Gr 10, Gr 11 & Gr 12 Mathematics

EXEMPLAR PAPER 2s

(memos follow)

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GRADE 10 EXEMPLAR PAPER 2

Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining the answers.

Answers only will NOT necessarily be awarded full marks.

You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

If necessary, round off answers to ***TWO*** *decimal places, unless stated otherwise.*

▶ STATISTICS [15]

QUESTION 1

A baker keeps a record of the number of scones that he sells each day. The data for 19 days is shown below.

31	36	62	74	65	63	60	34	46	56
37	46	40	52	48	39	43	31	66	

1.1 Determine the mean of the given data. (2)

1.2 Rearrange the data in ascending order and then determine the median. (2)

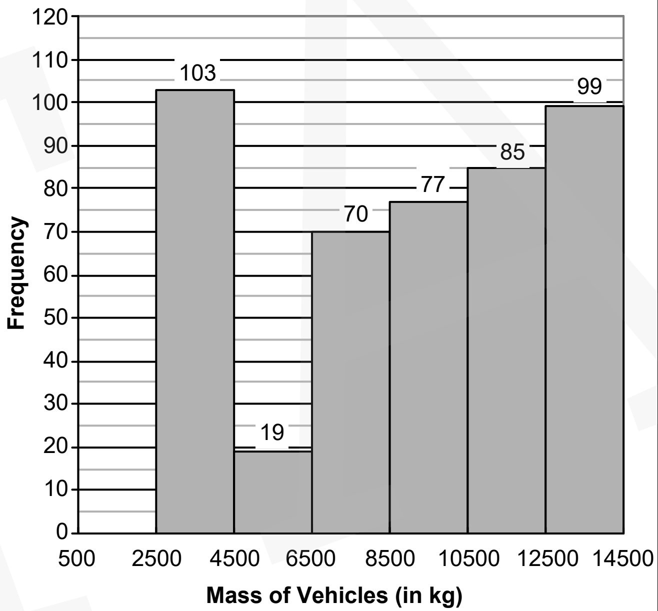
1.3 Determine the lower and upper quartiles for the data. (2)

1.4 Draw a box and whisker diagram to represent the data. (2) [8]

QUESTION 2

Traffic authorities are concerned that heavy vehicles (trucks) are often overloaded. In order to deal with this problem, a number of weighbridges have been set up along the major routes in South Africa. The gross (total) vehicle mass is measured at these weigh bridges. The histogram below shows the data collected at a weighbridge over a month.

Histogram showing Gross Vehicle Mass



2.1 Write down the modal class of the data. (1)

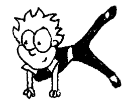
2.2 Estimate the mean gross vehicle mass for the month. (5)

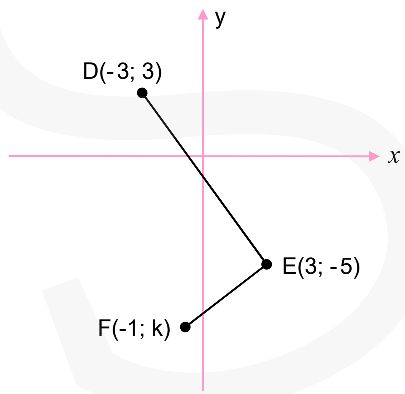
2.3 Which of the measures of central tendency, the modal class or the estimated mean, will be most appropriate to describe the data set? Explain your choice. (1) [7]

▶ ANALYTICAL GEOMETRY [18]

QUESTION 3

3.1 In the diagram below, D(-3; 3), E(3; -5) and F(-1; k) are three points in the Cartesian plane.





3.1.1 Calculate the length of DE. (2)

3.1.2 Calculate the gradient of DE. (2)

3.1.3 Determine the value of k if $D\hat{E}F = 90°$. (4)

3.1.4 If k = -8, determine the coordinates of M, the midpoint of DF. (2)

3.1.5 Determine the coordinates of a point G such that the quadrilateral DEFG is a rectangle. (4)

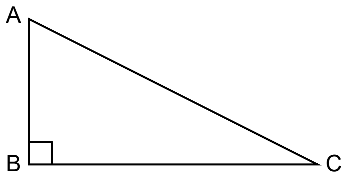
3.2 C is the point (1; -2). The point D lies in the second quadrant and has coordinates $(x; 5)$.

If the length of CD is $\sqrt{53}$ units, calculate the value of x. (4) [18]

TRIGONOMETRY [36]

QUESTION 4

4.1 In the diagram below, ΔABC is right-angled at B.



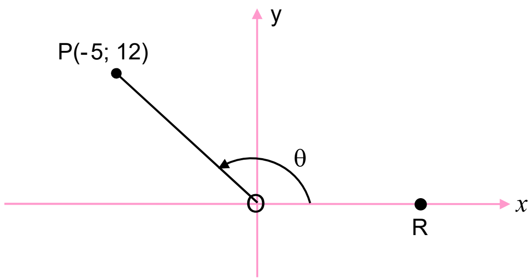
Complete the following statements:

4.1.1 $\sin C = \frac{AB}{\dots}$ (1)

4.1.2 $\dots A = \frac{AB}{BC}$ (1)

4.2 **Without using a calculator**, determine the value of: $\frac{\sin 60^\circ \cdot \tan 30^\circ}{\sec 45^\circ}$ (4)

4.3 In the diagram, P(-5; 12) is a point in the Cartesian plane and $R\hat{O}P = \theta$.



Determine the value of:

4.3.1 $\cos \theta$ (3)

4.3.2 $\text{cosec}^2 \theta + 1$ (3) [12]

QUESTION 5

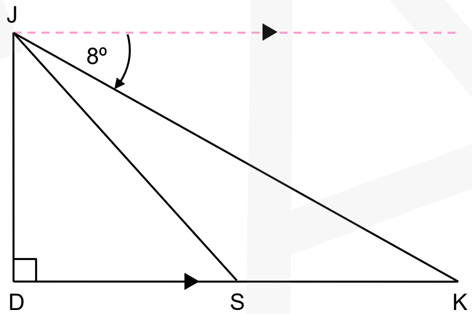
5.1 Solve for x, correct to ONE decimal place, in each of the following equations where $0^\circ \leq x < 90^\circ$.

5.1.1 $5 \cos x = 3$ (2)

5.1.2 $\tan 2x = 1{,}19$ (3)

5.1.3 $4 \sec x - 3 = 5$ (4)

5.2 An aeroplane at J is flying directly over a point D on the ground at a height of 5 kilometres. It is heading to land at point K. The angle of depression from J to K is 8°. S is a point along the route from D to K.



5.2.1 Write down the size of $J\hat{K}D$. (1)

5.2.2 Calculate the distance DK, correct to the nearest metre. (3)

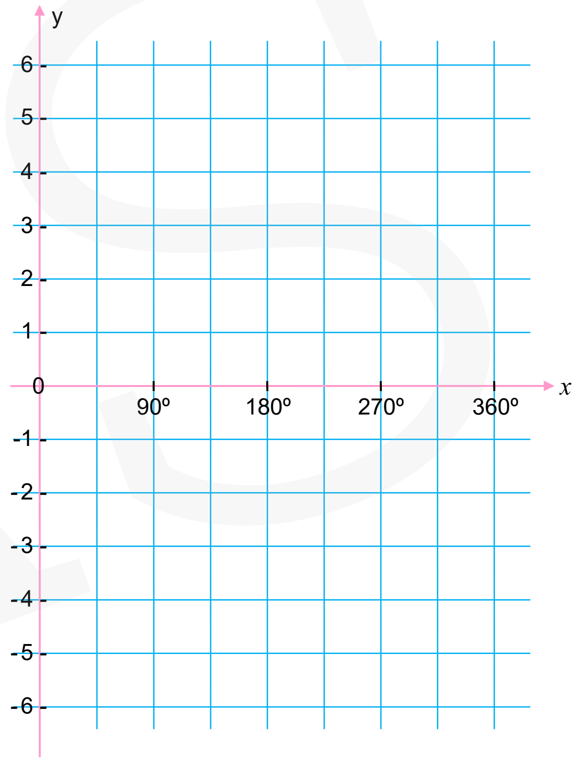
5.2.3 If the distance SK is 8 kilometres, calculate the distance DS. (1)

5.2.4 Calculate the angle of elevation from point S to J, correct to ONE decimal place. (2) [16]

QUESTION 6

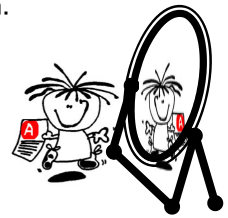
6.1 Consider the function $y = 2 \tan x$.

6.1.1 Make a neat sketch of $y = 2 \tan x$ for $0^\circ \leq x \leq 360^\circ$ on the axes provided below. Clearly indicate on your sketch the intercepts with the axes and the asymptotes.

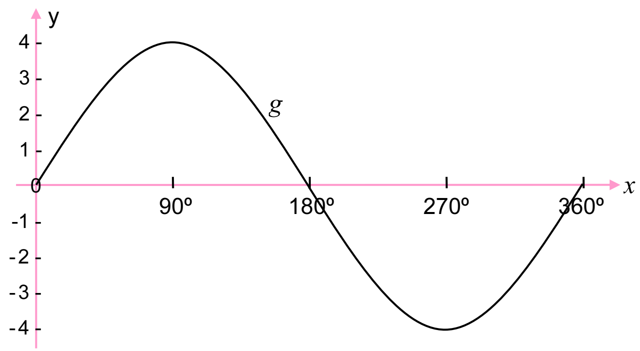


(4)

6.1.2 If the graph of $y = 2 \tan x$ is reflected about the x-axis, write down the equation of the new graph obtained by this reflection. (1)



6.2 The diagram below shows the graph of $g(x) = a \sin x$ for $0° \leq x \leq 360°$.



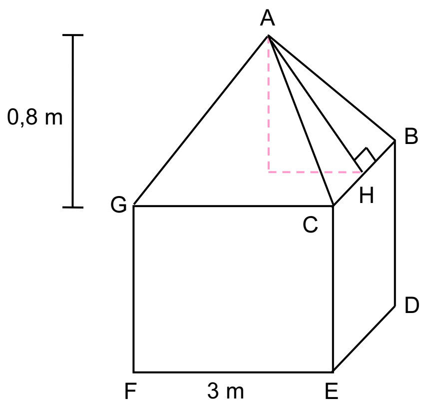
6.2.1 Determine the value of a. (1)

6.2.2 If the graph of g is translated 2 units upwards to obtain a new graph h, write down the range of h. (2) [8]

▸ MEASUREMENT [12]

QUESTION 7

7.1 The roof of a canvas tent is in the shape of a right pyramid having a perpendicular height of 0,8 metres on a square base. The length of one side of the base is 3 metres.



7.1.1 Calculate the length of AH. (2)

7.1.2 Calculate the surface area of the roof. (2)

7.1.3 If the height of the walls of the tent is 2,1 metres, calculate the total amount of canvas required to make the tent if the floor is excluded. (2)

7.2 A metal ball has a radius of 8 millimetres.

7.2.1 Calculate the volume of metal used to make this ball, correct to TWO decimal places. (2)

> The volume of a sphere $= \frac{4}{3}\pi r^3$

7.2.2 If the radius of the ball is doubled, write down the ratio of

the new volume : the original volume. (2)

7.2.3 You would like this ball to be silver plated to a thickness of 1 millimetre.

What is the volume of silver required?

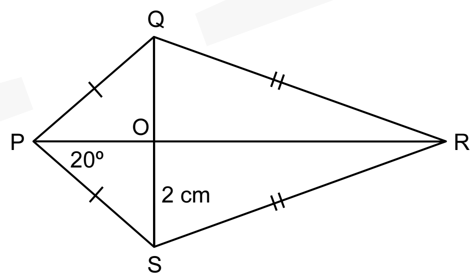
Give your answer correct to TWO decimal places. (2) [12]

▸ EUCLIDIAN GEOMETRY [19]

Give reasons for your statements in the answers to QUESTIONS 8 and 9.

QUESTION 8

PQRS is a kite such that the diagonals intersect in O.
OS = 2 cm and $O\hat{P}S = 20°$.



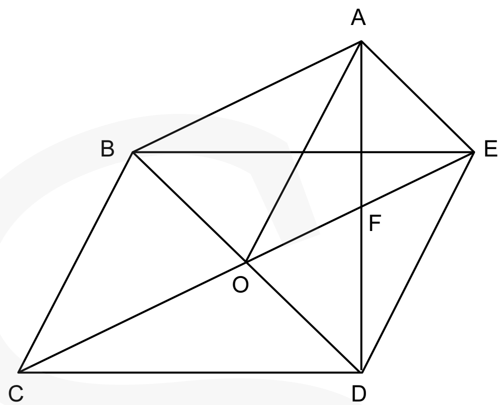
8.1 Write down the length of OQ. (2)

8.2 Write down the size of $P\hat{O}Q$. (2)

8.3 Write down the size of $Q\hat{P}S$. (2) [6]

QUESTION 9

In the diagram, BCDE and AODE are parallelograms.



9.1 Prove that OF || AB. (4)

9.2 Prove that ABOE is a parallelogram. (4)

9.3 Prove that $\Delta ABO \equiv \Delta EOD$. (5) [13]

TOTAL: 100

GRADE 11 EXEMPLAR PAPER 2

You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

If necessary, round off answers to ***TWO*** *decimal places, unless stated otherwise.*

▶ STATISTICS [23]

QUESTION 1

The data below shows the number of people visiting a local clinic per day to be vaccinated against measles.

5	12	19	29
35	23	15	33
37	21	26	18
23	18	13	21
18	22	20	

1.1 Determine the mean of the given data. (2)

1.2 Calculate the standard deviation of the data. (2)

1.3 Determine the number of days that the number of people vaccinated against measles lies within ONE standard deviation of the mean. (2)

1.4 Determine the interquartile range for the data. (3)

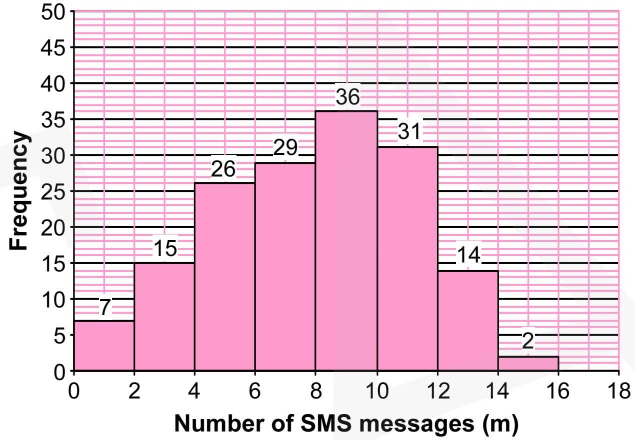
1.5 Draw a box and whisker diagram to represent the data. (3)

1.6 Identify any outliers in the data set. Substantiate your answer. (2) [14]

QUESTION 2

A group of Grade 11 learners were interviewed about using a certain application to send SMS messages. The number of SMS messages, m, sent by each learner was summarised in the histogram below.

Histogram showing the number of SMS messages sent by learners

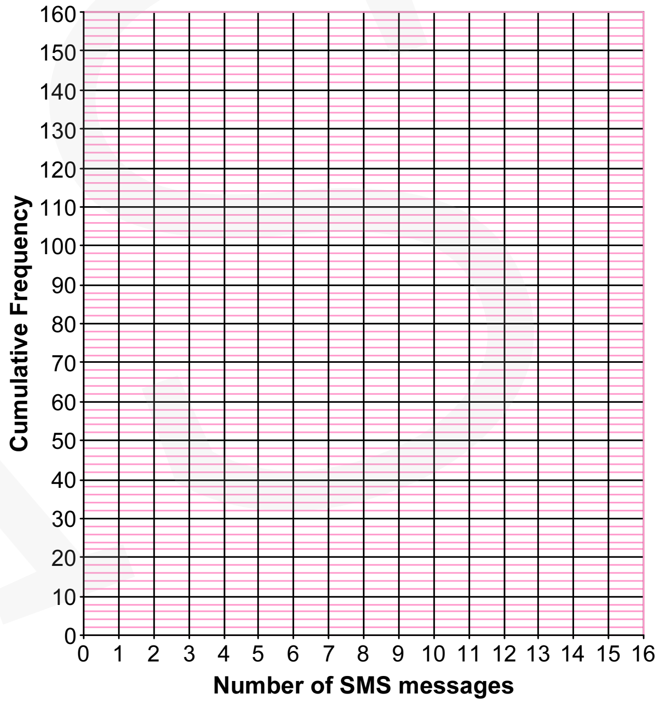


2.1 Complete the cumulative frequency table. (2)

CLASS	FREQUENCY	CUMULATIVE FREQUENCY
0 ≤ m < 2		
2 ≤ m < 4		
4 ≤ m < 6		
6 ≤ m < 8		
8 ≤ m < 10		
10 ≤ m < 12		
12 ≤ m < 14		
14 ≤ m < 16		

2.2 Use the grid to draw an ogive (cumulative frequency curve) to represent the data. (3)

Cumulative Frequency



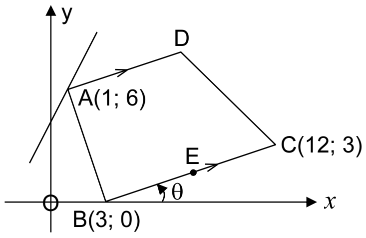
2.3 Use the ogive to identify the median for the data. (1)

2.4 Estimate the percentage of the learners who sent more than 11 messages using this application. (2)

2.5 In which direction is the data skewed? (1) [9]

ANALYTICAL GEOMETRY [29]

QUESTION 3



A(1; 6), B(3; 0), C(12; 3) and D are the vertices of a trapezium with AD || BC.

E is the midpoint of BC.

The angle of inclination of the straight line BC is θ, as shown in the diagram.

3.1 Calculate the coordinates of E. (2)

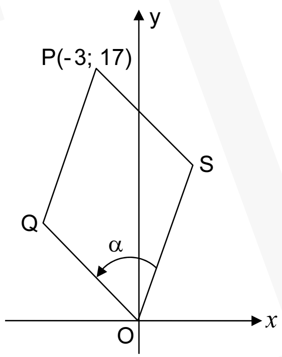
3.2 Determine the gradient of the line BC. (2)

3.3 Calculate the magnitude of θ. (2)

3.4 Prove that AD is perpendicular to AB. (3)

3.5 A straight line passing through vertex A does not pass through any of the sides of the trapezium. This line makes an angle of 45° with side AD of the trapezium. Determine the equation of this straight line. (5) [14]

QUESTION 4



In the diagram alongside, P(-3; 17), Q, O and S are the vertices of a parallelogram.

The sides OS and OQ are defined by the equations $y = 6x$ and $y = -x$ respectively.

$Q\hat{O}S = \alpha$.

4.1 Determine the equation of QP in the form $y = mx + c$. (3)

4.2 Hence, determine the coordinates of Q. (4)

4.3 Calculate the length of OQ. Leave your answer in simplified surd form. (2)

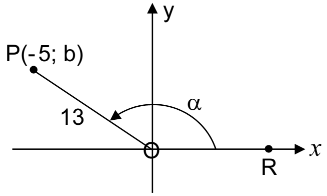
4.4 Calculate the size of α. (3)

4.5 If OS = $\sqrt{148}$ units, calculate the length of QS. (3) [15]

TRIGONOMETRY [52]

QUESTION 5

5.1 In the figure alongside, the point P(-5; b) is plotted on the Cartesian plane.



OP = 13 units and $R\hat{O}P = \alpha$.

Without using a calculator, determine the value of the following:

5.1.1 $\cos\alpha$ 5.1.2 $\tan(180° - \alpha)$ (1)(3)

5.2 Consider: $\dfrac{\sin(\theta - 360°)\sin(90° - \theta)\tan(-\theta)}{\cos(90° + \theta)}$

5.2.1 Simplify $\dfrac{\sin(\theta - 360°)\sin(90° - \theta)\tan(-\theta)}{\cos(90° + \theta)}$ to a single trigonometric ratio. (5)

5.2.2 Hence, or otherwise, **without using a calculator**, solve for θ if $0° \leq \theta \leq 360°$:

$\dfrac{\sin(\theta - 360°)\sin(90° - \theta)\tan(-\theta)}{\cos(90° + \theta)} = 0{,}5$ (3)

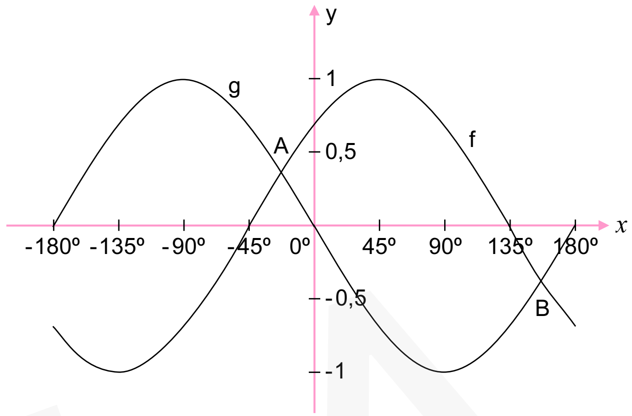
5.3.1 Prove that $\dfrac{8}{\sin^2 A} - \dfrac{4}{1 + \cos A} = \dfrac{4}{1 - \cos A}$. (5)

5.3.2 For which value(s) of A in the interval $0° \leq A \leq 360°$ is the identity in QUESTION 5.3.1 undefined? (3)

5.4 Determine the general solution of $8\cos^2 x - 2\cos x - 1 = 0$. (6) [26]

QUESTION 6

In the diagram below, the graphs of $f(x) = \cos(x + p)$ and $g(x) = q\sin x$ are shown for the interval $-180° \leq x \leq 180°$.



6.1 Determine the values of p and q. (2)

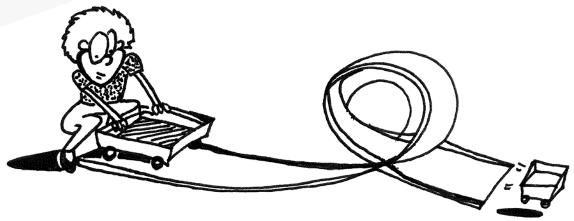
6.2 The graphs intersect at A(-22,5°; 0,38) and B. Determine the coordinates of B. (2)

6.3 Determine the value(s) of x in the interval $-180° \leq x \leq 180°$ for which $f(x) - g(x) < 0$. (2)

6.4 The graph f is shifted 30° to the left to obtain a new graph h.

6.4.1 Write down the equation of h in its simplest form. (2)

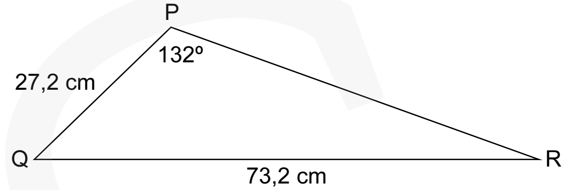
6.4.2 Write down the value of x for which h has a minimum in the interval $-180° \leq x \leq 180°$. (1) [9]



QUESTION 7

7.1 Prove that in any acute-angled $\triangle ABC$, $\dfrac{\sin A}{a} = \dfrac{\sin C}{c}$. (5)

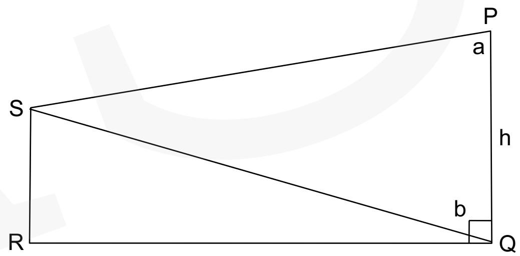
7.2 In $\triangle PQR$, $\hat{P} = 132°$, PQ = 27,2 cm and QR = 73,2 cm.



7.2.1 Calculate the size of $\hat{R}$. (3)

7.2.2 Calculate the area of $\triangle PQR$. (3)

7.3 In the figure below, $S\hat{P}Q = a$, $P\hat{Q}S = b$ and PQ = h. PQ and SR are perpendicular to RQ.



7.3.1 Determine the distance SQ in terms of a, b and h. (3)

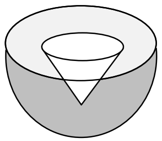
7.3.2 Hence show that $RS = \dfrac{h\sin a.\cos b}{\sin(a + b)}$. (3) [17]

2 scenarios

▶ MEASUREMENT [6]

QUESTION 8

A solid metallic hemisphere has a radius of 3 cm. It is made of metal A. To reduce its weight a conical hole is drilled into the hemisphere (as shown in the diagram) and it is completely filled with a lighter metal B. The conical hole has a radius of 1,5 cm and a depth of $\frac{8}{9}$ cm.



Calculate the ratio of the volume of metal A to the volume of metal B. [6]

▶ EUCLIDIAN GEOMETRY [40]

QUESTION 9

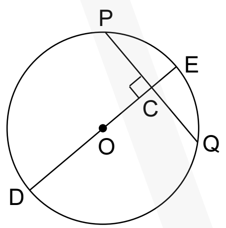
9.1 Complete the statement so that it is valid:

The line drawn from the centre of the circle perpendicular to the chord . . . (1)

9.2 In the diagram, O is the centre of the circle.

The diameter DE is perpendicular to the chord PQ at C.

DE = 20 cm and CE = 2 cm.

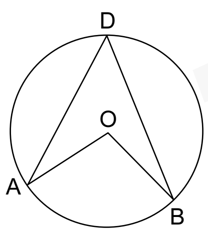


Calculate the length of the following with reasons:

9.2.1 OC 9.2.2 PQ (2)(4) [7]

QUESTION 10

10.1 In the diagram, O is the centre of the circle and A, B and D are points on the circle.

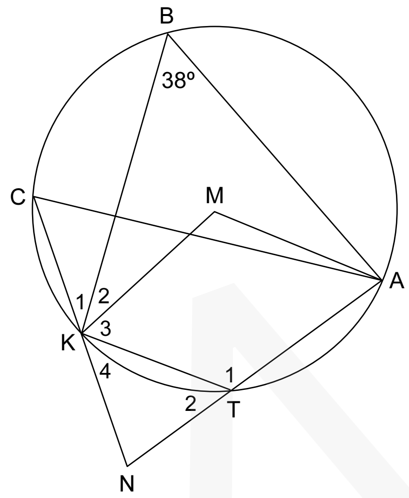


Use Euclidean geometry methods to prove the theorem which states that $A\hat{O}B = 2A\hat{D}B$. (5)

10.2 In the diagram, M is the centre of the circle. A, B, C, K and T lie on the circle.

AT produced and CK produced meet in N.

Also NA = NC and $\hat{B} = 38°$.



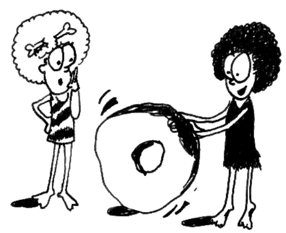
10.2.1 Calculate, with reasons, the size of the following angles:

(a) $K\hat{M}A$ (b) $\hat{T}_2$ (2)(2)

(c) $\hat{C}$ (d) $\hat{K}_4$ (2)(2)

10.2.2 Show that NK = NT. (2)

10.2.3 Prove that AMKN is a cyclic quadrilateral. (3) [18]



QUESTION 11

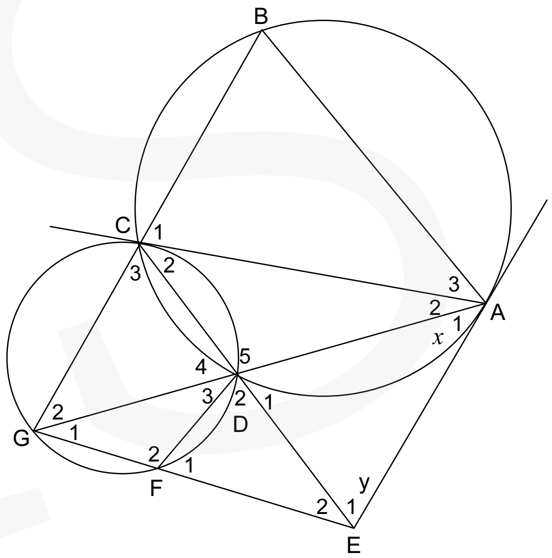
11.1 Complete the following statement so that it is valid:

The angle between a chord and a tangent at the point of contact is . . . (1)

11.2 In the diagram, EA is a tangent to circle ABCD at A.

AC is a tangent to circle CDFG at C.

CE and AG intersect at D.



If $\hat{A}_1 = x$ and $\hat{E}_1 = y$, prove the following with reasons:

11.2.1 BCG || AE (5)

11.2.2 AE is a tangent to circle FED (5)

11.2.3 AB = AC (4) [15]

TOTAL: 150

GRADE 12 EXEMPLAR PAPER 2

You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

*If necessary, round off answers to **TWO** decimal places, unless stated otherwise.*

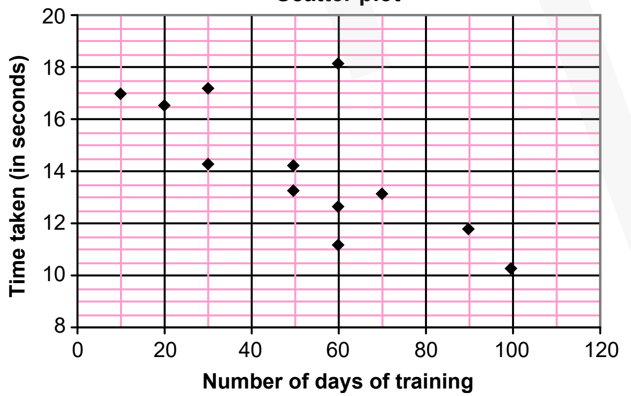
▶ STATISTICS [21]

QUESTION 1

Twelve athletes trained to run the 100 m sprint event at the local athletics club trials. Some of them took their training more seriously than others. The following table and scatter plot shows the number of days that an athlete trained and the time taken to run the event. The time taken, in seconds, is rounded to one decimal place.

Number of days of training	50	70	10	60	60	20	50	90	100	60	30	30
Time taken (in seconds)	12,9	13,1	17,0	11,3	18,1	16,5	14,3	11,7	10,2	12,7	17,2	14,3

Scatter plot



1.1 Discuss the trend of the data collected. (1)

1.2 Identify any outlier(s) in the data. (1)

1.3 Calculate the equation of the least squares regression line. (4)

1.4 Predict the time taken to run the 100 m sprint for an athlete training for 45 days. (2)

1.5 Calculate the correlation coefficient. (2)

1.6 Comment on the strength of the relationship between the variables. (1) [11]

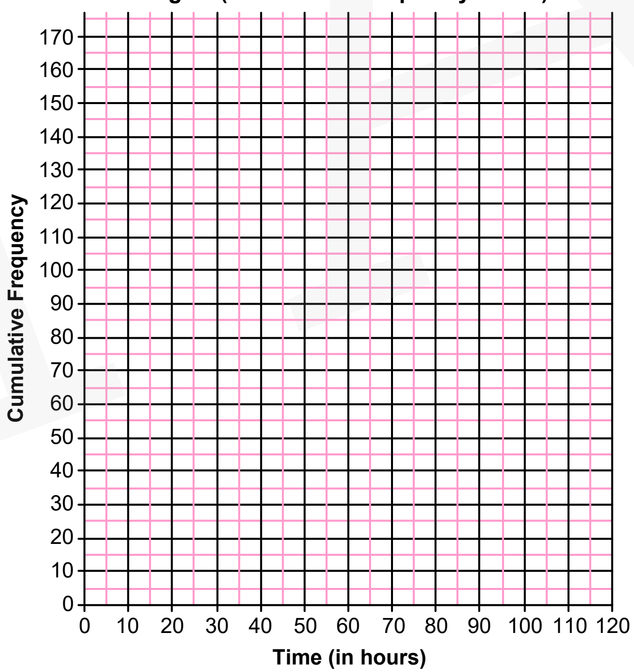
QUESTION 2

The table below shows the amount of time (in hours) that learners aged between 14 and 18 spent watching television during 3 weeks of the holiday.

Time (hours)	Cumulative frequency
$0 \leq t < 20$	25
$20 \leq t < 40$	69
$40 \leq t < 60$	129
$60 \leq t < 80$	157
$80 \leq t < 100$	166
$100 \leq t < 120$	172

2.1 Draw an ogive (cumulative frequency curve) on the grid provided below to represent the given data. (3)

Ogive (Cumulative Frequency Curve)



2.2 Write down the modal class of the data. (1)

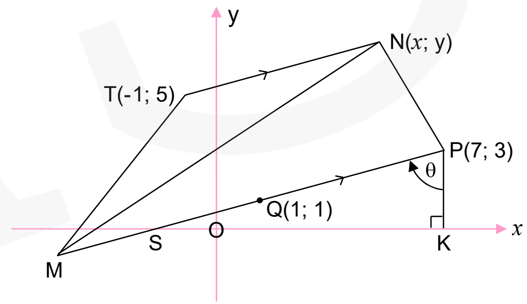
2.3 Use the ogive (cumulative frequency curve) to estimate the number of learners who watched television more than 80% of the time. (2)

2.4 Estimate the mean time (in hours) that learners spent watching television during 3 weeks of the holiday. (4) [10]

▶ ANALYTICAL GEOMETRY [37]

QUESTION 3

In the diagram below, M, T(-1; 5), N(x; y) and P(7; 3) are vertices of trapezium MTNP having TN || MP. Q(1; 1) is the midpoint of MP. PK is a vertical line and $\hat{SPK} = \theta$. The equation of NP is $y = -2x + 17$.



3.1 Write down the coordinates of K. (1)

3.2 Determine the coordinates of M. (2)

3.3 Determine the gradient of PM. (2)

3.4 Calculate the size of θ. (3)

3.5 Hence, or otherwise, determine the length of PS. (3)

3.6 Determine the coordinates of N. (5)

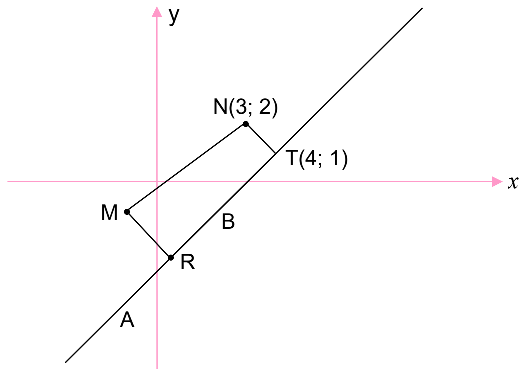
3.7 If A(a; 5) lies in the Cartesian plane:

3.7.1 Write down the equation of the straight line representing the possible positions of A. (1)

3.7.2 Hence, or otherwise, calculate the value(s) of a for which $\hat{TAQ} = 45°$. (5) [22]

QUESTION 4

In the diagram below, the equation of the circle having centre M is $(x + 1)^2 + (y + 1)^2 = 9$. R is a point on chord AB such that MR bisects AB. ABT is a tangent to the circle having centre N(3; 2) at point T(4; 1).



4.1 Write down the coordinates of M. (1)

4.2 Determine the equation of AT in the form y = mx + c. (5)

4.3 If it is further given that MR = $\frac{\sqrt{10}}{2}$ units, calculate the length of AB. Leave your answer in simplest surd form. (4)

4.4 Calculate the length of MN. (2)

4.5 Another circle having centre N touches the circle having centre M at point K. Determine the equation of the new circle. Write your answer in the form $x^2 + y^2 + Cx + Dy + E = 0$ (3) [15]

TRIGONOMETRY [41]

QUESTION 5

5.1 Given that $\sin\alpha = -\frac{4}{5}$ and $90° < \alpha < 270°$.

WITHOUT using a calculator, determine the value of each of the following in its simplest form:

5.1.1 $\sin(-\alpha)$ (2)

5.1.2 $\cos\alpha$ (2)

5.1.3 $\sin(\alpha - 45°)$ (3)

5.2 Consider the identity:

$$\frac{8\sin(180° - x)\cos(x - 360°)}{\sin^2 x - \sin^2(90° + x)} = -4\tan 2x$$

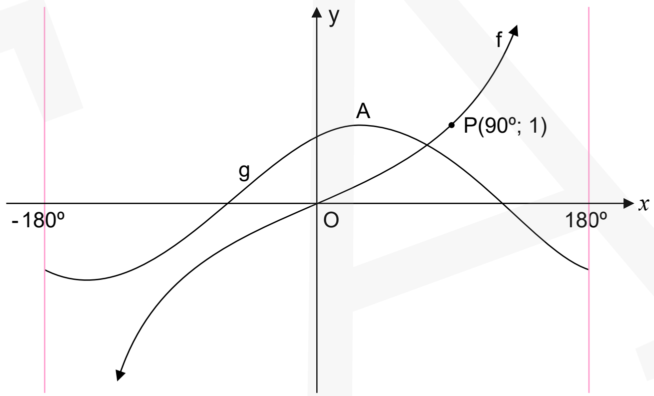
5.2.1 Prove the identity. (6)

5.2.2 For which value(s) of x in the interval $0° < x < 180°$ will the identity be undefined? (2)

5.3 Determine the general solution of $\cos 2\theta + 4\sin^2\theta - 5\sin\theta - 4 = 0$. (7) [22]

QUESTION 6

In the diagram below, the graphs of $f(x) = \tan bx$ and $g(x) = \cos(x - 30°)$ are drawn on the same system of axes for $-180° \leq x \leq 180°$. The point P(90°; 1) lies on f. Use the diagram to answer the following questions.

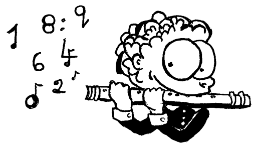


6.1 Determine the value of b. (1)

6.2 Write down the coordinates of A, a turning point of g. (2)

6.3 Write down the equation of the asymptote(s) of $y = \tan b(x + 20°)$ for $x \in [-180°; 180°]$. (1)

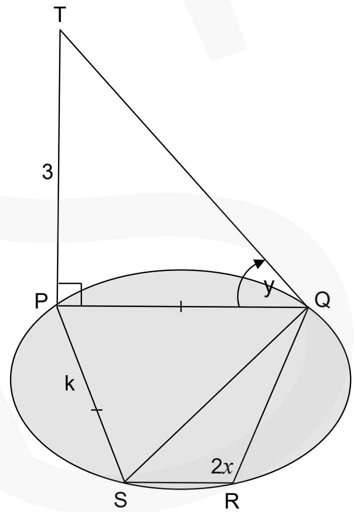
6.4 Determine the range of h if $h(x) = 2g(x) + 1$. (2) [6]



QUESTION 7

7.1 Prove that in any acute-angled $\triangle ABC$, $\frac{\sin A}{a} = \frac{\sin B}{b}$. (5)

7.2 The framework for a construction consists of a cyclic quadrilateral PQRS in the horizontal plane and a vertical post TP as shown in the figure. From Q the angle of elevation of T is y°. PQ = PS = k units, TP = 3 units and $\hat{SRQ} = 2x°$.



7.2.1 Show, giving reasons, that $\hat{PSQ} = x$. (2)

7.2.2 Prove that SQ = 2k cos x. (4)

7.2.3 Hence, prove that SQ = $\frac{6\cos x}{\tan y}$. (2) [13]

EUCLIDEAN GEOMETRY AND MEASUREMENT [51]

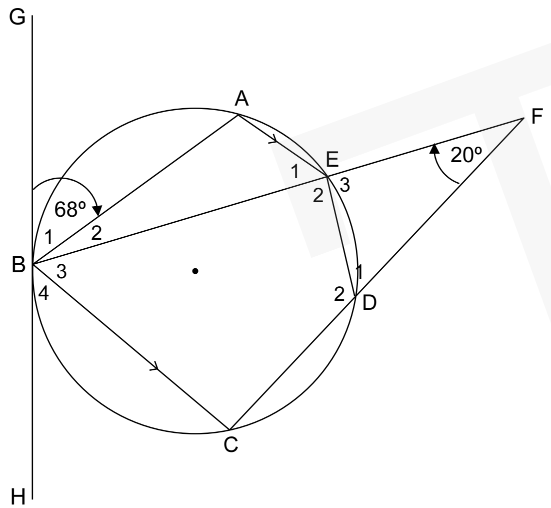
Give reasons for your statements in QUESTIONS 8, 9 and 10.

QUESTION 8

8.1 Complete the following statement:

The angle between the tangent and the chord at the point of contact is equal to . . . (1)

8.2 In the diagram, A, B, C, D and E are points on the circumference of the circle such that AE || BC. BE and CD produced meet in F. GBH is a tangent to the circle at B. $\hat{B}_1 = 68^\circ$ and $\hat{F} = 20^\circ$.



Determine the size of each of the following:

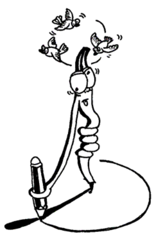
8.2.1 $\hat{E}_1$ (2)

8.2.2 $\hat{B}_3$ (1)

8.2.3 $\hat{D}_1$ (2)

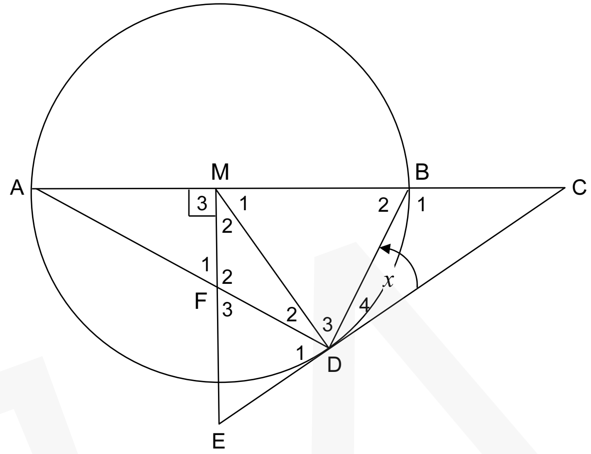
8.2.4 $\hat{E}_2$ (1)

8.2.5 $\hat{C}$ (2) [9]



QUESTION 9

In the diagram, M is the centre of the circle and diameter AB is produced to C. ME is drawn perpendicular to AC such that CDE is a tangent to the circle at D. ME and chord AD intersect at F. MB = 2BC.



9.1 If $\hat{D}_4 = x$, write down, with reasons, TWO other angles each equal to x. (3)

9.2 Prove that CM is a tangent at M to the circle passing through M, E and D. (4)

9.3 Prove that FMBD is a cyclic quadrilateral. (3)

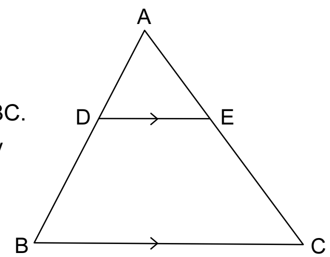
9.4 Prove that $DC^2 = 5BC^2$. (3)

9.5 Prove that ΔDBC ||| ΔDFM. (4)

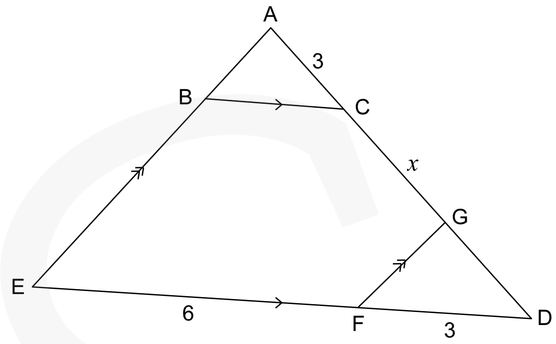
9.6 Hence, determine the value of $\frac{DM}{FM}$. (2) [19]

QUESTION 10

10.1 In the diagram, points D and E lie on sides AB and AC respectively of ΔABC such that DE || BC. Use Euclidean Geometry methods to prove the theorem which states that $\frac{AD}{DB} = \frac{AE}{EC}$. (6)



10.2 In the diagram, ADE is a triangle having BC || ED and AE || GF. It is also given that AB : BE = 1 : 3, AC = 3 units, EF = 6 units, FD = 3 units and CG = x units.



Calculate, giving reasons:

10.2.1 the length of CD (3)

10.2.2 the value of x (4)

10.2.3 the length of BC (5)

10.2.4 the value of $\frac{\text{area } \Delta ABC}{\text{area } \Delta GFD}$ (5) [23]

TOTAL: 150

EXEMPLAR MEMOS

Gr 10, 11 & 12

GRADE 10 EXEMPLAR PAPER 2 MEMO

1.1 The mean,

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \quad \ldots \frac{\textit{total number of scores}}{\textit{total number of days}}$$

$$= \frac{929}{19}$$

$$\approx 48{,}89 \;\checkmark$$

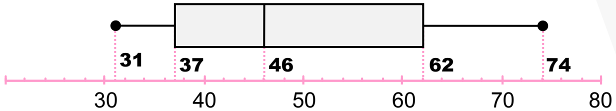
1.2 31; 31; 34; 36; **(Q_1)** 37; 39; 40; 43; 46; **(Q_2)** 46; 48; 52; 56; 60; **(Q_3)** 62; 63; 65; 66; 74

The median **(Q_2)** = 46 ✓

1.3 The lower quartile **(Q_1)** = 37 ✓

The upper quartile **(Q_3)** = 62 ✓

1.4 Min value = 31 & Max value = 74



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2.1 $2\,500 \leq x < 4\,500$

The **sum** of . . .
the **products** of the **frequency**
and the **mid-value** for each interval

2.2 Estimated mean, $\bar{X}$

$$= \frac{103 \times 3\,500 + 19 \times 5\,500 + 70 \times 7\,500 + 77 \times 9\,500 \ldots^*}{103 + 19 + 70 + 77 + 85 + 99}$$

The sum of the frequencies

$^*\ldots + 85 \times 11\,500 + 99 \times 13\,500$

$$= \frac{4\,035\,500}{453}$$

$$\approx 8\,908{,}39 \text{ kg} \;\checkmark$$

$$\bar{X} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$$

2.3 The estimated mean ✓

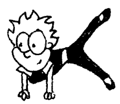
This value is at the centre of the set, whereas the modal class is an extreme situation in relation to the other intervals. ✓

3.1.1 $DE^2 = (3 + 3)^2 + (-5 - 3)^2$

$= 36 + 64$

$= 100$

$\therefore$ DE = 10 units ✓



3.1.2 Gradient of DE,

$$m_{DE} = \frac{-5 - 3}{3 + 3} = \frac{-8}{6} = -\frac{4}{3} \;\checkmark$$

3.1.3 $$m_{EF} = \frac{k + 5}{-1 - 3} = \frac{k + 5}{-4}$$

$D\hat{E}F = 90^\circ$ ➡ $m_{EF} = +\frac{3}{4}$ . . . *EF ⊥ DE*

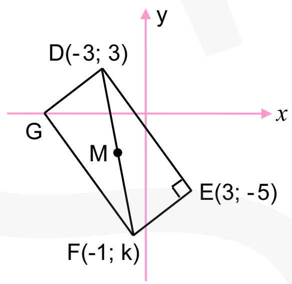
$$\therefore \frac{k + 5}{-4} = \frac{3}{4}$$

× (-4) $\therefore k + 5 = -3$

$\therefore k = -8$ ✓

3.1.4 $$M\left(\frac{-3 + (-1)}{2}\,;\ \frac{3 + (-8)}{2}\right)$$

$$\therefore M\left(-2\,;\ -\frac{5}{2}\right) \;\checkmark$$



3.1.5 DEFG will be a $||^m$ if M is the midpoint of EG too.

& Since $D\hat{E}F = 90^\circ$, DEFG will be a rectangle.

. . . *if one ∠ of a $||^m$ is a right ∠, then the $||^m$ is a rectangle.*

$$\frac{x_G + 3}{2} = -2 \quad \text{and} \quad \frac{y_G + (-5)}{2} = -\frac{5}{2}$$

× 2) $\therefore x_G + 3 = -4$ $\qquad \therefore y_G - 5 = -5$

$\therefore x_G = -7$ $\qquad \therefore y_G = 0$

$\therefore$ G(-7; 0) ✓

OR: The translation F to G equals that of E to D

$\therefore$ G(-1 - 6; -8 + 8)

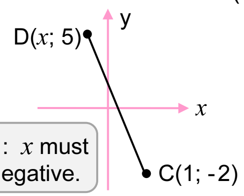
$\therefore$ G(-7; 0) ✓

OR: The translation D to G equals that of E to F

$\therefore$ G(-3 - 4; 3 - 3)

$\therefore$ G(-7; 0) ✓

3.2 $CD^2 = (x - 1)^2 + (5 + 2)^2 = \left(\sqrt{53}\right)^2$



Note: x must be negative.

$\therefore (x - 1)^2 + 49 = 53$

$\therefore (x - 1)^2 = 4$

$\therefore x - 1 = \pm 2$

$\therefore x = 3$ or -1

But $x < 0$ in the second quadrant

$\therefore x = -1$ ✓ . . . *only the neg. value of x is valid*

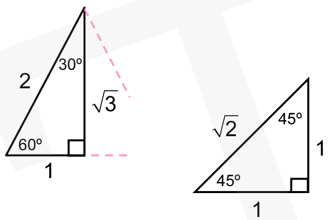
4.1.1 $\sin C = \dfrac{AB}{\mathbf{AC}}$ ✓

4.1.2 **cot** $A = \dfrac{AB}{BC}$

Note: $\tan A = \dfrac{BC}{AB}$; $\cot A = \dfrac{1}{\tan A}$

4.2 The expression

$= \dfrac{\dfrac{\sqrt{3}}{2} \times \dfrac{1}{\sqrt{3}}}{\sqrt{2}}$

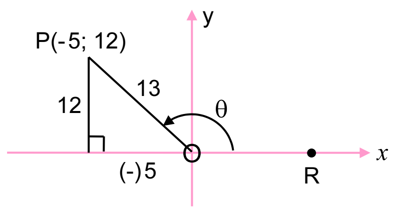


$= \dfrac{1}{2} \times \dfrac{1}{\sqrt{2}}$

$= \dfrac{1}{2\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}}$ . . . *The denominator must be rationalised*

$= \dfrac{\sqrt{2}}{4}$ ✓ . . . $\sqrt{2} \times \sqrt{2} = 2$

4.3.1 OP = 13 units . . . *5 : 12 : 13 Δ; Pythagoras*



$\therefore \cos\theta = \dfrac{-5}{13} = -\dfrac{5}{13}$ ✓ . . . $\cos\theta = \dfrac{x}{r}$

4.3.2 $\sin\theta = \dfrac{12}{13}$ ➨ $\operatorname{cosec}\theta = \dfrac{13}{12}$

$\therefore \operatorname{cosec}^2\theta + 1 = \left(\dfrac{13}{12}\right)^2 + 1 = \dfrac{169}{144} + 1$

$= \dfrac{169 + 144}{144} = \dfrac{313}{144}$ ✓ $\left(= 2\dfrac{25}{144}\right)$ ✓

5.1.1 $5\cos x = 3$

$\div 5)$ $\therefore \cos x = \dfrac{3}{5}$ $(= 0{,}6)$

$\therefore x \approx 53{,}1°$ ✓ . . . $\cos^{-1}\left(\dfrac{3}{5}\right) =$

5.1.2 $\tan 2x = 1{,}19$

$\therefore 2x = 49{,}958\ldots°$ . . . $\tan^{-1} 1{,}19 =$

$\div 2)$ $\therefore x \approx 25{,}0°$ ✓

5.1.3 $4\sec x - 3 = 5$

$+ 3)$ $\therefore 4\sec x = 8$

$\div 4)$ $\therefore \sec x = 2$

$\therefore \cos x = \dfrac{1}{2}$

$x = 60°$ ✓ . . . $\cos^{-1}\left(\dfrac{1}{2}\right) =$

5.2.1 $J\hat{K}D = 8°$ ✓ . . . *alternate ∠'s; || lines*

5.2.2 In ΔJDK: $\dfrac{DK}{5} = \cot 8°$ . . . $= \dfrac{1}{\tan 8°}$

$\times 5)$ $\therefore DK = \dfrac{5}{\tan 8°}$

$= 35{,}5768\ldots$ km

$= 35\,576{,}8$ metres

$\approx 35\,577$ metres ✓

. . . *correct to the nearest metre*

5.2.3 DS = DK – SK

= 35,58 km – 8 km

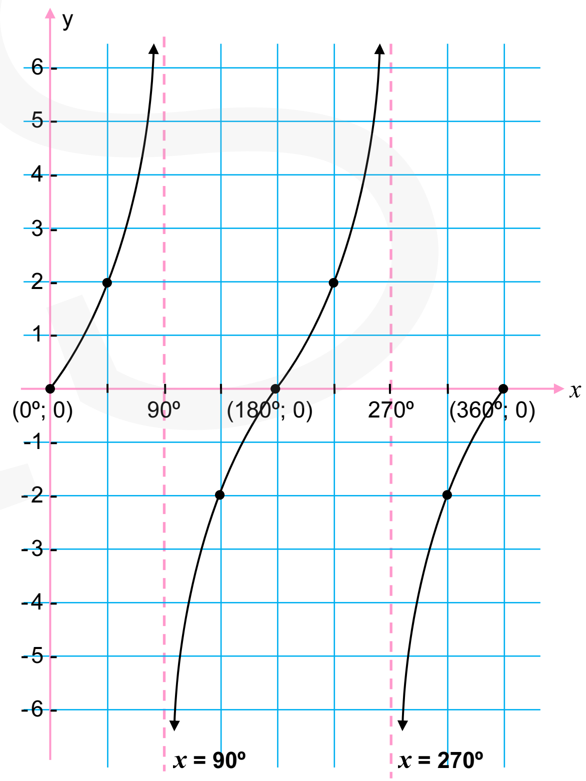
= 27,58 km ✓

5.2.4 $\tan J\hat{S}D = \dfrac{5}{27{,}58}$

$\therefore J\hat{S}D \approx 10{,}3°$ ✓ . . . $\tan^{-1}\left(\dfrac{5}{27{,}58}\right) =$

correct to 1 dec. place

6.1.1



6.1.2 $y = -2\tan x$ ✓

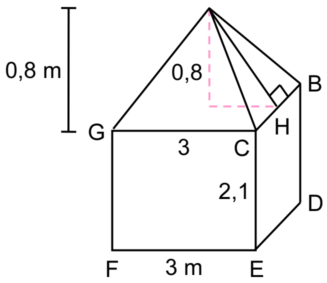
6.2.1 a = 4 ✓

$g(x) = a\sin x$ ➨ $g(90°) = a\sin 90°$
➨ $4 = a$

6.2.2 The range of h:

$-2 \leq y \leq 6$ ✓ . . . *the values of y*

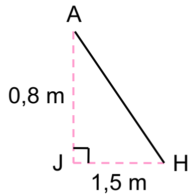
7.



7.1.1 $AH^2 = 0{,}8^2 + 1{,}5^2$

$= 2{,}89$

$\therefore AH \approx 1{,}7$ m ✓



OR: Pythag. triple: 8 : 15 : 17

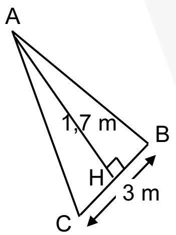
➡ 0,8 : 1,5 : **1,7** ✓

7.1.2 Surface area of roof

$= 4 \times$ area of ΔABC

$= 4 \times \frac{1}{2}(3)(1{,}7)$

$= 10{,}2\ m^2$ ✓



7.1.3 Surface area of the walls

$= 4 \times$ area of GFEC

$= 4 \times (3)(2{,}1)$

$= 25{,}2\ m^2$ ✓

$\therefore$ The total surface area of the tent

$= 10{,}2 + 25{,}2$

$= 35{,}4\ m^2$ ✓

7.2.1 Volume $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(8)^3 \approx 2\ 144{,}66\ mm^3$ ✓

7.2.2 $2^3 : 1$

$= 8 : 1$ ✓

$$\frac{\text{New volume}}{\text{Original volume}} = \frac{\frac{4}{3}\pi(2r)^3}{\frac{4}{3}\pi r^3} = \frac{2^3 r^3}{r^3} = \frac{8}{1}$$

7.2.3 Volume of silver

$= \frac{4}{3}\pi(8 + 1)^3 - \frac{4}{3}\pi(8)^3$ . . . *The volume of silver covering the ball*

$= 908{,}967\ldots$

$\approx 908{,}97\ mm^3$ ✓

8.1 OQ = 2 cm ✓ . . . *the longer diagonal of a kite bisects the shorter diagonal*

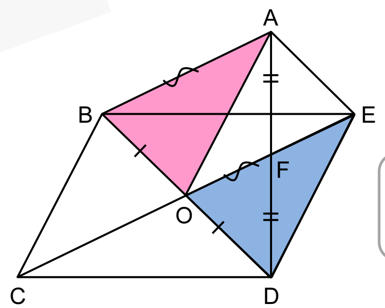
8.2 $P\hat{O}Q = 90°$ ✓ . . . *the diagonals of a kite intersect at right angles*

8.3 $Q\hat{P}O = 20°$ . . . *the longer diagonal of a kite bisects the (opposite) angles of a kite*

$\therefore Q\hat{P}S = 40°$ ✓

9.

Hint:
Use hiliters to mark the various $||^{ms}$ and Δ^s



The hilited Δ^s (and their sides) refer to Question 9.3.

9.1 In ΔDBA:

O is the midpt of BD . . . *diagonals of $||^m$ BCDE bisect each other*

& F is the midpt of AD . . . *diagonals of $||^m$ AODE bisect each other*

$\therefore$ OF || AB ✓ . . . *the line joining the midpoints of two sides of a Δ is || to the 3rd side*

9.2 AE || OD . . . *opp. sides of $||^m$ AODE*

$\therefore$ AE || BO

and OF || AB . . . *proven above*

$\therefore$ OE || AB

$\therefore$ ABOE is a $||^m$ . . . *both pairs of opposite sides are parallel*

OR: In $||^m$ AODE: AE = and || OD . . . *opp. sides of $||^m$*

But OD = and || BO . . . *O proved midpt of BD in 9.1*

$\therefore$ AE = and || BO

$\therefore$ ABOE is a $||^m$ ✓ . . . *1 pr of opp. sides = and ||*

9.3 In Δ^s ABO and EOD

1) AB = EO . . . *opposite sides of $||^m$ ABOE*

2) BO = OD . . . *proved in 9.1*

3) AO = ED . . . *opposite sides of $||^m$ AODE*

$\therefore \Delta ABO \equiv \Delta EOD$ ✓ . . . *SSS*

GRADE 11 EXEMPLAR PAPER 2 MEMO

You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

*If necessary, round off answers to **TWO** decimal places, unless stated otherwise.*

STATISTICS [23]

Calculator instructions to find:
- the mean, and
- standard deviation (for ungrouped data)

Casio fx-82ES

- [MODE] [2 : STAT] [1 : 1 - VAR]
- Enter each value, followed by [=] after the last value: [=] **[AC]**
- To find the **mean**: [SHIFT] [STAT] [5 : VAR] [2 : $\bar{x}$] [=]
- To find the **S.D.**: [SHIFT] [STAT] [5 : VAR] [3 : $x\sigma n$] [=]

You'll see:

	X	FREQ
1	...	*automatically*
2	...	*entered*
3	...	*as 1*

1.1 The mean, $\bar{x} \approx$ **21,47**

1.2 The standard deviation, $\sigma \approx$ **7,81**

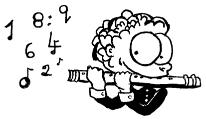
1.3 $\bar{x} + 1\sigma = 29,28$ . . . *the upper limit*

$\bar{x} - 1\sigma = 13,66$ . . . *the lower limit*

∴ The number of people vaccinated per day must lie between 13,66 and 29,28.

The numbers within the range are:
19 29 23 15 21 26
18 23 18 21 18 22 20

This occurs on 13 days.



1.4 The 19 scores must be arranged from smallest to biggest.

A stem and leaf diagram:

This diagram is a useful tool when determining quartiles.

The stems	The leaves
0	5
1	2 3 5 [8] (Q_1) 8 8 9
2	0 [1] (Q_2) 1 2 3 3 [6] (Q_3) 9
3	3 5 7

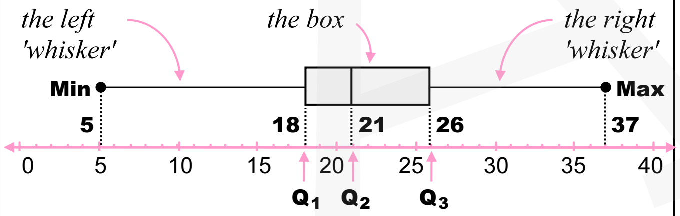
Q_1 = the 5th score = 18

(Q_2, the median = the 10th score = 21)

Q_3 = the 15th score = 26

∴ **The IQR = $Q_3 - Q_1$ = 26 - 18 = 8**

1.5



1.6 **5 is an outlier** . . . *see the stem and leaf diagram*

All the other scores are close to one another. They differ by no more than 3, whereas the score 5 is 7 less than the next score (12).

An outlier is a score which does not fit the trend of the data. As a matter of interest, a formula (not specified in the curriculum) exists to identify outliers:

If a score lies further away from 'the box' than 1,5 times the IQR, then it is an outlier.

In our example: 1,5 times the IQR = 1,5 × 8 = 12

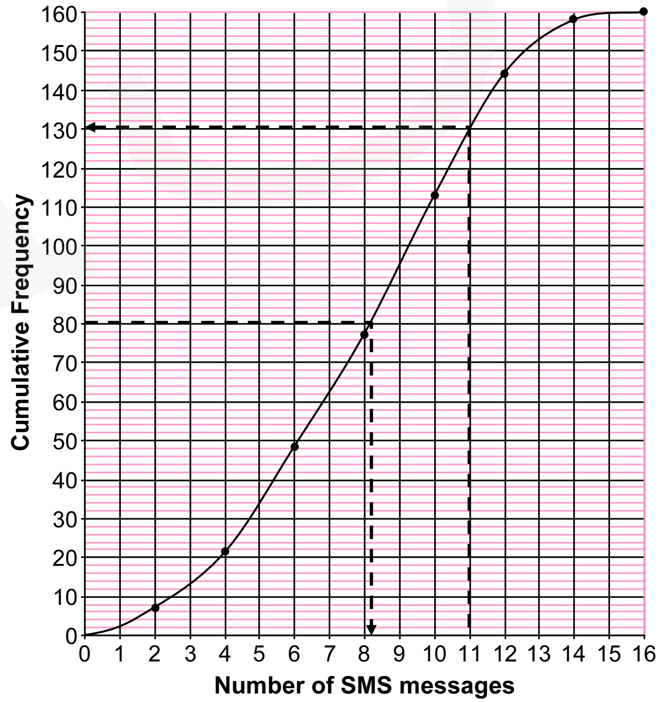
$Q_1 - 12 = 6$ ∴ 5 is an outlier

$Q_3 + 12 = 38$ ∴ 37 is not an outlier

2.1

CLASS	FREQUENCY	CUMULATIVE FREQUENCY
0 ≤ m < 2	7	7
2 ≤ m < 4	15	22
4 ≤ m < 6	26	48
6 ≤ m < 8	29	77
8 ≤ m < 10	36	113
10 ≤ m < 12	31	144
12 ≤ m < 14	14	158
14 ≤ m < 16	2	160

2.2



2.3 **The median is approximately 8 messages**

Read off the ogive from 80 on the y-axis to find the (middlemost) value on the x-axis.

2.4 The number of learners who sent less than 11 messages = 130

$\therefore$ The number of learners who sent more than 11 messages = 30

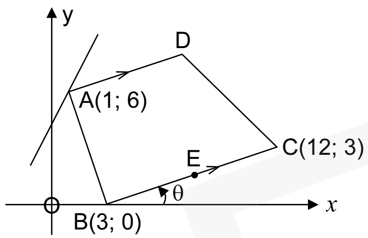
$\therefore$ The fraction of learners who sent more than 11 messages = $\frac{30}{160}$ (= 0,1875)

$\therefore$ The % is $\frac{30}{160} \times 100\%$ = **18,75%** ✓

2.5 There is no significant skewedness

ANALYTICAL GEOMETRY [29]

3.



3.1 Point E is $\left(\frac{3+12}{2}; \frac{0+3}{2}\right)$, ... $\left(\frac{x_1+x_2}{2}; \frac{y_1+y_2}{2}\right)$

$\therefore$ **E$\left(7\frac{1}{2}; 1\frac{1}{2}\right)$** ✓

3.2 $m_{BC} = \frac{3-0}{12-3} = \frac{3}{9} = \mathbf{\frac{1}{3}}$ ✓ ... $m = \frac{y_2 - y_2}{x_1 - x_2}$

3.3 $\tan\theta = \frac{1}{3}$ ➡ $\theta \approx$ **18,43°** ✓

3.4 $m_{AD} = m_{BC} = \frac{1}{3}$ ... *AD || BC*

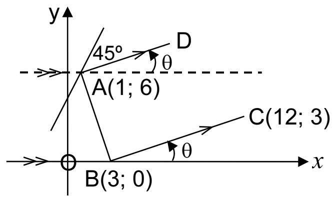
& $m_{AB} = \frac{0-6}{3-1} = \frac{-6}{2} = -3$

$\therefore m_{AD} \times m_{AB} = \left(\frac{1}{3}\right)(-3) = -1$

$\therefore$ **AD ⊥ AB** ✓

3.5 We need the gradient of the line.
$\therefore$ We need the ∠ of inclination.

Draw a horizontal line (|| x-axis) through point A.



The ∠ of inclination of the line is θ + 45°, i.e. 63,43°.

$\therefore$ The gradient of the line is tan 63,43° ≈ 2

AD || BC
∴ They have equal ∠s of inclination.

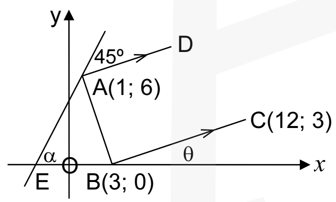
The equation: $\mathbf{y - y_1 = m(x - x_1)}$

Substitute pt. A(1; 6): $y - 6 = 2(x - 1)$

$\therefore$ **y = 2x + 4** ✓

or **y = mx + c**
$\therefore$ 6 = (2)(1) + c
$\therefore$ 4 = c
$\therefore$ Eqn.: **y = 2x + 4** ✓

OR: Extend the line to cut the x-axis (at E)



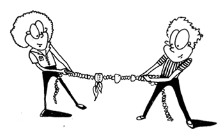
$A\hat{B}C$ = 90° ... *co-int. ∠s; AD || BC*

$\therefore A\hat{B}X$ = 90° + 18,43°
= 108,43°

$\therefore E\hat{A}B$ = 45° ... *∠s on a straight line*

$\therefore$ α = 108,43° - 45° ... *ext. ∠ of ΔAEB*
= 63,43°

m_{EA} = tan 63,43° ≈ 2, etc.



4.1 $m_{QP} = m_{OS} = 6$ ... *QP || OS in ||m*

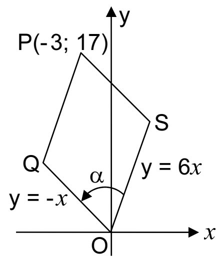
& Substitute point P(-3; 17):

$y - 17 = 6(x + 3)$

$\therefore$ **y = 6x + 35** ✓

OR: 17 = (6)(-3) + c
$\therefore$ 35 = c
$\therefore$ Eqn.: **y = 6x + 35** ✓

4.2 At Q: y = 6x + 35 and y = -x

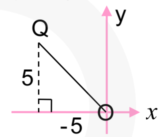


$\therefore$ 6x + 35 = -x
$\therefore$ 7x = -35
$\therefore$ x = -5
& $\therefore$ y = 5

$\therefore$ **Q(-5; 5)** ✓

4.3 $OQ^2 = 5^2 + 5^2$ ... *Thm. of Pythag.*
= 50

$\therefore OQ = \sqrt{50}$
= **5$\sqrt{2}$ units** ✓



$\sqrt{50} = \sqrt{25 \times 2} = \sqrt{25}\sqrt{2} = 5\sqrt{2}$

4.4 $\tan Q\hat{O}X = -1$ ... $m_{OQ} = -1$

$\therefore Q\hat{O}X$ = 135°

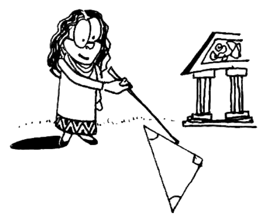
$\tan S\hat{O}X = 6$ ... $m_{OS} = 6$

$\therefore S\hat{O}X$ = 80,54°

$\therefore$ α = 135° - 80,54°
= **54,46°** ✓

4.5 In ΔQOS: $QS^2 = OQ^2 + OS^2 - 2OQ.OS \cos\alpha$
$= 50 + 148 - 2\sqrt{50}\sqrt{148}.\cos 54,46°$
= 97,994...

$\therefore$ **QS ≈ 9,90 units** ✓



▸ TRIGONOMETRY [52]

5.1.1 $\cos\alpha = \frac{\mathbf{x}}{\mathbf{r}} = \frac{-5}{13} = -\frac{\mathbf{5}}{\mathbf{13}}$ ◄

5.1.2 $\tan(180^\circ - \alpha) = -\tan\alpha$ . . .

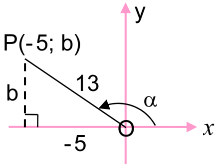
this is an identity ∴ true for ALL values of α.

$b = 12$ … *5 : 12 : 13 Δ ; Pythag.*

$\therefore \tan\alpha = \frac{12}{-5}$

$\therefore \tan(180^\circ - \alpha) = -\left(\frac{12}{-5}\right)$

$= \frac{\mathbf{12}}{\mathbf{5}}$ ◄



5.2.1 Expression $= \frac{\sin\theta \cdot \cos\theta \cdot (-\tan\theta)}{-\sin\theta}$

$= +\cos\theta \times \frac{\sin\theta}{\cos\theta}$ . . .

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$= \sin\theta$ ◄

5.2.2 The equation: $\sin\theta = 0{,}5$ . . . $0^\circ \leq \theta \leq 360^\circ$

$\therefore \theta = \mathbf{30^\circ}$ ◄

or $\theta = 180^\circ - 30^\circ$

$= \mathbf{150^\circ}$ ◄

5.3.1 **LHS** $= \frac{8}{1 - \cos^2 A} - \frac{4}{1 + \cos A}$

$= \frac{8}{(1 + \cos A)(1 - \cos A)} - \frac{4}{1 + \cos A}$

$= \frac{8 - 4(1 - \cos A)}{(1 + \cos A)(1 - \cos A)}$

$= \frac{8 - 4 + 4\cos A}{(1 + \cos A)(1 - \cos A)}$

$= \frac{4 + 4\cos A}{(1 + \cos A)(1 - \cos A)}$

$= \frac{4(1 + \cos A)}{(1 + \cos A)(1 - \cos A)}$

$= \frac{4}{1 - \cos A}$

$=$ **RHS**

OR: The identity to be proved is **equivalent** to:

$\frac{8}{\sin^2 A} = \frac{4}{1 + \cos A} + \frac{4}{1 - \cos A}$

RHS $= \frac{4(1 - \cos A) + 4(1 + \cos A)}{(1 + \cos A)(1 - \cos A)}$

$= \frac{4 - 4\cos A + 4 + 4\cos A}{1 - \cos^2 A}$

$= \frac{8}{\sin^2 A}$

$=$ **LHS**

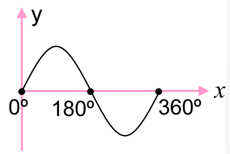
This identity is true.

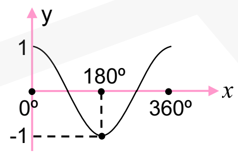
∴ The original identity is true.

5.3.2 The identity is undefined if any denominator = 0

∴ for: $\sin A = 0$ or $\cos A = -1$ or $\cos A = 1$

Refer to your well-known basic sine and cosine graphs.





∴ The identity is undefined for:

A = 0° ; 180° or **360°** ◄

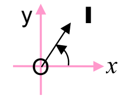
5.4 $8\cos^2 x - 2\cos x - 1 = 0$

$\therefore (2\cos x - 1)(4\cos x + 1) = 0$

$\therefore \cos x = \frac{1}{2}$ or $\cos x = -\frac{1}{4}$

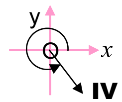
$\cos x = \frac{1}{2}$

➡ $\boldsymbol{x = 60^\circ + n(360^\circ)}$ ◄



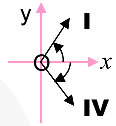
or $x = 360^\circ - 60^\circ + n(360^\circ)$

$= \mathbf{300^\circ + n(360^\circ), n \in \mathbb{Z}}$ ◄



OR:

$\boldsymbol{x = \pm 60^\circ + n(360^\circ), n \in \mathbb{Z}}$ ◄

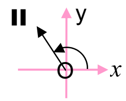


Which is the simpler option?

& $\cos x = -\frac{1}{4}$ (= -0,25)

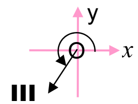
➡ $x = 180^\circ - 75{,}52^\circ + n(360^\circ)$

$= \mathbf{104{,}48^\circ + n(360^\circ)}$ ◄



or $x = 180^\circ + 75{,}52^\circ + n(360^\circ)$

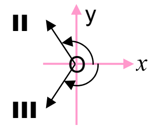
$= \mathbf{255{,}52^\circ + n(360^\circ), n \in \mathbb{Z}}$ ◄



OR:

$x = \pm(180^\circ - 75{,}52^\circ) + n(360^\circ)$

$\therefore \boldsymbol{x = \pm 104{,}48^\circ + n(360^\circ), n \in \mathbb{Z}}$ ◄



These 2 options are equivalent - they yield the same ∠s.

6.1 **p = -45°** ✓

. . . *f is y = cos x moved 45° to the right. Substitute to check: e.g. y = cos(45°* **- 45°**) *= cos 0° = 1*

q = -1 ✓

. . . *g is y = sin x inverted. Note: y = -sin 90° = -1*

6.2 $x_B = 180° - 22{,}5° = 157{,}5°$

& $y_B = -0{,}38$

∴ **B(157,5°; -0,38)** ✓

6.3 $f(x) - g(x) < 0$

➡ $f(x) < g(x)$

(i.e. the values of x for which f is below g)

$\mathbf{-180° \leq x < -22{,}5°}$ or $\mathbf{157{,}5° < x \leq 180°}$ ✓

6.4.1 $h(x) = \cos(x - 45° + 30°)$

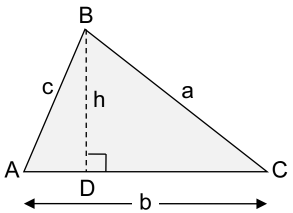
∴ $\mathbf{h(x) = \cos(x - 15°)}$ ✓

6.4.2 f has a minimum at $x = -135°$

∴ **h has a minimum at $x = -165°$** ✓ . . . *30° left of -135°*

7.1 Construction:

Draw BD ⊥ AC



Proof:

In ΔBAD: $\frac{h}{c} = \sin A$ & In ΔBCD: $\frac{h}{a} = \sin C$

∴ $h = c \sin A$ ∴ $h = a \sin C$

∴ $c \sin A = a \sin C$

÷ ac ∴ $\mathbf{\frac{\sin A}{a} = \frac{\sin C}{c}}$ ✓

7.2.1 $\frac{\sin R}{27{,}2} = \frac{\sin 132°}{73{,}2}$

∴ $\sin R = \frac{27{,}2 \sin 132°}{73{,}2}$

$= 0{,}276...$

∴ $\mathbf{\hat{R} = 16{,}03°}$ ✓

The sine rule says: The sine of an ∠ divided by the side opposite it, equals the sine of either of the two ∠s divided by the side opposite it. (OR, in inverted form.)

7.2.2 The area of a Δ = $\frac{1}{2}$ the product of 2 sides × the sine of the included ∠.

$\hat{Q} = 180° - (132° + 16{,}03°)$ . . . Area = $\frac{1}{2}$rp sin Q ; So, we need $\hat{Q}$.

$= 31{,}97°$

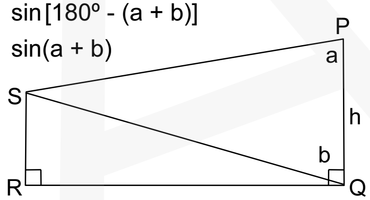
∴ Area of ΔPQR $= \frac{1}{2}(27{,}2)(73{,}2) \sin 31{,}97°$

$= \mathbf{527{,}10\ cm^2}$ ✓

7.3.1 In ΔPSQ: $P\hat{S}Q = 180° - (a + b)$

∴ $\sin P\hat{S}Q = \sin[180° - (a + b)]$

$= \sin(a + b)$



& $\frac{SQ}{\sin a} = \frac{h}{\sin P\hat{S}Q}$

∴ $\mathbf{SQ = \frac{h \sin a}{\sin(a + b)}}$ ✓ . . . ❶

7.3.2 In ΔSRQ: $S\hat{Q}R = 90° - b$

∴ $\sin S\hat{Q}R = \sin(90° - b)$

$= \cos b$

& $\frac{RS}{SQ} = \sin S\hat{Q}R$

× SQ) ∴ $RS = SQ \cos b$ . . . ❷

Substitute ❶ in ❷: ∴ $RS = \frac{h \sin a}{\sin(a + b)} . \cos b$

∴ $\mathbf{RS = \frac{h \sin a . \cos b}{\sin(a + b)}}$ ✓

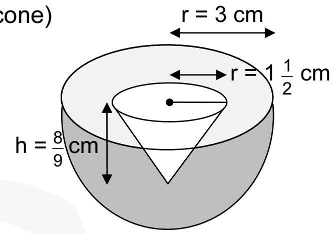
▸ MEASUREMENT [6]

8. Volume of metal B (the cone)

$= \frac{1}{3}\pi r^2 . h$

$= \frac{1}{3}\pi(1{,}5)^2 . \frac{8}{9}$

$= \frac{2}{3}\pi$



Volume of the hemisphere $= \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$= \frac{2\pi}{3} . 3^3$

$= 18\pi$

∴ Volume of metal A $= 18\pi - \frac{2}{3}\pi = 17\frac{1}{3}\pi$

∴ The ratio:

Volume of metal A : Volume of metal B

$= 17\frac{1}{3}\pi : \frac{2}{3}\pi$

× 3) $= 52\pi : 2\pi$

÷ 2π) $=$ **26 : 1** ✓

▸ EUCLIDEAN GEOMETRY [40]

9.1 . . . **bisects the chord** ✓

9.2.1 $OE = OD = \frac{1}{2}(20) = 10$ cm . . . *radii = $\frac{1}{2}$ diameter*

∴ **OC = 8 cm** ✓ . . . *CE = 2 cm*

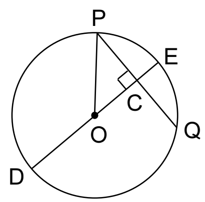
9.2.2 In ΔOPC:

$PC^2 = OP^2 - OC^2$ . . . *Pythagoras*

$= 10^2 - 8^2$

$= 36$

∴ PC = 6 cm



∴ **PQ = 12 cm** ✓ . . . *OC ⊥ chord PQ bisects PQ*

10.1 Construction: Join DO and produce it to C

Proof:

Let $\hat{D}_1 = x$

then $\hat{A} = x$ . . .

$\therefore \hat{O}_1 = 2x$

. . . *ext. ∠ of ΔDAO*

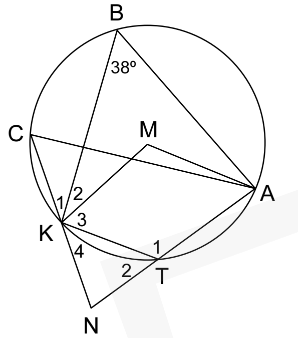
Similarly, $\hat{O}_2 = 2y$

$\therefore A\hat{O}B = 2x + 2y$

$= 2(x + y)$

$=$ **$2\,A\hat{D}B$** ◀

10.2



10.2.1 (a) $K\hat{M}A = 2(38º)$ . . . *∠ at centre = 2 × ∠ at circumference*

$=$ **76º** ◀

(b) $\hat{T}_2 =$ **38º** ◀ . . . *ext. ∠ of cyclic quad. BKTA*

(c) $\hat{C} =$ **38º** ◀ . . . *same segment; arc KA subtends or, ext. ∠ of cyclic quad. CKTA*

(d) $N\hat{A}C = 38º$ . . . *base ∠s of isos. Δ, NA = NC*

$\therefore \hat{K}_4 =$ **38º** ◀ . . . *ext. ∠ of c.q. CKTA*

10.2.2 In ΔNKT: $\hat{K}_4 = \hat{T}_2$ . . . *both = 38º in 10.2.1*

∴ **NK = NT** ◀ . . . *equal base ∠s*

10.2.3 $K\hat{M}A = 2(38º)$ . . . *see 10.2.1(a)*

& $\hat{N} = 180º - 2(38º)$ . . . *sum of ∠s of ΔNKT (see 10.2.2)*

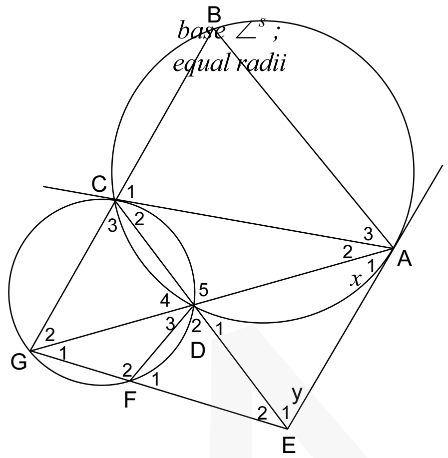
$\therefore K\hat{M}A + \hat{N} = 180º$

∴ **AMKN is a cyclic quadrilateral** ◀

. . . *opposite ∠s supplementary*

11.1 . . . **equal to the angle subtended by the chord in the alternate segment.** ◀

11.2



11.2.1 $\hat{A}_1 = x$ . . . *given*

$\therefore \hat{C}_2 = x$ . . . *tangent EA; chord AD*

$\therefore \hat{G}_2 = x$ . . . *tangent AC; chord CD*

$\therefore \hat{A}_1 =$ alternate $\hat{G}_2$

∴ **BCG || AE** ◀

11.2.2 $\hat{F}_1 = \hat{C}_3$ . . . *ext. ∠ of cyclic quad. CGFD*

$= \hat{E}_1$ (= y) . . . *alternate ∠s ; BCG || AE*

∴ **AE is a tangent to ⊙FED** ◀

. . . *converse of tan-chord theorem*

11.2.3 $\hat{C}_1 = C\hat{A}E$ . . . *alternate ∠s ; BCG || AE*

$= \hat{B}$ . . . *tangent EA ; chord AC*

∴ **AB = AC** ◀ . . . *equal base ∠s in ΔABC*

GRADE 12 EXEMPLAR PAPER 2 MEMO

STATISTICS [21]

1.1 **The more the number of days of training, the less the time taken to run the event** ✓

OR: As the number of days of training increased, so the time taken to run the event decreased. ✓

OR: The fewer the days of training, the longer the time taken to run the event. ✓

1.2 **(60; 18,1)** ✓

1.3 Equation of the regression line: $y = A + Bx$

where $A = 17{,}8193\ldots$ & $B = -0{,}0706\ldots$ *(Calculator)*

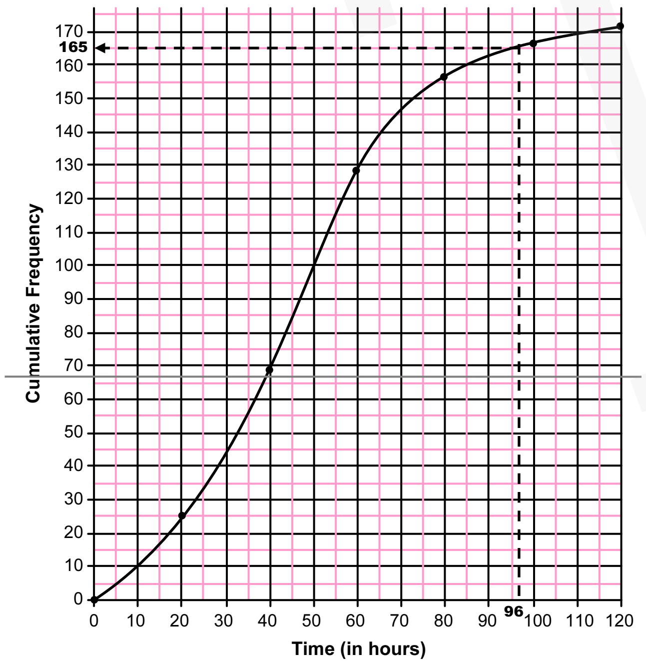
$\therefore$ **The equation: $y = 17{,}82 - 0{,}07x$** ✓

1.4 Time taken $= 17{,}82 - 0{,}07(45) =$ **14,67 seconds** ✓

1.5 The correlation coefficient, **$r \simeq -0{,}74$** ✓

1.6 **The relationship between variables is moderately strong.** ✓

2.1 **Ogive (Cumulative Frequency Curve)**



2.2 **$40 \le t < 60$** ✓ . . . *the steepest curve over this interval indicates the biggest number of learners*

2.3 80% of the time $=$ 80% of 120 h $=$ 96 h

The number of learners who spent ≤ 96 h is 165 . . . *see graph*

$\therefore$ The number of learners who spent > 96 h is $172 - 165 =$ **7** ✓

2.4

Time (hours)	Cumulative frequency	Frequency per interval
$0 \le t < 20$	25	25
$20 \le t < 40$	69	44
$40 \le t < 60$	129	60
$60 \le t < 80$	157	28
$80 \le t < 100$	166	9
$100 \le t < 120$	172	6

The mean time

$$= \frac{25\times10 + 44\times30 + 60\times50 + 28\times70 + 9\times90 + 6\times110}{172}$$

$$= \frac{8\,000}{172} \simeq \textbf{46,51 hours}$$ ✓ *(or, by calculator)*

ANALYTICAL GEOMETRY [37]

3.1 **K(7; 0)** ✓

3.2 **M(-5; -1)** ✓ . . . *Q midpoint of MP*

3.3 $m_{PM} = \frac{3-1}{7-1} = \frac{2}{6} = \mathbf{\frac{1}{3}}$ ✓

3.4 $\tan P\hat{S}K = m_{PM} = \frac{1}{3} \Rightarrow P\hat{S}K = 18{,}43°$

$\therefore$ **$\theta = 71{,}57°$** ✓ . . . *$\angle^s$ of ΔPSK*

3.5 In ΔPSK: $\cos\theta = \frac{PK}{PS}$

$\therefore \cos 71{,}57° = \frac{3}{PS}$

$\therefore$ **PS** $= \frac{3}{\cos 71{,}57°}$ . . . $a = \frac{k}{b} \Rightarrow b = \frac{k}{a}$

$\simeq$ **9,49 units** ✓

OR: $\sin 18{,}43° = \frac{3}{PS}$, etc.

3.6 $N(x; y)$ on the line $y = -2x + 17$

$\Rightarrow$ Point N is $(x; -2x + 17)$

$m_{NT} = m_{PM}$ . . . *NT || PM in trapezium*

$\therefore \frac{-2x + 17 - 5}{x - (-1)} = \frac{1}{3}$

$\therefore \frac{-2x + 12}{x + 1} = \frac{1}{3}$

$\therefore -6x + 36 = x + 1$

$\therefore -7x = -35$

$\therefore x = 5$ & $y = -2(5) + 17 = 7$

$\therefore$ **N(5; 7)** ✓

OR: Find the equation of TN:

Substitute $m = \frac{1}{3}$ and $(-1; 5)$ in

$y - y_1 = m(x - x_1)$ OR $y = mx + c$

Equation is $y = \frac{1}{3}x + 5\frac{1}{3}$.

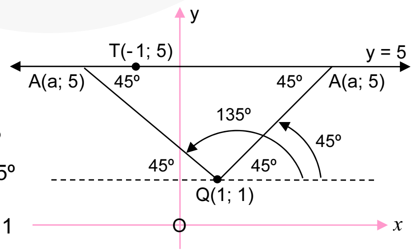
N is the point of intersection of TN and NP

$\therefore$ Solve the equations.

3.7.1 The equation:

y = 5 ✓

3.7.2 The gradient of AQ = tan 45° or tan 135°



$\therefore \frac{5-1}{a-1} = 1$ or -1

$\therefore \frac{4}{a-1} = \pm 1$

$\therefore a - 1 = \pm 4$ $\therefore$ **a = 5** or **-3** ✓

4.1 **M(-1; -1)** ✓

4.2 NT $\perp$ AT . . . *tangent $\perp$ radius*

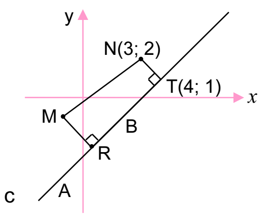
$m_{NT} = \frac{1-2}{4-3} = -1 \Rightarrow m_{AT} = 1$

Substitute $m = 1$ and $T(4; 1)$ in

$y - y_1 = m(x - x_1)$ or $y = mx + c$

$\therefore y - 1 = 1(x - 4)$ $\therefore 1 = (1)(4) + c$, etc.

$\therefore$ **$y = x - 3$** ✓



4.3 MR ⊥ AB . . . *MR is the line from the centre to the midpoint of chord AB*

$\therefore$ In ΔMRA: $AR^2 = MA^2 - MR^2$ . . . *Theorem of Pythagoras*

$= 9 - \left(\frac{\sqrt{10}}{2}\right)^2$ . . . $r^2 = 9$

$= 9 - \frac{10}{4}$

$= \frac{13}{2}$

$\therefore$ AR $= \sqrt{\frac{13}{2}}$

$\therefore$ AB $= 2\sqrt{\frac{13}{2}}$ . . . *AB = 2AR*

$= \sqrt{26}$ **units** ◄ . . . $\sqrt{4}\sqrt{\frac{13}{2}} = \sqrt{4 \times \frac{13}{2}}$

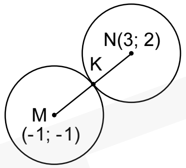
4.4 $MN^2 = (-1-3)^2 + (-1-2)^2 = 25$

$\therefore$ **MN = 5 units** ◄

4.5 MN = 5 units . . . *in 4.4*

& MK = 3 units . . . *radius of ⊙M*

$\therefore$ KN = 2 units



$\therefore$ Equation of 'new' ⊙N:

$(x-3)^2 + (y-2)^2 = 2^2$

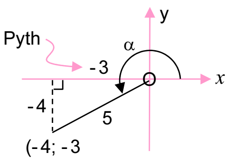
$\therefore x^2 - 6x + 9 + y^2 - 4y + 4 = 4$

$\therefore$ $\boldsymbol{x^2 + y^2 - 6x - 4y + 9 = 0}$ ◄

▸ TRIGONOMETRY [41]

5.1 $\sin\alpha = -\frac{4}{5}$ & $90° < \alpha < 270°$

$\therefore$ α is in the 3rd Quadrant



5.1.1 $\sin(-\alpha) = -\sin\alpha = -\left(-\frac{4}{5}\right)$

$= \frac{4}{5}$ ◄

5.1.2 $\cos\alpha = \frac{-3}{5} = -\frac{3}{5}$ ◄

5.1.3 $\sin(\alpha - 45°) = \sin\alpha\cos 45° - \cos\alpha\sin 45°$

$= \left(-\frac{4}{5}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(-\frac{3}{5}\right)\left(\frac{1}{\sqrt{2}}\right)$

$= -\frac{4}{5\sqrt{2}} + \frac{3}{5\sqrt{2}}$

$= -\frac{4}{5\sqrt{2}}\left(\times\frac{\sqrt{2}}{\sqrt{2}}\right)$

$= -\frac{\sqrt{2}}{10}$ ◄

5.2.1 **LHS** $= \frac{8\sin x \cdot \cos x}{\sin^2 x - \cos^2 x}$

$= \frac{4 \cdot 2\sin x\cos x}{-(\cos^2 x - \sin^2 x)}$

$= \frac{4 \cdot 2\sin x}{-\cos 2x}$

$= -4\tan 2x$

= **RHS** ◄

5.2.2 It will be undefined if $\cos 2x = 0$

$\therefore$ when $2x = 90° + n(180°)$

$\therefore x = 45° + n(90°)$

$\therefore$ $\boldsymbol{x = 45°}$ OR **135°** ◄

OR: When $\tan 2x$ is undefined. Same solution.

5.3 $(1 - 2\sin^2\theta) + 4\sin^2\theta - 5\sin\theta - 4 = 0$

$\therefore 2\sin^2\theta - 5\sin\theta - 3 = 0$

$\therefore (2\sin\theta + 1)(\sin\theta - 3) = 0$

$\therefore \sin\theta = -\frac{1}{2}$ . . . *sin θ ≠ 3 ∵ sin θ can only have values between -1 and 1*

$\therefore \theta = 210° + n(360°)$

or $\boldsymbol{\theta = 330° + n(360°),\ n \in \mathbb{Z}}$ ◄

6.1 $\mathbf{b = \frac{1}{2}}$ ◄ . . . $\tan\frac{1}{2}(90°) = \tan 45° = 1$ . . . *see pt. P*

6.2 **A(30; 1)** . . . $\cos(\mathbf{30°} - 30°) = \cos 0° = \mathbf{1}$

6.3 The asymptotes of f: $x = -180°$ and $x = 180°$

$\therefore$ **The required asymptote is** $\boldsymbol{x = 160°}$ ◄ . . . The asymptotes move 20° to the left.

Note that $x = -200°$ falls outside the domain

6.4 $-1 \leq g(x) \leq 1$ . . . *The range of g*

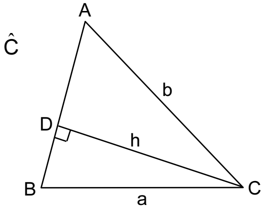
×2) $\therefore -2 \leq 2g(x) \leq 2$

+1 $\therefore -1 \leq 2g(x) + 1 \leq 3$

$\therefore$ **The range of h: $\mathbf{-1 \leq y \leq 3}$** ◄

7.1 Construction:

Draw the altitude h or CD from $\hat{C}$ *(the angle not involved in the formula)*



Proof:

In ΔADC: $\frac{h}{b} = \sin A$

$\therefore h = b\sin A$ . . . ❶

& In ΔBDC: $\frac{h}{a} = \sin B$

$\therefore h = a\sin B$ . . . ❷

From ❶ & ❷: $b\sin A = a\sin B$ . . . *both equal h*

÷ab) $\therefore \frac{\sin A}{a} = \frac{\sin B}{b}$ ◄

7.2.1 $S\hat{P}Q = 180° - 2x$ . . . *opposite ∠s of c.q.*

$\therefore P\hat{S}Q + P\hat{Q}S = 2x$ . . . *sum of ∠s in Δ*

$\therefore$ $\boldsymbol{P\hat{S}Q = P\hat{Q}S = x}$ ◄ . . . *∠s opposite equal sides*

7.2.2 In ΔSPQ: $\frac{SQ}{\sin(180° - 2x)} = \frac{h}{\sin x}$

$\therefore SQ = \frac{k\sin 2x}{\sin x}$ . . . $\sin(180° - 2x) = \sin 2x$

$= \frac{k \cdot 2\sin x\cos x}{\sin x}$

OR: Could use cosine rule

$= \mathbf{2k\cos x}$ ◄ . . . ❶

7.2.3 In ΔTPQ: $\frac{3}{PQ} = \tan y$

$\therefore \frac{k}{3} = \frac{1}{\tan y}$ . . . *k = PQ*

×3) $\therefore k = \frac{3}{\tan y}$ . . . ❷

❷ in ❶: $\therefore SQ = 2 \cdot \frac{3}{\tan y} \cdot \cos x$

$= \frac{\mathbf{6\cos x}}{\mathbf{\tan x}}$ ◄

EUCLIDEAN GEOMETRY AND MEASUREMENT [51]

8.1 . . . the angle subtended by the chord in the alternate segment.

8.2.1 $\hat{E}_1 = \hat{B}_1$ . . . *tan-chord theorem*
$= \mathbf{68^\circ}$ ➤

8.2.2 $\hat{B}_3 = \hat{E}_1$ . . . *alt. ∠ˢ ; AE || BC*
$= \mathbf{68^\circ}$ ➤

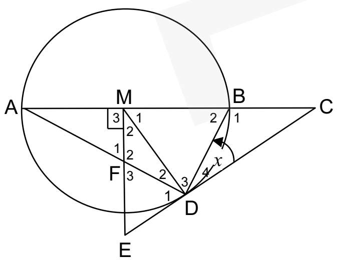
8.2.3 $\hat{D}_1 = \hat{B}_3$ . . . *ext. ∠ of cyclic quad.*
$= \mathbf{68^\circ}$ ➤

8.2.4 $\hat{E}_2 = \hat{D}_1 + 20^\circ$ . . . *ext. ∠ of Δ*
$= \mathbf{88^\circ}$ ➤

8.2.5 $\hat{C} = 180^\circ - \hat{E}_2$ . . . *opp. ∠ˢ of cyclic quad.*
$= \mathbf{92^\circ}$ ➤

9.1 $\hat{A} = x$ . . . *tan-chord theorem*
$\hat{D}_2 = x$ . . . *∠ˢ opp. equal sides*

9.2



$\hat{M}_1 = \hat{A} + \hat{D}_2$ . . . *ext. ∠ of Δ*
$= 2x$
$\therefore \hat{M}_2 = 90^\circ - 2x$ . . . *ME ⊥ AC*
& $M\hat{D}E = 90^\circ$ . . . *radius MD ⊥ tangent CDE*
$\therefore \hat{E} = 2x$ . . . *sum of ∠ˢ of ΔMED*
$\therefore \hat{M}_1 = \hat{E}$
∴ **CM is a tangent at M to ⊙MED** ➤

9.3 $A\hat{D}B = 90^\circ$ . . . *∠ in semi-⊙*
& $\hat{M}_3 = 90^\circ$ . . . *ME ⊥ AC*
$\therefore \hat{M}_3 = A\hat{D}B$
∴ **FMBD is a cyclic quad.** ➤ . . . *ext. ∠ = int. opp. ∠*

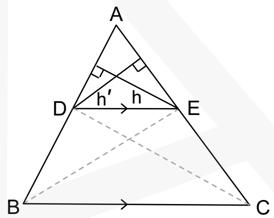
9.4 Let BC = a ; then MB = 2a
∴ MD = 2a . . . *radii*

In ΔMDC: $M\hat{D}C = 90^\circ$ . . . *radius ⊥ tangent*
$\therefore DC^2 = MC^2 - MD^2$
$= (3a)^2 - (2a)^2$
$= 9a^2 - 4a^2$
$= 5a^2$
$= \mathbf{5BC^2}$ ➤

9.5 In Δˢ DBC and DFM
(1) $\hat{B}_1 = \hat{F}_2$ . . . *ext. ∠ of c.q. FMBD = int. opp. ∠*
(2) $\hat{D}_4 = \hat{D}_2$ . . . *both = x*
∴ **ΔDBC ||| ΔDFM** ➤ . . . ∠∠∠

9.6 $\therefore \frac{DM}{FM} = \frac{DC}{BC}$ . . . *proportional sides*
$= \frac{\sqrt{5}\,BC}{BC}$ . . . *see 9.4*
$= \mathbf{\sqrt{5}}$ ➤

10.1 **Construction:**
Join DC and EB
and heights h and h′



Proof:

$\frac{\text{area of }\Delta ADE}{\text{area of }\Delta DBE} = \frac{\frac{1}{2}AD.h}{\frac{1}{2}DB.h}$
$= \frac{AD}{DB}$ . . . *equal heights*

& $\frac{\text{area of }\Delta ADE}{\text{area of }\Delta EDC} = \frac{\frac{1}{2}AE.h'}{\frac{1}{2}EC.h'} = \frac{AE}{EC}$ . . . *equal heights*

But, area of ΔDBE = area of ΔEDC . . . *same base DE & betw. same || lines, i.e. same height*

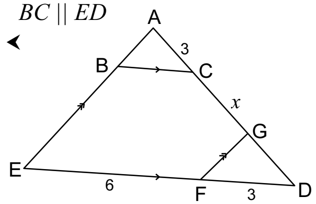
$\therefore \frac{\text{area of }\Delta ADE}{\text{area of }\Delta DBE} = \frac{\text{area of }\Delta ADE}{\text{area of }\Delta EDC}$

$\therefore \mathbf{\frac{AD}{DB} = \frac{AE}{EC}}$ ➤

10.2.1 Let AB = p ; then BE = 3p

In ΔAED: $\frac{CD}{3} = \frac{3p}{p}$ . . . *prop. thm.; BC || ED*

× 3) ∴ **CD = 9 units** ➤



10.2.2 CG = x ; so GD = 9 - x

In ΔDAE: $\frac{9-x}{x+3} = \frac{3}{6}$ . . . *prop. thm. ; AE || GF*
$\therefore 54 - 6x = 3x + 9$
$\therefore -9x = -45$
$\therefore \mathbf{x = 5}$ ➤

10.2.3 In Δˢ ABC and AED
(1) $\hat{A}$ is common
(2) $A\hat{B}C = \hat{E}$ . . . *corr. ∠ˢ; BC || ED*
∴ ΔABC ||| ΔAED . . . ∠∠∠
$\therefore \frac{BC}{ED} = \frac{AB}{AE}$ . . . *prop. sides*
$\therefore \frac{BC}{9} = \frac{p}{4p}$
× 9) ∴ **BC = $\mathbf{\frac{9}{4}}$ units** ➤

10.2.4 $\frac{\text{area of }\Delta ABC}{\text{area of }\Delta GFD} = \frac{\frac{1}{2}AC.BC\sin A\hat{C}B}{\frac{1}{2}DG.DF\sin\hat{D}}$

$= \frac{\frac{1}{2}.3.\frac{9}{4}.\sin\hat{D}}{\frac{1}{2}.4.3.\sin\hat{D}}$ . . . *corr. ∠ˢ; BC || ED*

$= \frac{\frac{9}{4}}{4}$

$= \mathbf{\frac{9}{16}}$ ➤

OR: $\frac{\text{area of }\Delta ABC}{\text{area of }\Delta AED} = \frac{\frac{1}{2}.p.3.\sin\hat{A}}{\frac{1}{2}.4p.12.\sin\hat{A}} = \frac{1}{16}$

$\therefore$ area of ΔABC $= \frac{1}{16}$ area of ΔAED . . . ❶

& $\frac{\text{area of }\Delta GFD}{\text{area of }\Delta AED} = \frac{\frac{1}{2}.4.3.\sin\hat{D}}{\frac{1}{2}.12.9.\sin\hat{D}} = \frac{1}{9}$

$\therefore$ area of ΔGFD $= \frac{1}{9}$ area of ΔAED . . . ❷

❶ ÷ ❷: $\therefore \frac{\text{area of }\Delta ABC}{\text{area of }\Delta GFD} = \frac{\frac{1}{16}\text{ area of }\Delta AED}{\frac{1}{9}\text{ area of }\Delta AED}$

$= \mathbf{\frac{9}{16}}$ ➤