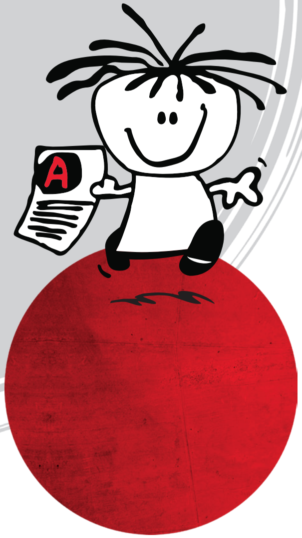
EUCLIDEAN GEOMETRY



CONTENT FRAMEWORK

- LINES
- TRIANGLES
- QUADRILATERALS
- CIRCLES (Gr 11)

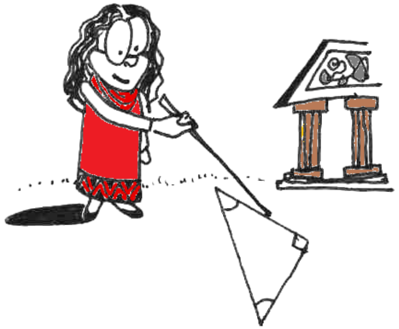
(Gr 8 → 10)

Gr 12?

Gr 12: Theorem of Pythagoras (Gr 8)

Similar Δ^s (Gr 9)

Midpoint Theorem (Gr 10)



& The Proposition Theorem

Ratio Proportion Area

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CIRCLE GEOMETRY

The Language (Vocabulary)

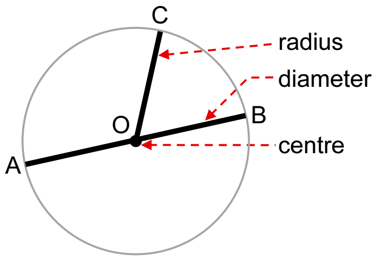
GROUP I : Circles with centre

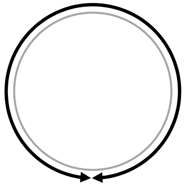
GROUP II : Circles with no centre

Centre

Diameter

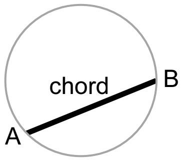
Radius



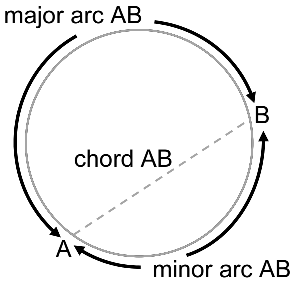


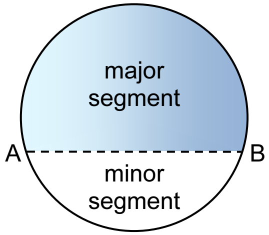
Circumference

Chords



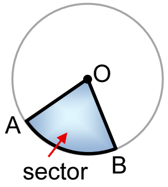
Arcs (major & minor)





Segments (major & minor)

Sectors



'SUBTEND' . . . Understand the word!

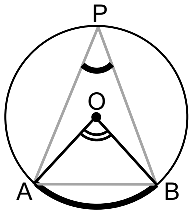


Figure 1

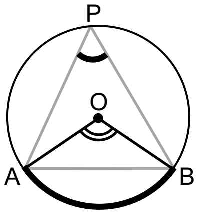


Figure 2

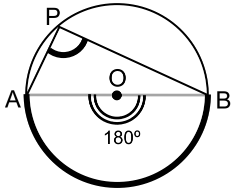


Figure 3

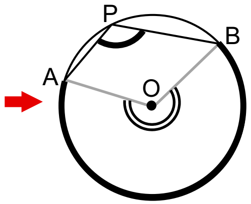


Figure 4

Central and Inscribed angles

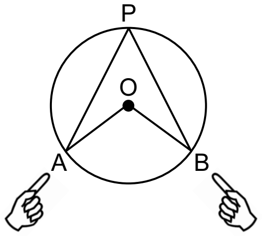
In all the figures, arc AB ($\overset{\frown}{AB}$), or chord AB, **subtends**:

- a **central $A\hat{O}B$** at the **centre** of the circle, and
- an **inscribed $A\hat{P}B$** at the **circumference** of the circle.

Consider that **subtend** means **support**.

To ensure that you grasp the meaning of the word 'subtend':

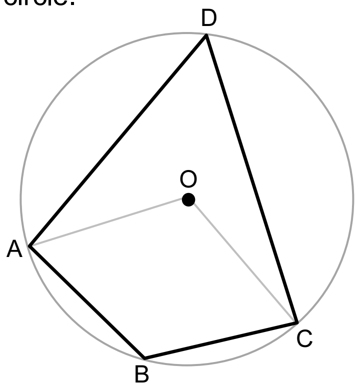
- Take **each** of the figures:
  - Place your index fingers on A & B;
  - move along the radii to meet at O and back; then,
  - move to meet at P on the circumference and back.



- Turn your book upside down and sideways.
  You need to recognise different views of these situations.
- Take note of whether the angles are acute, obtuse, right, straight or reflex.
- Redraw figures 1 to 4 leaving out the chord AB completely and **observe the arc** subtending the central and inscribed angles in each case.

GROUP III : Cyclic Quadrilaterals

A **cyclic quadrilateral** is a quadrilateral which has all 4 vertices on the circumference of a circle.



Points A, B, C and D are **concyclic**, i.e. they lie on the same circle.

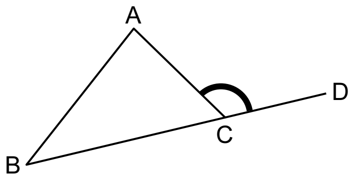
Note: Quadrilateral AOCB is **not** a cyclic quadrilateral because point O is **not** on the circumference! (A, **O**, C and B are **not** concyclic)

We name *quadrilaterals* by going around, either way, using consecutive vertices, i.e. ABCD or ADCB, **not** ADBC.

Exterior angles of polygons

The **exterior angle** of any polygon is an angle which is formed between one side of the polygon and another side *produced*.

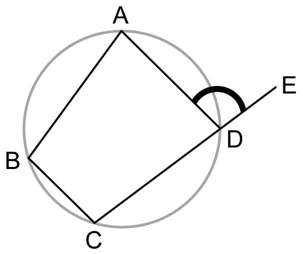
e.g. A triangle



$A\hat{C}D$ is an **exterior** ∠ of ΔABC.

[NB: BCD is a straight line!]

e.g. A quadrilateral/cyclic quadrilateral



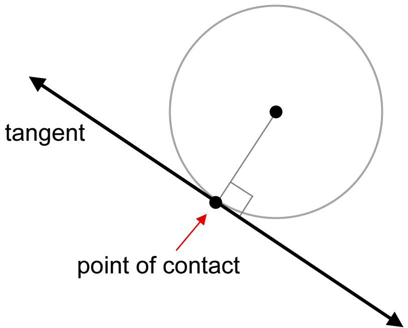
$A\hat{D}E$ is an **exterior** ∠ of c.q. ABCD.

[NB: CDE is a straight line!]

GROUP IV : Tangents

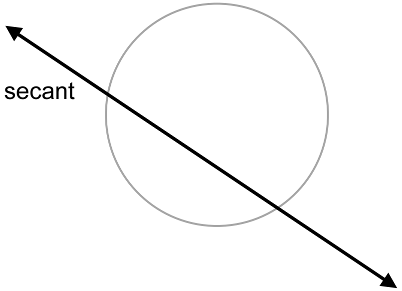
Special lines

- A **tangent** is a line which *touches* a circle at a point.



NB:
It is assumed that the tangent is perpendicular to the radius (or diameter) at the point of contact.

- A **secant** is a line which *cuts* a circle (in two points).



The 4 Circle Geometry Groups

We divide the Circle Geometry theorems into 4 groups, making it easier to grasp and recall all the statements systematically. (See the summary on the next page)

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GROUPING/LINKING CIRCLE GEOMETRY THEOREMS

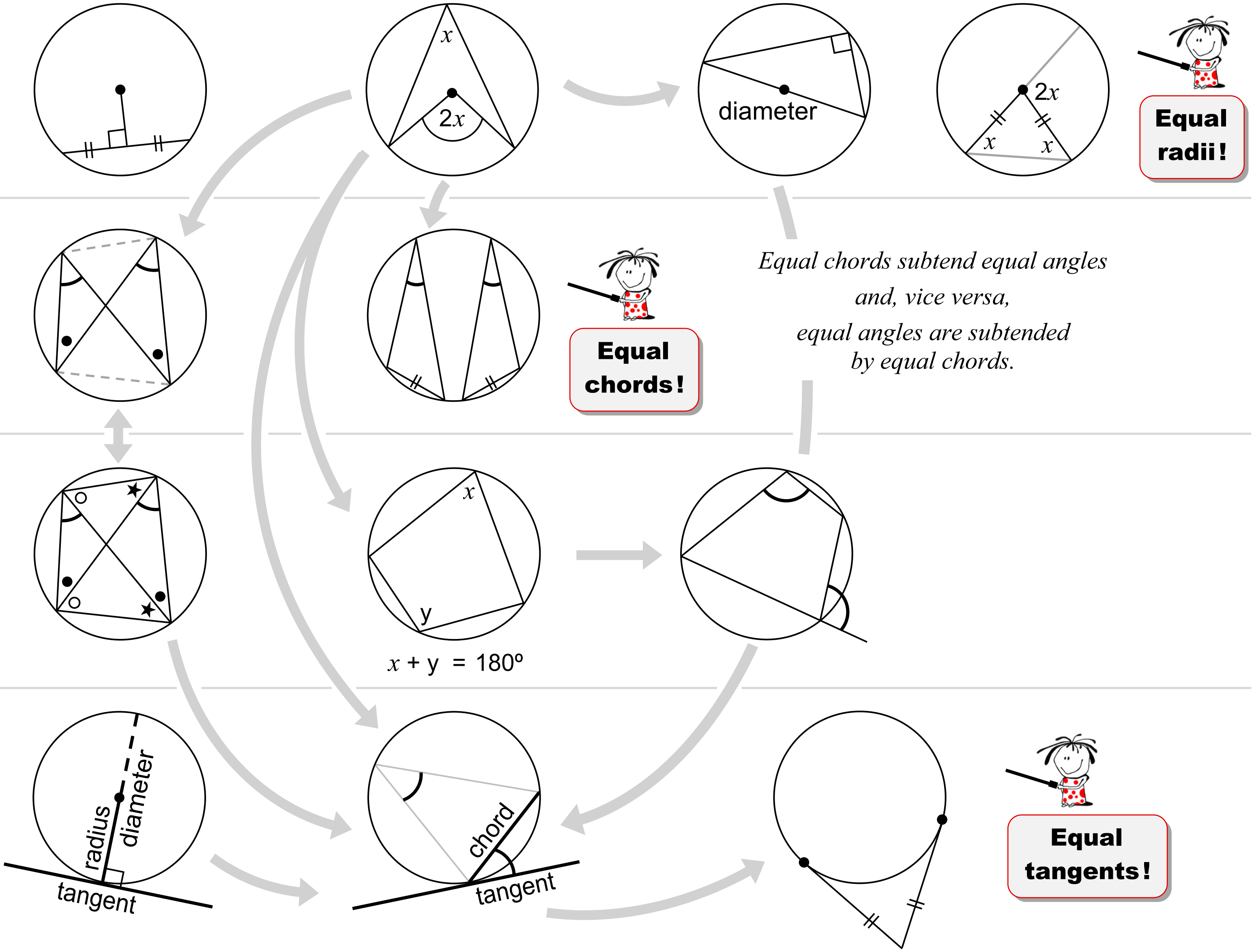
The grey arrows indicate how various theorems are used to prove subsequent ones

I **The 'Centre' group**

II **The 'No Centre' group**

III **The 'Cyclic Quad.' group**

IV **The 'Tangent' group**



Equal radii!

Equal chords!

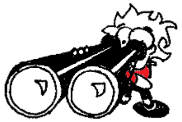
Equal chords subtend equal angles
and, vice versa,
equal angles are subtended
by equal chords.

Equal tangents!

VISUALISING THEOREM PROOFS

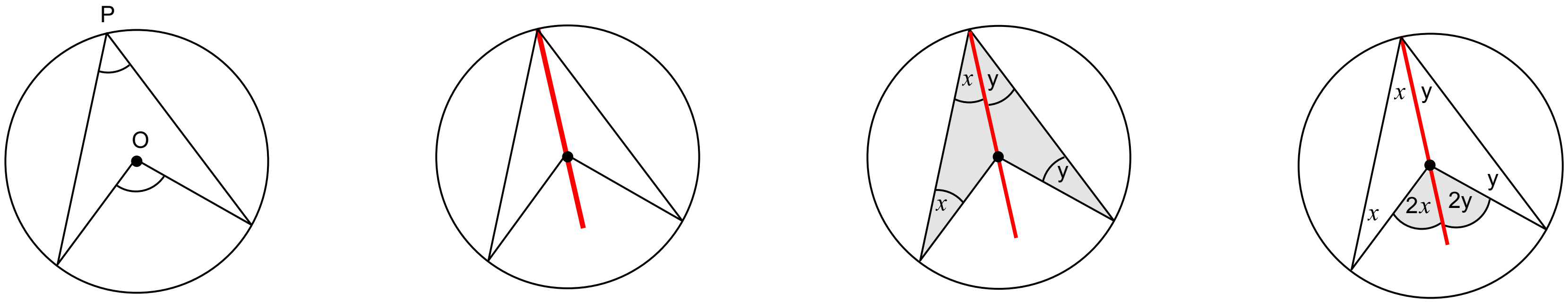
Using VISUALISATION to understand and master theorem proofs . . .

Proving theorem statements with understanding is critical for succeeding in geometry.



Text vs Visuals?

Proving the theorem statement: Angle at centre is 2 times angle at circumference (Thm 3 on p. 1.5)

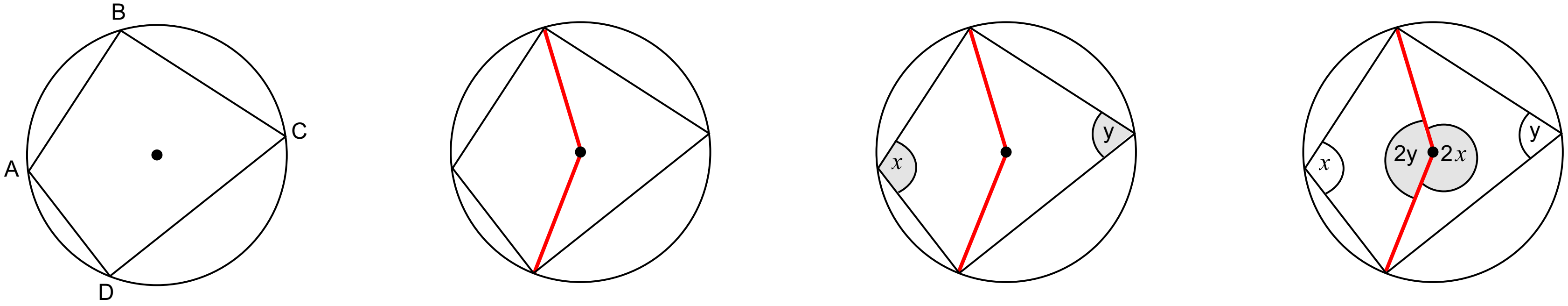


$2x + 2y$

$= 2(x + y)$

Done!

Proving the theorem statement: Opposite angles of a cyclic quad are supplementary (Thm 4 on p. 1.5)



$2x + 2y = 360°$ . . . *revolution*

$\therefore\ x + y = 180°$

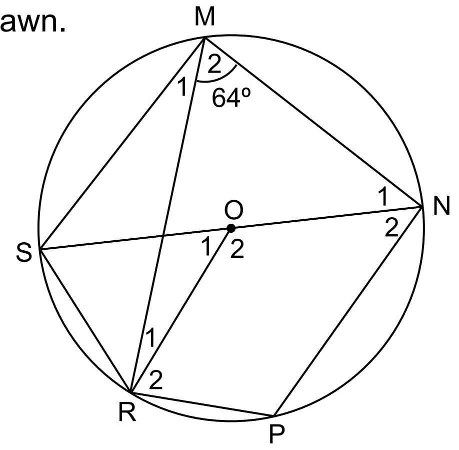
Done!

EUCLIDEAN GEOMETRY (36,7%): DBE NOVEMBER 2022

QUESTION 8 55%

8.1 In the diagram, O is the centre of the circle. MNPR is a cyclic quadrilateral and SN is a diameter of the circle.
71%

Chord MS and radius OR are drawn.
$\hat{M}_2 = 64^\circ$.



Determine, giving reasons, the size of the following angles:

8.1.1 $\hat{P}$ (2)

8.1.2 $\hat{M}_1$ (2)

8.1.3 $\hat{O}_1$ (2)

MEMOS

8.1.1 $\hat{P} = 180^\circ - 64^\circ$ . . . **opp ∠s of cyclic quad**
$= \mathbf{116^\circ}$ ✓

8.1.2 $S\hat{M}N = 90^\circ$ . . . **∠ in semi-⊙**
$\therefore \hat{M}_1 = 90^\circ - 64^\circ$
$= \mathbf{26^\circ}$ ✓

8.1.3 $\hat{O}_1 = 2\hat{M}_1$ . . . **∠ at centre = 2 × ∠ at circumference**
$= \mathbf{52^\circ}$ ✓

Common Errors and Misconceptions

(a) In **Q8.1.1** some candidates incorrectly stated that $\hat{P} = \hat{M}_2$ with the reason that the opposite angles of a cyclic quadrilateral are equal. Other candidates did not state adequate information in **the reason**. Opposite angles and opposite angles are supplementary were not accepted as correct.

(b) When answering **Q8.1.2**, some candidates incorrectly stated that $\hat{M}_1 = \hat{O}_1$ with the reason that they were angles in the same segment. Some candidates gave the reason right-angled triangle. This was not accepted as correct.

(c) In **Q8.1.3** some candidates failed to see the relationship between $\hat{M}_1$ and $\hat{O}_1$ as being angle at centre is equal to twice the angle at the circumference.

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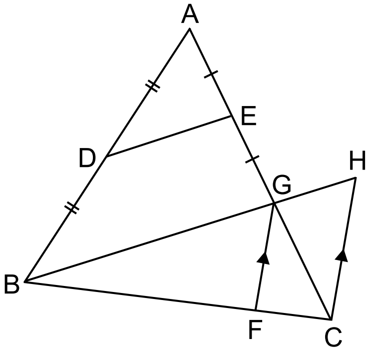
QUESTION 8 (cont.)

42% 8.2 In the diagram, ΔABG is drawn. D and E are midpoints of AB and AG respectively. AG and BG are produced to C and H respectively. F is a point on BC such that FG || CH.



8.2.1 Give a reason why DE || BH. (1)

8.2.2 If it is further given that $\frac{FC}{BF} = \frac{1}{4}$, DE = $3x - 1$ and GH = $x + 1$, calculate, giving reasons, the value of x. (6)

[13]

Common Errors and Misconceptions

(d) In **Q8.2.1** many candidates were unable to give the correct reason for the lines being parallel. They confused the theorem with its converse. Answers given were the *proportionality theorem* instead of the ***converse** proportionality theorem*; or the ***converse** midpoint theorem* instead of the *midpoint theorem*.

MEMO

8.2.1 **CONVERSE** Midpoint Theorem

For those not familiar with **the Midpoint Theorem**, one could use the converse of the Proportion Theorem.

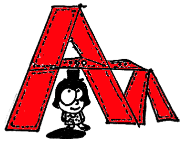
Common Errors and Misconceptions

(e) In **Q8.2.2** many candidates **assumed** that BG = DE even though this was not the case in the diagram given. Some candidates made **algebraic errors**, e.g. $4(x + 1) = 4x + 1$.

Algebra!

Other candidates incorrectly wrote down $\frac{BF}{FC} = \frac{BG}{DE}$ instead of $\frac{BF}{FC} = \frac{BG}{GH}$. Some candidates did not **mention the parallel lines in the reason**.

MEMO



8.2.2 In ΔABG:

$BG = 2(3x - 1)$ . . . *midpoint theorem*

$\therefore BG = 6x - 2$

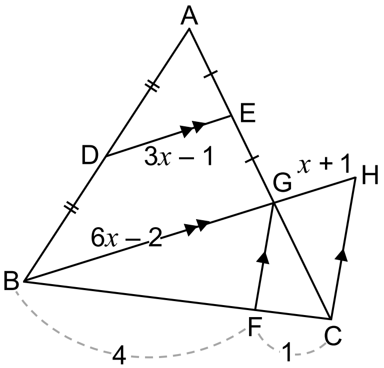
& In ΔBCH: $\frac{GH}{BG} = \frac{1}{4}$ . . . *prop thm;* **FG || CH**

$\therefore \frac{x + 1}{6x - 2} = \frac{1}{4}$

$\therefore 6x - 2 = 4x + 4$

$\therefore 2x = 6$

$\therefore \boldsymbol{x = 3}$



QUESTION 8: Suggestions for Improvement

(a) The key to answering Euclidean Geometry successfully is to be fully conversant with the **terminology** in this section. To this end, teachers should explain the meaning of *chord*, *tangent*, *cyclic quadrilateral*, etc. so that learners will be able to use them correctly.

(b) Teachers must cover the **basic work** thoroughly. An explanation of the theorem should be accompanied by showing the relationship in a diagram.

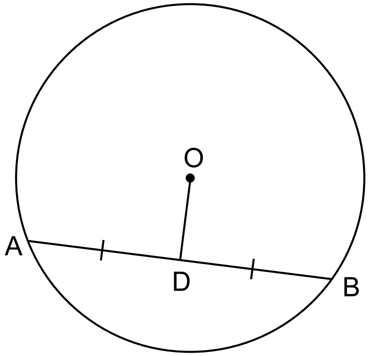
(c) Teachers are encouraged to use the **'Acceptable Reasons'** in the *Examination Guidelines* when teaching. This should start from as early as Grade 8.

(d) Learners should be encouraged to scrutinise the given information and the diagram for **clues** about **which theorems** could be used when answering the question.

(e) Learners should be taught that **all statements must be accompanied by reasons**. It is essential that the **parallel lines** be mentioned when stating that corresponding angles are equal, alternate angles are equal, the sum of the co-interior angles is 180º or when stating the proportional intercept theorem

QUESTION 9 37%

9.1 In the diagram, O is the centre of a circle. OD bisects chord AB.
56%



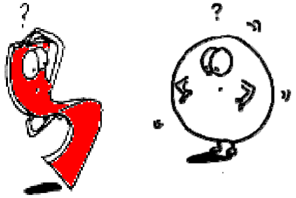
Prove the theorem that states that the line from the centre of a circle that bisects a chord is perpendicular to the chord, i.e. OD $\perp$ AB. (5)

Common Errors and Misconceptions

(a) In **Q9.1** many candidates omitted the reason that angles on a straight line were supplementary. Some candidates used ***similarity*** instead of ***congruency*** to prove this theorem. Many candidates **stated that OD was perpendicular to AB** in the proof. This resulted in a **breakdown** as these candidates were unclear about what information was given and what they had to prove.

MEMOS

9.1 Theorem proof

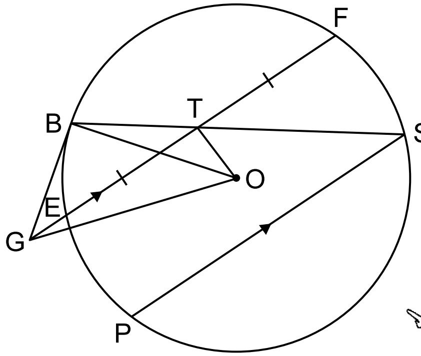


QUESTION 9 (cont.)

9.2 In the diagram, E, B, F, S and P are points on the circle centred at O. GB is a tangent to the circle at B. FE is produced to meet the tangent at G. OT is drawn such that T is the midpoint of EF. GO and BO are drawn. BS is drawn through T. PS || GF.

26%

Sketch redrawn.



Prove, giving reasons, that:

9.2.1 OTBG is a cyclic quadrilateral (5)

9.2.2 $G\hat{O}B = \hat{S}$ (4)

[14]

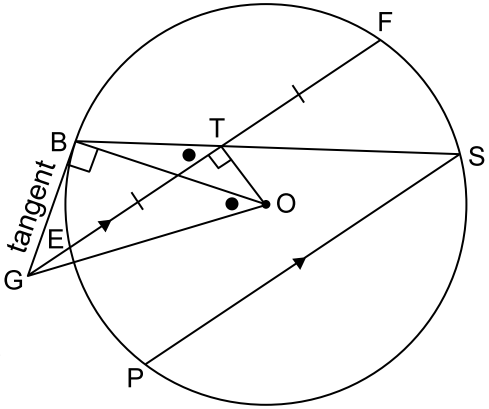
Common Errors and Misconceptions

(b) In **Q9.2.1** many candidates could not identify that the radius was drawn to the midpoint of the chord. Those who could prove that OTBG was a *cyclic quadrilateral* gave the incorrect reason that angles in the same segment instead of **converse** angles in the same segment.

(c) When answering **Q9.2.2**, some candidates stated that BG and OG were equal because they were tangents from a common point. This was incorrect because OG was not a tangent to the circle. A big challenge in this question was the poor labelling of angles. Candidates would refer to $\hat{T}$ while there are a number of angles around point T.

MEMOS

9.1 Theorem proof



9.2.1 $G\hat{B}O = 90^\circ$ . . . *tan ⊥ radius*

$O\hat{T}G = 90^\circ$ . . . *line from centre to midpt of chord*

∴ $G\hat{B}O = O\hat{T}G$

∴ **OTBG is a cyclic quadrilateral** ◄ . . . **converse** *∠ˢ in the same segment*

9.2.2 $G\hat{O}B = G\hat{T}B$ . . . *∠ˢ in the same segment*

$= \hat{S}$ ◄ . . . *corresp ∠ˢ; PS || GF*

QUESTION 9: Suggestions for Improvement

(a) Learners should be taught that a **construction** is required in order to prove a theorem. If the construction is not shown, then the proof is regarded as **a breakdown** and they get no marks. Teachers should **reinforce theory** in short tests and assignments.

(b) Teachers should focus on developing learners' skills to analyse the question and the diagram for **clues** on **which theorems** are required to answer the questions correctly.

(c) Learners should be forced to use **acceptable reasons** in Euclidean Geometry. Teachers should explain the difference between a ***theorem*** **and its** ***converse***. They should also explain the **conditions for which theorems are applicable** and when the converse will apply.

(d) Learners need to be exposed to questions in Euclidean Geometry that include the theorems and the converses. When proving that a quadrilateral is cyclic, no circle terminology may be used when referring to the quadrilateral.

(e) Learners should be discouraged from writing correct statements that are not related to the solution. No marks are awarded for statements that do not lead to solving the problem.

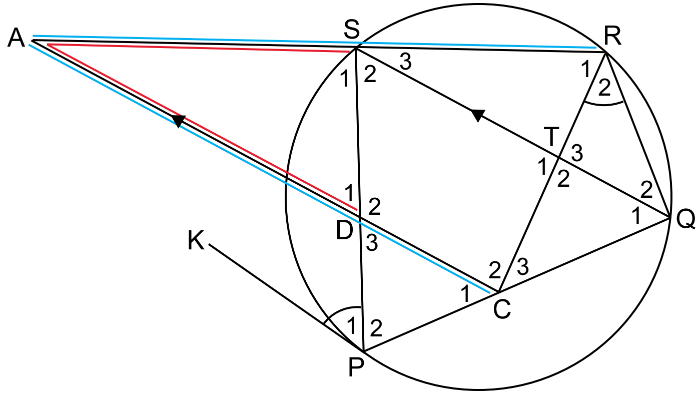
(f) Learners need to be told that success in answering Euclidean Geometry comes from regular practice, starting off with the easy and progressing to the difficult.

(g) Teachers should take some time to discuss the naming of angles. The acceptable methods are $\hat{T}$ or $\hat{T}_1$ or $O\hat{T}S$. Teachers should also clarify when it is acceptable to refer to an angle as $\hat{T}$ and when to refer to it as $\hat{T}_1$.

QUESTION 10 18%

In the diagram, PQRS is a cyclic quadrilateral. KP is a tangent to the circle at P. C and D are points on chords PQ and PS respectively and CD produced meets RS produced at A.

CA || QS. RC is drawn. $\hat{P}_1 = \hat{R}_2$.



Prove, giving reasons, that:

19% 10.1 $\hat{S}_1 = \hat{T}_2$ (4)

25% 10.2 $\frac{AD}{AR} = \frac{AS}{AC}$ (5)

7% 10.3 $AC \times SD = AR \times TC$ (4)

[13]

Common Errors and Misconceptions

(a) A fair number of candidates made **incorrect assumptions** when answering **Q10.1**. Among them were that: $\hat{S}_1 = 90°$ and $\hat{C}_2 = 90°$, $\hat{P}_1 = \hat{R}_1 + \hat{R}_2$ with the reason *exterior angle of cyclic quadrilateral*, $\hat{P}_1 = \hat{C}_1$ with the reason *tan-chord theorem* and $\hat{S}_2 = \hat{R}_2$ with the reason *angles in the same segment*.

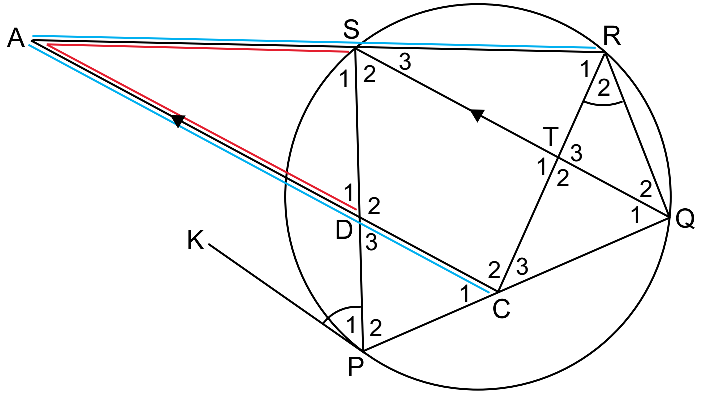
(b) In **Q10.2** some candidates attempted to prove the ratios equal by using the ***proportionality theorem*** instead of **similar triangles**. A common error made by candidates attempting to prove that ΔASD was similar to ΔACR was to merely state that $\hat{S}_1 = \hat{C}_2$ without any proof or reasons. This was seen as a **breakdown** in the answer.

(c) **Q10.3** required candidates to obtain a proportion from the **similar triangles** in **Q10.2**, using the ***proportional intercept theorem*** in ΔRAC to establish a second proportion and then to combine the two. Many candidates failed to establish one or the other proportion and therefore could not make the conclusion.

QUESTION 10

In the diagram, PQRS is a cyclic quadrilateral. KP is a tangent to the circle at P. C and D are points on chords PQ and PS respectively and CD produced meets RS produced at A.

CA || QS. RC is drawn. $\hat{P}_1 = \hat{R}_2$.



Prove, giving reasons, that:

10.1 $\hat{S}_1 = \hat{T}_2$ (4)

10.2 $\dfrac{AD}{AR} = \dfrac{AS}{AC}$ (5)

10.3 $AC \times SD = AR \times TC$ (4)

[13]

TOTAL: 150

MEMOS

10.1 Let $\hat{P}_1 = \hat{R}_2 = x$

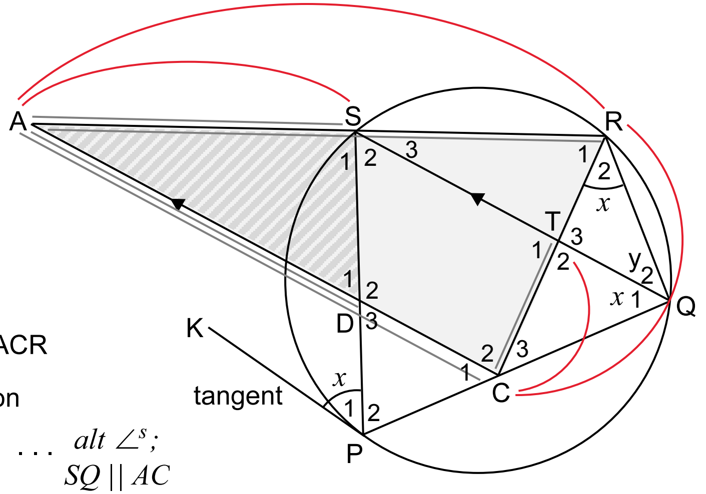
$\hat{Q}_1 = x$ . . . *tan chord theorem*

Let $\hat{Q}_2 = y$

$\therefore \hat{T}_2 = x + y$ . . . *ext ∠ of ΔRTQ*

$\therefore \hat{S}_1 = x + y$ . . . *ext ∠ of cyclic quad*

$\therefore \hat{S}_1 = \hat{T}_2$



10.2 In Δ^s ASD and ACR

(1) $\hat{A}$ is common

(2) $\hat{C}_2 = x + y$ . . . *alt ∠^{s}; SQ || AC*

$\therefore \hat{S}_1 = \hat{C}_2$

$\therefore \Delta ASD \,|||\, \Delta ACR$ . . . ∠∠∠

$\therefore \dfrac{AD}{AR} = \dfrac{AS}{AC}$ ◄ . . . $= \dfrac{SD}{CR}$

10.3 $\dfrac{AS}{AC} = \dfrac{SD}{CR}$ . . . *similar Δ^s in 10.2*

$\therefore AS.CR = AC.SD$ . . . ❶

& In ΔACR: $\dfrac{AS}{AR} = \dfrac{CT}{CR}$ . . . *prop thm; CA || TS*

$\therefore AS.CR = AR.CT$ . . . ❷

From ❶ & ❷:

$\therefore$ **AC.SD = AR.TC** ◄

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QUESTION 10: Suggestions for Improvement

(a) **More time needs to be spent on the teaching of Euclidean Geometry in all grades**. More practice in Grade 11 and 12 Euclidean Geometry will help learners to understand theorems and diagram analysis. They should read the given information carefully without making any **assumptions**. This work covered in class must include different activities and all levels of the taxonomy.

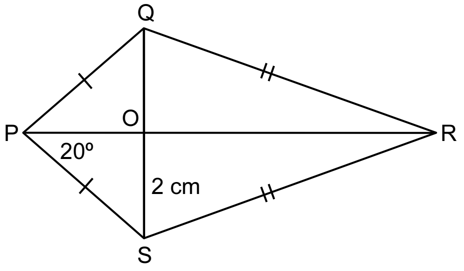
(b) Teachers should require learners to make **use** of the **diagrams** in the Answer Book to **indicate angles and sides** that are equal and **record information** that has been calculated.

(c) Learners need to be made aware that writing correct, but **irrelevant statements**, **will not earn them any marks** in an examination.

GR 10 – 12 EXEMPLAR GEOMETRY

GRADE 10: QUESTIONS

1. PQRS is a kite such that the diagonals intersect in O. OS = 2 cm and $O\hat{P}S$ = 20º.

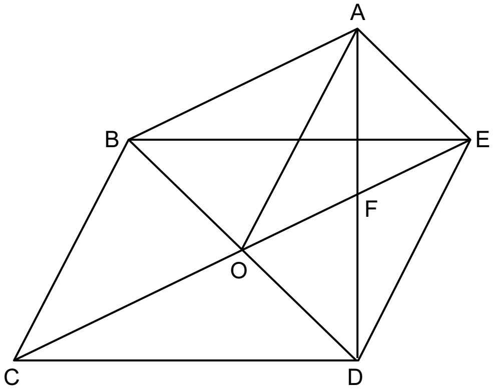


1.1 Write down the length of OQ. (2)

1.2 Write down the size of $P\hat{O}Q$. (2)

1.3 Write down the size of $Q\hat{P}S$. (2) [6]

2. In the diagram, BCDE and AODE are parallelograms.



2.1 Prove that OF || AB. (4)

2.2 Prove that ABOE is a parallelogram. (4)

2.3 Prove that ΔABO ≡ ΔEOD. (5) [13]

GRADE 10: MEMOS

1.1 OQ = 2 cm ◄ . . . *the longer diagonal of a kite bisects the shorter diagonal*

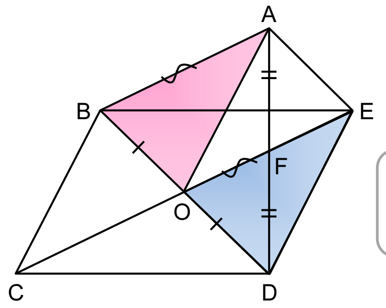
1.2 $P\hat{O}Q$ = 90º ◄ . . . *the diagonals of a kite intersect at right angles*

1.3 $Q\hat{P}O$ = 20º . . . *the longer diagonal of a kite bisects the (opposite) angles of a kite*

∴ $Q\hat{P}S$ = 40º ◄

2.

Hint:
Use highlighters to mark the various $||^{ms}$ and Δ^s



The highlighted Δ^s (and their sides) refer to Question 2.3.

2.1 In ΔDBA:

O is the midpt of BD . . . *diagonals of $||^m$ BCDE bisect each other*

& F is the midpt of AD . . . *diagonals of $||^m$ AODE bisect each other*

∴ OF || AB ◄ . . . *the line joining the midpoints of two sides of a Δ is || to the 3rd side*

2.2 AE || OD . . . *opp. sides of $||^m$ AODE*

∴ AE || BO

and OF || AB . . . *proven above*

∴ OE || AB

∴ ABOE is a $||^m$ . . . *both pairs of opposite sides are parallel*

OR: In $||^m$ AODE: AE = and || OD . . . *opp. sides of $||^m$*

But OD = BO . . . *O proved midpt of BD of BD in 2.1*

∴ AE = and || BO

∴ ABOE is a $||^m$ ◄ . . . *1 pr of opp. sides = and ||*

2.3 In Δ^s ABO and EOD

1) AB = EO . . . *opposite sides of $||^m$ ABOE*

2) BO = OD . . . *proved in 2.1*

3) AO = ED . . . *opposite sides of $||^m$ AODE*

∴ ΔABO ≡ ΔEOD ◄ . . . *SSS*

GRADE 11: QUESTIONS

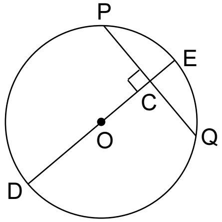
1.1 Complete the statement so that it is valid:

The line drawn from the centre of the circle perpendicular to the chord . . . (1)

1.2 In the diagram, O is the centre of the circle.

The diameter DE is perpendicular to the chord PQ at C.

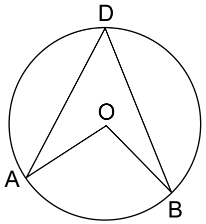
DE = 20 cm and CE = 2 cm.



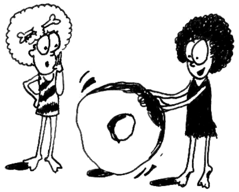
Calculate the length of the following with reasons:

1.2.1 OC 1.2.2 PQ (2)(4) [7]

2.1 In the diagram, O is the centre of the circle and A, B and D are points on the circle.



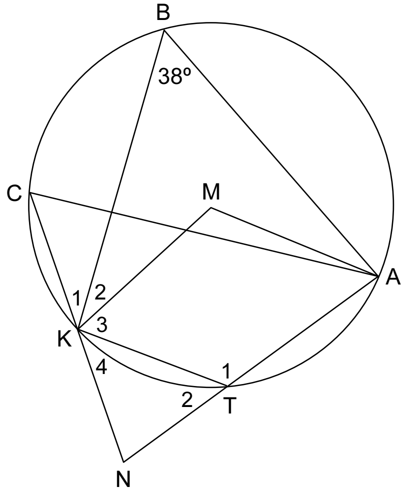
Use Euclidean geometry methods to prove the theorem which states that $A\hat{O}B = 2A\hat{D}B$. (5)



2.2 In the diagram, M is the centre of the circle. A, B, C, K and T lie on the circle.

AT produced and CK produced meet in N.

Also NA = NC and $\hat{B} = 38°$.



2.2.1 Calculate, with reasons, the size of the following angles:

(a) $K\hat{M}A$ (b) $\hat{T}_2$ (2)(2)

(c) $\hat{C}$ (d) $\hat{K}_4$ (2)(2)

2.2.2 Show that NK = NT. (2)

2.2.3 Prove that AMKN is a cyclic quadrilateral. (3) [18]

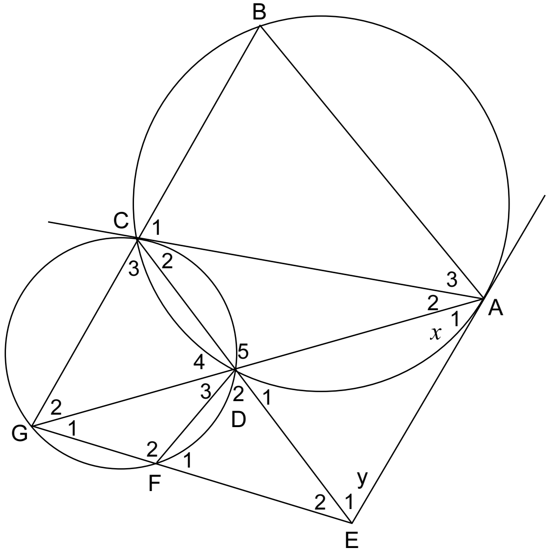
3.1 Complete the following statement so that it is valid:

The angle between a chord and a tangent at the point of contact is . . . (1)

3.2 In the diagram, EA is a tangent to circle ABCD at A.

AC is a tangent to circle CDFG at C.

CE and AG intersect at D.



If $\hat{A}_1 = x$ and $\hat{E}_1 = y$, prove the following with reasons:

3.2.1 BCG || AE (5)

3.2.2 AE is a tangent to circle FED (5)

3.2.3 AB = AC (4) [15]

GRADE 11: MEMOS

1.1 . . . **bisects the chord** ◄

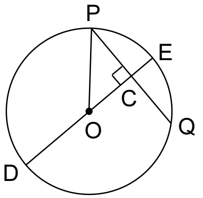
1.2.1 OE = OD = $\frac{1}{2}$(20) = 10 cm

radii
$= \frac{1}{2}$ *diameter*

∴ **OC = 8 cm** ◄ . . . *CE = 2 cm*

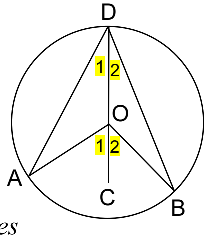
1.2.2 In ΔOPC:

$PC^2 = OP^2 - OC^2$ . . . *Pythagoras*
$= 10^2 - 8^2$
$= 36$
∴ PC = 6 cm



∴ **PQ = 12 cm** ◄ . . . *line from centre ⊥ chord*

2.1 Construction: Join DO and produce it to C



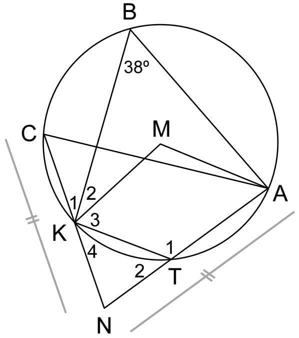
Proof:

Let $\hat{D}_1 = x$
then $\hat{A} = x$ . . . *radii; ∠^{s} opp = sides*
∴ $\hat{O}_1 = 2x$
. . . *ext. ∠ of ΔDAO*

Similarly: Let $\hat{D}_2$ = y
then, $\hat{O}_2$ = 2y

∴ $A\hat{O}B = 2x + 2y$
$= 2(x + y)$
$= \mathbf{2\ A\hat{D}B}$ ◄

2.2



2.2.1 (a) $K\hat{M}A = 2(38°)$ . . . *∠ at centre = 2 × ∠ at circumference*
$=$ **76°** ◄

(b) $\hat{T}_2$ = **38°** ◄ . . . *ext. ∠ of cyclic quad. BKTA*

(c) $\hat{C}$ = **38°** ◄ . . . *∠^{s} in the same segment or, ext. ∠ of cyclic quad. CKTA*

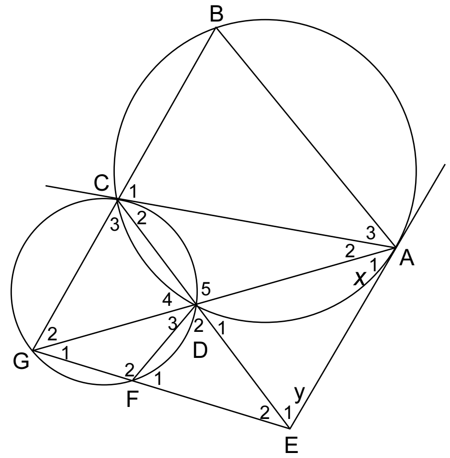
(d) $N\hat{A}C$ = 38° . . . *∠^{s} opp = sides*
∴ $\hat{K}_4$ = **38°** ◄ . . . *ext. ∠ of c.q. CKTA*

2.2.2 In ΔNKT: $\hat{K}_4 = \hat{T}_2$ . . . *both = 38° in 2.2.1*
∴ **NK = NT** ◄ . . . *sides opp equal ∠^{s}*

2.2.3 $K\hat{M}A = 2(38°)$ . . . *see 2.2.1(a)*
& $\hat{N} = 180° - 2(38°)$ . . . *sum of ∠^{s} in ΔNKT (see 2.2.2)*
∴ $K\hat{M}A + \hat{N} = 180°$
∴ **AMKN is a cyclic quadrilateral** ◄
. . . *opposite ∠^{s} supplementary*

3.1 . . . **equal to the angle subtended by the chord in the alternate segment.** ◄

3.2



3.2.1 $\hat{A}_1 = x$ . . . *given*
∴ $\hat{C}_2 = x$ . . . *tan chord theorem*
∴ $\hat{G}_2 = x$ . . . *tan chord theorem*
∴ $\hat{A}_1$ = (alternate) $\hat{G}_2$
∴ **BCG || AE** ◄ . . . *(alternate ∠^{s} equal)*

3.2.2 $\hat{F}_1 = \hat{C}_3$ . . . *ext. ∠ of cyclic quad. CGFD*
$= \hat{E}_1$ (= y) . . . *alternate ∠^{s}; BCG || AE*
∴ **AE is a tangent to ⊙FED** ◄
. . . *converse of tan chord theorem*

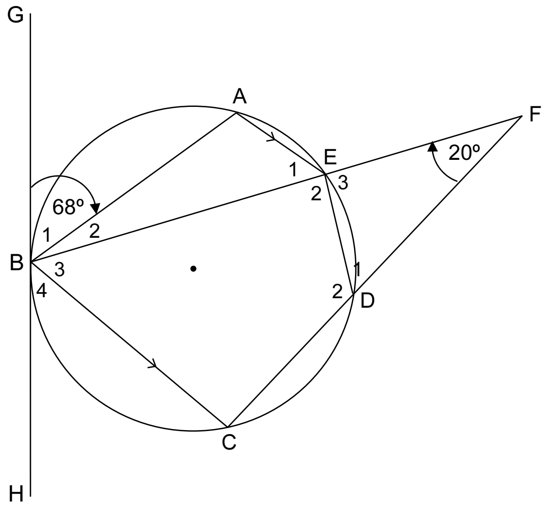
3.2.3 $\hat{C}_1 = C\hat{A}E$ . . . *alternate ∠^{s}; BCG || AE*
$= \hat{B}$ . . . *tan chord theorem*
∴ **AB = AC** ◄ . . . *sides opposite equal ∠^{s}*

GRADE 12: QUESTIONS

1.1 Complete the following statement:

The angle between the tangent and the chord at the point of contact is equal to . . . (1)

1.2 In the diagram, A, B, C, D and E are points on the circumference of the circle such that AE || BC. BE and CD produced meet in F. GBH is a tangent to the circle at B. $\hat{B}_1 = 68^\circ$ and $\hat{F} = 20^\circ$.



Determine the size of each of the following:

1.2.1 $\hat{E}_1$ (2)

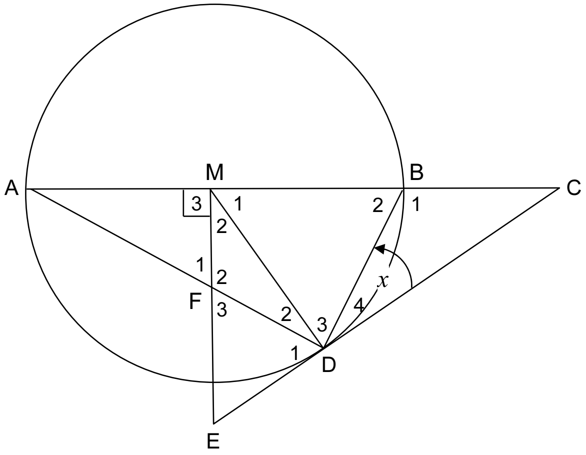
1.2.2 $\hat{B}_3$ (1)

1.2.3 $\hat{D}_1$ (2)

1.2.4 $\hat{E}_2$ (1)

1.2.5 $\hat{C}$ (2) [9]

2. In the diagram, M is the centre of the circle and diameter AB is produced to C. ME is drawn perpendicular to AC such that CDE is a tangent to the circle at D. ME and chord AD intersect at F. MB = 2BC.



2.1 If $\hat{D}_4 = x$, write down, with reasons, TWO other angles each equal to x. (3)

2.2 Prove that CM is a tangent at M to the circle passing through M, E and D. (4)

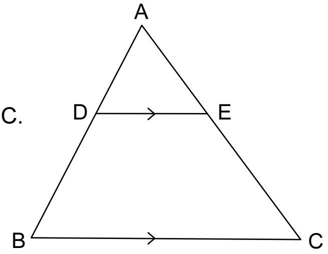
2.3 Prove that FMBD is a cyclic quadrilateral. (3)

2.4 Prove that $DC^2 = 5BC^2$. (3)

2.5 Prove that $\Delta DBC \;|||\; \Delta DFM$. (4)

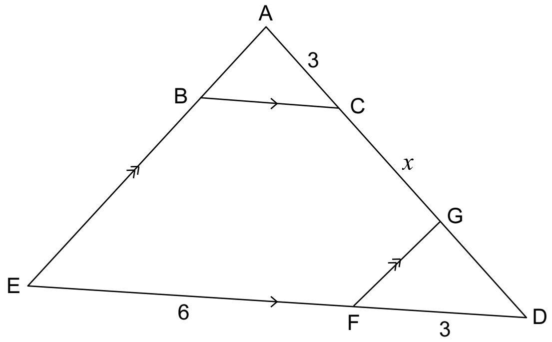
2.6 Hence, determine the value of $\frac{DM}{FM}$. (2) [19]

3.1 In the diagram, points D and E lie on sides AB and AC respectively of ΔABC such that DE || BC. Use Euclidean Geometry methods to prove the theorem which states that $\frac{AD}{DB} = \frac{AE}{EC}$.



(6)

3.2 In the diagram, ADE is a triangle having BC || ED and AE || GF. It is also given that AB : BE = 1 : 3, AC = 3 units, EF = 6 units, FD = 3 units and CG = x units.



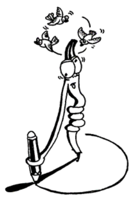
Calculate, giving reasons:

3.2.1 the length of CD (3)

3.2.2 the value of x (4)

3.2.3 the length of BC (5)

3.2.4 the value of $\frac{\text{area } \Delta ABC}{\text{area } \Delta GFD}$ (5) [23]



1.1 . . . the angle subtended by the chord in the alternate segment.

1.2.1 $\hat{E}_1 = \hat{B}_1$ . . . *tan chord theorem*

$= \mathbf{68^\circ}$ ◄

1.2.2 $\hat{B}_3 = \hat{E}_1$ . . . *alt.* $\angle^s$ *; AE || BC*

$= \mathbf{68^\circ}$ ◄

1.2.3 $\hat{D}_1 = \hat{B}_3$ . . . *ext.* $\angle$ *of cyclic quad.*

$= \mathbf{68^\circ}$ ◄

1.2.4 $\hat{E}_2 = \hat{D}_1 + 20^\circ$ . . . *ext.* $\angle$ *of* Δ

$= \mathbf{88^\circ}$ ◄

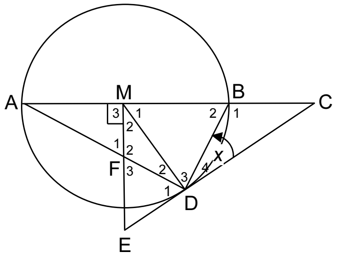
1.2.5 $\hat{C} = 180^\circ - \hat{E}_2$ . . . *opp.* $\angle^s$ *of cyclic quad.*

$= \mathbf{92^\circ}$ ◄

2.1 $\hat{A} = x$ . . . *tan chord theorem*

$\hat{D}_2 = x$ . . . $\angle^s$ *opp. equal sides*

2.2



$\hat{M}_1 = \hat{A} + \hat{D}_2$ . . . *ext.* $\angle$ *of* Δ

$= 2x$

$\therefore \hat{M}_2 = 90^\circ - 2x$ . . . *ME* $\perp$ *AC*

& $M\hat{D}E = 90^\circ$ . . . *radius MD* $\perp$ *tangent CDE*

$\therefore \hat{E} = 2x$ . . . *sum of* $\angle^s$ *in* Δ*MED*

$\therefore \hat{M}_1 = \hat{E}$

$\therefore$ **CM is a tangent at M to ⊙MED** ◄ . . . *converse tan chord theorem*

2.3 $A\hat{D}B = 90^\circ$ . . . $\angle$ *in semi-*⊙

& $\hat{M}_3 = 90^\circ$ . . . *ME* $\perp$ *AC*

$\therefore \hat{M}_3 = A\hat{D}B$

$\therefore$ **FMBD is a cyclic quad** ◄ . . . *converse ext.* $\angle$ *of cyclic quad*

2.4 Let BC = a ; then MB = 2a

$\therefore$ MD = 2a . . . *radii*

In ΔMDC : $M\hat{D}C = 90^\circ$ . . . *radius* $\perp$ *tangent*

$\therefore DC^2 = MC^2 - MD^2$ . . . *theorem of Pythagoras*

$= (3a)^2 - (2a)^2$

$= 9a^2 - 4a^2$

$= 5a^2$

$= \mathbf{5BC^2}$ ◄

2.5 In Δ^s DBC and DFM

(1) $\hat{B}_1 = \hat{F}_2$ . . . *ext* $\angle$ *of c.q. FMBD*

(2) $\hat{D}_4 = \hat{D}_2$ . . . *both = x*

$\therefore$ **ΔDBC ||| ΔDFM** ◄ . . . *equiangular* Δ^s

2.6 $\therefore \dfrac{DM}{FM} = \dfrac{DC}{BC}$ . . . ||| Δ^s

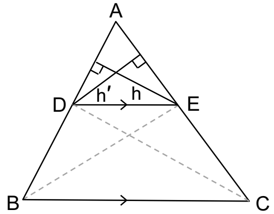
$= \dfrac{\sqrt{5}\,BC}{BC}$ . . . *see 2.4*

$= \mathbf{\sqrt{5}}$ ◄

3.1 **Construction :**

Join DC and EB

and heights h and h′



Proof :

$$\frac{\text{area of }\Delta ADE}{\text{area of }\Delta DBE} = \frac{\cancel{\tfrac{1}{2}}AD.\cancel{h}}{\cancel{\tfrac{1}{2}}DB.\cancel{h}}$$

$$= \frac{AD}{DB} \quad \text{. . . equal heights}$$

$$\& \quad \frac{\text{area of }\Delta ADE}{\text{area of }\Delta EDC} = \frac{\cancel{\tfrac{1}{2}}AE.\cancel{h'}}{\cancel{\tfrac{1}{2}}EC.\cancel{h'}} = \frac{AE}{EC} \quad \text{. . . equal heights}$$

But, area of ΔDBE = area of ΔEDC . . . *same base DE & betw. same || lines, i.e. same height*

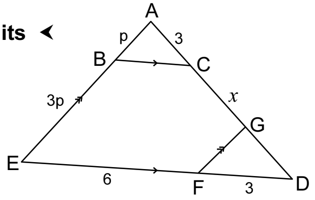
$$\therefore \frac{\text{area of }\Delta ADE}{\text{area of }\Delta DBE} = \frac{\text{area of }\Delta ADE}{\text{area of }\Delta EDC}$$

$$\therefore \mathbf{\frac{AD}{DB} = \frac{AE}{EC}} \text{ ◄}$$

3.2.1 Let AB = p ; then BE = 3p

In ΔAED : $\dfrac{CD}{3} = \dfrac{3\cancel{p}}{\cancel{p}}$ . . . *proportion thm; BC || ED*

$\times 3)$ $\therefore$ **CD = 9 units** ◄



3.2.2 CG = x ; so GD = 9 - x

In ΔDAE : $\dfrac{9 - x}{x + 3} = \dfrac{3}{6}$ . . . *prop. thm. ; AE || GF*

$\therefore 54 - 6x = 3x + 9$

$\therefore -9x = -45$

$\therefore \mathbf{x = 5}$ ◄

3.2.3 In Δ^s ABC and AED

(1) $\hat{A}$ is common

(2) $A\hat{B}C = \hat{E}$ . . . *corr.* $\angle^s$*; BC || ED*

$\therefore$ ΔABC ||| ΔAED . . . *equiangular* Δ^s

$\therefore \dfrac{BC}{ED} = \dfrac{AB}{AE}$ . . . ||| Δ^s

$\therefore \dfrac{BC}{9} = \dfrac{\cancel{p}}{4\cancel{p}}$

$\times 9)$ $\therefore$ **BC = $\mathbf{\frac{9}{4}}$ units** ◄

3.2.4

$$\frac{\text{area of }\Delta ABC}{\text{area of }\Delta GFD} = \frac{\tfrac{1}{2}AC.BC\sin A\hat{C}B}{\tfrac{1}{2}DG.DF\sin\hat{D}}$$

$$= \frac{\cancel{\tfrac{1}{2}}.\cancel{3}.\tfrac{9}{4}.\sin\hat{D}}{\cancel{\tfrac{1}{2}}.4.\cancel{3}.\sin\hat{D}} \quad \text{. . . corr. } \angle^s\text{; BC || ED}$$

$$= \frac{\tfrac{9}{4}}{4}$$

$$= \mathbf{\frac{9}{16}} \text{ ◄}$$

OR : $\dfrac{\text{area of }\Delta ABC}{\text{area of }\Delta AED} = \dfrac{\cancel{\tfrac{1}{2}}.\cancel{p}.\cancel{3}.\sin\hat{A}}{\cancel{\tfrac{1}{2}}.4\cancel{p}.\underset{4}{\cancel{12}}.\sin\hat{A}} = \dfrac{1}{16}$

$\therefore$ area of ΔABC $= \dfrac{1}{16}$ area of ΔAED . . . ❶

& $\dfrac{\text{area of }\Delta GFD}{\text{area of }\Delta AED} = \dfrac{\cancel{\tfrac{1}{2}}.\cancel{4}.\cancel{3}.\cancel{\sin D}}{\cancel{\tfrac{1}{2}}.\underset{3}{\cancel{12}}.\underset{3}{\cancel{9}}.\cancel{\sin D}} = \dfrac{1}{9}$

$\therefore$ area of ΔGFD $= \dfrac{1}{9}$ area of ΔAED . . . ❷

❶ ÷ ❷ : $\therefore \dfrac{\text{area of }\Delta ABC}{\text{area of }\Delta GFD} = \dfrac{\tfrac{1}{16}\ \cancel{\text{area of }\Delta AED}}{\tfrac{1}{9}\ \cancel{\text{area of }\Delta AED}}$

$= \mathbf{\dfrac{9}{16}}$ ◄

Euclidean Geometry: Theorem Statements & Acceptable Reasons

LINES

The adjacent angles on a straight line are supplementary.	$\angle^s$ on a str line
If the adjacent angles are supplementary, the outer arms of these angles form a straight line.	adj $\angle^s$ supp
The adjacent angles in a revolution add up to 360º.	$\angle^s$ round a pt **OR** $\angle^s$ in a rev
Vertically opposite angles are equal.	vert opp $\angle^s$
If AB || CD, then the alternate angles are equal.	alt $\angle^s$; AB || CD
If AB || CD, then the corresponding angles are equal.	corresp $\angle^s$; AB || CD
If AB || CD, then the co-interior angles are supplementary.	co-int $\angle^s$; AB || CD
If the alternate angles between two lines are equal, then the lines are parallel.	alt $\angle^s$ =
If the corresponding angles between two lines are equal, then the lines are parallel.	corresp $\angle^s$ =
If the co-interior angles between two lines are supplementary, then the lines are parallel.	co-int $\angle^s$ supp

TRIANGLES

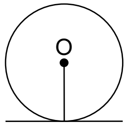
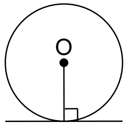
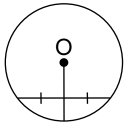
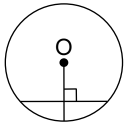
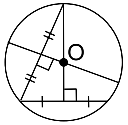
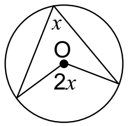
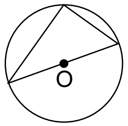
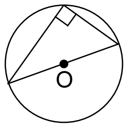
The interior angles of a triangle are supplementary.	$\angle$ sum in Δ **OR** sum of $\angle^s$ **OR** int $\angle^s$ in Δ
The exterior angle of a triangle is equal to the sum of the interior opposite angles.	ext $\angle^s$ of Δ
The angles opposite the equal sides in an isosceles triangle are equal.	$\angle^s$ opp equal sides
The sides opposite the equal angles in an isosceles triangle are equal.	sides opp equal $\angle^s$
In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.	Pythagoras **OR** Theorem of Pythagoras
If the square of the longest side in a triangle is equal to the sum of the squares of the other two sides then the triangle is right-angled.	**Converse** Pythagoras **OR Converse** Theorem of Pythagoras
If three sides of one triangle are respectively equal to three sides of another triangle, the triangles are congruent.	SSS
If two sides and an included angle of one triangle are respectively equal to two sides and an included angle of another triangle, the triangles are congruent.	SAS **OR** S$\angle$S
If two angles and one side of one triangle are respectively equal to two angles and the corresponding side in another triangle, the triangles are congruent.	AAS **OR** $\angle\angle$S
If in two right angled triangles, the hypotenuse and one side of one triangle are respectively equal to the hypotenuse and one side of the other, the triangles are congruent.	RHS **OR** 90ºHS
The line segment joining the midpoints of two sides of a triangle is parallel to the third side and equal to half the length of the third side.	Midpt Theorem
The line drawn from the midpoint of one side of a triangle, parallel to another side, bisects the third side.	line through midpt || to 2^{nd} side
A line drawn parallel to one side of a triangle divides the other two sides proportionally.	line || one side of Δ **OR** prop theorem; name || lines
If a line divides two sides of a triangle in the same proportion, then the line is parallel to the third side.	line divides two sides of Δ in prop
If two triangles are equiangular, then the corresponding sides are in proportion (and consequently the triangles are similar).	||| Δ^s **OR** equiangular Δ^s
If the corresponding sides of two triangles are proportional, then the triangles are equiangular (and consequently the triangles are similar).	sides of Δ in prop
If triangles (or parallelograms) are on the same base (or on bases of equal length) and between the same parallel lines, then the triangles (or parallelograms) have equal areas.	same base; same height **OR** equal bases; equal height

QUADRILATERALS

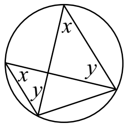
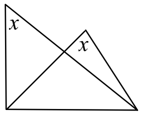
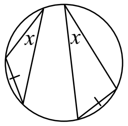
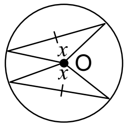
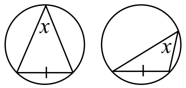
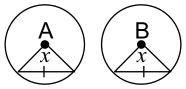
The interior angles of a quadrilateral add up to 360º.	sum of $\angle^s$ in quad
The opposite sides of a parallelogram are parallel.	opp sides of ||m
If the opposite sides of a quadrilateral are parallel, then the quadrilateral is a parallelogram.	opp sides of quad are || **OR** **converse** opp sides of ||m
The opposite sides of a parallelogram are equal in length.	opp sides of ||m
If the opposite sides of a quadrilateral are equal, then the quadrilateral is a parallelogram.	opp sides of quad are = **OR** **converse** opp sides of a parm
The opposite angles of a parallelogram are equal.	opp $\angle^s$ of ||m
If the opposite angles of a quadrilateral are equal then the quadrilateral is a parallelogram.	opp $\angle^s$ of quad are = **OR** **converse** opp angles of a parm
The diagonals of a parallelogram bisect each other.	diag of ||m
If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.	diags of quad bisect each other **OR** **converse** diags of a parm
If one pair of opposite sides of a quadrilateral are equal and parallel, then the quadrilateral is a parallelogram.	pair of opp sides = and ||
The diagonals of a parallelogram bisect its area.	diag bisect area of ||m
The diagonals of a rhombus bisect at right angles.	diags of rhombus
The diagonals of a rhombus bisect the interior angles.	diags of rhombus
All four sides of a rhombus are equal in length.	sides of rhombus
All four sides of a square are equal in length.	sides of square
The diagonals of a rectangle are equal in length.	diags of rect
The diagonals of a kite intersect at right-angles.	diags of kite
A diagonal of a kite bisects the other diagonal.	diag of kite
A diagonal of a kite bisects the opposite angles.	diag of kite

CIRCLES

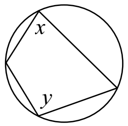
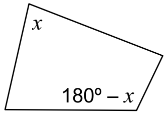
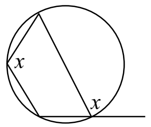
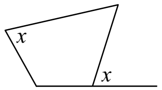
GROUP I

	The tangent to a circle is perpendicular to the radius/diameter of the circle at the point of contact.	tan $\perp$ radius tan $\perp$ diameter
	If a line is drawn perpendicular to a radius/diameter at the point where the radius/diameter meets the circle, then the line is a tangent to the circle.	line $\perp$ radius **OR** **converse** tan $\perp$ radius **OR** **converse** tan $\perp$ diameter
	The line drawn from the centre of a circle to the midpoint of a chord is perpendicular to the chord.	line from centre to midpt of chord
	The line drawn from the centre of a circle perpendicular to a chord bisects the chord.	line from centre $\perp$ to chord
	The perpendicular bisector of a chord passes through the centre of the circle.	perp bisector of chord
	The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre)	$\angle$ at centre $= 2 \times \angle$ at circumference
	The angle subtended by the diameter at the circumference of the circle is 90º.	$\angle^s$ in semi circle **OR** diameter subtends right angle **OR** $\angle$ in ½ ⊙
	If the angle subtended by a chord at the circumference of the circle is 90°, then the chord is a diameter.	chord subtends 90º **OR** **converse** $\angle^s$ in semi circle

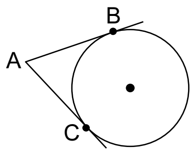
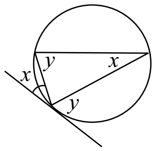
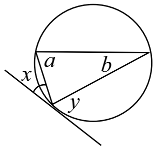
GROUP II

	Angles subtended by a chord of the circle, on the same side of the chord, are equal.	$\angle^s$ in the same seg
	If a line segment joining two points subtends equal angles at two points on the same side of the line segment, then the four points are concyclic. (This can be used to prove that the four points are concyclic).	line subtends equal $\angle^s$ **OR** **converse** $\angle^s$ in the same seg
	Equal chords subtend equal angles at the circumference of the circle.	equal chords; equal $\angle^s$
	Equal chords subtend equal angles at the centre of the circle.	equal chords; equal $\angle^s$
	Equal chords in equal circles subtend equal angles at the circumference of the circles.	equal circles; equal chords; equal $\angle^s$
	Equal chords in equal circles subtend equal angles at the centre of the circles. (A and B indicate the centres of the circles)	equal circles; equal chords; equal $\angle^s$

GROUP III

	The opposite angles of a cyclic quadrilateral are supplementary (i.e. x and y are supplementary)	opp $\angle^s$ of cyclic quad
	If the opposite angles of a quadrilateral are supplementary then the quadrilateral is cyclic.	opp $\angle^s$ quad sup **OR** **converse** opp $\angle^s$ of cyclic quad
	The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.	ext $\angle$ of cyclic quad
	If the exterior angle of a quadrilateral is equal to the interior opposite angle of the quadrilateral, then the quadrilateral is cyclic.	ext $\angle$ = int opp $\angle$ **OR** **converse** ext $\angle$ of cyclic quad

GROUP IV

	Two tangents drawn to a circle from the same point outside the circle are equal in length (AB = AC)	Tans from common pt **OR** Tans from same pt
	The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment.	tan chord theorem
	If a line is drawn through the end-point of a chord, making with the chord an angle equal to an angle in the alternate segment, then the line is a tangent to the circle. (If $x = b$ or if $y = a$ then the line is a tangent to the circle)	**converse** tan chord theorem **OR** $\angle$ between line and chord

Euclidean Geometry

References to TAS Maths books

THE ANSWER SERIES Your Key to Exam Success

Gr 10 Maths 3-in-1 (Module 7)

# 1:	Lines, angles & triangles: revision • vocabulary & facts	7.1 ⟶ 7.7
# 2:	Quadrilaterals: revision • definitions • theorems • areas	7.8 ⟶ 7.15
# 3:	Midpoint theorem	7.16 ⟶ 7.17
# 4:	Polygons: definitions & types • interior angles • exterior angles	7.18

Note: The Gr 10 Exemplar Exams and Memos are at the end of the book

Gr 11 Maths 3-in-1 (Module 9)

# 1:	Revision from earlier grades	9.1 ⟶ 9.5
# 2:	Circle Geometry	9.6 ⟶ 9.26

Note: The Gr 11 Exemplar Exams and Memos are at the end of the book

Gr 12 Maths 2-in-1 (Module 10)

See Challenging Questions booklet
pages 29 ⟶ 38

# 1:	Circle Geometry	36 ⟶ 40
# 2:	Proportion Theorem	40 ⟶ 42
# 3:	Similar Triangles	42 ⟶ 43
# 4:	Mixed	43
Back pages:	Circle Geometry, Proportion and Similar Triangles Theorem PROOFS	i ⟶ iii
	Grouping of Circle Geometry Theorems	viii
	Converse Theorems in Circle Geometry	ix
	Theorem Statements & Acceptable Reasons	x ⟶ xii

See the Topic Guide on p. 148
for further exam practice.

Gr 12 Maths Past Papers Toolkit

Back pages:	Circle Geometry, Proportion and Similar Triangles Theorems PROOFS	i ⟶ iii
	Grouping of Circle Geometry Theorems	xiii
	Theorem Statements & Acceptable Reasons	xiv ⟶ xvi

See the Topic Guides: DBE: p. 2 & IEB: p. 40