Mathematics

TEST & EXAM PREPARATION

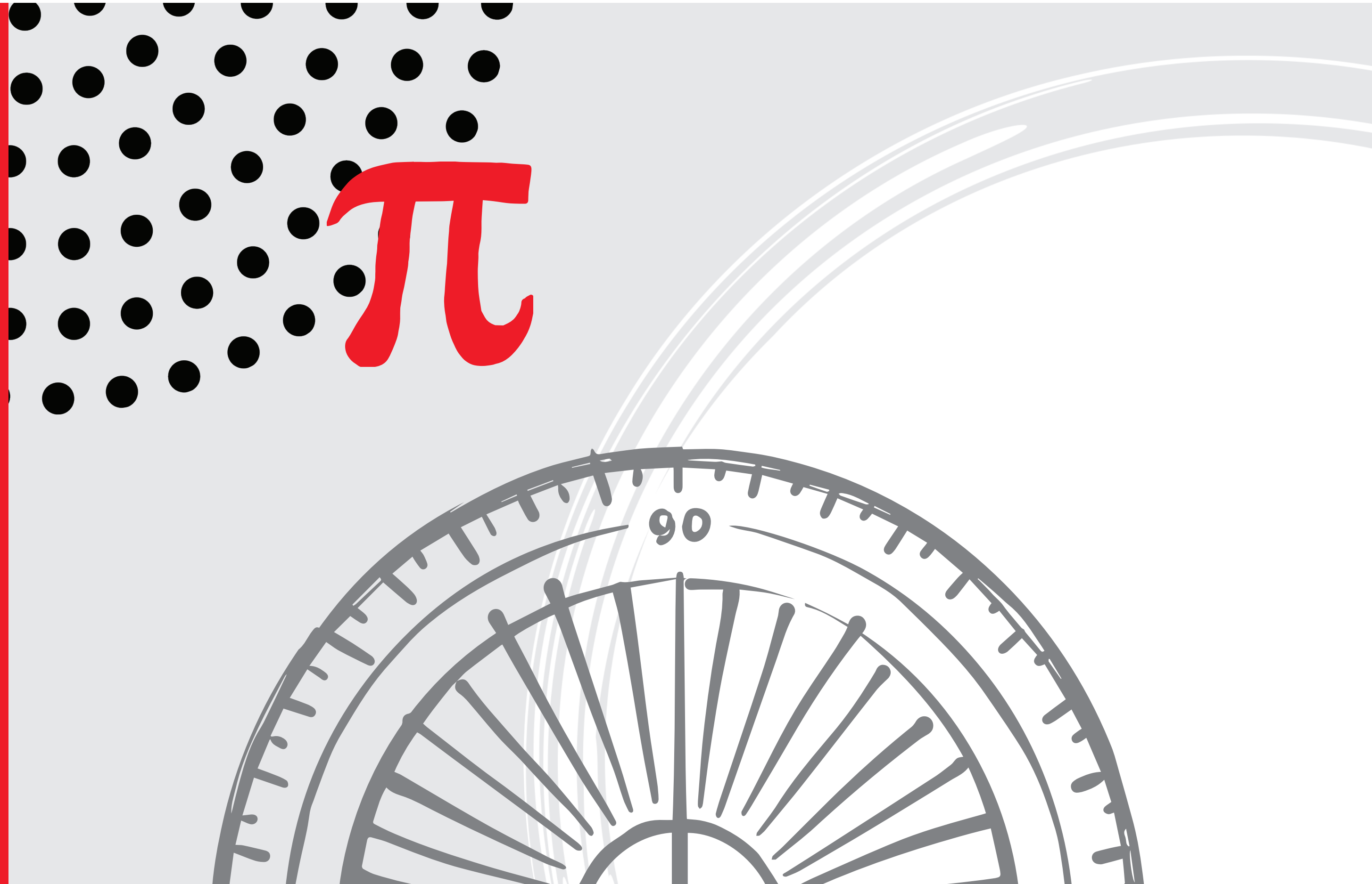
Anne Eadie & Gretel Lampe

GRADE

12

CAPS

2-in-1



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(in separate booklet)

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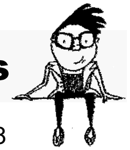

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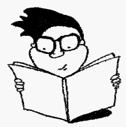


3 FUNCTIONS & INVERSE FUNCTIONS

EXERCISE 3.1

Characteristics of Graphs & Functions

The focus in this section is *mainly* on the parabola and the straight line and, further on, on their inverses.
See Topic 4 for exponential and log functions and their inverses.
See the **Topic Guide** on p. 147 & 148 for extensive revision of the hyperbola (no inverse required).

Identifying different types of graphs is very important!

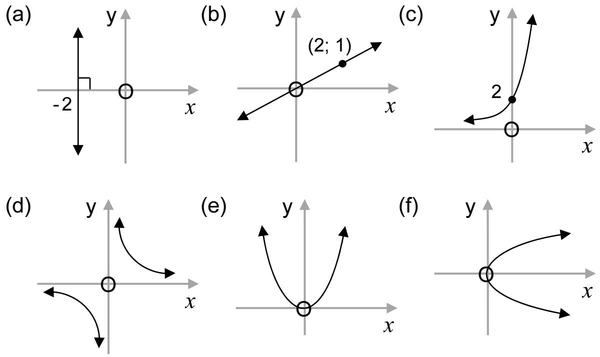
1. On a separate set of axes, for each, draw graphs of:

1.1 $y = 4 - x^2$ 1.2 $y = \frac{1}{x-4}$ 1.3 $y = 4 - x$

1.4 $y = -\frac{4}{x}$ 1.5 $y = \frac{x}{4}$ 1.6 $y = 4^x$

(6 × 3 = 18)

2.1 Six graphs named (a) → (f) are sketched below.
They are followed by 10 equations. Match the graphs with the equations. Write down (a) → (f) and alongside these, the number selected from (1) → (10) that is the equation of the graph.



List of possible equations

(1) $xy = 2$ (2) $xy = -2$
(3) $y = -2$ (4) $x = -2$
(5) $y = x^2$ (6) $x = y^2$
(7) $y = 2x$ (8) $y = \frac{x}{2}$
(9) $y = 2^{x+1}$ (10) $y = 2^{x-1}$ (10)

2.2 Write down (a) → (f) and say whether the graph represents a one-to-one, a many-to-one or a one-to-many relationship between the values of x (the domain) and the values of y (the range). (6)

2.3 Which of the graphs (a) → (f) are not functions?
Why not? (4)

Hint: If a vertical line cuts a graph more than once, it is not a function.
If all vertical lines will cut a graph once (only), then the graph is a function.

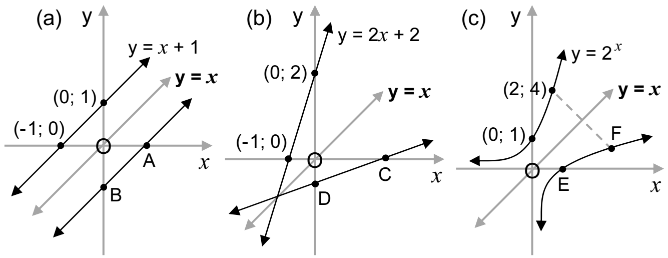
2.4 Write down the domain and range of graphs (a) → (f). (12)

2.5 Write down the equations of the asymptotes in (c) and (d). (3)

3.1 Draw sketches to show the reflections of point P(5; 2)
(a) in the y-axis (b) in the x-axis (c) in the line $y = x$ (6)

3.2 Describe the change in the coordinates in each case. (3)

3.3 Note the reflections of the graphs in the line $y = x$ in the following cases:

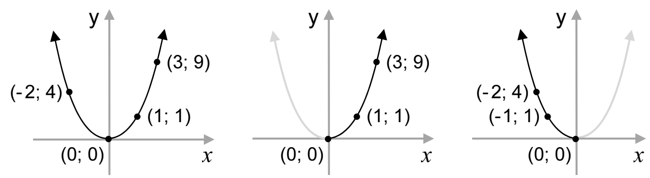


(a) Write down the coordinates of the points A to F which are reflections of the given points in the line $y = x$. (6)

(b) Determine the equations of the reflected graphs in (i), (ii) and (iii) by inspection. (3)

4.1 Draw the reflections of the following graphs in the line $y = x$. (6)

(a) $\mathbf{y = x^2}$ (b) $\mathbf{y = x^2 ; x \geq 0}$ (c) $\mathbf{y = x^2 ; x \leq 0}$



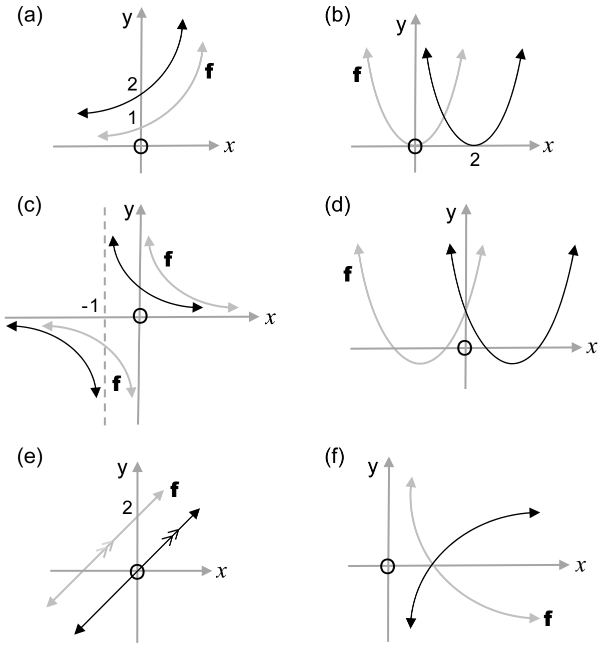
4.2 Are the reflections drawn in 4.1 functions? (3)

4.3 Determine the equations of the reflections drawn in 4.1. (6)

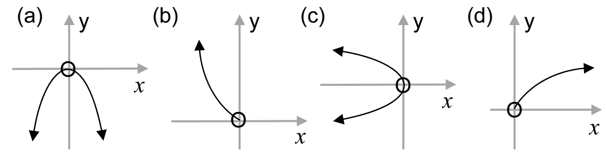
5.1 Given any function, $y = \mathbf{f}(x)$, line, hyperbola, parabola or exponential, describe the transformation required for the following images of **f** to be obtained:

A $y = \mathbf{f}(x) + 1$ B $y = \mathbf{f}(x) - 2$ C $y = \mathbf{f}(x + 1)$
D $y = \mathbf{f}(x - 2)$ E $y = \mathbf{f}(-x)$ F $y = -\mathbf{f}(x)$ (6)

5.2 Match the black graph in each of these sketches to the equations A, B, C, . . . in 5.1. (The grey graph is the original graph **f** in each case.) (6)



6.1 Four graphs (a) → (d) are sketched below. Are any of these graphs functions? Give reasons. (2)



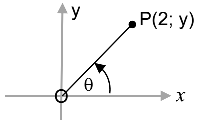
6.2 Match graphs (a) → (d) with the equations (1) → (6) below. Write down (a) → (d) and alongside these the number selected from (1) to (6) that is the equation of the graph.

(1) $y = x^2$ (2) $y = x^2 \; ; \; x \leq 0$
(3) $y = -x^2$ (4) $y = -x^2 \; ; \; x \geq 0$
(5) $x = -y^2$ (6) $y = \sqrt{x} \; ; \; x \geq 0$ (4)

6.3 Draw the graph defined by $y = \pm\sqrt{x}$. (2)

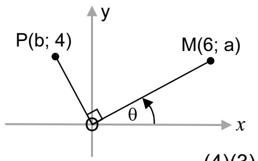
Solutions on p. 72 – 73

6. In the accompanying figure, P(2; y) is a point in the first quadrant and $OP = \sqrt{13}$. $X\hat{O}P = \theta$.



   6.1 Calculate the value of y. (2)
   6.2 Write down the numerical value of $\cos^2\theta$. (1)
   6.3 Calculate the value of: $13\sin^2(180° + \theta) - \tan\theta$. (4)

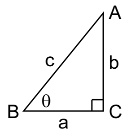
7. In the diagram alongside, $P\hat{O}M = 90°$, $X\hat{O}M = \theta$, M(6; a), P(b; 4) and $\sqrt{5}\cos\theta - 2 = 0$.



   Determine, without the use of a calculator, the numerical value of:
   7.1 a   7.2 b (4)(3)

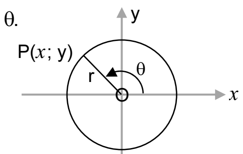
Bookwork

8. In the sketch alongside, prove that:



   8.1 $\frac{\sin\theta}{\cos\theta} = \tan\theta$ 8.2 $\sin^2\theta + \cos^2\theta = 1$
   8.3 $\sin(90° - \theta) = \cos\theta$

9. In the accompanying figure, P(x; y) is a point on the circle with radius r, centre (0; 0) and $X\hat{O}P = \theta$.



   9.1 Prove that:
   (a) $\tan\theta = \frac{\sin\theta}{\cos\theta}$
   (b) $\sin^2\theta + \cos^2\theta = 1$
   9.2 Find the value of θ if $x = -\sqrt{3}$ and y = 1.

Given Ratios, calculate . . .

Note: *'without a calculator' means 'with a sketch'*

10. If $5\sin x = 3$ and $x + y = 90°$, calculate without a calculator the value of:
   10.1 cos y   10.2 $\tan x + \tan y$ (3)(3)

11. If $5\tan\theta = 12$ and $\theta \in [90°; 360°]$, determine, without using a calculator, but using a suitable sketch, the value of $\sin\theta + \cos\theta$. (5)

12. If $\tan A = -\frac{5}{12}$ and $180° \leq A \leq 360°$, draw a sketch and calculate (without determining the value of A) the value of: $13\sin^2 A$ (5)

13. $\sin A = \frac{3}{5}$ and $90° < A < 270°$. Determine, by means of a sketch, the value of: $\frac{\cos A + \sin A}{1 - \frac{1}{3}\tan A}$ (6)

14. $\tan\alpha = -\frac{3}{4}$; $\alpha \in (0°; 180°)$ and $13\cos\beta - 12 = 0$; $\beta \in (180°; 360°)$
   Calculate the value of $\sin\alpha\sin\beta$, without using a calculator. (5)

15. If $\sin x = -\frac{5}{13}$ and $90° < x < 270°$, calculate $\tan^2 x.\cos^2 x$ without using a calculator. (7)

16. If $\cos x = t$ and $\hat{x}$ is acute, express
   16.1 $\sin x$, and   16.2 $\tan^2 x$ in terms of t (1)(2)

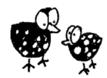
17. If $\tan 27° = p$, express each of the following in terms of p:
   17.1 sin 27°   17.2 cos 27°   17.3 sin 63°
   17.4 tan 153°   17.5 tan(–27°)   17.6 $\cos^2 387°$
   (6×2 = 12)

EXERCISE 6.2

Identities

Special $\angle^s$; 180°/360° & 90° rules

Special angles



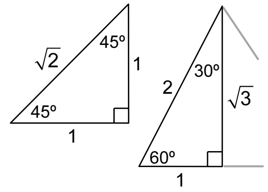
Simplify WITHOUT using a calculator:

1.1 $\frac{\cos^2 45°}{\sin 30°} \cdot \frac{\cos 0°}{\tan 60°}$ 1.2 $\frac{\cos 330°.\tan 150°}{\tan 315°}$ (5)(4)

1.3 $\frac{\cos 240°.\sin 330°.\tan 120°}{\sin 150°.\tan 210°.\cos 120°}$ 1.4 $\frac{\cos 300°.\tan^2 330°}{\sin^2 315°}$ (9)(5)

2.1 $\frac{1}{\sqrt{3}}\sin^2 45°.\sin(-300°) - \frac{1}{2}\tan(-45°).\cos^2 585° - \sin(-30°)$ (9)

2.2 $\frac{\tan 135°.\sin 230°.\tan(-60°)}{\cos 140°.\tan 300°.\sin 150°}$ (8)



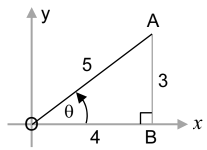
3. $\frac{\tan(-330°).\sin 480°.\sin 260°}{\cos 225°.\sin 315°.\cos 350°}$ (8)

4. Determine, without the use of a calculator the value of:
   4.1 $\frac{\sin 137°}{\cos 133°}$ 4.2 $\frac{\cos 10°.\cos 120°}{\sin 80°.\sin 150°}$ (4)(4)

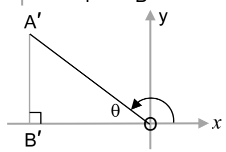
180°/360° Rule (no ratio changes!)

Basic Gr 9 transformations **(CAPS)** are used here to develop identities

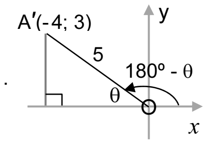
5.1 (a) In the figure alongside, the coordinates of point A are . . .
   (b) Determine the values of sin θ, cos θ and tan θ.



5.2 (a) A′ is a reflection of point A in the . . . and the coordinates of point A′ are . . .
   (b) Write $X\hat{O}A'$ in terms of θ.

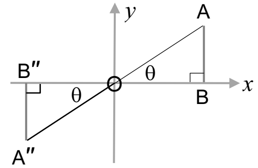


   (c) Determine, first numerically, and then in terms of θ:

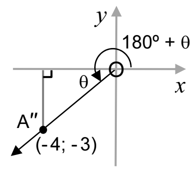


   sin(180° – θ) = $\left(\frac{y}{r} =\right)$ . . . . = ...
   cos(180° – θ) = $\left(\frac{x}{r} =\right)$ . . . . = ...
   tan(180° – θ) = $\left(\frac{y}{x} =\right)$ . . . . = ...

5.3 (a) A″ is a reflection of point A in the . . . and the coordinates of point A″ are . . .
   (b) Write reflex $X\hat{O}A''$ in terms of θ.

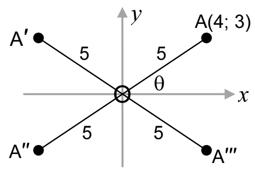


   (c) Determine, first numerically, and then in terms of θ:

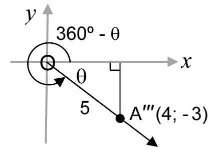


   sin(180° + θ) = $\left(\frac{y}{r} =\right)$ . . . . = ...
   cos(180° + θ) = $\left(\frac{x}{r} =\right)$ . . . . = ...
   tan(180° + θ) = $\left(\frac{y}{x} =\right)$ . . . . = ...

5.4 (a) A‴ is a reflection of point A in the . . . and the coordinates of point A‴ are . . .
   (b) Write reflex $X\hat{O}A'''$ in terms of θ & write acute $X\hat{O}A'''$ in terms of θ.

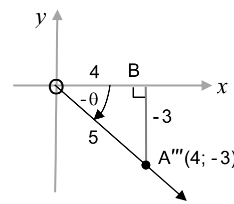


   (c) Determine, first numerically, and then in terms of θ:



   sin(360° – θ) = $\left(\frac{y}{r} =\right)$ . . . . = ...
   cos(360° – θ) = $\left(\frac{x}{r} =\right)$ . . . . = ...
   tan(360° – θ) = $\left(\frac{y}{x} =\right)$ . . . . = ...

   (d) Determine, first numerically, and then in terms of θ:

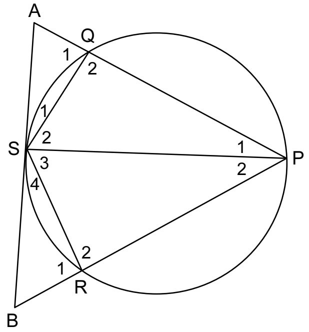


   sin(–θ) = $\left(\frac{y}{r} =\right)$ . . . . = ...
   cos(–θ) = $\left(\frac{x}{r} =\right)$ . . . . = ...
   tan(–θ) = $\left(\frac{y}{x} =\right)$ . . . . = ...

MARKS HAVE NOT BEEN ALLOCATED IN 'THEORY' Q5 & 6.

4.3 The diagram alongside shows a circle with chords QP and RP. Chord SP bisects $Q\hat{P}R$.

The tangent at S meets PQ produced at A and PR produced at B. Q and S and also S and R are joined.

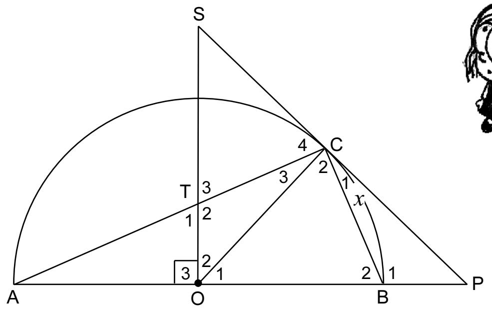


(a) Give, with reasons, three angles each of which is equal to $\hat{S}_1$. (5)

(b) Give the reason why $\hat{R}_1 = \hat{Q}_2$. (1)

(c) Prove that $\hat{S}_3 = \hat{A}$. (5)

5. In the figure below, AB is the diameter of a semicircle with centre O. P is a point on AB produced. PCS is a tangent touching the circle at C, and SO is perpendicular to AB. SO and AC intersect at T. BC and OC are drawn.



5.1 If $\hat{C}_1 = x$, give, with reasons, two other angles each of which is equal to x. (2)

5.2 Prove that $P\hat{C}T = \hat{T}_2$. (3)

5.3 Give, with reasons, the magnitude of the following angles in terms of x:

(a) $C\hat{S}T$ (b) $C\hat{O}B$ (c) $\hat{P}$ (4)(2)(2)

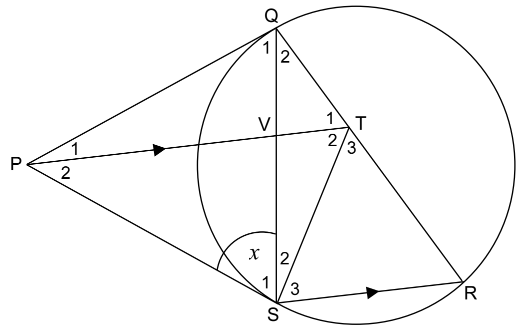
Apply basic Gr 9 knowledge of similar Δ^s and the Theorem of Pythagoras in 5.4 and 5.5.

5.4 Name (without giving reasons) TWO triangles which are similar to ΔCTO. (2)

5.5 Prove that $PA.PB = OP^2 - OA^2$. (4)

See **p. ix** for the **Summary of the Converse Theorems** in ⊙ Geometry.

6. PQ and PS are tangents to the circle at the points Q and S. PT || SR with T on QR. $P\hat{S}Q = x$.



6.1 Name, with reasons, three other angles each equal to x. (3)

6.2 Prove that PQTS is a cyclic quadrilateral. (2)

6.3 Prove that ΔTSR is isosceles. (4)

6.4 If TQ is a tangent to circle QVP, prove that QSR is a right-angled triangle. (6)

7.1 (a) B and C are points on a circle and the tangents at these points meet at A. Then . . . . . (1)

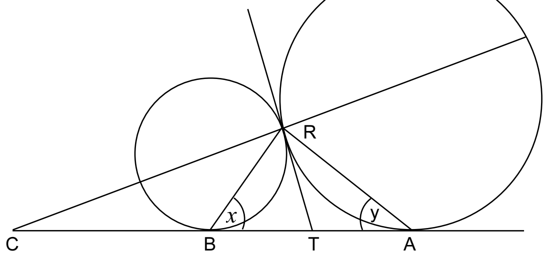
(b) The angle between a tangent and a chord drawn from the point of contact is . . . . (1)

7.2 In the figure, the two circles touch externally at R.

The straight line passing through R and the centre of the smaller circle meets the common tangent AB produced at the point C.

The common tangent at R meets AB at T.

$R\hat{B}T = x$ and $R\hat{A}T = y$.



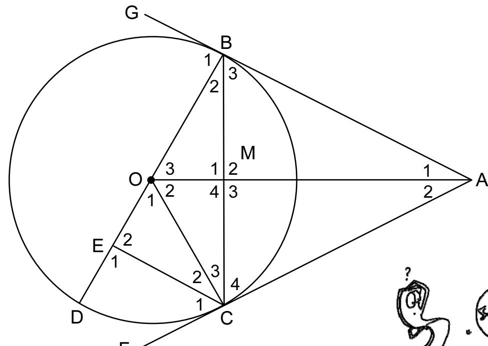
(a) Prove that $A\hat{R}B = 90°$. (5)

(b) Prove that CR is a tangent to the circle which passes through A, R and B. (3)

8. In the following figure, AB and AC are tangents to a circle with centre O.

BD is a diameter and CE ⊥ BD.

BC and CO are drawn. AO cuts BC at M.



Prove that:

8.1 ABOC is a cyclic quadrilateral. (4)

8.2 CEOM is a cyclic quadrilateral. (6)

8.3 CB bisects $E\hat{C}A$. (4)

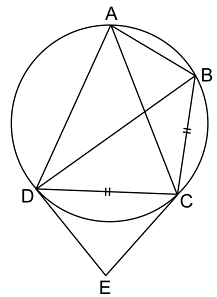
The following question involves similar Δ^s. See more combined similar triangles and Circle Geometry examples in the last section.

9.1 If PQ and RS are chords of a circle, and PQ = RS then PQ and RS subtend equal angles in the same segment at the circumference of the circle.

State the converse of this useful fact. (2)

9.2 Cyclic quadrilateral ABCD has DC = CB.

The tangents at D and C meet at E.



(a) If $B\hat{A}C = x$, find, giving reasons, five other angles each equal to x. (10)

(b) Prove that $CB^2 = EC.DB$. (4)

Know your theory!

Each topic has definitions, vocabulary, facts, laws, theorems . . . a 'blueprint'! Be sure to study all the concepts involved, as all the calculations require you to have and apply this knowledge. Do so with confidence!

Solutions on p. 128 – 129

2.3 (a) and (f) are not functions;
A graph is only a function if for each x-value there is only one y-value. In the case of (a) and (f), each x-value has more than one y-value.

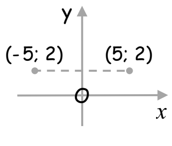
The vertical line test: A vertical line would cut these graphs more than once.

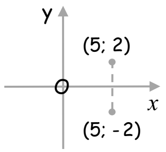
2.4

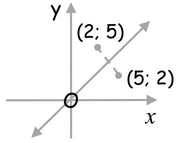
	Domain	Range
(a)	$x = -2$	$y \in \mathbb{R}$
(b)	$x \in \mathbb{R}$	$y \in \mathbb{R}$
(c)	$x \in \mathbb{R}$	$y > 0\,;\ y \in \mathbb{R}$
(d)	$x \neq 0\,;\ x \in \mathbb{R}$	$y \neq 0\,;\ y \in \mathbb{R}$
(e)	$x \in \mathbb{R}$	$y \geq 0\,;\ y \in \mathbb{R}$
(f)	$x \geq 0\,;\ x \in \mathbb{R}$	$y \in \mathbb{R}$

2.5 (c) $\mathbf{y = 0}$ ◄ ... *the x-axis*
(d) $\mathbf{y = 0}$ ◄ ... *the x-axis*
& $\mathbf{x = 0}$ ◄ ... *the y-axis*

3.1 (a) reflection in the **y-axis** (b) reflection in the **x-axis** (c) reflection in the line $\mathbf{y = x}$







3.2 (a) $x \to -x$, $\mathbf{y \to y}$ ◄ (b) $x \to x$, $\mathbf{y \to -y}$ ◄ (c) $x \to y$, $\mathbf{y \to x}$ ◄ i.e. x & y swop

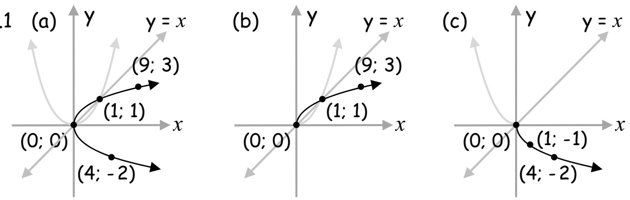
3.3 (a) A(1; 0), B(0; −1), C(2; 0), D(0; −1), E(1; 0), F(4; 2)

(b) (i) $\mathbf{y = x - 1}$ ◄ (ii) $\mathbf{y = \frac{1}{2}x - 1}$ ◄ (iii) $\mathbf{x = 2^y}$ ◄

Note:
- In (c), understandably, x and y are swopped in the equation to get the reflection in $y = x$.
- Now swop x & y in the given equations in (a) & (b):

Given: (a) $y = x + 1$ (b) $y = 2x + 2$
The reflection: $x = y + 1$ $x = 2y + 2$
Now make y the subject:
$y + 1 = x$ $\therefore 2y + 2 = x$
$\therefore \mathbf{y = x - 1}$ ◄ $\therefore 2y = x - 2$
$\therefore y = \mathbf{\frac{x}{2} - 1}$ ◄
These are the equations determined by inspection above.

4.1 (a) (b) (c)



Locate the 'critical points' by swopping x and y.

4.2 **(b) and (c) are, but (a) is not.** ◄

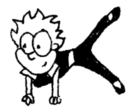
Note: Restricting the domains ($x \geq 0$ or $x \leq 0$) ensured that the reflections are functions.

4.3 (a) $x = y^2$
$\therefore y^2 = x$
$\therefore \mathbf{y = \pm\sqrt{x}}$ ◄

(b) $\mathbf{y = (+)\sqrt{x}}$ ◄ (c) $\mathbf{y = -\sqrt{x}}$ ◄

Note: The graph $y = \pm\sqrt{x}$ is split into 2 graphs: $y = +\sqrt{x}$ and $y = -\sqrt{x}$

5.1 A **translated up 1 unit** ◄
B **translated down 2 units** ◄
C **translated 1 unit to the left** ◄
D **translated 2 units to the right** ◄
E **reflection in the y-axis** ◄
F **reflection in the x-axis** ◄



5.2 (a) A (b) D (c) C (d) E (e) B (f) F

6.1 **All except (c), because in (c), there are 2 values of y for each x-value (except for x = 0).** ◄

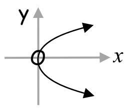
Note: This graph will be cut twice by a vertical line. (All other graphs will only be cut once.)

6.2 (a) (3) (b) (2) (c) (5) *(d) (6)

*Note: (d) • One has to have $x \geq 0$ in $y = \sqrt{x}$
... $\sqrt{\text{a negative number}}$ is imaginary
• $y = +\sqrt{x}$ ➡ $y \geq 0$
• $y = \sqrt{x}$ ➡ $y^2 = x$ ... $\therefore$ Only the 'top arm' of the parabola.
i.e. $x = y^2$, **but y ≥ 0** (& $x > 0$)

6.3 $y = \pm\sqrt{x}$ ➡ $y^2 = x$ and y can be + or −.

$\therefore$ The sketch:



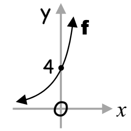
... 'Both arms' of the parabola.
$x = y^2$ or $y = \pm\sqrt{x}$

7. **B is not a function** ◄
For each value of x (in the domain) there is not only one y-value. (A vertical line would cut this graph twice.)

8.1 Equation of **f**: $\mathbf{y = 2^{x+a}}$

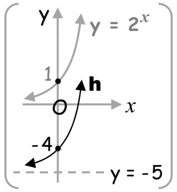
If a point lies on a graph, its co-ords make the eqn. true!

Subst. pt. (1; 8): $8 = 2^{1+a}$
$\therefore 1 + a = 3$
$\therefore \mathbf{a = 2}$ ◄



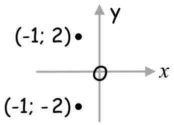
Eqn. of **f**: $y = 2^{x+2}$

8.2 **h**: $y = 2^{x+2-2} - 5$ ...
$\therefore \mathbf{y = 2^x - 5}$ ◄

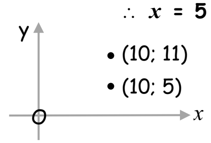


8.3 $\mathbf{y = -5}$ ◄

9.1 No;
For $x = -1$,
y can be 2 or −2. ◄

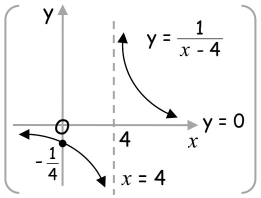


9.2 Not a function if $x_P = x_Q$
i.e. if $2x = x + 5$
$\therefore \mathbf{x = 5}$ ◄



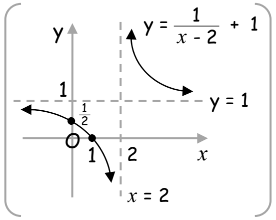
A vertical line will cut the graph more than once.

10.1 $\mathbf{x \neq 4\,;\ x \in \mathbb{R}}$ ◄
(*In* $y = \frac{1}{x-4}$: $x - 4 \neq 0$ $\therefore x \neq 4$)



10.2 $\mathbf{x = 4}$ & $\mathbf{y = 0}$ ◄

10.3 (a) $y = \frac{1}{x - 4 \mathbf{+ 2}} \mathbf{+ 1}$
$\therefore \mathbf{y = \frac{1}{x-2} + 1}$ ◄
(b) $\mathbf{x = 2}$ ◄ and $\mathbf{y = 1}$ ◄



11.1 **x-intercepts:** $\mathbf{f}(x) = 0$ ➡ $x^2 - 4x - 5 = 0$
$\therefore (x - 5)(x + 1) = 0$
$\therefore \mathbf{x = 5}$ **or** $\mathbf{-1}$ ◄

1.3 $\dfrac{(-\cos 60^\circ)(-\sin 30^\circ)(-\tan 60^\circ)}{(+\sin 30^\circ)(+\tan 30^\circ)(-\cos 60^\circ)}$

$= +\dfrac{\tan 60^\circ}{\tan 30^\circ}$

$= \dfrac{\sqrt{3}}{\frac{1}{\sqrt{3}}}$ . . . $\left(= \sqrt{3} \times \frac{\sqrt{3}}{1}\right)$

$= \mathbf{3}$ ◄

1.4 $\dfrac{(+\cos 60^\circ)(-\tan 30^\circ)^2}{(-\sin 45^\circ)^2}$

$= \dfrac{\left(\frac{1}{2}\right)\left(-\frac{1}{\sqrt{3}}\right)^2}{\left(-\frac{1}{\sqrt{2}}\right)^2}$

$= \dfrac{\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)}{\left(\frac{1}{2}\right)} = \mathbf{\frac{1}{3}}$ ◄

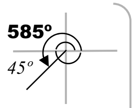
2.1 $\dfrac{1}{\sqrt{3}} \cdot \left(\dfrac{1}{\sqrt{2}}\right)^2 (\sin 60^\circ) - \dfrac{1}{2}(-\tan 45^\circ)(-\cos 45^\circ)^2 - (-\sin 30^\circ)$

$= \dfrac{1}{\sqrt{3}} \cdot \dfrac{1}{2} \cdot \left(\dfrac{\sqrt{3}}{2}\right) - \dfrac{1}{2}(-1)\left(-\dfrac{1}{\sqrt{2}}\right)^2 - \left(-\dfrac{1}{2}\right)$

$= \dfrac{1}{4} + \dfrac{1}{2}\left(\dfrac{1}{2}\right) + \dfrac{1}{2}$

$= \mathbf{1}$ ◄

360° + 225°: 585°



cos 585° = cos 225° = – cos 45°

2.2 $\dfrac{(-\tan 45^\circ)(-\sin 50^\circ)(-\tan 60^\circ)}{(-\cos 40^\circ)(-\tan 60^\circ)(+\sin 30^\circ)}$

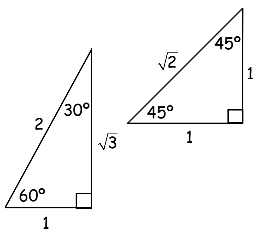
$= \dfrac{(-1)(-\sin 50^\circ)}{(-\cos 40^\circ)\left(\frac{1}{2}\right)}$

$= -\dfrac{\cos 40^\circ}{(\cos 40^\circ)\left(\frac{1}{2}\right)}$ . . . *sin 50° = cos 40°* *sin(90° – θ) = cos θ!*

$= \mathbf{-2}$ ◄

3. $\dfrac{(\tan 30^\circ)(+\sin 60^\circ)(-\sin 80^\circ)}{(-\cos 45^\circ)(-\sin 45^\circ)(+\cos 10^\circ)}$

$= \dfrac{\left(\frac{1}{\sqrt{3}}\right)\left(\frac{\sqrt{3}}{2}\right)(-\sin 80^\circ)}{\left(-\frac{1}{\sqrt{2}}\right)\left(-\frac{1}{\sqrt{2}}\right)(\sin 80^\circ)}$



$= -\dfrac{\frac{1}{2}}{\frac{1}{2}} = \mathbf{-1}$ ◄

4.1 $\dfrac{+\sin 43^\circ}{-\cos 47^\circ}$

$= -\dfrac{\cos 47^\circ}{\cos 47^\circ}$ . . . *43° + 47° = 90°!*

$= \mathbf{-1}$ ◄

4.2 $\dfrac{(\sin 80^\circ)(-\cos 60^\circ)}{(\sin 80^\circ)(+\sin 30^\circ)}$

$= \dfrac{\left(-\frac{1}{2}\right)}{\left(\frac{1}{2}\right)}$ $\left[\textbf{OR}: \dfrac{-\sin 30^\circ}{\sin 30^\circ}\right]$

$= \mathbf{-1}$ ◄

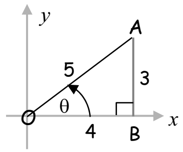
180°/360° Rule (no ratio changes!)

5.1 (a) **A(4; 3)** ◄

(b) $\sin\theta = \mathbf{\frac{3}{5}}$ ◄

$\cos\theta = \mathbf{\frac{4}{5}}$ ◄

$\tan\theta = \mathbf{\frac{3}{4}}$ ◄



5.2 (a) **y-axis** ◄ ; **A'(–4; 3)** ◄

(b) $X\hat{O}A' = \mathbf{180^\circ - \theta}$ ◄

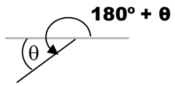
(c) $\mathbf{\sin(180^\circ - \theta)} = \dfrac{3}{5} = \mathbf{\sin\theta}$ ◄

$\mathbf{\cos(180^\circ - \theta)} = -\dfrac{4}{5} = \mathbf{-\cos\theta}$ ◄ . . . *see 5.1(b) above*

$\mathbf{\tan(180^\circ - \theta)} = -\dfrac{3}{4} = \mathbf{-\tan\theta}$ ◄

5.3 (a) **the origin** ◄ ; **A''(–4; –3)** ◄

(b) $X\hat{O}A'' = \mathbf{180^\circ + \theta}$ ◄



(c) $\mathbf{\sin(180^\circ + \theta)} = -\dfrac{3}{5} = \mathbf{-\sin\theta}$ ◄

$\mathbf{\cos(180^\circ + \theta)} = -\dfrac{4}{5} = \mathbf{-\cos\theta}$ ◄

$\mathbf{\tan(180^\circ - \theta)} = \dfrac{-3}{-4} = \dfrac{3}{4} = \mathbf{\tan\theta}$ ◄

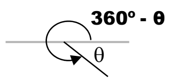
5.4 (a) ***x*-axis** ◄ ; **A'''(4; –3)** ◄

(b) refl. $X\hat{O}A''' = \mathbf{360^\circ - \theta}$ ◄ & acute $X\hat{O}A''' = \mathbf{-\theta}$ ◄

(c) $\mathbf{\sin(360^\circ - \theta)} = -\dfrac{3}{5} = \mathbf{-\sin\theta}$ ◄

$\mathbf{\cos(360^\circ - \theta)} = \dfrac{4}{5} = \mathbf{\cos\theta}$ ◄

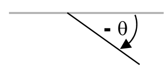
$\mathbf{\tan(360^\circ - \theta)} = -\dfrac{3}{4} = \mathbf{-\tan\theta}$ ◄



(d) $\mathbf{\sin(-\theta)} = -\dfrac{3}{5} = \mathbf{-\sin\theta}$ ◄

$\mathbf{\cos(-\theta)} = \dfrac{4}{5} = \mathbf{\cos\theta}$ ◄

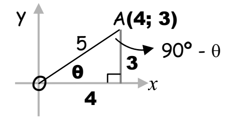
$\mathbf{\tan(-\theta)} = -\dfrac{3}{4} = \mathbf{-\tan\theta}$ ◄



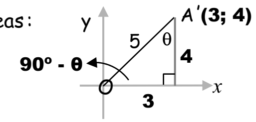
90° Rule (co-ratios!!!)

6.1 (a) $O\hat{A}B = \mathbf{90^\circ - \theta}$ ◄ (b) **y = x** ◄ (c) **A'(3; 4)** ◄

NB: The acute ∠s in the Δ, θ and 90° - θ, have swopped positions! As well as the two right-angled sides (4 & 3).



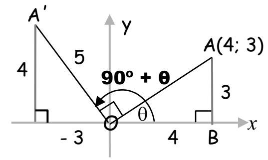
whereas:

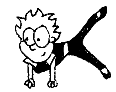


(d) $\mathbf{\sin(90^\circ - \theta)} = \dfrac{4}{5} = \mathbf{\cos\theta}$ ◄

$\mathbf{\cos(90^\circ - \theta)} = \dfrac{3}{5} = \mathbf{\sin\theta}$ ◄

6.2 (a) **A'(–3; 4)** ◄





(b) $\mathbf{B\hat{O}A'} = \mathbf{\theta} + 90^\circ$ or $\mathbf{90^\circ + \theta}$ ◄

(c) $\mathbf{\sin(90^\circ + \theta)} = \dfrac{4}{5} = \mathbf{\cos\theta}$ ◄

$\mathbf{\cos(90^\circ + \theta)} = -\dfrac{3}{5} = \mathbf{-\sin\theta}$ ◄

180°/360° & 90° Rule: Mixed

7. (a) – sin 45° ◄ (b) – cos 30° ◄ (c) + tan 60° ◄
(d) + sin 30° ◄ (e) – cos 60° ◄ (f) + tan 45° ◄
(g) + tan 60° ◄ (h) + sin 45° ◄ (i) + cos 60° ◄

8.1 $\dfrac{(+\sin x)(-\cos x)(+\tan x)}{(-\sin x)(+\cos x)}$

$= \mathbf{+\tan x}$ ◄

8.2 $\dfrac{(\cos\theta)(-\tan\theta)}{(+\cos\theta)(-\sin\theta)}$

$= +\dfrac{\sin\theta}{\cos\theta} \times \dfrac{1}{\sin\theta}$

$= \mathbf{\dfrac{1}{\cos\theta}}$ ◄

8.3 $\dfrac{(+\sin x)(\sin x)}{(+\cos x)(-\sin x)}$

$= \mathbf{-\tan x}$ ◄

8.4 $\dfrac{(+\cos\theta).(-\tan\theta)}{(-\sin\theta)} + \dfrac{(+\cos\theta)}{(+\cos\theta)}$ . . . $\cos(\theta - 360^\circ)$

$= +\dfrac{\cos\theta}{\sin\theta} \times \dfrac{\sin\theta}{\cos\theta} + 1$ $= \cos[-(360^\circ - \theta)]$

$= 1 + 1$ $= \cos(360^\circ - \theta)$

$= \mathbf{2}$ ◄ $= +\cos\theta$

9.1 $\mathbf{x = 35^\circ}$ ◄ . . . *35° + 55° = 90°*

9.2 $\mathbf{y = 40^\circ}$ ◄ . . . *cos(180° – θ) = –cos θ*

(b) $A\hat{C}B = y$ ... *alternate* $\angle^s$ *; AD || BC*

$\therefore D\hat{C}B = x + y$

$\therefore A\hat{B}H = x + y$ ... *corresponding* $\angle^s$ *; AB || CD*

But $\hat{F} = x + y$ in 14.2 (a)

$\therefore A\hat{B}H = \hat{F}$

$\therefore$ HBEF is a cyclic quad. ... *ext.* $\angle$ *= int. opp.* $\angle^s$

$\therefore A\hat{H}B = B\hat{E}F$ (= 90°) ... *ext.* $\angle$ *of cyclic. quad.*

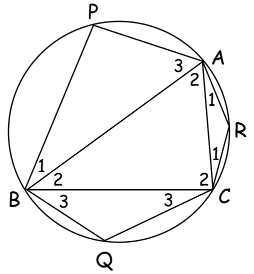
i.e. **$A\hat{H}C = 90°$** ◄

15. $B\hat{P}A = 180° - \hat{C}_2$

$B\hat{Q}C = 180° - \hat{A}_2$

& $A\hat{R}C = 180° - \hat{B}_2$

opp. $\angle^s$ *of cyclic quads. BPAC, BQCA, ARCB (respectively)*



$\therefore$ **$B\hat{P}A + B\hat{Q}C + A\hat{R}C$** $= 3 \times 180° - (\hat{C}_2 + \hat{A}_2 + \hat{B}_2)$

$= 3 \times 180° - 180°$ ... $\angle$ *sum in* Δ

$= 2 \times 180°$

$=$ **360°** ◄

EXERCISE 10.2

Circle Geometry: Including Tangents

1.1 Theorem

See **BOOKWORK: EXAMINABLE PROOFS** on p. i (at the back of this book).

1.2 (a) **$\hat{A}_2 = x$** ◄ ... *tan chord theorem*

(b) $A\hat{E}D = 90°$ ... *diameter* $\perp$ *tangent*

$\therefore$ **$\hat{E}_1 = 90° - x$** ◄

(c) In ΔEFC, $\hat{F}_3 = 90°$ ... *line from centre to midpoint of chord*

$\therefore$ **$\hat{C}_2 = x$** ◄ ... $\hat{E}_1 = 90^o - x$ *;* $\angle$ *sum in* Δ

(d) $\hat{A}_1 = \hat{C}_2 = x$ ◄ ... $\angle^s$ *in same segment*

2.1 **$\hat{D}_1 = x$** ◄ ... *tan chord theorem*

$\therefore$ **$\hat{A}_3 = x$** ◄ ... *alt* $\angle^s$*; AB || ED*

$\therefore$ **$\hat{C}_1 = x$** ◄ ... *ext* $\angle$ *of cyclic quad*

2.2 **$\hat{E} = y$** ◄ ... *tan chord theorem tangent NAT; chord AD*

3.1 **$D\hat{B}E$** $= B\hat{A}E$ ... *tan chord theorem*

$= C\hat{A}E$ ... *given*

$=$ **$C\hat{B}E$** ... $\angle^s$ *in same segment*

i.e. **BE bisects $C\hat{B}D$** ◄

3.2 **Yes**;

$A\hat{B}F = A\hat{E}B$ ... *tan chord theorem*

$\therefore C\hat{A}E = A\hat{E}B$

$\therefore$ AC || BE ... *alt* $\angle^s$ *=*

4.1 (a) $B\hat{A}O$ = **38°** ◄ ... $\angle^s$ *opp = radii*

(b) $A\hat{O}D$ = **76°** ◄ ... $\angle$ *at centre = 2 ×* $\angle$ *at circumference or: ext.* $\angle$ *of* ΔAOB

(c) $A\hat{C}D$ = **38°** ◄ ... $\angle^s$ *in same segment*

(d) $A\hat{C}B$ = **52°** ◄ ... $B\hat{C}D$ *in semi-*⊙

(e) $D\hat{C}E$ = **25°** ◄ ... *tan chord theorem*

$\therefore A\hat{C}E = 25° + 38° =$ **63°** ◄

4.2 (a) **$Q\hat{P}R = x$** ◄ ... *tan chord theorem*

$\therefore$ **$P\hat{R}T = x$** ◄ ... *alt* $\angle^s$ *; QP || RT*

$\therefore$ **$Q\hat{S}R = x$** ◄ ... *tan chord theorem or* $\angle^s$ *in same segment*

(b) **Equal chords subtend equal $\angle^s$.** ◄

(c) $K\hat{Q}R = Q\hat{P}R + P\hat{R}Q$ ... *exterior* $\angle$ *of* ΔPQR

$= x + y$ ... *in 4.2(a) and 4.2(b)*

& $M\hat{N}R = Q\hat{S}R + S\hat{R}T$ ... *exterior* $\angle$ *of* ΔNRS

$= x + y$ ... *in 4.2(a) and 4.2(b)*

$\therefore$ **$K\hat{Q}R = M\hat{N}R$** ◄

4.3 (a) **$\hat{P}_1 = x$** ◄ ... *tan chord theorem*

$\therefore$ **$\hat{P}_2 = x$** ◄ ... *given*

$\therefore$ **$\hat{S}_4 = x$** ◄ ... *tan chord theorem*

(b) An exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.

(c) $\hat{S}_3 = \hat{R}_1 - \hat{P}_2$ ... *exterior* $\angle$ *of* ΔSRP

& $\hat{A} = \hat{Q}_2 - \hat{S}_1$ ... *exterior* $\angle$ *of* ΔASQ

But $\hat{R}_1 = \hat{Q}_2$ ... *proved in 4.3(b)*

& $\hat{P}_2 = \hat{S}_1$ ... *proved in 4.3(a)*

$\therefore$ **$\hat{S}_3 = \hat{A}$** ◄

5.1 **$\hat{A} = x$** ◄ ... *tan chord theorem*

$\therefore$ **$\hat{C}_3 = x$** ◄ ... $\angle^s$ *opp = radii*

5.2 $B\hat{C}A = 90°$ ... $\angle$ *in semi-*⊙

$\therefore P\hat{C}T = 90° + x$ ... $\hat{C}_1 = x$

OR: $O\hat{C}P = 90°$ ... *radius* $\perp$ *tangent*

$\therefore P\hat{C}T = 90° + x$ ... $\hat{C}_3 = x$

& $\hat{T}_2 = \hat{O}_3 + \hat{A}$ ... *exterior* $\angle$ *of* ΔTAO

$= 90° + x$ ... *SO* $\perp$ *AB,* $\hat{A} = x$

$\therefore P\hat{C}T = \hat{T}_2$

5.3 (a) In ΔCAP: $\hat{P} = 180° - (x + 90° + x)$ ... $\angle$ *sum in* Δ

$= 90° - 2x$

$\therefore$ In ΔSOP: $\hat{S} = 2x$ ... $S\hat{O}P = 90^o$*;* $\angle$ *sum in* Δ

i.e. **$C\hat{S}T = 2x$** ◄

(b) $\hat{O}_1 = \hat{C}_3 + \hat{A}$ ... *ext.* $\angle$ *of* ΔCAO

i.e. **$C\hat{O}B = 2x$** ◄ ... $\hat{C}_3 = \hat{A} = x$ *in 5.1*

(c) **$\hat{P} = 90° - 2x$** ◄ ... *see 5.3(a)*

5.4 **ΔACP & ΔCBP** (||| ΔCTO [x; 90 + x]) ◄

5.5 $OP^2 - OA^2 = (OP + OA)(OP - OA)$

$= PA.(OP - OB)$ ... *OA = OB = radius*

$= PA.PB$

OR:

In ΔOCP: $O\hat{C}P = 90°$ ... *radius* $\perp$ *tangent*

$\therefore PC^2 = OP^2 - OC^2$ ... *Pythagoras*

$\therefore PC^2 = OP^2 - OA^2$ ... ❶ ... *OC = OA = rad.*

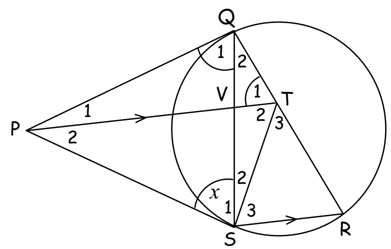
ΔACP ||| ΔCBP ... *both ||| ΔCTO in 5.4*

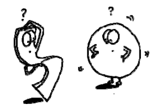
$\therefore \frac{PA}{PC} = \frac{PC}{PB}$ ... *proportional sides*

$\therefore PC^2 = PA.PB$ ... ❷

$\therefore PA.PB = OP^2 - OA^2$ ... *from* ❶ *&* ❷

6.





6.1 $\hat{S}_1 (= x) = \hat{Q}_1$ ... *tans from same point*

$= \hat{R}$ ... *tan chord theorem*

$= \hat{T}_1$ ... *corresponding* $\angle^s$ *; PT || SR*

$\therefore$ **$\hat{Q}_1$, $\hat{R}$ and $\hat{T}_1$ each equal x** ◄

2 EXAM PAPERS

PAPER A1

You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.

Answers only will not necessarily be awarded full marks.

*If necessary, round off answers to **TWO** decimal places, unless stated otherwise.*

QUESTION 1

1.1 Solve for x in each of the following:

1.1.1 $(2x - 1)(x + 4) = 0$ (2)

1.1.2 $3x^2 - x = 5$
(Leave your answer correct to TWO decimal places.) (4)

1.1.3 $x^2 + 7x - 8 < 0$ (4)

1.2 Given: $4y - x = 4$ and $xy = 8$

1.2.1 Solve for x and y simultaneously. (6)

1.2.2 The graph of $4y - x = 4$ is reflected across the line having equation $y = x$. What is the equation of the reflected line? (2)

1.3 The solutions of a quadratic equation are given by

$$x = \frac{-2 \pm \sqrt{2p + 5}}{7}.$$

For which value(s) of p will this equation have:

1.3.1 Two equal solutions, i.e. only 1 root (2)

1.3.2 No real roots (1)

1.4 Solve for x: $\sqrt{5 - x} - x = 1$ (5) [26]

QUESTION 2

2.1 $3x + 1$; $2x$; $3x - 7$ are the first three terms of an arithmetic sequence. Calculate the value of x. (2)

2.2 The first and second terms of an arithmetic sequence are 10 and 6 respectively.

2.2.1 Calculate the 11th term of the sequence. (2)

2.2.2 The sum of the first n terms of this sequence is –560. Calculate n. (5) [9]

QUESTION 3

3.1 Given the geometric sequence: 27 ; 9 ; 3 . . .

3.1.1 Determine a formula for T_n, the nth term of the sequence. (2)

3.1.2 Why does the sum to infinity for this sequence exist? (1)

3.1.3 Determine S_∞. (2)

3.2 The nth term of a sequence is given by $T_n = -2(n - 5)^2 + 18$.

3.2.1 Write down the first THREE terms of the sequence. (3)

3.2.2 Which term of the sequence will have the greatest value? (1)

3.2.3 What is the second difference of this quadratic sequence? (2)

3.2.4 Determine ALL values of n for which the terms of the sequence will be less than –110. (6) [17]

QUESTION 4

4.1 Consider the function $f(x) = 3.2^x - 6$.

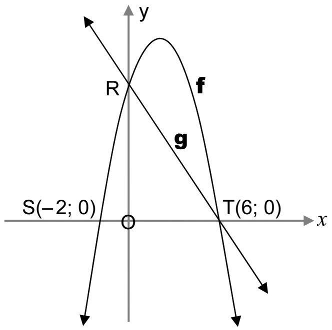
4.1.1 Calculate the coordinates of the y-intercept of the graph of f. (1)

4.1.2 Calculate the coordinates of the x-intercept of the graph of f. (2)

4.1.3 Sketch the graph of f. Clearly show ALL asymptotes and intercepts with the axes. (3)

4.1.4 Write down the range of f. (1)

4.2 S(–2; 0) and T(6; 0) are the x-intercepts of the graph of $f(x) = ax^2 + bx + c$ and R is the y-intercept. The straight line through R and T represents the graph of $g(x) = -2x + d$.



4.2.1 Determine the value of d. (2)

4.2.2 Determine the equation of f in the form $f(x) = ax^2 + bx + c$. (4)

4.2.3 If $f(x) = -x^2 + 4x + 12$, calculate the coordinates of the turning point of f. (2)

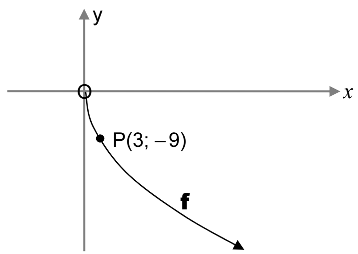
4.2.4 For which values of k will $f(x) = k$ have two distinct roots? (2)

4.2.5 Determine the maximum value of $h(x) = 3^{f(x) - 12}$. (3) [20]

QUESTION 5

The graph of $f(x) = -\sqrt{27x}$ for $x \geq 0$ is sketched alongside.

The point P(3; –9) lies on the graph of f.



5.1 Use the graph to determine the values of x for which $f(x) \geq -9$. (2)

5.2 Write down the equation of f^{-1} in the form y = . . .
Include ALL restrictions. (3)

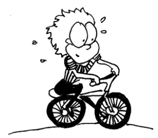
5.3 Sketch f^{-1}, the inverse of f on the graph above. Indicate the intercept(s) with the axes and coordinates of ONE other point. (3)

5.4 Describe the transformation from f to g if $g(x) = \sqrt{27x}$, where $x \geq 0$. (1) [9]

QUESTION 6

The graph of a hyperbola with equation $y = f(x)$ has the following properties:

- Domain: $x \in \mathbb{R}, x \neq 5$
- Range: $y \in \mathbb{R}, y \neq 1$
- Passes through the point (2; 0)



Determine $f(x)$. [4]

Note:

TOPIC GUIDES on pp. 147 & 148 can guide revision of specific sections throughout these papers.

Solutions on p. 178 – 179

3.2 $T_n = -2(n - 5)^2 + 18$

3.2.1 $T_1 = -2(1 - 5)^2 + 18 = -32 + 18 = \mathbf{-14}$ ◄

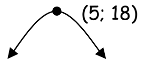
$T_2 = -2(2 - 5)^2 + 18 = -18 + 18 = \mathbf{0}$ ◄

$T_3 = -2(3 - 5)^2 + 18 = -8 + 18 = \mathbf{10}$ ◄

3.2.2 If one drew a graph of $T_n = -2(n - 5)^2 + 18$,

Compare to: $y = -2(x - 5)^2 + 18$

then the turning point would be (5; 18)



∴ The maximum value of T_n (which is 18) would occur when n = 5.

∴ **The 5th term** ◄

3.2.3

	T_1		T_2		T_3
	-14		0		10
1st differences:		14		10	
2nd differences:			-4		

∴ **The second difference = -4** ◄

3.2.4 $T_n < -110$ ➡ $-2(n - 5)^2 + 18 < -110$

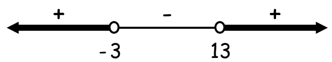
$\therefore\ -2(n^2 - 10n + 25) + 128 < 0$

$\therefore\ -2n^2 + 20n - 50 + 128 < 0$

$\therefore\ -2n^2 + 20n + 78 < 0$

$\div(-2)$ $\therefore\ n^2 - 10n - 39 > 0$

$\therefore\ (n + 3)(n - 13) > 0$



$\therefore\ n < -3$ or $n > 13$

n is the number of terms $\therefore\ n \geq 0$ and $n \in \mathbb{N}_0$

$\therefore$ **$n > 13$; $n \in \mathbb{N}$** ◄

4.1 $f(x) = 3.2^x - 6$

4.1.1 On the y-axis, $x = 0$:

$f(0) = 3.2^0 - 6 = 3 - 6 = -3$ ... *$2^0 = 1$*

∴ The y-intercept: **(0; -3)** ◄

4.1.2 On the x-axis, y = 0, i.e. $f(x) = 0$

$3.2^x - 6 = 0$

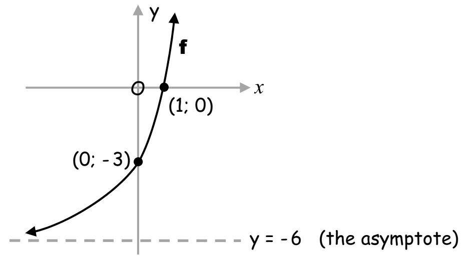
$\therefore\ 3.2^x = 6$

$\therefore\ 2^x = 2$

$\therefore\ x = 1$

∴ The x-intercept: **(1; 0)** ◄

4.1.3



4.1.4 **$y > -6$; $y \in \mathbb{R}$** ◄

4.2.1 **By inspection, d = 12** ◄ ... *grad., m = -2 & x-int. (6; 0)*

OR: Substitute (6; 0) into $y = -2x + d$

$\therefore\ 0 = -2(6) + d$

$\therefore\ 12 = d$

4.2.2 $f(x) = a(x + 2)(x - 6)$ ... *roots -2 and 6*

$\therefore\ f(x) = a(x^2 - 4x - 12)$

The y-intercept: $-12a = 12$

$\therefore\ a = -1$

$\therefore\ f(x) = -(x^2 - 4x - 12)$

$\therefore\ \mathbf{f(x) = -x^2 + 4x + 12}$ ◄

Note: Candidates are penalised for not showing ALL their working.

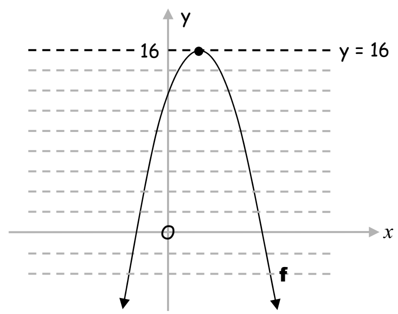
4.2.3 The x-coordinate of the turning point is $x = \frac{-b}{2a} = \frac{-(4)}{2(-1)} = 2$

or $x = 2$ ... *halfway between the roots -2 and 6*

& $f(2) = -(2)^2 + 4(2) + 12 = 16$

∴ **The turning point is (2; 16)** ◄

4.2.4 **k < 16** ◄



all these lines cut f twice ∴ there will be 2 distinct roots

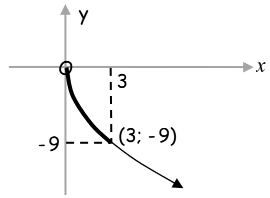
4.2.5 h(x) has a maximum value when f(x) has a maximum value.

Maximum value of $f(x) = 16$

∴ Maximum value of $f(x) - 12 = 16 - 12 = 4$

∴ Maximum value of $h(x) = 3^4 = \mathbf{81}$ ◄

5.1 **$0 \leq x \leq 3$** ◄



5.2 The equation of f: $y = -\sqrt{27x}$ for $x \geq 0$

∴ The equation of f^{-1}: $x = -\sqrt{27y}$ for $y \geq 0$ ... *we swop x and y*

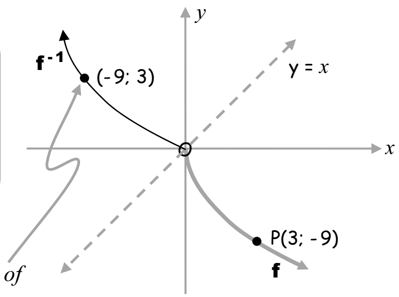
$\therefore\ x^2 = 27y$; but remember: $x \leq 0$

$\div 27)$ $\therefore\ y = \frac{x^2}{27}$ ... *or $y = \frac{1}{27}x^2$*

$\therefore$ $\mathbf{y = \frac{x^2}{27}}$ **for $x \leq 0$** ◄

5.3

Note: f^{-1} is the reflection of f in the line $y = x$



This point is a reflection of (3; -9) in the dashed line

5.4 **A reflection in the x-axis** ◄ ... *$(x; y) \to (x; -y)$*

---

6. The equation of the hyperbola: $y = \frac{a}{x - p} + q$

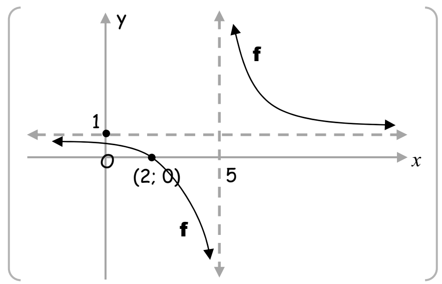
$\therefore\ y = \frac{a}{x - 5} + 1$

Substitute (2; 0): $\therefore\ 0 = \frac{a}{(2 - 5)} + 1$

$\therefore\ -1 = \frac{a}{-3}$

$\times(-3)$ $\therefore\ 3 = a$

$\therefore$ $\mathbf{f(x) = \frac{3}{x - 5} + 1}$ ◄



THE ANSWER SERIES
Your Key to Exam Success

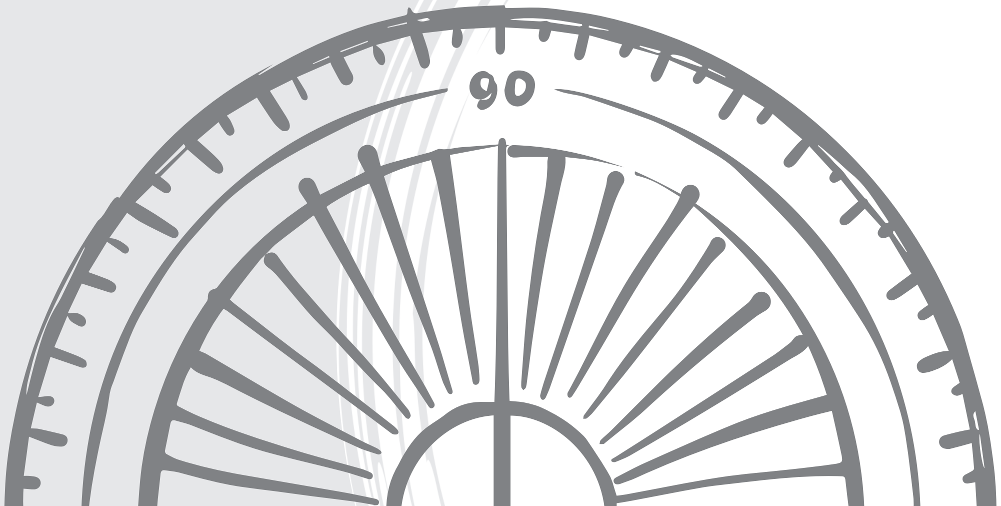
2-in-1

Mathematics

LEVEL 3 & 4 CHALLENGING QUESTIONS WITH SOLUTIONS

Anne Eadie & Gretel Lampe

GRADE **12** CAPS



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Independent Events vs Mutually Exclusive Events

First study The Probability Rules on the previous page.

QUESTION 2

A survey concerning their holiday preferences was done with 180 staff members. The options they could choose from were to:

- Go to the coast
- Visit a game park
- Stay at home

The results were recorded in the table below:

	Coast	Game Park	Home	Total
Male	46	24	13	83
Female	52	38	7	97
Total	98	62	20	180

2.1 Determine the probability that a randomly selected staff member:

2.1.1 is male (1)

2.1.2 does not prefer visiting a game park (2)

2.2 Are the events 'being a male' and 'staying at home' independent events? Motivate your answer with relevant calculations. (4) [7]

Solutions

NB: $P(E) = \frac{n(E)}{n(S)}$

2.1.1 $P(\text{male}) = \mathbf{\frac{83}{180}}$ ◄

2.1.2 $P(\text{not game park}) = \frac{98 + 20}{180}$

$= \frac{118}{180}$

$= \mathbf{\frac{59}{90}}$ ◄ . . . *P(coast or at home)*

2.2 $P(\text{M and at home}) = \frac{13}{180} = 0{,}07\dot{2}$ ◄

$P(\text{male}) = \frac{83}{180}$ and $P(\text{at home}) = \frac{20}{180}$

$\therefore\ P(\text{male}) \times P(\text{at home}) = \frac{83}{180} \times \frac{20}{180}$

$= 0{,}051\ldots$

$\therefore$ **The events are not independent** ◄ . . . P(M & at home) ≠ P(M) × P(at home)

QUESTION 3

For two events, A and B, it is given that:

P(A) = 0,2

P(B) = 0,63

P(A and B) = 0,126

Are the events, A and B, independent?
Justify your answer with appropriate calculations. (3)

Solution

Independent events:

For 2 events **A** and **B** to be INDEPENDENT:

P(A and B) must be equal to **P(A) X P(B)**

. . . *called the* **PRODUCT** *Rule*

So, calculate the value of each of these expressions to determine whether they are equal or not.

P(A) x P(B) $= 0{,}2 \times 0{,}63 = 0{,}126$

But, **P(A and B)** = 0,126 also . . . *given*

$\therefore$ **P(A) x P(B) = P(A and B)**

$\therefore$ The 2 events A and B are independent

Note the layout of the PROOF;
i.e. the answer must be JUSTIFIED!

NB: Do **not** confuse **independent** events with **mutually exclusive** events!

Mutually exclusive events:

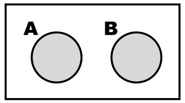
For 2 events **A** and **B** to be MUTUALLY EXCLUSIVE:

P(A or B) must be equal to **P(A) + P(B)**

. . . *called the* **SUM** *rule*

So, necessarily: $\mathbf{P(A \cap B) = 0}$ . . .

(**A** and **B** do not overlap)



QUESTION 4

Given: P(A) = 0,45; P(B) = y and P(A or B) = 0,74

Determine the value(s) of y if A and B are mutually exclusive. (3)

Solution

P(A or B) = P(A) + P(B) . . . *A and B are mutually exclusive events.*

$\therefore\ 0{,}74 = 0{,}45 + y$

$\therefore$ **y = 0,29** ◄

QUESTION 5

Events A and B are mutually exclusive. It is given that:

- P(B) = 2P(A)
- P(A or B) = 0,57

Calculate P(B). (3)

Solution

P(A or B) = P(A) + P(B) . . . *A & B are mutually exclusive*

$\therefore\ 0{,}57 = \frac{1}{2}P(B) + P(B)$ . . . $2P(A) = P(B)$ ➡ $P(A) = \frac{1}{2}P(B)$

$\therefore\ 1{,}5\,P(B) = 0{,}57$

$\therefore$ **P(B) = 0,38** ◄

Use the **TOPIC GUIDE** on p. 147 (in the study guide) to select and practise further questions on this section.

CHALLENGING QUESTIONS & SOLUTIONS: PAPER 2

Trigonometry

Identities & Compound Angles

QUESTION 1

1.1 Given: $\sin 16^\circ = p$

Determine the following in terms of p, **without using a calculator**.

1.1.1 $\sin 196^\circ$ 1.1.2 $\cos 16^\circ$ (2)(2)

1.2 Given: $\cos(A - B) = \cos A\cos B + \sin A\sin B$

Use the formula for cos(A – B) to derive a formula for sin(A + B). (3)

1.3 Simplify $\dfrac{\sqrt{1 - \cos^2 2A}}{\cos(-A)\,.\,\cos(90^\circ + A)}$ completely, given that $0^\circ < A < 90^\circ$. (5)

1.4 Given: $\cos 2B = \dfrac{3}{5}$ and $0^\circ \leq B \leq 90^\circ$

Determine, **without using a calculator**, the value of EACH of the following in its simplest form:

1.4.1 cos B 1.4.2 sin B (3)(2)

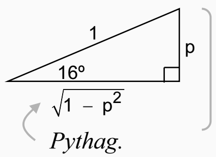
1.4.3 $\cos(B + 45^\circ)$ (4) [21]

Solutions

1.1.1 $\sin 196^\circ = \sin(180^\circ + 16^\circ) = -\sin 16^\circ = \mathbf{-p}$ ◂

1.1.2 $\cos^2 16^\circ = 1 - \sin^2 16^\circ$
$= 1 - p^2$
$\therefore \cos 16^\circ = \mathbf{\sqrt{1 - p^2}}$ ◂

OR: $\sin 16^\circ = \dfrac{p}{1}$
$\therefore \cos 16^\circ = \mathbf{\sqrt{1 - p^2}}$ ◂



Refer to Formulae & Derivations of **Compound and Double Angles** on p. v (in the study guide).

1.2 Given: $\cos(A - B) = \cos A\cos B + \sin A\sin B$

$\sin(A + B) = \cos[90^\circ - (A + B)]$
$= \cos[(90^\circ - A) - B]$
$= \cos(90^\circ - A)\cos B + \sin(90^\circ - A)\sin B$
$= \mathbf{\sin A\cos B + \cos A\sin B}$ ◂

1.3 $\dfrac{\sqrt{1 - \cos^2 2A}}{\cos(-A)\,.\,\cos(90^\circ + A)} = \dfrac{\sqrt{\sin^2 2A}}{\cos A\,.\,(-\sin A)}$
$= \dfrac{\sin 2A}{-\sin A\cos A}$
$= \dfrac{2\sin A\cos A}{-\sin A\cos A}$
$= \mathbf{-2}$ ◂

1.4 Given: $\cos 2B = \dfrac{3}{5}$

1.4.1 $2\cos^2 B - 1 = \cos 2B$
$\therefore 2\cos^2 B = \cos 2B + 1$
$= \dfrac{3}{5} + 1$
$= \dfrac{8}{5}$
$\therefore \cos^2 B = \dfrac{4}{5}$
$\therefore \cos B = \mathbf{\dfrac{2}{\sqrt{5}}}$ ◂ . . . *$0^\circ \leq B \leq 90^\circ$*

1.4.2 $\sin^2 B = 1 - \cos^2 B$
$= 1 - \dfrac{4}{5}$
$= \dfrac{1}{5}$
$\therefore \sin B = \mathbf{\dfrac{1}{\sqrt{5}}}$ ◂

1.4.3 $\cos(B + 45^\circ) = \cos B\cos 45^\circ - \sin B\sin 45^\circ$
$= \left(\dfrac{2}{\sqrt{5}}\right)\left(\dfrac{1}{\sqrt{2}}\right) - \left(\dfrac{1}{\sqrt{5}}\right)\left(\dfrac{1}{\sqrt{2}}\right)$
$= \dfrac{2}{\sqrt{5}\sqrt{2}} - \dfrac{1}{\sqrt{5}\sqrt{2}}$
$= \mathbf{\dfrac{1}{\sqrt{10}}}$ ◂

QUESTION 2

2.1 Prove the identity:
$\cos^2(180^\circ + x) + \tan(x - 180^\circ)\sin(720^\circ - x)\cos x = \cos 2x$ (5)

2.2 Use $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ to derive the formula for $\sin(\alpha - \beta)$. (3)

2.3 If $\sin 76^\circ = x$ and $\cos 76^\circ = y$, show that $x^2 - y^2 = \sin 62^\circ$, without using a calculator. (4) [12]

Solutions

2.1 LHS $= (-\cos x)^2 + (+\tan x)(-\sin x)\cos x$
$= \cos^2 x - \left(\dfrac{\sin x}{\cos x}\right)\left(\dfrac{\sin x}{1}\right)\left(\dfrac{\cos x}{1}\right)$
$= \cos^2 x - \sin^2 x$
$= \cos 2x$
$=$ **RHS** ◂

2.2 $\sin(\alpha - \beta) = \cos[90^\circ - (\alpha - \beta)]$
$= \cos[90^\circ - \alpha + \beta]$
$= \cos[(90^\circ - \alpha) + \beta]$
$= \cos(90^\circ - \alpha)\cos\beta - \sin(90^\circ - \alpha)\sin\beta$ . . . *from the formula provided*
$= \mathbf{\sin\alpha\cos\beta - \cos\alpha\sin\beta}$ ◂

2.3 $x^2 - y^2$
$= \sin^2 76^\circ - \cos^2 76^\circ$
$= \cos^2 14^\circ - \sin^2 14^\circ$ . . .
$= \cos 2(14^\circ)$
$= \cos 28^\circ$
$= \sin(90^\circ - 28^\circ)$
$= \mathbf{\sin 62^\circ}$ ◂

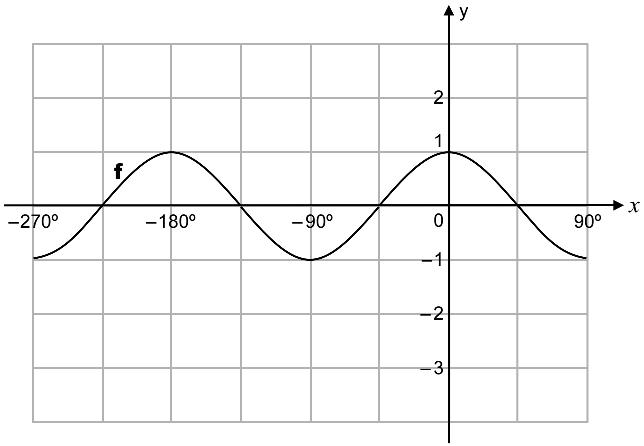
OR $= -(\cos^2 76^\circ - \sin^2 76^\circ)$
$= -\cos 2(76^\circ)$
$= -\cos 152^\circ$
$= -(-\cos 28^\circ)$
$= \cos 28^\circ$, etc.

Use the **Trig Summary** – see p. vi (in the study guide) – to master this topic!

Graphs & Equations & Compound Angles

QUESTION 3

In the diagram, the graph of $f(x) = \cos 2x$ is drawn for the interval $x \in [-270^\circ; 90^\circ]$.



3.1 Draw the graph of $g(x) = 2\sin x - 1$ for the interval $x \in [-270^\circ; 90^\circ]$ on the grid. Show ALL the intercepts with the axes, as well as the turning points. (4)

3.2 Let A be a point of intersection of the graphs of **f** and **g**.

Show that the x-coordinate of A satisfies the equation $\sin x = \frac{-1 + \sqrt{5}}{2}$. (4)

3.3 Hence, calculate the coordinates of the points of intersection of graphs of **f** and **g** for the interval $x \in [-270^\circ; 90^\circ]$. (4)

[12]

Remember double angle formulae:

There are **3 possible expansions** for **cos 2x**.
In this question, we choose the one which has (only) **sin x** in it so that we can arrive at a quadratic equation in $\sin x$.
Also see Q5.2 on the next page & Q10.2 on p. 28.

Solutions

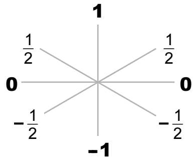
3.1

When sketching a graph, bear in mind:

- **the shape** . . . *sine & cosine graphs are wave-shaped*

- **the critical points** (best found using point by point plotting).

$\sin x$:



Angle x	–270º	–180º	–90º	0º	90º
$\sin x$	1	0	–1	0	1
$2\sin x$	2	0	–2	0	2
2 sin x – 1	**1**	**–1**	**–3**	**–1**	**1**

↑ Study & understand these critical values. You will be able to do the table mentally!

Take note of the given domain: $-270^\circ \leq x \leq 90^\circ$

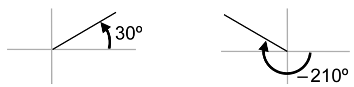
- **x-intercepts:**

$2\sin x - 1 = 0$ . . . *where $y = 0$*

$\therefore\ 2\sin x = 1$

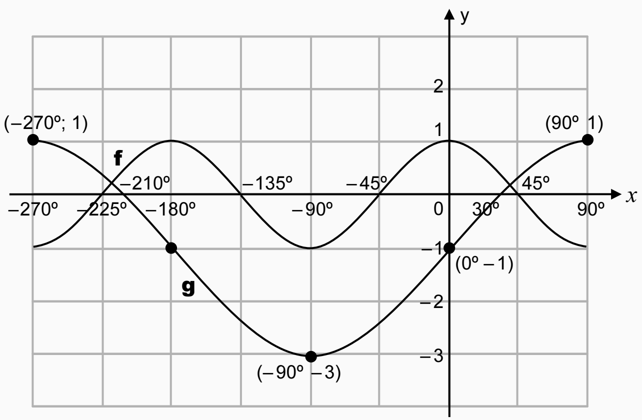
$\therefore\ \sin x = \frac{1}{2}$

$\therefore\ x = 30^\circ$ or -210°



- **Parameters:** $y = \mathbf{a}\sin \mathbf{k}(x + \mathbf{p}) + \mathbf{q}$

The effects of **a**, **k**, **p** and **q** are useful as a checking tool, but tricky for accuracy as an initial method.



3.2 At the points of intersection, $f(x) = g(x)$

cos 2x $= 2\sin x - 1$ . . . *the solutions of this eqn. are the x-coordinates of the pts. of intersection of* **f** & **g**.

$\therefore$ **1 – 2 sin²x** $= 2\sin x - 1$ . . . *double angle formula*

$\therefore\ -2\sin^2 x - 2\sin x + 2 = 0$ . . . *a quadratic equation in sin x*

$\div(-2)\ \therefore\ \sin^2 x + \sin x - 1 = 0$ . . . *quadratic formula required!*

$\therefore\ \sin x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ . . . *$a = 1;\ b = 1;\ c = -1$*

$= \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(-1)}}{2(1)}$ . . . *it is safest to use brackets when substituting*

$= \frac{-1 + \sqrt{5}}{2}$ or $\frac{-1 - \sqrt{5}}{2}$

But $\frac{-1 - \sqrt{5}}{2}$ is < -1 . . . *sin x only has values from –1 to +1. We write: For all values of x, $-1 \leq \sin x \leq 1$.*

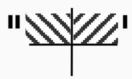
$\therefore$ this solution is invalid

$\therefore$ The x-coordinate of (any) point of intersection of **f** and **g** will satisfy the equation $\mathbf{\sin x = \frac{-1 + \sqrt{5}}{2}}$ ◄

3.3 Now, to establish the **x-coordinates** of the 2 points of intersection of **f** and **g over the given domain**:

$\sin x = \frac{-1 + \sqrt{5}}{2}$

$= +0{,}62$ . . . *sin x is* **pos.** *in quadrants* **I** & **II**



$\therefore\ x = 38{,}17^\circ + n(360^\circ)$ or $x = 180^\circ - 38{,}17^\circ + n(360^\circ)$, $n \in \mathbb{Z}$

$\therefore\ x = 38{,}17^\circ$ or $-218{,}17^\circ$ ← *$n = -1$ gives a value in the domain.*

And now, the **y-coordinates** . . .

$y = \cos 2x$ or $2\sin x - 1$ . . . *$f(x)$ or $g(x)$*

$= 0{,}24$ (for both values of x)

$\therefore$ The points of intersection are:

(–218,17º; 0,24) and (38,17º; 0,24) ◄

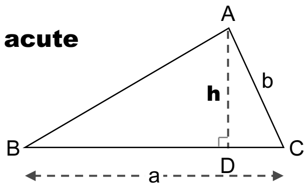
PAPER 2: TRIGONOMETRY

Area, Sine & Cosine Rules: PROOFS

① The Area rule

Area of $\triangle ABC = \frac{1}{2}$ ab sin C

$\hat{C}$ acute



Construction: Draw $AD \perp BC$

Proof: Area of $\triangle ABC = \frac{1}{2}a\mathbf{h}$ . . . ❶

But, in $\triangle ACD$: $\frac{\mathbf{h}}{b} = \sin C$

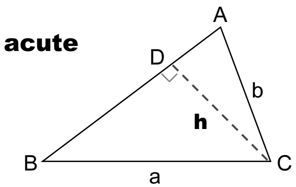
$\therefore\ \mathbf{h} = b \sin C$ . . . ❷

❷ in ❶: $\therefore$ **Area of $\triangle ABC = \frac{1}{2}$ab sin C**

② The Sine rule

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$\hat{B}$ acute



Construction: Draw $CD \perp AB$

Proof: In $\triangle ADC$: $\frac{\mathbf{h}}{b} = \sin A$

$\therefore\ \mathbf{h} = b \sin A$ . . . ❶

In $\triangle BDC$: $\frac{\mathbf{h}}{a} = \sin B$

$\therefore\ \mathbf{h} = a \sin B$ . . . ❷

Equating ❶ & ❷: $\therefore\ b \sin A = a \sin B$

$\div ab)\quad \therefore\ \frac{\sin A}{a} = \frac{\sin B}{b}$

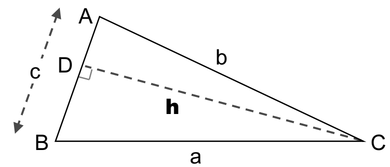
Similarly, by drawing a perpendicular from **B**, one can prove: $\frac{\sin A}{a} = \frac{\sin C}{c}$

$\therefore\ \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

③ The Cosine rule

$a^2 = b^2 + c^2 - 2bc \cos A$

$\hat{A}$ acute:



Construction: Draw $CD \perp BA$

Proof: $a^2 = BD^2 + h^2$ . . . *Pythagoras*

$\therefore\ a^2 = (c - AD)^2 + h^2$

$= c^2 - 2c\,.\,AD + \underbrace{AD^2 + h^2}$

$= c^2 - 2c\,.\,AD + \quad b^2$ . . . *Pythagoras*

$= b^2 + c^2 - 2c\,.\,AD$ . . . ❶

In $\triangle ADC$: $\frac{AD}{b} = \cos A$

$\therefore\ AD = b \cos A$ . . . ❷

❷ in ❶:

$\therefore$ **$a^2 = b^2 + c^2 - 2bc \cos A$**

For CONSTRUCTIONS in all 3 proofs,

Area Rule, Sine Rule & Cosine Rule

Always construct the height from a vertex not involved in the formula

- To prove: Area = $\frac{1}{2}$ab sin **C**, draw a height from **A** or **B**, not **C**.
- To prove: $\frac{\sin \mathbf{A}}{a} = \frac{\sin \mathbf{B}}{b}$, draw a height from **C**, not **A** or **B**.
- To prove: $a^2 = b^2 + c^2 - 2bc \cos \mathbf{A}$, draw a height from **B** or **C**, not **A**.

ANALYTICAL GEOMETRY TOOLKIT

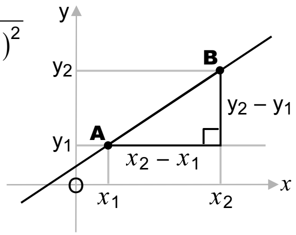
FORMULAE

Consider two points A(x_1; y_1) and B(x_2; y_2):

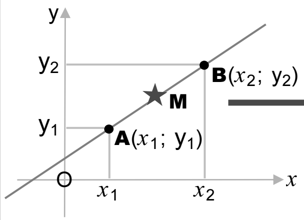
DISTANCE

$$\mathbf{AB}^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$ . . . *Thm. of Pythagoras*

$$\therefore\ \mathbf{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



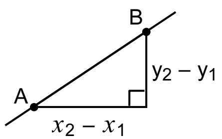
MIDPOINT



$$\mathbf{M}\left(\frac{x_1 + x_2}{2};\ \frac{y_1 + y_2}{2}\right)$$

The co-ordinates of the midpoint, M, are the **averages** *of the co-ordinates of the endpoints, A and B.*

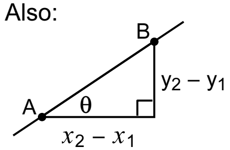
GRADIENT



$$\mathbf{m} = \frac{\text{change in y}}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

. . . *the* **gradient** *of the line*

Also:

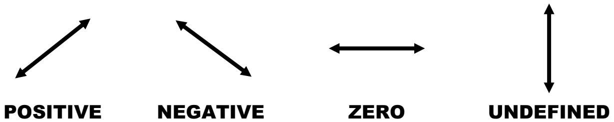


$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{y_2 - y_1}{x_2 - x_1}$$

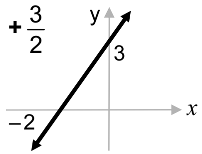
. . . *where θ is the* **angle of inclination** *of the line*

The Gradient of a line

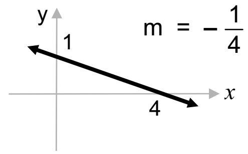
Values



$m = +\frac{3}{2}$

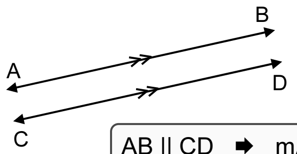


$m = -\frac{1}{4}$



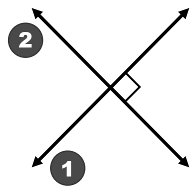
Parallel lines

Parallel lines have **equal** gradients.



AB || CD ➡ $m_{AB} = m_{CD}$

Perpendicular lines

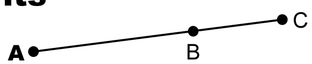


If the gradient of line ❶ is $\frac{2}{3}$, then the gradient of line ❷ will be $-\frac{3}{2}$.

Note: $m_1 \times m_2 = \left(+\frac{2}{3}\right)\left(-\frac{3}{2}\right) = -1$

i.e. The **product** of the gradients of ⊥ lines is **−1**.

Collinear points



Three points A, B & C are collinear if the gradients of **A**B & **A**C are equal. (Note: Point **A** is common.)

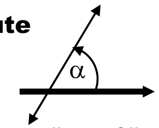
$m_{AB} = m_{AC}$ ⬌ A, B & C are collinear

The Inclination of a line

Angles α and β below are **angles of inclination**.

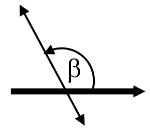
The inclination of a line is the **angle** which the line makes with the positive direction of the x-axis.

α acute



gradient of line is positive

β obtuse



gradient of line is negative

Gradient, **m = tan** α or **tan** β where α and β are the ∠**s of inclination**.

Graphs in general

3 Basic facts

❶ : **Axis intercepts**

Every point on the **y-axis** has **x = 0**.
Every point on the **x-axis** has **y = 0**.

❷ : **The equation**

The **equation** of a graph is true for **all** points on the graph.

∴ The **equation** of the **y-axis** is **x = 0**;
& the **equation** of the **x-axis** is **y = 0**.

❸ : **Types of graph**

Different **types/patterns** are indicated by various equations.

e.g. $\mathbf{y = mx + c}$ indicates a straight line
$\mathbf{x^2 + y^2 = r^2}$ indicates a circle

Important Facts

FACT ❶ : Points on Graphs

If a point lies on a graph, the equation is true for its coordinates, i.e. the coordinates of the point satisfy the equation . . . so, substitute!

and, conversely,

If a point (i.e. its coordinates) satisfies the equation of a graph (i.e. 'makes it true'), then it lies on the graph.

FACT ❷ : Point(s) of Intersection

The coordinates of the point(s) of intersection of two graphs 'obey the conditions' of both graphs, i.e. they satisfy both equations simultaneously.

They are found:

- **'algebraically'** by solving the 2 equations, or
- **'graphically'** by reading from the graph.

THESE 2 FACTS ARE CRUCIAL!

STRAIGHT LINE GRAPHS & their equations

Standard forms

Standard forms of the equation of a straight line:

- **y = mx + c:**
  where **m** = the gradient & **c** = the y-intercept
  When m = 0: **y = c** *… a line* || **x-axis**
  When c = 0: **y = mx** *… a line through the* **origin**
  Also: **x = k** *… a line* || **y-axis**

- **$y - y_1 = m(x - x_1)$:**
  where **m** = the gradient & **$(x_1; y_1)$** is a fixed point.

General form

The **general form** of the equation of a straight line is
ax + by + c = 0, e.g. $2x + 3y + 6 = 0$

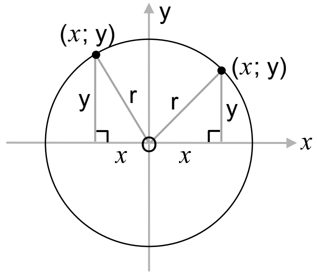
CIRCLES & their equations

Circles with the origin as centre

True of **any** point **(x; y)** on a circle with centre **(0; 0)** and radius **r** is that:

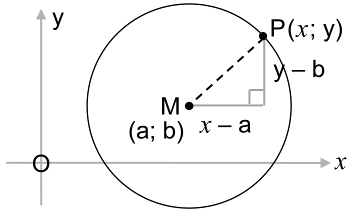
$$x^2 + y^2 = r^2$$

Thm. of Pythag.!



Circles with any given centre

True of any point **(x; y)** on a circle with centre **(a; b)** and radius **r** is that:



$$(x - a)^2 + (y - b)^2 = r^2$$

Distance formula! (Thm. of Pythag.)

Converting the equation of a circle

General form: $Ax^2 + Bx + Cy^2 + Dy + E = 0$

to **Standard form:** $(x - a)^2 + (y - b)^2 = r^2$

(using completion of squares)

e.g. $x^2 - 6x + y^2 + 8y - 25 = 0$

$\therefore x^2 - 6x + y^2 + 8y = 25$

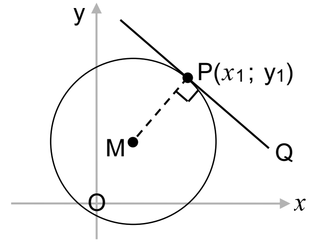
$\therefore x^2 - 6x + \mathbf{3^2} + y^2 + 8y + \mathbf{4^2} = 25 + \mathbf{9} + \mathbf{16}$

$\therefore (x - 3)^2 + (y + 4)^2 = 50$

This is the equation of a circle with:
centre (3; –4) & **radius**, $r = \sqrt{50}$ $(= 5\sqrt{2})$ units

A Tangent to a circle . . .

is **perpendicular** to the **radius** of the circle at the **point** of **contact**.



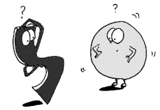
To find the **equation** of a tangent, use **'m and 1 point'** in the straight line equation:

$y - y_1 = m(x - x_1)$

e.g. $m_{MP} = 2 \Rightarrow m_{PQ} = -\frac{1}{2}$

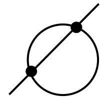
($\because$ radius MP $\perp$ tangent PQ)

Point(s) of intersection of a Line and a Circle

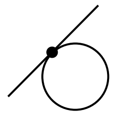


A line and a circle **either**

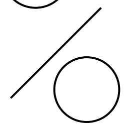
❶ **'cut'** (twice!) [secant]
(2 points in common)



or ❷ **'touch'** (once!) [tangent]
(1 point in common)



or ❸ **don't cut or touch**
(no points in common)



If we substitute $y = mx + c$ into the equation of the ⊙, there will **either** be: ❶ **2 solutions**

or ❷ **1 solution**

or ❸ **no solutions**

for x, resulting in one of the above scenarios.

FINAL ADVICE

Use common sense

& ALWAYS DRAW A PICTURE!!!