Hands on Grade 7 Problem Solving with TAS

1. The sum of a square number and a cube number is equal to a square number.

   List the natural numbers less than or equal to 10 that satisfy this condition.

2. The difference between a cube number and a square number is a square number.

   List the integers less than or equal to 10 that satisfy this condition.

3. Find two pairs of square numbers that have a sum of 130.

4. Calculate the value of the expression below:

$$\left(1-\frac{1}{2}\right)\times\left(1-\frac{1}{3}\right)\times\left(1-\frac{1}{4}\right)\times\left(1-\frac{1}{5}\right)\times\ldots\times\left(1-\frac{1}{500}\right)$$

5. Without doing any calculations, arrange the fractions $\frac{9}{10}$; $\frac{11}{12}$; $\frac{14}{15}$ in descending order.

6. There are some rabbits and some rabbit hutches.

   If seven rabbits are put into each hutch, one rabbit is left without a hutch.

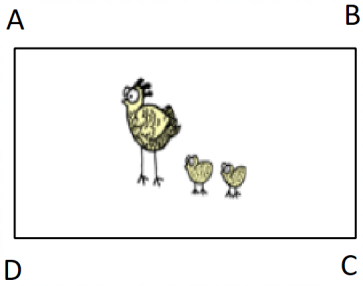
   If 9 rabbits are put into each hutch, one hutch is left empty.

   How many rabbit hutches and how many rabbits are there?

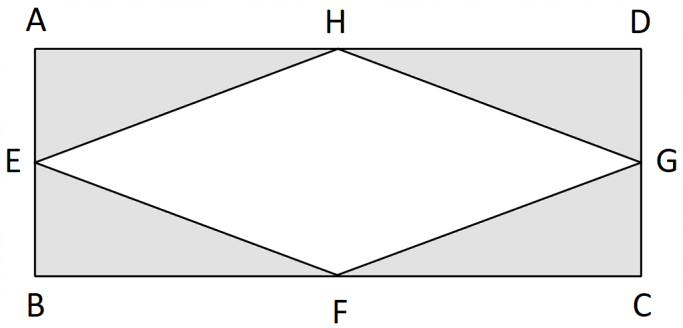
7. Mpho is 5 times as old as Thapelo and half as old as Dumisani.

   The product of their ages is 400. Calculate the sum of their ages.

8. Bongani has 40 m of fencing and plans to build a rectangular enclosure for his chickens. Determine the dimensions of the rectangle for which the chicken enclosure will have maximum area, given that the dimensions are natural numbers.



9.



ABCD is a rectangle with E, F, G and H midpoints of the sides as shown.

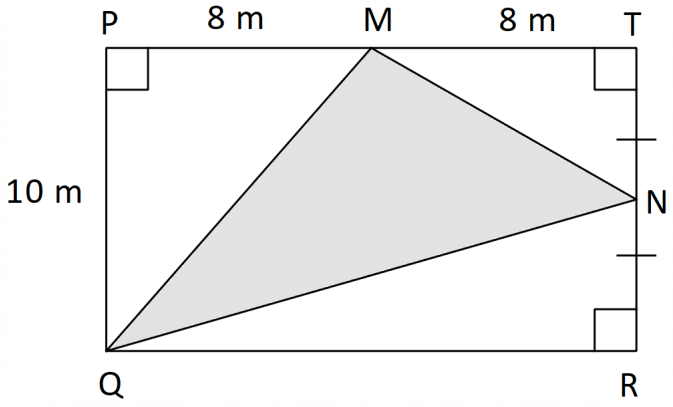
An arrow is shot at random onto the rectangle.

What is the probability that the arrow strikes

9.1 Δ EBF?

9.2 the unshaded area of the rectangle?

10.



10.1 What is the ratio of the shaded area to the unshaded area?

10.2 Calculate the area of Δ MNQ in two different ways.

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–ó Solutions –ó

1. $1^2=1$ $2^2=4$ $3^2=9$ $4^2=16$ $5^2=25$

$6^2=36$ $7^2=49$ $8^2=64$ $9^2=81$ $10^2=100$

Numbers that end in 2, 3, 7 or 8 cannot be perfect squares.

	Numbers	Calculations	
$1^2=1$	1	$1^2+1^3=1+1=2$	x
$2^2=4$	2	$2^2+2^3=4+8=12$	x
$3^2=9$	3	$3^2+3^3=9+27=36$	✓
$4^2=16$	4	$4^2+4^3=16+64=80$	x
$5^2=25$	5	$5^2+5^3=25+125=150$	x
$6^2=36$	6	$6^2+6^3=36+216=252$	x
$7^2=49$	7	$7^2+7^3=49+343=392$	x
$8^2=64$	8	$8^2+8^3=64+512=576$	✓
$9^2=81$	9	$9^2+9^3=81+729=810$	x
$10^2=100$	10	$10^2+10^3=100+1\,000=1\,100$	x

2	**5**	**7**	**6**
2	2	8	8
2	1	4	4
2		7	2
2		3	6
2		1	8
3			9
3			3
			1

Rule out 2, 12, 252, 392

- each of these numbers end in 2

$80=16\times5$

$150=10\times5$

$810=81\times10$

$1\,100=11\times100$

Is 576 a square number?

$20^2=400$

21^2 ends in a 1

22^2 ends in a 4

23^2 ends in a 9

24^2 ends in 6

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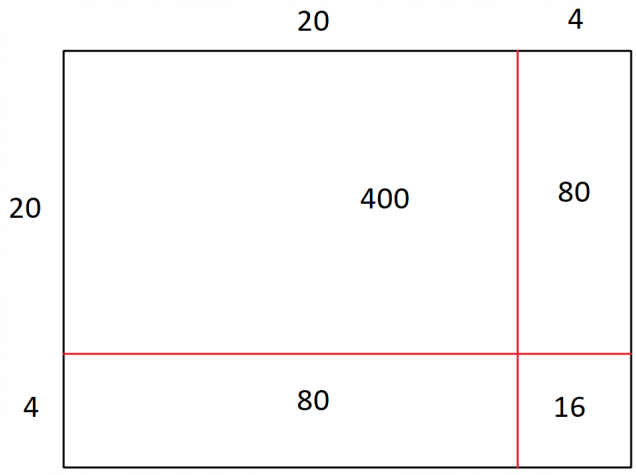
Find the product of 24×24 :



24×24

$= (20+4) \times (20+4)$

$= 400+80+80+16$

$= 400+160+16$

$= 576$

If you use the same number, there are two possible answers, 3 or 8.

(Go back to Page 1 to find the solutions in the table.)

If you used different numbers, then 1 and 2 must be used together as follows:

$1^2+2^3=1+8=9$

Why these numbers?

If the number is *x*, then

x^2+x^3

$= x^2+x^2 \times x$

$= x^2(1+x)$

x^2 is a perfect square, so $x+1$ must also be a perfect square.

This means that x must be 1 less than a square number.

3 is 1 less than 4 and 8 is 1 less than 9.

2. A number than ends in 2, 3, 7 or 8 cannot be a perfect square.

Number	Calculations	
1	$1^3-1^2=1-1=0$	✓
2	$2^3-2^2=8-4=4$	✓
3	$3^3-3^2=27-9=18$	x
4	$4^3-4^2=64-16=48$	x
5	$5^3-5^2=125-25=100$	✓
6	$6^3-6^2=216-36=180$	x
7	$7^3-7^2=343-49=294$	x
8	$8^3-8^2=512-64=448$	x
9	$9^3-9^2=729-81=648$	x
10	$10^3-10^2=1\,000-100=900$	✓

1 ; 2 ; 5 ; 10

Why these numbers?

If the number is *x*, then

x^3-x^2

$=x^2\times x-x$

$=x^2(x-1)$

x^2 is a perfect square, so $x-1$ must also be a perfect square.

This means that x must be 1 more than a square number.

1 is 1 more than 0

2 is 1 more than 1

5 is 1 more than 4

10 is 1 more than 9

3. $1^2 = 1$ $2^2 = 4$ $3^2 = 9$ $4^2 = 16$ $5^2 = 25$ $6^2 = 36$

$7^2 = 49$ $8^2 = 64$ $9^2 = 81$ $10^2 = 100$ $11^2 = 121$

Sum of square numbers must end in 0, so the last digit combinations could be:

- 1 and 9
- 4 and 6
- 5 and 5

Two pairs of square numbers with a sum of 130 :

- **9** & **121** (9 + 121 = 130)
- **49** & **81** (49 + 81 = 130)

4. $\left(1-\frac{1}{2}\right)\times\left(1-\frac{1}{3}\right)\times\left(1-\frac{1}{4}\right)\times\left(1-\frac{1}{5}\right)\times\ldots\times\left(1-\frac{1}{500}\right)$

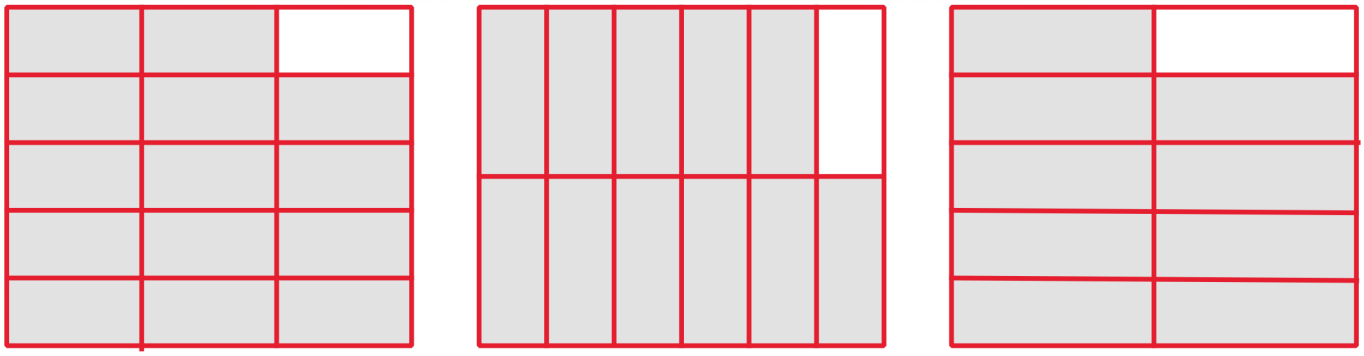
$=\left(\frac{1}{2}\right)\times\left(\frac{2}{3}\right)\times\left(\frac{3}{4}\right)\times\left(\frac{4}{5}\right)\times\ldots\times\left(\frac{499}{500}\right)$

$=\frac{1}{500}$

5. Descending order: $\frac{14}{15}$; $\frac{11}{12}$; $\frac{9}{10}$

Explanation:

- the larger the denominator, the smaller the part of the whole



- $\frac{1}{15} < \frac{1}{12} < \frac{1}{10}$
- the smaller the part is that is removed, the bigger the part is that is left

Check: $\frac{14}{15} \times \frac{4}{4} = \frac{56}{60}$; $\frac{11}{12} \times \frac{5}{5} = \frac{55}{60}$; $\frac{9}{10} \times \frac{6}{6} = \frac{54}{60}$

6.

Number of hutches	9 rabbits per hutch with 1 spare empty hutch	7 rabbits per hutch with 1 extra rabbit left over	
2	$1 \times 9 + 0 = 9$	$2 \times 7 + 1 = 15$	x
3	$2 \times 9 + 0 = 18$	$3 \times 7 + 1 = 22$	x
4	$3 \times 9 + 0 = 27$	$4 \times 7 + 1 = 29$	x
5	$4 \times 9 + 0 = 36$	$5 \times 7 + 1 = 36$	✓

There are 5 rabbit hutches and 36 rabbits.

7. **Using a table:**

Mpho	Thapelo	Dumisani	Product	
5	1	10	$5\times1\times10=50$	**x**
10	2	20	$10\times2\times20=400$	✓

$10+2+20=32$

The sum of their ages is 32 years.

Using algebra:

Let Thapelo be x years old

Then Mpho is $5x$ years old and Dumisani is $10x$ years old.

$$x\times5x\times10x=400$$

$$\therefore 50\boxed{x^3}=400$$

$$50\times\boxed{8}=400$$

$$\therefore x^3=8=2^3$$

$$\therefore x=2$$

Thapelo is 2 years old, Mpho is 10 years old and Dumisani is 20 years old.

The sum of their ages is 32 years.

8. 20 m make up one length and one breadth.

Possible combinations:

1 & 19 ; 2 & 18 ; 3 & 17 ; 4 & 16 ; 5 & 15 ; 6 & 14 ; 7 & 13 ; 8 & 12 ; 9 & 11 ; 10 & 10

$1 \times 19 = 19$

$2 \times 18 = 36$

$3 \times 17 = 51$

$4 \times 16 = 64$

$5 \times 15 = 75$

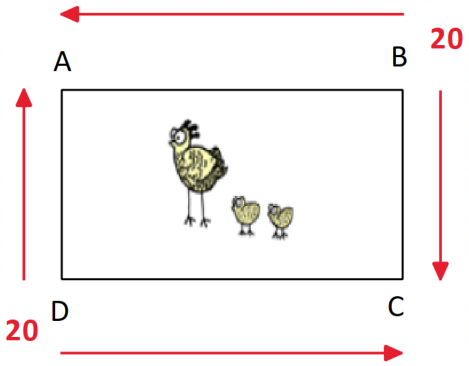
$6 \times 14 = 84$

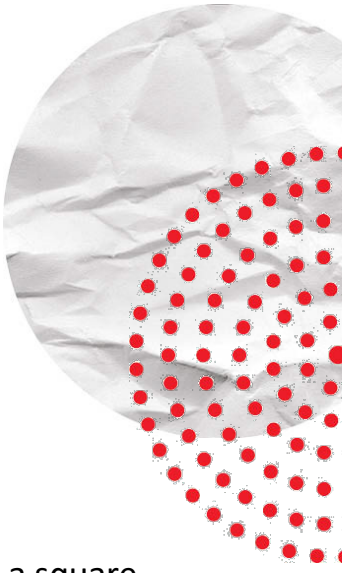
$7 \times 13 = 91$

$8 \times 12 = 96$

$9 \times 11 = 99$

$10 \times 10 = 100$



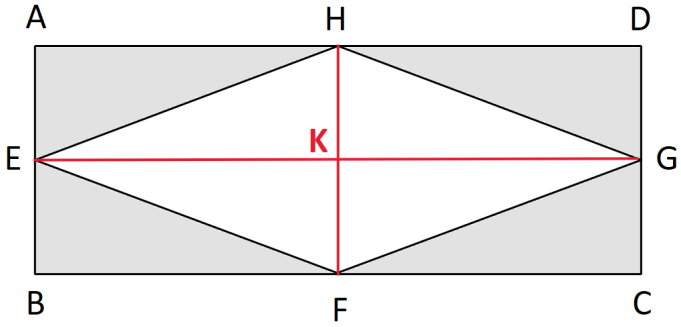


The chicken enclosure has a maximum area of $100\ m^2$ when ABCD is a square.

The dimensions are 10 m by 10 m.

9. EG and FH are symmetry lines of ABCD.

AEKH, BEKF, DGKH and CGKH are congruent (identical) and each of these rectangles is divided into 2 congruent triangles by their respective diagonals.



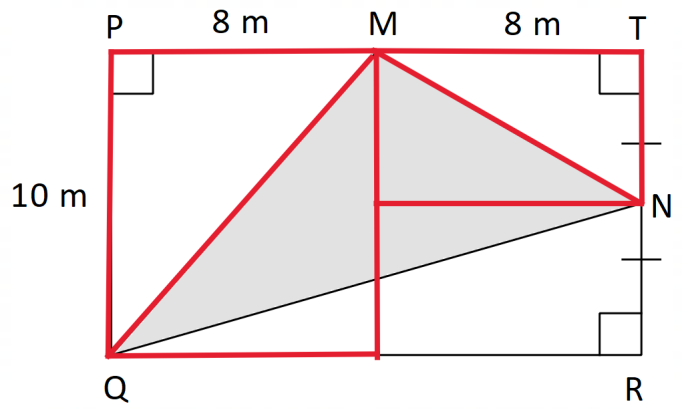
9.1 The area of Δ EBF $= \frac{1}{8}$ of the area of ABCD

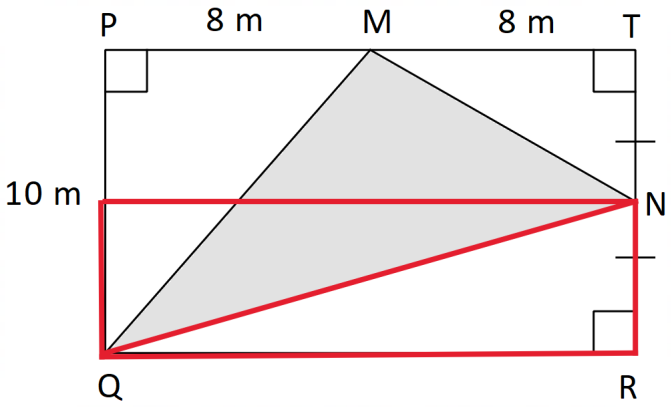
The probability that a randomly shot arrow lands on Δ EBF $= \frac{1}{8}$.

9.2 unshaded area = shaded area

The probability that a randomly shot arrow lands on the unshaded area $= \frac{1}{2}$.

10.1





Fraction of total unshaded area

$=$ Fraction of area total area represented by (Area Δ PQM + Area Δ MNT + Area Δ QRN)

$$=\frac{1}{4}\times\frac{2}{2}+\frac{1}{8}+\frac{1}{4}\times\frac{2}{2}$$

$$=\frac{2+1+2}{8}$$

$$=\frac{5}{8}$$

The shaded area $=1-\frac{5}{8}=\frac{3}{8}$ of the total area

Shaded area : Unshaded area

$$=\frac{3}{8}:\frac{5}{8}$$

$$=3:5$$

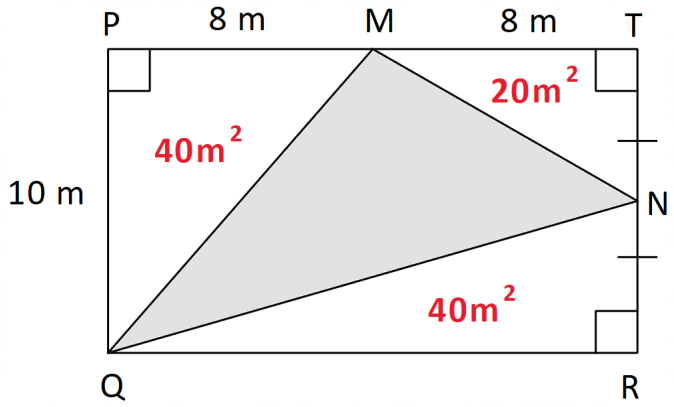
10.2 Area PQRT $= 10\text{ m} \times 16\text{ m} = 160\text{ m}^2$

$\therefore$ Area Δ MNQ

$= \frac{3}{8} \times \frac{160}{1}$

$= 60\text{ m}^2$

or



Area Δ PQM + Area Δ MNT + Area Δ QRN

$= \frac{1}{2} \times 10 \times 8 + \frac{1}{2} \times 8 \times 5 + \frac{1}{2} \times 16 \times 5$

$= 40 + 20 + 40$

$= 100\text{ m}^2$

Area PQRT

$= 10\text{ m} \times 16\text{ m}$

$= 160\text{ m}^2$

Area Δ MNQ

$= 160\text{ m}^2 - 100\text{ m}^2$

$= 60\text{ m}^2$