necessary resources
  - explain the thinking processes to solve the problem
  - make recommendations
  - take a decision regarding the most suitable course of action
  - implement the plan of action and
  - evaluate the effectiveness of the process.

Problem-solving includes considering the ethical and environmental implications of proposed plans of action. A key aspect of problem-solving is the ability to apply known concepts in unfamiliar contexts.

- **Discern values and respect diversity:** the student is able to demonstrate an understanding of the values that are represented in her/his own and others' value systems; to recognise the ethical and/or moral issues that present themselves as part of personal decisions as well as in societal, national and global ones, and to take personal responsibility where this is required/possible.
- **Contribute to one's society:** this outcome, in conjunction with others, serves to develop active citizens who are committed to upholding democratic values of social justice and fundamental human rights, the assumption of responsibility for ensuring the wellbeing of all people and the environment. For this outcome the student is able to identify significant community issues and consider their own knowledge, skills and attitudes before making informed choices about volunteering for and participating in community activities/civic issues.
- **Develop a global consciousness:** this outcome, in conjunction with others, serves to develop active citizens who are committed to upholding democratic values of social justice and fundamental human rights, the assumption of responsibility for ensuring the wellbeing of all people and the environment. For this outcome the student is able to identify the factors that influence his/her own opinions; explore the notions of diversity, empathy and inter-connectedness – ecological, economic, social, political; recognise the need for understanding the views of others, and begin to articulate new global perspectives, informed by the disciplines being studied.
- **Engage with creative activity:** the student is able to demonstrate the ability to make informed choices, considering aesthetics and interpretive options, and to provide a reasoned explanation for those choices. During the process of study these choices become refined by engaging with the theory of the discipline and a broader awareness of the relevant social and cultural frameworks. Creativity in approach and exposure to unusual ways of seeing informs the learning and assessment of all aspects of the curriculum.
- **Manage oneself and one's work:** the student is able to provide a thoughtful growing awareness of their own identity, of the importance of self-management, of being mindful and healthy and of the identities and needs of others.

1.4 Linking the levels of the curriculum

The links between the different levels inherent in the qualification are as follows:

- the Intentional Educational Beliefs are embedded in the Broad Learning Outcomes;
- the Broad Learning Outcomes are in turn embedded in the Qualification Specific Outcomes;
- the Qualification Specific Outcomes are embedded in the Discipline and Subject Specific Outcomes.

The embedding of the Intentional Educational Beliefs in the Broad Learning Outcomes could be seen as follows:

- Broad Learning Outcome 1: enable a thorough understanding and caring for self and others; this outcome accommodates developing the following traits in the Intentional Educational Beliefs
  - ethical reasoners
  - creative and reflective thinkers
  - society members respectful of diversity
  - active citizens who are committed to upholding democratic principles and the wellbeing of all people.
- Broad Learning Outcome 2: enable and encourage life-long learning; this outcome accommodates developing the following traits in the Intentional Educational Beliefs
  - lifelong students
  - critical users of information
  - problem solvers
  - creative and reflective thinkers
- Broad Learning Outcome 3: enable contribution to the well-being of society through work; this outcome accommodates developing the following traits in the Intentional Educational Beliefs
  - ethical reasoners
  - critical users of information
  - problem solvers
  - creative and reflective thinkers
- Broad Learning Outcome 4: enable and encourage responsible and constructive civic participation; this outcome accommodates the following traits in the Intentional Educational Beliefs
  - society members respectful of diversity
  - active citizens who are committed to upholding democratic principles and the wellbeing of all people.

All the Broad Learning Outcomes are embedded in each of the Qualification Specific Outcomes:

- Qualification Specific outcome 1: Communicate
- Qualification Specific outcome 2: Co-operate
- Qualification Specific outcome 3: Think critically, analyse, and evaluate
- Qualification Specific outcome 4: Solve problems
- Qualification Specific outcome 5: Discern values and respect diversity
- Qualification Specific outcome 6: Contribute to one's society
- Qualification Specific outcome 7: Develop a global consciousness
- Qualification Specific outcome 8: Engage with creative activity
- Qualification Specific outcome 9: Manage oneself and one's work

Representation of the embedded outcomes at each level of the Curriculum Framework for the ISC

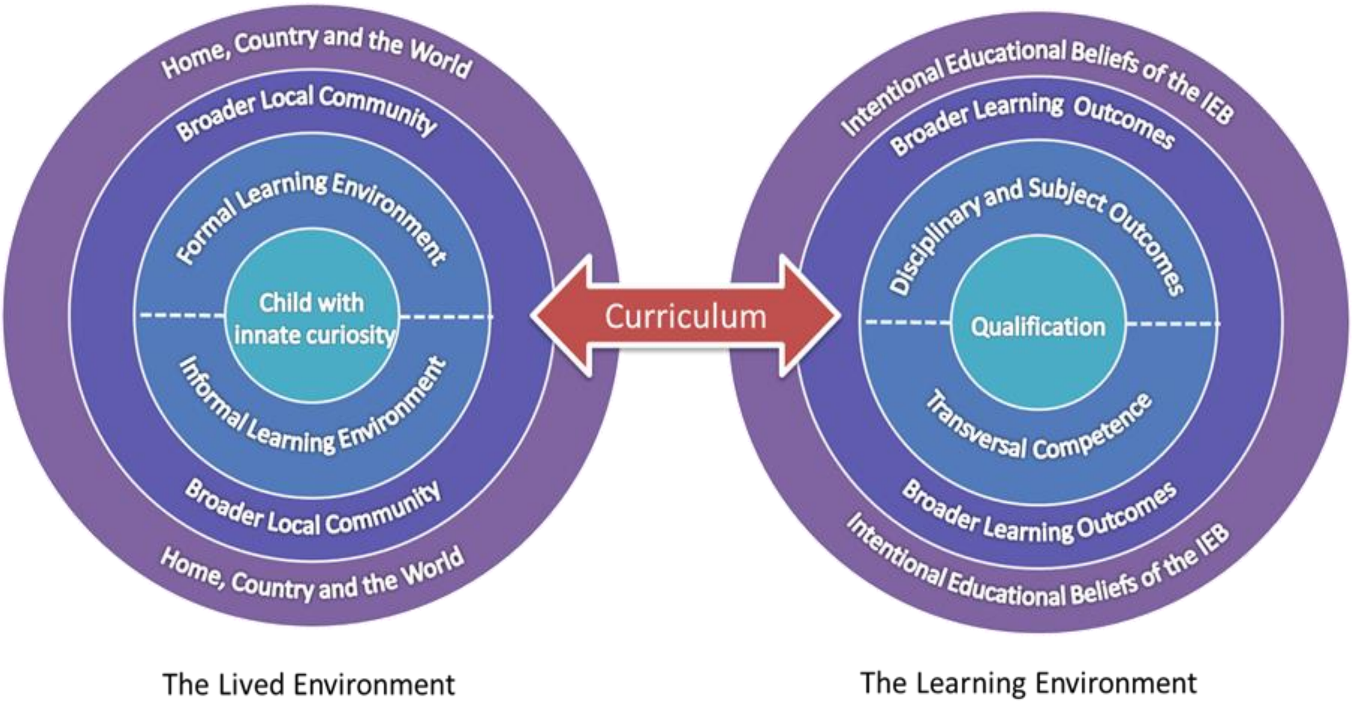
The ISC as a qualification assesses, as far as possible, the extent to which the intentions of its underpinning curriculum have been met. Details of how the learning outcomes expected in the qualification are met through the subject syllabi are explained in the individual subject syllabi and assessment guidelines.

Individual teachers may see a different alignment of the outcomes. However, the key point is that the rigour and rationale for each discipline and subject takes cognisance of the tight-knit cohesion between the educational philosophy espoused by the IEB, broad learning outcomes and the specific intentions of the qualification and the curriculum.

2. THE CURRICULUM FRAMEWORK

The Curriculum Framework maps the relationship between a child's lived environment and experiences and the learning environment.

Representation of the Curriculum Framework for the ISC



Learning begins at birth. Children are born with innate curiosity and hence learn through constantly questioning the world in which they find themselves.

Opportunities to find answers to their questions are provided for by various aspects of the lived environment of the child and also the learning environment of the child.

The informal learning environment is where the students are exposed to learning opportunities in a less structured way among people who are close to them e.g. the family, extended family, close friends and community.

The formal learning environment is where students formally experience teaching and learning in a purposeful structured way.

The formal learning environment is primarily where the responsibility for disciplinary and subject learning outcomes lies while it is within both the informal learning environment and the formal learning environment where transversal competences are developed. Hence insofar as specific subjects contribute to the development of transversal competence, teachers are obliged to create overt learning opportunities to actively emphasise relevant transversal competences. It is not sufficient to leave the development of these competences to the informal learning environment.

The Broader Local Community lived environment is where the child experiences learning outside the home, e.g. sports clubs, hobbies, community events, societies such as Boy Scouts/Girl Guides, extra-mural activities at the school, home or place of religious observance.

The Home Country and the World lived environment is where the child is exposed to national and global issues, outside of their local community e.g. universal human commonalities and differences, global and national problems, features of agreement and disagreement on a global level.

The responsibility for the development of broader learning outcomes is shared between the formal learning environment and the community within which the child lives. It is in the formal learning environment where learning opportunities are provided to raise awareness and find strategies to develop the broader learning outcomes. The broader local community provides the environment within which the broader learning outcomes are practiced. Hence insofar as specific subjects contribute to raising awareness and developing strategies to develop these broader educational outcomes, teachers are obliged to create overt learning opportunities to actively emphasise the broader learning outcomes. While the community within which the child lives may also provide learning opportunities for the child to develop the broader learning outcomes, the local community provides the environment within which the broader learning outcomes are practised.

Both the learning environment and the home country and the world at large, within which the child lives provide learning opportunities for the child to develop the outcomes detailed in the Intentional Educational Beliefs of the IEB. The child's own experiences or awareness of global issues that are locally relevant provide the stimulus for experience of and engagement with the complexities inherent in the Intentional Educational Beliefs of the IEB.

The learning environment itself consists of the qualification which the child achieves. The qualification incorporates Discipline and Subject Specific Outcomes and more generic competences. These in turn incorporate the Broader Learning Outcomes which finally incorporate the Intentional Educational Beliefs of the IEB.

The curriculum is the vehicle through which a child's lived environment is connected to the learning environment, the formal learning pathway through which a child's learning achievements are developed and finally recognised by the award of a qualification.

The documentation that informs the learning chain is as follows:

The International Secondary Certificate qualification	describes the qualification that is awarded at the culmination of formal study
The Curriculum Framework	describes the intended outcomes of the learning
Individual Subject Syllabi	describes the discipline and subject-specific outcomes for subjects in the Curriculum Framework
Teaching Plan for each class	describes the progression of lessons to ensure subject syllabus and curriculum coverage
Assessment Plan for each class	describes the formal assessment activities that will inform feedback on the progress of the student

3. APPROACH TO ASSESSMENT

The IEB believes that it is through the assessment of the curriculum that its true worth and value as an educating tool can be experienced. The way in which a question is asked will either encourage the 'opening up' of young minds to see new ideas and possibilities or it will focus the mind narrowly onto the facts taught and presumably learnt during the year. The approach in assessment can either offer students an opportunity to express their own opinions and show that they have used sound thinking skills and a reliable knowledge base to come to their conclusions or at the other end of the scale, it will encourage the re-gurgitation of learnt facts and opinions. The IEB aims to harness the positive impact of good assessment techniques on learning and teaching and through its instruments, open the minds of teachers and students to the higher order thinking skills required in our complex world. In-depth study and rigorous assessment prepare students for the demands of tertiary study, including the development of the skills of prioritisation and perseverance.

Assessment in all its forms is a fundamental aspect of learning and teaching and hence should serve to inform:

- students of the degree to which they have mastered the skills and knowledge that has been taught;
- students of the degree to which they have mastered transversal skills;
- teachers of the strengths and gaps of the group of students as a whole thereby providing the teacher with information about where teaching of a skill or concept may need a different approach and hence plan an appropriate intervention;
- teachers of the strengths and gaps of individual students thereby enabling a targeted intervention for specific students.

Traditional assessment methods including tests and examinations are able to serve these purposes. However, the term assessment encompasses a range of other methodologies which serve to provide opportunities for students to display:

- the skills and knowledge they have acquired over a period of time in a variety of ways thereby exposing students to a range of experiences and ways of communicating their competence[1];

- skills and, as appropriate, knowledge that cannot be assessed in a written test or formal examination but is, however, critical to the discipline. These assessment methodologies include oral assessment particularly in the study of languages, but is also a useful assessment method to build confidence in speaking in a more formal environment in other subjects; practical competence in the arts, e.g. visual, dramatic, dance, in the sciences, in semi-vocational subjects; digital competence and time and inspiration for creativity including creative writing;

While assessment of these skills may well form part of the final assessment, their development is the result of ongoing experiences, opportunities and exposure to the challenges, a necessary part of preparation for the final assessment.

Constant exposure to a range of assessment methodologies, including tests and examinations, is essential so that students are well-prepared to face examinations and are also given ample opportunity to develop and display their competence.

To ensure that students are absolutely clear about what is expected of them, each assessment task ought to identify the criteria being assessed in order to create a transparent learning environment. Furthermore, the associated marking guidelines or rubric, should be shared with students. They should be encouraged to reflect on their achievement in order to improve.

Assessment at school level, unless expressly required by the Means of Assessment in the Subject Curricula, is a professional activity and should serve a formative function; hence it will not contribute towards the final examination mark. However, the IEB will require a mark to be submitted by the school for each student in order for the IEB to ascertain whether an educational institution's standards align with the standards expected by the IEB's assessments. The Means of Assessment provide a suggested assessment plan which may be used to arrive at the mark to be submitted to the IEB. This plan underscores the fact that the mark to be submitted should not consist of only the result in a preliminary examination.

[1] Competence constitutes mastery of the knowledge and skills that are required

The IEB follows an authentic approach to assessment, setting as far as possible, contextualised problems that require the application of skills and knowledge, encouraging the application of the 'known' within an unfamiliar context. Examination papers are developed by experienced examiners, moderated internally by very experienced people who have had examining experience previously and the papers are then externally moderated. The marking is done by qualified teachers in the IEB teaching community, with appropriate structures to ensure moderation of the marking process. Appropriate structures are in place to oversee standardisation of results, as necessary, and any irregularities that may have occurred. Normal appeal processes are available.

The assessment tasks in all subjects align with the aims and objectives of the curriculum and the assessment takes cognisance of the explicit outcomes for the subjects in the ISC qualification itself. This naturally leads to a range of assessment tasks and activities which may look rather different from more conventional forms of assessment since their focus is to develop, instil and assess a more searching range of thinking skills, processes and values. Guidelines for each subject provide the weighting of the cognitive skills as they are reflected in the final assessments as well as the weighting of the various topics, which broadly aligns with the amount of time allocated to the teaching and learning process. As is the IEB's practice, exemplars of the kinds of assessment are made available as part of its commitment to a publicly recognisable standard.

The assessment requirements detail the expectations in the written external examination, which may consist of one or more papers, in each subject. Depending on the subject area and the curriculum requirements, students will need to develop oral competence, performance competence and/or practical competence which will be assessed and contribute towards their final achievement of the qualification. Finally, guidance is provided for an assessment program and calculations of the mark to be submitted to the IEB, which provide an opportunity to explore assessment methodologies that do not fit neatly into tests and examination-type assessment.

Best practice requires that teachers and students are fully aware of the criteria against which their performance will be judged. Hence the IEB provides teachers with the analysis grids, which detail the cognitive demand, the level of difficulty and the curriculum coverage of each question in the examination paper, as well as the marking guidelines. The IEB also provides detailed reports per institution of how their students performed in each question, thus encouraging the development of focussed interventions at institution level. The IEB advocates that the criteria associated with all site-based assessments be shared with students, encouraging not only a transparent learning environment but also providing valuable feedback for students when they reflect on their efforts.

4. RESOURCES

4.1 What kind of teacher does the ISC need

The teachers who facilitate the learning required for the ISC must

- be appropriately qualified to teach the subject they are teaching and as professionals, keep up to date with developments in education generally and within their discipline and subject areas specifically;
- be mindful of the Intentional Educational Beliefs as well as the broad outcomes of the qualification and build these outcomes into the lessons they prepare;
- be mediators of learning, mindful of the range of theories of learning and provide opportunities in the classroom that harness the positive aspects of each theory;
- be mindful of the contention that may arise during discussion in the classroom and hence be sensitive and caring in managing different points of view and ideas;
- have competence in managing the learning process including planning, assessment and monitoring progress;
- have competence in developing assessment tasks that are appropriate for the outcomes being assessed and appropriately judging the evidence of learning to positively impact on further learning;
- be mindful of the power they hold and the impact they have in influencing the learning experience of each student.

Obviously, the teacher should operate in a functional, well-managed school where there are clear teaching and assessment plans that overtly address the curriculum and specific syllabus requirements, thereby providing a bridge between the intended outcomes of the curriculum and syllabus and the implemented curriculum, i.e. it is the 'how' in the learning chain.

In respect of class size, Andreas Schleicher noted as follows:

The highest performing education systems in PISA tend to systematically prioritise the quality of teachers over the size of classes, that is, whenever they have to make c choice between a smaller class and a better teacher, they go for the latter. Rather than in small classes, they invest in attractive teacher working conditions and careers, ongoing professional development and a balance in working time that allows teachers to contribute to their profession and to grow in their careers.[2]

[2] Debunking Education Myths, Andreas Schleicher at https://www.teachermagazine.com.au/columnists/andreas-schleicher/debunking-education-myths

This is a decision that needs to be made based on the context in which the educational institution operates. The use of teacher aids is a useful strategy in multi-lingual classrooms.

4.2 Textbooks and teacher support

It is important for the IEB to clarify its position in respect of textbooks. The IEB has never prescribed textbooks for any subject and has no intention of doing so in the foreseeable future.

The primary reason for this stance is that the IEB believes that teachers, as professionals, should have the ability to determine which books, available on the market, are an appropriate complement to their teaching style and approach to the subject. Furthermore, the IEB encourages teachers to make use of a range of textbooks and learning resources to ensure that their students are exposed to alternate approaches to a concept, a variety of interpretations and theories about an issue and a range of questioning techniques.

These are educational principles on which the IEB bases its position. There are numerous financial and ethical issues that could be aligned to this position. However, the point is that the IEB has taken a firm educational stand on the matter.

Despite the IEB having no prescribed textbooks, there are a number of publishers that have developed textbooks that align with the IEB Subject Assessment Guidelines. There is hence no shortage of resource material and textbooks.

The IEB hosts teacher conferences annually and also has well-run cluster groups which provide ample opportunity for networking among teachers. They share their experiences of and opinions about various textbooks. This networking is encouraged further through cluster groups where a group of schools, generally in close geographical proximity, meet regularly with the intention for professional development. In addition, the assessment specialists are able to provide guidance. The IEB website has resources including past examination papers and marking guidelines, for teachers to use.

4.3 Information Communication Technology (ICT)

Just as the IEB encourages teachers to make use of a variety of textbooks, so too the teacher ought to make use of a variety of other learning resources that support specific learning activities. These include a range of sources, e.g. the media – newspapers, sound clips; the internet – Khan Academy, TED talks, Siyavula; video or digital material. The intention of every teacher should be to develop a bank of resources that make learning interesting and sufficiently varied to cater for individual learning approaches in the particular context within which the learning takes place.

The child of this century is often referred to as "tech-savvy". They know what information communication technology (ICT) is available and inevitably know how to use each one.

However, technical proficiency does not necessarily mean digital literacy. Digital literacy is defined by the American Library Association as "the ability to use information and communication technologies to find, evaluate, create, and communicate information, requiring both cognitive and technical skills".

Key learning outcomes include the ability to judge the appropriateness of tone in online communication and the ability to use ICT as a useful educational tool. Digital literacy encompasses digital citizenship, i.e. the consideration of ethical and social impacts of using technologies, identification of "fake news" and inappropriate information sources.

Clearly students of today need to understand the tools and skills necessary to process the enormous quantity of information they encounter on a daily basis. This is a transversal skill. However, the relevance of ICT and digital literacy varies from subject area to subject area. Hence embedded in each subject syllabus is an awareness of how ICT impacts on the subject area and how it can be used to advance a positive and effective learning culture. Ideally the learning environment should provide students with access to computers and the internet.

4.4 Time allocation for the ISC

The time allocations below are *indicative* of the amount of time suggested per subject. Educational institutions, however, have some latitude regarding how the timetabling of the subjects is achieved, given the consideration of the stipulated length of the school day and the number of teaching weeks available within a country. Independent institutions normally have leeway to extend beyond the minimum requirements.

(a) The recommended instructional time for the ISC in Grades 10–12 is as follows:

Subject	Minimum time allocation (hours) per week (5-day cycle)
Primary Language	4,5
Additional Language	4,5
CLL, e.g. Active Citizenry	2
A minimum of any four subjects selected from Table A2 (Annexure A) of the policy document, *International Secondary Certificate (IEB),* subject to the provisos stipulated in section 2.6 of the said policy document.	16 (4 × 4h)
Total	27

(b) The recommended instructional time for the Further Studies Programmes in the ISC in Grades 10–12 is as follows:

Subject	Minimum time allocation (hours) per week (5-day cycle)
Further Studies Mathematics (Standard level) (Extended level)	 5 6
Further Studies Language together with Primary Language	6 (1,5 + 4,5)
Further Studies Physics and Further Studies Chemistry	6
The offering of Further Studies Programmes is subject to the provisos stipulated in section 2.6 of the policy document, *International Secondary Certificate (IEB).*	

5. FURTHER STUDIES MATHEMATICS: DEFINITION AND PURPOSE

DEFINITION

Further Studies Mathematics is an extension of Mathematics and is similarly based on the following view of the nature of the discipline.

Further Studies Mathematics enhances mathematical creativity and logical reasoning about problems in the physical and social world and in the context of Mathematics itself. All mathematics is a distinctly human activity developed over time as a well-defined system with a growing number of applications in our world. Knowledge in the mathematical sciences is constructed through the establishment of descriptive, numerical and symbolic relationships. Further Studies Mathematics also observes patterns and relationships, leading to additional conjectures and hypotheses and developing further theories of abstract relations through rigorous logical thinking. Mathematical problem solving in Further Studies Mathematics enables us to understand the world in greater depth and make use of that understanding more extensively in our daily lives.

PURPOSE

In a society that values diversity and equality, and a nation that has a globally competitive economy, it is imperative that within the Further Education and Training band students who perform well in mathematics or who have a significant enthusiasm for mathematics are offered an opportunity to increase their knowledge, skills, values and attitudes associated with mathematics, and so put them into a position to contribute more significantly as citizens of South Africa.

Further Studies Mathematics is aimed at increasing the number of students who through competence and desire enter Higher Education to pursue careers in mathematics, engineering, technology and the sciences. Further Studies Mathematics is an extension and challenge for students who demonstrate a greater than average ability in, or enthusiasm for mathematics. The greater breadth of mathematical knowledge gained, and depth of mathematical processes developed through being exposed to advanced mathematics ideas enhances the student's understanding of mathematics both as a discipline and as a tool in society. This broadens the student's perspective on possible careers in mathematics and develops a passion for and a commitment to the continued learning of mathematics amongst mathematically talented students.

AIMS

Further Studies Mathematics enables students to:

- experience more of the joy and beauty of mathematics;
- extend their mathematical knowledge to solve new problems in the world around them and grow in confidence in this ability;
- use sophisticated mathematical processes to solve and pose problems creatively and critically;
- demonstrate the patience and perseverance to work both independently and cooperatively on problems that require more time to solve;
- contribute to quantitative arguments relating to local, national and global issues;
- focus on the processes of mathematics, rather than only correct answers;
- become more self-reliant and validate their own answers;
- learn to value mathematics and its role in the development of our contemporary society and explore relationships among mathematics and the disciplines it serves;
- communicate mathematical problems, ideas, explorations and solutions through reading, writing and mathematical language;
- enable students to become problem solvers and users of mathematics in the real world.

The study of Further Studies Mathematics should encourage students to talk about mathematics, use the language and symbols of mathematics, communicate, discuss problems and problem solving, and develop competence and confidence in themselves as mathematics students.

EDUCATIONAL AND CAREER LINKS

Further Studies Mathematics is valuable in the curriculum of any student who intends to pursue a career in the physical, mathematical, financial, computer, life, earth, space and environmental sciences or in technology. Further Studies Mathematics also supports the pursuance of careers in the economic, management and social sciences. The knowledge and skills attained in Further Studies Mathematics provide more appropriate tools for creating, exploring and expressing theoretical and applied aspects of the sciences.

The subject Further Studies Mathematics provides the ideal platform for linkages to Mathematics in Higher Education institutions. Students proceeding to institutions of Higher Education with Further Studies Mathematics, will be in a strong position to progress effectively in whatever mathematically related discipline they decide to follow. The added exposure to modelling encountered in Further Studies Mathematics provides students with deeper insights and skills when solving problems related to modern society, commerce and industry.

Further Studies Mathematics, although not required for the study of mathematics, engineering, technology or the sciences in Higher Education, is intended to provide talented mathematics students an opportunity to advance their potential, competence, enthusiasm and success in mathematics so that it is more likely that they will follow mathematically related careers.

In particular the following are some of the career fields that demand the use of high-level mathematics:

- Actuarial science
- Operations research
- Mathematical modelling
- Economic and Industrial sciences
- Movie and video game special effects
- Engineering
- Computational mathematics
- Theoretical and applied physics
- Statistical applications
- Academic research and lecturing in mathematics, applied mathematics, actuarial science and statistics
- Data Scientist

MODULES

PAPER I (STANDARD)

Module 1A: Calculus

The student is able to establish, define, manipulate, determine and represent the derivative and integral, both as an anti-derivative and as the area under a curve, of various algebraic and trigonometric functions, and solve related problems.

Module 1B: Algebra

The student is able to represent, investigate, analyse, manipulate and prove conjectures about numerical and algebraic relationships and functions, and solve related problems.

PAPER II (EXTENDED)

Module 2: Statistics and Probability

The student is able to represent, manipulate, analyse and interpret statistical and probability models, and solve related problems.

OR

Module 3: Finance and Mathematical Modelling

The student is able to investigate, represent and model growth and decay problems using formulae, difference equations and series.

OR

Module 4: Matrices and Graph Theory

The student is able to identify, represent and manipulate discrete variables using graphs and matrices, applying algorithms in modeling finite systems.

COURSE REQUIREMENTS

	Compulsory Modules		**Elective Modules (pick one)**		
Grade 10–12	Calculus	Algebra	Statistics & Probability	Matrices & Graph Theory	Finance & Modelling

6. FURTHER STUDIES MATHEMATICS: MODULES AND LEARNING OUTCOMES

Module 1A: Calculus

The student is able to establish, define, manipulate, determine and represent the derivative and integral, both as an anti-derivative and as the area under a curve, of various algebraic and trigonometric functions, and solve related problems.

	Grade 10		Grade 11		Grade 12
	We know this when the student is able to:		We know this when the student is able to:		We know this when the student is able to:
		11.1.1	(a) Sketch the graphs of mathematical functions including split domain and composite functions (comprised of linear, quadratic, hyperbolic, absolute value and exponential functions). (b) Manipulate and analyse split domain and composite functions using the definition of a function and the graph of the function.		
		11.1.2	(a) Define and use trigonometric and reciprocal trigonometric functions to: • manipulate trigonometric statements. • solve trigonometric problems in realistic and mathematical contexts. • find the general solution of trigonometric equations. (b) convert between angles measured in degrees and radians. (c) use trigonometric functions defined in terms of a real variable (angle in radians) and the *x*, *y* and r-definition to: • calculate the lengths of arcs of circles, • calculate the area of sectors and segments of circles.		

	Grade 10		Grade 11		Grade 12
	We know this when the student is able to:		We know this when the student is able to:		We know this when the student is able to:
		11.1.3	(a) Compare the graphical, numerical and symbolic representations of the limit of a function. (b) Determine the limit of an algebraic function at a point, including from the right and left, and to infinity algebraically. (Note: The limit at a point is defined as the limit from the left and from the right.) (c) Illustrate the continuity of a function graphically and apply the definition of continuity at a point to simple algebraic functions, including split domain functions.	12.1.3	(a) Use first principles and graphs to determine the continuity and differentiability at a given point of algebraic functions, including split domain functions. (b) Without proof, apply the theorem and its converse, "A function that is differentiable at a point is continuous at that point", and deal with examples to indicate that the converse is not valid. (c) Demonstrate the derivative of a function at a point as the rate of change, by graphical, numerical and symbolic representations.
		11.1.4	(a) Illustrate the differentiability of a function graphically and determine the derivative as the gradient of a function at a point using limits. (b) Establish the derivatives of functions of the form $ax^2 + bx + c$ & $mx + c$, from first principles.	12.1.4	(a) Establish the derivatives of functions of the form $\sqrt{x}$, $\sqrt{ax+b}$, $\frac{1}{ax+b}$, $\frac{1}{\sqrt{ax+b}}$ from first principles. (b) Use the following rules of differentiation: $\frac{d}{dx}\left[f(x).(gx)\right] = g(x).\frac{d}{dx}\left[f(x)\right] + f(x).\frac{d}{dx}\left[g(x)\right]$ $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x).\frac{d}{dx}\left[f(x)\right] - f(x).\frac{d}{dx}\left[g(x)\right]}{\left[g(x)\right]^2}$ $\frac{d}{dx}\left[f(g(x))\right] = f'(g(x)).g'(x)$

	Grade 10		Grade 11		Grade 12
	We know this when the student is able to:		We know this when the student is able to:		We know this when the student is able to:
					applied to: • polynomial, rational, radical, natural logarithm, ef(x) and trigonometric functions, (Students may assume without proof that $\frac{d}{dx}\sin x = \cos x$), • higher order derivatives, but the n-th derivative is not examinable, and • functions in two variables using implicit differentiation. (c) Newton's method. (d) Calculate maximum and minimum values of a function using calculus methods. Determine both absolute and relative maximum and minimum values of a function over a given interval. (e) Use calculus methods to sketch the curves of polynomial and rational functions determining: • intervals over which a function is increasing or decreasing. • *y*-intercepts and *x*-intercepts, using Newton's Method if necessary. • the coordinates and nature of stationary points. • any vertical, horizontal or oblique asymptotes. (f) Use methods learned above to solve practical problems involving optimisation and rates of change in real, realistic and abstract mathematical contexts. Verify the results of the calculus modelling by referring to the practical context.

	Grade 10		Grade 11		Grade 12
	We know this when the student is able to:		We know this when the student is able to:		We know this when the student is able to:
10.1.5	(a) Approximate the area between familiar curves, such as straight lines, parabolas, hyperbolae and exponential graphs, and the *x*-axis using the Rectangle Method on an interval of the *x*-axis. (b) Estimate the margin of error of the approximate area determined by the upper and lower sums method. (c) Experiment with the accuracy of the approximation by varying the width and number of rectangles. (Not examinable in grade 12)	11.1.5	(a) Investigate and develop a formula for the upper and lower sums method of approximating area under the curve of $y = x^n$, for $n \in$ N and $x \geq 0$ on the interval [a; b] (i.e. $\int_a^b x^n dx = \left[\frac{1}{n+1}x^{n+1}\right]_a^b$) (b) Use available technology to manipulate the width of sub-intervals and the accuracy of the approximate area under a polynomial function. (c) Investigate and intuitively develop the Riemann (definite) integral as the approximating rectangles are made narrower and the number of rectangles, *n*, increases. (Generating and simplifying the formula $Area = \lim_{n\to\infty}\left(\frac{b-a}{n}\right)\sum_{i=1}^{n} f\left(x_i\right)$ is not examinable. Students may be required to interpret the formula.)	12.1.5	(a) Appreciate the Fundamental Theorem of Calculus and its significance in recognising anti-differentiation as the reverse of differentiation. (b) Manipulate and then integrate algebraic, natural logarithm, $e^{f(x)}$ and trigonometric functions of the form: • $\int ax^n dx$, *a* is a constant and $n \in$ Q, • $\int p(x)\,dx$ and $\int \frac{p(x)}{q(x)}\,dx$, *p*(*x*) and *q*(*x*) are polynomials or radicals, • $\int \frac{f(x)}{g(x)}\,dx$, leading to partial fractions • $\int f(g(x)).g'(x)\,dx$ • where the anti-derivative of a trigonometric function can be directly determined from the derivative (e.g. $\int \cos x dx$, $\int \sec^2 x dx$, $\int \sec x \tan x dx$) • $\int \sin^n x dx$ and $\int \cos^n x dx$ where n $\in$ N and n $\leq$ 3 • $\int \sin mx\, dx$; $\int \sin mx \cos nx dx$ and similar functions

	Grade 10		Grade 11		Grade 12
	We know this when the student is able to:		We know this when the student is able to:		We know this when the student is able to:
					using only the following methods: • direct anti-derivatives • simplification of trigonometric functions using appropriate squares, compound angle and given product-sum formulae • integration by t-substitution • integration with a given substitution • integration by parts (c) Find the definite integral of any function using a calculator. (d) Apply the definite integral and techniques of integration to solve area and volume problems by: • Calculating the area under or between curves using the manipulation of intervals. • Calculating the volume generated by rotating a function about the x-axis in mathematical and real-world contexts.

Module 1B: Algebra

The student is able to represent, investigate, analyse, manipulate and prove conjectures about numerical and algebraic relationships and functions, and solve related problems.

	Grade 10		Grade 11		Grade 12
	We know this when the student is able to:		We know this when the student is able to:		We know this when the student is able to:
10.1.6	(a) Characterise and discuss the nature and relevance of the roots of $x^2+1=0$. (b) Classify numbers using sub-fields of the Complex numbers. (c) Determine the roots of equations of the form $ax^2+bx+c=0$ and classify the roots as real or imaginary. Determine the real and complex roots of quadratic and cubic equations using: • factorisation, • the quadratic formula, and • the factor theorem to find the first real root of cubic equations. (d) Perform the four basic operations (+, –, /, x) on Complex numbers and their conjugates without the use of a calculator. (e) Illustrate a Complex number on an Argand diagram.	11.1.6	Solve: (a) equations containing multi-term algebraic fractions using algebraic methods. (b) polynomial and rational inequalities. (c) absolute value equations.	12.1.6	Solve exponential equations using the laws of exponents, algebraic manipulation and logs.
10.1.7	Simplify compound fractions.	11.1.7	Decompose algebraic fractions into partial fractions when the denominator is of the form: • $(a_1x+b_1)(a_2x+b_2)\ \dots\ (a_nx+b_n)$, using the 'cover up' method. • $(ax+b)^2(cx+d)^2$, by comparing coefficients.	12.1.7	(a) Simplify and manipulate algebraic expressions using the laws of exponents and logarithms. (b) Demonstrate an understanding of *e* and its role in exponents and logarithms by using *e* freely in problem solving.

	Grade 10		Grade 11		Grade 12
	We know this when the student is able to:		We know this when the student is able to:		We know this when the student is able to:
10.1.8	(a) Demonstrate an understanding of the absolute value of an algebraic expression as a distance from the origin. $\lvert x\rvert = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$ (b) Solve linear absolute value equations. (c) Draw graphs of the form $y = a\lvert x\rvert + q$.	11.1.8	(a) Draw graphs of $y = a\lvert x-p\rvert + q$, $y = \lvert f(x)\rvert$ and $y = f(\lvert x\rvert)$ by inference where $f(x)$ is a simple function (e.g. see 11.1.1). (b) Find the equation of the absolute value function, in the form $y = a\lvert x-p\rvert + q$, given the graph and necessary points on the graph. (c) Interpret the graphs of absolute value functions to determine the: • domain and range; • intercepts with the axes; • turning points, minima and maxima; • shape and symmetry; • transformations. (d) Solve absolute value inequalities using graphical representations of the associated functions.	12.1.8	(a) Draw exponential graphs, including $y = e^x$. (b) Draw logarithmic graphs, including y = ln x. (c) Find the equation for the reflections of the exponential or logarithmic functions in the lines x = 0, y = 0 and y = x, the inverse of the function. (d) transformations of ln(x) / e^x graphs.
				12.1.9	Use mathematical induction to prove: (a) statements of summation of series. (b) statements about factors and factoring with Natural numbers.

Module 2: Statistics and probability

The student is able to organise, summarise, analyse and interpret data to identify, formulate and test statistical and probability models, and solve related problems.

	Grade 10		**Grade 11**		**Grade 12**
	We know this when the student is able to:		We know this when the student is able to:		We know this when the student is able to:
10.2.1	(a) Use Venn diagrams as an aid to solving probability problems of random events. (b) Use Tree diagrams as an aid to solving probability problems of random events. (c) Use contingency tables to solve probability problems. (d) Recognise and then determine the probability of conditional events using diagrams and the formula: $P(A\mid B)=\dfrac{P(A\cap B)}{P(B)}$ (e) Identify and determine the probability of mutually exclusive and independent events. (f) Use the Laws of Probability to evaluate simple random events.	11.2.1	(a) Identify, apply and calculate the probability of the following distributions of discrete random events: • Hypergeometric distribution model • Binomial distribution model The mean and variance of the binomial distribution should be known and applied. (b) Formulate and/or apply a probability mass function (including simple continuous models) and find the expected value and variance. (c) Identify and apply the Normal distribution model to the probability of continuous random events, using statistical tables and calculations as necessary.	12.2.1	(a) Understand how the Central Limit Theorem is applied when a large (n > 30) sample is taken from any distribution. (b) Apply the Normal distribution model to a sample to estimate a confidence interval for the population mean or proportion, using statistical tables to deal with various confidence levels. (c) Apply the normal approximation to the binomial distribution, utilising continuity corrections, as appropriate. (d) Formulate a probability mass or density function for a: • Hypergeometric distribution • Binomial distribution • Simple continuous probability models

	Grade 10		Grade 11		Grade 12
	We know this when the student is able to:		We know this when the student is able to:		We know this when the student is able to:
10.2.2	Count arrangements using permutations (including those where repetition occurs).	11.2.2	Count arrangements and choices using permutations (including those where repetition occurs) and combinations.	12.2.2	–
10.2.3	(a) Calculate the standard deviation. (b) Understand the mean and standard deviation of a population as applied to the normal distribution. The effect of these parameters on the shape of the curve should be understood. (c) The percentage of values that lie within 1, 2 and 3 standard deviations of the mean should be known.	11.2.3		12.2.3	Perform a one-tail and/or two-tail hypothesis test on a mean or difference of means. This includes being able to: • Distinguish between one-tail and two-tail events. • Establish a null hypothesis based on the prevalent condition.

Module 3: Finance and Mathematics Modelling

The student is able to investigate, represent and model growth and decay problems using formulae, difference equations and series.

	Grade 10		Grade 11		Grade 12
	We know this when the student is able to:		We know this when the student is able to:		We know this when the student is able to:
10.3.1	(a) Generalise number patterns using first order linear difference equations of the form $u_n = k.u_{n-1} + c$. (b) Use appropriate technology to solve higher terms in first order linear difference equations. (c) Convert first order linear difference equations into a general solution in explicit form. (d) With the aid of appropriate technology, use first order linear difference equations to solve future and present value annuities.	11.3.1	(a) Generalise or produce number patterns using second order homogenous linear difference equations $(u_n = p.u_{n-1} + q.u_{n-2} + c)$. (b) Use appropriate technology to solve higher terms in second order homogenous linear difference equations. (c) Model simple population growth and decay problems using a discrete Malthusian population model of the form $P_{n+1} = (1+r).P_n + c$.	12.3.1	(a) Model simple population growth and decay problems using • a discrete Logistic population model of the form $P_{n+1} = P_n + a(1-b.P_n).P_n$ • a discrete two species Lotka-Volterra predator-prey population model written in difference equation form $R_{n+1} = R_n + a.R_n(1- R_n / k) - b.R_n.F_n$ $F_{n+1} = F_n + e.b.R_n.F_n - c.F_n$ and interpret characteristics of the parameters and attributes of the models (e.g. carrying capacity; equilibrium populations) (b) Evaluate a realistic population scenario and apply the most suitable model for a given scenario.

	Grade 10		**Grade 11**		**Grade 12**
	We know this when the student is able to:		We know this when the student is able to:		We know this when the student is able to:
10.3.2	(a) Use simple and compound growth formulae to solve problems in various contexts including but not limited to: • simple interest and straight-line depreciation, • compound interest and reducing balance depreciation, • compound growth and decay problems (b) Investigate and derive the future value annuity formula using first order linear difference equations in explicit form.	11.3.2	Formulate timelines and apply future and present value annuity formulae to: (a) Calculate the present value or future value of an annuity, or the termly payment. (b) Calculate the balance outstanding on a loan at a specified point in the amortisation period. (c) Establish a sinking fund in a given context. (d) Calculate the value of a deferred annuity. (e) Convert effective and nominal interest rates to solve problems with different accumulation periods.	12.3.2	Formulate timelines and apply future and present value annuity formulae to: (a) Determine the number of repayment periods using logarithms. (b) Calculate the number of payments and the final payment when a loan is repaid by fixed instalments. (c) Solve annuity problems involving changing circumstances such as changes to time periods, repayments (including missed payments), withdrawals and interest rates.

Module 4: Matrices and Graph Theory

The student is able to identify, represent and manipulate discrete variables using graphs and matrices, applying algorithms in modeling finite systems.

	Grade 10		Grade 11		Grade 12
	We know this when the student is able to:		We know this when the student is able to:		We know this when the student is able to:
10.4.1	(a) Arrange numbers in a suitable rectangular array or matrix to facilitate problem solving. (b) Knowing when a matrix operation is possible, perform the following operations on a matrix or matrices: • addition, • multiplication of a matrices, and • multiplication by a scalar. (c) Solve systems of two variable linear equations using the method of diagonalisation. (d) Determine the inverse of 2 × 2 matrices by a sequence of row transformation $[A : I_{nxn}] = [I_{nxn} : A^{-1}]$. (e) Solve systems of linear equations using the inverse matrix.	11.4.1	Use 2×2 matrices to transform points and figures in the Cartesian Plane by: (a) A translation, given in the form $\begin{pmatrix} a \\ b \end{pmatrix}$. (b) Rotation through any given angle about the origin. (c) Reflection in any given line through the origin. (d) Enlargement, using construction, with positive or negative scale factors and the centre of enlargement at the origin. (e) Shear and stretch with the *x* or *y* axis as the invariant line using negative or positive shear/stretch factors.	12.4.1	Use matrices to: (a) Solve systems of three variable linear equations using the method of diagonalisation (i.e. Gaussian Reduction). (b) Determine the inverse matrix by a sequence of row transformations using $[A : I_{nxn}] = [I_{nxn} : A^{-1}]$. (c) Calculate the determinant of the matrix. (d) Solve systems of linear equations using the inverse matrix up to a 4 by 4 matrix.

	Grade 10		Grade 11		Grade 12
	We know this when the student is able to:		We know this when the student is able to:		We know this when the student is able to:
10.4.2	(a) Define simple, regular and connected graphs, their vertices, edges and the degree of the graph. (b) Identify complete, complementary and isomorphic graphs. (c) Define walks, paths and circuits. (d) Evaluate and determine Eulerian paths within a graph. (e) Evaluate and classify graphs, intuitively and algorithmically, as Eulerian Circuits or Hamiltonian Circuits. (f) Use Euler's, Fleury's and Dirac's algorithms to test the nature of the paths and circuits in a graph.	11.4.2	(a) Determine the number of different graphs that can be drawn on $n \leq 6$ vertices. (b) Represent simple network problems using a weighted graph. (c) Solve simple optimisation problems using weighted graphs. (d) Determine the shortest path of a network or weighted graph using Dijkstra's shortest path algorithm. (e) Optimise route inspection (Chinese postman) problems using Eulerian circuits, paths and the shortest path algorithm.	12.4.2	(a) Solve minimum connector problems using graphs, matrices and the Kruskal and Prim algorithms. (b) Solve by finding an upper bound for simple travelling salesman problems using graphs and matrices and the nearest-neighbour algorithm. (c) Solve simple travelling salesman problems using simple algorithms researched in the literature. (d) Use matrices to represent graphs and to solve optimisation problems.

MATRICES: TERMINOLOGY AND NOTATION

M^{-1}:	inverse of a matrix
M^{T}:	transpose of a matrix
|M| or det M:	determinant of a matrix
Leading/main diagonal:	running from top left to bottom right
Trace:	the sum of the leading diagonal of a matrix
Order:	number of vertices in a graph
Size:	number of edges in a graph

Degree:	number of edges leading out of a vertex
Neighbourhood:	vertices directly connected a to given vertex
Singular matrix:	a matrix that has no inverse
Non-singular matrix:	a matrix that has an inverse
Row Echelon Form:	lower half of matrix below leading diagonal consists only of zero elements
Reduced Row Echelon Form:	matrix reduced to only zero elements, except main diagonal which is only ones

GRAPH THEORY: TERMINOLOGY

A **graph** is a set of **vertices** and **edges;** every edge starts and finishes at a vertex.

A **connected** graph has all vertices directly or indirectly connected to each other.

A **complete** graph is a graph in which each pair of vertices is connected by an edge.

The **complement** of a graph consists of the same set of vertices but the edges in the complement are the edges not present in the original graph.

Simple graphs have no loops or multiple edges.

A **undirected** graph means that the distance $A \rightarrow B = B \rightarrow A$

The **order** of a graph is the number of vertices in the graph.

The **degree** of a vertex is the number of edges leading to/from that vertex.

The **size** of a graph is the total number of edges in the graph.

A **regular graph** has all vertices of the same degree.

Adjacent vertices are directly connected to each other (share a common edge).

The **neighbourhood** of a vertex are those vertices to which it is directly connected.

In a simple graph a **walk** is a sequence of vertices and edges in a graph. such that the finishing vertex of an edge becomes the starting vertex of the next edge. The edges and vertices need not distinct.

A **path** is a route in a graph so that no edge is used more than once; not all the edges of the graph need be used and the starting point need not be the endpoint.

A path that starts and ends at the same vertex is a **closed path, circuit.**

A **tree** is a connected graph, with **no circuits.**

The **weight** of an edge is the distance, time or cost factor ascribed to it.

Graphs can also be represented in **adjacency matrices** or as **geometric figures**.

Isomorphic graphs have the same size, order and neighbourhoods (circuitry).

GRAPH THEORY: AFRIKAANS TERMINOLOGY

Chinese Postman Problem	Chinese Posmanprobleem
Circuit/Ring	Kring
Coincident Planes	Samevallende Vlakke
Complete Graph	Volledige Grafiek
Complementary Graph	Komplimentêre Grafiek
Connected Graphs	Samehangende Grafieke
Edges of a Graph	Skakels van 'n Grafiek
Eulerian Circuit/Path	Eulerkring of -pad
Gaussian Reduction	Gaussherleiding
Girth of a Circuit	Gordellengte van 'n Kring
Hamiltonian Circuit	Hamiltonkring
Loop at Vertex A	Lus by Nodus A
Multigraph	Veelvoudige Grafiek
Nearest Neighbour Algorithm	Naaste Buurpunt Algoritme
Path between Vertices A and B	Pad tussen Nodusse A en B
Simple Graph	Enkelvoudige Grafiek
Spanning Tree	Spanboom
Travelling Salesman Problem	Reisende Verkoopsmanprobleem
Undirected Graph/Digraph	Ongerigte Grafiek
Upper and Lower Bounds	Bo- en Ondergrense

Weighted Graphs	Geweegde Grafieke
Adjoint Matrix	Adjunk Matriks
Adjacency Matrix	Nodus Matriks
Augmented Matrix	Aangevulde Matriks
Enlargement	Vergroting
Invariant Line for Transformations	Invariantelyn vir Transformasies
Inverse Matrix	Inverse Matriks
Leading Diagonal of Matrix	Hoofdiagonal van Matriks
Matrix of Cofactors	Kofaktormatriks
Matrix of Minors	Minormatriks
Reflection	Refleksie
Rotation	Rotasie
Row Reduction	Ry Reduksie
Shear	Dwarsdruk
Stretch	Rek
Translation	Translasie
Transpose of Matrix	Transponeer van Matriks
Vertex/Node of a Graph	Nodus van 'n Grafiek

7. A. MEANS OF ASSESSMENT

Paper 1 (Standard)	2 hours	[200]
Paper 2 (Extended)	1 hour	[100]

B. REQUIREMENTS

The Learning Outcomes of the Further Studies Mathematics National Curriculum Statement are divided into Core Learning Outcomes (LO 1: Calculus & LO 2 Algebra) and Elective Learning Outcomes (LO 3: Statistics, LO 4: Mathematical Modelling & LO 5: Matrices and Graph Theory). Students will be examined on the Core Learning Outcomes (LO 1 & LO 2) and in addition, one of the Elective Outcomes (LO 3 or LO 4 or LO 5).

GRADE 12

EXAMINATION MARK ALLOCATION

Learning Outcome	Marks	Time
1	130–160	80–90 minutes
2	40–70	30–40 minutes
Elective	100	60 minutes
Total	300	3 hours

WEIGHTING ACCORDING TO TAXONOMY OF COGNITIVE LEVEL

Level		%
1	Knowledge	12–18
2	Routine procedures	37–43
3	Complex procedures	30–36
4	Problem-solving	7–13

DISTRIBUTION OF MARKS FOR CORE LEARNING OUTCOMES: FURTHER STUDIES (STANDARD)

Learning outcome	**Topic**	**Mark distribution** (±5)
1	Functions and limits	20
	Trigonometry	15
	Differentiation	35
	Integration	30
	Drawing functions	20
	Applications (max/min; rates of change; volume & area)	20
	Total	140
2	Real and complex roots	15
	Exponents and logarithms	15
	Absolute value	20
	Induction	10
	Total	60

DISTRIBUTION OF MARKS FOR ELECTIVE LEARNING OUTCOMES: FURTHER STUDIES (EXTENDED)

Learning Outcome	**Topic**	**Percentage**
3	Probability fundamentals	15–30
	Probability functions and applications	50–60
	Inferential statistics	15–30
	Total	100
4	Graph theory	40–60
	Matrices	40–60
	Total	100
5	Financial models	40–60
	Recursive models	40–60
	Total	100

C. INTERPRETATION OF REQUIREMENTS

INFORMATION BOOKLET

Algebra

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$|x| = \begin{cases} x & ; \quad x \geq 0 \\ -x & ; \quad x < 0 \end{cases}$$

$$\sum_{i=1}^{n} 1 = n$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} = \frac{n^2}{2} + \frac{n}{2}$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_n = \frac{a\left(1 - r^n\right)}{1 - r}$$

$z = a + bi$ $\qquad$ $z^* = a - bi$

$\ln A + \ln B = \ln (AB)$ $\qquad$ $\ln A - \ln B = \ln\left(\frac{A}{B}\right)$

$\ln A^n = n \ln A$ $\qquad$ $\log_a x = \dfrac{\log_b x}{\log_b a}$

Calculus

$$\text{Area} = \lim_{n \to \infty}\left(\frac{b-a}{n}\right)\sum_{i=1}^{n} f\left(x_i\right)$$

$$\int_a^b x^n dx = \left[\frac{x^{n+1}}{n+1}\right]_a^b$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\int f'\left(g(x)\right).g'(x)\,dx = f(g(x)) + c$$

$$\int f(x).g'(x)\,dx = f(x).g(x) - \int g(x).f'(x)\,dx + c$$

$$x_{r+1} = x_r - \frac{f(x_r)}{f'(x_r)}$$

$$V = \pi\int_a^b y^2 dx$$

Function	**Derivative**
x^n	nx^{n-1}
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\text{cosec}^2 x$
$\sec x$	$\sec x.\tan x$
$\text{cosec}\, x$	$-\text{cosec}\, x.\cot x$
e^x	e^x
$\ln x$	$\frac{1}{x}$
$f(g(x))$	$f'(g(x)).g'(x)$
$f(x).g(x)$	$g(x).f'(x)+f(x).g'(x)$
$\frac{f(x)}{g(x)}$	$\frac{g(x).f'(x)-f(x).g'(x)}{[g(x)]^2}$

$A=\frac{1}{2}r^2\theta$ $\qquad$ $s=r\theta$

In $\triangle ABC$: $\qquad \frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$

$$a^2=b^2+c^2-2bc.\cos A$$

$$Area=\frac{1}{2}ab.\sin C$$

$\sin^2 A+\cos^2 A=1$ $\qquad$ $1+\tan^2 A=\sec^2 A$ $\qquad$ $1+\cot^2 A=\text{cosec}^2 A$

$\sin(A\pm B)=\sin A.\cos B\pm\cos A\sin B$ $\qquad$ $\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$

$\sin 2A=2\sin A\cos A$ $\qquad$ $\cos 2A=\begin{cases}\cos^2 A - \sin^2 A\\ 2\cos^2 A-1\\ 1-2\sin^2 A\end{cases}$

$$\sin A.\cos B=\frac{1}{2}\left[\sin(A+B)+\sin(A-B)\right]$$

$$\sin A.\sin B=\frac{1}{2}\left[\cos(A-B)-\cos(A+B)\right]$$

$$\cos A.\cos B=\frac{1}{2}\left[\cos(A-B)+\cos(A+B)\right]$$

Matrix Transformations

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \qquad \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$$

Finance & Modelling

$$F = P(1+in) \qquad F = P(1-in) \qquad F = P(1+i)^n \qquad F = P(1-i)^n$$

$$F = x\left[\frac{(1+i)^n - 1}{i}\right] \qquad P = x\left[\frac{1-(1+i)^{-n}}{i}\right] \qquad r_{eff} = \left(1+\frac{r}{k}\right)^k - 1$$

$$P_{n+1} = P_n + rP_n\left(1-\frac{P_n}{K}\right)$$

$$R_{n+1} = R_n + aR_n\left(1-\frac{R_n}{K}\right) - bR_nF_n \qquad F_{n+1} = F_n + f.bR_nF_n - cF_n$$

Statistics

$$P(A) = \frac{n(A)}{n(S)} \qquad P(B \mid A) = \frac{P(B \cap A)}{P(A)} \qquad P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$^nP_r = \frac{n!}{(n-r)!} \qquad ^nC_r = \binom{n}{r} = \frac{n!}{(n-r)!r!} \qquad P(X=x) = \binom{n}{x}p^x(1-p)^{n-x}$$

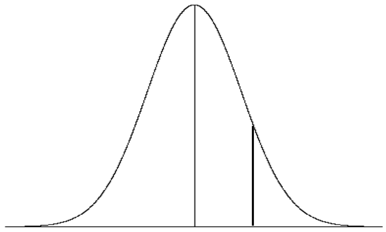
$$P(R=r) = \frac{\binom{p}{r}\binom{N-p}{n-r}}{\binom{N}{n}} \qquad E[X] = n \cdot p \qquad Var[X] = n \cdot p(1-p)$$

$$z = \frac{X-\mu}{\sigma} \qquad z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} \qquad z = \frac{(\bar{x}-\bar{y})-(\mu_x-\mu_y)}{\sqrt{\frac{\sigma_x^2}{n_x}+\frac{\sigma_y^2}{n_y}}}$$

$$\bar{x} \pm z\frac{\sigma}{\sqrt{n}} \qquad p \pm z\sqrt{\frac{p(1-p)}{n}} \qquad E[X] = \sum x \cdot P(X=x)$$

$$Var[X] = E\left[X^2\right] - \left(E[X]\right)^2$$

NORMAL DISTRIBUTION TABLE



Areas under the Normal Curve

$$H(z) = \frac{1}{\sqrt{2\pi}} \int_0^z e^{-\frac{1}{2}x^2}\, dx$$

$H(-z) = H(z)$, $H(\infty) = \frac{1}{2}$

Entries in the table are values of H(z) for $z \geq 0$.

z	,00	,01	,02	,03	,04	,05	,06	,07	,08	,09
0,0	,0000	,0040	,0080	,0120	,0160	,0199	,0239	,0279	,0319	,0359
0,1	,0398	,0438	,0478	,0517	,0557	,0596	,0636	,0675	,0714	,0753
0,2	,0793	,0832	,0871	,0910	,0948	,0987	,1026	,1064	,1103	,1141
0,3	,1179	,1217	,1255	,1293	,1331	,1368	,1406	,1443	,1480	,1517
0,4	,1554	,1591	,1628	,1664	,1700	,1736	,1772	,1808	,1844	,1879
0,5	,1915	,1950	,1985	,2019	,2054	,2088	,2123	,2157	,2190	,2224
0,6	,2257	,2291	,2324	,2357	,2389	,2422	,2454	,2486	,2517	,2549
0,7	,2580	,2611	,2642	,2673	,2704	,2734	,2764	,2794	,2823	,2852
0,8	,2881	,2910	,2939	,2967	,2995	,3023	,3051	,3078	,3106	,3133
0,9	,3159	,3186	,3212	,3238	,3264	,3289	,3315	,3340	,3365	,3389
1,0	,3413	,3438	,3461	,3485	,3508	,3531	,3554	,3577	,3599	,3621
1,1	,3643	,3665	,3686	,3708	,3729	,3749	,3770	,3790	,3810	,3830
1,2	,3849	,3869	,3888	,3907	,3925	,3944	,3962	,3980	,3997	,4015
1,3	,4032	,4049	,4066	,4082	,4099	,4115	,4131	,4147	,4162	,4177
1,4	,4192	,4207	,4222	,4236	,4251	,4265	,4279	,4292	,4306	,4319
1,5	,4332	,4345	,4357	,4370	,4382	,4394	,4406	,4418	,4429	,4441
1,6	,4452	,4463	,4474	,4484	,4495	,4505	,4515	,4525	,4535	,4545
1,7	,4554	,4564	,4573	,4582	,4591	,4599	,4608	,4616	,4625	,4633
1,8	,4641	,4649	,4656	,4664	,4671	,4678	,4686	,4693	,4699	,4706
1,9	,4713	,4719	,4726	,4732	,4738	,4744	,4750	,4756	,4761	,4767
2,0	,4772	,4778	,4783	,4788	,4793	,4798	,4803	,4808	,4812	,4817
2,1	,4821	,4826	,4830	,4834	,4838	,4842	,4846	,4850	,4854	,4857
2,2	,4861	,4864	,4868	,4871	,4875	,4878	,4881	,4884	,4887	,4890
2,3	,48928	,48956	,48983	,49010	,49036	,49061	,49086	,49111	,49134	,49158
2,4	,49180	,49202	,49224	,49245	,49266	,49286	,49305	,49324	,49343	,49361
2,5	,49379	,49396	,49413	,49430	,49446	,49461	,49477	,49492	,49506	,49520
2,6	,49534	,49547	,49560	,49573	,49585	,49598	,49609	,49621	,49632	,49643
2,7	,49653	,49664	,49674	,49683	,49693	,49702	,49711	,49720	,49728	,49736
2,8	,49744	,49752	,49760	,49767	,49774	,49781	,49788	,49795	,49801	,49807
2,9	,49813	,49819	,49825	,49831	,49836	,49841	,49846	,49851	,49856	,49861
3,0	,49865	,49869	,49874	,49878	,49882	,49886	,49889	,49893	,49896	,49900
3,1	,49903	,49906	,49910	,49913	,49916	,49918	,49921	,49924	,49926	,49929
3,2	,49931	,49934	,49936	,49938	,49940	,49942	,49944	,49946	,49948	,49950
3,3	,49952	,49953	,49955	,49957	,49958	,49960	,49961	,49962	,49964	,49965
3,4	,49966	,49968	,49969	,49970	,49971	,49972	,49973	,49974	,49975	,49976
3,5	,49977									
3,6	,49984									
3,7	,49989									
3,8	,49993									
3,9	,49995									
4,0	,49997									