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TOTAL
MARKS

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INTERNATIONAL SECONDARY CERTIFICATE EXAMINATION
NOVEMBER 2022

FURTHER STUDIES MATHEMATICS (STANDARD): PAPER I

EXAMINATION NUMBER

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Time: 2 hours

200 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 28 pages and an Information Booklet of 4 pages (i–iv). Please check that your question paper is complete.
2. **Answer ALL the questions on the question paper and hand it in at the end of the examination. Remember to write your examination number in the space provided.**
3. Non-programmable and non-graphical calculators may be used, unless otherwise indicated.
4. All necessary calculations must be clearly shown and writing must be legible.
5. Diagrams have not been drawn to scale.
6. Round off your answers to 4 decimal digits, unless otherwise indicated.
7. FIVE blank pages (pages 24–28) are included at the end of the question paper. If you run out of space for an answer, use these pages. Clearly indicate the number of your answer should you use this extra space.

FOR OFFICE USE ONLY: MARKER TO ENTER MARKS

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	TOTAL
36	20	22	20	22	12	10	8	22	16	12	/200

QUESTION 1

1.1 Solve for $x \in \mathbb{R}$, showing all working:

(a) $|\ln(x-2)| = 1$

(6)

(b) $e^{x+1} - e^x = 12$

(5)

(c) $3 - 2|2x + 4| \geq -13$

(7)

1.2 Determine a if: $\frac{15 - 5ai}{a + 2i} = -1 - 7i$

(8)

1.3 Consider $f(x) = x^4 - 3x^3 - 19x^2 + 85x - 100$ which has $f(2 + i) = 0$.

(a) Factorise f fully in \mathbb{R} .

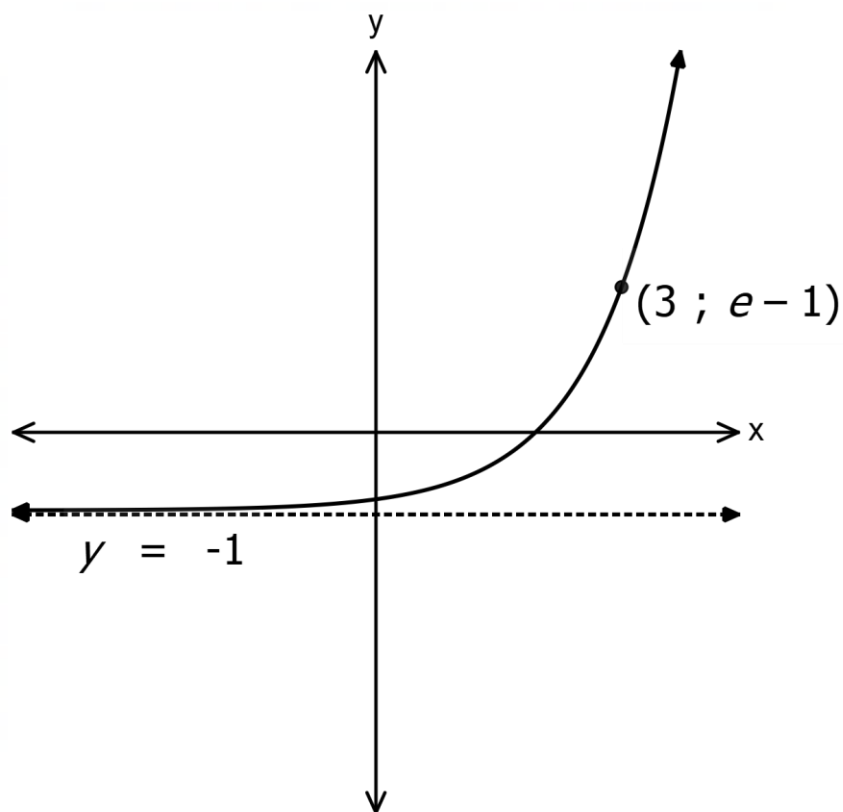
(7)

(b) Hence, or otherwise, solve $f(x) = 0$ for $x \in \mathbb{C}$.

(3)
[36]

QUESTION 2

2.1 Consider the function $f(x) = e^{x+p} + c$, which has been drawn below:

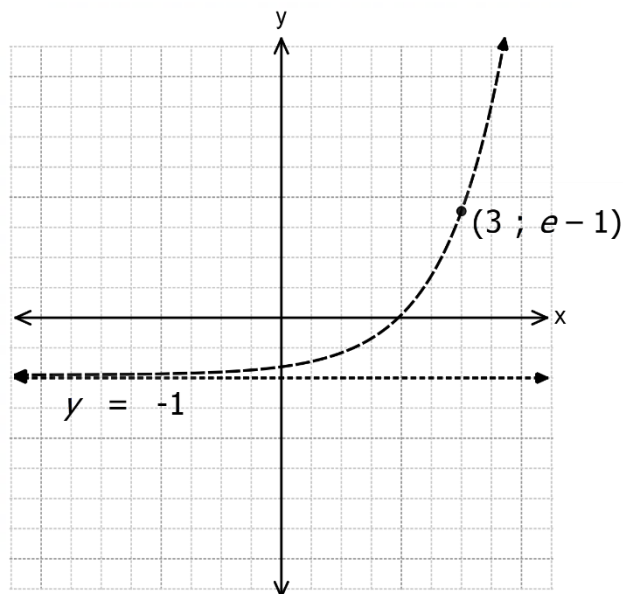


- (a) Determine the values of p and c .

(6)

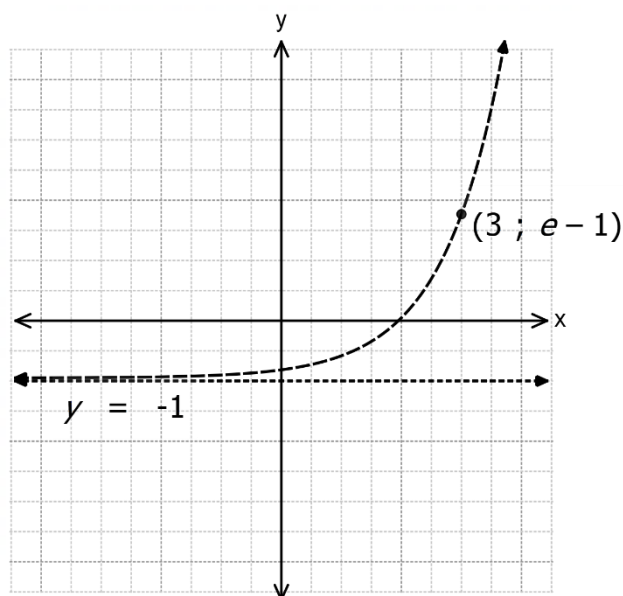
- (b) The graph of $f(x)$ has been reproduced below for each question using a dashed line. On each set of axes sketch the graph of the required function, showing asymptotes where necessary. Also indicate by means of a cross on your curve any points at which the function is NOT differentiable.

(i) $y = |f(x)|$



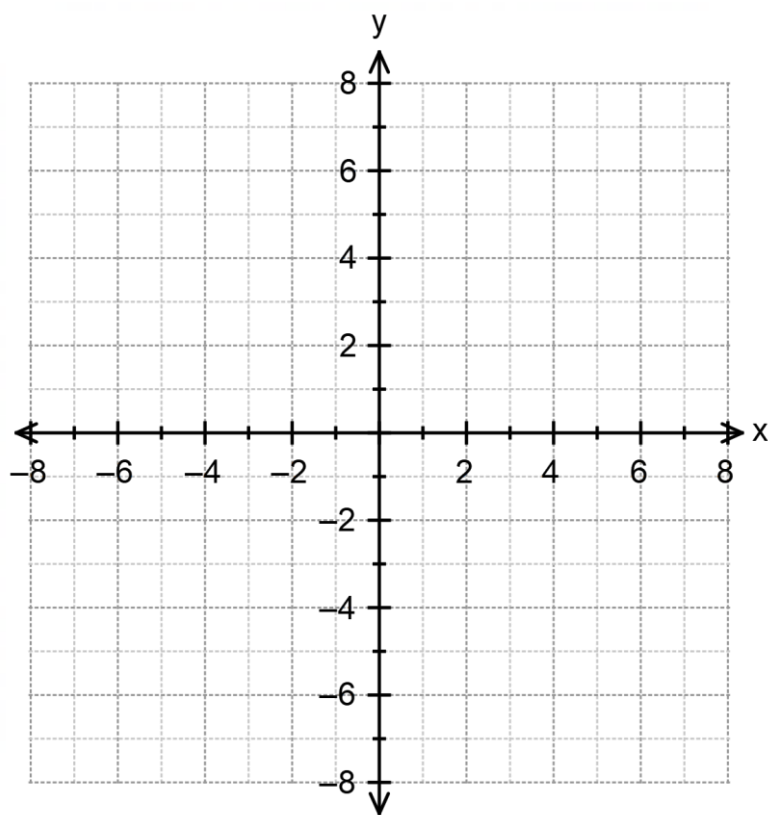
(4)

(ii) $y = f(|x|)$



(4)

- 2.2 Draw the graph of $y = -|x - 4| + 2$ on the axes provided, showing all points of interest.



(6)
[20]

QUESTION 3

3.1 Consider the function: $g(x) = \frac{2x^2 + bx - 10}{ax - 8}$ with the following asymptotes:

$$x = 2 \text{ and } y = \frac{1}{2}x + 4$$

Determine the values of a and b .

(6)

3.2 Write down the equation of a rational function which has:

- A y -intercept of 3
- x -intercepts of -2 and 3
- Two vertical asymptotes, one of which must be $x = 1$
- A horizontal asymptote of $y = -2$

(8)

3.3 Determine the coordinates of the stationary points of:

$$g(x) = \frac{x^2 + 3x - 10}{3x - 13}$$

QUESTION 4

4.1 Given the function defined as follows:

$$f(x) = \begin{cases} mx + c & x \leq -2 \\ -x^2 + 5 & -2 < x < 2 \\ p & x = 2 \\ 2x - 3 & x > 2 \end{cases}$$

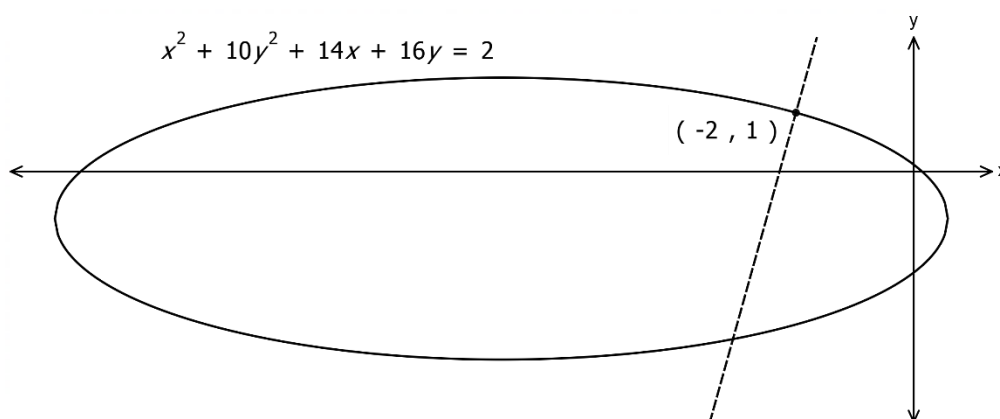
- (a) State the value of p which makes f continuous at $x = 2$.

(2)

- (b) Determine, paying careful attention to notation and providing your reasoning, the values of m and c which will make f differentiable at $x = -2$.

(8)

4.2 Given the ellipse with equation $x^2 + 10y^2 + 14x + 16y = 2$ drawn below.



- (a) Find an expression for $\frac{dy}{dx}$.

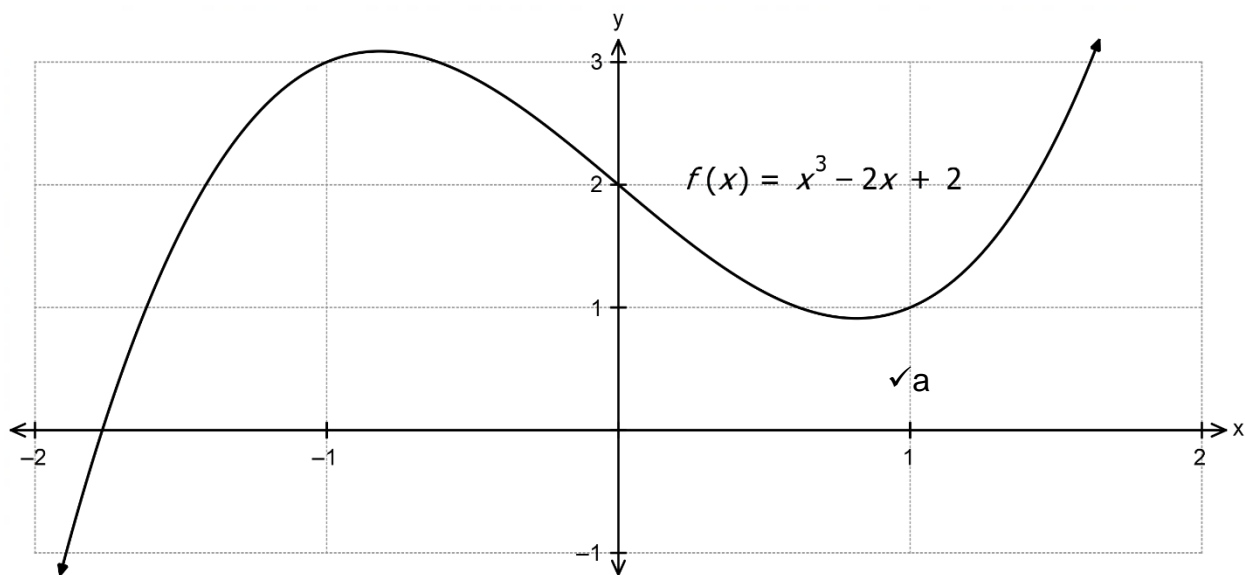
(6)

- (b) Hence find the equation of the **normal** to the ellipse at the point $(-2; 1)$ as indicated by means of the dashed line. A **normal** is a line perpendicular to the tangent to the curve at a point.

(4)
[20]

QUESTION 5

5.1 Consider the graph of the function $f(x) = x^3 - 2x + 2$ as shown below:



Letlotlo is trying to find the x -intercept using the Newton-Raphson method.

(a) Give the iterative formula he will use.

(6)

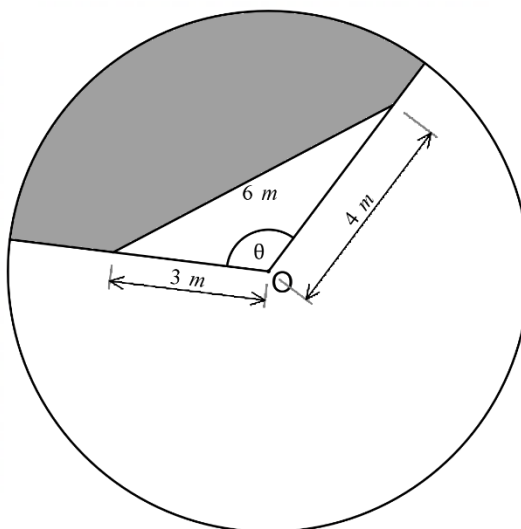
- (b) When he starts with an initial guess of $x_0 = 1$ the method fails to deliver the required result. Illustrate on the above diagram what is happening. Include a sentence to assist you in your explanation.

(4)

- (c) He decides to begin again with an initial guess of $x_0 = -1$.
Give his answer to 5 decimal places.

(2)

- 5.2 Consider the diagram below with measurements as shown. O is the centre of the circle with radius 5 metres.



- (a) Determine the size of θ in radians to 3 decimal places.

(4)

- (b) Hence, or otherwise, determine the area of the shaded region.

(6)
[22]

QUESTION 6

Use mathematical induction to prove that:

$$\sum_{i=1}^n (3i-1)(3i+2) = 3n^3 + 6n^2 + n$$

QUESTION 7

The incidence of a disease grows at a rate proportional to the number of cases according to the following formula:

$$y = y_0 e^{kt}$$

Where y_0 is the initial number of cases, t is the time in months and k is a constant.

- 7.1 If there are currently 10 000 cases and the number of cases grows by 2% per month then determine the value of k to 4 decimal places.

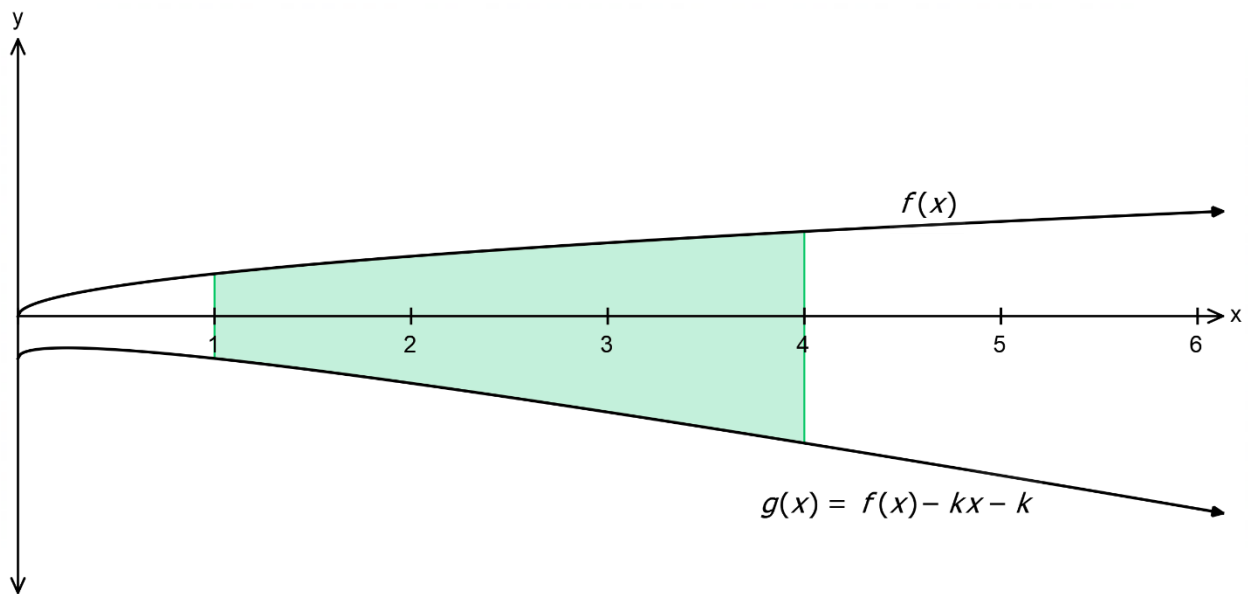
(4)

- 7.2 By first making t the subject of the formula, determine how long it will take before the number of cases first exceeds 100 000.

(6)
[10]

QUESTION 8

The diagram below shows the graphs of the functions $f(x)$ and $g(x) = f(x) - kx - k$.



If $\int_1^4 f(x) dx = \frac{14}{3}$ and the shaded area is $\frac{21}{2}$ units² then determine k showing all working details.

QUESTION 9

Determine the following integrals:

9.1 $\int -\operatorname{cosec}^2 \theta \cot \theta d\theta$

(5)

9.2 $\int \frac{3x}{\sqrt{2x^2 + 5}} dx$

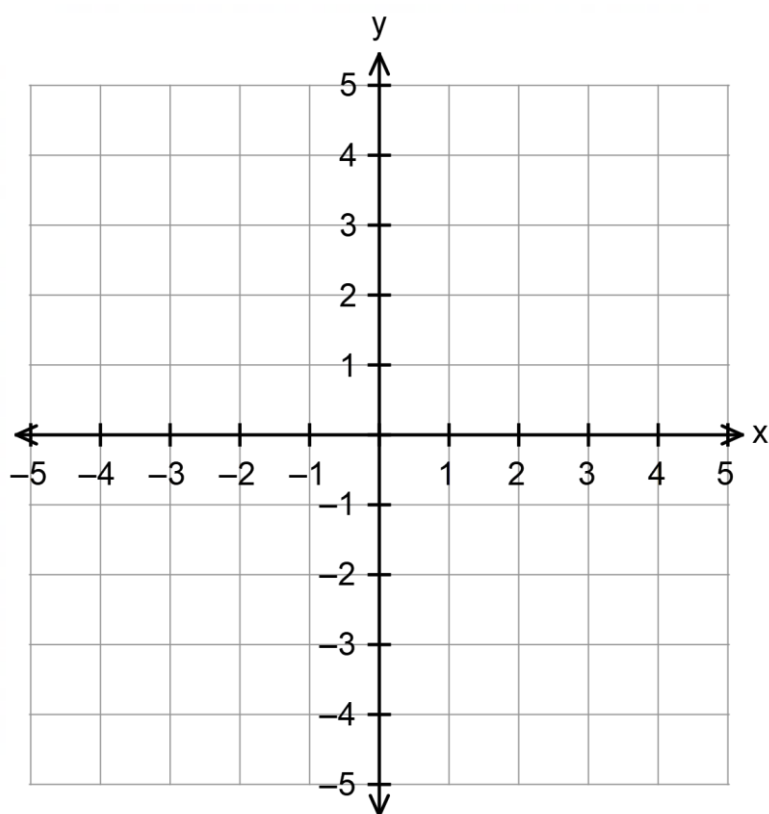
(7)

9.3 $\int \frac{2x^2 + 3x + 8}{x^2 - x - 6} dx$

QUESTION 10

10.1 Use the information in the table below to draw a rough sketch of f on the axes provided:

	$-5 < x < -3$	$x = -3$	$-3 < x < 1$	$x = 1$	$1 < x < 5$
$f(x)$	+'ve	1	+'ve	3	+'ve
$f'(x)$	- 've	d.n.e.	+'ve	0	- 've
$f''(x)$	+'ve		- 've	0	- 've



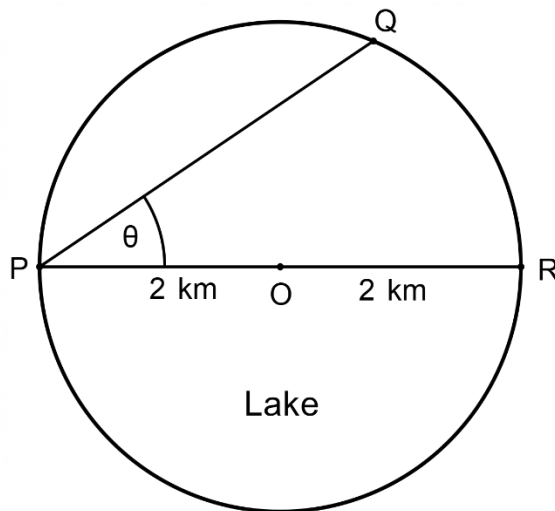
10.2 Determine the coordinates of the stationary point of $g(x) = \sin x \tan x$ on the interval $(0; 4]$.

QUESTION 11

Nonhlanhla and her boyfriend wish to get from P to the opposite side R of a circular lake of radius 2 km, centre O.

When they row, they cover 3 km/h and when they walk around the edge they cover 5 km/h.

In the diagram below chord PQ represents the distance that will be rowed, while arc QR represents the distance that will be walked.



Nonhlanhla has studied calculus and wishes to calculate the angle θ at which she should row across the lake to maximise their time together.

11.1 Show that the time (t) they will have together is given by the formula:

$$t = \frac{4}{3} \cos \theta + \frac{4\theta}{5}$$

11.2 Hence, or otherwise, determine the size of $\theta \in [0; 2\pi]$ that will result in the maximum time t . Express your answer as an angle in radians to 3 decimal places.

(4)
[12]

Total: 200 marks

ADDITIONAL SPACE (ALL QUESTIONS)

REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.

