# Mathematics

## CLASS TEXT & STUDY GUIDE

**Anne Eadie & Gretel Lampe**

GRADE
**10**
CAPS

**3-in-1**

THE ANSWER SERIES
*Your Key to Exam Success*

# Grade 10 Mathematics 3-in-1 CAPS

## CLASS TEXT & STUDY GUIDE

The Answer Series Grade 10 Maths 3-in-1 study guide uses simple, logical steps to explore the CAPS curriculum in great depth, from first principles all the way up to final mastery. It addresses gaps in your memory from previous grades, before inviting you to tackle new work through carefully selected graded exercises.

**Key features:**

- Comprehensive, explanatory notes and worked examples
- Graded exercises to promote logic and develop a technique for each topic
- Detailed solutions for all exercises
- An exam with fully explained solutions (paper 1 and paper 2) for thorough consolidation and final exam preparation.

This study guide has proven to be a great companion to Grade 10 Maths learners. Not only that, it builds confidence and also lays the foundations for success in Grade 11 and 12.

THE ANSWER SERIES *Your Key to Exam Success*

GRADE

# 10

CAPS

3-in-1

# Mathematics

Anne Eadie & Gretel Lampe

**THIS CLASS TEXT & STUDY GUIDE INCLUDES**

1. Comprehensive Notes
2. Exercises
3. Full Solutions

eBook available

*Plus: Exam Papers and Memos*
*& Problem Solving Questions and Memos*

THE ANSWER SERIES *Your Key to Exam Success*

2012 publication | 2021 edition | ISBN: 978-1-920297-79-4

# DETAILED CONTENTS



**LAW 1** **the PRODUCT of POWERS:** $a^m \times a^n = a^{m+n}$

*(same bases)*

**We know:** $a^3 \times a^2 = a \times a \times a \times a \times a = a^{3+2} = a^5$ . . . *we add the exponents*

$\therefore\ a^4 \times a^2 = \ldots ?$ $\quad a^x \times a^y = \ldots ?$ $\quad a^x \times a^{x^2} = \ldots ?$

*Answers:* $a^6$ ; $a^{x+y}$ ; $a^{x+x^2}$ . . . *not always that intuitive!*

Do you see that it becomes less and less intuitive?
Keep referring to the law to keep on track!

***& the law reversed:*** $a^{m+n} = a^m \times a^n$

i.e. $2^{n+1} = 2^n \times 2^1$ ; even $2^{n-1} = 2^{n+(-1)} = 2^n \times 2^{-1}$

## EXERCISE 2.1 – An exercise on LAW 1

*(Answers on page 2.9)*

These are **EXPRESSIONS** to be simplified.

Complete:

1. $x^3 \times x^4 =$
2. $2^p \times 2^q =$
3. $x^a \times x^b =$
4. $2^{\frac14} \times 2^{\frac34} =$
5. $2^{-3} \times 2^{-4} =$
6. $x^n \times x^n =$
7. $a^{m+n} . a^{m-n} =$
8. $(x+y)^2 . (x+y)^3 =$
9. $3^{2x+4} . 3^{-2x-3} =$
10. $4^{x+1} . 7^{x+1} . 4^{3-x} =$
11. $a^b \times c^d =$
12. $3^{x^2} \times 3^x =$
13. $\left(a^{\frac23} b^{\frac12}\right)\left(a^{\frac13} b^{\frac{-1}{2}}\right) =$
14. $\sqrt[3]{2^2 . 4^5} =$

Factorise:

15. $2^{n+3} = \ldots . \times \ldots .$ ; $\quad 2^{n-5} = \ldots . \times \ldots .$ *(law 1 reversed)*

Simplify:

16. $\dfrac{5^{x+2} - 5^{x+1}}{5^{x+1}}$
17. $\dfrac{3^x - 3^{x-2}}{8.3^x}$
18. $\dfrac{2.2^{n+1} - 2^n}{3.2^{n-1}}$

These are **EQUATIONS** to be solved.

Solve for $x$:

$a^m = a^n$
$\Rightarrow m = n$

19. $2^{x+1} = 2^3$
20. $3^{2x}.3^x = 9$
21. $7^{4x} = 49$
22. $a^x.a^{x+5}.a = 1$
23. $2^{x+1} + 2^x + 2^{x-1} = 28$
24. $5^{x+1} + 5^{x-1} = \dfrac{26}{25}$

**NOTE:** There is no law for $a^m + a^n$ – only for the **PRODUCT** of powers, $a^m \times a^n$

**LAW 2** **the QUOTIENT of POWERS:** $\dfrac{a^m}{a^n} = a^{m-n}$

*(same bases)*

The 4 factors below cancel 4 of the 7 above.

**We know:** $2^7 \div 2^4 = \dfrac{2^7}{2^4} = \dfrac{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2 \times 2} = 2^3$

So, $\dfrac{2^7}{2^4} = 2^{7-4}$ . . . *we subtract the exponents!*

***& the law reversed:*** $a^{m-n} = \dfrac{a^m}{a^n}$ i.e. $2^{5-a} = \dfrac{2^5}{2^a}$; $2^{n-1} = \dfrac{2^n}{2}$

## EXERCISE 2.2 – An exercise on LAW 2

*(Answers on page 2.9)*

These are **EXPRESSIONS** to be simplified.

Complete:

1. $a^5 \div a^2 =$
2. $x^{1\frac14} \div x^{\frac14} =$
3. $p^6 \div p^2 =$
4. $\dfrac{2^{3x}}{2^{2x}} =$
5. $x^{16} \div x^4 =$
6. $\dfrac{7^{1-n}}{7^n} =$
7. $a^7 \div a =$
8. $\dfrac{a^{\frac32}}{a^{-\frac12}} =$
9. $\dfrac{p^{21}}{p^{20}} =$
10. $\dfrac{a^b}{c^d} =$
11. $\dfrac{b}{b^{-2}} =$

Simplify:

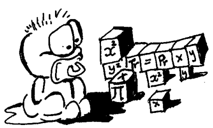
12. $\dfrac{7x^{-2}y^5}{14x^{-1}y^8}$
13. $\dfrac{15^{4n}}{9^{2n+1} . 25^{2n}}$
14. $\dfrac{a^{-3}}{a^{-5}}$

## Answers

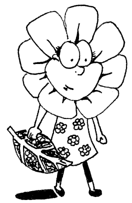
### EXERCISE 2.1 – LAW 1

*(Questions on page 2.4)*

1. $x^7$
2. $2^{p+q}$
3. $x^{a+b}$
4. $2^{\frac{1}{4}+\frac{3}{4}} = 2^1 = 2$
5. $2^{-3-4} = 2^{-7}$
6. $x^{n+n} = x^{2n}$
7. $a^{m+n+m-n} = a^{2m}$
8. $(x+y)^{2+3} = (x+y)^5$
9. $3^{(2x+4)+(-2x-3)} = 3^1 = 3$
10. $4^{x+1+3-x}.7^{x+1} = 4^4.7^{x+1}$
11. cannot be simplified
12. $3^{x^2}\times 3^x = 3^{x^2+x}$
13. $\left(a^{\frac{2}{3}}b^{\frac{1}{2}}\right)\left(a^{\frac{1}{3}}b^{\frac{-1}{2}}\right)$
$= a^{\frac{2}{3}+\frac{1}{3}}.b^{\frac{1}{2}-\frac{1}{2}}$
$= a^1b^0$
$= a$
14. $\sqrt[3]{2^2.4^5} = \sqrt[3]{2^2.2^{10}}$
$= \sqrt[3]{2^{12}}$
$= 2^4$
$= 16$
15. $2^{n+3} = 2^n\times 2^3$;
$2^{n-5} = 2^n\times 2^{-5}$
16. $\dfrac{5^x.5^2 - 5^x.5}{5^x.5}$
$= \dfrac{\cancel{5^x}(25-5)}{\cancel{5^x}.5}$
$= \dfrac{20}{5}$
$= 4$
17. $\dfrac{3^x - 3^x.3^{-2}}{8.3^x}$
$= \dfrac{\cancel{3^x}\left(1-\frac{1}{9}\right)}{8.\cancel{3^x}}$
$= \dfrac{8}{9}\times\dfrac{1}{8}$
$= \dfrac{1}{9}$
18. $\dfrac{2.2^n.2 - 2^n}{3.2^n.2^{-1}}$
$= \dfrac{\cancel{2^n}(4-1)}{\cancel{2^n}.\frac{3}{2}}$
$= 3\times\dfrac{2}{3}$
$= 2$
19. $2^{x+1} = 2^3$
$\therefore x+1 = 3$
$\therefore x = 2$
20. $3^{2x}.3^x = 9$
$\therefore 3^{2x+x} = 3^2$
$\therefore 3x = 2$
$\therefore x = \dfrac{2}{3}$
21. $7^{4x} = 49$
$\therefore 7^{4x} = 7^2$
$\therefore 4x = 2$
$\therefore x = \dfrac{1}{2}$
22. $a^{x+x+5+1} = 1$
$\therefore a^{2x+6} = 1$
$\therefore 2x+6 = 0$
$\therefore 2x = -6$
$\therefore x = -3$
23. $2^x.2 + 2^x + 2^x.2^{-1} = 28$
$\therefore 2^x\left(2+1+\frac{1}{2}\right) = 28$
$\therefore 2^x\left(\frac{7}{2}\right) = 28$
$\therefore 2^x = {}^4\cancel{28}\times\dfrac{2}{\cancel{7}}$
$\therefore 2^x = 8$
$\therefore 2^x = 2^3$
$\therefore x = 3$



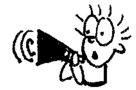
24. $5^{x+1} + 5^{x-1} = \dfrac{26}{25}$
$\therefore 5^x.5 + 5^x.5^{-1} = \dfrac{26}{25}$
$\therefore 5^x\left(5+\frac{1}{5}\right) = \dfrac{26}{25}$
$\therefore 5^x\left(\frac{26}{5}\right) = \dfrac{26}{25}$
$\times\frac{5}{26})$ $\therefore 5^x = \dfrac{\cancel{26}}{25}\times\dfrac{5}{\cancel{26}}$
$\therefore 5^x = \dfrac{1}{5}$
$\therefore 5^x = 5^{-1}$
$\therefore x = -1$



### EXERCISE 2.2 – LAW 2

*(Questions on page 2.4)*

1. $a^3$
2. $x^{1\frac{1}{4}-\frac{1}{4}} = x^1 = x$
3. $p^{6-2} = p^4$
4. $2^{3x-2x} = 2^x$
5. $x^{16-4} = x^{12}$
6. $7^{1-n-n} = 7^{1-2n}$
7. $a^{7-1} = a^6$
8. $a^{\frac{3}{2}-\left(-\frac{1}{2}\right)} = a^{\frac{4}{2}} = a^2$
9. $p^{21-20} = p$
10. cannot be simplified
11. $b^{1-(-2)} = b^3$
12. $\dfrac{\cancel{7}x^{-2}y^5}{{}_2\cancel{14}x^{-1}y^8} = \dfrac{1}{2xy^3}$
13. $\dfrac{15^{4n}}{9^{2n+1}.25^{2n}}$
$\dfrac{(3.5)^{4n}}{(3^2)^{2n+1}.(5^2)^{2n}}$
$= \dfrac{3^{4n}.\cancel{5^{4n}}}{3^{4n+2}.\cancel{5^{4n}}} = \dfrac{1}{3^2} = \dfrac{1}{9}$
14. $\dfrac{a^{-3}}{a^{-5}} = a^{-3+5} = a^2$
15. $3^{2x} = \dfrac{3^3}{3^{-x}}$
$\therefore 3^{2x} = 3^{3+x}$
$\therefore 2x = 3+x$
$\therefore x = 3$
16. $2^{4x} = \dfrac{2^3}{2^{2x-2}}$
$\therefore 2^{4x} = 2^{3-2x+2}$
$\therefore 2^{4x} = 2^{5-2x}$
$\therefore 4x = 5-2x$
$\therefore 6x = 5$
$\therefore x = \dfrac{5}{6}$
17. $4\times 3^{2x} = 9\times 2^{2x}$
$\div 4\times 2^{2x})$ $\therefore \dfrac{3^{2x}}{2^{2x}} = \dfrac{9}{4}$
$\therefore \left(\dfrac{3}{2}\right)^{2x} = \left(\dfrac{3}{2}\right)^2$ ... $\dfrac{a^n}{b^n} = \left(\dfrac{a}{b}\right)^n$
$\therefore 2x = 2$
$\therefore x = 1$



Law 5 reversed, to be compared to law 2.

18. $3^{n-2} = \dfrac{3^n}{3^2}$ ; $5^{3-p} = \dfrac{5^3}{5^p}$

EXPONENTS 2

# FACTORISING – SOME GOOD ADVICE

***How many terms do I have?***
***The number of terms determines my options.***

| NO. OF TERMS | OPTIONS | |
|---|---|---|
| 2 terms | Always look out for a **common factor** FIRST! | **The difference of two squares OR**<br>**The sum of two cubes or the difference of two cubes** |
| 3 terms | | **Trinomial** |
| 4 terms | **Grouping**<br>... watch out for switchrounds! | 2 - 2 – **for common 'brackets',** or<br>3 - 1 or<br>1 - 3 – **leading to difference of squares** |
| 5 terms | | 3 - 2 or<br>2 - 3 – **for common 'brackets'** |
| 6 terms | | 3 - 3 – **for common 'brackets'** or **difference of squares** or<br>2 - 2 - 2 – **for common 'brackets'** |

And then, two good habits . . .

Once you've factorised:

- **CHECK your factors by multiplying out**.

Remember: factorising ⇔ multiplying out . . . **REVERSE PROCESSES!**

- **Ask yourself: 'Have I finished?'**
  Double-check for any simplification or any further factorisation *(e.g. common factor or difference of squares).*

e.g. (1) $2x(a + b) + 4(a + b)$
$= (a + b)(2x + 4)$
$= 2(a + b)(x + 2)$

(2) $x^4 - y^4$
$= (x^2 + y^2)(x^2 - y^2)$
$= (x^2 + y^2)(x + y)(x - y)$

Knowing that there are guidelines shifts the mind into a more confident way of thinking.

# FIVE FACTORISATION TESTS

*(approximately ½ hour each)*

## A

1. $x^2 - x - 12$ 2. $8ax - 12ay - 10x + 15y$ 3. $(x + 5)(x + 3) + k(3 + x)$ (2)(4)(2)
4. $p^2 - 14p - 32$ 5. $4m - pm + 8 - 2p$ 6. $12x^2 - 19xy - 21y^2$ (2)(2)(2)
7. $(x - y)^3 - 3(x - y)^2$ 8. $2a^2 - 18$ 9. $28ab + 4a^2 - 15b^2$ (3)(2)(2)
10. $ac + yd - ad - yc$ 11. $3k(2m - 3n) + 5t(3n - 2m)$ 12. $(a - b)^2 - 49$ (2)(3)(2)
13. $12x^3 + 11x^2 - x$ 14. $x^6 - 64y^6$ 15. $x(x - 1)(x - 2) - (x - 1)^2$ (3)(3)(4)
16. $1 - 16a^{16}$ 17. $-6m^2 + 11m + 10$ 18. $4x - ax + ay - 4y$ (4)(2)(2)
19. Write down the simplest expression (in factorised form) into which the expressions in questions 6, 7 & 18 can divide (i.e. the lowest common multiple). (1)
20. Calculate the value of $109^2 - 9^2$ in the shortest possible way, without using a calculator. (3) [50]

## B

1. $pa + pb + qa + qb$ 2. $x^2 + 5x + 6$ 3. $4x^2 - 9$ (2)(1)(1)
4. $5at + 9 + 3a + 15t$ 5. $4x^2 - 20x + 25$ 6. $5 - 20a^2$ (3)(2)(2)
7. $3ac + 2bc - 2bd - 3ad$ 8. $3y^2 + 15y - 108$ 9. $ac + 6b - 3ab - 2c$ (2)(2)(3)
10. $p^3 - 8$ 11. $52^2 - 50^2$ (evaluate) 12. $132x^2 + 96xy - 36y^2$ (2)(3)(3)
13. $9x^2 - 5y - 3x - 25y^2$ 14. $8m^2 - 50mn + 33n^2$ 15. $x^4 - x^3 + x - 1$ (3)(3)(3)
16. $12mb + 9a^2 - 4m^2 - 9b^2$ 17. $x^2 - 2xy - a^2 + y^2$ 18. $k^4 - 37k^2 + 36$ (3)(3)(3)
19. $16(2a + b)^2 - 9(a - 2b)^2$ (6) [50]

## C

1. $x^2 - 25xy + 144y^2$ 2. $x^2 - 24xy + 144y^2$ 3. $2ac - 3ad - 2bc + 3bd$ (2)(2)(3)
4. $k(a - b) + n(b - a)$ 5. $122^2 - 120^2$ (evaluate) 6. $2a^2 - 2a - 12$ (2)(3)(3)
7. $ax - b^2 - bx + ab$ 8. $a^2x^2 + 5ax - 24$ 9. $x^2 - 2\frac{1}{4}$ (3)(2)(3)
10. $20m^2n + 62mn^2 - 28n^3$ 11. $ax^2 + 3by^2 - 3bxy - axy$ 12. $125x^3 + y^3$ (3)(3)(2)
13. $20x^2 - 45y^2$ 14. $3a^3 + 12a^2b + 9ab^2$ 15. $x^2 - 2x + 2y - y^2$ (3)(3)(3)
16. Factorise $a^2 - b^2$ and then write down the factors of $(2x^2 - 4x + 1)^2 - (x^2 - 3x + 3)^2$ in the simplest form. (10) [50]

## D

1. $2x^2 - 8$ 2. $a^2 - b^2 + a - b$ 3. $x^2 - 12x + 36$ (3)(3)(2)
4. $\left(r + \frac{1}{r}\right)^2 - \left(r - \frac{1}{r}\right)^2$ 5. $10x^2 + 38xy - 8y^2$ 6. $2x^3 - x^2 + 4 - 8x$ (3)(3)(3)
7. $40ap^2 + 82a^2p + 40a^3$ 8. $12ab + 8b^2 - 6af - 4bf$ 9. $-3x^2 + 21x - 30$ (3)(3)(3)
10. $3 - 3(x - y)^2$ 11. $x^2 + 8 + \frac{16}{x^2}$ 12. $k(x^3 - 1) - k(x - 1)^3$ (3)(2)(5)
13. $4p^2(3p - 1) - 5p$ 14. $x^2 - y^2 + 4x + 4$ 15. $3a^3 - 24b^3$ (4)(4)(3)
16. If $P = 3x + 2$ and $Q = 2x - 1$, express $P^2 - 2PQ + Q^2$ in terms of $x$. (3) [50]

## E

1. $9a^2 - 49b^2$ 2. $xy + 6x + 2y + 12$ 3. $6a^2 - 5ab - 6b^2$ (2)(2)(2)

4. $18a^2 - 8b^2$ 5. $-6x^3 + 5x^2 + 25x$ 6. $x^4 + 24x^2y^2 + 108y^4$ (3)(3)(2)

7. $21a^2 + 26a - 15$ 8. $-0{,}81 + c^2$ 9. $4b^3 - 8b^2 - ab + 2a$ (2)(2)(4)

10. $\frac{3}{4} - 3x^2$ 11. $y^3 - y^2 + y - 1$ 12. $x^3 + 4x^2y + 3xy^2$ (3)(3)(3)

13. $a^2 + c^2 - b^2 - 2ac$ 14. $(x^2 - 2x - 8)^2 + 5(x^2 - 2x - 8)$ (3)(4)

15. $3a^2 + 6ab + 3b^2 + 9a^2y + 18aby + 9b^2y$ 16. $2{,}7^2 - 2{,}3^2$ (evaluate) (5)(2)

17. Factorise $x^3 + \frac{1}{x^3}$ and then determine the value of this expression if $x + \frac{1}{x} = 2$. (5) [50]

## Answers to Factorisation Tests

### A

1. $(x - 4)(x + 3)$

2. $4a(2x - 3y) - 5(2x - 3y)$
 $= (2x - 3y)(4a - 5)$

3. $(x + 3)(x + 5 + k)$

4. $(p - 16)(p + 2)$

5. $m(4 - p) + 2(4 - p)$
 $= (4 - p)(m + 2)$

6. $(4x + 3y)(3x - 7y)$

7. $(x - y)^2[(x - y) - 3]$
 $= (x - y)^2(x - y - 3)$

8. $2(a^2 - 9)$
 $= 2(a + 3)(a - 3)$

9. $4a^2 + 28ab - 15b^2$
 $= (2a - b)(2a + 15b)$

10. $ac - ad - yc + yd$
 $= a(c - d) - y(c - d)$
 $= (c - d)(a - y)$

11. $3k(2m - 3n) - 5t(2m - 3n)$
 $= (2m - 3n)(3k - 5t)$

12. $(a - b + 7)(a - b - 7)$

13. $x(12x^2 + 11x - 1)$
 $= x(12x - 1)(x + 1)$

14. $(x^3)^2 - (8y^3)^2$ . . . *the difference of two squares*
 $= (x^3 + 8y^3)(x^3 - 8y^3)$
 $= (x + 2y)(x^2 - 2xy + 4y^2)(x - 2y)(x^2 + 2xy + 4y^2)$

 OR $(x^2)^3 - (4y^2)^3$ . . . *the difference of two cubes*
 $= (x^2 - 4y^2)(x^4 + 4x^2y^2 + 16y^4)$
 $= (x + 2y)(x - 2y)(x^4 + 4x^2y^2 + 16y^4)$

15. $(x - 1)[x(x - 2) - (x - 1)]$
 $= (x - 1)(x^2 - 2x - x + 1)$
 $= (x - 1)(x^2 - 3x + 1)$

16. $(1 + 4a^8)(1 - 4a^8)$
 $= (1 + 4a^8)(1 + 2a^4)(1 - 2a^4)$

17. $-(6m^2 - 11m - 10)$
 $= -(2m - 5)(3m + 2)$
 OR $10 + 11m - 6m^2$
 $= (5 - 2m)(2 + 3m)$

18. $4x - ax - 4y + ay$
 $= x(4 - a) - y(4 - a)$
 $= (4 - a)(x - y)$

20. $109^2 - 9^2 = (109 + 9)(109 - 9)$
 $= (118)(100)$
 $= 11\ 800$

19. $(4x + 3y)(3x - 7y)(x - y)^2(x - y - 3)(4 - a)$

### B

1. $p(a + b) + q(a + b)$
 $= (a + b)(p + q)$

2. $(x + 2)(x + 3)$

3. $(2x + 3)(2x - 3)$

4. $5at + 15t + 3a + 9$
 $= 5t(a + 3) + 3(a + 3)$
 $= (a + 3)(5t + 3)$

5. $(2x - 5)^2$

6. $5(1 - 4a^2)$
 $= 5(1 + 2a)(1 - 2a)$

7. $3ac + 2bc - 3ad - 2bd$
 $= c(3a + 2b) - d(3a + 2b)$
 $= (3a + 2b)(c - d)$

8. $3(y^2 + 5y - 36)$
 $= 3(y + 9)(y - 4)$

9. $ac - 3ab - 2c + 6b$
 $= a(c - 3b) - 2(c - 3b)$
 $= (c - 3b)(a - 2)$

10. $(p - 2)(p^2 + 2p + 4)$

11. $(52 + 50)(52 - 50)$
 $= (102)(2)$
 $= 204$

12. $12(11x^2 + 8xy - 3y^2)$
 $= 12(11x - 3y)(x + y)$

13. $(9x^2 - 25y^2) - 3x - 5y$
 $= (3x + 5y)(3x - 5y) - (3x + 5y)$
 $= (3x + 5y)(3x - 5y - 1)$

14. $(4m - 3n)(2m - 11n)$

15. $x^3(x - 1) + (x - 1)$
 $= (x - 1)(x^3 + 1)$
 $= (x - 1)(x + 1)(x^2 - x + 1)$

16. $9a^2 - 4m^2 + 12mb - 9b^2$
 $= 9a^2 - (4m^2 - 12mb + 9b^2)$
 $= (3a)^2 - (2m - 3b)^2$
 $= [3a + (2m - 3b)][3a - (2m - 3b)]$
 $= (3a + 2m - 3b)(3a - 2m + 3b)$

17. $x^2 - 2xy + y^2 - a^2$
 $= (x - y)^2 - a^2$
 $= (x - y + a)(x - y - a)$

18. $(k^2 - 36)(k^2 - 1)$
 $= (k + 6)(k - 6)(k + 1)(k - 1)$

19. $[4(2a + b) + 3(a - 2b)][4(2a + b) - 3(a - 2b)]$
 $= (8a + 4b + 3a - 6b)(8a + 4b - 3a + 6b)$
 $= (11a - 2b)(5a + 10b)$
 $= 5(11a - 2b)(a + 2b)$

### C

1. $(x - 16y)(x - 9y)$

2. $(x - 12y)^2$

3. $a(2c - 3d) - b(2c - 3d)$
 $= (2c - 3d)(a - b)$

4. $k(a - b) - n(a - b)$
 $= (a - b)(k - n)$

5. $(122 + 120)(122 - 120)$
 $= (242)(2) = 484$

6. $2(a^2 - a - 6)$
 $= 2(a - 3)(a + 2)$

7. $ax - bx + ab - b^2$
 $= x(a - b) + b(a - b)$
 $= (a - b)(x + b)$

8. $(ax + 8)(ax - 3)$

9. $x^2 - \frac{9}{4}$
 $= \left(x + \frac{3}{2}\right)\left(x - \frac{3}{2}\right)$

10. $2n(10m^2 + 31mn - 14n^2)$
 $= 2n(5m - 2n)(2m + 7n)$

11. $ax^2 - axy - 3bxy + 3by^2$
 $= ax(x - y) - 3by(x - y)$
 $= (x - y)(ax - 3by)$

12. $(5x + y)(25x^2 - 5xy + y^2)$

# SIMULTANEOUS LINEAR EQUATIONS

This refers to solving problems where you have **TWO** equations and **TWO** unknowns.
To build up our understanding of how to solve these, let us start with **ONE** equation and **TWO** unknowns.

## ❑ Single Equation, Two Unknowns – Infinite Solutions!

Until now we have dealt with equations with only **ONE** unknown

e.g. $3x + 8 = 14$

$\therefore\ x = 2$

> Linear equations with **ONE** unknown have **ONE** solution *(mostly)*

▶ Now consider a linear equation with **TWO** unknowns:

e.g. $\mathbf{x + y = 10}$ . . . *A RELATIONSHIP between x and y*

| | | | |
|---|---|---|---|
| If $x = 1$, | y = . . . . ? | or | (1; . . . ?) |
| If $x = 6$, | y = . . . . ? | or | (6; . . . ?) |
| If $x = 2{,}96$, | y = . . . . ? | or | (2,96; . . . ?) |
| If $x = -2$, | y = . . . . ? | or | (-2; . . . ?) |

$\therefore$ **Possible solutions:** (1; 9), (6; 4), (2,96; 7,04), (-2; 12), etc. ⬅ **A**

` These are all 'solutions' to this equation since, in each case, the PAIR of values of $x$ & y makes the equation true. And we can just keep going, i.e. there appears to be an INFINITE set of answers!

> A linear equation with **TWO** unknowns has an **INFINITE** number of solutions!

▶ Now consider: $\mathbf{x - y = 2}$

This equation also has an infinite set of solutions of which some examples are:

**Possible solutions**: (20; 18), (12; 10), (6; 4), (-4; -6), etc. ⬅ **B**

## ❑ The Meaning of 'Simultaneous' Equations – Two Equations, Two Unknowns

Simultaneous means **TOGETHER** or **AT THE SAME TIME**. So when we have 'simultaneous equations', it means we need to find a solution that solves TWO equations **AT THE SAME TIME**.

Let us take the two equations we have just been looking at, *simultaneously . . .*

$\mathbf{x + y = 10}$ . . . **(1)**

$\mathbf{x - y = 2}$ . . . **(2)**

> *We always number the equations on the right*

Can both of these equations be true at the same time?

In other words, are there values of $x$ and y that will make both equations true?

In words, let us ask ourselves:
Can *you think of two numbers which add up to ten* ***AND*** *have a difference of two?*

> **Hint:** Look at the list of possible solutions for each equation above – see **A** & **B**

## ❑ Solving Simultaneous Equations

If we are able to find the answer (6; 4) by just looking at the two equations and thinking about it, this is termed: solving the equations **BY INSPECTION**.

Now we will learn to find the answers to simultaneous equations *algebraically*, so that we can solve more complex problems.

***Equations can be added or subtracted . . .***

> *because . . .* **if a = b**
> **and c = d**
> **then a + c = b + d or a - c = b - d**

> **LOGIC is essential in Maths!**

In our example above we had: $x + y = 10$ . . . **(1)**
and $x - y = 2$ . . . **(2)**

Let us add equation **(1)** to equation **(2)** to find the answer algebraically . . .

**(1) + (2):** $\therefore\ x + y + x - y = 10 + 2$

> *We always explain on the left which operation we are doing.*

$\therefore\ 2x = 12$ . . .

$\therefore\ x = 6$

Substitute $x = 6$ in **(1):** $\therefore\ 6 + y = 10$

$\therefore\ y = 4$

> **Notice that by adding the equations we have eliminated (lost) y!**

$\therefore$ **The two numbers are 6 and 4** ◀

> *We also say: (6; 4) is the SOLUTION to the simultaneous linear equations*
> $x + y = 10$ *and* $x - y = 2$
> *as these values of x & y make both the equations true.*

Sometimes one can't *eliminate* a variable by adding or subtracting straight away . . .

*What do we mean by that?*

### Worked example

Solve for $x$ and y: $2x + 3y = -1$ **. . . (1)**

$3x - 6y = -12$ **. . . (2)**

We first need to alter equation number **(1)** so that adding it to or subtracting it from equation number **(2)** will eliminate one of the variables:

**(1)** $\times$ 2: $4x + 6y = -2$ **. . . (3)**

*Now, observe the coefficient of y in* **(2)** *and* **(3)**

**(2)** + **(3)**: $\therefore 7x = -14$ . . . y has been **eliminated!**

$\therefore x = -2$

**OR:** Eliminate $x$ – see below

Subst. $x = -2$ in **(1)**: $\therefore -4 + 3y = -1$

$\therefore 3y = 3$

$\therefore y = 1$

$\therefore$ Solution: **(-2; 1)** ◄ . . . .

This is an **ORDERED PAIR**
– *x first, then y second!*
– a good way to give the solution of 2 equations with 2 unknowns.

CHECK THIS ANSWER BY substituting the values into the equations **(1)** and **(2)** to see whether they hold true!

We could've gone for eliminating $x$ (instead of y):

**(1)** $\times$ 3**:** $6x + 9y = -3$ **. . . (3)**

**(2)** $\times$ 2**:** $6x - 12y = -24$ **. . . (4)**

**(3)** - **(4):** $\therefore 21y = 21$ . . . $x$ has been **eliminated**

$\therefore y = 1$

Subst. y = 1 in **(1):** $2x + 3 = -1$

$\therefore 2x = -4$

$\therefore x = -2$, etc.

### Can you solve *these* equations?

$ar^6 = 162$ **. . . (1)**

$ar^2 = 2$ **. . . (2)**

Let us try addition . . .

**(1) + (2):** $ar^6 + ar^2 = 162 + 2$

$a(r^6 + r^2) = 164$ *and now what ? ? ? – no good!*

Addition and subtraction don't work, do they! What else can we try?
Equations can also be multiplied or divided:

*because,* **if a = b**
**and c = d**
**then a.c = b.d** *and* $\frac{a}{c} = \frac{b}{d}$

*Which of these is best to use in our example to eliminate one of the variables?*

### Answer

**(1) ÷ (2):** $\frac{ar^6}{ar^2} = \frac{162}{2}$ . . . Division seems to be the way to **eliminate a**

$\therefore r^4 = 81$

$\therefore r = \pm 3$

Subst. $r = \pm 3$ in **(2):** $a \times 9 = 2$

$\therefore a = \frac{2}{9}$ ◄

Even though this sum **looks** different (and is!) the **LOGIC** is the same!

### EXERCISE 4.6

**Remember:**
**It is possible to check your answers!**

Solve the following pairs of simultaneous equations:

1. $x + y = 12$
   $x - y = 4$

   Do this one by inspection first **and** then algebraically.

2. $a + 7b = 49$
   $a + 3b = 9$

3. $p + 2q = 1$
   $3p - q = 10$

4. $2x + 3y = 8$
   $3x + 4y = 11$

5. $2x = 3y - 4$
   $y = 3 - x$

6. $\frac{y}{2} + 1 = \frac{x}{5}$
   and $\frac{1}{4}x + \frac{1}{2} = \frac{1}{3}y$

7. $\frac{x+y}{2} = 7 - \frac{2x-y}{3}$ . . . ❶
   and $\frac{x-y}{4} - \frac{x+y}{3} + 4\frac{1}{2} = 0$ . . . ❷

8. The length of a rectangle is a mm and the breadth is b mm. The area of the rectangle is unchanged if the length is increased by 6 mm and the breadth is diminished by 2 mm. The area is also unchanged if the length is decreased by 6 mm and the breadth is increased by 3 mm. Find the length and breadth of the original rectangle.

## Answers

1. **'By inspection'**: The sum of 2 numbers is 12 and their difference is 4. What are the numbers?

**Algebraically**:

$x + y = 12 \quad \ldots$ **(1)**
$x - y = 4 \quad \ldots$ **(2)**

**(1) + (2):** $2x = 16$
$\therefore x = 8$

**(1):** $8 + y = 12$
$\therefore y = 4$

$\therefore$ Solution: $(8; 4)$

*Did you get this by inspection?*

2. $a + 7b = 49 \quad \ldots$ **(1)**
$a + 3b = 9 \quad \ldots$ **(2)**

**(1) - (2):** $4b = 40$
$\therefore b = 10$

**(2):** $a + 30 = 9$
$\therefore a = -21$

$\therefore$ Solution: $(-21; 10)$

3. $p + 2q = 1 \quad \ldots$ **(1)**
$3p - q = 10 \quad \ldots$ **(2)**

**(2)** $\times$ 2: $6p - 2q = 20 \quad \ldots$ **(3)**

**(1) + (3)**: $7p = 21$
$\therefore p = 3$

**(2):** $9 - q = 10$
$q = -1$

$\therefore$ Solution: $(3; -1)$

4. $2x + 3y = 8 \quad \ldots$ **(1)**
$3x + 4y = 11 \quad \ldots$ **(2)**

**(1)** $\times$ 3: $6x + 9y = 24 \quad \ldots$ **(3)**

**(2)** $\times$ 2: $6x + 8y = 22 \quad \ldots$ **(4)**

**(3) - (4):** $\therefore y = 2$

**(3):** $\therefore 6x + 18 = 24$
$\therefore 6x = 6$
$\therefore x = 1$

$\therefore$ Solution: $(1; 2)$

5. $2x = 3y - 4 \quad \ldots$ **(1)**
$y = 3 - x \quad \ldots$ **(2)**

**(2)** in **(1):** $2x = 3(3 - x) - 4$
$\therefore 2x = 9 - 3x - 4$
$\therefore 2x + 3x = 9 - 4$
$\therefore 5x = 5$
$\therefore x = 1$

**(2):** $y = 3 - 1$
$\therefore y = 2$

$\therefore$ Solution: $(1; 2)$

6. $\frac{y}{2} + 1 = \frac{x}{5}$

$\times 10)$ $5y + 10 = 2x$
$\therefore 5y - 2x = -10$
$\times (-1)$: $2x - 5y = 10 \quad \ldots$ **(1)**

$\frac{x}{4} + \frac{1}{2} = \frac{y}{3}$

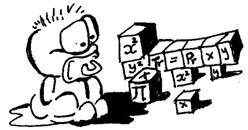
$\times 12)$ $3x + 6 = 4y$
$\therefore 3x - 4y = -6 \quad \ldots$ **(2)**

**(1)** $\times$ 3: $6x - 15y = 30 \quad \ldots$ **(3)**
**(2)** $\times$ 2: $6x - 8y = -12 \quad \ldots$ (4)
**(3)** - **(4)**: $\therefore -7y = 42$
$\div (-7)$: $\therefore y = -6$

From **(1)**: $2x = 5y + 10$
$= 5(-6) + 10$
$= -30 + 10$
$= -20$
$\therefore x = -10$ and $y = -6$

$\therefore$ Solution: $(-10; -6)$

7. **(1)** $\times$ 6: $3(x + y) = 42 - 2(2x - y)$
$\therefore 3x + 3y = 42 - 4x + 2y$
$\therefore 7x + y = 42 \quad \ldots$ **(3)**



**(2)** $\times$ 12: $3(x - y) - 4(x + y) + 54 = 0$
$\therefore 3x - 3y - 4x - 4y + 54 = 0$
$\therefore -x - 7y = -54$
$\times (-1)$: $\therefore x + 7y = 54 \quad \ldots$ **(4)**

**(3)** $\times$ 7: $\therefore 49x + 7y = 294 \quad \ldots$ **(5)**

**(4)** - **(5)**: $\therefore -48x = -240$
$\therefore x = 5$

**(3)**: $\therefore y = 42 - 7x$
$= 42 - 35$
$\therefore y = 7$

$\therefore$ Solution: $(5; 7)$

8. $(a + 6)(b - 2) = ab$
$\therefore ab - 2a + 6b - 12 = ab$
$\therefore -2a + 6b = 12$
$\div 2)$ $\therefore -a + 3b = 6 \quad \ldots$ **(1)**

and

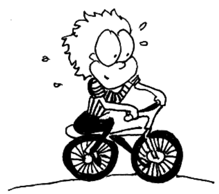
$(a - 6)(b + 3) = ab$
$\therefore ab + 3a - 6b - 18 = ab$
$\therefore 3a - 6b = 18$
$\div 3)$ $\therefore a - 2b = 6 \quad \ldots$ **(2)**

(1) + (2): $b = 12$

From **(2)**: $a = 2(b) + 6$
$= 2(12) + 6$
$= 24 + 6$
$= 30$

$\therefore$ length = 30 mm and breadth = 12 mm

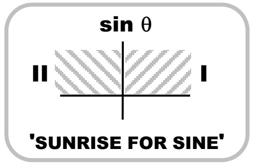
$\therefore$ Solution: $(30; 12)$



## The SIGNS of the trig ratios IN A FLASH!

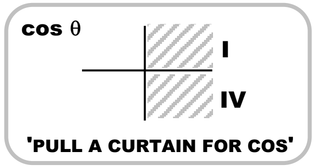
sin θ = $\frac{y}{r}$ ← and **y is positive** in **I** and **II**

∴ **sin θ is POSITIVE** in quadrants 1 & 2
*(and negative in 3 & 4)*




cos θ = $\frac{x}{r}$ ← and **x is positive** in **I** and **IV**

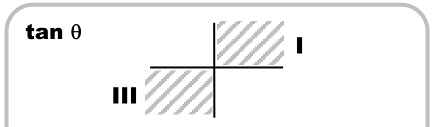
∴ **cos θ is POSITIVE** in quadrants 1 & 4
*(and negative in 2 & 3)*




tan θ = $\frac{y}{x}$ ←←

and **x & y** have the same sign in **I** and **III**

∴ **tan θ is POSITIVE** in quadrants 1 & 3
*(and negative in 2 & 4)*



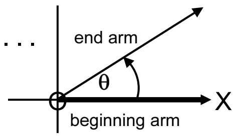

Apparently, this is **THE BMW SYMBOL**!!!

Learn these easy PICTURES so that you know the SIGNS of your trig ratios IN A FLASH !

**NO MORE CAST RULE!!!**

## The 4 steps to find the trig ratios of <u>any</u> angle:

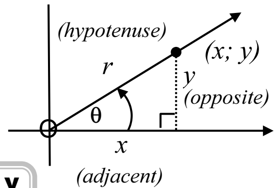
1. **Place the ∠ in STANDARD POSITION** (starting at $\overrightarrow{OX}$) . . .




2. **Pick a point (x; y) on the end arm of the ∠**
   – we'll call its distance from the origin **r**

3. **Write down x = y = r =**

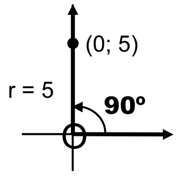
4. **Apply the DEFINITIONS**




**sin θ = $\frac{y}{r}$**  **cos θ = $\frac{x}{r}$**  **tan θ = $\frac{y}{x}$**

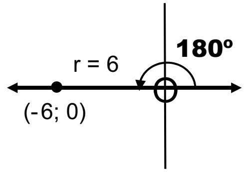
## The trig ratios of 90º and multiples of 90º

Use this procedure to find the trig ratios of 90º ; 180º ; 270º & 360º (& 0º)



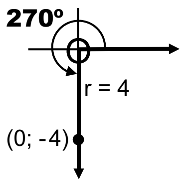

sin 90º = $\frac{y}{r}$ = $\frac{5}{5}$ = 1
cos 90º = $\frac{x}{r}$ = $\frac{0}{5}$ = 0
tan 90º = $\frac{y}{x}$ = $\frac{5}{0}$ = ∞

$x$ = 0 ; y = 5 ; r = 5



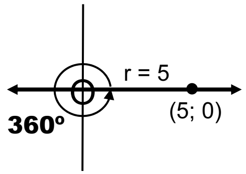

sin 180º = $\frac{y}{r}$ = $\frac{0}{6}$ = 0
cos 180º = $\frac{x}{r}$ = $\frac{-6}{6}$ = -1
tan 180º = $\frac{y}{x}$ = $\frac{0}{-6}$ = 0

$x$ = -6 ; y = 0 ; r = 6




sin 270º = $\frac{y}{r}$ = $\frac{-4}{4}$ = -1
cos 270º = $\frac{x}{r}$ = $\frac{0}{4}$ = 0
tan 270º = $\frac{y}{x}$ = $\frac{-4}{0}$ ➡ ∞

$x$ = 0 ; y = -4 ; r = 4




sin 360º = $\frac{y}{r}$ = $\frac{0}{5}$ = 0
cos 360º = $\frac{x}{r}$ = $\frac{5}{5}$ = 1
tan 360º = $\frac{y}{x}$ = $\frac{0}{5}$ = 0

$x$ = 5 ; y = 0 ; r = 5

Note: The results for 0º and 360º are the same.

**SUMMARY**

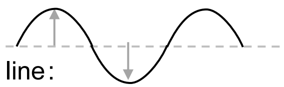
| θ: | 0° | ⟶ | 90° | ⟶ | 180° | ⟶ | 270° | ⟶ | 360° |
|---|---|---|---|---|---|---|---|---|---|
| sin θ: | 0 | - - -→ | 1 | - - -→ | 0 | - - -→ | -1 | - - -→ | 0 |
| cos θ: | 1 | - - -→ | 0 | - - -→ | -1 | - - -→ | 0 | - - -→ | 1 |
| tan θ: | 0 | - - -→ | ± ∞ | - - -→ | 0 | - - -→ | ± ∞ | - - -→ | 0 |

## Trigonometric graphs

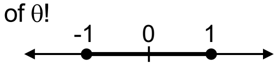
We will learn how to sketch the graphs y = sin θ, y = cos θ and y = tan θ for 0º ≤ θ ≤ 360º. We will use the critical values of these ratios to make it easy. But first, some terminology . . .

### Terminology

The sine and cosine graphs are WAVE-shaped.

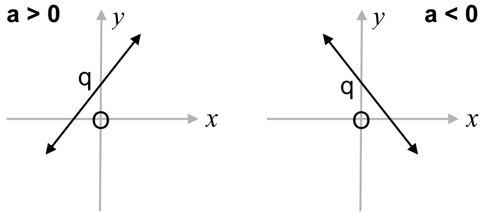


- The **amplitude** of a WAVE is the deviation from its centre line:
- The **period** of a graph is the number of degrees spanning a FULL WAVE.
- The **range** is the set of all the possible y-values.
  Our investigations of the trig ratios have shown us that the range of values of sines and cosines is very small - only between **-1** and **1**.
  We write: **-1** ≤ **sin** θ ≤ **1** and **-1** ≤ **cos** θ ≤ **1** for all values of θ!
  By contrast, the range of tan values is from -∞ to +∞!



# A Summary of Algebraic Functions

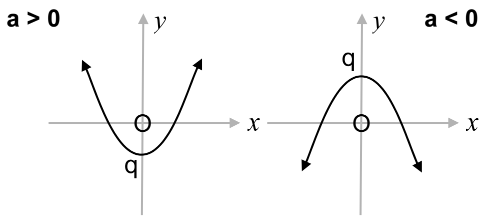
**Straight lines: y = ax + q**




- $x$-intercept (y = 0) & y-intercept ($x$ = 0)
- Domain: $x \in \mathbb{R}$
- Range: $y \in \mathbb{R}$

*(No axes of symmetry; no asymptotes)*

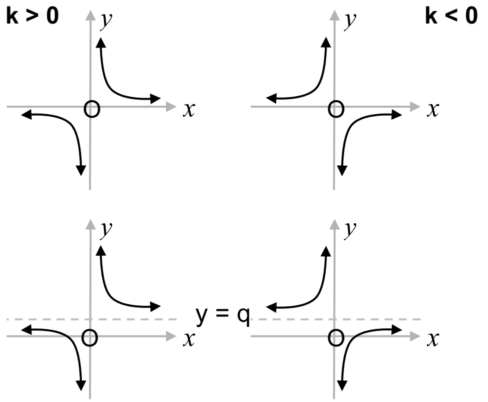
**Parabolas: y = ax² + q**




- $x$-intercept(s) (y = 0) & y-intercept ($x$ = 0)
- Domain: $x \in \mathbb{R}$
- Range: $y \geq q$ if $a > 0$
  $y \leq q$ if $a < 0$
- Axis of symmetry: $x = 0$ (the y-axis)

*(No asymptotes)*

**Hyperbolas: $y = \frac{k}{x} + q$**




- $x$-intercept (y = 0) only when $q \neq 0$
- no y-intercept ($x \neq 0$)
- Domain: $x \in \mathbb{R}$, $x \neq 0$
- Range: $y \in \mathbb{R}$, $y \neq q$
- Axes of symmetry:

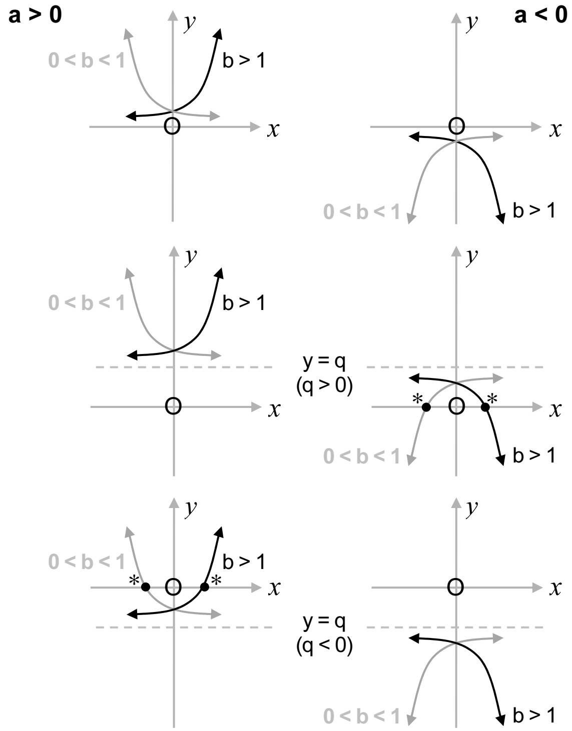
  for $y = \frac{k}{x}$ ➡ $y = x$ & $y = -x$

  & for $y = \frac{k}{x} + q$ ➡ $y = x + q$

  & $y = -x + q$
- Asymptotes: $x = 0$ (y-axis) & $y = q$

a > 0 means: **a** is positive
a < 0 means: **a** is negative

**Exponential functions: $y = ab^x + q$**




- *$x$-intercept (y = 0)
  only if $a > 0$ & $q < 0$ or
  $a < 0$ & $q > 0$
- y-intercept ($x$ = 0)
  ➡ (0; a) if q = 0
  otherwise ➡ (0; a + q)
- Domain: $x \in \mathbb{R}$
- Range:
  for $a > 0$, $y > q$ &
  for $a < 0$, $y < q$
- Asymptote: $y = q$
- Axes of symmetry:
  *there are no axes of symmetry*

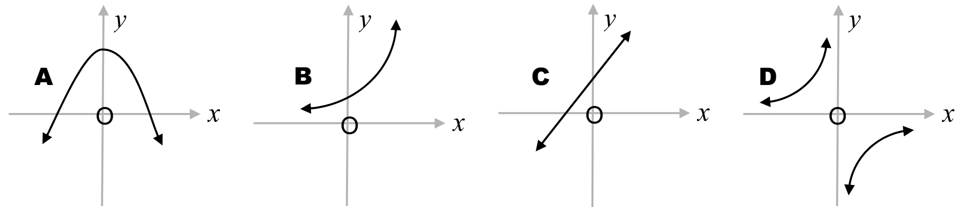
## EXERCISE 6.9 – Algebraic graphs

*(Answers on page 6.25)*

1. Given:

**y = ax + q ;** **y = ax² + q ;** **$y = \frac{a}{x} + q$ ;** **y = abˣ + q** *(b > 0)*

1.1 Choose which one of the above equations suits each graph best.
Give a reason for your answer.




1.2 State in each case if a > 0 or a < 0. Motivate your answer.

1.3 State in each case if q > 0; q < 0 or q = 0.
Motivate your answer.

⬅ **Note:** q is not necessarily the y-intercept!

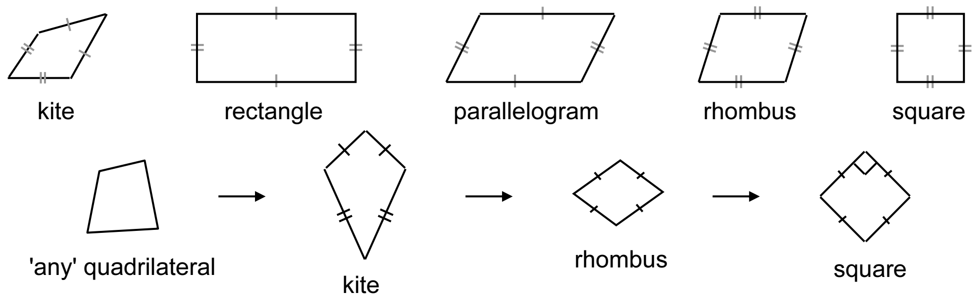
FUNCTIONS & GRAPHS 6

# QUADRILATERALS

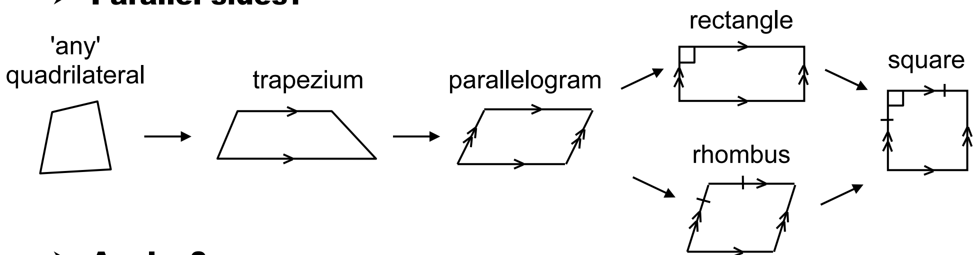
## ❑ Revision of Properties of Quadrilaterals

- Recall all the quadrilaterals ... *(kite, trapezium, parallelogram, rectangle, rhombus, square)*.
- What *properties* do they have?

### ➤ Equal sides?



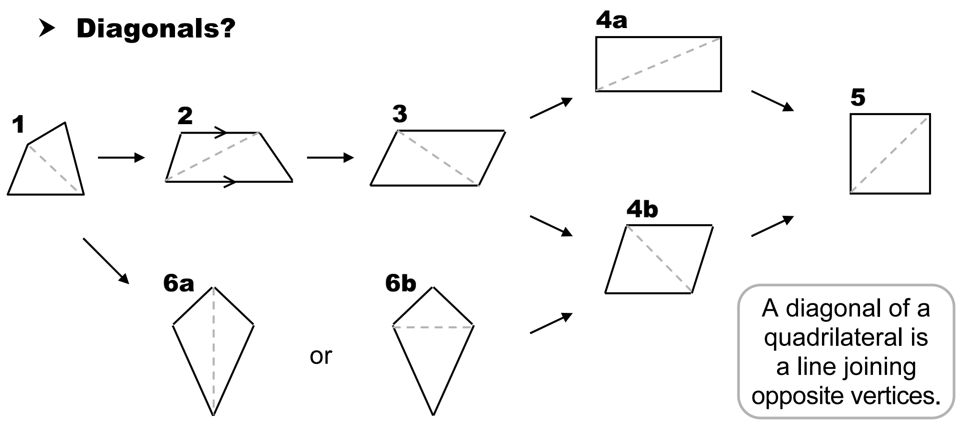

### ➤ Parallel sides?




### ➤ Angles?

Which are equal? Which are supplementary? Which are right angles?

### ➤ Diagonals?




A diagonal of a quadrilateral is a line joining opposite vertices.

**Investigating quadrilaterals, using diagonals:**

**fig. 1:** Use a diagonal to determine the sum of the interior angles of a quadrilateral.

**fig. 2:** Use a diagonal to find the area of a trapezium.

*How would you find the sum of the interior angles of a pentagon? A hexagon?*

**fig. 3 - 6:** Which of these quadrilaterals have their areas bisected by the diagonal?

**fig. 3 - 6:** Draw in the second diagonal. For each figure, establish whether the diagonals are:

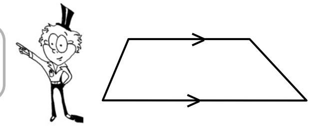
- equal
- bisect each other
- intersect at right angles
- bisect the angles of the quadrilateral

**fig. 6:** Find the area of a kite in terms of its diagonals. Could this formula apply to a rhombus? A square?

## ❑ Defining Quadrilaterals

### ➤ A trapezium

**Definition:** A trapezium is a quadrilateral with ONE PAIR OF OPPOSITE SIDES parallel.



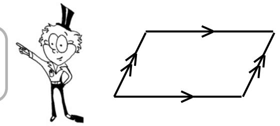
### ➤ A parallelogram

We have observed the *properties* of a parallelogram:

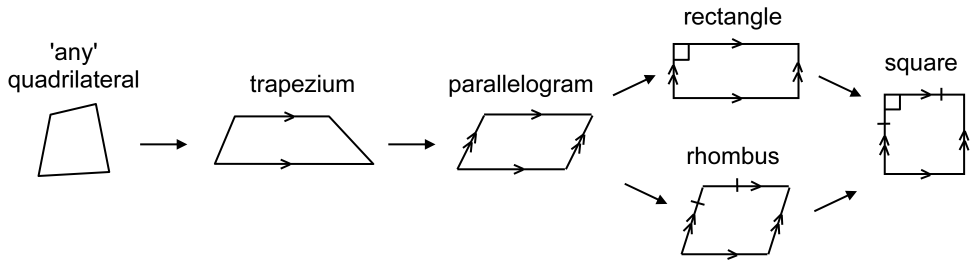
- both pairs of opposite sides parallel
- both pairs of opposite sides equal
- both pairs of opposite angles equal
- diagonals which bisect one another.

We will, however, *define* the parallelogram in terms of its parallel lines.

**Definition:** A parallelogram is a quadrilateral with TWO PAIRS OF OPPOSITE SIDES parallel.



Observe the progression of quadrilaterals below as we discuss further definitions:

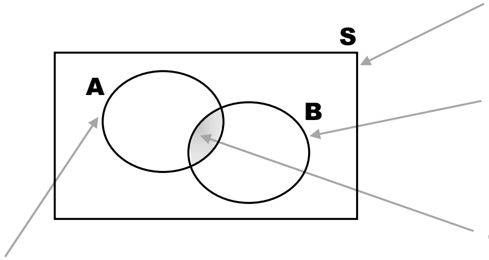


## ❑ Venn Diagrams

A graphical method to visually represent the outcomes of two or more different events (by means of circles); together with common elements of the events (by means of overlapping circles); as well as the sample space of all the events (by means of a rectangle).

### ▶ Sample Space & Events

**Possible Venn Diagram layout:**




**Sample Space**
– the set of all possible outcomes

**Event B**
– an event which contains all the possible outcomes of B

**Common elements** (shaded)
– from event A and event B [should they exist]

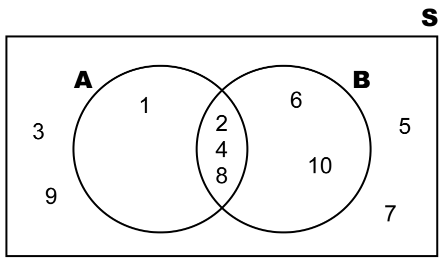
**Event A**
– an event which contains all the possible outcomes of A

**Worked Example 1**

Set up a Venn Diagram to illustrate the following:

- A sample space from 1 to 10 (whole numbers only)
- Event A: the factors of 8
- Event B: the multiples of 2

**Answer**




Event A: Factors of 8 = {1; 2; 4; 8}
$\therefore$ n(**A**) = 4 elements

Event B: Multiples of 2 = {2; 4; 6; 8; 10}
$\therefore$ n(**B**) = 5 elements

Common elements = {2: 4: 8}

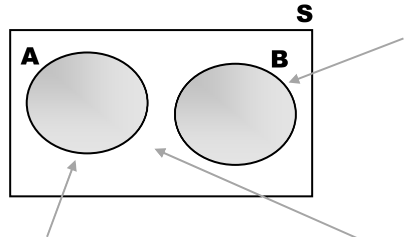
**Remember!**

- ❖ n(event) = number of favourable outcomes in the event
- ❖ P(event) = probability of the number of favourable outcomes n(E) divided by the total number of elements in the sample space n(S)

  i.e. $P(E) = \dfrac{\text{the number of favourable outcomes that exist } n(E)}{\text{the total number of possible outcomes } n(S)}$

- ❖ The sum of the probabilities in the Venn diagram must be 1 or 100%

### ▶ Mutually Exclusive / Disjoint Events




**Event B**
– contains all the possible outcomes of B

Events **A** & **B** are . . .

**Event A**
– contains all the possible outcomes of A

**Mutually exclusive**
– the events do not overlap
$\therefore$ **they have no common elements**

**Worked Example 2**

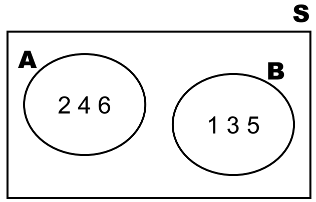
Given that **S** = {1; 2; 3; 4; 5; 6} and that event **A** is all the even numbers and event **B** is all the odd numbers:

(a) Draw a Venn Diagram to illustrate the situation.

(b) Are these events **A** and **B** mutually exclusive? Give a reason for your answer.

**Answers**

(a)

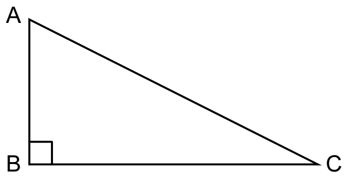



(b) Yes, these events are mutually exclusive as an even number can never be an odd number.

## TRIGONOMETRY [36]

### QUESTION 4

4.1 In the diagram below, $\Delta$ABC is right-angled at B.



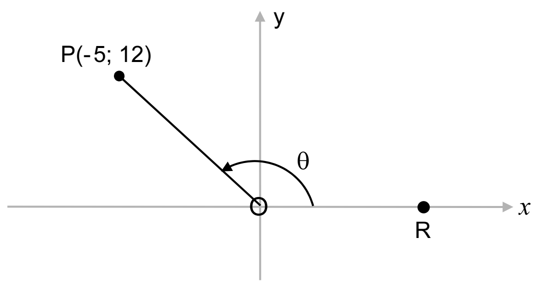

Complete the following statements:

4.1.1 $\sin C = \frac{AB}{...}$ (1)

4.1.2 $... A = \frac{AB}{BC}$ (1)

4.2 **Without using a calculator**, determine the value of: $\frac{\sin 60° \, . \, \tan 30°}{\sec 45°}$ (4)

4.3 In the diagram, P(-5; 12) is a point in the Cartesian plane and $R\hat{O}P = \theta$.




Determine the value of:

4.3.1 $\cos \theta$ (3)

4.3.2 $\text{cosec}^2 \theta + 1$ (3) [12]

### QUESTION 5

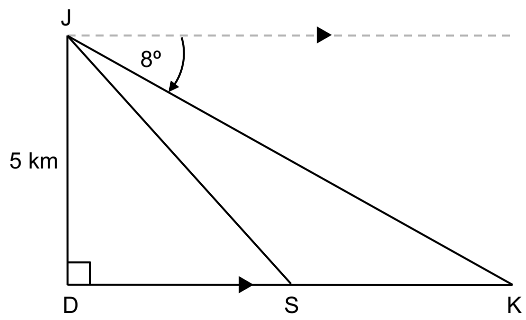
5.1 Solve for $x$, correct to ONE decimal place, in each of the following equations where $0° \leq x < 90°$.

5.1.1 $5 \cos x = 3$ (2)

5.1.2 $\tan 2x = 1{,}19$ (3)

5.1.3 $4 \sec x - 3 = 5$ (4)

5.2 An aeroplane at J is flying directly over a point D on the ground at a height of 5 kilometres. It is heading to land at point K. The angle of depression from J to K is 8º. S is a point along the route from D to K.




5.2.1 Write down the size of $J\hat{K}D$. (1)

5.2.2 Calculate the distance DK, correct to the nearest metre. (3)

5.2.3 If the distance SK is 8 kilometres, calculate the distance DS. (1)

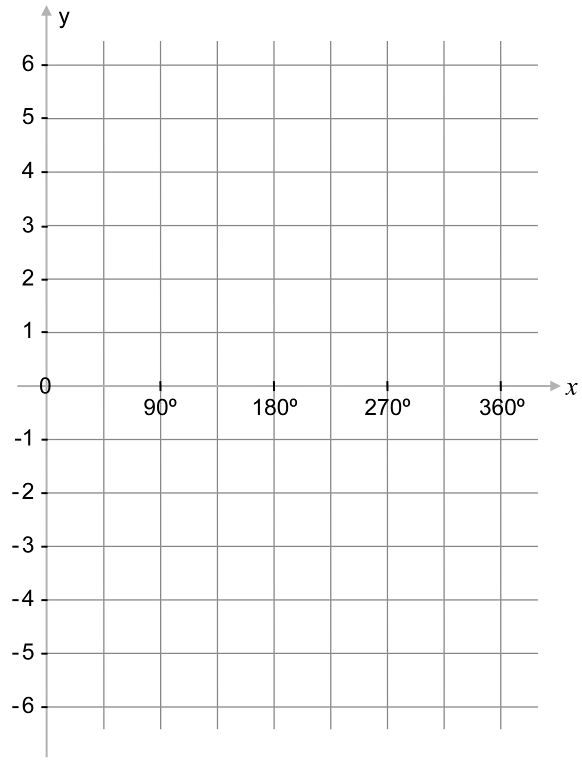
5.2.4 Calculate the angle of elevation from point S to J, correct to ONE decimal place. (2) [16]

### QUESTION 6

6.1 Consider the function $y = 2 \tan x$.

6.1.1 Make a neat sketch of $y = 2 \tan x$ for $0° \leq x \leq 360°$ on the axes provided below.

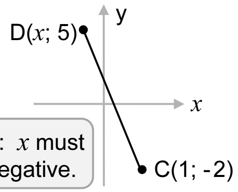
Clearly indicate on your sketch the intercepts with the axes and the asymptotes.




(4)

6.1.2 If the graph of $y = 2 \tan x$ is reflected about the $x$-axis, write down the equation of the new graph obtained by this reflection. (1)

3.2 $CD^2 = (x - 1)^2 + (5 + 2)^2 = \left(\sqrt{53}\right)^2$




**Note**: $x$ must be negative.

$\therefore\ (x - 1)^2 + 49 = 53$

$\therefore\ (x - 1)^2 = 4$

$\therefore\ x - 1 = \pm 2$

$\therefore\ x = 3$ or $-1$

But $x < 0$ in the second quadrant

$\therefore\ x = -1$ ◂ . . . *only the neg. value of x is valid*

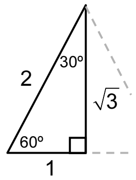
---

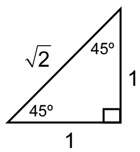
4.1.1 $\sin C = \dfrac{AB}{\mathbf{AC}}$ ◂

4.1.2 **cot** A $= \dfrac{AB}{BC}$

**Note**: $\tan A = \dfrac{BC}{AB}$; $\cot A = \dfrac{1}{\tan A}$

4.2 The expression



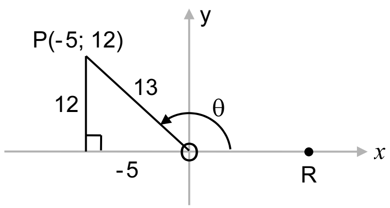

$= \dfrac{\dfrac{\sqrt{3}}{2} \times \dfrac{1}{\sqrt{3}}}{\sqrt{2}}$

$= \dfrac{1}{2} \times \dfrac{1}{\sqrt{2}}$

$= \dfrac{1}{2\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}}$ . . . *The denominator must be rationalised*

$= \dfrac{\sqrt{2}}{4}$ ◂ . . . $\sqrt{2} \times \sqrt{2} = 2$

4.3.1 OP = 13 units . . . *5 : 12 : 13 Δ; Pythagoras*




$\therefore\ \cos\theta = \dfrac{-5}{13} = -\dfrac{5}{13}$ ◂ . . . $\cos\theta = \dfrac{x}{r}$

4.3.2 $\sin\theta = \dfrac{12}{13}$ ➡ $\operatorname{cosec}\theta = \dfrac{13}{12}$

$\therefore\ \operatorname{cosec}^2\theta + 1 = \left(\dfrac{13}{12}\right)^2 + 1 = \dfrac{169}{144} + 1$

$= \dfrac{169 + 144}{144} = \dfrac{313}{144}$ ◂ $\left(= 2\dfrac{25}{144}\right)$ ◂

---

5.1.1 $5\cos x = 3$

$\div 5)$ $\therefore\ \cos x = \dfrac{3}{5}$ $(= 0{,}6)$

$\therefore\ x \approx 53{,}1^\circ$ ◂ . . . $\cos^{-1}\left(\dfrac{3}{5}\right) =$

5.1.2 $\tan 2x = 1{,}19$

$\therefore\ 2x = 49{,}958...^\circ$ . . . $\tan^{-1} 1{,}19 =$

$\div 2)$ $\therefore\ x \approx 25{,}0^\circ$ ◂

5.1.3 $4\sec x - 3 = 5$

$+ 3)$ $\therefore\ 4\sec x = 8$

$\div 4)$ $\therefore\ \sec x = 2$

$\therefore\ \cos x = \dfrac{1}{2}$

$x = 60^\circ$ ◂ . . . $\cos^{-1}\left(\dfrac{1}{2}\right) =$

5.2.1 $J\hat{K}D = 8^\circ$ ◂ . . . *alternate ∠'s; || lines*

5.2.2 In ΔJDK: $\dfrac{DK}{5} = \cot 8^\circ$ . . . $= \dfrac{1}{\tan 8^\circ}$

$\times 5)$ $\therefore\ DK = \dfrac{5}{\tan 8^\circ}$

$= 35{,}5768\ ...$ km

$= 35\ 576{,}8$ metres

$\approx 35\ 577$ metres ◂

. . . *correct to the nearest metre*

5.2.3 DS = DK – SK

= 35,58 km – 8 km

= 27,58 km ◂

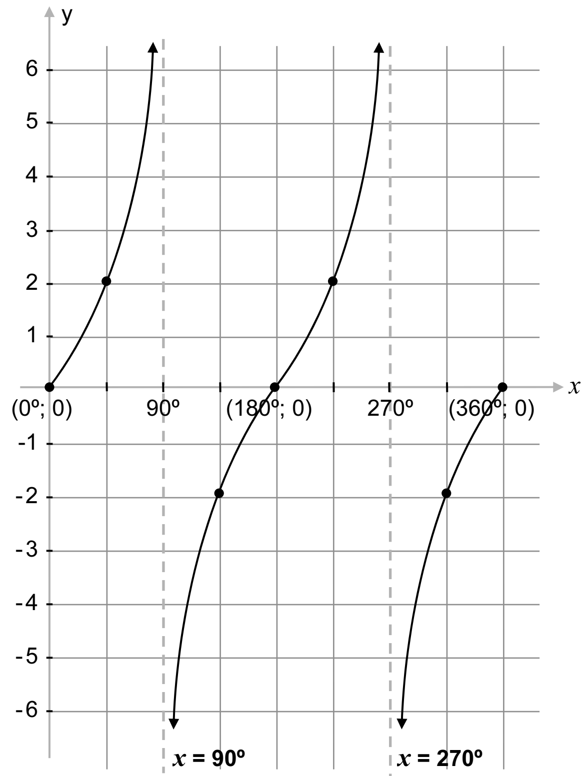
5.2.4 $\tan J\hat{S}D = \dfrac{5}{27{,}58}$

$\therefore\ J\hat{S}D \approx 10{,}3^\circ$ ◂ . . . $\tan^{-1}\left(\dfrac{5}{27{,}58}\right) =$

*correct to 1 dec. place*

---

6.1.1




6.1.2 $y = -2\tan x$ ◂

6.2.1 $a = 4$ ◂

$g(x) = a\sin x$ ➡ $g(90^\circ) = a\sin 90^\circ$
➡ $4 = a$

6.2.2 The range of h:

$-2 \leq y \leq 6$ ◂ . . . *the values of y*

# EUCLIDEAN GEOMETRY: THEOREM STATEMENTS & ACCEPTABLE REASONS

## LINES

| | |
|---|---|
| The adjacent angles on a straight line are supplementary. | ∠s on a str line |
| If the adjacent angles are supplementary, the outer arms of these angles form a straight line. | adj ∠s supp |
| The adjacent angles in a revolution add up to 360º. | ∠s around a pt **OR** ∠s in a rev |
| Vertically opposite angles are equal. | vert opp ∠s |
| If AB \|\| CD, then the alternate angles are equal. | alt ∠s; AB \|\| CD |
| If AB \|\| CD, then the corresponding angles are equal. | corresp ∠s; AB \|\| CD |
| If AB \|\| CD, then the co-interior angles are supplementary. | co-int ∠s; AB \|\| CD |
| If the alternate angles between two lines are equal, then the lines are parallel. | alt ∠s = |
| If the corresponding angles between two lines are equal, then the lines are parallel. | corresp ∠s = |
| If the co-interior angles between two lines are supplementary, then the lines are parallel. | co-int ∠s supp |

## TRIANGLES

| | |
|---|---|
| The interior angles of a triangle are supplementary. | ∠ sum in Δ **OR** sum of ∠s in Δ **OR** int ∠s in Δ |
| The exterior angle of a triangle is equal to the sum of the two interior opposite angles. | ext ∠ of Δ |
| The angles opposite the equal sides in an isosceles triangle are equal. | ∠s opp equal sides |
| The sides opposite the equal angles in an isosceles triangle are equal. | sides opp equal ∠s |
| In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. | Pythagoras **OR** Theorem of Pythagoras |
| If the square of the longest side in a triangle is equal to the sum of the squares of the other two sides then the triangle is right-angled. | **Converse** Pythagoras **OR Converse** Theorem of Pythagoras |
| If three sides of one triangle are respectively equal to three sides of another triangle, the triangles are congruent. | SSS |
| If two sides and an included angle of one triangle are respectively equal to two sides and an included angle of another triangle, the triangles are congruent. | SAS **OR** S∠S |
| If two angles and one side of one triangle are respectively equal to two angles and the corresponding side in another triangle, the triangles are congruent. | AAS **OR** ∠∠S |