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TOTAL MARKS

INTERNATIONAL SECONDARY CERTIFICATE EXAMINATION
MAY 2023

FURTHER STUDIES MATHEMATICS (EXTENDED): PAPER II

EXAMINATION NUMBER

Time: 1 hour — 100 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 32 pages and an Information Booklet of 4 pages (i–iv). Please check that your question paper is complete.
2. This question paper consists of THREE modules.

   Choose **ONE** of the **THREE** modules:
   MODULE 2: STATISTICS (100 marks) OR
   MODULE 3: FINANCE AND MODELLING (100 marks) OR
   MODULE 4: MATRICES AND GRAPH THEORY (100 marks)
3. **Answer the questions on the question paper and hand it in at the end of the examination. Remember to write your examination number in the space provided.**
4. Non-programmable and non-graphical calculators may be used, unless otherwise indicated.
5. All necessary calculations must be clearly shown and writing must be legible.
6. Diagrams have not been drawn to scale.
7. **Rounding of final answers.**
   MODULE 2: Four decimal places, unless otherwise stated.
   MODULE 3: Two decimal places, unless otherwise stated.
   MODULE 4: Two decimal places, unless otherwise stated.
8. TWO blank pages (pages 31 and 32) are included at the end of the question paper. If you run out of space for an answer, use these pages. Clearly indicate the number of your answer should you use this extra space.

FOR MARKER'S USE ONLY

Module 2	Q1	Q2	Q3	Q4	Q5	**Total**
Marks	**25**	**14**	**22**	**15**	**24**	**100**

Module 3	Q1	Q2	Q3	Q4	Q5	Q6	**Total**
Marks	**17**	**23**	**10**	**14**	**23**	**13**	**100**

Module 4	Q1	Q2	Q3	Q4	Q5	Q6	**Total**
Marks	**25**	**18**	**13**	**10**	**28**	**6**	**100**

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MODULE 2 STATISTICS

QUESTION 1

At Barnes-burns candle factory, the probability that a candle chosen at random has a lavender scent is 0,1. Each box of candles contains 24 scented candles.

1.1 Define a suitable distribution to model the number of candles, X, that have a lavender scent in the form $X \sim ...$

(2)

1.2 A box is selected at random. Show that the probability that there are at least two lavender-scented candles in this box is 0,7075.

(8)

1.3 A sample of 15 **boxes** is taken. Find the probability that there are exactly nine boxes with at least two lavender-scented candles.

(5)

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1.4 A random sample of 150 candles is taken.

(a) Using a suitable approximation, find the probability that at most 20 of the candles have a lavender scent.

(8)

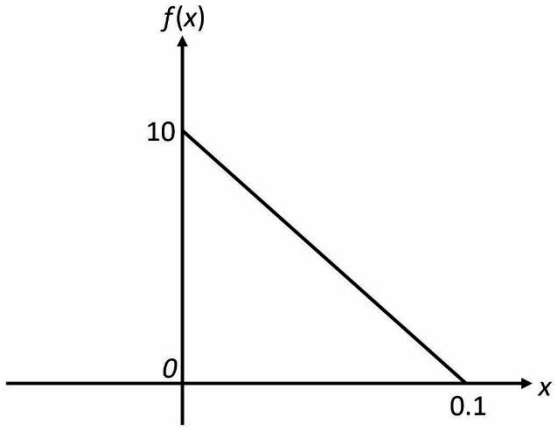
(b) Justify that this is a suitable approximation.

(2)
[25]

QUESTION 2

Determine, giving a reason, whether each of the following graphs can be considered a probability density function.

2.1



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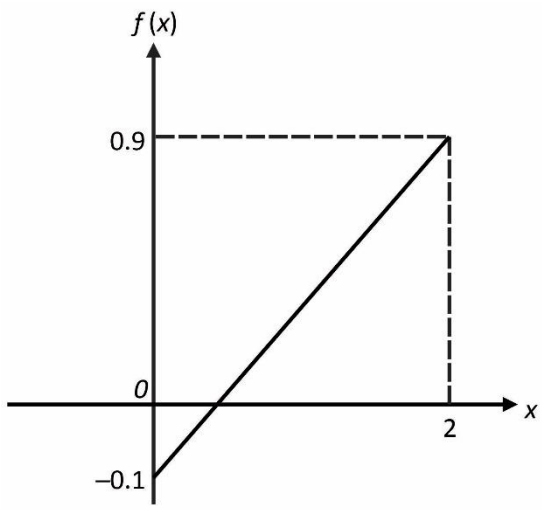
2.2



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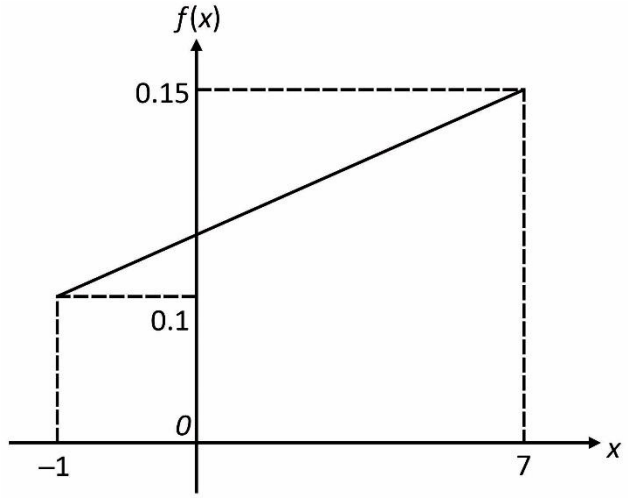
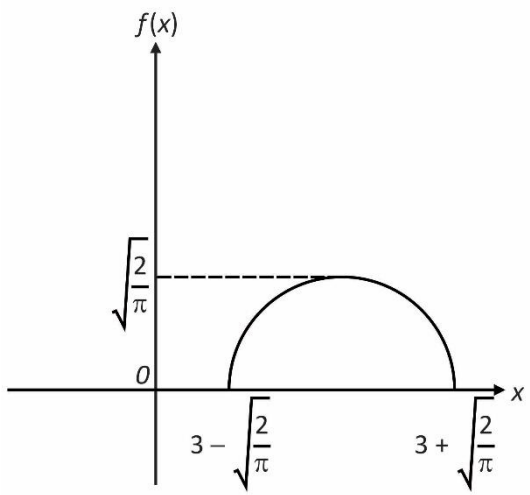
2.3



(4)

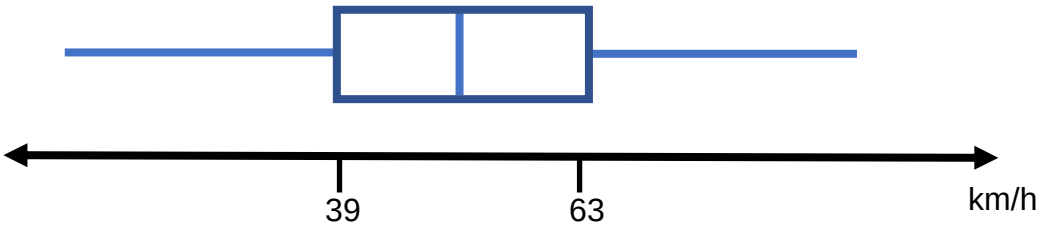
2.4 The semi-circle below.



(4)
[14]

QUESTION 3

3.1 Becky measured the speed of various cars in a residential suburb over a period of one year. The data are displayed in the box-and-whisker plot below and the values of the first and third quartiles are shown. Assume that the speed follows a normal distribution with a mean μ km/h and standard deviation σ km/h.



(a) Calculate values for μ and σ.

(8)

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(b) Two of these cars from this residentail suburb go through a speed trap. Find the probability that exactly one of them is travelling faster than 63 km/h.

(5)

3.2 A nutritionist is interested in the mean fat content of a particular type of cheese. She takes a random sample of 20 equal mass blocks of cheese and sends them to be analysed. A summary of results of the percentage of fat in the 20 blocks of cheese is as follows.

$$\sum x = 620 \qquad \sigma = 3$$

Assume that the percentage of fat is normally distributed.

(a) Calculate a 98% confidence interval for the population mean percentage of fat.

(7)

(b) The manufacturer claims that the mean percentage of fat in this type of cheese block is 30%. Use your answer to part (a) to determine whether the nutritionist should accept this claim.

(2)
[22]

QUESTION 4

A random sample of 25 observations from the random variable $X \sim N\left(\mu;60^2\right)$ is used to test the null hypothesis $H_0 : \mu = 400$ against an alternative hypotheses.

4.1 Given that $H_1 : \mu > 400,$ find the values of $\bar{X}$ that would result in the null hypothesis being rejected at the 5% significance level.

(5)

4.2 Given that $H_1 : \mu < 400$ and that the values of $\bar{X} < 382$ result in the null hypothesis being rejected. Determine at what significance level this occurs.

(5)

4.3 Given that $H_1 : \mu \neq 400,$ determine the conclusion to the hypothesis test when using a 5% significance level if the sample mean is $\bar{X} = 379.$

(5)
[15]

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QUESTION 5

5.1 Jonty plays a game. He can score 0, 1, 2, 3 or 4 points each time the game is played. The random variable X, representing his score, has the following probability distribution where a, b and c are constants.

x	0	1	2	3	4
$P(X = x)$	a	b	c	0,1	0,15

- The probability of scoring less than 2 points is twice the probability of scoring at least 2 points.
- Each game is independent of previous games played.
- Jonty plays the game twice and adds the two scores to get a total.

Calculate the probability that the total is 6 points.

(8)

5.2 Neo is an avid baker. She is planning to bake 14 cakes for a function. She plans to bake 3 identical cheesecakes, 2 identical carrot cakes, 4 identical chocolate cakes, 2 identical sponge cakes and 3 identical fruit cakes.

(a) The 14 cakes will be equally divided between two venues. Find the number of possible arrangements if all the cheesecakes and chocolate cakes are in one venue and all the rest are in the other venue.

(8)

(b) Four of the 14 cakes will be selected for judging. Determine the probability that of the four selected, exactly one cake will be a cheesecake and one will be a chocolate cake.

(8)
[24]

Total for Module 2: 100 marks

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MODULE 3 FINANCE AND MODELLING

QUESTION 1

Rekesh wants to save for an overseas holiday in 4 years' time. He saves R5 500 each month, starting immediately. The initial interest rate is 7,8% per annum compounded monthly. After 30 months, he is able to increase his payment to R7 000 per month. Then, for the last year, the interest rate increases to 9,2% per annum compounded monthly.

1.1 Determine the balance in his savings account after:

(a) 30 months

(4)

(b) 36 months

(6)

1.2 Hence, determine the final amount after 4 years.

(7)
[17]

QUESTION 2

Sicelo takes a loan for R20 000 000 in order to buy a large property in Cape Town. He repays the loan by means of equal monthly payments starting at 8 months, from when the loan was secured, and finishing after 15 years. In addition, he is able to pay in an annual lump sum of R750 000 at the end of each year. The effective interest rate on the loan is 11,5% per annum.

2.1 Calculate the nominal interest rate with monthly compounding. Give the percentage of the interest rate accurate to 2 decimal places.

(3)

2.2 Calculate the total value of the annual lump sum payments over the course of the loan.

(6)

2.3 Calculate Sicelo's monthly payment and, hence, determine the total interest paid over the course of the loan.

(14)
[23]

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QUESTION 3

Marion invests a lump sum of R20 000, where interest is compounded quarterly. At the same time, Jarryd starts an investment of R50 per day at 8% per annum compounded daily. Determine the effective interest rate that Marion must receive so that the two investments are equal in value after 10 years. (Assume 365 days in a year).

[10]

QUESTION 4

A population of weasels has a birth rate of b per annum and a death rate of d per annum. The average lifespan of a weasel is 3 years. The female produces a litter of 5, three times per year. About 50% are female, of which 30% produce young, and there is a 60% mortality rate at birth. A certain number are killed by poachers.

4.1 Determine the intrinsic per annum growth rate of the weasels.

(7)

4.2 State, with reasoning, the assumption you would make about which population growth model might be appropriate.

(2)

4.3 If it is found that an initial population of 90 remains constant, given the level of poaching. Determine the number of weasels lost to poaching each year.

(5)
[14]

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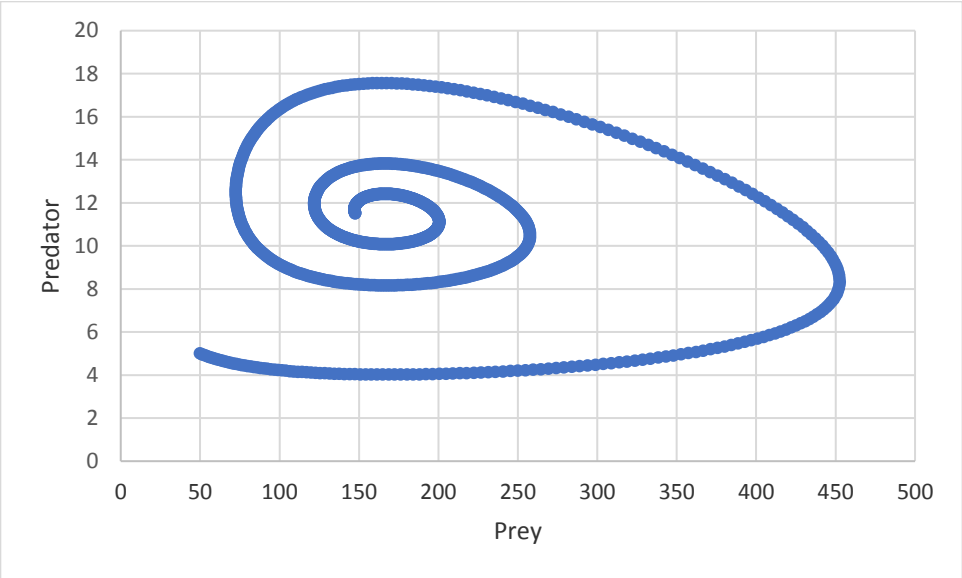
QUESTION 5

The following information is given about a predator-prey system and its related phase plot:

a = 0,04 b = 0,003 c = 0,01 f = 0,02 K = 1200



5.1 (a) Estimate, from the diagram, the initial numbers of predator and prey.

(2)

(b) Calculate, using the given parameters, the equilibrium numbers of predator and prey.

(7)

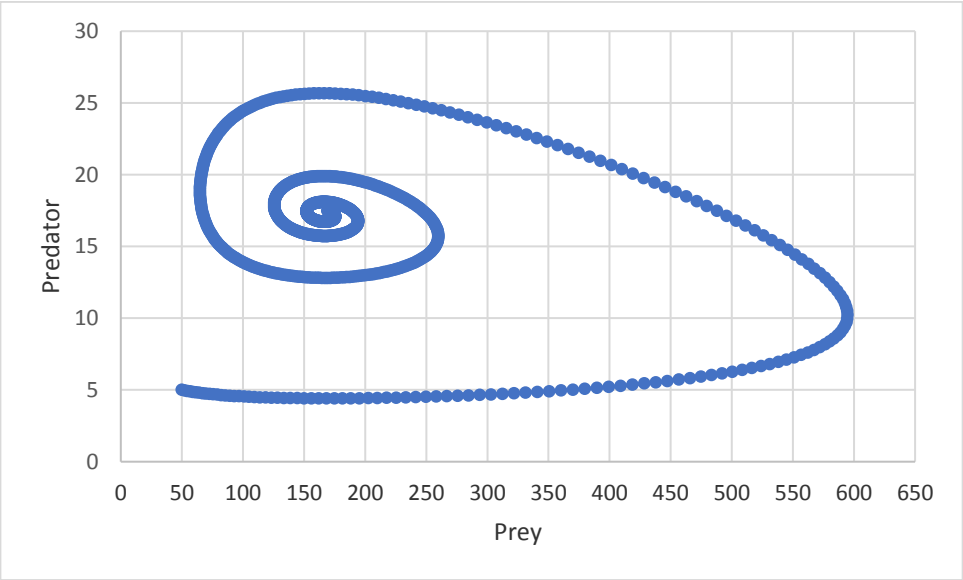
(c) Estimate, from the diagram, the number of predators and prey when the predator is first declining at its most rapid rate.

(2)

5.2 In each case below, the given parameter has changed.

- State whether it has increased or decreased.
- Describe the effect of the model reaching the equilibrium point, giving reasons.

(a) Parameter a



(3)

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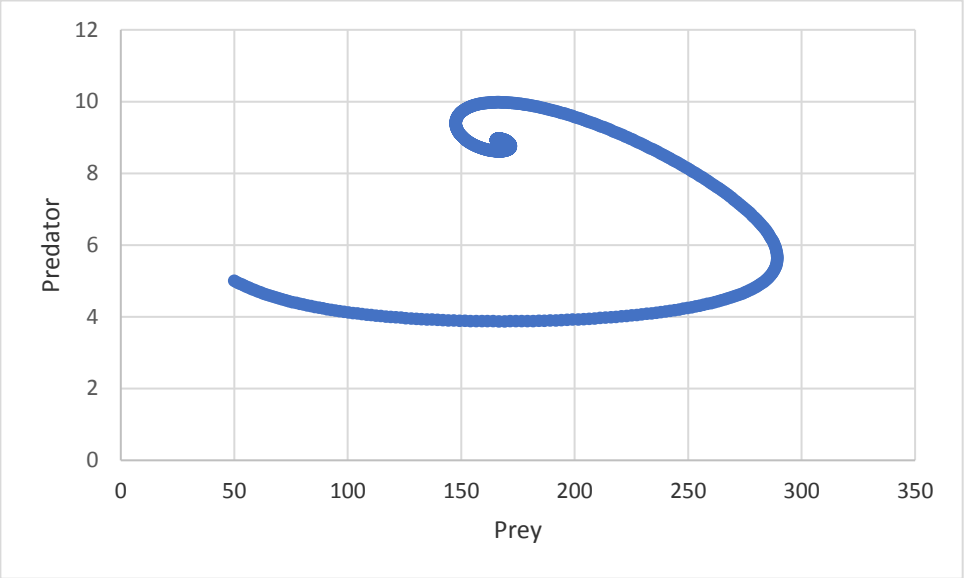
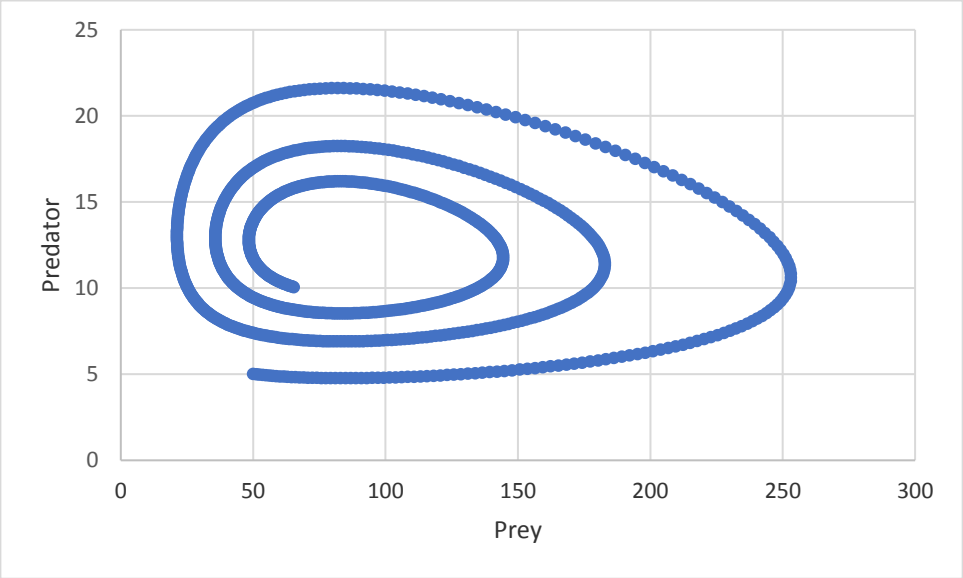
(b) Parameter K



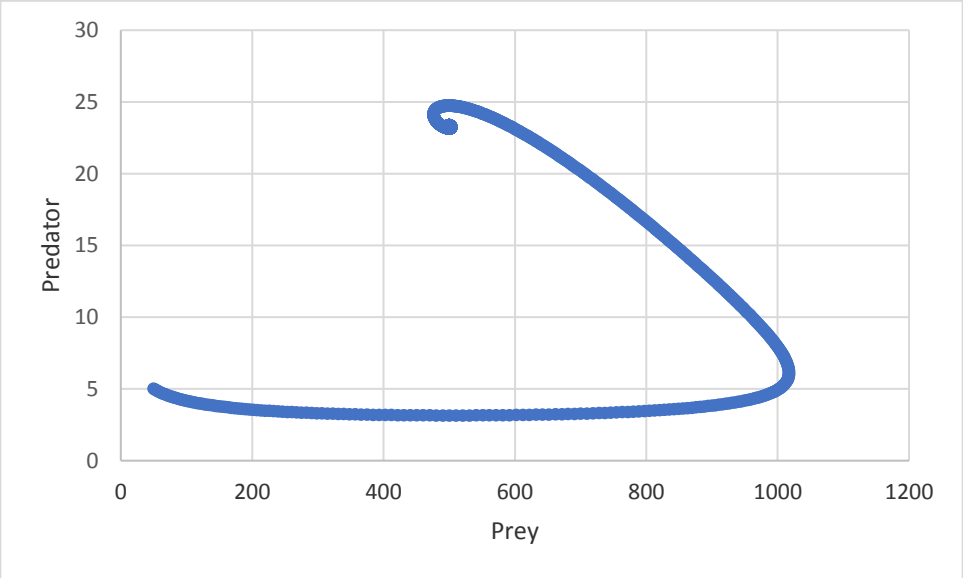
(3)

(c) Parameter *f*



(3)

(d) Parameter *b*



(3)
[23]

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QUESTION 6

A recursive rule is given: $T_{n+1} = aT_n - \frac{T_n^2}{b}$; n = 0, 1, 2, 3, ….

6.1 Assume a = 1,6 and b = 5 000.

Given that $T_1 = 158$, calculate the possible value(s) of T_0.

(4)

6.2 Given that $T_0 = 50$, $T_1 = \frac{175}{2}$ and $\lim_{n\to\infty} T_n = 800$

Calculate a and b.

(9)
[13]

Total for Module 3: 100 marks

MODULE 4 MATRICES AND GRAPH THEORY

QUESTION 1

1.1 $2x+6y+1z=7$, $1x+2y-1z=-1$ and $5x+7y-4z=9$, are equations in a 3-dimensional plane.

Use matrix algebra and/or row-reduction to solve for x, y and z.

(10)

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1.2 Suppose $A = \begin{pmatrix} 2 & 1 \\ 6 & 3 \\ 2 & 5 \end{pmatrix}$, show that $A.A^T$, is a symmetric matrix.

(7)

1.3 If matrix $A = \begin{pmatrix} 5 & 1 & 2 & 4 \\ -1 & 0 & 2 & 3 \\ 1 & 1 & 6 & 1 \\ 1 & 0 & 0 & -4 \end{pmatrix}$, show that the determinant is 34.

(8)
[25]

QUESTION 2

2.1 A triangle PQR has vertices $P(-1;-1)$, $Q(3;1)$ and $R(0;3)$.

For each transformation, show the matrix transformation and state the vertices of the image, in matrix form.

(a) Enlargement by a factor of $\frac{3}{2}$.

(3)

(b) Reflection across the x-axis.

(3)

(c) Reflection about the line $y = 3x$.

(6)

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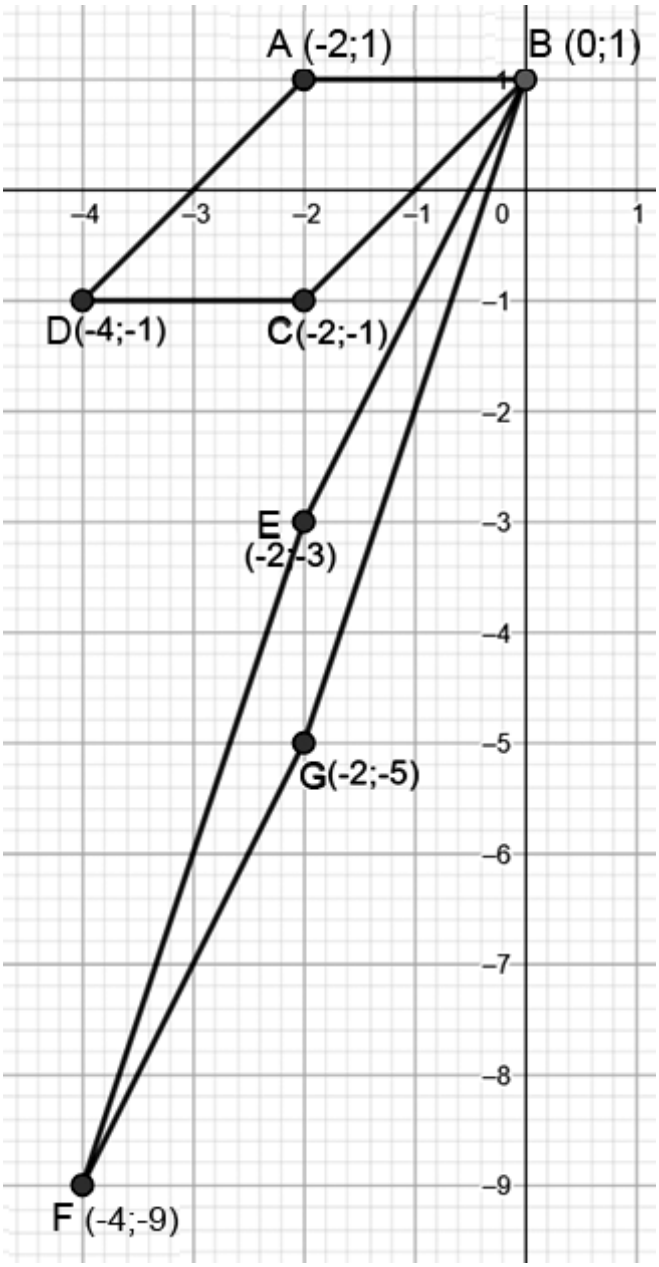
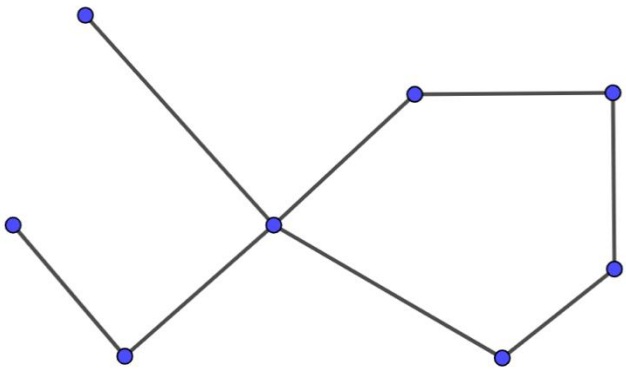
2.2 The shape ABCD, is mapped to EBGF, state the transformation(s) and find the matrix that transformed the shape.



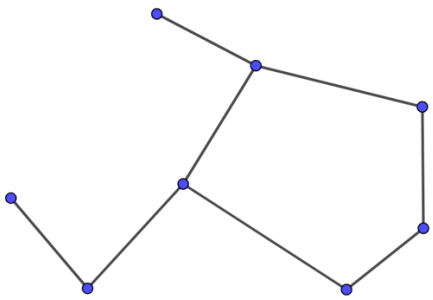
(6)
[18]

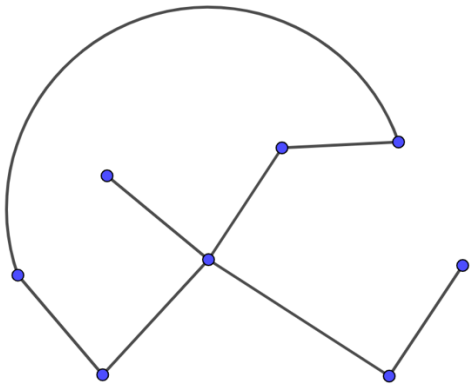
QUESTION 3

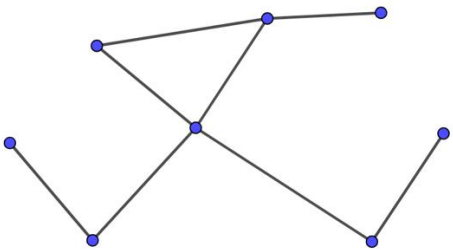
3.1 Given the graph.

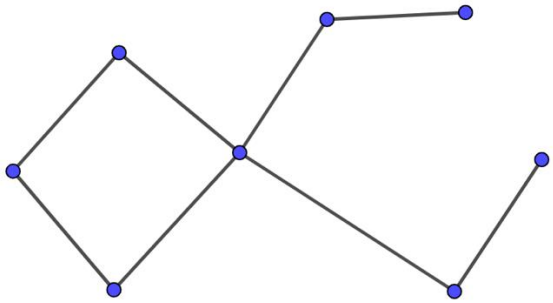


Which of the following graph(s) below are isomorphs of the graph above.

1. 

2. 

3. 

4. 

(4)

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3.2 (a) Let a graph G be a complete graph, with n vertices, where $n > 2$. Then the number of unique Hamiltonian circuits of G is:

1 ☐ n!

2 ☐ (n – 1)!

3 ☐ 1

4 ☐ $\frac{(n-1)!}{2}$

(2)

(b) A simple connected graph G has an Eulerian circuit if and only if all the vertices are of:

1 ☐ the same degree

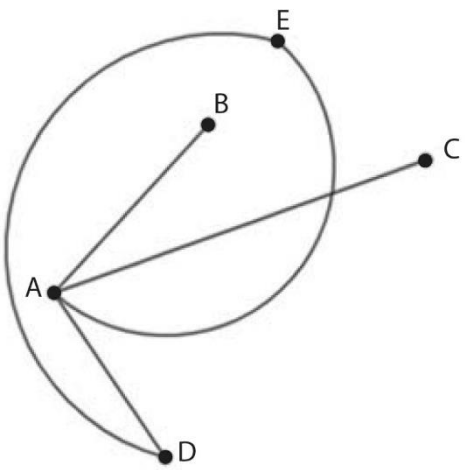
2 ☐ even degree

3 ☐ odd degree

4 ☐ different degree

(2)

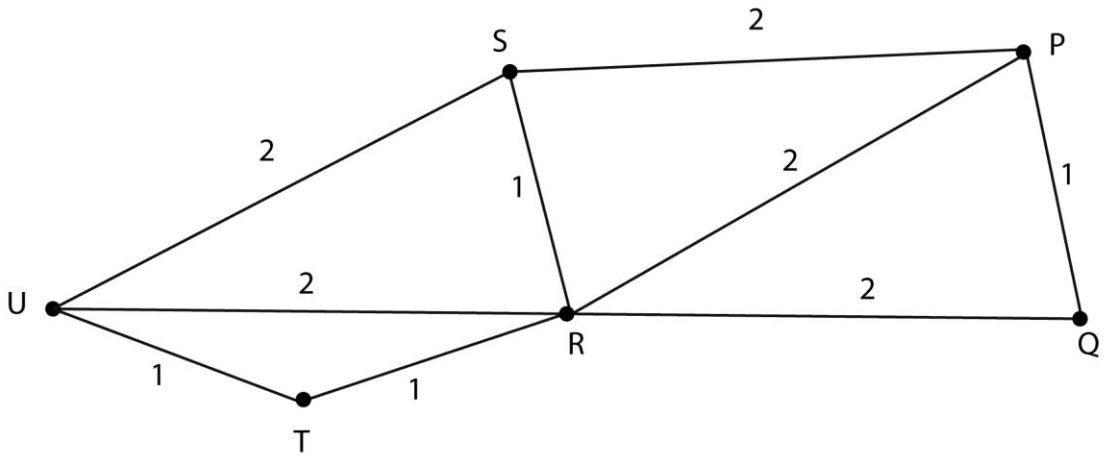
3.3 Draw the complement of:



(5)
[13]

QUESTION 4

Find all the unique minimum spanning trees for the graph below and write down the weight.



[10]

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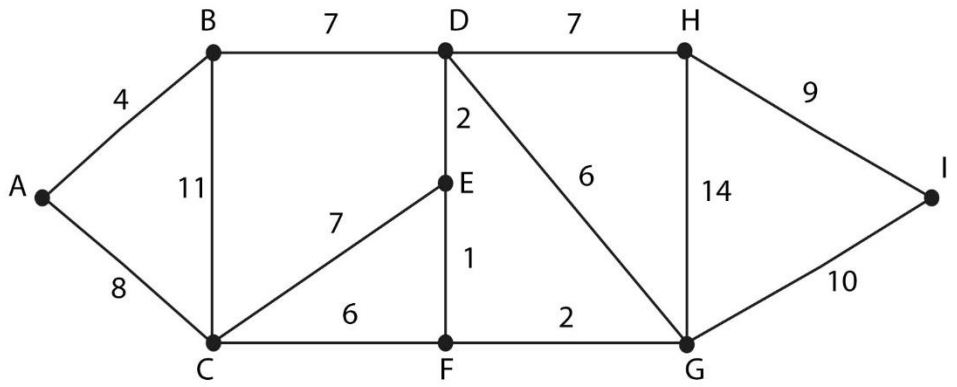
QUESTION 5

5.1 A maze is set up to test the spread of a bacteria. The weights in the graph represent energy used to traverse the edge. The bacteria can use at most 13 units of energy to traverse the first two edges if it is going to have enough energy to traverse the maze. Use Dijkstra's algorithm to find the route that uses the least energy to traverse the graph starting at A and ending at I. Show clear evidence of your working including the termination of non-viable routes. Write down the route and its weight.



(12)

5.2 Mark is wanting to travel in a new city. He identifies locations J, K, L, M and N, and notes the time taken (in minutes) on public transport, to travel to and tour each location.

	J	K	L	M	N
J	–	165	185	65	160
K	165	–	155	115	275
L	185	155	–	205	125
M	65	115	205	–	225
N	160	275	125	225	–

(a) Use the nearest neighbour algorithm, starting and ending at J, to find the upper boundary for Mark to tour the five locations.

(8)

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(b) By removing J, find the lower bound of Mark's tour.

(6)

(c) Mark has to meet friends after his tour. The tour starts at 10:00 a.m. By using your answers from the above questions recommend a time that Mark and his friends should meet.

(2)
[28]

QUESTION 6

Assume the determinants in this question have no zeros and no repeated rows or columns.

- To find a 3×3 determinant, you have to find three, 2×2 determinants.
- To find a 4×4 determinant, you have to find four, 3×3 determinants. Each of which become three, 2×2 determinants for a total of twelve, 2×2 determinants.

Using the pattern above, how many 2×2 determinants will be required for a 7×7 determinant?

[6]

Total for Module 4: 100 marks

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ADDITIONAL SPACE (ALL questions)

REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.

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