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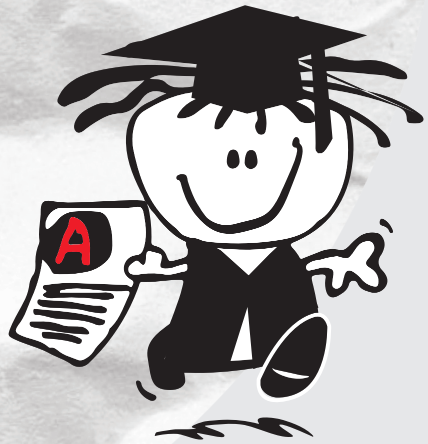
Mathematics

OFFICIAL DBE/IEB EXAMS & MEMOS

Anne Eadie, Gretel Lampe, Jenny Campbell & Susan Carletti

GRADE
12
CAPS



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This low-priced product, offering both theory and practice, is perfect for 'remote' exam preparation for matrics, particularly during an extremely challenging time, following the loss of teaching and learning countrywide.

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CONTENTS: Gr 12 Exams for DBE & IEB

NOTE:
The questions marked with an asterisk (*) are **Level 3 & 4 questions** – identified as being those that learners struggled with in the exam!

We are grateful to the Department of Basic Education and the IEB for granting their permission for the inclusion of these exam papers.

USEFUL REMINDERS

A helpful reference for what to study before a test or exam

PAPER 1

Linear & Quadratic Equations

Solve using . . .

- Factorising
- Substitution method or the k-method
- Quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Nature of Roots

- Use Δ (the discriminant) to classify roots: $x = \dfrac{-b \pm \sqrt{\Delta}}{2a}$, where $\Delta = b^2 - 4ac$

Simultaneous Equations

Fractions

- Denominators and/or numerators may need to be factorised
- Check for zero denominators & invalid solutions

Linear & Quadratic Inequalities

- Number lines
- Interval and inequality notation

Exponents & Surds & Logs

- Exponent, Surd and Log Laws
- Surd equations must be checked for extraneous answers
- Logs . . . Definition: $x = b^a \Leftrightarrow \log_b x = a$
- Solve log equations & inequalities using graphs

Patterns & Sequences

See Sum Formulae on p. i

Linear Patterns (APs): $T_n = an + b$ or $T_n = a + (n-1)d$ & $S_n = \dfrac{n}{2}(a + T_n)$;

$S_n = \dfrac{n}{2}[2a + (n-1)d]$

think **+** and **–** for **APs**

- constant first difference: $d = T_n - T_{n-1}$ . . . Def: $T_2 - T_1 = T_3 - T_2$

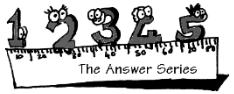
Exponential Patterns (GPs): $T_n = ar^{n-1}$ & $S_n = \dfrac{a(r^n - 1)}{r - 1}$; $S_n = \dfrac{a(1 - r^n)}{1 - r}$;

- Sum to infinity: $S_\infty = \dfrac{a}{1 - r}$ for $-1 < r < 1$
- constant ratio: $r = \dfrac{T_n}{T_{n-1}}$ . . . Def: $\dfrac{T_2}{T_1} = \dfrac{T_3}{T_2}$

think **×** and **÷** for **GPs**

Quadratic Patterns: $T_n = an^2 + bn + c$

- constant second difference



Sigma: $\sum_{k=1}^{n} T_k = S_n$ . . . Note: $T_n = S_n - S_{n-1}$

Finance

Simple Interest Growth & Decay $A = P(1 \pm in)$

- Application of SI Growth involving hire purchase: Find interest rate, no. of years or principle amount
- Simple Interest Decay = Straight line Depreciation

Compound Interest Growth & Decay $A = P(1 \pm i)^n$

- Applications involving inflation, population growth, exchange rates
- Find P, i, or n (using logs)
- The effect of different compounding intervals
- Compound Interest Decay = Depreciation on a Reducing Balance

Effective and Nominal Interest Rates

Convert fluently between **nominal** and **effective** interest rates for: monthly, quarterly, half-yearly/semi-annual compounding periods

Time lines

Annuities

Present Value Annuity: $P_v = \dfrac{x\left[1 - (1 + i)^{-n}\right]}{i}$

&

Future Value Annuity: $F_v = \dfrac{x\left[(1 + i)^n - 1\right]}{i}$

. . . where payment commences 1 time period from the present and ends at n.

- Interest must be compounded at the same rate as the payments
- Calculate the value of any of the variables in the above formulae except i
- Keep an eye out for deferred payments, early payments, missed payments
- Interest
- Balance Outstanding

DBE P1: TOPIC GUIDE	2014	2015	2016	2017	2018	2019	2020
➤ **Algebra: [25]**							
Quadratic equations & theory	1.1.1, 1.1.2, 1.4	1.1.1, 1.1.2, 1.3*	1.1.1, 1.1.2, 1.2.1	1.1.1, 1.1.2	1.1.1, 1.1.2	1.1.1, 1.1.2	1.1.1, 1.1.2
Quadratic inequalities	1.3	1.1.5	1.2.2	1.3.1	1.1.3	1.1.3	1.1.3,
Simultaneous equations	1.2	1.2*	1.3	1.2	1.2	1.2	1.2
Expressions							
➤ **Exponents:**							
Expressions					1.3*		
Equations & inequalities	1.1.3	1.1.3	1.1.4			1.3*	1.3*
➤ **Surds:**							
Expressions							
Equations		1.1.4	1.1.3	1.1.3	1.1.4	1.1.4	1.1.4
➤ **Logs (Application):**							
➤ **Patters & Sequences: [25]**							
Quadratic	3.1		3.1*	2.1		2.1	2.2
Arithmetic	2.1, 2.2, 2.4, 2.5	3.1 – 3.3, 3.4*	2.1 – 2.3, 2.4*	2.2			2.1
Geometric	3.2	2.1 – 2.4	3.2*		3.1, 3.2	2.2	11.3*
Σ	2.3			3*	3.3, 3.4*	3.1*	3.1, 3.2*
Mixed / General	3.3				2.1 – 2.3	3.2	
➤ **Finance, growth & decay: [15]**							
Simple & compound growth & decay	7.1	7.1 – 7.3		6.1		6.1	6.2
Annuities	7.2	7.4*	7.1 – 7.3, 7.4*	6.2*	7.2	6.2	6.1, 6.3*
Time line					7.1*		
➤ **Functions & Graphs: [35]**							
Straight line and/or parabola		5.1, 6.1.1 – 6.1.3		1.3*, 4.1 – 4.4, 4.5*, 4.6, 4.7*	6.1 – 6.3, 6.4*, 6.6*		
Hyperbola	4	6.2			5.1 – 5.3,		4.1
Exponent. & log function (incl. Inverses)	5	4.1 – 4.3, 5.2*, 5.3, 5.4	4.1 – 4.4, 4.5*				
Inverse functions					4.1 – 4.3, 4.4*	5.1 – 5.3, 5.4*, 5.5*	5.1, 5.2, 5.3*, 5.4*, 5.5
Mixed	6*	5.5*	5.1, 5.2*, 5.3, 5.4*, 5.5*, 6.1, 6.2, 6.3*, 6.4	5.1 – 5.5, 5.6*		4.1 – 4.6, 4.7*	4.2
➤ **Differential Calculus: [35]**							
Finding the derivative: 1st principles	8.1	8.1	8.1, 8.2*	7.1	8.1	7.1	7.1
Finding the derivative: using the rules	8.2, 8.3	8.2	8.3	7.2	8.2	7.2, 7.3	7.2, 8.4
(or) Finding the average gradient		9.2					
Tangent: the gradient & the equation		9.5*	8.4*			7.4*	
Curve sketching & f'' & concavity	8.4, 9.1 – 9.3	5.6*, 6.1.4, 9.1, 9.3*, 9.4*	5.6*, 9.1, (9.2 – 9.4)*	8*	5.4*, 6.5*, 9.1*, 9.2	9.1, 9.2*, 9.3, 9.4*	8.1, 8.2, 8.3*
Practical application (incl. Max/min)	10*	10*	1.2.3, 10.1, 10.2*, 10.3*	9*	10*	8.1, 8.2*, 8.3	8.5*, 9.1*, 9.2*
➤ **Probability: [15]**							
Probability rules		11.1			12.1		
Venn diagrams				10.1, 10.2*, 10.3*		11.1*	
Tree diagrams		11.3*			12.2*		11.1*, 11.2
2-way contingency tables	11*		11.1, 11.2*, 11.3				
Fundamental Counting Principle	12*	11.2*	12*	11*	11*	10*, 11.2	10*

Questions marked with an asterisk (*) are **Level 3 & 4**/Challenging Questions.

DBE NOV 2015 PAPER 1

Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.

Answers only will NOT necessarily be awarded full marks.

You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

*If necessary, round off answers to **TWO** decimal places, unless stated otherwise.*

▸ ALGEBRA AND EQUATIONS AND INEQUALITIES [26]

QUESTION 1

Answers on p. A9

1.1 Solve for x:

1.1.1 $x^2 - 9x + 20 = 0$ (3)

1.1.2 $3x^2 + 5x = 4$ *(correct to TWO decimal places)* (4)

1.1.3 $2x^{\frac{-5}{3}} = 64$ . . . *No calculator!* (4)

1.1.4 $\sqrt{2 - x} = x - 2$ (4)

1.1.5 $x^2 + 7x < 0$ (3)

1.2* Given: $(3x - y)^2 + (x - 5)^2 = 0$
Solve for x and y. (4)

1.3* For which value(s) of k will the equation $x^2 + x = k$ have no real roots? (4) **[26]**

▸ PATTERNS AND SEQUENCES [22]

QUESTION 2

. . . *Geometric Sequence* *Answers on p. A9*

The following geometric sequence is given:

10 ; 5 ; 2,5 ; 1,25 ; . . .

2.1 Calculate the value of the 5th term, T_5, of this sequence. (2)

2.2 Determine the n th term, T_n, in terms of n. (2)

2.3 Explain how you know that the infinite series
10 + 5 + 2,5 + 1,25 + . . . converges. (2)

2.4 Determine $S_\infty - S_n$ in the form ab^n, where S_n is the sum of the first n terms of the sequence. (4) **[10]**

QUESTION 3

Answers on p. A10

Consider the series: $S_n = -3 + 5 + 13 + 21 + \ldots$ to n terms.

3.1 Determine the general term of the series in the form $T_k = bk + c$. (2)

3.2 Write S_n in sigma notation. (2)

3.3 Show that $S_n = 4n^2 - 7n$. (3)

3.4* Another sequence is defined as:

$Q_1 = -6$
$Q_2 = -6 - 3$
$Q_3 = -6 - 3 + 5$
$Q_4 = -6 - 3 + 5 + 13$
$Q_5 = -6 - 3 + 5 + 13 + 21$

3.4.1 Write down a numerical expression for Q_6. (2)

3.4.2 Calculate the **value** of Q_{129}. (3) **[12]**

▸ FUNCTIONS AND GRAPHS [37]

QUESTION 4

Answers on p. A10

Given: $f(x) = 2^{x+1} - 8$

4.1 Write down the equation of the asymptote of **f**. (1)

4.2 Sketch the graph of **f**. Clearly indicate ALL intercepts with the axes as well as the asymptote. (4)

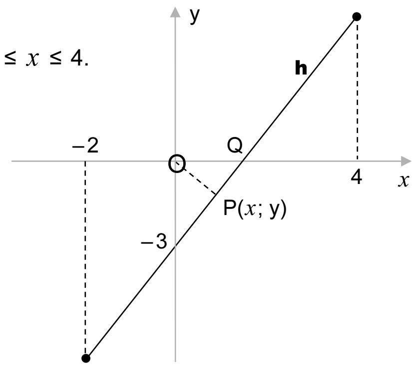
4.3 The graph of **g** is obtained by reflecting the graph of **f** in the y-axis. Write down the equation of **g**. (1) **[6]**

QUESTION 5

Answers on p. A10

Given:
$h(x) = 2x - 3$ for $-2 \le x \le 4$.

The x-intercept of h is at Q.



5.1 Determine the coordinates of Q. (2)

5.2* Write down the domain of $\mathbf{h^{-1}}$. (3)

5.3 Sketch the graph of $\mathbf{h^{-1}}$, clearly indicating the y-intercept and the end points. (3)

5.4 For which value(s) of x will $h(x) = h^{-1}(x)$? (3)

5.5* P(x; y) is the point on the graph of **h** that is closest to the origin. Calculate the distance OP. (5)

5.6* Given: $h(x) = f'(x)$ where **f** is a function defined for $-2 \le x \le 4$.

5.6.1 Explain why **f** has a local minimum. (2)

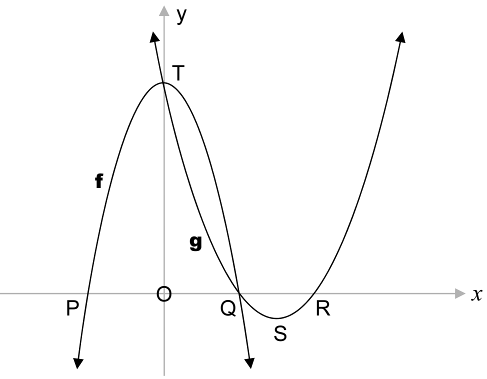
5.6.2 Write down the value of the maximum gradient of the tangent to the graph of **f**. (1) **[19]**

QUESTION 6

Answers on p. A11

6.1 The graphs of $f(x) = -2x^2 + 18$ and $g(x) = ax^2 + bx + c$ are sketched below.

Points P and Q are the x-intercepts of **f**. Points Q and R are the x-intercepts of **g**. S is the turning point of **g**. T is the y-intercept of both **f** and **g**.



6.1.1 Write down the coordinates of T. (1)

6.1.2 Determine the coordinates of Q. (3)

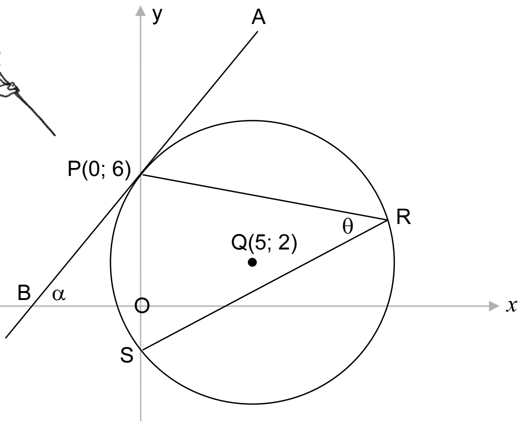
6.1.3 Given that $x = 4{,}5$ at S, determine the coordinates of R. (2)

6.1.4 Determine the value(s) of x for which $g''(x) > 0$. (2)

QUESTION 4

Answers on p. A17

In the diagram below, Q(5; 2) is the centre of a circle that intersects the y-axis at P(0; 6) and S. The tangent APB at P intersects the x-axis at B and makes the angle α with the positive x-axis. R is a point on the circle and $P\hat{R}S = \theta$.



4.1 Determine the equation of the circle in the form $(x - a)^2 + (y - b)^2 = r^2$. (3)

4.2 Calculate the coordinates of S. (3)

4.3 Determine the equation of the tangent APB in the form $y = mx + c$. (4)

4.4 Calculate the size of α. (2)

4.5 Calculate, with reasons, the size of θ. (4)

4.6 Calculate the area of ΔPQS. (4) **[20]**

▸ TRIGONOMETRY [42]

Feeling rusty or confused?
Refer to the Trig Summary on p. vii.

QUESTION 5

Answers on p. A17

5.1 Given that $\sin 23° = \sqrt{k}$, determine, in its simplest form, the value of each of the following in terms of k, WITHOUT using a calculator:

5.1.1 sin 203°   5.1.2 cos 23° (2)(3)

5.1.3 tan(–23°) (2)

Need help – go to pp. v & vi to master Compound and Double Angle Formulae.

5.2* Simplify the following expression to a single trigonometric function:

$$\frac{4\cos(-x)\,.\,\cos(90° + x)}{\sin(30° - x)\,.\,\cos x \;+\; \cos(30° - x)\,.\,\sin x}$$ (6)

5.3 Determine the general solution of $\cos 2x - 7\cos x - 3 = 0$. (6)

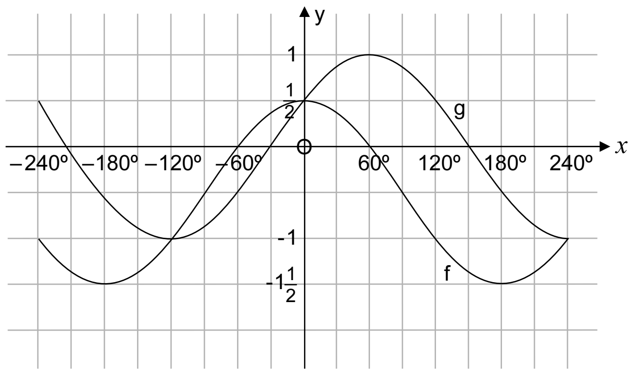
5.4* Given that $\sin\theta = \frac{1}{3}$, calculate the numerical value of $\sin 3\theta$, WITHOUT using a calculator. (5) **[24]**

QUESTION 6

Answers on p. A18

In the diagram below, the graphs of $f(x) = \cos x + q$ and $g(x) = \sin(x + p)$ are drawn on the same system of axes for $-240° \leq x \leq 240°$.

The graphs intersect at $\left(0°; \frac{1}{2}\right)$, (–120°; –1) and (240°; –1).



6.1 Determine the values of p and q. (4)

6.2 Determine the values of x in the interval $-240° \leq x \leq 240°$ for which $f(x) > g(x)$. (2)

6.3* Describe a transformation that the graph of **g** has to undergo to form the graph of **h**, where $h(x) = -\cos x$. (2) **[8]**

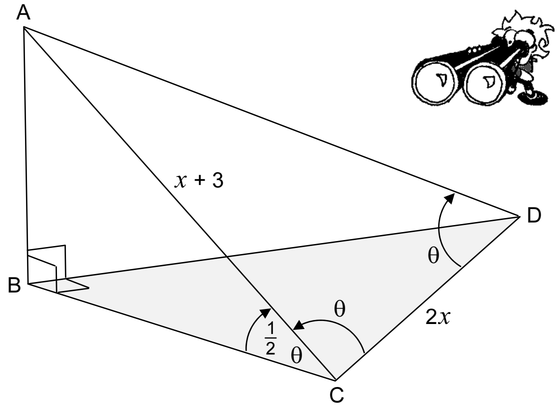
QUESTION 7*

Answers on p. A18

A corner of a rectangular block of wood is cut off and shown in the diagram below.

The inclined plane, that is, ΔACD, is an isosceles triangle having $A\hat{D}C = A\hat{C}D = \theta$.

Also $A\hat{C}B = \frac{1}{2}\theta$, AC = $x + 3$ and CD = $2x$.



7.1 Determine an expression for $C\hat{A}D$ in terms of θ. (1)

7.2 Prove that $\cos\theta = \frac{x}{x + 3}$. (4)

7.3 If it is given that $x = 2$, calculate AB, the height of the piece of wood. (5) **[10]**

Your tools . . .

RIGHT ANGLED Δ^S	NON-RIGHT ANGLED Δ^S
❶ Regular trig ratios	❶ Sine rule
❷ Theorem of Pythagoras	❷ Cos rule

Also: Area of a $\Delta = \frac{1}{2}bh$ *or* $\frac{1}{2}ab\sin C$

See the Paper 2 Topic Guides (on pp. 2 & 40) to select and practice more examples.

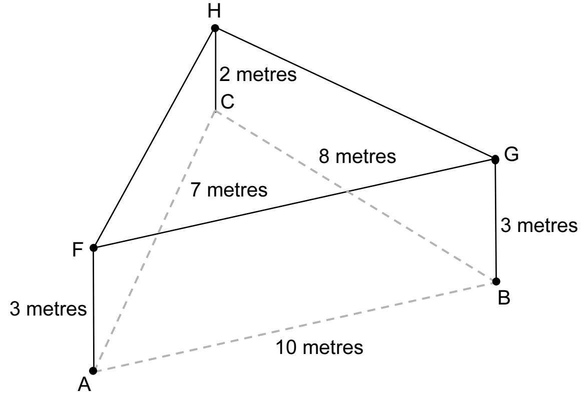
Also see p. 23 of the **EXTENSION Booklet** on CHALLENGING QUESTIONS accompanying our **Gr 12 Maths 2-in-1** study guide (the booklet also forms part of the Gr 12 Maths 2-in-1 eBook).

QUESTION 9

Answers on p. A84

9.1 A metal frame is built to help provide some shade to a triangular piece of land ABC.

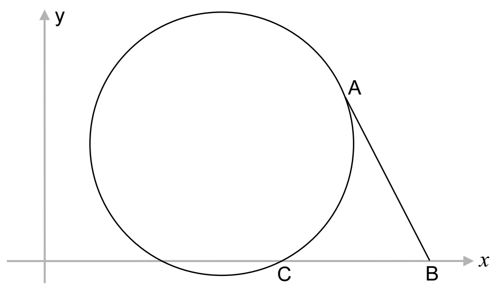
- A, B and C are on the same horizontal plane.
- AC = 7 metres; CB = 8 metres and AB = 10 metres.
- AF, BG and CH are vertical metal poles.
- AF = BG = 3 metres and CH = 2 metres.
- HF, FG and GH are metal poles that complete the metal frame.



Calculate the area of ΔFGH. (The area of canvas required.) (7)

9.2 In the diagram below, C and A are points that lie on the circle.

- C and B lie on the x-axis.
- AB is a tangent at point A(5; 3).
- The equation of the circle is $x^2 + y^2 - 6x - 4y + 8 = 0$.



9.2.1 Find the coordinates of C. (2)

9.2.2 Calculate the length of CB. (8)

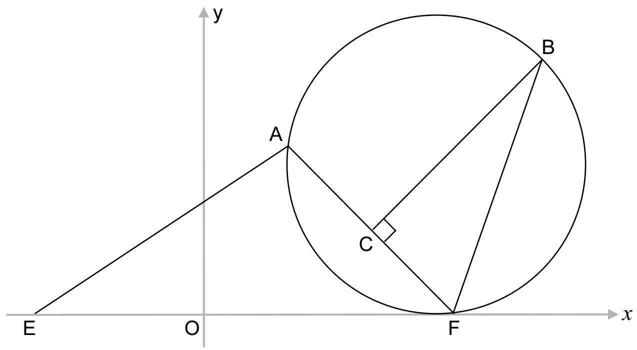
[17]

QUESTION 10

Answers on p. A84

In the diagram below, A; B and F lie on the circle.

- The equation of line EA is $3y - 2x = 8$.
- The gradient of line AF is -1.



10.1 Calculate the size of $E\hat{A}F$. (5)

10.2 If EA = $\sqrt{52}$ and FB = $\sqrt{40}$ then calculate the length of CB if the centre of the circle lies on CB and CB $\perp$ AF. (7)

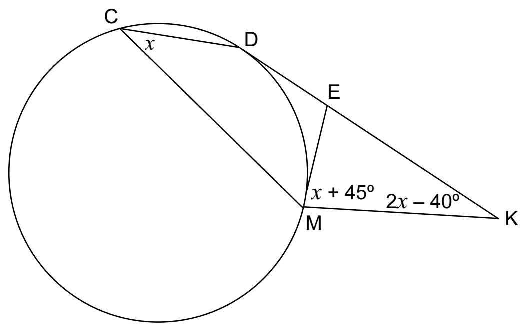
[12]

QUESTION 11

Answers on p. A85

In the diagram below, C, D and M are points on the circle.

- $M\hat{C}D = x$.
- KD is a tangent to the circle at D.
- E is a point on DK.
- EM is another tangent to the circle at M.
- $K\hat{M}E = x + 45°$ and $E\hat{K}M = 2x - 40°$



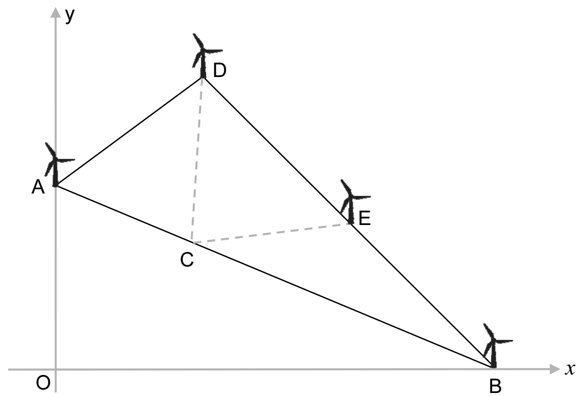
Determine the value of x. **[7]**

QUESTION 12

Answers on p. A85

The diagram below is an aerial view of four wind turbines placed at A, D, E and B.

- Line AB has equation $5x + 12y = 60$.
- A lies on the y-axis.
- B lies on the x-axis.
- E is the midpoint of DB.
- C lies on AB and represents the control station.
- The area of ΔADC : ΔECD is 8 : 9.



12.1 Calculate the distance of AB. (2)

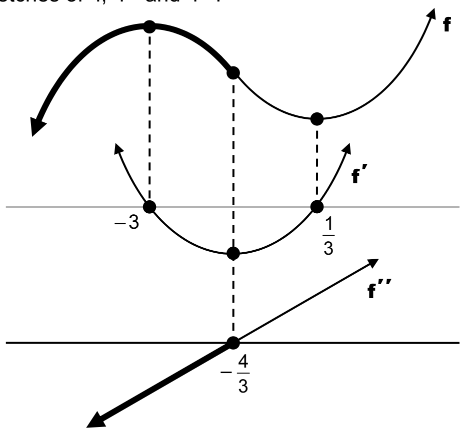
12.2 Find the coordinates of C. (8)

[10]

TOTAL SECTION B: 75

TOTAL: 150

9.1 Sketches of f, f′ and f″:



At the stationary points of f:

$f'(x) = 0 \quad ➡ \quad 3x^2 + 8x - 3 = 0$

$\therefore (3x-1)(x+3) = 0$

$\therefore \boldsymbol{x = \frac{1}{3}}$ **or** $\boldsymbol{-3}$ ➤

9.2 At the point of inflection:

$f''(x) = 0$

$\therefore 6x + 8 = 0$

$\therefore 6x = -8$

$\therefore x = -\frac{4}{3}$

f is concave down for $\boldsymbol{x < -\frac{4}{3}}$ ➤ . . . *See the sketch of f and f″.*

OR: x is halfway between $\frac{1}{3}$ & -3

$\therefore x = \dfrac{\frac{1}{3} + (-3)}{2}$

$= \dfrac{-2\frac{2}{3}}{2}$

$= \boldsymbol{-1\frac{1}{3}}$ ➤

OR: $f''(x) < 0$

$\therefore 6x + 8 < 0$

$\therefore 6x < -8$

$\therefore \boldsymbol{x < -\frac{4}{3}}$ ➤

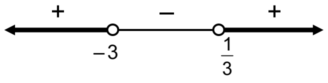
9.3 f strictly increasing ➡ $f'(x) > 0$

$\therefore x < -3$ or $x > \frac{1}{3}$ . . . *See the sketches of f and f′.*

OR: $3x^2 + 8x - 3 < 0$

$\therefore (3x-1)(x+3) < 0$

f′(x):
(x):



$\therefore \boldsymbol{x < -3}$ **or** $\boldsymbol{x > \frac{1}{3}}$ ➤

9.4 $f(x) = ax^3 + bx^2 + cx + d$

$\therefore f(0) = -18 \quad ➡ \quad d = -18$

& $f'(x) = 3ax^2 + 2bx + c$

But, $f'(x) = 3x^2 + 8x - 3$ . . . *given*

$\therefore 3a = 3$; $2b = 8$; $c = -3$

$\therefore a = 1$; $\therefore b = 4$

$\therefore \boldsymbol{f(x) = x^3 + 4x^2 - 3x - 18}$ ➤

---

10. Read the information very carefully, so that you know that:

M(t) = the **number of molecules** after time t hours

& **t** = the **number of hours** after the drug has been taken

$M(t) = -t^3 + 3t^2 + 72t, \quad 0 < t < 10$

10.1 After **3 hours** (t = 3), the **number of molecules**:

$\boldsymbol{M(3)} = -3^3 + 3(3)^2 + 72(3)$

$= -27 + 27 + 216$

$=$ **216 molecules** ➤

10.2 The **'rate of change'** of M(t) vs t **at** time t = 2 is the derivative:

as opposed to the **'average rate of change'** which would be $\frac{M(2) - M(0)}{2 - 0}$ **during** the first 2 hours

$M'(t) = -3t^2 + 6t + 72$

$\therefore M'(2) = -3(2)^2 + 6(2) + 72$

$= -12 + 12 + 72$

$=$ **72 molecules per hour** ➤

10.3 The **rate** at which the number of molecules, M(t), is changing is: $M'(t) = -3t^2 + 6t + 72$

. . . *a quadratic expression*

& it will be a maximum at the turning point, i.e. when

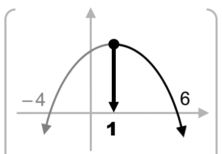
$t = \frac{-b}{2a}$ or $M''(t) = 0$

$= \frac{-6}{2(-3)}$ $\quad \therefore -6t + 6t = 0$

$= 1$ $\quad \therefore -6t = -6t$

$\therefore t = 1$

$\therefore$ **After 1 hour** ➤



OR:
t = the average of −4 & 6

$= \frac{-4+6}{2}$

$= 1$

▸ PROBABILITY [13]

11.

	WATCHED TV DURING EXAMINATIONS	DID NOT WATCH TV DURING EXAMINATIONS	TOTALS
Males	80	a = 20	100
Females	48	12	60
Total	b = 128	32	160

11.1 **a** = 100 − 80 = **20** ➤

& **b** = 80 + 48 or 160 − 32 = **128** ➤

5.4.2 $f(x) = \sin(x + 10°)$ . . . *see above*

$\therefore$ Minimum value (of –1) when

$x + 10° = 270° + n(360°)$

$\therefore\ x = 260° + n(360°)$

$-1 \leq \sin\theta \leq 1$ for all θ;
$\therefore$ min. value = –1

$\therefore$ In the given interval: $\mathbf{x = 260°}$ ◂

6.1 The range of f: $\mathbf{-2 \leq y \leq 0}$ ◂

6.2 $\mathbf{90° < x < 270°}$ ◂

6.3 $PQ = g(x) - f(x)$

$= \cos 2x - (\sin x - 1)$

$= 1 - 2\sin^2 x - \sin x + 1$

$= -2\sin^2 x - \sin x + 2$

Maximum PQ when $\sin x = -\frac{-1}{2(-2)} = -\frac{1}{4}$

$\therefore\ x = 180° + 14{,}48°$ . . . *(III)* *Reference $\angle$ = 14,48°*

$= \mathbf{194{,}48°}$ ◂

or $x = 360° - 14{,}48°$ . . . *(IV)*

$= \mathbf{345{,}52°}$ ◂

PQ must lie between A & B, so one cannot include $x = -14{,}48°$

7.1 In right-angled ΔADK: $\frac{AK}{x} = \sin 60°$

$\therefore\ AK = x\sin 60°$

$= \mathbf{\frac{\sqrt{3}x}{2}}$ ◂

7.2 $\mathbf{K\hat{C}F = 120°}$ ◂ . . . *DE || CF in rhombus; co-int. $\angle^s$ are supplementary*

7.3 The area of ΔAKF $= \frac{1}{2}$AK.KF sin y . . . ❶

Area of a Δ $= \frac{1}{2}ab\sin C$

AK $= \frac{\sqrt{3}x}{2}$ units . . . *see 7.1*

& In ΔKFC: KC $= \frac{1}{2}$DC $= \frac{1}{2}x$ & CF $= x$

$\therefore\ KF^2 = KC^2 + CF^2 - 2KC.CF\cos K\hat{C}F$ . . . cos-rule

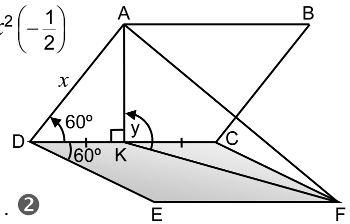
$= \left(\frac{1}{2}x\right)^2 + x^2 - 2\left(\frac{1}{2}x\right)(x)\cos 120°$

$= \frac{1}{4}x^2 + x^2 - x^2\left(-\frac{1}{2}\right)$

$= 1\frac{1}{4}x^2 + \frac{1}{2}x^2$

$= \frac{7}{4}x^2$

$\therefore$ KF $= \frac{\sqrt{7}}{2}x$ units . . . ❷

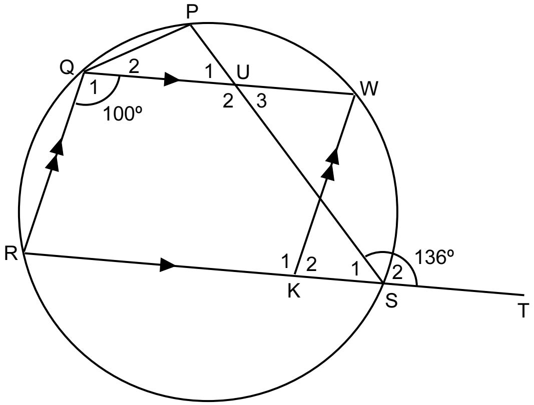


$\therefore$ The Area of ΔAKF $= \frac{1}{2}\left(\frac{\sqrt{3}x}{2}\right)\left(\frac{\sqrt{7}x}{2}\right)\sin y$

$= \mathbf{\frac{\sqrt{21}x^2}{8}}$ **sin y square units** ◂

▸ EUCLIDEAN GEOMETRY & MEASUREMENT [48]

8.1



8.1.1 $\hat{R} = 180° - 100°$ . . . *QW || RK in ||^m; co-int. $\angle^s$ supplementary*

$= \mathbf{80°}$ ◂

8.1.2 $\hat{P} = 180° - 80°$ . . . *opposite $\angle^s$ of c.q. PQRS are supplementary*

$= \mathbf{100°}$ ◂

8.1.3 $P\hat{Q}W + \hat{Q}_1 = 136°$ . . . *exterior $\angle$ of c.q. PQRS = int. opposite $\angle$*

$\therefore\ \mathbf{P\hat{Q}W = 36°}$ ◂

8.1.4 $\hat{U}_2 = \hat{S}_2$ . . . *alternate $\angle^s$; QW || RS*

$= \mathbf{136°}$ ◂

or: $\hat{U}_2 = P\hat{Q}W + \hat{P}$ . . . *ext. $\angle$ of ΔPQU*

$= 36° + 100°$

$= \mathbf{136°}$ ◂

8.2.1 In Δ^s FTE and CTD:

$\frac{FT}{CT} = \frac{TE}{TD} = \frac{FE}{CD} = \frac{1}{2}$

. . . $\frac{5}{10} = \frac{7}{14} = \frac{9}{18}$

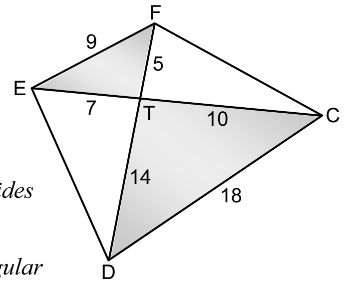
$\therefore$ ΔFTE ||| ΔCTD

. . . *proportional sides*

$\therefore\ T\hat{F}E = T\hat{C}D$

. . . *Δ^s are equiangular*

i.e. $\mathbf{E\hat{F}D = E\hat{C}D}$ ◂



8.2.2 Quadrilateral CDEF is a cyclic quadrilateral . . .

ED subtends equal $\angle^s$ at F and C . . . proved in 8.2.1

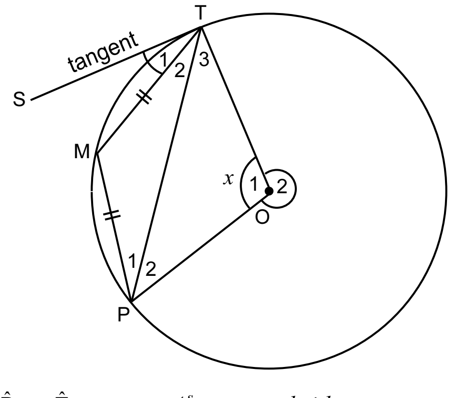
(i.e. converse of same segment thm.)

$\therefore\ \mathbf{D\hat{F}C = D\hat{E}C}$ ◂ . . . *$\angle^s$ in the same segment*

---

9. $\hat{O}_2 = 360° - x$ . . . *$\angle^s$ about point O*

$\therefore\ \hat{M} = 180° - \frac{1}{2}x$ . . . *$\angle$ at centre = 2 × $\angle$ at circumf.*



$\hat{P}_1 = \hat{T}_2$ . . . *$\angle^s$ opp equal sides*

$\therefore\ \hat{P}_1 = \frac{1}{2}\left[180° - \left(180° - \frac{1}{2}x\right)\right]$ . . . *$\angle$ sum of Δ*

$= \frac{1}{2}\left(\frac{1}{2}x\right)$

$= \frac{1}{4}x$

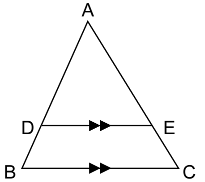
$S\hat{T}M = \hat{P}_1$ . . . *tan chord theorem*

$= \mathbf{\frac{1}{4}x}$ ◂

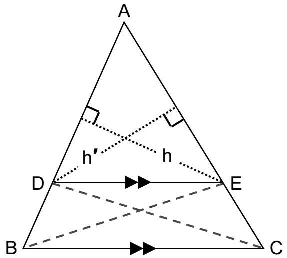
▶ The Proportion Theorem

6 A line parallel to one side of a triangle divides the other two sides proportionally.

i.e. DE || BC ➡ $\frac{AD}{DB} = \frac{AE}{EC}$



Given: $\triangle$ABC with DE || BC,
D & E on AB & AC respectively.



To prove: $\frac{AD}{DB} = \frac{AE}{EC}$

Construction: Join DC & BE

Proof: $\frac{\text{Area of } \triangle ADE}{\text{Area of } \triangle DBE} = \frac{\frac{1}{2}AD.h}{\frac{1}{2}DB.h} = \frac{AD}{DB}$

h is the height of $\triangle^s$ ADE and DBE

h′ is the height of $\triangle^s$ ADE and EDC

Similarly: $\frac{\text{Area of } \triangle ADE}{\text{Area of } \triangle EDC} = \frac{AE}{EC}$ $\left(\frac{\frac{1}{2}AE.h'}{\frac{1}{2}EC.h'}\right)$

But: $\triangle$DBE = $\triangle$EDC, in area . . . *on the same base DE ; between || lines, DE & BC*

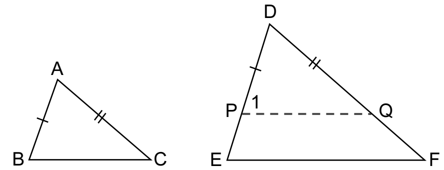
and: $\triangle$ADE is common

$\therefore \frac{\text{Area of } \triangle ADE}{\text{Area of } \triangle DBE} = \frac{\text{Area of } \triangle ADE}{\text{Area of } \triangle EDC}$

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ ◀

▶ The Similar $\triangle^s$ Theorem

7 If two triangles are equiangular, then their sides are proportional and, therefore, they are similar.



Given: $\triangle$ABC & $\triangle$DEF with $\hat{A} = \hat{D}$ $\hat{B} = \hat{E}$ & $\hat{C} = \hat{F}$

To prove: $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$

Construction: Mark P & Q on DE & DF such that DP = AB & DQ = AC

Proof: In $\triangle^s$ DPQ & ABC

(1) DP = AB . . . *construction*
(2) DQ = AC . . . *construction*
(3) $\hat{D} = \hat{A}$ . . . *given*

$\therefore$ $\triangle DPQ \equiv \triangle ABC$ . . . *S∠S*

stage 1: congruency

$\therefore$ $\hat{P}_1 = \hat{B}$
$= \hat{E}$ . . . *given*

stage 2: corresponding $\angle^s$

The focal point

$\therefore$ PQ || EF . . . *corresponding $\angle^s$ equal*

stage 3: parallel lines

$\therefore \frac{DP}{DE} = \frac{DQ}{DF}$ . . . *proportion theorem ; PQ || EF*

But DP = AB and
DQ = AC . . . *construction*

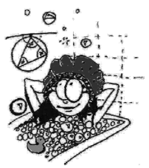
$\therefore \frac{AB}{DE} = \frac{AC}{DF}$

stage 4: proportions

Similarly, by marking S and T on DE and EF such that SE = AB and ET = BC, it can be proved that: $\frac{AB}{DE} = \frac{BC}{EF}$

$\therefore \frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$ ◀

$\therefore$ $\triangle$ABC and $\triangle$DEF are similar.



Similar $\triangle^s$

$\triangle^s$ are similar if: **A:** they are equiangular, and
B: their sides are proportional

In this proof, we show that **A** ➡ **B**

$\therefore$ The $\triangle^s$ are similar . . . Both conditions, **A** and **B**, apply

The converse statement says: **B** ➡ **A**
$\therefore$ *The $\triangle^s$ are similar*

Compound Angle Formulae

1. $\sin(A + B) = \sin A \cos B + \cos A \sin B$

2. $\sin(A - B) = \sin A \cos B - \cos A \sin B$

Sign stays the same sine & cosine of A and B mixed

3. $\cos(A + B) = \cos A \cos B - \sin A \sin B$

4. $\cos(A - B) = \cos A \cos B + \sin A \sin B$

Sign changes cosine of A and B first, then sine of A & B

We will prove formula no. 4 (see alongside) and then derive the other 3 from it.

Double Angle Formulae

5. $\sin 2A = 2 \sin A \cos A$ . . . This formula will be derived from the formula no. 1.

6. $\cos 2A = \cos^2 A - \sin^2 A$ . . . This formula will be derived from the formula no. 3.

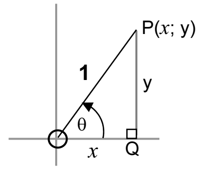
or $\cos 2A = 1 - 2\sin^2 A$

or $\cos 2A = 2\cos^2 A - 1$

Proof of the Formula:

$\cos(A - B) = \cos A \cos B + \sin A \sin B$

First, an important concept!



NOTE: **If OP = 1 unit!**

then: $\frac{x}{1} = \cos\theta$ and $\frac{y}{1} = \sin\theta$

i.e. $x = \cos\theta$ and $y = \sin\theta$

i.e. **P is the point $(\cos\theta;\ \sin\theta)$**

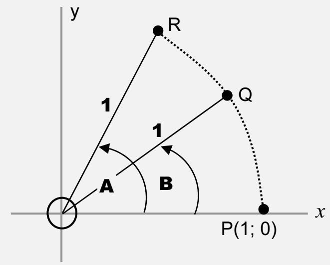
In the sketch alongside, $\hat{A}$ and $\hat{B}$ have been placed in standard position.

$R\hat{O}Q = \hat{A} - \hat{B}$.

The coordinates of the points **R** and **Q**, both **1 unit** from the origin, are:

R(cos A ; sin A) & **Q(cos B ; sin B)**

. . . See NOTE above



▶ **Determine 2 expressions for RQ^2**

$RQ^2 = 1^2 + 1^2 - 2(1)(1)\cos(A - B)$ . . . **COSINE RULE**

$= 2 - 2\cos(A - B)$ . . . ❶

& $RQ^2 = (\cos A - \cos B)^2 + (\sin A - \sin B)^2$ . . . **DISTANCE FORMULA**

$= \cos^2 A - 2\cos A \cos B + \cos^2 B + \sin^2 A - 2\sin A \sin B + \sin^2 B$

$= 2 - 2\cos A \cos B - 2\sin A \sin B$ ◂ . . . ❷ . . . $\sin^2\theta + \cos^2\theta = 1$

▶ **Equate the two expressions for RQ^2 above:**

❶ = ❷ $\therefore\ 2 - 2\cos(A - B) = 2 - 2\cos A \cos B - 2\sin A \sin B$

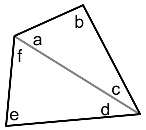
▶ **Subtract 2:** $\therefore\ -2\cos(A - B) = -2\cos A \cos B - 2\sin A \sin B$

▶ **Divide by -2** $\left(\text{or} \times \text{by } -\frac{1}{2}\right)$: $\therefore$ **$\cos(A - B) = \cos A \cos B + \sin A \sin B$** ◂

QUADRILATERALS - definitions, areas & properties

All you need to know

'Any' Quadrilateral



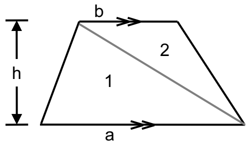
Sum of the $\angle^s$ of *any quadrilateral* = 360º

Sum of the interior angles
= (a + b + c) + (d + e + f)
= 2 × 180º . . . (2 Δ^s)
= 360º

The arrows indicate various '**pathways**' from 'any' quadrilateral to the square (the 'ultimate quadrilateral'). These pathways, which combine logic and fact, are essential to use when proving specific types of quadrilaterals.

See how the properties accumulate as we move from left to right, i.e. the first quad has no special properties and each successive quadrilateral has all preceding properties.

A Trapezium

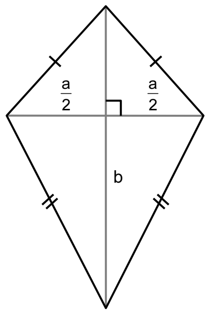


DEFINITION:
Quadrilateral with **1 pair of opposite sides ||**

Area = Δ 1 + Δ 2
= $\frac{1}{2}$ah + $\frac{1}{2}$bh
= $\mathbf{\frac{1}{2}(a + b).h}$

'Half the sum of the || sides × the distance between them.'

A Kite



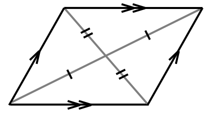
DEFINITION:
Quadrilateral with **2 pairs of adjacent sides equal**

Given diagonals a and b
Area = $2\Delta^s = 2\left(\frac{1}{2}b.\frac{a}{2}\right) = \frac{ab}{2}$

'Half the product of the diagonals'

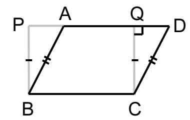
THE DIAGONALS
- *cut perpendicularly*
- ***ONE DIAGONAL** bisects the other diagonal, the opposite angles and the area of the kite*

A Parallelogram



DEFINITION:
Quadrilateral with **2 pairs opposite sides ||**

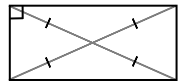
Area = base × height



$||^m$ ABCD = ABCQ + ΔQCD
rect. PBCQ = ABCQ + ΔPBA
where ΔQCD ≡ ΔPBA . . . *RHS*
∴ $||^m$ ABCD = rect. PBCQ (in area)
= BC × QC

Properties:
2 pairs opposite sides equal
2 pairs opposite angles equal
*& **DIAGONALS BISECT ONE ANOTHER***

A Rectangle

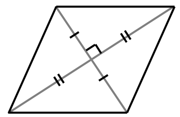


DEFINITION:
A $||^m$ with one right $\angle$

Area = ℓ × b

***DIAGONALS** are EQUAL*

A Rhombus

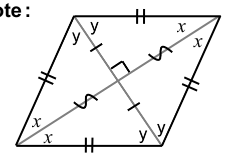


DEFINITION:
A $||^m$ with one pair of adjacent sides equal

Area
= $\mathbf{\frac{1}{2}}$ **product of diagonals** (as for a kite)
or
= **base × height** (as for a parallelogram)

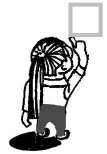
THE DIAGONALS
- *bisect one another PERPENDICULARLY*
- *bisect the angles of the rhombus*
- *bisect the area of the rhombus*

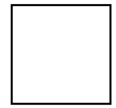
Note:



2x + 2y = 180º . . . $\angle^s$ *of* Δ *or*
➡ *x* + *y* = 90º *co-int.* $\angle^s$ *suppl.*

The Square



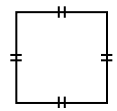


the 'ultimate' quadrilateral!

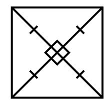
Area = s²

Properties:
It's all been said 'before'!
Since a square is a rectangle, a rhombus, a parallelogram, a kite, . . . ALL the properties of these quadrilaterals apply.

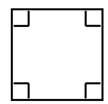
sides



diagonals



angles



Quadrilaterals play a prominent role in both Euclidean & Analytical Geometry right through to Grade 12!

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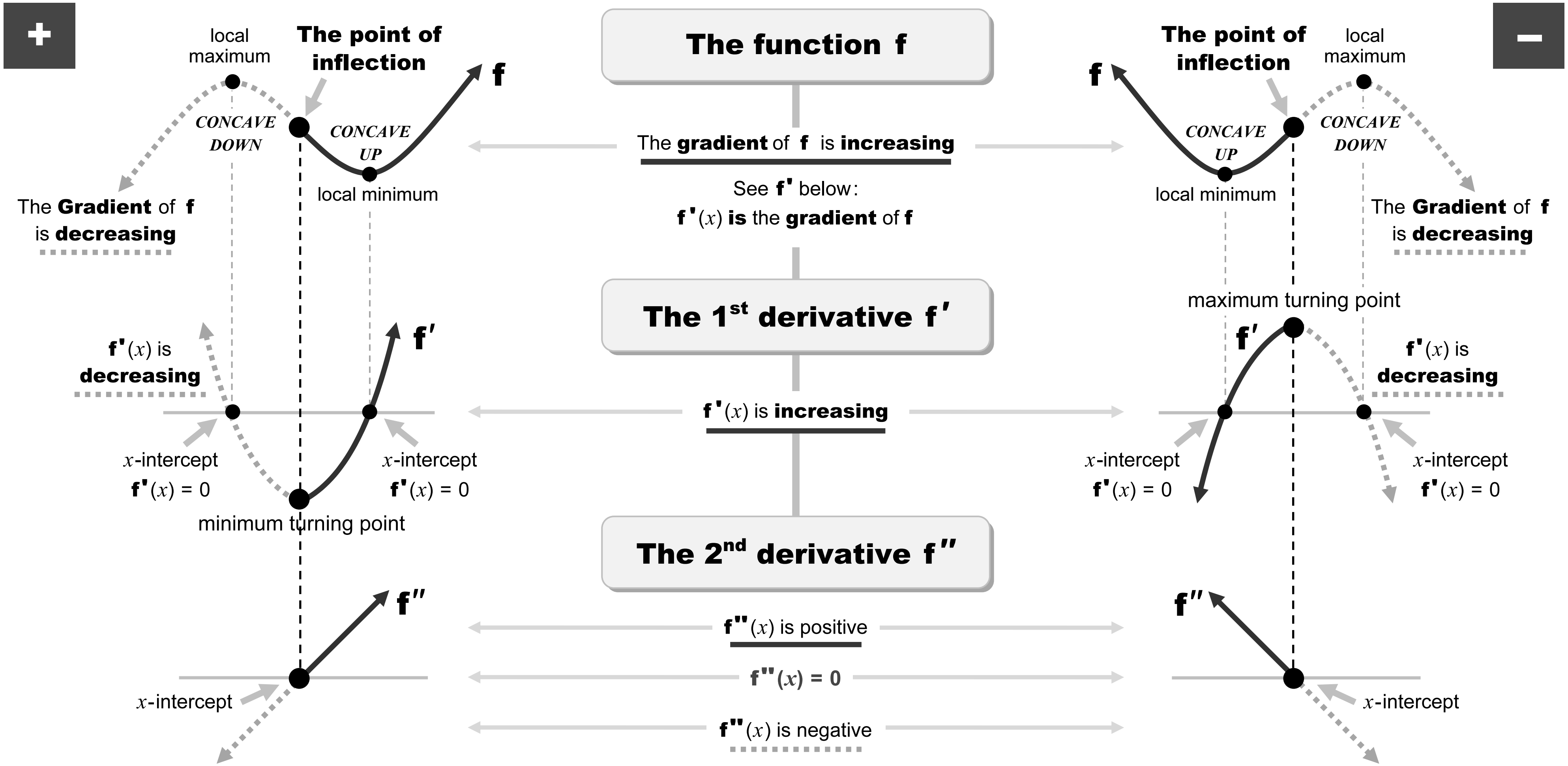
CONCAVITY & THE POINT OF INFLECTION

The **Concavity** of cubic graphs: **Concave up** or **Concave down**, changes at the point of inflection:

As x increases (i.e. from left to right) . . .



$f''(x) = 0$ **at the point of inflection of f**

$f''(x) < 0$ **where f is concave down**

$f''(x) > 0$ **where f is concave up**

Note: For cubic graphs with 2 stationary points, the coordinates of the point of inflection are the averages of the x- and y-coordinates of the stationary points.

GROUPING OF CIRCLE GEOMETRY THEOREMS

The grey arrows indicate how various theorems are used to prove subsequent ones