Gr 10, Gr 11 & Gr 12 Mathematics

EXEMPLAR
PAPER 1s

(memos follow)

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GRADE 10 EXEMPLAR PAPER 1

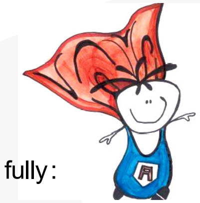
Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.

Answers only will NOT necessarily be awarded full marks.

You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

*If necessary, round off answers to **TWO** decimal places, unless stated otherwise.*

▶ ALGEBRA [32]



QUESTION 1

1.1 Simplify the following expressions fully:

1.1.1 $(m - 2n)(m^2 - 6mn - n^2)$ (3)

1.1.2 $\dfrac{x^3 + 1}{x^2 - x + 1} - \dfrac{4x^2 - 3x - 1}{4x + 1}$ (5)

1.2 Factorise the following expressions fully:

1.2.1 $6x^2 - 7x - 20$ (2)

1.2.2 $a^2 + a - 2ab - 2b$ (3)

1.3 Determine, **without the use of a calculator**, between which two consecutive integers $\sqrt{51}$ lies. (2)

1.4 Prove that $0{,}\dot{2}\dot{4}\dot{5}$ is rational. (4) [19]

QUESTION 2

2.1 Determine, **without the use of a calculator**, the value of x in each of the following:

2.1.1 $x^2 - 4x = 21$ (3)

2.1.2 $96 = 3x^{\frac{5}{4}}$ (3)

2.1.3 $R = \dfrac{2\sqrt{x}}{3S}$ (2)

2.2 Solve for p and q simultaneously if:

$6q + 7p = 3$
$2q + p = 5$ (5) [13]

▶ NUMBERS & NUMBER PATTERNS [11]

QUESTION 3

3.1 $3x + 1$; $2x$; $3x - 7$ ….. are the first three terms of a linear number pattern.

3.1.1 If the value of x is three, write down the FIRST THREE terms. (3)

3.1.2 Determine the formula for T_n, the general term of the sequence. (2)

3.1.3 Which term in the sequence is the first to be less than -31? (3)

3.2 The multiples of three form the number pattern: 3 ; 6 ; 9 ; 12 ; . . .

Determine the 13th number in this pattern that is even. (3) [11]

▶ FINANCE & GROWTH [14]

QUESTION 4

4.1 Thando has R4 500 in his savings account. The bank pays him a compound interest rate of 4,25% p.a. Calculate the amount Thando will receive if he decides to withdraw the money after 30 months. (3)

4.2 The following advertisement appeared with regard to buying a bicycle on a hire-purchase agreement loan:

Purchase price	*R5 999*
Required deposit	*R600*
Loan term	*Only 18 months, at 8% p.a. simple interest*

4.2.1 Calculate the monthly amount that a person has to budget for in order to pay for the bicycle. (6)

4.2.2 How much interest does one have to pay over the full term of the loan? (1)

4.3 The following information is given:

1 ounce = 28,35 g
\$1 = R8,79

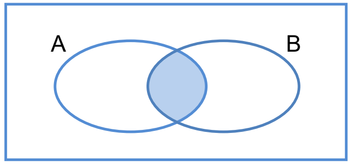
Calculate the rand value of a 1 kg gold bar, if 1 ounce of gold is worth \$978,34. (4) [14]

▶ PROBABILITY [13]

QUESTION 5

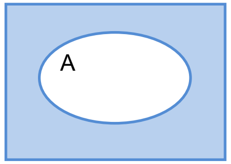
5.1 What expression BEST represents the shaded area of the following Venn diagrams?

5.1.1



(1)

5.1.2



(1)

5.2 State which of the following sets of events is mutually exclusive:

A Event 1: The learners in Grade 10 in the swimming team

Event 2: The learners in Grade 10 in the debating team

B Event 1: The learners in Grade 8

Event 2: The learners in Grade 12

C Event 1: The learners who take Mathematics in Grade 10

Event 2: The learners who take Physical Sciences in Grade 10 (1)

5.3 In a class of 40 learners the following information is TRUE:

- 7 learners are left-handed
- 18 learners play soccer
- 4 learners play soccer and are left-handed
- All 40 learners are either right-handed or left-handed

Let L be the set of all left-handed people and S be the set of all learners who play soccer.

5.3.1 How many learners in the class are right-handed and do NOT play soccer? (1)

5.3.2 Draw a Venn diagram to represent the above information. (4)

5.3.3 Determine the probability that a learner is:

(a) left-handed or plays soccer (3)

(b) right-handed and plays soccer (2) [13]

▶ FUNCTIONS & GRAPHS [30]

QUESTION 6

Given: $f(x) = \frac{3}{x} + 1$ and $g(x) = -2x - 4$

6.1 Sketch the graphs of f and g on the same set of axes. (4)

6.2 Write down the equations of the asymptotes of f. (2)

6.3 Write down the domain of f. (2)

6.4 Solve for x if $f(x) = g(x)$. (5)

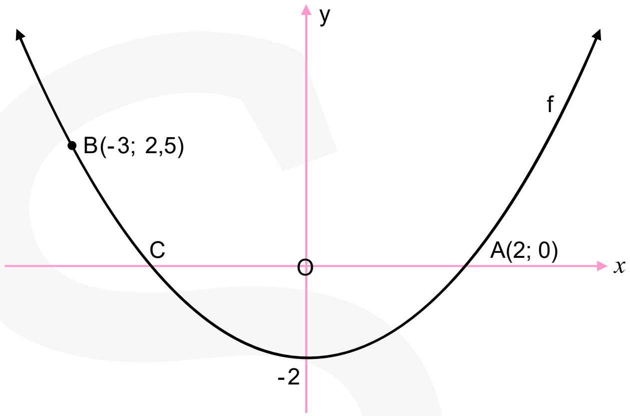
6.5 Determine the values of x for which $-1 \leq g(x) < 3$. (3)

6.6 Determine the y-intercept of k if $k(x) = 2g(x)$ (2)

6.7 Write down the coordinates of the x- and y-intercepts of h if h is the graph of g reflected about the y-axis. (2) [20]

QUESTION 7

The graph of $f(x) = ax^2 + q$ is sketched below.
Points A(2; 0) and B(-3; 2,5) lie on the graph of f.
Points A and C are x-intercepts of f.



7.1 Write down the coordinates of C. (1)

7.2 Determine the equation of f. (3)

7.3 Write down the range of f. (1)

7.4 Write down the range of h, where $h(x) = -f(x) - 2$. (2)

7.5 Determine the equation of an exponential function, $g(x) = b^x + q$, with range $y > -4$ and which passes through the point A. (3) [10]

TOTAL: 100

GRADE 11 EXEMPLAR PAPER 1

You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

*If necessary, round off answers to **TWO** decimal places, unless stated otherwise.*

ALGEBRA AND EQUATIONS AND INEQUALITIES [47]

QUESTION 1

1.1 Solve for x:

1.1.1 $(2x - 1)(x + 5) = 0$ (2)

1.1.2 $2x^2 - 4x + 1 = 0$ (Leave your answer in simplest surd form.) (3)

1.2 Simplify, without the use of a calculator, the following expressions fully:

1.2.1 $125^{\frac{2}{3}}$ (2)

1.2.2 $(3\sqrt{2} - 12)(2\sqrt{2} + 1)$ (3)

1.3 Given: $\dfrac{x^2 - x - 6}{3x - 9}$

1.3.1 For which value(s) of x will the expression be undefined? (2)

1.3.2 Simplify the expression fully. (3) [15]

Algebra includes exponents & surds

QUESTION 2

2.1 Given: $(x + 2)(x - 3) < -3x + 2$

2.1.1 Solve for x if: $(x + 2)(x - 3) < -3x + 2$ (4)

2.1.2 Hence or otherwise, determine the sum of all the integers satisfying the inequality $x^2 + 2x - 8 < 0$. (3)

2.2 Given: $\dfrac{4^{x-1} + 4^{x+1}}{17.12^x}$

2.2.1 Simplify the expression fully. (4)

2.2.2 If $3^{-x} = 4t$, express $\dfrac{4^{x-1} + 4^{x+1}}{17.12^x}$ in terms of t. (1)

2.3 Solve for x and y from the given equations:

$3^y = 81^x$ and $y = x^2 - 6x + 9$ (7) [19]

QUESTION 3

3.1 The solution to a quadratic equation is

$x = \dfrac{3 \pm \sqrt{4 - 8p}}{4}$ where $p \in Q$.

Determine the value(s) of p such that:

3.1.1 The roots of the equation are equal. (2)

3.1.2 The roots of the equation are non-real. (2)

3.2 Given: $\sqrt{5 - x} = x + 1$

3.2.1 Without solving the equation, show that the solution to the above equation lies in the interval $-1 \leq x \leq 5$. (3)

3.2.2 Solve the equation. (5)

3.2.3 Without any further calculations, solve the equation $-\sqrt{5 - x} = x + 1$. (1) [13]

FINANCE, GROWTH AND DECAY [18]

QUESTION 4

4.1 Melissa has just bought her first car. She paid R145 000 for it. The car's value depreciates on the straight-line method at a rate of 17% per annum. Calculate the value of Melissa's car 5 years after she bought it. (2)

4.2 An investment earns interest at a rate of 8% per annum compounded quarterly.

4.2.1 At what rate is interest earned each quarter of the year? (1)

4.2.2 Calculate the effective annual interest rate on this investment. (2)

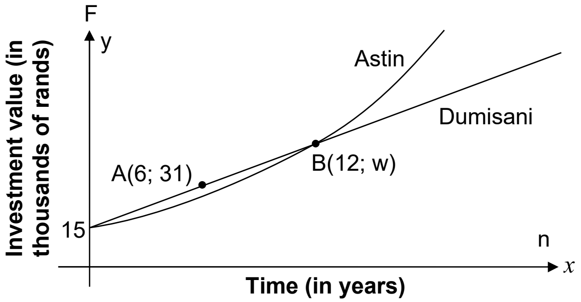
4.3 R14 000 is invested in an account.

The account earns interest at a rate of 9% per annum compounded semi-annually for the first 18 months and thereafter 7,5% per annum compounded monthly.

How much money will be in the account exactly 5 years after the initial deposit? (5) [10]

QUESTION 5

The graphs below represent the growth of two investments, one belonging to Dumisani and one belonging to Astin. Both investments earn interest annually (only).



5.1 What is the value of both initial investments? (1)

5.2 Does Dumisani's investment earn simple or compound interest? (1)

5.3 Determine Dumisani's interest rate. (2)

5.4 Hence or otherwise, calculate the interest rate on Astin's investment. Give your answer correct to ONE decimal place. (4) [8]

▸ PATTERNS AND SEQUENCES [23]

QUESTION 6

6.1 Given: $\frac{1}{2}; \frac{1}{4}; \frac{1}{8}; \ . \ . \ . \ ; \frac{1}{1\ 024}$

6.1.1 Explain how you will determine the 4th term of the sequence. (2)

6.1.2 Write a formula for the nth term of the sequence. (2)

6.1.3 Determine the number of terms in the sequence. (2)

6.2 Given the linear pattern: 156 ; 148 ; 140 ; 132 ; . . .

6.2.1 Write down the 5th term of this number pattern. (1)

6.2.2 Determine a general formula for the nth term of this pattern. (2)

6.2.3 Which term of this linear number pattern is the first term to be negative? (3)

6.2.4 The given linear number pattern forms the sequence of first differences of a quadratic number pattern $T_n = an^2 + bn + c$ with $T_5 = -24$. Determine a general formula for T_n. (5) [17]

Higher order

QUESTION 7

A question asked differently

A quadratic pattern $T_n = an^2 + bn + c$ has $T_2 = T_4 = 0$ and a second difference of 12.

Determine the value of the 3rd term of the pattern. [6]

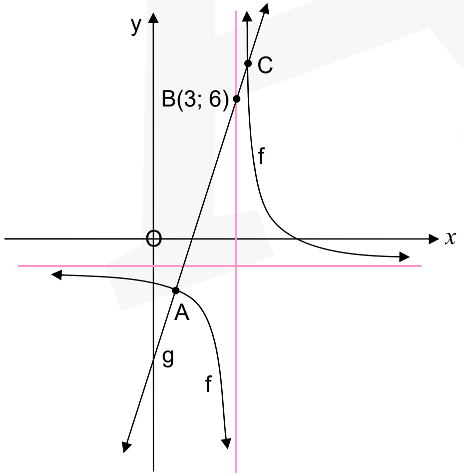
▸ FUNCTIONS AND GRAPHS [43]

QUESTION 8

The sketch below represents the graphs of

$f(x) = \frac{2}{x - 3} - 1$ and $g(x) = dx + e$.

Point B(3; 6) lies on the graph of g and the two graphs intersect at points A and C.



8.1 Write down the equations of the asymptotes of f. (2)

8.2 Write down the domain of f. (2)

8.3 Determine the values of d and e, correct to the nearest integer, if the graph of g makes an angle of 76° with the x-axis. (3)

8.4 Determine the coordinates of A and C. (6)

8.5 For what values of x is $g(x) \geq f(x)$? (3)

8.6 Determine an equation for the axis of symmetry of f which has a positive slope. (3) [19]

QUESTION 9

Given: $f(x) = -x^2 + 2x + 3$ and $g(x) = 1 - 2^x$

9.1 Sketch the graphs of f and g on the same set of axes. (9)

9.2 Determine the average gradient of f between $x = -3$ and $x = 0$. (3)

9.3 For which value(s) of x is $f(x).g(x) \geq 0$? (3)

9.4 Determine the value of c such that the x-axis will be a tangent to the graph of h, where $h(x) = f(x) + c$. (2)

9.5 Determine the y-intercept of t if $t(x) = -g(x) + 1$. (2)

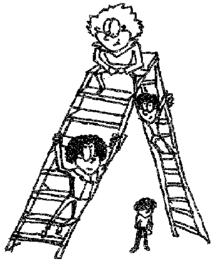
9.6 The graph of k is a reflection of g about the y-axis. Write down the equation of k. (1) [20]

QUESTION 10

Also asked differently

Sketch the graph of $f(x) = ax^2 + bx + c$ if it is also given that:

- the range of f is $(-\infty; 7]$
- $a \neq 0$
- $b < 0$
- one root of f is positive and the other root of f is negative. [4]



PROBABILITY [19]

QUESTION 11

Given: $P(W) = 0,4$ $P(T) = 0,35$ $P(T \text{ and } W) = 0,14$

11.1 Are the events W and T mutually exclusive? Give reasons for your answer. (2)

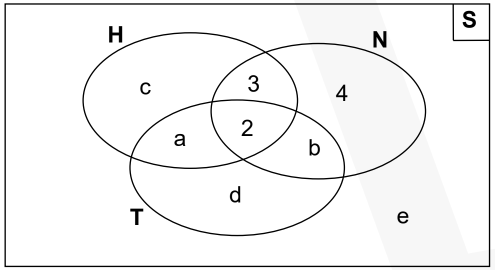
11.2 Are the events W and T independent? Give reasons for your answer. (3) [5]

QUESTION 12

12.1 A group of 33 learners was surveyed at a school. The following information from the survey is given:

- 2 learners play tennis, hockey and netball
- 5 learners play hockey and netball
- 7 learners play hockey and tennis
- 6 learners play tennis and netball
- A total of 18 learners play hockey
- A total of 12 learners play tennis
- 4 learners play netball ONLY

12.1.1 A Venn diagram representing the survey results is given below. Use the information provided to determine the values of a, b, c, d and e. (5)



12.1.2 How many of these learners do not play any of the sports on the survey (that is netball, tennis or hockey)? (1)

12.1.3 Write down the probability that a learner selected at random from this sample plays netball ONLY. (1)

12.1.4 Determine the probability that a learner selected at random from this sample plays hockey or netball. (1)

12.2 In all South African schools, EVERY learner must choose to do either Mathematics or Mathematical Literacy.

At a certain South African school, it is known that 60% of the learners are girls. The probability that a randomly chosen girl at the school does Mathematical Literacy is 55%. The probability that a randomly chosen boy at the school does Mathematical Literacy is 65%.

Determine the probability that a learner selected at random from this school does Mathematics. (6) [14]

TOTAL: 150

NATIONAL GRADE 11 EXAMINATIONS

Recommended weighting for Paper 1 & Paper 2

Description	Grade 11
PAPER 1	
Algebra and Equations (and inequalities)	**45 ± 3**
Patterns and Sequences	**25 ± 3**
Finance, Growth and Decay	**15 ± 3**
Functions and Graphs	**45 ± 3**
Probability	**20 ± 3**
TOTAL	**150**
PAPER 2: Theorems and/or trigonometric proofs: maximum 12 marks	
Statistics	**20 ± 3**
Analytical Geometry	**30 ± 3**
Trigonometry	**50 ± 3**
Euclidian Geometry and Measurement	**50 ± 3**
TOTAL	**150**

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GRADE 12 EXEMPLAR PAPER 1

You may use an approved scientific calculator
(non-programmable and non-graphical),
unless stated otherwise.

*If necessary, round off answers to **TWO** decimal places,*
unless stated otherwise.

ALGEBRA AND EQUATIONS AND INEQUALITIES [23]

QUESTION 1

1.1 Solve for x:

1.1.1 $3x^2 - 4x = 0$ (2)

1.1.2 $x - 6 + \frac{2}{x} = 0;\ x \neq 0.$ (Leave your answer correct to TWO decimal places.) (4)

1.1.3 $x^{\frac{2}{3}} = 4$ (2)

1.1.4 $3^x(x - 5) < 0$ (2)

1.2 Solve for x and y simultaneously:

$y = x^2 - x - 6$ **and** $2x - y = 2$ (6)

1.3 Simplify, without the use of a calculator:

$\sqrt{3}.\sqrt{48} - \frac{4^{x+1}}{2^{2x}}$ (3)

1.4 Given: $f(x) = 3(x - 1)^2 + 5$ and $g(x) = 3$

1.4.1 Is it possible for $f(x) = g(x)$? Give a reason for your answer. (2)

1.4.2 Determine the value(s) of k for which $f(x) = g(x) + k$ has TWO unequal real roots. (2) [23]

PATTERNS AND SEQUENCES [26]

QUESTION 2

2.1 Given the arithmetic series: 18 + 24 + 30 + . . . + 300

2.1.1 Determine the number of terms in this series. (3)

2.1.2 Calculate the sum of this series. (2)

2.1.3 Calculate the sum of all the whole numbers up to and including 300 that are NOT divisible by 6. (4)

2.2 The first three terms of an infinite geometric sequence are 16, 8 and 4 respectively.

2.2.1 Determine the nth term of the sequence. (2)

2.2.2 Determine all possible values of n for which the sum of the first n terms of this sequence is greater than 31. (3)

2.2.3 Calculate the sum to infinity of this sequence. (2) [16]

QUESTION 3

3.1 A quadratic number pattern $T_n = an^2 + bn + c$ has a first term equal to 1. The general term of the first differences is given by $4n + 6$.

3.1.1 Determine the value of a. (2)

3.1.2 Determine the formula for T_n. (4)

3.2 Given the series:

$(1 \times 2) + (5 \times 6) + (9 \times 10) + (13 \times 14) + \ \ldots \ + (81 \times 82)$

Write the series in sigma notation. (It is not necessary to calculate the value of the series.) (4) [10]

FUNCTIONS AND GRAPHS [37]

QUESTION 4

4.1 Given: $f(x) = \frac{2}{x + 1} - 3$

4.1.1 Calculate the coordinates of the y-intercept of f. (2)

4.1.2 Calculate the coordinates of the x-intercept of f. (2)

4.1.3 Sketch the graph of f, showing clearly the asymptotes and the intercepts with the axes. (3)

4.1.4 One of the axes of symmetry of f is a decreasing function. Write down the equation of this axis of symmetry. (2)

4.2 The graph of an increasing exponential function with equation $f(x) = a.b^x + q$ has the following properties:

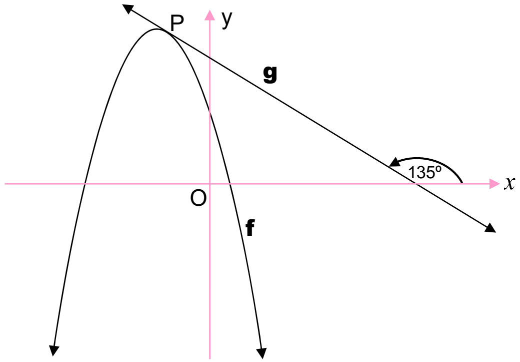
- Range: $y > -3$
- The points $(0; -2)$ and $(1; -1)$ lie on the graph of f.

4.2.1 Determine the equation that defines f. (4)

4.2.2 Describe the transformation from $f(x)$ to $h(x) = 2.2^x + 1$ (2) [15]

QUESTION 5

The sketch below shows the graphs of $f(x) = -2x^2 - 5x + 3$ and $g(x) = ax + q$. The angle of inclination of graph g is 135º in the direction of the positive x-axis. P is the point of intersection of f and g such that g is a tangent to the graph of f at P.



5.1 Calculate the coordinates of the turning point of the graph of f. (3)

5.2 Calculate the coordinates of P, the point of contact between f and g. (4)

5.3 Hence or otherwise, determine the equation of g. (2)

5.4 Determine the values of d for which the line $k(x) = -x + d$ will not intersect the graph of f. (1) [10]

QUESTION 6

The graph of g is defined by the equation $g(x) = \sqrt{ax}$.
The point (8; 4) lies on g.

6.1 Calculate the value of a. (2)

6.2 For what values of x will g be defined? (1)

6.3 Determine the range of g. (1)

6.4 Write down the equation of g^{-1}, the inverse of g, in the form y = . . . (2)

6.5 If $h(x) = x - 4$ is drawn, determine ALGEBRAICALLY the point(s) of intersection of h and g. (4)

6.6 Hence, or otherwise, determine the values of x for which $g(x) > h(x)$. (2) [12]

▶ FINANCE, GROWTH AND DECAY [16]

QUESTION 7

Siphokazi bought a house. She paid a deposit of R102 000, which is equivalent to 12% of the selling price of the house. She obtained a loan from the bank to pay the balance of the selling price. The bank charges her interest of 9% per annum, compounded monthly.

7.1 Determine the selling price of the house. (1)

7.2 The period of the loan is 20 years and she starts repaying the loan one month after it was granted. Calculate her monthly instalment. (4)

7.3 How much interest will she pay over the period of 20 years? Round your answer correct to the nearest rand. (2)

7.4 Calculate the balance of her loan immediately after her 85th instalment. (3)

7.5 She experienced financial difficulties after the 85th instalment and did not pay any instalments for 4 months (that is months 86 to 89). Calculate how much Siphokazi owes on her bond at the end of the 89th month. (2)

7.6 She decides to increase her payments to R8 500 per month from the end of the 90th month. How many months will it take to repay her bond after the new payment of R8 500 per month? (4) [16]

▶ DIFFERENTIAL CALCULUS [32]

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x) = 3x^2 - 2$. (5)

8.2 Determine $\frac{dy}{dx}$ if $y = 2x^{-4} - \frac{x}{5}$. (2) [7]

QUESTION 9

Given: $f(x) = x^3 - 4x^2 - 11x + 30$

9.1 Use the fact that $f(2) = 0$ to write down a factor of $f(x)$. (1)

9.2 Calculate the coordinates of the x-intercepts of f. (4)

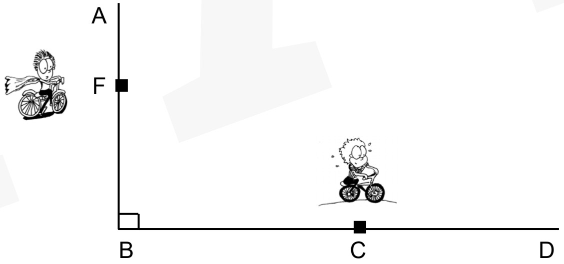
9.3 Calculate the coordinates of the stationary points of f. (5)

9.4 Sketch the curve of f. Show all intercepts with the axes and turning points clearly. (3)

9.5 For which value(s) of x will $f'(x) < 0$? (2) [15]

QUESTION 10

Two cyclists start to cycle at the same time. One starts at point B and is heading due north towards point A, whilst the other starts at point D and is heading due west towards point B. The cyclist starting from B cycles at 30 km/h while the cyclist starting from D cycles at 40 km/h. The distance between B and D is 100 km. After time t (measured in hours), they reach points F and C respectively.



10.1 Determine the distance between F and C in terms of t. (4)

10.2 After how long will the two cyclists be closest to each other? (4)

10.3 What will the distance between the cyclists be at the time determined in Question 10.2? (2) [10]

▶ PROBABILITY [16]

QUESTION 11

11.1 Events A and B are mutually exclusive. It is given that:

- P(B) = 2P(A)
- P(A or B) = 0,57

Calculate P(B). (3)

11.2 Two identical bags are filled with balls. Bag A contains 3 pink and 2 yellow balls. Bag B contains 5 pink and 4 yellow balls. It is equally likely that Bag A or Bag B is chosen. Each ball has an equal chance of being chosen from the bag. A bag is chosen at random and a ball is then chosen at random from the bag.

11.2.1 Represent the information by means of a tree diagram. Clearly indicate the probability associated with each branch of the tree diagram and write down all the outcomes. (4)

11.2.2 What is the probability that a yellow ball will be chosen from Bag A? (1)

11.2.3 What is the probability that a pink ball will be chosen? (3) [11]

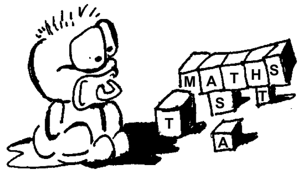
QUESTION 12

Consider the word M A T H S.

12.1 How many different 5-letter arrangements can be made using all the above letters? (2)

12.2 Determine the probability that the letters S and T will always be the first two letters of the arrangements in Question 12.1. (3) [5]

TOTAL: 150



EXEMPLAR MEMOS

Gr 10, 11 & 12

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GRADE 10 EXEMPLAR PAPER 1 MEMO

1.1.1 $(m - 2n)(m^2 - 6mn - n^2)$

$= m^3 - 6m^2n - mn^2$

$\quad - 2m^2n + 12mn^2 + 2n^3$

$= m^3 - 8m^2n + 11mn^2 + 2n^3$ ◄

1.1.2 $\dfrac{x^3 + 1}{x^2 - x + 1} - \dfrac{4x^2 - 3x - 1}{4x + 1}$

$= \dfrac{(x + 1)\cancel{(x^2 - x + 1)}}{\cancel{(x^2 - x + 1)}} - \dfrac{\cancel{(4x + 1)}(x - 1)}{\cancel{(4x + 1)}}$

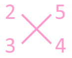
$= (x + 1) - (x - 1)$

$= x + 1 - x + 1$

$= 2$ ◄

1.2.1 $6x^2 - 7x - 20$

$= (2x - 5)(3x + 4)$ ◄



1.2.2 $a^2 + a - 2ab - 2b$

$= a(a + 1) - 2b(a + 1)$

$= (a + 1)(a - 2b)$ ◄

1.3 $49 < 51 < 64$ . . . *i.e. 51 lies between 49 and 64*

$\therefore\ 7 < \sqrt{51} < 8$ . . . *taking the square root*

i.e. $\sqrt{51}$ lies between 7 and 8 ◄

1.4 Let $x = 0,\dot{2}\dot{4}\dot{5}$

$\therefore\ x = 0,245\,245\ldots$ . . . ❶

× 1 000) $\therefore\ 1\,000x = 245,\,245\,245\ldots$ . . . ❷

❷ - ❶: $\therefore\ 999x = 245$

$\therefore\ x = \dfrac{245}{999}$

. . . *i.e. x can be expressed as $\dfrac{a}{b}$ where $a\ \&\ b \in \mathbb{Z}$*

$\therefore$ x is a rational number

2.1.1 $x^2 - 4x = 21$

$\therefore\ x^2 - 4x - 21 = 0$

$\therefore\ (x + 3)(x - 7) = 0$

$\therefore\ x + 3 = 0$ or $x - 7 = 0$

$\therefore\ x = -3$ ◄ $\therefore\ x = 7$ ◄

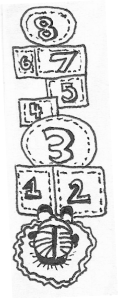
2.1.2 $3x^{\frac{5}{4}} = 96$

÷ 3) $\therefore\ x^{\frac{5}{4}} = 32$

$\therefore\ \left(x^{\frac{5}{4}}\right)^{\frac{4}{5}} = (2^5)^{\frac{4}{5}}$

$\therefore\ x = 2^4$

$\therefore\ x = 16$ ◄



2.1.3 $\dfrac{2\sqrt{x}}{3S} = R$

× 3S) $\therefore\ 2\sqrt{x} = 3SR$

÷ 2) $\therefore\ \sqrt{x} = \dfrac{3SR}{2}$

Square: $\therefore\ x = \dfrac{9S^2R^2}{4}$ ◄

2.2 $6q + 7p = 3$ . . . ❶

$2q + p = 5$ . . . ❷

❷ × 3: $6q + 3p = 15$ . . . ❸

❶ - ❸: $\therefore\ 4p = -12$

$\therefore\ p = -3$ ◄

❷: $\therefore\ 2q - 3 = 5$

$\therefore\ 2q = 8$

$\therefore\ q = 4$ ◄

3.1.1 The 1st 3 terms:

$3(3) + 1\ ;\ 2(3)\ ;\ 3(3) - 7$

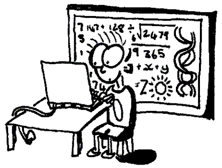
$\therefore\ 10\ ;\ 6\ ;\ 2$ ◄

3.1.2 The difference is -4

$\therefore$ In $T_n = an + b$: $a = -4$

& $T_0 = b = 14$ . . . *the term before the 1st term*

$\therefore\ T_n = -4n + 14$ ◄



3.1.3 n? if $T_n < -31$

$\therefore\ -4n + 14 < -31$

$\therefore\ -4n < -45$

÷ (-4) $\therefore\ n > 11\frac{1}{4}$

∴ The 12th term ◄

3.2 The even numbers: 6 ; 12 ; 18 . . .

∴ The 13th even number $= 13 \times 6 = 78$ ◄

OR: The 13th even number

= the 26th term of the pattern

$= 26 \times 3$

$= 78$

4.1 $P = 4\,500;\ i = \frac{4,25}{100} = 0,0425;\ n = \frac{30}{12} = 2\frac{1}{2};\ A?$

$A = P(1 + i)^n = 4\,500(1 + 0,0425)^{2,5} = R4\,993,47$ ◄

4.2.1 The loan amount = R5 999 - R600 = R5 399

The accumulated amount, $A = P(1 + in)$

where $P = 5\,399;\ i = 8\% = 0,08;\ n = 1\frac{1}{2}$ years; A?

$\therefore\ A = 5\,399\left[1 + (0,08)\left(\frac{3}{2}\right)\right]$

= R6 046,88

∴ The monthly amount to be paid $= \frac{6\,046,88}{18}$

= R335,94 ◄

4.2.2 The amount of interest

= The total amount paid over the 18 months – the loan amount

= R6 046,88 – R5 399

= R647,88

4.3 28,35 g is worth \$978,34 $= R978,34 \times 8,79$

= R8 599,61

∴ 1 g is worth $\frac{R8\,599,61}{28,35}$

∴ 1 kg is worth $R\frac{8\,599,61}{28,35} \times 1\,000$ . . . *1 kg = 1 000 g*

≈ R303 337,16 ◄

5.1.1 $A \cap B$ ◄ OR: A and B ◄

5.1.2 A′ ◄ OR: not A ◄

5.2 Set B ◄

5.3.1 Of the 40 learners, 7 are left-handed

∴ 40 - 7 = 33 are right-handed

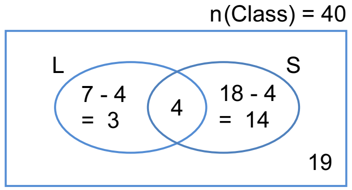
Of the 18 learners who play soccer, 4 are left-handed

∴ 14 learners who play soccer are right-handed

∴ The number of learners who are right-handed and DON'T play soccer

= 33 - 14 = 19 ◄

5.3.2



5.3.3 (a) n(L or S) = 3 + 4 + 14 = 21

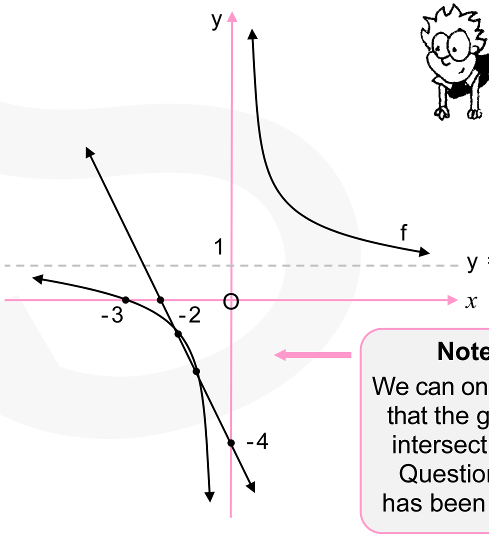
$\therefore$ P(L or S) $= \frac{21}{40}$ ◄

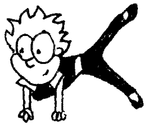
(b) n(R and S) = 14 . . . *where R is the set of all right-handed people*

$\therefore$ P(R and S) $= \frac{14}{40}$

$= \frac{7}{20}$ ◄

6.1





Note:
We can only know that the graphs intersect once Question 6.4 has been done!

f: $y = \frac{3}{x} + 1$

y-intercept ($x = 0$): none

x-intercept (y = 0): $\frac{3}{x} + 1 = 0$

$\therefore\ \frac{3}{x} = -1$

$\therefore\ x = -3$

g: $y = -2x - 4$

y-intercept ($x = 0$): y = -4

x-intercept (y = 0): $-2x - 4 = 0$

$\therefore\ -2x = 4$

$\therefore\ x = -2$

6.2 Asymptotes: $y = 1$ ◄

& $x = 0$ (the y-axis) ◄

6.3 Domain of f: $x \neq 0;\ x \in \mathbb{R}$ ◄

... $(-\infty;\ 0) \cup (0;\ \infty)$

6.4 $f(x) = g(x)$ ➡ $\frac{3}{x} + 1 = -2x - 4$

$\times x)$ $\therefore\ 3 + x = -2x^2 - 4x$

$\therefore\ 2x^2 + 5x + 3 = 0$

$\therefore\ (2x + 3)(x + 1) = 0$

$\therefore\ 2x + 3 = 0$ or $x + 1 = 0$

$\therefore\ 2x = -3$ $\quad\therefore\ x = -1$ ◄

$\therefore\ x = -\frac{3}{2}$ ◄

Note: These are the x-coordinates of the points of intersection of f and g:

$\left(\mathbf{-1\frac{1}{2}};\ -1\right)$ & $(\mathbf{-1};\ -2)$

6.5 $-1 \leq g(x) < 3$

$\therefore\ -1 \leq -2x - 4 < 3$ ... *$g(x) = -2x - 4$*

add 4: $\therefore\ 3 \leq -2x < 7$

÷ (-2): $\therefore\ -\frac{3}{2} \geq x > -\frac{7}{2}$ ... *When one divides by a negative number, the direction of the 'inequality' changes.*

$\therefore\ -\frac{7}{2} < x \leq -\frac{3}{2}$ ... *The inequality has been rewritten with the smaller value on the left.*

i.e. $-3\frac{1}{2} < x \leq -1\frac{1}{2}$ ◄ OR: $\left(-3\frac{1}{2};\ -1\frac{1}{2}\right]$ ◄

(means excluding;] means including

6.6 $k(x) = 2g(x) = 2(-2x - 4) = -4x - 8$

$\therefore$ The equation of k: $y = -4x - 8$

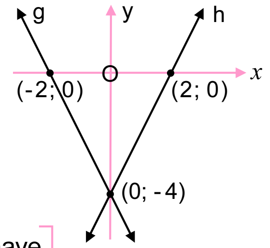
$\therefore$ The y-intercept of k: $(0; -8)$ ◄ ... *substitute $x = 0$*

6.7 x-intercept of g: $(-2; 0)$

& x-intercept of h: $(2; 0)$ ◄

y-intercept of g: $(0; -4)$

& y-intercept of h: $(0; -4)$ ◄



Notice: The reflected points have the same y-coordinate, but the x-coordinates are opposite in sign.

7.1 C(-2; 0) ◄ ... *symmetrical about the y-axis*

7.2 The equation of f: $y = a(x + 2)(x - 2)$ ... *roots -2 & 2*

$\therefore\ y = a(x^2 - 4)$

Subst. $B\left(-3;\ \frac{5}{2}\right)$: $\therefore\ \frac{5}{2} = a\left[(-3)^2 - 4\right]$

$\therefore\ \frac{5}{2} = a(5)$

÷ 5) $\therefore\ a = \frac{1}{2}$

$\therefore$ The equation of f: $y = \frac{1}{2}(x^2 - 4)$

$\therefore\ y = \frac{1}{2}x^2 - 2$ ◄

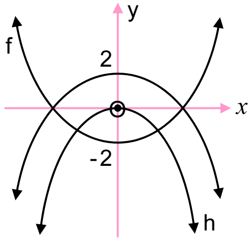
7.3 The y-intercept of f is (0; -2)

$\therefore$ The range of f: $y \geq -2$ ◄ OR: $[-2;\ \infty)$ ◄

7.4 The graph of h is obtained by flipping f ... *$-f(x)$*

then, shifting down 2 units ... *-2*

$\therefore$ The range of h: $y \leq 0$ ◄

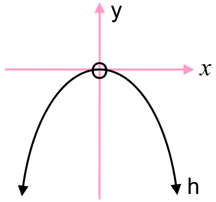


OR: $(-\infty;\ 0]$ ◄

OR: $h(x) = -\left(\frac{1}{2}x^2 - 2\right) - 2$

$\therefore\ h(x) = -\frac{1}{2}x^2 + 2 - 2$

$\therefore\ h(x) = -\frac{1}{2}x^2$

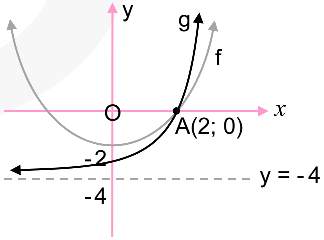


$\therefore$ The range of h: $y \leq 0$ ◄

7.5 $q = -4$ ... range $y > -4$ ➡ $y = -4$ is an asymptote

$\therefore$ Equation of g:

$y = b^x - 4;\ b > 0$



Substitute A(2; 0):

$0 = b^2 - 4$

$\therefore\ b^2 = 4$

$\therefore\ b = 2$ ... *$b \neq -2\ \because\ b > 0$*

$\therefore$ Equation of g:

$y = 2^x - 4$ ◄

$\therefore$ means therefore

$\because$ means because

GRADE 11 EXEMPLAR PAPER 1 MEMO

ALGEBRA AND EQUATIONS AND INEQUALITIES [47]

1.1.1 $(2x - 1)(x + 5) = 0$

$\Rightarrow 2x - 1 = 0$ **or** $x + 5 = 0$

$\therefore 2x = 1 \qquad \therefore \mathbf{x = -5}$ $\blacktriangleleft$

$\therefore \mathbf{x = \frac{1}{2}}$ $\blacktriangleleft$

> $a \times b = 0$
> $\Rightarrow a = 0$
> **or** $b = 0$

1.1.2 $2x^2 - 4x + 1 = 0$

$$\therefore x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(1)}}{2(2)}$$

$$= \frac{4 \pm \sqrt{16 - 8}}{4}$$

$$= \frac{4 \pm \sqrt{8}}{4}$$

$$= \frac{4 \pm 2\sqrt{2}}{4} \quad \ldots$$

$$= \mathbf{\frac{2 \pm \sqrt{2}}{2}} \blacktriangleleft \quad \ldots \frac{\cancel{2}(2 \pm \sqrt{2})}{\cancel{4}\,2}$$

> **The formula:**
> $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
>
> **Note:** No formula sheet is supplied for the Grade 11 exam.

> $\sqrt{8} = \sqrt{4 \times 2} = \sqrt{4} \times \sqrt{2}$
> $= 2\sqrt{2}$

1.2.1 $125^{\frac{2}{3}}$

$= (5^3)^{\frac{2}{3}}$

$= 5^2 \quad \ldots (a^m)^n = a^{mn}$

$= \mathbf{25}$ $\blacktriangleleft$

> OR: $125^{\frac{2}{3}}$
> $= 5^2 \quad \ldots$ *cubed root of 125 is 5*
> $= \mathbf{25}$ $\blacktriangleleft$

1.2.2 $(3\sqrt{2} - 12)(2\sqrt{2} + 1) \quad \ldots$

> *'FOIL': Firsts, Outers, Inners, Lasts!*

$= 6.\mathbf{2} + 3\sqrt{2} - 24\sqrt{2} - 12$

$= 12 - 21\sqrt{2} - 12$

$= \mathbf{-21\sqrt{2}}$ $\blacktriangleleft$

> $\sqrt{2}\sqrt{2} = \mathbf{2}$

1.3.1 The expression is undefined for $3x - 9 = 0$

$\therefore 3x = 9$

$\therefore \mathbf{x = 3}$ $\blacktriangleleft$

> Division by zero is undefined.

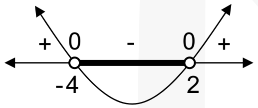
1.3.2 $\frac{x^2 - x - 6}{3x - 9} = \frac{(x - 3)(x + 2)}{3(x - 3)}$

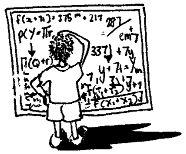
$= \mathbf{\frac{x + 2}{3}}$ **for** $\mathbf{x \neq 3}$ $\blacktriangleleft$

2.1.1 $x^2 - x - 6 < -3x + 2$

$\therefore x^2 + 2x - 8 < 0$

$\therefore (x + 4)(x - 2) < 0$





$\therefore \mathbf{-4 < x < 2}$ $\blacktriangleleft$

2.1.2 The integers between -4 and 2 are: -3 ; -2 ; -1 ; 0 and 1

> See $x^2 + 2x - 8 < 0$ in line 2 in 2.1.1.

$\therefore$ The sum of the integers $= (-3) + (-2) + (-1) + 0 + 1$

$= \mathbf{-5}$ $\blacktriangleleft$

2.2.1 $\frac{4^x.4^{-1} + 4^x.4}{17.4^x.3^x} \quad \ldots$

> $a^{m+n} = a^m.a^n$
> $(ab)^n = a^n.b^n$

$= \frac{\cancel{4^x}\left(\frac{1}{4} + 4\right)}{17.\cancel{4^x}.3^x} \quad \ldots$

> **OR:** $= \frac{4^{x-1}(1 + 4^2)}{17.4^x.3^x}$
> $= \frac{4^{x-1-x}.17}{17.3^x}$

$= \frac{\frac{17}{4}}{17.3^x}$

$= \frac{\cancel{17}}{4} \times \frac{1}{\cancel{17}.3^x}$

$= \mathbf{\frac{1}{4.3^x}} \quad \ldots \frac{1}{4}.3^{-x}$ $\blacktriangleleft$

2.2.2 The expression $= \frac{1}{4}.3^{-x} = \frac{1}{4}.4t = \mathbf{t}$ $\blacktriangleleft$

2.3 $3^y = (3^4)^x \quad \& \quad y = x^2 - 6x + 9 \quad \ldots$ ❷

$\therefore 3^y = 3^{4x}$

$\therefore y = 4x \quad \ldots$ ❶

Equating ❷ & ❶: $\therefore x^2 - 6x + 9 = 4x$

$\therefore x^2 - 10x + 9 = 0$

$\therefore (x - 1)(x - 9) = 0$

$\therefore x = 1$ or $x = 9$

If $x = 1$: $y = 4(1) = 4$

If $x = 9$: $y = 4(9) = 36$

$\therefore$ The solutions: **(1; 4)** or **(9; 36)** $\blacktriangleleft$

3.1 The roots of a quadratic equation: $x = \frac{3 \pm \sqrt{4 - 8p}}{4}$

i.e. The roots are $\frac{3 \mathbf{+} \sqrt{4 - 8p}}{4}$ and $\frac{3 \mathbf{-} \sqrt{4 - 8p}}{4}$

3.1.1 The roots will be **EQUAL** if

$4 - 8p = 0$

$\therefore -8p = -4$

$\div(-8) \quad \therefore \mathbf{p = \frac{1}{2}}$ $\blacktriangleleft$

> *Compare the roots . . .* $\mathbf{+}\sqrt{\ }$ *and* $\mathbf{-}\sqrt{\ }$ *is the only part that is different.*

3.1.2 The roots will be **NON-REAL** if

$4 - 8p < 0$

$\therefore -8p < -4$

$\div(-8) \quad \therefore \mathbf{p > \frac{1}{2}}$ $\blacktriangleleft$

> $\sqrt{\textbf{a negative no.}}$ *is non-real*

3.2.1 Both the following conditions must hold:

- $5 - x \geq 0 \quad \ldots \sqrt{a}$ *only real if* $a \geq 0$

  $\therefore -x \geq -5$

  $\times(-1) \quad \therefore x \leq 5$

AND

- $x + 1 \geq 0 \quad \ldots$

  $\therefore x \geq -1$

> $\sqrt{a}$ *defined as* $\mathbf{+}\sqrt{a}$ *for all* $a \geq 0$
>
> $\therefore$ *For equation to be true RHS* ≥ 0

$\therefore \mathbf{-1 \leq x \leq 5}$ $\blacktriangleleft$

3.2.2 $\sqrt{5-x} = x + 1$

$\therefore \left(\sqrt{5-x}\right)^2 = (x+1)^2$

$\therefore 5 - x = x^2 + 2x + 1$

$\therefore 0 = x^2 + 3x - 4$

$\therefore (x+4)(x-1) = 0$

$\therefore x = -4$ or 1

But $-1 \leq x \leq 5$ . . . *see 3.2.1*

$\therefore$ **Only $x = 1$** ◄

OR: Test . . .

For $x = -4$:

LHS $= \sqrt{9} = 3$ & RHS $= -3$ $\therefore x \neq -4$

For $x = 1$: LHS = RHS = 2 $\therefore x = 1$ ✓

3.2.3 The solution: **$x = -4$** ◄

Note: This is the rejected answer in 3.2.2! Squaring the equation $-\sqrt{5-x} = x + 1$ will yield the identical calculation as in 3.2.2 except, when we test, $x + 1$ must be *negative*.

▶ FINANCE, GROWTH AND DECAY [18]

4.1 **A = P(1 - in)** . . . Formula for **depreciation** on the **straight-line** method.

A? ; **P** = R145 000 ; **i** = 17% = $\frac{17}{100}$ = 0,17 ; **n** = 5

$\therefore$ A = 145 000[1 - (0,17)(5)]

= **R21 750** ◄

4.2.1 The rate earned quarterly, **i** = $\frac{8\%}{4}$ = **2%** = **0,02** ◄

4.2.2 $\mathbf{1 + i_{eff}} = \left(\mathbf{1 + \frac{i_{nom}}{4}}\right)^4$ . . . *Note: i_{nom} = 8%*

$= (1 + 0{,}02)^4$

$= (1{,}02)^4$

$= 1{,}08243\ldots$

$\therefore \mathbf{i_{eff}} = 0{,}08243\ldots$

$\approx$ **8,24% per annum** ◄

Note:

$A = P(\mathbf{1 + i_{eff}})^1$

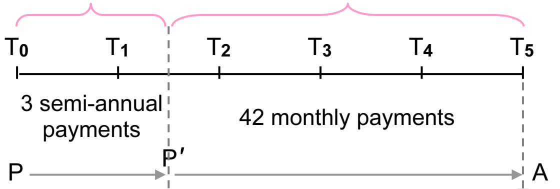
and

$A = P\left(\mathbf{1 + \frac{i_{nom}}{4}}\right)^4$

4.3 semi-annually | monthly

$\mathbf{i} = \frac{9\%}{2} = \frac{0{,}09}{2}$ $\qquad$ $\mathbf{i} = \frac{7{,}5\%}{12} = \frac{0{,}075}{12}$

$\mathbf{n} = 3$ $\qquad$ $\mathbf{n} = 42$



$P' = P\left(1 + \frac{0{,}09}{2}\right)^3$ $\qquad$ $A = P'\left(1 + \frac{0{,}075}{12}\right)^{42}$

P = R14 000

$\therefore$ The accumulated amount, A

$= \text{R}14\ 000\left(1 + \frac{0{,}09}{2}\right)^3\left(1 + \frac{0{,}075}{12}\right)^{42}$

$\approx$ **R20 755,08** ◄

5.1 The value (of both investments) at the start (i.e. at $x = 0$) = **R15 000** ◄

5.2 **Simple interest** ◄ . . . *straight-line appreciation*

5.3 **i?** ; **P** = R15 000 ; **n** = 6 ; **A** = R31 000

See point A

A = P(1 + in)

$\therefore 31\ 000 = 15\ 000[1 + (i)(6)]$

÷ 15 000) $\therefore 1 + 6i = 2{,}0\dot{6}$

$\therefore 6i = 1{,}0\dot{6}$

$\therefore i = 0{,}1\dot{7}$

$\therefore$ ***i* = 17,78%** ◄

5.4 Determine w:

(12; w) is a point on Dumisani's graph.

$\therefore$ Substitute n = 12 ; P = R15 000 ; i = 17,777... in

A = P(1 + in) . . . *Dumisani's formula*

$\therefore w = 15[1 + (0{,}1\dot{7})(12)]$

≈ 47

Note: A, P and w represent 'thousands of rands'

Substitute point B(12; 47) in

$\mathbf{A = P(1 + i)^n}$ . . . *Astin's formula*

$\therefore 47 = 15(1 + i)^{12}$

$\therefore (1 + i)^{12} = 3{,}1\dot{3}$

$\therefore 1 + i = 1{,}09985\ldots$

$\therefore i = 0{,}09985\ldots$

= **10,0%** ◄

▶ PATTERNS AND SEQUENCES [23]

6.1 $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{8}$; . . . ; $\frac{1}{1\ 024}$

6.1.1 Multiply $\frac{1}{8}$ by $\frac{1}{2}$:

$\mathbf{T_4 = \frac{1}{16}}$ ◄ . . . $\left(\frac{1}{2}\right)^1 ; \left(\frac{1}{2}\right)^2 ; \left(\frac{1}{2}\right)^3 ; \ldots$

OR: The terms are: 2^{-1} ; 2^{-2} ; 2^{-3} ; . . . ; 2^{-10}

$\therefore \mathbf{T_4} = 2^{-4}$ . . . *the fourth term = 2^{-four}*

$\mathbf{= \frac{1}{16}}$ ◄

6.1.2 The nth term, $\mathbf{T_n} = \left(\mathbf{\frac{1}{2}}\right)^{\mathbf{n}}$ or $\mathbf{2^{-n}}$ ◄ . . . *see 6.1.1*

6.1.3 $1\ 024 = 2^{10}$ . . . *trial and error!*

$\therefore \frac{1}{1\ 024} = \left(\frac{1}{2}\right)^{10}$ or 2^{-10}

$\therefore$ The number of terms in the sequence, **n = 10** ◄

6.2 156 ; 148 ; 140 ; 132 ; . . .

6.2.1 The 5th term, $\mathbf{T_5}$ = 132 - 8 **= 124** ◄

6.2.2 The general term of a linear pattern is $T_n = an + b$

This sequence has a common 1st difference of -8

$\therefore a = -8$

and $T_1 = a + b = 156$ . . . *$T_1 = a(1) + b$*

$\therefore -8 + b = 156$

$\therefore b = 164$

$\therefore$ A general formula: $\mathbf{T_n = -8n + 164}$ ◄

6.2.3 T_n negative, i.e. $T_n < 0$

➡ $-8n + 164 < 0$

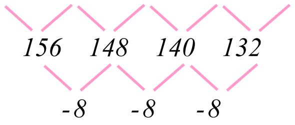
$\therefore -8n < -164$

÷ (-8) $\therefore n > 20\frac{1}{2}$

$\therefore$ **The 1st term to be negative is the 21st term** ◄

6.2.4 ▶ 1^{st} difference (between T_1 and T_2 of the quadratic pattern)

$= 3a + b = 156$



▶ The 2^{nd} difference,

$2a = -8$

$\therefore\ a = -4$

$\therefore\ 3(-4) + b = 156$

$\therefore\ b = 168$

▶ $T_5 = a(5)^2 + b(5) + c = -24$ . . . *given $T_5 = -24$*

$\therefore\ 25a + 5b + c = -24$

$\therefore\ 25(-4) + 5(168) + c = -24$

$\therefore\ -100 + 840 + c = -24$

$\therefore\ c = -764$

$\therefore\ T_n = -4n^2 + 168n - 764$

There are various other methods!

---

7. $T_n = an^2 + bn + c$

$T_2 = a(2)^2 + b(2) + c = 4a + 2b + c = 0$ . . . ❶

$T_4 = a(4)^2 + b(4) + c = 16a + 4b + c = 0$ . . . ❷

❷ - ❶: $12a + 2b = 0$

$\therefore\ 6a + b = 0$

& 2^{nd} difference, $2a = 12$

$\therefore\ a = 6$

$\therefore\ 36 + b = 0$

$\therefore\ b = -36$

❶: $4(6) + 2(-36) + c = 0$

$\therefore\ c = -24 + 72$

$\therefore\ c = 48$

$T_3 = a(3)^2 + b(3) + c$

$= 9a + 3b + c$

$= 9(6) + 3(-36) + 48$

$= \mathbf{-6}$ ◄

There are various other methods!

▶ FUNCTIONS AND GRAPHS [43]

8.1 $\boldsymbol{x = 3}$ *(vertical asymptote)* ◄

& $\boldsymbol{y = -1}$ *(horizontal asymptote)* ◄

8.2 $x \in \mathbb{R}\ ;\ \boldsymbol{x \neq 3}$ ◄

8.3 $\mathbf{d} = \tan 76° \approx \mathbf{4}$ ◄

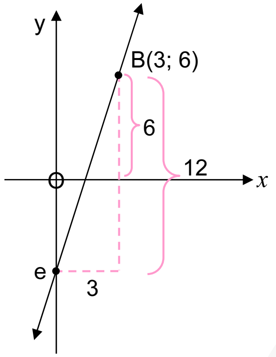
& $\mathbf{e = -6}$ ◄

OR: Find e by substituting d = 4 and B(3; 6) into $g(x) = dx + e$:

$6 = (4)(3) + e$

$6 = 12 + e$

$-6 = e$



8.4 At A & C:

$$\frac{2}{x-3} - 1 = 4x - 6$$

$\times (x-3)$ $\therefore\ 2 - (x-3) = (4x-6)(x-3)$

$\therefore\ 2 - x + 3 = 4x^2 - 18x + 18$

$\therefore\ 0 = 4x^2 - 17x + 13$

$\therefore\ 0 = (4x - 13)(x - 1)$

$\therefore\ x = 1$ at A and $x = \frac{13}{4}$ at C

$g(1) = 4(1) - 6 = -2$ and $g\left(\frac{13}{4}\right) = 4\left(\frac{13}{4}\right) - 6 = 7$

$\therefore\ \mathbf{A(1; -2)}$ **and** $\mathbf{C\left(\frac{13}{4}; 7\right)}$ ◄

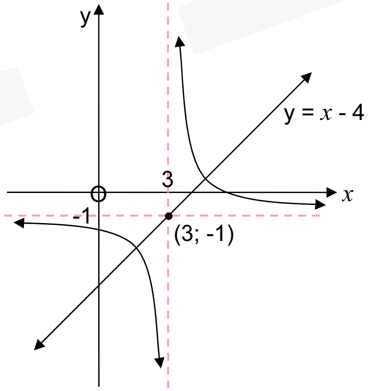
8.5 $\mathbf{1 \leq x < 3}$ or $\mathbf{x \geq \frac{13}{4}}$ ◄ . . . *g is above or on f*

↖

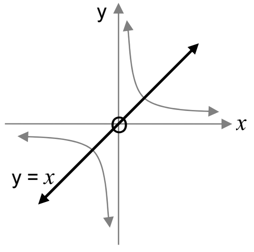
[Note: $x \neq 3$ $\because$ f(x) is undefined at $x = 3$]

8.6 $y = (x - 3) - 1$ . . .

$\therefore\ \mathbf{y = x - 4}$ ◄



The axis of symmetry, $y = x$, of $y = \frac{k}{x}$



moves 3 units right & 1 unit down

OR: Axis of symmetry: $y = x + c$

Substitute (3; -1): $-1 = 3 + c$

$\therefore\ -4 = c$

$\therefore$ Equation: $\boldsymbol{y = x - 4}$ ◄

9.1 ▶ $f(x) = -x^2 + 2x + 3$

- y-intercept: (0; 3) . . . *$x = 0$*
- x-intercepts: Substitute y = 0

$-x^2 + 2x + 3 = 0$

$\times (-1)$ $\therefore\ x^2 - 2x - 3 = 0$

$\therefore\ (x - 3)(x + 1) = 0$

$\therefore\ x = 3$ or -1

- Turning point: Axis of symmetry: $x = 1$

(Halfway between the roots)

& Maximum $y = -(1)^2 + 2(1) + 3 = 4$

$\therefore$ Turning point is (1; 4)

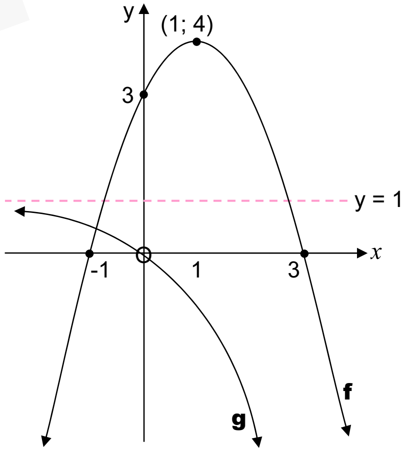
▶ $g(x) = 1 - 2^x$

- y-intercept: Substitute $x = 0$

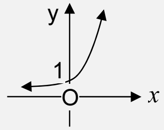
$\therefore\ y = 1 - 2^0 = 1 - 1 = 0$

$\therefore$ (0; 0) . . . $\therefore$ *x-int. too!*

- equation of asymptote: y = 1

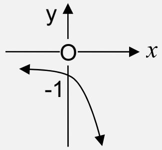


Consider $\mathbf{y = 2^x}$



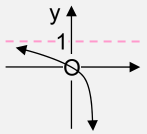
then, $y = \mathbf{-}2^x$

. . . *flip*



then, $y = \mathbf{+1} - 2^x$:

. . . *move 1 unit up*



9.2 $f(-3) = -(-3)^2 + 2(-3) + 3 = -9 - 6 + 3 = -12$

& $f(0) = -(0)^2 - 2(0) + 3 = 3$

∴ Average gradient between $x = -3$ and $x = 0$

$= \frac{f(0) - f(-3)}{0 - (-3)}$

$= \frac{3 - (-12)}{3}$

$= \frac{15}{3}$

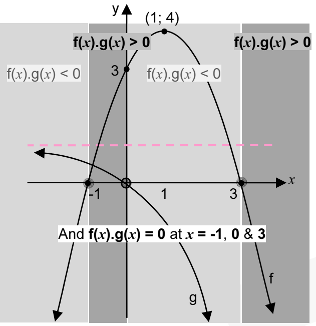
= 5 ◄

Average gradient

$= \frac{\text{change in y}}{\text{change in } x}$

Also: $\frac{y_2 - y_1}{x_2 - x_1}$

9.3 **$-1 \leq x \leq 0$ or $x \geq 3$** ◄

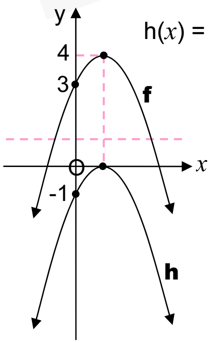


Observe f(x) and g(x), the y-values of f and g.

The question is asking for which values of x, moving from left to right, is the product of the graphs positive or zero?

i.e. for which values of x do the graphs have the same sign, either both positive or both negative and for which values of x are either of the graphs zero.

9.4 **c = -4** ◄



h(x) = f(x) + a number.

For f to just *touch* the x-axis, it must move down 4 units.

∴ 'the number' must be -4.

9.5 $t(x) = -(1 - 2^x) + 1$

$= -1 + 2^x + 1$

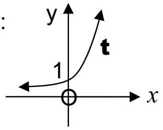
$= 2^x$

At the y-intercept, $x = 0$

∴ $y = 2^0 = 1$

∴ **(0; 1)** ◄

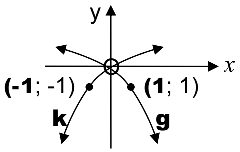
y = 2^x:



9.6 **$k(x) = 1 - 2^{-x}$** ◄

When points (or graphs) are reflected about the y-axis, x is replaced by $-x$.

e.g. (**1**; -1) on g becomes (**-1**; -1) on k.



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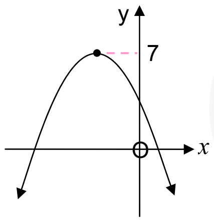
10. The range, (-∞; 7], indicates the y-values.

➡ Max f(x) = 7 and a < 0 ;

Axis of symmetry:

$x = -\frac{b}{2a} = -\frac{-}{-}$ is negative;

(. . . it is given that b < 0 and concluded that a < 0.)



One root positive & one negative

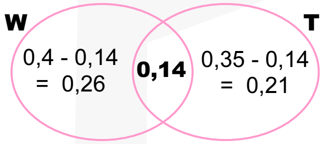
➡ roots on opposite sides of y-axis.

Note: Range notation:

(means excluding & **]** means including

► PROBABILITY [19]

11.



Note:

W includes the part that overlaps with T (just as T includes the part that overlaps with W).

∴ The overlap needs to be subtracted to find W without T and T without W.

▸ Events A and B are mutually exclusive if:

- **P(A∩B) = 0, or** if:
- **P(A or B) = P(A) + P(B)**

▸ Events A and B are independent if:

- **P(A and B) = P(A) × P(B)**

11.1 **Method 1**

P(W and T) = 0,14 . . . *given*

∴ P(W∩T) ≠ 0

∴ **W and T are not mutually exclusive events** ◄

Method 2

P(W or T) = 0,26 + 0,14 + 0,21 = 0,61

P(W) + P(T) = 0,4 + 0,35 = 0,75 . . . *≠ 0,61*

∴ P(W or T) ≠ P(W) + P(T)

∴ **W and T are not mutually exclusive events** ◄

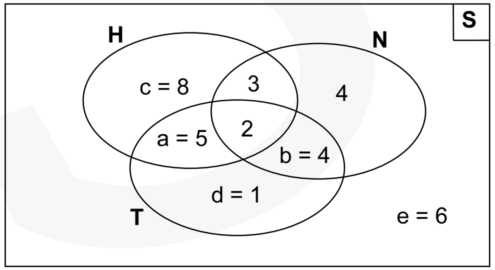
11.2 P(W and T) = 0,14 . . . *given*

P(W) × P(T) = (0,4)(0,35) = 0,14

∴ P(W and T) = P(W) × P(T)

∴ **W and T are independent events** ◄

12.



12.1.1 **a = 5** ◄ . . . *line 3*

b = 4 ◄ . . . *line 4*

c = 8 ◄ . . . *line 5, but after a is determined*

d = 1 ◄ . . . *line 6*

e = 6 ◄ . . . *e = n(S) - n(H∪T∪N)*

= 33 - 27 . . . *33 learners were surveyed*

Lines 1, 2 and 7 were not required for finding values a to e.

Note: n(H∪T∪N) = 18 + 1 + 4 + 4 = 27

12.1.2 **6** ◄ . . . *the value of e, the number not in H, T or N*

Note: In Question 12.1.2, the **number** of learners is required.

In Question 12.1.3 & 12.1.4, the **probability** is required.

NB: The probability of an event (E) occurring

$$= \frac{\text{the number of ways E can occur}}{\text{the total number of outcomes}}$$

12.1.3 The **number** of learners playing netball ONLY = 4

∴ The **probability** that a learner plays netball only

$$= \frac{\text{the number that play netball only}}{\text{the total number of learners}}$$

$= \mathbf{\frac{4}{33}}$ ◄

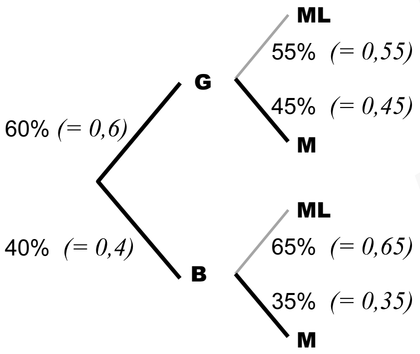
$(\simeq 0{,}12)$

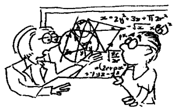
12.1.4 The **number** of learners playing hockey or netball (or both) = 26 . . . *n(H∪N)*

∴ The **probability** that a learner plays hockey or netball (or both)

$= \frac{n(H \cup N)}{n(S)} = \mathbf{\frac{26}{33}}$ **(≈ 0,78)** ◄

12.2





P(a learner does Maths)

= P(a girl doing Maths) + P(a boy doing Maths)

= (60% × 45%) + (40% × 35%)

= 0,27 + 0,14

= **0,41** ◄ . . . *= 41%*

OR: Using decimals only:

P(M) = P(G and M) + P(B and M)

= (0,6 × 0,45) + (0,4 × 0,35)

= 0,27 + 0,14

= **0,41** ◄

GRADE 12 EXEMPLAR PAPER 1 MEMO

▸ ALGEBRA AND EQUATIONS AND INEQUALITIES [23]

1.1.1 $3x^2 - 4x = 0$

$\therefore x(3x - 4) = 0$

$\therefore \boldsymbol{x = 0}$ ◀ or $3x - 4 = 0$

$\therefore 3x = 4$

$\therefore \boldsymbol{x = \frac{4}{3}}$ ◀

1.1.2 $x - 6 + \frac{2}{x} = 0$

$\times x)$ $\therefore x^2 - 6x + 2 = 0$

$\therefore x = \frac{-(-6) \pm \sqrt{(1-6)^2 - 4(1)(2)}}{2(1)}$

$\therefore x = \frac{6 \pm \sqrt{28}}{2}$

$\therefore \boldsymbol{x \simeq 5,65}$ **or** **0,35** ◀

1.1.3 $x^{\frac{2}{3}} = 4$

Take $\sqrt{\ }$: $\therefore x^{\frac{1}{3}} = \pm 2$

Raise to power 3: $\therefore \boldsymbol{x = \pm 8}$ ◀

OR: $\left(x^{\frac{2}{3}}\right)^{\frac{3}{2}} = 4^{\frac{3}{2}}$

$\therefore x = 2^3$

$\therefore x = 8$

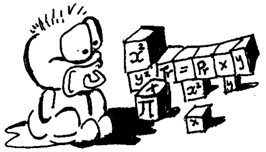
But, **remember** that an *eventh* root was taken

$\therefore \boldsymbol{x = \pm 8}$ ◀

1.1.4 $3^x(x - 5) < 0$

➡ $x - 5 < 0$ . . . because 3^x is positive for all real values of x.

$\therefore \boldsymbol{x < 5}$ ◀ . . . (+)(-) = (-)



1.2 $2x - y = 2$ ➡ $2x - 2 = y$ . . . ❶

$y = x^2 - x - 6$ . . . ❷

Equate ❶ and ❷:

$\therefore x^2 - x - 6 = 2x - 2$

$\therefore x^2 - 3x - 4 = 0$

$\therefore (x + 1)(x - 4) = 0$

$\therefore x = -1$ or $x = 4$

OR: Substitute ❷ in ❶

$\therefore 2x - (x^2 - x - 6) = 2$, etc.

❶: If $x = -1$: $y = 2(-1) - 2 = -4$

If $x = 4$: $y = 2(4) - 2 = 6$

$\therefore$ The solution: **(-1; -4)** **or** **(4; 6)** ◀

1.3 $\sqrt{3}\sqrt{16 \times 3} - \frac{(2^2)^{x+1}}{2^{2x}}$

$= \sqrt{3}\sqrt{16}\sqrt{3} - \frac{2^{2x+2}}{2^{2x}}$

$= \left(\sqrt{3}\right)^2 . 4 - 2^{2x+2-2x}$

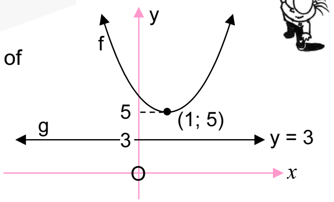
$= 3 . 4 - 2^2$

$= 12 - 4$

$= \mathbf{8}$ ◀

1.4 **Note:** Each of the 2 questions requires a 2 mark answer only! Lengthy algebraic calculations (see the alternative methods) would not be appropriate!

A rough sketch of f and g:



1.4.1 **No** ◀ ; The MINIMUM value of $f(x) = 5$

$\therefore$ **f and g have no points of intersection** ◀

1.4.2 **k > 2** ◀ . . . $g(x) + k$ must be > 5 so that a line $y = g(x) + k$ (parallel to the x-axis) will cut f twice.

OR: Algebraic methods, requiring more time!

1.4.1 **No** ◀ ; $f(x) = g(x)$ ➡ $3(x - 1)^2 + 5 = 3$

$\therefore 3(x - 1)^2 = -2$

$\therefore (x - 1)^2 = -\frac{2}{3}$,

which is impossible because a square cannot be negative.

OR: $3(x^2 - 2x + 1) + 5 = 3$

$\therefore 3x^2 - 6x + 3 + 5 = 3$

$\therefore 3x^2 - 6x + 5 = 0$

$\therefore x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(5)}}{2(3)}$

$= \frac{6 \pm \sqrt{\mathbf{-24}}}{6}$ ⬅ $\left\{ \begin{array}{l} \Delta = -24 \\ \therefore \sqrt{\Delta} \text{ is non-real} \end{array} \right\}$

$\therefore$ There are no solutions to the equation $f(x) = g(x)$.

1.4.2 $f(x) = g(x) + k$ ➡ $3(x - 1)^2 + 5 = 3 + k$

$\therefore 3(x^2 - 2x + 1) + 5 - 3 - k = 0$

$\therefore 3x^2 - 6x + (5 - k) = 0$

$\Delta = (-6)^2 - 4(3)(5 - k)$

$= 36 - 60 + 12k$

$= 12k - 24$

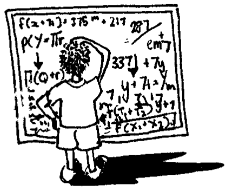
If we want 2 (real & unequal) roots, then Δ must be positive:

$\therefore 12k - 24 > 0$

$\therefore 12k > 24$

$\therefore \mathbf{k > 2}$ ◀

The sketch is much easier.



▶ PATTERNS AND SEQUENCES [26]

2.1 $18 + 24 + 30 + \ldots + 300$

2.1.1 The series is arithmetic: $a = 18$; $d = 6$; n?

$$T_n = a + (n-1)d \Rightarrow 300 = 18 + (n-1)(6)$$
$$\therefore 282 = 6(n-1)$$
$$\div 6) \quad \therefore n - 1 = 47$$
$$\therefore n = 48$$

$\therefore$ **48 terms** ◄

OR: This is a linear series

$\therefore$ The general term, $T_n = an + b$ where

a = the 1st difference = 6 & $b = T_0 = 12$

$\therefore T_n = 6n + 12$

$$\therefore \text{Let } 6n + 12 = 300$$
$$\therefore 6n = 288$$
$$\therefore n = 48$$

$\therefore$ **48 terms** ◄

2.1.2 The sum, $S_n = \frac{n}{2}(a + T_n)$

where $n = 48$ (from 2.1.1) ; $a = 18$ & $T_{48} = 300$

$$\therefore S_{48} = \frac{48}{2}(18 + 300)$$
$$= \mathbf{7\,632} \blacktriangleleft$$

OR: $S_n = \frac{n}{2}[2a + (n-1)d]$

where $n = 48$; $a = 18$ & $d = 6$

$$\therefore S_n = \frac{48}{2}[2(18) + (48-1)(6)]$$
$$= \mathbf{7\,632} \blacktriangleleft$$

2.1.3 The sum of all the whole numbers up to and including 300

$$= (0+)\ 1 + 2 + 3 + \ldots + 300$$
$$= \frac{300}{2}(1 + 300) \qquad \ldots\ S_n = \frac{n}{2}(a + T_n)$$
$$= 45\,150$$

from 2.1.2

$$\therefore \text{The required sum} = 45\,150 - (6 + 12 + 7\,632)$$
$$= \mathbf{37\,500} \blacktriangleleft$$

2.2 G.S.: 16; 8; 4; ...

2.2.1 $T_n = ar^{n-1}$ where $a = 16$ & $r = \frac{8}{16}$ or $\frac{4}{8} = \frac{1}{2}$

$$\therefore T_n = 16.\left(\frac{1}{2}\right)^{n-1} = 2^4.(2^{-1})^{n-1}$$
$$= 2^4.2^{-n+1}$$
$$= 2^{4-n+1}$$
$$= \mathbf{2^{5-n}} \blacktriangleleft$$

2.2.2 Consider $16 + 8 + 4 + 2 + 1 = 31$

i.e. $S_5 = 31$

$\therefore S_n > 31 \Rightarrow \mathbf{n > 5} \blacktriangleleft$

OR:

$$S_n = \frac{a(1-r^n)}{1-r} \quad \text{where } a = 16 \ \& \ r = \frac{1}{2}$$
$$= \frac{16\left[1 - \left(\frac{1}{2}\right)^n\right]}{1 - \frac{1}{2}}$$
$$= \frac{16\left[1 - \left(\frac{1}{2}\right)^n\right]}{\frac{1}{2}}$$
$$= 32$$

$$S_n > 31 \Rightarrow 32\left[1 - \left(\frac{1}{2}\right)^n\right] > 31$$
$$\therefore 1 - \left(\frac{1}{2}\right)^n > \frac{31}{32}$$
$$\therefore -\left(\frac{1}{2}\right)^n > -\frac{1}{32}$$
$$\times(-1) \quad \therefore \left(\frac{1}{2}\right)^n < \left(\frac{1}{2}\right)^5$$
$$\therefore \mathbf{n > 5} \blacktriangleleft$$

Note: It is acceptable to write: $n \geq 6$ because $n \in \mathbb{N}$.

2.2.3 $$S_\infty = \frac{a}{1-r} = \frac{16}{1 - \frac{1}{2}} = \frac{16}{\frac{1}{2}} = \mathbf{32} \blacktriangleleft$$

3.1.1 The terms: $-5 \qquad 1 \qquad 11$

1st differences: $4(0) + 6 = 6 \qquad 4(1) + 6 = 10 \qquad 4(2) + 6 = 14$ ◄ $\boxed{4n + 6}$

2nd differences: $4 \qquad 4$

$$\therefore 2a = 4$$
$$\therefore \mathbf{a = 2} \blacktriangleleft$$

3.1.2 $T_n = an^2 + bn + c$

$$\therefore T_0 = c = -5$$
$$T_1 = a + b + c = 1$$
$$\therefore 2 + b - 5 = 1$$
$$\therefore b = 4$$
$$\therefore T_n = 2n^2 + 4n - 5 \blacktriangleleft$$

... OR: First 1st diff:

$$3a + b = 10$$
$$\therefore 3(2) + b = 10$$
$$\therefore b = 4$$

3.2 ▶ The first factors of each term:

$1 ; 5 ; 9 ; 13 ; \ldots ; 81$

is a linear sequence

$T_n = an + b$ ...

OR: A.S. $\therefore T_n = a + (n-1)d$, etc.

where $a = 4$ and $b = T_0 = -3$

$\therefore$ General term: $T_n = 4n - 3$

The nth term, $T_n = 81$

$$\therefore 4n - 3 = 81$$
$$\therefore 4n = 84$$
$$\therefore n = 21$$

▶ The second factors of each term:

$2 ; 6 ; 10 ; 14 ; \ldots$

Each term is just 1 more than the above sequence

$\therefore T_n = 4n - 2$ up to $n = 21$

$\therefore$ Sigma notation: $\displaystyle\sum_{n=1}^{21}(4n-3)(4n-2)$

This question could have been done entirely by inspection!

▶ FUNCTIONS AND GRAPHS [37]

4.1 $f(x) = \frac{2}{x+1} - 3$

4.1.1 **y-int.**:

Substitute $x = 0$

then $y = \frac{2}{0+1} - 3 = -1 \qquad \ldots\ y = f(0)$

$\therefore$ **(0; -1)** ◄

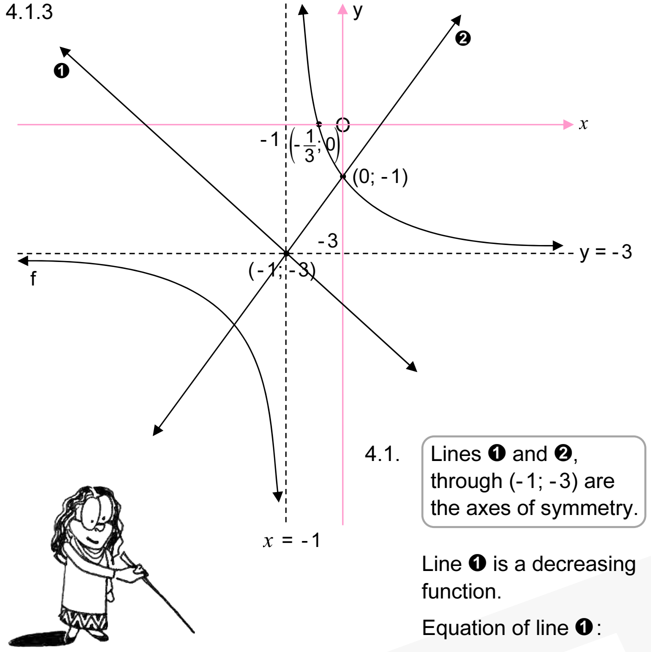
4.1.2 **x-int.**:

Substitute $y = 0 \qquad \ldots\ f(x) = 0$

$$\text{then} \quad 0 = \frac{2}{x+1} - 3$$
$$\therefore 3 = \frac{2}{x+1}$$
$$\therefore 3x + 3 = 2$$
$$\therefore 3x = -1$$
$$\therefore x = -\frac{1}{3}$$

$\therefore \left(-\frac{1}{3}; 0\right) \blacktriangleleft$

4.1.3



4.1. Lines ❶ and ❷, through (-1; -3) are the axes of symmetry.

Line ❶ is a decreasing function.

Equation of line ❶:

$y = -(x + 1) - 3$

$\therefore$ **$y = -x - 4$** ◄

The equation of line ❷ is

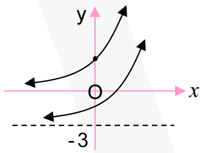
$y = x + 1 - 3$

$\therefore y = x - 2$

OR: Substitute (-1; -3) in

$y = -x + c$

4.2 $f(x) = a.b^x + q$



4.2.1 $q = -3$ . . . range: $y > -3$

$\therefore$ Equation: $y = a.b^x - 3$

Substitute (0; -2):

$\therefore -2 = a.b^0 - 3$

$\therefore 1 = a$

$\therefore$ Equation: $y = b^x - 3$

Substitute (1; -1):

$\therefore -1 = b^1 - 3$

$\therefore 2 = b$

$\therefore$ Equation: **$y = 2^x - 3$** ◄

Note:

There are 3 unknowns to be determined.

The order of the process is important: asymptote, y-intercept, then the other point.

4.2.2 $h(x) = 2^{x+1} + 1$ . . . $2^1.2^x = 2^{x+1}$

$\therefore$ **Shift f 1 unit left and 4 units up** ◄

OR: $h(x) = 2.2^x + 1$

$= 2(2^x - 3) + 7$ . . .

$\therefore$ **Dilate f by a factor of 2, then shift it 7 units up** ◄

To relate h to f, the whole f(x) must be dilated.

5.1 $f(x) = -2x^2 - 5x + 3$

Max occurs when $x = -\frac{b}{2a} = -\frac{-5}{2(-2)} = -\frac{5}{4}$

OR: when $f'(x) = 0$, i.e. $-4x - 5 = 0$

$\therefore -4x = 5$

$\therefore x = -\frac{5}{4}$

$\therefore$ Maximum $= -2\left(-\frac{5}{4}\right)^2 - 5\left(-\frac{5}{4}\right) + 3 = \frac{49}{8}$

$\therefore$ Turning point: $\left(\mathbf{-\frac{5}{4}; \frac{49}{8}}\right)$ ◄

OR: $f(x) = -2\left(x^2 + \frac{5}{2}x \qquad - \frac{3}{2}\right)$

$= -2\left[x^2 - \frac{5}{2}x \mathbf{+ \left(\frac{5}{4}\right)^2} - \frac{3}{2} \mathbf{- \frac{25}{16}}\right]$

$= -2\left[\left(x - \frac{5}{4}\right)^2 - \frac{49}{16}\right]$

$= -2\left(x - \frac{5}{4}\right)^2 + \frac{49}{8}$

$\therefore$ Turning point $\left(\mathbf{-\frac{5}{4}; \frac{49}{8}}\right)$ ◄

5.2 At P, the gradient of f, $f'(x)$, equals the gradient of the tangent (g)

$\therefore f'(x) = \tan 135°$

$\therefore -4x - 5 = -1$

$\therefore -4x = 4$

$\therefore x = -1$

& $f(-1) = -2(-1)^2 - 5(-1) + 3$

$= -2 + 5 + 3$

$= 6$

$\therefore$ **P(-1; 6)** ◄

5.3 Equation of g: $y = ax + q$

a = the gradient of g = -1

$\therefore y = -x + q$

Substitute P(-1; 6):

$\therefore 6 = -(-1) + q$

$\therefore 5 = q$

$\therefore$ **Eqn. of g: $y = -x + 5$** ◄

OR: Substitute the gradient = -1 and the point (-1; 6) in $y - y_1 = m(x - x_1)$:

$\therefore y - 6 = -1(x + 1)$

$\therefore y - 6 = -x - 1$

$\therefore$ **$y = -x + 5$** ◄

5.4 **d > 5** ◄ . . . *y-intercept, d must be > 5*

6.1 Equation of g: $y = \sqrt{ax}$

(8; 3) on g ➡ $4 = \sqrt{a(8)}$ . . .

Square both sides: $\therefore 16 = 8a$

$\therefore$ **a = 2** ◄

If a point lies on a graph, the coordinates of the point satisfy the equation of the graph.

6.2 **$g(x) = \sqrt{2x}$ is defined for $x \geq 0$** ◄

6.3 **$y \geq 0$** ◄

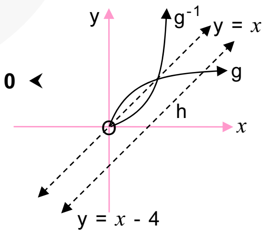
6.4 Equation of g: $y = \sqrt{2x}$

$\therefore$ Eqn. of g^{-1}: $x = \sqrt{2y}$

$\therefore x^2 = 2y$

$\therefore$ **$y = \frac{1}{2}x^2$; $x \geq 0$** ◄

See the sketches of g and g^{-1} below:



It is important to understand the reflections of the inverse functions, g and g^{-1}, in the line $y = x$.

6.5 $h(x) = g(x)$ ➡ $x - 4 = \sqrt{2x}$ . . . **Note:** $x - 4 \geq 0$ $\therefore x \geq 4$

$\therefore (x - 4)^2 = 2x$

$\therefore x^2 - 8x + 16 = 2x$

$\therefore x^2 - 10x + 16 = 0$

$\therefore (x - 2)(x - 8) = 0$

$\therefore x = 2$ or $x = 8$

BUT, for $x = 2$: LHS $= h(x) = -2$ and RHS $= g(x) = +2$

$\therefore$ Only $x = 8$ . . . See the sketch: The point (2; -4) cannot lie on g.

and $y = 8 - 4$ or $\sqrt{2(8)} = 4$

$\therefore$ **The point of intersection is (8; 4)** ◄

6.6 **$0 \leq x < 8$** ≺

Although you obtained the point of intersection algebraically, it is important to understand this entire Q6 graphically too.

▶ FINANCE, GROWTH AND DECAY [16]

7.1 12% of the selling price = R102 000

∴ 1% of the selling price = R102 000 ÷ 12

∴ 100% of the selling price = (R102 000 ÷ 12) × 100

= **R850 000** ≺

7.2 The balance of the selling price = R748 000 (= the loan)

Method 1: Present value

This is the quicker method!

$$P_v = \frac{x\left[1-(1+i)^{-n}\right]}{i} \quad \text{where } P_v = R748\,000;\ x?$$

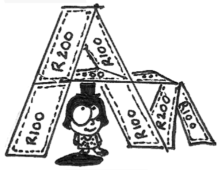
$$i = \frac{9\%}{12} = \frac{0{,}09}{12};\ n = 20 \times 12 = 240$$

$$\therefore\ 748\,000 = \frac{x\left[1-\left(1+\frac{0{,}09}{12}\right)^{-240}\right]}{\frac{0{,}09}{12}} = x\,.\,\mathbf{A}$$

STOre 111,144954 in **A**

$$\therefore\ x = \frac{748\,000}{\mathbf{A}}$$

$\simeq$ **R6 729,25** ≺



Method 2: Future value

The Future value of the loan:

$$\therefore\ F_v = P_v(1+i)^n \text{ where } P_v = R748\,000;\ n = 20 \times 12 = 240$$

$$= 748\,000\left(1+\frac{0{,}09}{12}\right)^{240} \qquad \text{and } i = \frac{9\%}{12} = \frac{0{,}09}{12}$$

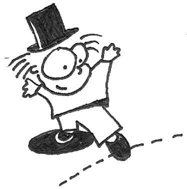
= R4 494 845,34 → **STO**re in **A**

$$\text{and } F_v = \frac{x\left[(1+i)^n - 1\right]}{i}$$

$$= \frac{x\left[\left(1+\frac{0{,}09}{12}\right)^{240} - 1\right]}{\frac{0{,}09}{12}}$$

$= x\,.\,\mathbf{B}$

$\therefore\ x = \frac{\mathbf{A}}{\mathbf{B}} \simeq$ **R6 729,05** ≺



7.3 The amount of interest

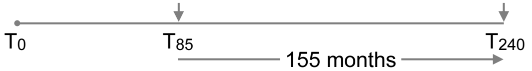
= The amount paid over 20 years - the original amount

= (240 × R6 729,95) - R748 000

= R1 615 188 - R748 000

= **R867 188** ≺

7.4



⬆ **The 'present'**

Method 1: Present value

After the 85th instalment,
the number of instalments remaining = 240 - 85 = 155
& the balance of the loan, then

$$= \frac{6\,729{,}95\left[1-\left(1+\frac{0{,}09}{12}\right)^{-155}\right]}{\frac{0{,}09}{12}}$$

= **R615 509,74** ≺

Method 2: Future value

At this stage:
The value of the loan,

- The amount owed ➡ . . . $A = 748\,000\left(1+\frac{0{,}09}{12}\right)^{85}$

  = 1 411 663,73 STOre in A

whereas:
The value of the annuity,

- The amount paid ➡ . . . $F_v = \frac{6\,729{,}95\left[\left(1+\frac{0{,}09}{12}\right)^{85} - 1\right]}{\frac{0{,}09}{12}}$

  = R796 153,96 STOre in B

The balance of the loan = A - Fv = R615 509,77 ≺

= **A - Fv = R615 509,77** ≺ . . . *the remaining amount to be paid*

7.5 The amount owed after month 89

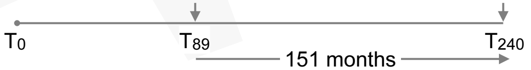
= The accrued amount for the months after month 85

$$= R615\,509{,}74\left(1+\frac{0{,}09}{12}\right)^4$$

Note: No payments were made, so there was nothing to subtract.

= **R634 183,81** ≺ . . . OR: R634 183,84 if the amount from Method 2 in 7.4 was used.

7.6



The present value of the annuity following month 89 must equal the amount owed at that stage.

$$\therefore\ \frac{8\,500\left[1-\left(1+\frac{0{,}09}{12}\right)^{-n}\right]}{\frac{0{,}09}{12}} = 634\,183{,}81$$

$$P_v = \frac{x\left[1-(1+i)^{-n}\right]}{i}$$
where $x = 8\,500$

$\times \frac{0{,}09}{12}$ and $\div$ 8 500:

$$\therefore\ 1 - \left(1+\frac{0{,}09}{12}\right)^{-n} = 0{,}55957\ldots.$$

$$\therefore\ 0{,}44042605 = \left(1+\frac{0{,}09}{12}\right)^{-n}$$

$$✱\ \therefore\ -n = \frac{\log 0{,}44042605}{\log\left(1+\frac{0{,}09}{12}\right)}$$

$= -109{,}744\ldots$

∴ **n ≃ 110 months** ≺

$a = b^x$
➡ $x = \log_b a$
$\therefore\ x = \frac{\log a}{\log b}$

✱ OR:

$$\log 0{,}44042605 = \log\left(1+\frac{0{,}09}{12}\right)^{-n} \quad \ldots$$

A = B
➡ log A = log B

$$\therefore\ \log 0{,}44042605 = -n\log\left(1+\frac{0{,}09}{12}\right) \quad \ldots \log A^x = x\log A$$

$$\therefore\ \frac{\log 0{,}44042605}{\log\left(1+\frac{0{,}09}{12}\right)} = -n$$

etc.

▶ DIFFERENTIAL CALCULUS [32]

8.1 $\text{f}(x) = 3x^2 - 2$

$\therefore\ \text{f}(x + \text{h}) = 3(x + \text{h})^2 - 2$

$= 3(x^2 + 2x\text{h} + \text{h}^2) - 2$

$= 3x^2 + 6x\text{h} + 3\text{h}^2 - 2$

$\therefore\ \text{f}(x + \text{h}) - \text{f}(x) = 6x\text{h} + 3\text{h}^2$

$\therefore\ \dfrac{\text{f}(x + \text{h}) - \text{f}(x)}{\text{h}} = 6x + 3\text{h}$

$\therefore\ \mathbf{f'(x) = \lim_{h \to 0} \dfrac{f(x + h) - f(x)}{h}}$

$= \lim_{\text{h} \to 0} (6x + 3\text{h})$

$= \mathbf{6x}$ ◄

OR:

$\mathbf{f'(x) = \lim_{h \to 0} \dfrac{f(x + h) - f(x)}{h}}$

$= \lim_{\text{h} \to 0} \dfrac{3(x + \text{h})^2 - 2 - (3x^2 - 2)}{\text{h}}$

$= \lim_{\text{h} \to 0} \dfrac{3(x^2 + 2x\text{h} + \text{h}^2) - 2 - 3x^2 + 2)}{\text{h}}$

$= \lim_{\text{h} \to 0} \dfrac{3x^2 + 6x\text{h} + 3\text{h}^2 - 3x^2}{\text{h}}$

$= \lim_{\text{h} \to 0} \dfrac{6x\text{h} + 3\text{h}^2}{\text{h}}$

$= \lim_{\text{h} \to 0} 6x + 3\text{h}$

$= \mathbf{6x}$ ◄

You must choose one or the other of these layouts.

Either you determine the components required for the definition of a derivative first and then apply the definition.

OR: Start with the definition, remembering to repeat $\lim_{\text{h} \to 0}$ on every line until you find the limit in the last line.

The most important thing is to understand the definition.

8.2 $\text{y} = 2x^{-4} - \frac{1}{5}x$

$\therefore\ \dfrac{\text{dy}}{\text{d}x} = 2 . -4x^{-5} - \frac{1}{5} . x^0$ . . .

If $\text{y} = \text{k}x^\text{n}$; then $\dfrac{\text{dy}}{\text{d}x} = \text{k} . \text{n}x^{\text{n}-1}$

$= \mathbf{-8x^{-5} - \frac{1}{5}}$ ◄ . . . *$x^0 = 1$*

$\left[= \mathbf{-\dfrac{8}{x^{-5}} - \dfrac{1}{5}}\right]$

---

9. $\text{f}(x) = x^3 - 4x^2 - 11x + 30$

9.1 **f(2) = 0 ➡ $\boldsymbol{x - 2}$ is a factor of f($\boldsymbol{x}$)** ◄

9.2 $\therefore\ \text{f}(x) = (x - 2)(x^2\ .\ .\ .\ .\ x - 15)$ . . . $\left[-2x^2 - 2x^2 = -4x^2\right]$

$= (x - 2)(x^2 - 2x - 15)$ . . . [Check: $-15x + 4x = -11x$ ✓]

$= (x - 2)(x - 5)(x + 3)$

$\text{f}(x) = 0 \Rightarrow x = -3$ or 2 or 5

$\therefore$ **Coordinates of $\boldsymbol{x}$-intercepts: (-3; 0), (2; 0) & (5; 0)** ◄

9.3 At the stationary points: $\text{f}'(x) = 0$

$\therefore\ 3x^2 - 8x - 11 = 0$

$\therefore\ (3x - 11)(x + 1) = 0$

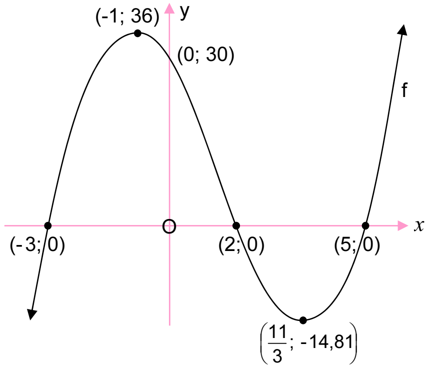
$\therefore\ x = \frac{11}{3}$ or -1

$\text{f}\left(\frac{11}{3}\right) = \left(\frac{11}{3}\right)^3 - 4\left(\frac{11}{3}\right)^2 - 11\left(\frac{11}{3}\right) + 30 \simeq -14{,}81$

& $\text{f}(-1) = (-1)^3 - 4(-1)^2 - 11(-1) + 30 = 36$

$\therefore$ Coordinates of stationary points: $(-1; 36)$ and $\left(\frac{11}{3}; -14{,}81\right)$

9.4



9.5 $\mathbf{-1 < x < \frac{11}{3}}$ ◄ . . . *for these values of x, the gradient of f is negative*

10.1 After t hours:

DC = 40t ; $\therefore$ BC = 100 - 40t ; BF = 30t

$\text{FC}^2 = \text{BF}^2 + \text{BC}^2$

$= (30\text{t})^2 + (100 - 40\text{t})^2$

$= 900\text{t}^2 + 10\ 000 - 8\ 000\text{t} + 1\ 600\text{t}^2$

$= 2\ 500\text{t}^2 - 8\ 000\text{t} + 10\ 000$

$\therefore\ \mathbf{FC = \sqrt{2\ 500t^2 - 8\ 000t + 10\ 000}}$ ◄

10.2 Min FC occurs when FC^2 is a minimum

$\therefore\ \text{t} = -\dfrac{\text{b}}{2\text{a}} = -\dfrac{-8\ 000}{2(2\ 500)}$

$= 1{,}6$ . . .

$\therefore$ **After 1 hr and 36 min** ◄

[OR: the derivative (of FC^2) = 0

$\therefore$ 5 000t - 8 000 = 0, etc.]

10.3 Min FC $= \sqrt{2\ 500(1{,}6)^2 - 8\ 000(1{,}6) + 10\ 000}$

$=$ **60 km** ◄

▶ PROBABILITY [16]

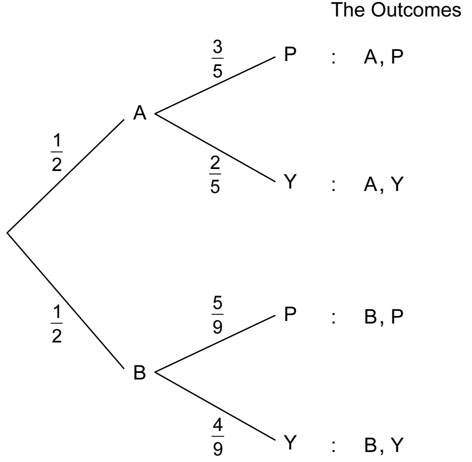
11.1 P(A or B) = P(A) + P(B) . . . *A & B are mutually exclusive*

$\therefore\ 0{,}57 = \frac{1}{2}\text{P(B)} + \text{P(B)}$ . . . *$2P(A) = P(B) \Rightarrow P(A) = \frac{1}{2}P(B)$*

$\therefore\ 1{,}5\ \text{P(B)} = 0{,}57$

$\therefore$ **P(B) = 0,38** ◄

11.2.1



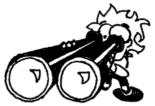
11.2.2 $P(A, Y) = \frac{1}{2} \times \frac{2}{5} = \mathbf{\frac{1}{5}}$ ◄

11.2.3 $P(\text{Pink}) = P(A, P) + P(B, P)$

$= \frac{1}{2} \times \frac{3}{5} + \frac{1}{2} \times \frac{5}{9}$

$= \frac{3}{10} + \frac{5}{18}$

$= \mathbf{\frac{26}{45}}$ ◄



12.1 5 choices 4 choices 3 choices 2 choices 1 choice

∴ The number of different 5-letter arrangements

$= 5 \times 4 \times 3 \times 2 \times 1$

$= 5!$

$= \mathbf{120}$ ◄

12.2 2 ways 1 way 3 ways 2 ways 1 way
S or T

∴ The number of 5-letter arrangements
starting ST _ _ _ or TS _ _ _

$= 2! \times 3!$

∴ The PROBABILITY of this

$= \frac{2! \times 3!}{120}$ . . . $P(E) = \frac{n(E)}{n(S)}$

$= \mathbf{\frac{1}{10}}$ ◄