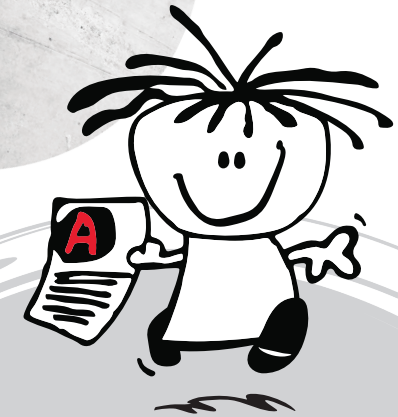

ABOUT TAS

WHO is The Answer Series?

- Family-run company with a focus on SA education
- Creators of learner-friendly study guides
- Written for high school learners and teachers
- Proven track record of over 45 years
- Authors are subject specialists

The Answer Series

What does TAS offer in Mathematics?

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- Gr 8 Learner Workbook 1
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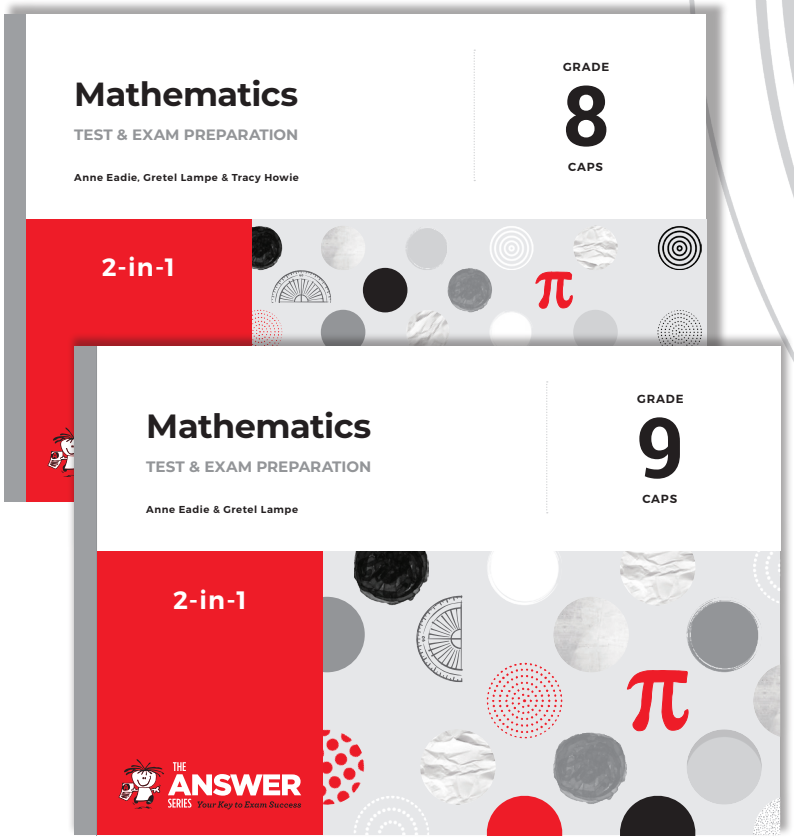
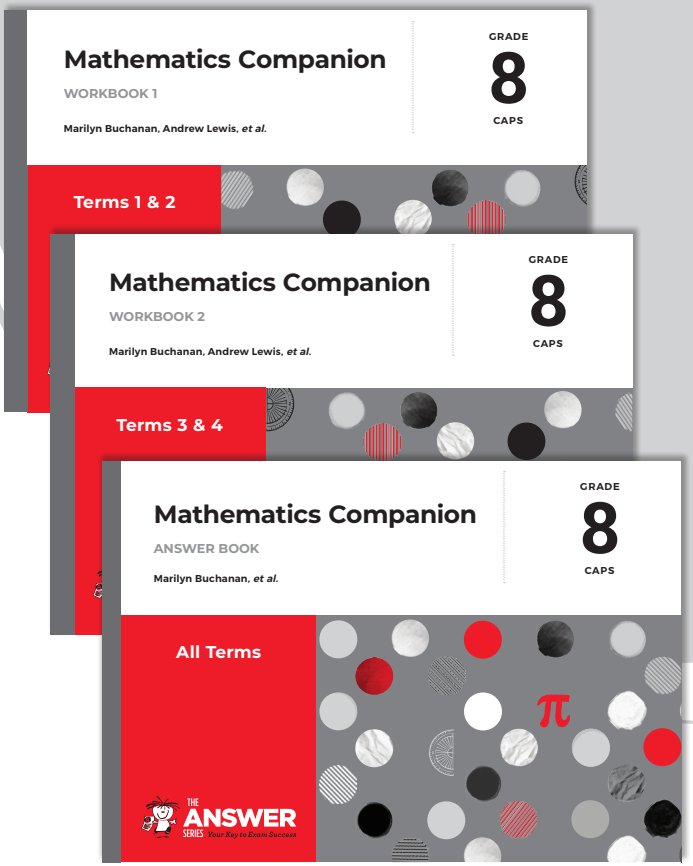
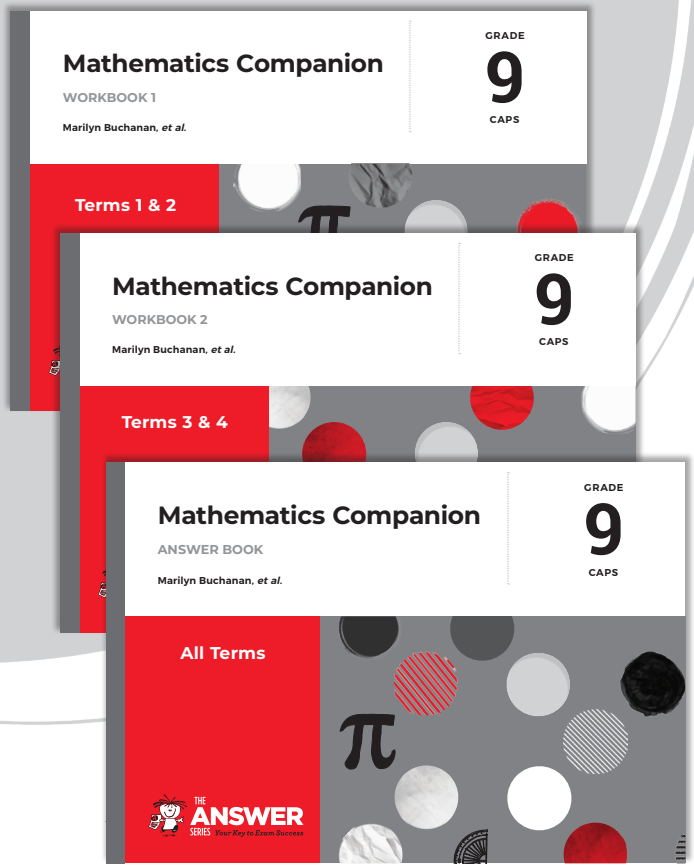
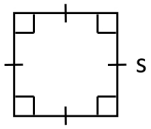
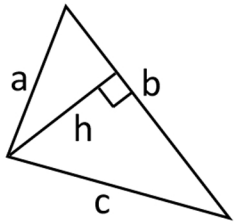
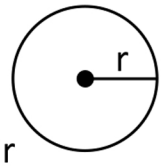
- Gr 9 Learner Workbook 1
- Gr 9 Learner Workbook 2
- Gr 9 Answer book







16 AREA & PERIMETER OF 2D SHAPES: FORMULAE

2D Shapes	Perimeter (P) The **sum** of the **outer boundary lengths** of a 2D shape.	Area (A) The **surface enclosed by** the **boundary lengths** of a 2D shape.
Square s = side 	Perimeter = 4 × side ∴ **P = 4s**	Area = side × side = (side)² ∴ **A = s²**
Triangle a = side₁ b = base c = side₂ h = ⊥ height 	Perimeter = side₁ + base + side₂ ∴ **P = a + b + c**	Area = $\frac{1}{2}$ base × ⊥h ∴ A = $\frac{1}{2}$ b × ⊥h OR Area = $\frac{\text{base} \times \text{height}}{2}$ ∴ **A = $\frac{\mathbf{b \times h}}{\mathbf{2}}$**
Circle r = radius d = diameter = 2r 	Circumference = 2 × π × radius OR = π × diameter (where π = $\frac{22}{7}$ or 3,14) ∴ **C = 2πr** **OR C = πd**	Area = π × (radius)² ∴ **A = πr²** 

SI Units & Conversions

Small unit ➡ *big unit :* ÷
Big unit ➡ *small unit :* ×

Perimeter

divide

÷ 1 000 ÷ 100 ÷ 10

km hm dam **m** dm **cm** **mm**

100 × 10

divide

(100)² ÷ (10)²

dm² **cm²** **mm²**

(100)² × (10)²

of **2D** *lengths,*
version factor)².

ys check
e same.

Solving proble
Area and Pe

Exercise 16.2

1. The area of the recta
alongside is 48 cm^2.

Determine the value

2. $2x + 3$

Find algebraic expressions for:

2.1 The area of the rectangle. (3)

2.2 The perimeter of the rectangle. (3)

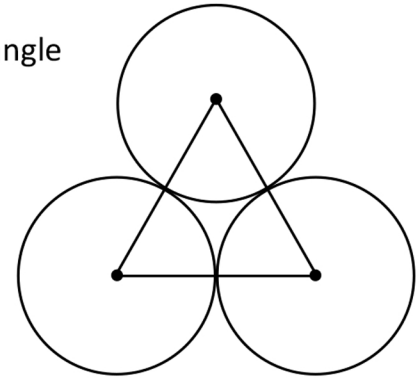
2.3 If the area of the rectangle is 60 cm^2, find the value of x. (4)

3. If the area of a Compact Disc (CD) is 10 568 mm^2, calculate the radius of the CD. (Ignore the hole in the middle.) (4)

4. A circular rotating water spray covers an area of 12 m^2. How far away from the spray would you have to stand if you don't want to get wet? Round off your answer to the nearest metre. (6)

5. The diagram below shows three circles, each with a diameter of 12 cm.

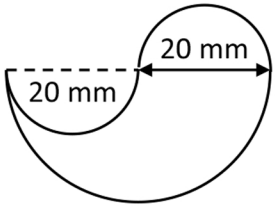
Each vertex of the triangle is at the centre of a circle.



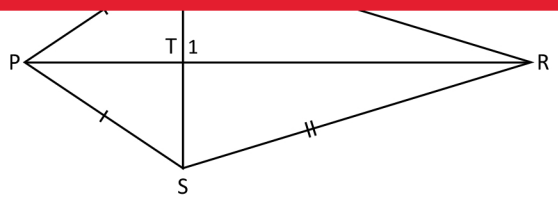
What is the perimeter of the triangle? (4)

7. The Yin-Yang symbol below is made up of a black and a white section. The black teardrop shape is given as a sketch with dimensions.

Determine the perimeter of this teardrop shape. Round the answer off to two decimal places.



(8)



Refer to p. 37 (Q1.7) to refresh your memory!

6.1 Name the quadrilateral PQRS, giving a reason for your answer. (2)

6.2 $\hat{T}_1$ = _____

Give a reason for your answer. (2)

6.3 If PT = 8 cm and QS = 12 cm, calculate the length of PQ giving a reason. (6)

6.4 Now, if TR = 2PT, calculate the perimeter of PQRS to the closest cm. (4)

6.5 Calculate the area of quadrilateral PQRS. (5)

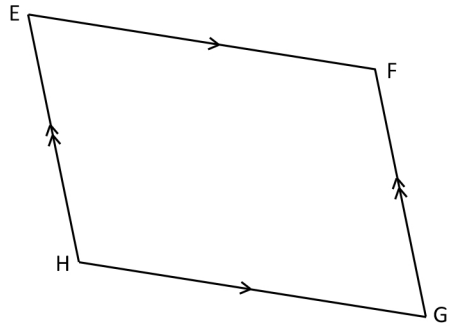
The length and breadth are given in the diagram.

What is the value of x in the diagram? (6)

9. The radius of a car's wheel is 42 cm.

What distance, in kilometres, has the car travelled after 2 000 revolutions of the wheel? (7)

10.



EF = 2EH (EF is twice the length of EH)

If the perimeter of EFGH is 30 cm, calculate the length of FG. (Let EH be x) (6)

PAPER E1

1½ hours
100 marks

Questions on p. E11

Remember: NO CALCULATOR

1.

	Natural	Integer	Rational	Irrational	Real	Imaginary
-3		✓	✓		✓	
4π				✓	✓	
$\sqrt{-7}$						✓
$\sqrt{36}$	✓	✓	✓		✓	

2.1 60 ◄ ... *10 = 2 × 5 and 12 = 2²*
∴ LCM = 2² × 3 × 5
*OR 10, 20, 30, 40, 50, **60***
*12, 24, 36, 48, **60**, 7*

2.2 **Note: No calculator** allowed!

$\sqrt{169} = 13$ ... $13^2 = 169$

$\therefore \sqrt{163} < 13$

$\therefore$ 13,2 is bigger than $\sqrt{163}$ ◄

2.3 $\sqrt{8} < \sqrt{9} = 3$ and $\sqrt{80} < \sqrt{81} = 9$

$\therefore$ The whole numbers between $\sqrt{8}$ and 3 ; 4 ; 5 ; 6 ; 7 ; 8

$\therefore$ The **number** of whole numbers **= 6** ◄ ... *Be sure the q*

2.4 The smallest product
$= (-7) \times 3 = -21$ ◄ ... *Trial & error*

The smallest will be the number furthest left on the number line!

2.5 $F_{18} = 1 ; 2 ; 3 ; 6 ; 9 ; 18$ ◄

2.6 $\frac{10^7}{5 \times 10^4} = \frac{10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10}{5 \times 10 \times 10 \times 10 \times 10}$

$= \frac{10^3}{5}$

3.1.1 $1\frac{1}{2} + 3\frac{2}{3}$

$= \frac{3}{2} + \frac{11}{3}$

$= \frac{9 + 22}{6}$

$= \frac{31}{6}$

$= 5\frac{1}{6}$ ◄

3.1.2 $1\frac{5}{16} \div 2\frac{11}{12}$

$= \frac{21}{16} \div \frac{35}{12}$

$= \frac{{}^{3}\cancel{21}}{\cancel{16}_{4}} \times \frac{\cancel{12}^{3}}{\cancel{35}_{5}}$

$= \frac{3 \times 3}{4 \times 5}$

$= \frac{9}{20}$ ◄

3.2

A: $\frac{1}{3} + \frac{1}{6} = \frac{2}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2} \neq \frac{1}{9}$

B: $\frac{1}{6} - \frac{1}{4} = \frac{2}{12} - \frac{3}{12} = -\frac{1}{12} \neq \frac{1}{2}$

C: $\frac{1}{2} \times \frac{1}{6} = \frac{1}{12} = 12^{-1}$ ✓

D: $\frac{1}{10} \div \frac{1}{5} = \frac{1}{10} \times \frac{5}{1} = \frac{1}{2} = 2^{-1}$ ✓

C and **D** are true ◄

4.1 The number of cats $= \frac{3}{2 + 3 + 30}$ of 385

$= \frac{3}{\cancel{35}_{1}} \times \frac{\cancel{385}^{11}}{1}$

$= \frac{3 \times 11}{1 \times 1}$

$= 33$ ◄

4.2 ***Hint:*** *Draw a diagram!*

44 potatoes to be peeled

	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	11th
y:											
	3	3	3	3	3	3	3	3	3	3	3
					5	**5**	**5**	**5**	5	5	5
eled	3	6	9	12	20	28	36	44			

er of potatoes which Charles peeled $= 4 \times 5 = 20$ ◄

Note: *The total of 44 potatoes were peeled by the 8th minute.*

mber of potatoes peeled
in the 1st 4 minutes: $4 \times 3 = 12$ ... *Matthew*
& thereafter:
3 + 5 = 8 per minute ... *Matthew & Charles*
for the remaining
44 – 12 = 32 potatoes
∴ 4 minutes ... $\frac{32 \text{ potatoes}}{8 \text{ per min}}$

Number of potatoes Charles peeled $= 4 \times 5 = 20$ ◄

$\times \frac{y}{z} = \frac{2}{3} \times \frac{7}{5}$ ... ***Note*** *the possibility of 'removing' y by cancelling.*

$\therefore \frac{x}{z} = \frac{14}{15}$

$\therefore \frac{z}{x} = \frac{15}{14}$ ◄ *If fractions are equal then their inverses are equal.*

Mixed Questions

A Summary of the Laws of Exponents using algebra (letters)

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

$$(a \times b)^n = a^n \times b^n$$

$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n} = \left(\frac{1}{a}\right)^n$$

5.1 Without the use of a calculator, determine which is larger: 2^{-5} or 5^{-2}? (3)

5.2 $-5^2 \div \sqrt[3]{-1}$ (3)

5.3 $\sqrt{3^2 + 3^1 + 3^0 + 3^{-1} + 3^{-2}}$ (4)

5.4 $\sqrt[3]{27} - \sqrt{49} + 4$ (3)

5.5 $\sqrt{9 + 16}$ (1)

5.6 $\sqrt{\frac{18}{2}} - (-2)^3$ (4)

5.7 $\sqrt[3]{\frac{27}{8}} + \sqrt{2\frac{1}{4}}$ (4)

Remember to use the exponent laws

Simplify, leaving the answer with positive exponents where applicable:

5.8 $\sqrt{36x^{36} + 64x^{36}}$ 5.9 $\sqrt{9x^{10} + 16x^{10}}$ (3)(3)

5.10 $\sqrt{144a^6b^{10}} \cdot (-2ab^2)$ (3)

5.11 $\sqrt{9(a+2b)^2x^4}$ 5.12 $\sqrt{169x^{14}y^{22}}$ (3)(3)

5.13 $\sqrt{\frac{4a^3}{a^9}}$ (2)

5.14 $\left(\frac{5x^{-1}}{2}\right)^{-1} \cdot \sqrt{25x^4y^{-2}}$ (3)

5.15 $\sqrt{\frac{(ab^2)^3}{a(-4c)^2}}$ (4)

5.16 $\frac{(3x^5y^{-3})^2}{-6y^5} \times \frac{2x^{-4}}{x^0}$ (6)

5.17 $\frac{2xy^2}{3m} \div \frac{4y^2}{9xm}$ (3)

5.18 $\frac{3x}{y^2z^3} \div \frac{9x^2}{2yz^4}$ (3)

5.19 $\frac{(2ac^2)^4}{4a^4} \times \frac{(a^2b^3)^2}{(b^2c)^3}$ (5)

5.20 $\frac{(5x^{-3}y)^2}{4xy \times (x^2y)^2} \div \left(\frac{5xy}{2}\right)^2$ (5)

5.21 $\frac{xy^3}{x^2} \div \frac{x^3y}{x^4} \times \frac{x^0}{x^2y}$ (6)

5.22 $\frac{x^2y^3}{x^4} \times \frac{x^3y^4}{y^7} \div \frac{x^2}{x^3}$ (6)

5.23 $\frac{x^4y}{y^0} \div \frac{xy^3}{x^2} \div \frac{x^2y^3}{x^{-3}y^4}$ (6)

5.24 $\frac{\sqrt{49a^6b^{-2}} \cdot (3a^2b^{-1})^{-2}}{2a^0b^{-3}}$ (4)

6.1 Determine which of the following has the largest value (you may use a calculator):

$\left(\frac{1}{7}\right)^{-7}$ $7^{\frac{10}{7}}$ $\sqrt[7]{777777}$ $777{,}777^{\frac{1}{7}}$ $0{,}7^{-0{,}7}$ (2)

6.2 Round off each of the following to three dec. places.

6.2.1 $\pi \times \sqrt[5]{237}$ 6.2.2 $\left(\frac{1}{7}\right)^{-7} \times 0{,}135$ (1)(1)

6.3 Express the number 32 as a power with base of 4. [Hint: Remember: $2^2 = 4$] (2)

9.1 For each of the following questions, four options have been given for the answer. Only one of the options provided is correct. In each case, write down the correct letter. (10)

$\frac{x^2}{y^2} = \ldots$

A: $\frac{x}{y}$ B: $\frac{y}{x}$

C: $x - y$ D: cannot be simplified

$\sqrt{36x^{16}} = \ldots$

A: $6x^8$ B: $6x^4$

C: $18x^8$ D: $18x^4$

9.3 $\sqrt{\frac{64a^{16}}{b^{36}}} = \ldots$

A: $\frac{8a^8}{b^{18}}$ B: $\frac{8a^4}{b^6}$

C: $\frac{32a^8}{b^{18}}$ D: $\frac{64^2a^{32}}{b^{72}}$

9.4 If $p = -\frac{1}{2}$ then $-p^2 = \ldots$

A: $\frac{1}{4}$ B: $-\frac{1}{4}$

C: -1 D: 1

9.5 $a^{-1} + b^{-1} = \ldots$

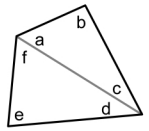
A: $\frac{1}{a+b}$ B: $\frac{1}{ab}$

C: $-a - b$ D: $\frac{1}{a} + \frac{1}{b}$

QUADRILATERALS - pathways of definitions, areas and properties - A Summary

All you need to know!

'Any' Quadrilateral



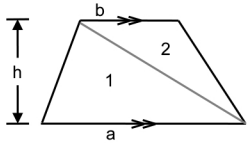
Sum of the $\angle^s$ of *any quadrilateral* = 360°

Sum of the interior angles
= (a + b + c) + (d + e + f)
= 2 × 180° . . . (2 Δ^s)
= 360°

The arrows indicate various 'pathways' from 'any' quadrilateral to the square (the 'ultimate quadrilateral'). These pathways, which combine logic and fact, are essential to use when proving specific types of quadrilaterals.

See how the properties accumulate as we move from left to right, i.e. the first quad has no special properties and each successive quadrilateral has all preceding properties.

A Trapezium

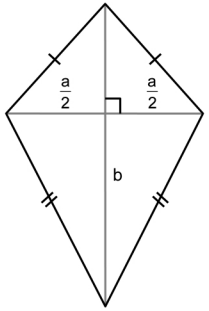


DEFINITION:
Quadrilateral with **1 pair of opposite sides ||**

Area = $\Delta 1 + \Delta 2$
= $\frac{1}{2}$ah + $\frac{1}{2}$bh
= $\frac{1}{2}$**(a + b).h**

'Half the sum of the || sides × the distance between them.'

A Kite



DEFINITION:
Quadrilateral with **2 pairs of adjacent sides equal**

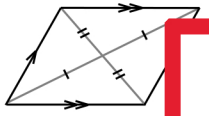
Given diagonals a and b

Area = $2\Delta^s = 2\left(\frac{1}{2}b.\frac{a}{2}\right) = \frac{ab}{2}$

'Half the product of the diagonals'

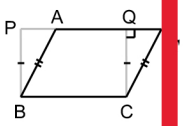
THE DIAGONALS
- *cut perpendicularly*
- ***ONE DIAGONAL** bisects the other diagonal, the opposite angles and the area of the kite*

A Parallelogram



DEFINITION:
Quadrilateral with **2 pairs op**

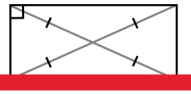
Area = base × height



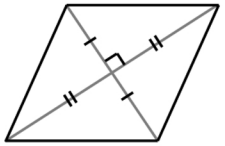
$||^m$ ABCD = ABCQ + ΔQCD
rect. PBCQ = ABCQ + ΔPB
where ΔQCD ≡ ΔPBA . . .
∴ $||^m$ ABCD = rect. PBCQ (
= BC × QC

Properties:
2 pairs opposite sides eq
2 pairs opposite angles e
& DIAGONALS BISECT ON

A Rectangle



A Rhombus

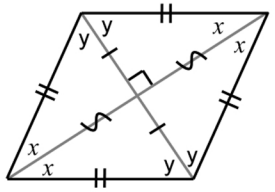


DEFINITION:
A $||^m$ with one pair of adjacent sides equal

Area
= $\frac{1}{2}$ **product of diagonals** (as for a kite)
or
= **base** × **height** (as for a parallelogram)

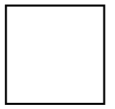
THE DIAGONALS
- *bisect one another PERPENDICULARLY*
- *bisect the angles of the rhombus*
- *bisect the area of the rhombus*

Note:



2x + 2y = 180° . . . $\angle^s$ *of* Δ *or*
➡ x + y = 90° *co-int.* $\angle^s$ *; || lines*

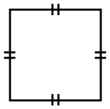
The Square

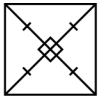


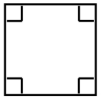
e 'ultimate' quadrilateral!

Area = s^2

Properties:
's all been said 'before'!
ce a square is a rectangle, hombus, a parallelogram, te, . . . ALL the properties hese quadrilaterals apply.

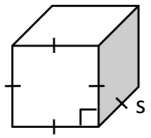
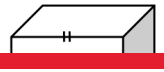
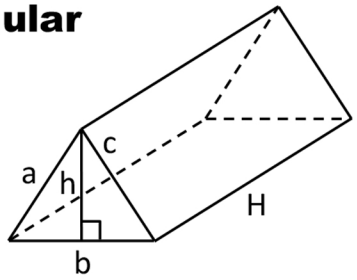
sides 

diagonals 

angles 

Quadrilaterals play a prominent role in both Euclidean and Analytical Geometry right through to Grade 12!

18 VOLUME & TOTAL SURFACE AREA OF 3D OBJECTS: FORMULAE

3D Objects	TSA = sum of areas of all surfaces	Volume = Area of base × Height
	The **total exterior area** of all the exposed surfaces of a 3D shape.	The **3D space** that a 3D object occupies.
Cube s = side 	Total Surface Area = (side × side) × 6 = 6(side)² ∴ **TSA = 6s²**	Volume = side × side × side = (side)³ ∴ **V = s³**
Rectangular Prism ℓ = length b = breadth 	Total Surface Area = 2 (length × breadth) + 2 (length × height) +	Volume = length × breadth × height ∴ **V = ℓ x b x h**
Triangular Prism a = side₁ b = side₂ (base of the triangle face) c = side₃ h = ⊥ height of Δ H = height of prism (distance between the 2 bases) 	Total Surface Area $= 2\left(\frac{b \times h}{2}\right)$ + (side₁ × prism height) + (side₂ × prism height) + (side₃ × prism height) ∴ **TSA** **$= 2\left(\frac{b \times h}{2}\right)$** **+ (a × H) + (b × H)** **+ (c × H)**	Volume = area of base × height of prism ∴ **$V = \left(\frac{b \times h}{2}\right) \times H$** **Remember:** Area of Δ $= \frac{1}{2} b \times h$ OR $\frac{b \times h}{2}$ 

SI Units & Conversions

Small unit ➡ *big unit:* ÷
Big unit ➡ *small unit:* ×

Total Surface Area (TSA)

divide ⬅

÷ (1 000)² ÷ (100)² ÷ (10)²

km² hm² dam² **m²** dm² **cm²** **mm²**

× (1 000)² × (100)² × (10)²

multiply ➡

*Since the area is the product of **2D** lengths, by the **(conversion factor)²**.*

divide ⬅

÷ (10)³

cm³ **mm³**

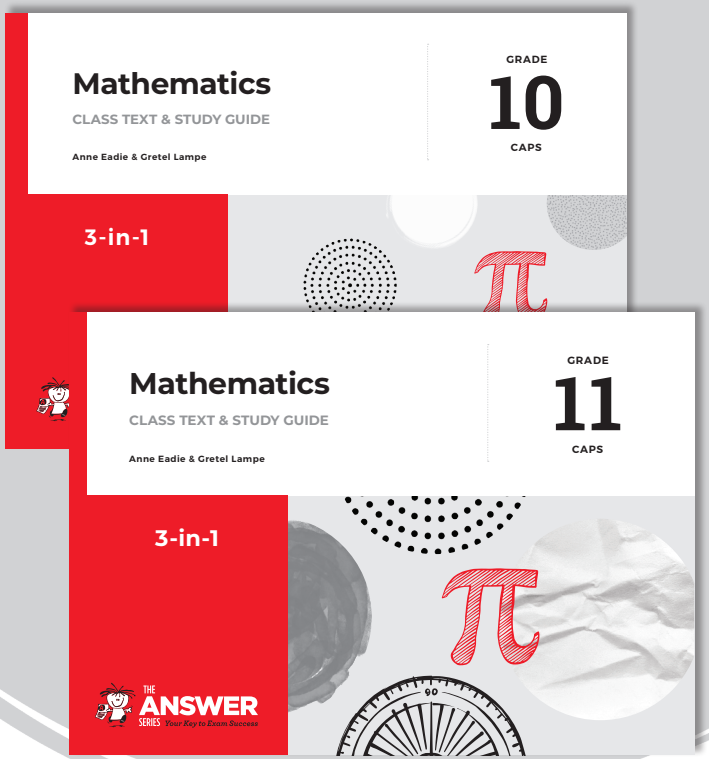
× (10)³

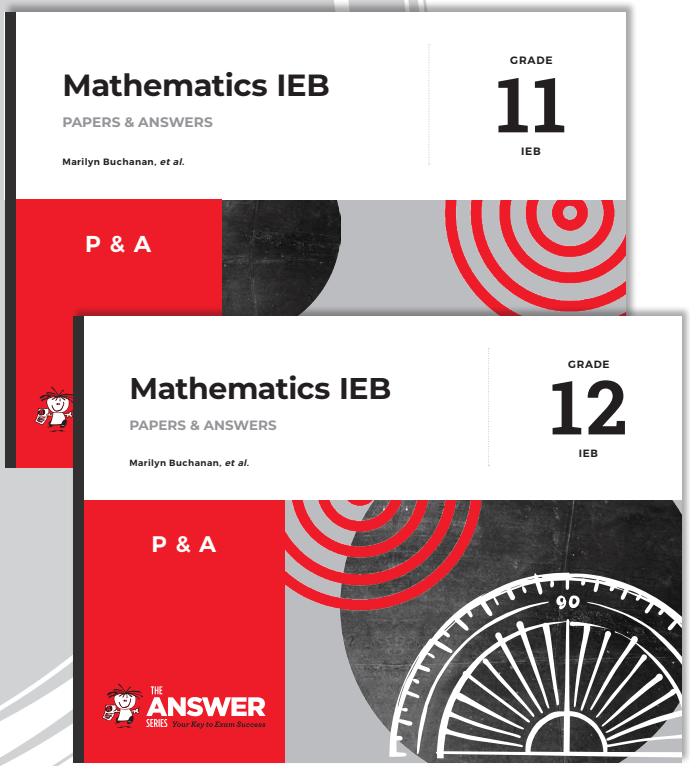
*the product of **3D** lengths, by the **(conversion factor)³**.*

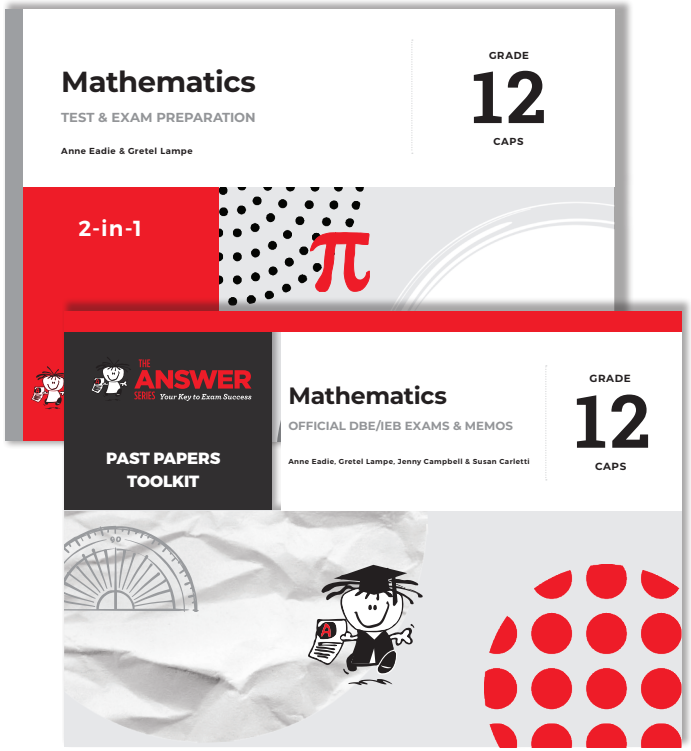
What does TAS offer in Mathematics?

Grade 10-12

- Gr 10 Maths 3-in-1
- Gr 11 Maths 3-in-1
- Gr 11 Maths P & A
- Gr 12 Maths P & A
- Gr 12 Maths 2-in-1
- Gr 12 Maths PAST PAPERS TOOLKIT







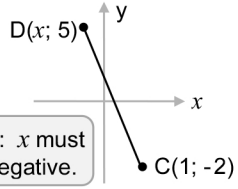
3.2 $CD^2 = (x - 1)^2 + (5 + 2)^2 = \left(\sqrt{53}\right)^2$

$\therefore (x - 1)^2 + 49 = 53$

$\therefore (x - 1)^2 = 4$

$\therefore x - 1 = \pm 2$

$\therefore x = 3$ or -1



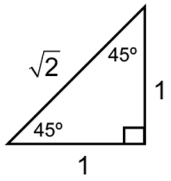
Note: x must be negative.

But $x < 0$ in the second quadrant

4.1.1 $\sin C = \frac{AB}{\mathbf{AC}}$ ◄

4.1.2 $\mathbf{cot}\ A = \frac{AB}{BC}$

Note: $\tan A = \frac{BC}{AB}$; $\cot A = \frac{1}{\tan A}$



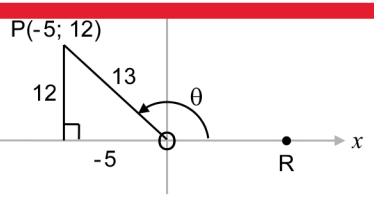
4.2 The expression

$= \frac{\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}}}{\sqrt{2}}$

$= \frac{1}{2} \times \frac{1}{\sqrt{2}}$

$= \frac{1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$ ... *The denominator must be rationalised*

$= \frac{\sqrt{2}}{4}$ ◄ ... $\sqrt{2} \times \sqrt{2} = 2$



$\therefore \cos\theta = \frac{-5}{13} = -\frac{5}{13}$ ◄ ... $\cos\theta = \frac{x}{r}$

4.3.2 $\sin\theta = \frac{12}{13}$ ➡ $\text{cosec}\ \theta = \frac{13}{12}$

$\therefore \text{cosec}^2\theta + 1 = \left(\frac{13}{12}\right)^2 + 1 = \frac{169}{144} + 1$

$= \frac{169 + 144}{144} = \frac{313}{144}$ ◄ $\left(= 2\frac{25}{144}\right)$ ◄

5.1.1 $5\cos x = 3$

÷ 5) $\therefore \cos x = \frac{3}{5}$ (= 0,6)

$\therefore x \approx 53{,}1°$ ◄ ... $\cos^{-1}\left(\frac{3}{5}\right) =$

$\tan 2x = 1{,}19$

$\therefore 2x = 49{,}958...°$ ... $\tan^{-1} 1{,}19 =$

÷ 2) $\therefore x \approx 25{,}0°$ ◄

$4\sec x - 3 = 5$

+ 3) $\therefore 4\sec x = 8$

÷ 4) $\therefore \sec x = 2$

$\therefore \cos x = \frac{1}{2}$

$x = 60°$ ◄ ... $\cos^{-1}\left(\frac{1}{2}\right) =$

$J\hat{K}D = 8°$ ◄ ... *alternate ∠'s; || lines*

In ΔJDK: $\frac{DK}{5} = \cot 8°$ ... $= \frac{1}{\tan 8°}$

× 5) $\therefore DK = \frac{5}{\tan 8°}$

$= 35{,}5768...$ km

$= 35\ 576{,}8$ metres

$\approx 35\ 577$ metres ◄

... *correct to the nearest metre*

5.2.3 DS = DK – SK

= 35,58 km – 8 km

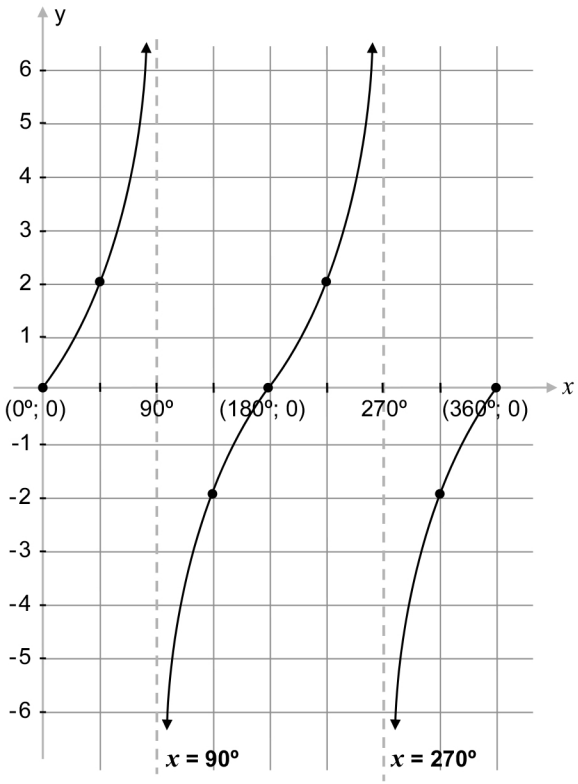
= 27,58 km ◄

5.2.4 $\tan J\hat{S}D = \frac{5}{27{,}58}$

$\therefore J\hat{S}D \approx 10{,}3°$ ◄ ... $\tan^{-1}\left(\frac{5}{27{,}58}\right) =$

correct to 1 dec. place

6.1.1



6.1.2 $y = -2\tan x$ ◄

6.2.1 $a = 4$ ◄

$g(x) = a\sin x$ ➡ $g(90°) = a\sin 90°$ ➡ $4 = a$

6.2.2 The range of h:

$-2 \le y \le 6$ ◄ ... *the values of y*

EXAM MEMOS: P

FACTORISING – SOME GOOD ADVICE

How many terms do I have?
The number of terms determines my options.

NO. OF TERMS	OPTIONS	
2 terms	Always look out for a **common factor** FIRST!	**The difference of two squares OR** **The sum of two cubes or the difference of two cubes**
3 terms		**Trinomial**
4 terms	**Grouping** ... watch out for switchrounds!	2 - 2 – **for common 'brackets',** or 3 - 1 or 1 - 3 – **leading to difference of squares**
5 terms		3 - 2 or 2 - 3 – **for common 'brackets'**
6 terms		3 - 3 – **for common 'brackets'** or **difference of squares** or 2 - 2 - 2 – **for common 'brackets'**

Remember: factorising ⇔ multiplying out . . . **REVERSE PROCESSES!**

- **Ask yourself: 'Have I finished?'**
  Double-check for any simplification or any further factorisation *(e.g. common factor or difference of squares).*

e.g. (1) $2x(a + b) + 4(a + b)$
$= (a + b)(2x + 4)$
$= 2(a + b)(x + 2)$

(2) $x^4 - y^4$
$= (x^2 + y^2)(x^2 - y^2)$
$= (x^2 + y^2)(x + y)(x - y)$

Knowing that there are guidelines shifts the mind into a more confident way of thinking.

VE FACTORISATION TESTS *(approximately ½ hour each)*

$x^2 - x - 12$	2.	$8ax - 12ay - 10x + 15y$	3.	$(x + 5)(x + 3) + k(3 + x)$	(2)(4)(2)
$p^2 - 14p - 32$	5.	$4m - pm + 8 - 2p$	6.	$12x^2 - 19xy - 21y^2$	(2)(2)(2)
$(x - y)^3 - 3(x - y)^2$	8.	$2a^2 - 18$	9.	$28ab + 4a^2 - 15b^2$	(3)(2)(2)
$ac + yd - ad - yc$	11.	$3k(2m - 3n) + 5t(3n - 2m)$	12.	$(a - b)^2 - 49$	(2)(3)(2)
$12x^3 + 11x^2 - x$	14.	$x^6 - 64y^6$	15.	$x(x - 1)(x - 2) - (x - 1)^2$	(3)(3)(4)
$1 - 16a^{16}$	17.	$-6m^2 + 11m + 10$	18.	$4x - ax + ay - 4y$	(4)(2)(2)

Write down the simplest expression (in factorised form) into which the expressions in questions 6, 7 & 18 can divide (i.e. the lowest common multiple). (1)

Calculate the value of $109^2 - 9^2$ in the shortest possible way, without using a calculator. (3) [50]

$pa + pb + qa + qb$	2.	$x^2 + 5x + 6$	3.	$4x^2 - 9$	(2)(1)(1)
$5at + 9 + 3a + 15t$	5.	$4x^2 - 20x + 25$	6.	$5 - 20a^2$	(3)(2)(2)
$3ac + 2bc - 2bd - 3ad$	8.	$3y^2 + 15y - 108$	9.	$ac + 6b - 3ab - 2c$	(2)(2)(3)
$p^3 - 8$	11.	$52^2 - 50^2$ (evaluate)	12.	$132x^2 + 96xy - 36y^2$	(2)(3)(3)
$9x^2 - 5y - 3x - 25y^2$	14.	$8m^2 - 50mn + 33n^2$	15.	$x^4 - x^3 + x - 1$	(3)(3)(3)
$12mb + 9a^2 - 4m^2 - 9b^2$	17.	$x^2 - 2xy - a^2 + y^2$	18.	$k^4 - 37k^2 + 36$	(3)(3)(3)
$16(2a + b)^2 - 9(a - 2b)^2$					(6) [50]

$x^2 - 25xy + 144y^2$	2.	$x^2 - 24xy + 144y^2$	3.	$2ac - 3ad - 2bc + 3bd$	(2)(2)(3)
$k(a - b) + n(b - a)$	5.	$122^2 - 120^2$ (evaluate)	6.	$2a^2 - 2a - 12$	(2)(3)(3)
$ax - b^2 - bx + ab$	8.	$a^2x^2 + 5ax - 24$	9.	$x^2 - 2\frac{1}{4}$	(3)(2)(3)
$20m^2n + 62mn^2 - 28n^3$	11.	$ax^2 + 3by^2 - 3bxy - axy$	12.	$125x^3 + y^3$	(3)(3)(2)
$20x^2 - 45y^2$	14.	$3a^3 + 12a^2b + 9ab^2$	15.	$x^2 - 2x + 2y - y^2$	(3)(3)(3)

16. Factorise $a^2 - b^2$ and then write down the factors of $(2x^2 - 4x + 1)^2 - (x^2 - 3x + 3)^2$ in the simplest form. (10) [50]

D

1.	$2x^2 - 8$	2.	$a^2 - b^2 + a - b$	3.	$x^2 - 12x + 36$	(3)(3)(2)
4.	$\left(r + \frac{1}{r}\right)^2 - \left(r - \frac{1}{r}\right)^2$	5.	$10x^2 + 38xy - 8y^2$	6.	$2x^3 - x^2 + 4 - 8x$	(3)(3)(3)
7.	$40ap^2 + 82a^2p + 40a^3$	8.	$12ab + 8b^2 - 6af - 4bf$	9.	$-3x^2 + 21x - 30$	(3)(3)(3)
10.	$3 - 3(x - y)^2$	11.	$x^2 + 8 + \frac{16}{x^2}$	12.	$k(x^3 - 1) - k(x - 1)^3$	(3)(2)(5)
13.	$4p^2(3p - 1) - 5p$	14.	$x^2 - y^2 + 4x + 4$	15.	$3a^3 - 24b^3$	(4)(4)(3)

16. If $P = 3x + 2$ and $Q = 2x - 1$, express $P^2 - 2PQ + Q^2$ in terms of x. (3) [50]

Worked example

Solve for x and y: $2x + 3y = -1$ **. . . (1)**

$3x - 6y = -12$ **. . . (2)**

We first need to alter equation number **(1)** so that adding it to or subtracting from equation number **(2)** will eliminate one of the variables:

(1) $\times 2$: $4x + 6y = -2$ **. . . (3)** *Now, observe the coe of y in* **(2)** *and* **(3)**

(2) + **(3)**: $\therefore 7x = -14$ . . . y has been **eliminated!**

$\therefore x = -2$

OR: Eliminate x – see belo

Subst. $x = -2$ in **(1)**: $\therefore -4 + 3y = -1$

$\therefore 3y = 3$

$\therefore y = 1$

$\therefore$ Solution: **(-2; 1)** ➤ . . . .

This is an **ORDERED PAIR**
– *x first, then y second!*
– a good way to give the sol of 2 equations with 2 unkn

CHECK THIS ANSWER BY substituting the values into the equations **(1)** and **(2)** to see whether they hold true!

We could've gone for eliminating x (instead of y):

(1) $\times 3$**:** $6x + 9y = -3$ **. . . (3)**

(2) $\times 2$**:** $6x - 12y = -24$ **. . . (4)**

(3) - **(4)**: $\therefore 21y = 21$ . . . x has been **eliminated**

$\therefore y = 1$

Subst. y = 1 in **(1)**: $2x + 3 = -1$

$\therefore 2x = -4$

$\therefore x = -2$, etc.

Can you solve *these* equations?

$ar^6 = 162$ **. . . (1)**

$ar^2 = 2$ **. . . (2)**

Let us try addition . . .

(1) + (2): $ar^6 + ar^2 = 162 + 2$

$a(r^6 + r^2) = 164$ *and now what ? ? ? – no good!*

Addition and subtraction don't work, do they! What else can we try?
Equations can also be multiplied or divided:

because, **if** $a = b$ **and** $c = d$

then $a.c = b.d$ ***and*** $\frac{a}{c} = \frac{b}{d}$

Which of these is best to use in our example to eliminate one of the variables?

Answer

(1) ÷ (2): $\frac{ar^6}{ar^2} = \frac{162}{2}$ . . . Division seems to be the way to **eliminate a**

$\therefore r^4 = 81$

$\therefore r = \pm 3$

Subst. $r = \pm 3$ in **(2):** $a \times 9 = 2$

$\therefore a = \frac{2}{9}$ ➤

Even though this sum **looks** different (and is!) the **LOGIC** is the same!

Remember:
It is possible to check your answers!

$3p - q = 10$

$3x + 4y = 11$

$y = 3 - x$

6. $\frac{y}{2} + 1 = \frac{x}{5}$ and $\frac{1}{4}x + \frac{1}{2} = \frac{1}{3}y$

7. $\frac{x+y}{2} = 7 - \frac{2x-y}{3}$ . . . ❶ and $\frac{x-y}{4} - \frac{x+y}{3} + 4\frac{1}{2} = 0$ . . . ❷

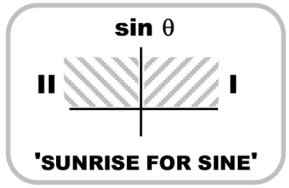
8. The length of a rectangle is a mm and the breadth is b mm. The area of the rectangle is unchanged if the length is increased by 6 mm and the breadth is diminished by 2 mm. The area is also unchanged if the length is decreased by 6 mm and the breadth is increased by 3 mm. Find the length and breadth of the original rectangle.

The SIGNS of the trig ratios IN A FLASH!

$\sin\theta = \frac{y}{r}$ ← and **y is positive** in **I** and **II**

∴ **sin θ is POSITIVE** in quadrants 1 & 2

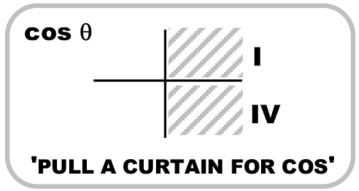
(and negative in 3 & 4)



$\cos\theta = \frac{x}{r}$ ← and **x is positive** in **I** and **IV**

∴ **cos θ is POSITIVE** in quadrants 1 & 4

(and negative in 2 & 3)

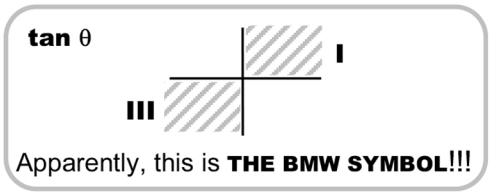


$\tan\theta = \frac{y}{x}$ ← ←

and **x & y** have the same sign in **I** and **III**

∴ **tan θ is POSITIVE** in quadrants 1 & 3

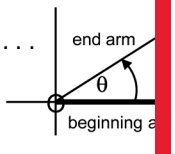
(and negative in 2 & 4)

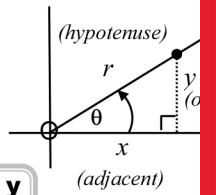


NO MORE CAST RULE!!!

The 4 steps to find the trig ratios of any angle

1. **Place the ∠ in STANDARD POSITION** (starting at $\overrightarrow{OX}$) . . .
2. **Pick a point (x; y) on the end arm of the ∠**
   – we'll call its distance from the origin **r**
3. **Write down x = y = r =**
4. **Apply the DEFINITIONS**

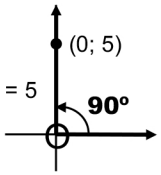




$\sin\theta = \frac{y}{r}$ $\quad \cos\theta = \frac{x}{r}$ $\quad \tan\theta = \frac{y}{x}$

The trig ratios of 90° and multiples of 90°

Use this procedure to find the trig ratios of 90° ; 180° ; 270° & 360° (& 0°)

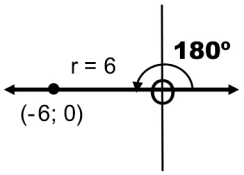


$\sin 90° = \frac{y}{r} = \frac{5}{5} = 1$

$\cos 90° = \frac{x}{r} = \frac{0}{5} = 0$

$\tan 90° = \frac{y}{x} = \frac{5}{0} = \infty$

$x = 0$; $y = 5$; $r = 5$

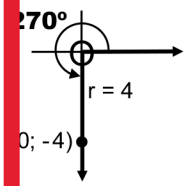


$\sin 180° = \frac{y}{r} = \frac{0}{6} = 0$

$\cos 180° = \frac{x}{r} = \frac{-6}{6} = -1$

$\tan 180° = \frac{y}{x} = \frac{0}{-6} = 0$

$x = -6$; $y = 0$; $r = 6$

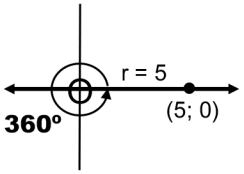


$\sin 270° = \frac{y}{r} = \frac{-4}{4} = -1$

$\cos 270° = \frac{x}{r} = \frac{0}{4} = 0$

$\tan 270° = \frac{y}{x} = \frac{-4}{0} \Rightarrow \infty$

$x = 0$; $y = -4$; $r = 4$



$\sin 360° = \frac{y}{r} = \frac{0}{5} = 0$

$\cos 360° = \frac{x}{r} = \frac{5}{5} = 1$

$\tan 360° = \frac{y}{x} = \frac{0}{5} = 0$

$x = 5$; $y = 0$; $r = 5$

Note: The results for 0° and 360° are the same.

SUMMARY

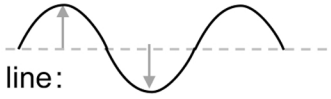
θ:	0°	⟶	90°	⟶	180°	⟶	270°	⟶	360°
sin θ:	0	- - - ⟶	1	- - - ⟶	0	- - - ⟶	-1	- - - ⟶	0
cos θ:	1	- - - ⟶	0	- - - ⟶	-1	- - - ⟶	0	- - - ⟶	1

Trigonometric graphs

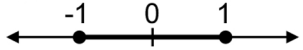
We will learn how to sketch the graphs $y = \sin\theta$, $y = \cos\theta$ and $y = \tan\theta$ for $0° \le \theta \le 360°$.
We will use the critical values of these ratios to make it easy. But first, some terminology . . .

Terminology

The sine and cosine graphs are WAVE-shaped.



- The **amplitude** of a WAVE is the deviation from its centre line:
- The **period** of a graph is the number of degrees spanning a FULL WAVE.
- The **range** is the set of all the possible y-values.
  Our investigations of the trig ratios have shown us that the range of values of sines and cosines is very small - only between **-1** and **1**.
  We write: $-1 \le \sin\theta \le 1$ and $-1 \le \cos\theta \le 1$ for all values of θ!
  By contrast, the range of tan values is from -∞ to +∞!

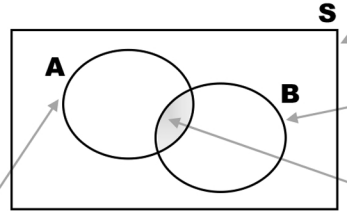


❑ Venn Diagrams

A graphical method to visually represent the outcomes of two or more different events (by means of circles); together with common elements of the events (by means of overlapping circles); as well as the sample space of all the events (by means of a rectangle)

▶ Sample Space & Events

Possible Venn Diagram layout:



Sample Space
– the set of all possible outcomes

Event B
– an event which contains all the possible outcomes of B

Common elements (shaded)
– from event A and event B [should they exist]

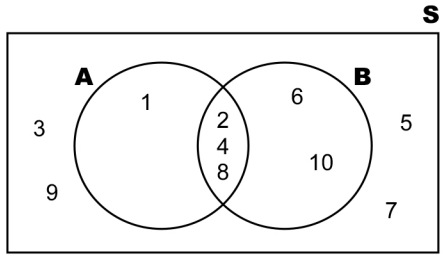
Event A
– an event which contains all the possible outcomes of A

Worked Example 1

Set up a Venn Diagram to illustrate the following:

- A sample space from 1 to 10 (whole numbers only)
- Event A: the factors of 8
- Event B: the multiples of 2

Answer



Event A: Factors of 8 = {1; 2; 4; 8}
∴ n(**A**) = 4 elements

Event B: Multiples of 2 = {2; 4; 6; 8; 10}
∴ n(**B**) = 5 elements

Common elements = {2: 4: 8}

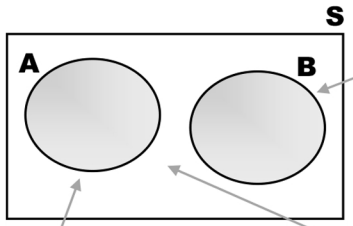
Remember!

- n(event) = number of favourable outcomes in the event
- P(event) = probability of the number of favourable outcomes n(E) divided by the total number of elements in the sample space n(S)

$$\text{i.e. } P(E) = \frac{\text{the number of favourable outcomes that exist } n(E)}{\text{the total number of possible outcomes } n(S)}$$

- The sum of the probabilities in the Venn diagram must be 1 or 100%

▶ Mutually Exclusive / Disjoint Events



Event B
– contains all the possible outcomes of B

Events **A** & **B** are . . .

Event A
– contains all the possible outcomes of A

Mutually exclusive
– the events do not overlap
∴ **they have no common elements**

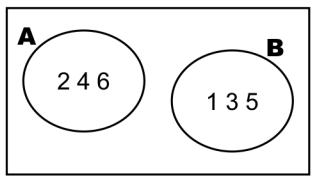
Given that **S** = {1; 2; 3; 4; 5; 6} and that event **A** is all the even numbers and event **B** is all the odd numbers:

(a) Draw a Venn Diagram to illustrate the situation.

(b) Are these events **A** and **B** mutually exclusive? Give a reason for your answer.

Answers

(a)



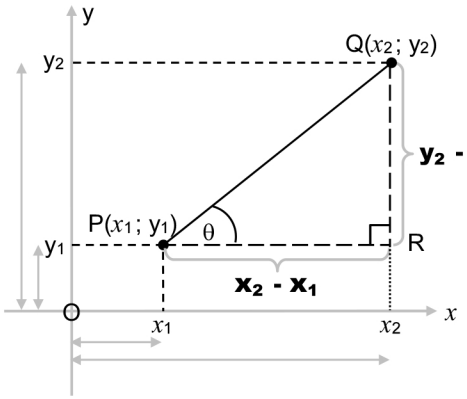
(b) Yes, these events are mutually exclusive as an even number can never be an odd number.

Checklist: The Drawers of Tools

Consider 4 'drawers' of tools - all BASIC FACTS. Use these to analyse the sketches, to reason, calculate, prove . . . .

Distance, Midpoint & Gradient

For any two fixed points, P(x_1; y_1) & Q(x_2; y_2)



Note:

Vertical length QR = $y_2 - y_1$

Horizontal length PR = $x_2 - x_1$

θ is the angle of inclination of the line PQ

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{y_2 - y_1}{x_2 - x_1}$$

1 Distance PQ . . .

$$PQ^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 \quad \Rightarrow \quad PQ = \sqrt{(\quad)^2 + (\quad)^2}$$

the sum of the squares! (Pythag.)

2 Gradient PQ . . .

$$m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1} \ (= \tan\theta)$$

change in y / change in x

3 Midpoint of PQ . . .

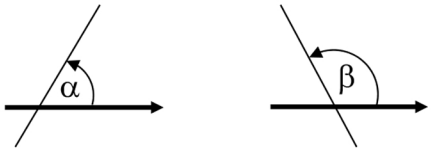
$$\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

Average of the x's & of the y's

Parallel lines, Perpendicular lines & Collinearity

- AB || CD $\Leftrightarrow$ $m_{AB} = m_{CD}$
- AB $\perp$ CD $\Leftrightarrow$ $m_{AB} = -\frac{1}{m_{CD}}$ . . . $m_{AB} = -\frac{1}{m_{CD}}$ also means: $m_{AB} \times m_{CD} = -1$
- A, B and C are collinear points $\Leftrightarrow$ $m_{AB} = m_{AC}$; $m_{AB} = m_{BC}$; $m_{AC} = m_{BC}$

The Angle of Inclination of a line



The ∠ of inclination of a line is the ∠ which the line makes with the positive direction of the x-axis.

the line = tan α or tan β (which is a ratio or number).

∴ Given α or β, one can find **the gradient**: . . . **a number**

Or, given the gradient, one can find α or β: . . . **an angle** (measured in degrees)

Equations of lines

NB: Bear in mind **Case 1**, **Case 2,** and **Case 3** on page 5.9.

- **Standard forms:**
  - General: **$y = mx + c$** or **$y - y_1 = m(x - x_1)$**
  - **$y = mx$** . . . when c = 0 . . . lines through the origin
  - **$y = c$** . . . when m = 0 . . . lines || x-axis
  - **$x = k$** . . . lines || y-axis

Finding the equation of a line: Special focus

- through 2 given points . . . find m first
- through 1 point and || or ⊥ to a given line . . . substitute m and the point.

the gradient

$y = m(x) + c$

the point

we need
- **the gradient** &
- **a point**

the gradient

$y - (y_1) = m(x - (x_1))$

the point

- **Y-cuts and X-cuts:** Put $x = 0$ and $y = 0$, respectively.
- **Point of intersection of 2 graphs:**
  Solve the equations of the graphs simultaneously.
- **If a point lies on a line, the equation is true for it,**
  and, vice versa . . .
  If a point satisfies the equation of a line, the point lies on the line.
  e.g. If a line has the equation $y = x + 1$, then all points on the line can be represented by $(x; x + 1)$.

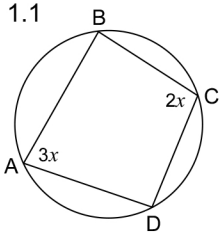
EXERCISE 9.6: Mixed Exercise

Answers on page A9.9

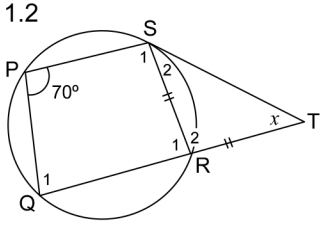
Q1 to Q8: Without tangents

1. Determine, with reasons, the value of x:

1.1

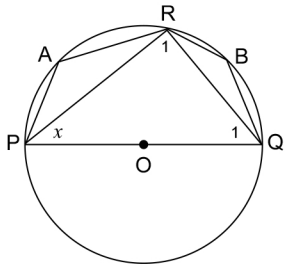


1.2



2.1 O is the centre of the circle. $x = 40°$

Determine, with reasons, the size of $\hat{Q}_1$, $\hat{B}$ and $\hat{A}$.

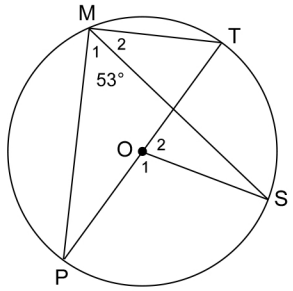


2.2 M, P, S and T are points on a circle with centre O.

PT is a diameter.

MP, MT, MS and OS are drawn. $\hat{M}_1 = 53°$.

Determine, with reasons, the size $\hat{O}_2$.

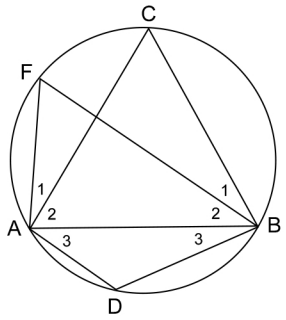


2.3 AB = BC and $A\hat{B}C = 50°$

Calculate, with reasons, the sizes of

2.3.1 $\hat{F}$ and

2.3.2 $\hat{D}$.



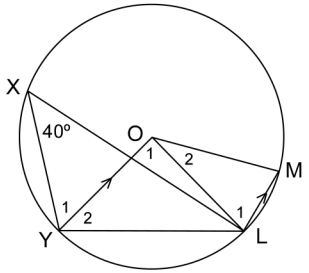
3. OY || ML and $\hat{X} = 40°$.

Calculate, with reasons, the sizes of the following:

3.1 $\hat{O}_1$

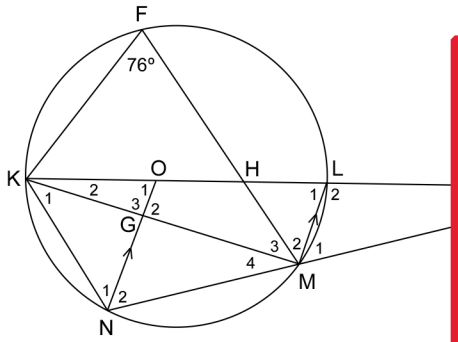
3.2 $\hat{Y}_2$

3.3 $\hat{O}_2$



4. O is the centre of the circle and diameter KL is produced to meet NM produced at P.

ON || LM and $\hat{F} = 76°$.



Calculate, giving reasons, the sizes

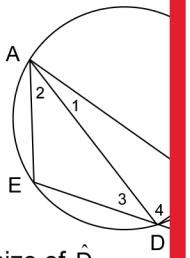
4.1 $\hat{L}_1$ 4.2 $\hat{O}_1$

4.3 $\hat{M}_4$ 4.4 $\hat{N}_1 + \hat{N}$

4.5 $\hat{M}_1$ 4.6 Prove

5. AB is a diameter of the circle.

The chord ED and the diameter AB are produced to meet at C.



5.1 Write down the size of $\hat{D}_4$. Give a reason for your answer.

5.2 If $\hat{A}_1 = 22°$ and $\hat{C} = 24°$, calc of $\hat{D}_5$ and deduce that DA bise

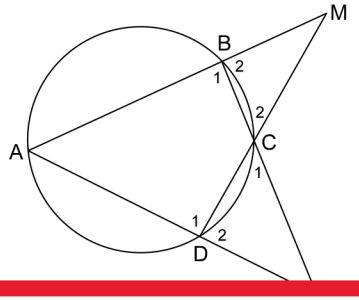
6.1 Complete the following by writing the appropriate missing word.

If a chord of a circle subtends a right angle on the circumference, then this chord is a . . . . . .

6.2 A, B, C and D are points on the circle.

BC produced and AD produced meet at N.

AB produced and



THEOREMS AND FACTS

ACT!

A: Be **A**ctive

C: Use all your **C**lues

T: Apply the **T**heory systematically, recalling each group, and fact, one at a time.

There are 4 groups of theorems and facts

- the **centre** group
- the **'no centre'** group
- the **cyclic quadrilateral** group, &
- the **tangent** group

12.6 The area of quadrilateral KPNQ

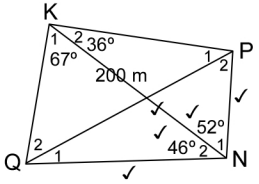
= the area of ΔKPN + the area of ΔKQN

$= \frac{1}{2}PN.KN.\sin 52^\circ + \frac{1}{2}QN.KN.\sin 46^\circ$

. . . where QN = KN = 200 m & PN = 118 m

= 9 298,526... + 14 386,796...

$\simeq$ **23 685,32 m²** ◄



13.1 $M\hat{P}N = 180^\circ - (52^\circ + 32^\circ)$ *. . . ∠ sum of ΔMPN*

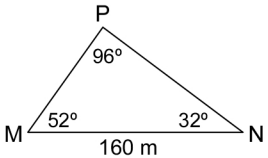
= **96°** ◄

13.2 In ΔMPN:

$\frac{MP}{\sin 32^\circ} = \frac{160}{\sin 96^\circ}$

$\therefore MP = \frac{160 \sin 32^\circ}{\sin 96^\circ}$

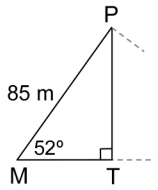
$\simeq$ **85 m** ◄



13.3 In ΔPMT: $\frac{PT}{85} = \sin 52^\circ$

$\therefore PT = 85 \sin 52^\circ$

$\simeq$ **67 m** ◄



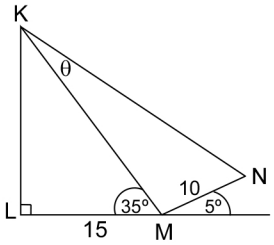
14.1 $c^2 = a^2 + b^2 - 2ab\cos C$

14.2.1 In ΔKLM:

$\frac{15}{KM} = \cos 35^\circ$

$\therefore \frac{15}{\cos 35^\circ} = KM$

$\therefore$ **KM $\simeq$ 18,3 m** ◄



14.2.2 In ΔKMN: $K\hat{M}N = 140^\circ$ *. . . ∠ sum of Δ*

& $KN^2 = 18{,}3^2 + 10^2 - 2(18{,}3)(10)\cos 140^\circ$

= 715,26...

$\therefore$ **KN $\simeq$ 26,7 m** ◄

14.2.3 In ΔKMN: ➡ $\frac{\sin\theta}{10} = \frac{\sin 140^\circ}{26{,}7}$

$\therefore \sin\theta = \frac{10 \sin 140^\circ}{26{,}7}$

= 0,2407...

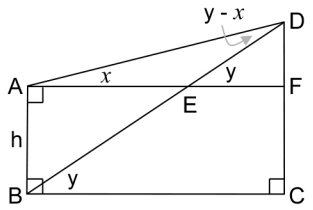
$\therefore \theta \simeq$ **13,9°** ◄

Note: θ is acute ∵ already KM̂N is obtuse

Place θ TOP posi

15.1 $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ ◄

15.2



15.2.1 $D\hat{E}F = y$ *. . . corresponding ∠s ; A*

$\therefore A\hat{D}B = y - x$ *. . . exterior ∠ of ΔDAE*

& $D\hat{A}B = 90^\circ + x$

In ΔDAB: $\frac{BD}{\sin(90^\circ + x)} = \frac{h}{\sin(y - x)}$ *. . . for in*

$\therefore BD = \frac{h \sin(90^\circ + x)}{\sin(y - x)}$

$\therefore$ **BD** $= \frac{h\cos x}{\sin(y - x)}$ ◄

15.2.2 $BD = \frac{8\cos 31^\circ}{\sin(61^\circ - 31^\circ)}$

$\therefore BD = \frac{8\cos 31^\circ}{\sin 30^\circ}$

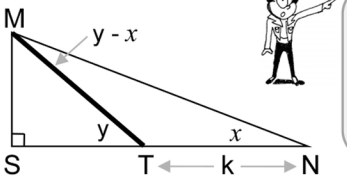
$\therefore BD = 13{,}71...$ m

In ΔDBC: $\frac{CD}{BD} = \sin y$

$\therefore CD = BD \sin 61^\circ$

$\therefore$ **CD = 12 m** ◄

16.



For Questions 16 and 17: Be sure to read the 'Advice for 2D problems' on page 10.13.

16.1 **Exterior ∠ of Δ = sum of the two int. opp. ∠s** ◄

16.2 In ΔMTN: $\frac{MT}{\sin x} = \frac{k}{\sin(y - x)}$

$\therefore MT = \frac{k \sin x}{\sin(y - x)}$ . . . ❶

It is important to place the required side in the **TOP LEFT** position.

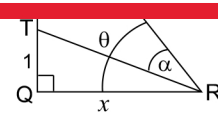
16.3 In right $\angle^d$ ΔMST: $\frac{MS}{MT} = \sin y$

$\therefore MS = MT \sin y$ . . . ❷

❶ in ❷: $\therefore MS = \frac{k \sin x . \sin y}{\sin(y - x)}$ ◄

Note: **MT** is the **LINK** between the 2 Δs: rt $\angle^d$ ΔMST and non-rt $\angle^d$ ΔMTN

$\therefore$ **TR** $= \frac{x}{\cos(\theta - \alpha)}$ ◄



17.2 $\hat{P} = 90^\circ - \theta$ ◄ *. . . ∠ sum of Δ*

17.3 In ΔPTR: $\frac{TR}{\sin(90^\circ - \theta)} = \frac{2}{\sin\alpha}$

$\therefore$ **TR** $= \frac{2\cos\theta}{\sin\alpha}$ ◄

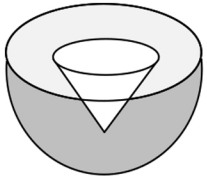
17.4 $\therefore \frac{x}{\cos(\theta - \alpha)} = \frac{2\cos\theta}{\sin\alpha}$ *. . . both = TR*

$\therefore$ **x** $= \frac{2\cos\theta.\cos(\theta - \alpha)}{\sin\alpha}$ ◄

17.5 **x** $= \frac{2\cos 50^\circ.\cos(50^\circ - 30^\circ)}{\sin 30^\circ} \simeq$ **2,4 metres** ◄

▶ MEASUREMENT [6]

QUESTION 8



A solid metallic hemisphere has a radius of 3 cm. It is made of metal A. To reduce its weight a conical hole is drilled into the hemisphere (as shown in the diagram) and it is completely filled with a lighter metal B. The conical hole has a radius of 1,5 cm and a depth of $\frac{8}{9}$ cm.

Calculate the ratio of the volume of metal A to the volume of metal B. [6]

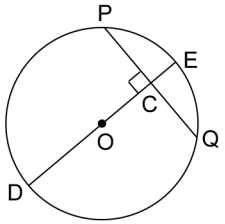
9.1 Complete the statement so that it is valid:

The line drawn from the centre of the circle perpendicular to the chord . . . (1)

9.2 In the diagram, O is the centre of the circle.

The diameter DE is perpendicular to the chord PQ at C.

DE = 20 cm and CE = 2 cm.

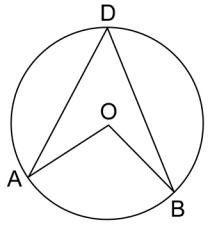


Calculate the length of the following with reasons:

9.2.1 OC 9.2.2 PQ (2)(4) [7]

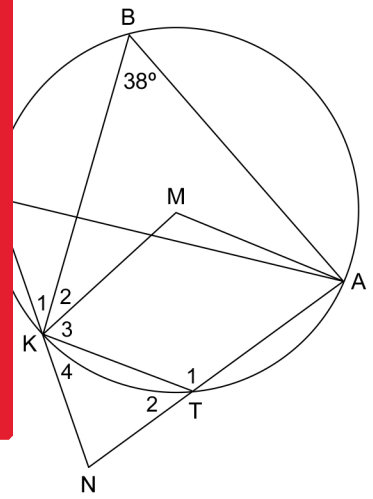
QUESTION 10

10.1 In the diagram, O is the centre of the circle and A, B and D are points on the circle.



Use Euclidean geometry methods to prove the theorem which states that $A\hat{O}B = 2A\hat{D}B$. (5)

agram, M is the centre of the circle.
K and T lie on the circle.
uced and CK produced meet in N.
= NC and $\hat{B} = 38°$.



10.2.1 Calculate, with reasons, the size of the following angles:

(a) $K\hat{M}A$ (b) $\hat{T}_2$ (2)(2)

(c) $\hat{C}$ (d) $\hat{K}_4$ (2)(2)

10.2.2 Show that NK = NT. (2)

10.2.3 Prove that AMKN is a cyclic quadrilateral. (3) [18]

QUESTION 11

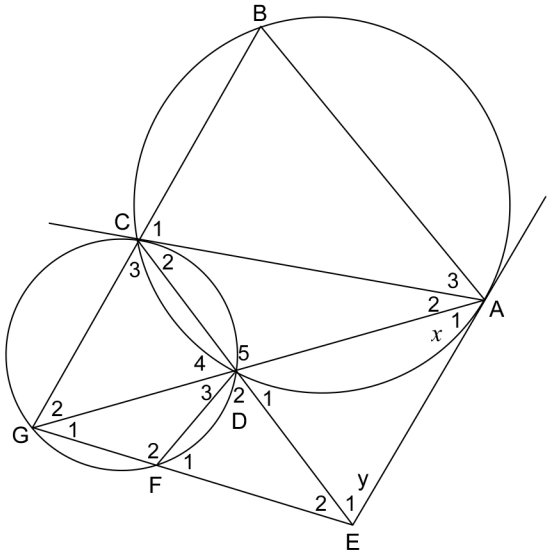
11.1 Complete the following statement so that it is valid:

The angle between a chord and a tangent at the point of contact is . . . (1)

11.2 In the diagram, EA is a tangent to circle ABCD at A.

AC is a tangent to circle CDFG at C.

CE and AG intersect at D.



If $\hat{A}_1 = x$ and $\hat{E}_1 = y$, prove the following with reasons:

11.2.1 BCG || AE (5)

11.2.2 AE is a tangent to circle FED (5)

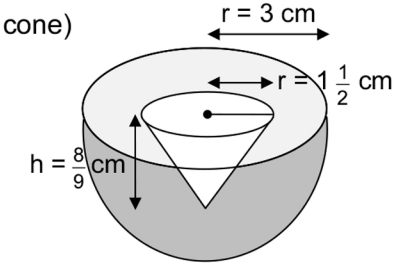
11.2.3 AB = AC (4) [15]

TOTAL: 150

EXAM PAPERS: PAPER 2

▶ MEASUREMENT [6]

8. Volume of metal B (the cone)



$= \frac{1}{3}\pi r^2.h$

$= \frac{1}{3}\pi(1,5)^2.\frac{8}{9}$

$= \frac{2}{3}\pi$

Volume of the hemisphere $= \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$= \frac{2\pi}{3}.3^3$

$= 18\pi$

$\therefore$ Volume of metal A $= 18\pi - \frac{2}{3}\pi = 17\frac{1}{3}\pi$

$\therefore$ The ratio:

Volume of metal A : Volume of metal B

$= 17\frac{1}{3}\pi : \frac{2}{3}\pi$

$\times 3)$ $= 52\pi : 2\pi$

$\div 2\pi)$ $=$ **26 : 1** ◄

9.2.1 OE = OD = $\frac{1}{2}$(20) = 10 cm

radii = $\frac{1}{2}$ *diameter*

$\therefore$ **OC = 8 cm** ◄ . . . *CE = 2 cm*

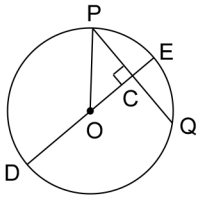
9.2.2 In ΔOPC:

$PC^2 = OP^2 - OC^2$ . . . *Pythagoras*

$= 10^2 - 8^2$

$= 36$

$\therefore$ PC = 6 cm



$\therefore$ **PQ = 12 cm** ◄ . . . *line from centre $\perp$ to chord*

Construction: Join DO and produce it to C

Proof:

Let $\hat{D}_1 = x$

then $\hat{A} = x$ . . . $\angle^s$ *opp = radii*

$\therefore \hat{O}_1 = 2x$

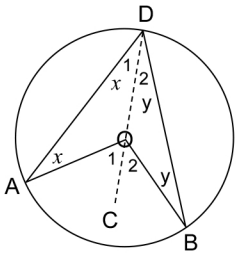
. . . *ext. $\angle$ of ΔDAO*

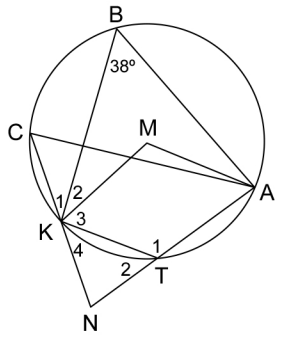
Similarly, $\hat{O}_2 = 2y$

$\therefore A\hat{O}B = 2x + 2y$

$= 2(x + y)$

$=$ **2 A$\hat{D}$B** ◄





(a) K$\hat{M}$A = 2(38°) . . . *$\angle$ at centre = 2 × $\angle$ at circumference*

= **76°** ◄

(b) $\hat{T}_2$ = **38°** ◄ . . . *ext. $\angle$ of cyclic quad. BKTA*

(c) $\hat{C}$ = **38°** ◄ . . . *$\angle^s$ in same segment or, ext. $\angle$ of c.q. CKTA*

(d) N$\hat{A}$C = 38° . . . *$\angle^s$ opposite = sides*

$\therefore \hat{K}_4$ = **38°** ◄ . . . *ext. $\angle$ of c.q. CKTA*

10.2.2 In ΔNKT: $\hat{K}_4 = \hat{T}_2$ . . . *both = 38° in 10.2.1*

$\therefore$ **NK = NT** ◄ . . . *sides opp = $\angle^s$*

10.2.3 K$\hat{M}$A = 2(38°) . . . *see 10.2.1(a)*

& $\hat{N}$ = 180° - 2(38°) . . . *sum of $\angle^s$ of ΔNKT (see 10.2.2)*

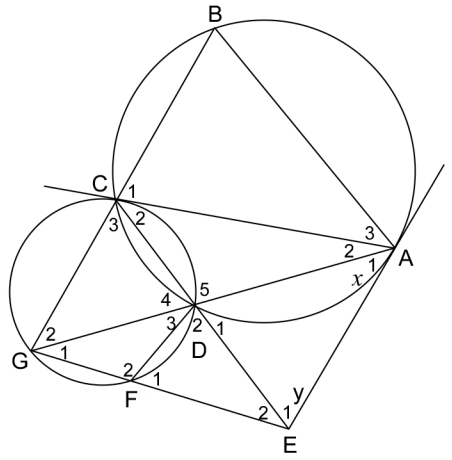
$\therefore$ K$\hat{M}$A + $\hat{N}$ = 180°

$\therefore$ **AMKN is a cyclic quadrilateral** ◄

. . . *opposite $\angle^s$ of quad supplementary or conv. opp $\angle^s$ of cyclic quad*

11.1 . . . **equal to the angle subtended by the chord in the alternate segment.** ◄

11.2



11.2.1 $\hat{A}_1 = x$ . . . *given*

$\therefore \hat{C}_2 = x$ . . . *tan chord theorem*

$\therefore \hat{G}_2 = x$ . . . *tan chord theorem*

$\therefore \hat{A}_1 = \hat{G}_2$

$\therefore$ **BCG || AE** ◄ . . . *alternate $\angle^s$ =*

11.2.2 $\hat{E}_1 = \hat{C}_3 = y$ . . . *alternate $\angle^s$; BG || AE*

$\therefore \hat{F}_1 = y$ . . . *exterior $\angle$ of cyclic quad*

$\therefore \hat{E}_1 = \hat{F}_1$

$\therefore$ **AE is a tangent to ⊙FED** ◄

. . . *converse tan-chord theorem*

11.2.3 $\hat{C}_1$ = C$\hat{A}$E . . . *alternate $\angle^s$; BCG || AE*

$= \hat{B}$ . . . *tan chord theorem*

$\therefore \hat{C}_1 = \hat{B}$

$\therefore$ **AB = AC** ◄ . . . *sides opposite = $\angle^s$*

EXAM MEMOS: PAPER 2

THE ANSWER SERIES *Your Key to Exam Success*

Gr 12 Maths 2 in 1 offers:

a UNIQUE 'question & answer method' of mastering maths

'a way of thinking'

Kilpatrick's interlinking strands of mathematical proficiency

To develop . . .

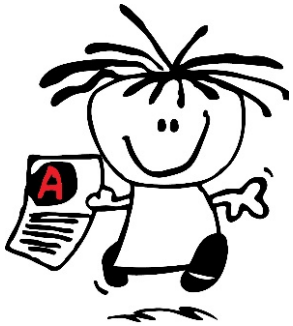
- conceptual understanding
- reasoning techniques
- procedural fluency & adaptability
- a variety of strategies for problem-solving

Our South African Maths Framework

The questions are designed to:

- transition from basic concepts through to the more challenging concepts
- include critical prior learning (Gr 10 & 11) when this foundation is required for mastering the entire FET curriculum
- engage learners eagerly as they participate and thrive on their maths journey
- accommodate all cognitive levels

The questions and detailed solutions have been provided in



SECTION 1: Separate topics

It is important that learners focus on and master one topic at a time BEFORE attempting 'past papers' which could be bewildering and demoralising. In this way they can develop confidence and a deep understanding.

SECTION 2: Exam Papers

THE ANSWER SERIES *Your Key to Exam Success*

When learners have worked through the topics and grown fluent, they can then move on to the exam papers to experience working through a variety of questions in one session, and to perfect their skills.

There are **TOPIC GUIDES** which enable learners to continue mastering one topic at a time, even when working through the exam papers.

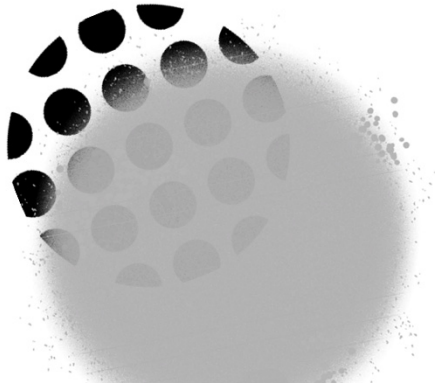
PLUS, . . .

NEW

EXTENSION SECTION

CHALLENGING QUESTIONS & MEMOS

These questions are Cognitive Level 3 & 4 questions, diagnosed as such following poor performance of learners in recent examinations.



Independent Events vs Mutually Exclusive Events

First do The Probability Rules: Q1 – 3 on p. 50.

QUESTION 2

A survey concerning their holiday preferences was done
180 staff members. The options they could choose from w

- Go to the coast
- Visit a game park
- Stay at home

The results were recorded in the table below:

	Coast	Game Park	Home	To
Male	46	24	13	8
Female	52	38	7	9
Total	98	62	20	1

2.1 Determine the probability that a randomly selected staff member:

2.1.1 is male

2.1.2 does not prefer visiting a game park

2.2 Are the events 'being a male' and 'staying at home independent events? Motivate your answer with relevant calculations.

Solutions

2.1.1 $P(\text{male}) = \frac{83}{180}$ ◄

NB: $P(E) = \frac{n(E)}{n(S)}$

2.1.2 $P(\text{not game park}) = \frac{98+20}{180}$

$= \frac{118}{180}$

$= \frac{59}{90}$ ◄ . . . *P(coast or at h*

2.2 $P(\text{M and at home}) = \frac{13}{180} = 0,07\dot{2}$ ◄

$P(\text{male}) = \frac{83}{180}$ and $P(\text{at home}) = \frac{20}{180}$

$\therefore P(\text{male}) \times P(\text{at home}) = \frac{83}{180} \times \frac{20}{180}$

$= 0,051\ldots$

$\therefore$ **The events are not independent** ◄ . . . $P(\text{M \& at home}) \neq P(\text{M}) \times P(\text{at home})$

QUESTION 3

For two events, A and B, it is given that:

$P(A) = 0,2$

$P(B) = 0,63$

$P(A \text{ and } B) = 0,126$

Are the events, A and B, independent?

Justify your answer with appropriate calculations. (3)

Solution

Independent events:

For 2 events **A** and **B** to be INDEPENDENT:

P(A and B) must be equal to **P(A) X P(B)**

. . . *called the* **PRODUCT** *Rule*

So, calculate the value of each of these expressions to determine whether they are equal or not.

P(A) x P(B) $= 0,2 \times 0,63 = 0,126$

But, **P(A and B)** $= 0,126$ also . . . *given*

$\therefore$ **P(A) x P(B) = P(A and B)**

$\therefore$ The 2 events A and B are independent

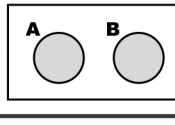
Note the layout of the PROOF;
i.e. the answer must be JUSTIFIED!

P(A or B) must be equal to **P(A) + P(B)**

. . . *called the* **SUM** *rule*

So, necessarily: **P(A∩B) = 0** . . .

(**A** and **B** do not overlap)



STION 4

P(A) = 0,45; P(B) = y and P(A or B) = 0,74

ine the value(s) of y if A and B are mutually
e. (3)

tion

P(A or B) = P(A) + P(B) . . . *A and B are mutually exclusive events.*

$\therefore$ 0,74 = 0,45 + y

$\therefore$ **y = 0,29** ◄

STION 5

A and B are mutually exclusive. It is given that:

= 2P(A)

r B) = 0,57

te P(B). (3)

tion

P(A or B) = P(A) + P(B) . . . *A & B are mutually exclusive*

$\therefore 0,57 = \frac{1}{2}P(B) + P(B)$ . . . $2P(A) = P(B)$ ➡

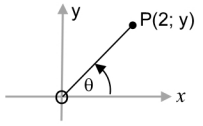
$P(A) = \frac{1}{2}P(B)$

$1,5\,P(B) = 0,57$

$\therefore$ **P(B) = 0,38** ◄

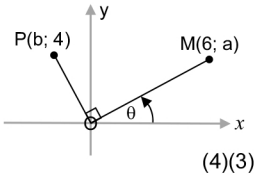
Use the **TOPIC GUIDE** on p. 147 to select and practise further questions on this section.

6. In the accompanying figure, P(2; y) is a point in the first quadrant and OP = $\sqrt{13}$. $\hat{XOP} = \theta$.



6.1 Calculate the value of y. (2)
6.2 Write down the numerical value of $\cos^2\theta$. (1)
6.3 Calculate the value of: $13\sin^2(180° + \theta) - \tan\theta$. (4)

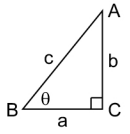
7. In the diagram alongside, $P\hat{O}M = 90°$, $\hat{XOM} = \theta$, M(6; a), P(b; 4) and $\sqrt{5}\cos\theta - 2 = 0$.



Determine, without the use of a calculator, the numerical value of:
7.1 a 7.2 b (4)(3)

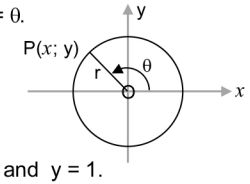
Bookwork

8. In the sketch alongside, prove that:



8.1 $\frac{\sin\theta}{\cos\theta} = \tan\theta$ 8.2 $\sin^2\theta + \cos^2\theta = 1$
8.3 $\sin(90° - \theta) = \cos\theta$

9. In the accompanying figure, P(x; y) is a point on the circle with radius r, centre (0; 0) and $\hat{XOP} = \theta$.



9.1 Prove that:
(a) $\tan\theta = \frac{\sin\theta}{\cos\theta}$
(b) $\sin^2\theta + \cos^2\theta = 1$
9.2 Find the value of θ if $x = -\sqrt{3}$ and y = 1.

Given Ratios, calculate . . .

Note: *'without a calculator' means 'with a sketch'*

10. If $5\sin x = 3$ and $x + y = 90°$, calculate without a calculator the value of:
10.1 cos y 10.2 $\tan x + \tan y$ (3)(3)

11. If $5\tan\theta = 12$ and $\theta \in [90°; 360°]$, determine, without using a calculator, but using a suitable sketch, the value of $\sin\theta + \cos\theta$. (5)

12. If $\tan A = -\frac{5}{12}$ and $180° \le A \le 360°$, draw a sketch and calculate (without determining the value of A) the value of: $13\sin^2 A$ (5)

13. $\sin A = \frac{3}{5}$ and $90° < A < 270°$. Determine, by means of a sketch, the value of: $\frac{\cos A + \sin A}{1 - \frac{1}{3}\tan A}$ (6)

14. $\tan\alpha = -\frac{3}{4}$; $\alpha \in (0°; 180°)$ and $13\cos\beta - 12 = 0$; $\beta \in (180°; 360°)$
Calculate the value of $\sin\alpha\sin\beta$, without using a calculator. (5)

15. If $\sin x = -\frac{5}{13}$ and $90° < x < 270°$, calculate $\tan^2 x.\cos^2 x$ without using a calculator. (7)

16. If $\cos x = t$ and $\hat{x}$ is acute, express
16.1 $\sin x$, and 16.2 $\tan^2 x$ in terms of t (1)(2)

17. If $\tan 27° = p$, express each of the following in terms of p:
17.1 sin 27° 17.2 cos 27° 17.3 sin 63°
17.4 tan 153° 17.5 tan(–27°) 17.6 $\cos^2 387°$
(6×2 = 12)

EXERCISE 6.2

Identities

Special $\angle^s$; 180°/360°

Special angles

Simplify WITHOUT using a calculator:

1.1 $\frac{\cos^2 45°}{\sin 30°} \cdot \frac{\cos 0°}{\tan 60°}$ 1.2

1.3 $\frac{\cos 240°.\sin 330°.\tan 120°}{\sin 150°.\tan 210°.\cos 120°}$ 1.4

2.1 $\frac{1}{\sqrt{3}}\sin^2 45°.\sin(-300°) - \frac{1}{2}\tan(-45°$

2.2 $\frac{\tan 135°.\sin 230°.\tan(-60°)}{\cos 140°.\tan 300°.\sin 150°}$

3. $\frac{\tan(-330°).\sin 480°.\sin 260°}{\cos 225°.\sin 315°.\cos 350°}$

4. Determine, without the use of a calc
4.1 $\frac{\sin 137°}{\cos 133°}$ 4.2

180°/360° Rule (no ratio chan

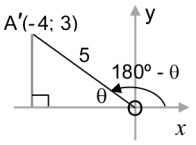
Basic Gr 9 transformatio are used here to develo

5.1 (a) In the figure alongside, the coordinates of point A are . . .

(b) Determine the values of $\sin\theta$, $\cos\theta$ and $\tan\theta$.

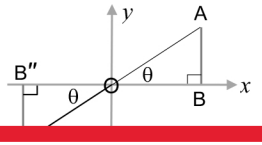
5.2 (a) A′ is a reflection of point A in the . . . and the coordinates of point A′ are . . .
(b) Write $\hat{XOA}'$ in terms of θ.

(c) Determine, first numerically, and then in terms of θ:

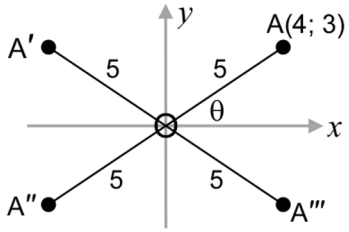


sin(180° – θ) = $\left(\frac{y}{r}=\right)$ . . . . = ...
cos(180° – θ) = $\left(\frac{x}{r}=\right)$ . . . . = ...
tan(180° – θ) = $\left(\frac{y}{x}=\right)$ . . . . = ...

5.3 (a) A″ is a reflection of point A in the . . . and the coordinates of point A″ are . . .

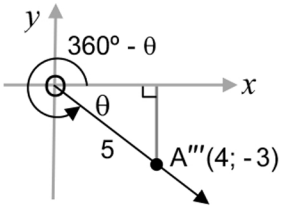


5.4 (a) A‴ is a reflection of point A in the . . . and the coordinates of point A‴ are . . .



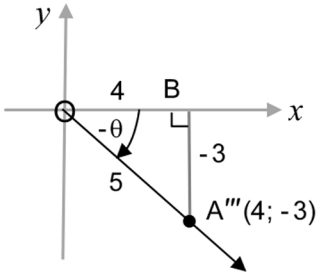
(b) Write reflex $\hat{XOA}'''$ in terms of θ & write acute $\hat{XOA}'''$ in terms of θ.

(c) Determine, first numerically, and then in terms of θ:



sin(360° – θ) = $\left(\frac{y}{r}=\right)$ . . . . = ...
cos(360° – θ) = $\left(\frac{x}{r}=\right)$ . . . . = ...
tan(360° – θ) = $\left(\frac{y}{x}=\right)$ . . . . = ...

(d) Determine, first numerically, and then in terms of θ:



sin(–θ) = $\left(\frac{y}{r}=\right)$ . . . . = ...
cos(–θ) = $\left(\frac{x}{r}=\right)$ . . . . = ...
tan(–θ) = $\left(\frac{y}{x}=\right)$ . . . . = ...

MARKS HAVE NOT BEEN ALLOCATED IN 'THEORY' Q5 & 6.

Solutions on p. 91 – 93

Trigonometry

Identities & Compound Angles

QUESTION 1

1.1 Given: $\sin 16^\circ = p$

Determine the following in terms of p, **without using a calculator**.

1.1.1 $\sin 196^\circ$ 1.1.2 $\cos 16^\circ$ (2)(2)

1.2 Given: $\cos(A - B) = \cos A \cos B + \sin A \sin B$

Use the formula for $\cos(A - B)$ to derive a formula for $\sin(A + B)$. (3)

1.3 Simplify $\dfrac{\sqrt{1 - \cos^2 2A}}{\cos(-A) \cdot \cos(90^\circ + A)}$ completely, given that $0^\circ < A < 90^\circ$. (5)

1.4 Given: $\cos 2B = \dfrac{3}{5}$ and $0^\circ \leq B \leq 90^\circ$

Determine, **without using a calculator**, the value of EACH of the following in its simplest form:

1.4.1 $\cos B$ 1.4.2 $\sin B$ (3)(2)

1.4.3 $\cos(B + 45^\circ)$ (4) [21]

Solutions

1.1.1 $\sin 196^\circ = \sin(180^\circ + 16^\circ) = -\sin 16^\circ = \mathbf{-p}$ ◄

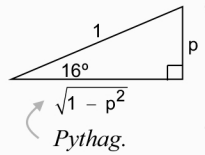
1.1.2 $\cos^2 16^\circ = 1 - \sin^2 16^\circ$

$= 1 - p^2$

$\therefore \cos 16^\circ = \mathbf{\sqrt{1 - p^2}}$ ◄

OR: $\sin 16^\circ = \dfrac{p}{1}$

$\therefore \cos 16^\circ = \mathbf{\sqrt{1 - p^2}}$ ◄



Refer to Formulae & Derivations of **Compound and Double Angles** on p. v (at the back of this book).

1.2 Given: $\cos(A - B) = \cos A \cos B + \sin A \sin B$

$\sin(A + B) = \cos[90^\circ - (A + B)]$

$= \cos[(90^\circ - A) - B]$

$= \cos(90^\circ - A)\cos B + \sin(90^\circ - A)\sin B$

$= \mathbf{\sin A \cos B + \cos A \sin B}$ ◄

1.3 $\dfrac{\sqrt{1 - \cos^2 2A}}{\cos(-A) \cdot \cos(90^\circ + A)} = \dfrac{\sqrt{\sin^2 2A}}{\cos A \cdot (-\sin A)}$

$= \dfrac{\sin 2A}{-\sin A \cos A}$

$= \dfrac{2 \sin A \cos A}{-\sin A \cos A}$

$= \mathbf{-2}$ ◄

1.4 Given: $\cos 2B = \dfrac{3}{5}$

1.4.1 $2\cos^2 B - 1 = \cos 2B$

$\therefore 2\cos^2 B = \cos 2B + 1$

$= \dfrac{3}{5} + 1$

$= \dfrac{8}{5}$

$\therefore \cos^2 B = \dfrac{4}{5}$

$\therefore \cos B = \mathbf{\dfrac{2}{\sqrt{5}}}$ ◄ ... $0^\circ \leq B \leq 90^\circ$

1.4.2 $\sin^2 B = 1 - \cos^2 B$

$= 1 - \dfrac{4}{5}$

$= \dfrac{1}{5}$

$\therefore \sin B = \mathbf{\dfrac{1}{\sqrt{5}}}$ ◄

1.4.3 $\cos(B + 45^\circ) = \cos B \cos 45^\circ - \sin B \sin 45^\circ$

$= \left(\dfrac{2}{\sqrt{5}}\right)\left(\dfrac{1}{\sqrt{2}}\right) - \left(\dfrac{1}{\sqrt{5}}\right)\left(\dfrac{1}{\sqrt{2}}\right)$

$= \dfrac{2}{\sqrt{5}\sqrt{2}} - \dfrac{1}{\sqrt{5}\sqrt{2}}$

$= \mathbf{\dfrac{1}{\sqrt{10}}}$ ◄

QUESTION 2

2.1 Prove the identity:

$\cos^2(180^\circ + x) + \tan(x - 180^\circ)\sin(720^\circ - x)\cos x = \cos 2x$ (5)

2.2 Use $\cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$ to derive the formula for $\sin(\alpha - \beta)$. (3)

2.3 If $\sin 76^\circ = x$ and $\cos 76^\circ = y$, show that $x^2 - y^2 = \sin 62^\circ$, without using a calculator. (4) [12]

Solutions

2.1 LHS $= (-\cos x)^2 + (+\tan x)(-\sin x)\cos x$

$= \cos^2 x - \left(\dfrac{\sin x}{\cos x}\right)\left(\dfrac{\sin x}{1}\right)\left(\dfrac{\cos x}{1}\right)$

$= \cos^2 x - \sin^2 x$

$= \cos 2x$

$= \mathbf{RHS}$ ◄

2.2 $\sin(\alpha - \beta) = \cos[90^\circ - (\alpha - \beta)]$

$= \cos[90^\circ - \alpha + \beta]$

$= \cos[(90^\circ - \alpha) + \beta]$

$= \cos(90^\circ - \alpha)\cos\beta - \sin(90^\circ - \alpha)\sin\beta$ ... *from the formula*

2.3 $x^2 - y^2$

$= \sin^2 76^\circ - \cos^2 76^\circ$

$= \cos^2 14^\circ - \sin^2 14^\circ$ ...

$= \cos 2(14^\circ)$

$= \cos 28^\circ$

$= \sin(90^\circ - 28^\circ)$

$= \mathbf{\sin 62^\circ}$ ◄

OR $= -(\cos^2 76^\circ - \sin^2 76^\circ)$

$= -\cos 2(76^\circ)$

$= -\cos 152^\circ$

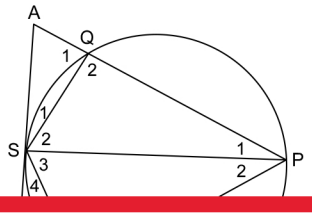
$= -(-\cos 28^\circ)$

$= \cos 28^\circ$, etc.

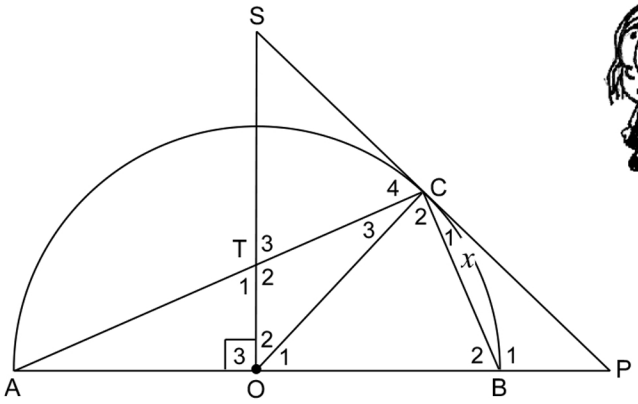
Use the **Trig Summary** – see p. vi (at the back of this book) – to master this topic!

4.3 The diagram alongside shows a circle with chords QP and RP. Chord SP bisects QP̂R.

The tangent at S



5. In the figure below, AB is the diameter of a semicircle with centre O. P is a point on AB produced. PCS is a tangent touching the circle at C, and SO is perpendicular to AB. SO and AC intersect at T. BC and OC are drawn.



5.1 If $\hat{C}_1 = x$, give, with reasons, two other angles each of which is equal to x. (2)

5.2 Prove that $P\hat{C}T = \hat{T}_2$. (3)

5.3 Give, with reasons, the magnitude of the following angles in terms of x:

(a) $C\hat{S}T$ (b) $C\hat{O}B$ (c) $\hat{P}$ (4)(2)(2)

Apply basic Gr 9 knowledge of similar Δ^s and the Theorem of Pythagoras in 5.4 and 5.5.

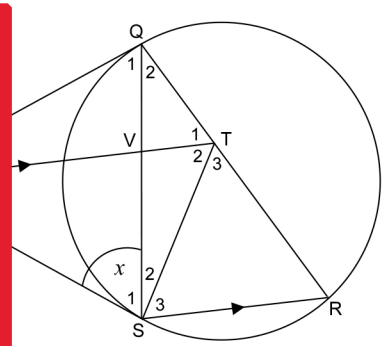
5.4 Name (without giving reasons) TWO triangles which are similar to ΔCTO. (2)

5.5 Prove that $PA.PB = OP^2 - OA^2$. (4)

Solutions on p. 128 – 129

See **p. ix** for the **Summary of the Converse Theorems** in ⊙ Geometry.

6. PQ and PS are tangents to the circle at the points Q and S. PT || SR with T on QR. $P\hat{S}Q = x$.



reasons, three other angles each (3)

QTS is a cyclic quadrilateral. (2)

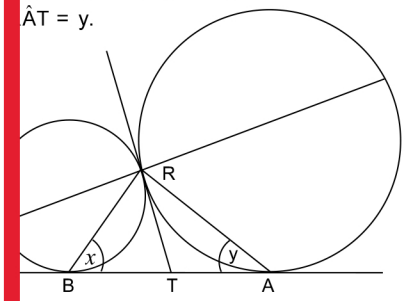
TSR is isosceles. (4)

ngent to circle QVP, prove that QSR is ed triangle. (6)

points on a circle and the tangents at meet at A. Then . . . . . (1)

etween a tangent and a chord drawn nt of contact is . . . . . (1)

two circles touch externally at R. passing through R and the centre of the smaller common tangent AB produced at the point C. gent at R meets AB at T. ÂT = y.



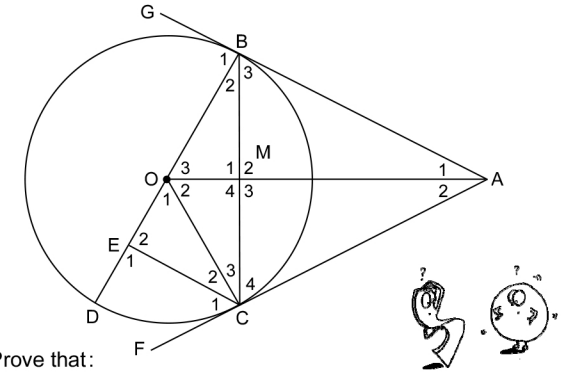
ARB = 90°. (5)

CR is a tangent to the circle which ugh A, R and B. (3)

8. In the following figure, AB and AC are tangents to a circle with centre O.

BD is a diameter and CE ⊥ BD.

BC and CO are drawn. AO cuts BC at M.



Prove that:

8.1 ABOC is a cyclic quadrilateral. (4)

8.2 CEOM is a cyclic quadrilateral. (6)

8.3 CB bisects EĈA. (4)

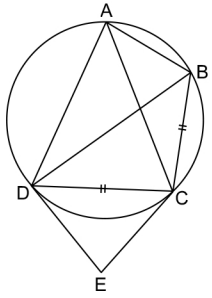
The following question involves similar Δ^s. See more combined similar triangles and Circle Geometry examples in the last section.

9.1 If PQ and RS are chords of a circle, and PQ = RS then PQ and RS subtend equal angles in the same segment at the circumference of the circle.

State the converse of this useful fact. (2)

9.2 Cyclic quadrilateral ABCD has DC = CB.

The tangents at D and C meet at E.



(a) If $B\hat{A}C = x$, find, giving reasons, five other angles each equal to x. (10)

(b) Prove that $CB^2 = EC.DB$. (4)

Know your theory!

Each topic has definitions, vocabulary, facts, laws, theorems . . . a 'blueprint'! Be sure to study all the concepts involved, as all the calculations require you to have and apply this knowledge.

Do so with confidence!

Important Facts

FACT 1 : Points on Graphs

If a point lies on a graph, the equation is true for its coordinates, i.e. the coordinates of the point satisfy the equation . . . so, substitute!

and, conversely,

If a point (i.e. its coordinates) satisfies the equation of a graph (i.e. 'makes it true'), then it lies on the graph.

FACT 2 : Point(s) of Intersection

The coordinates of the point(s) of intersection of two graphs 'obey the conditions' of both graphs, i.e. they satisfy both equations simultaneously.

They are found:

- **'algebraically'** by solving the 2 equations, or
- **'graphically'** by reading from the graph.

THESE 2 FACTS ARE CRUCIAL !

STRAIGHT LINE GRAPHS & their equations

Standard forms

Standard forms of the equation of a straight line:

- **$y = mx + c$:**
  where **m** = the gradient & **c** = the y-intercept
  When m = 0: **$y = c$** *... a line* || **x-axis**
  When c = 0: **$y = mx$** *... a line through the* **origin**
  Also: **$x = k$** *... a line* || **y-axis**

- **$y - y_1 = m(x - x_1)$:**
  where **m** = the gradient & **$(x_1; y_1)$** is a fixed point.

General form

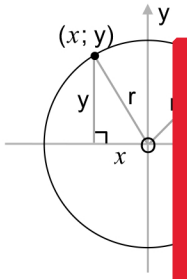
The **general form** of the equation of a straight line is **$ax + by + c = 0$**, e.g. $2x + 3y + 6 = 0$

CIRCLES & their equations

Circles with the origin as centre

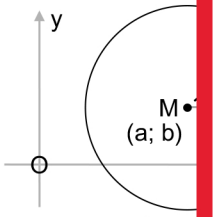
True of **any** point **(x; y)** on a circle with centre **(0; 0)** and radius **r** is that:

$x^2 + y^2 = r^2$
Thm. of Pythag.!



Circles with any given c

True of any point **(x; y)** on a circle with centre **(a; b)** and radius **r** is that:



$(x - a)^2 + (y - b)^2 = r^2$
Distance formula! (Thm. of Pythag.)

Converting the equation of a

General form: $Ax^2 + Bx + Cy^2 + Dy$

to **Standard form:** $(x - a)^2 + (y -$

(using completion of squares)

e.g. $x^2 - 6x + y^2 + 8y - 25 = 0$

$\therefore x^2 - 6x + y^2 + 8y = 2$

$\therefore x^2 - 6x + \mathbf{3^2} + y^2 + 8y + \mathbf{4^2} = 2$

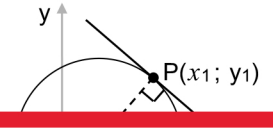
$\therefore (x - 3)^2 + (y + 4)^2 = 5$

This is the equation of a circle wit

centre $(3; -4)$ & **radius**, $r = \sqrt{50}$ (=

A Tangent to a circle . . .

is **perpendicular** to the **radius** of the circle at the **point** of **contact**.

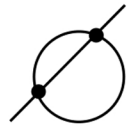


To find the **equation** of a tangent, use **'m and 1 point'**

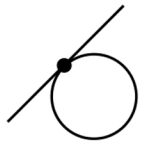
Point(s) of intersection of a Line and a Circle

A line and a circle **either**

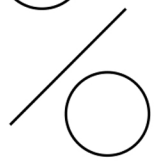
1 **'cut'** (twice!) [secant]
(2 points in common)



or 2 **'touch'** (once!) [tangent]
(1 point in common)



or 3 **don't cut or touch**
(no points in common)



If we substitute $y = mx + c$ into the equation of the ⊙,

there will **either** be: 1 **2 solutions**

or 2 **1 solution**

or 3 **no solutions**

for x, resulting in one of the above scenarios.

FINAL ADVICE

Use common sense

& ALWAYS DRAW A PICTURE !!!

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- Designed as stand-alone study guides for independent learning
- Meticulously structured for accessibility by different style learners
- Comprehensive in content
- Accurate and relevant material
- Inclusion of relevant material from earlier grades
- Accessible language and layout style

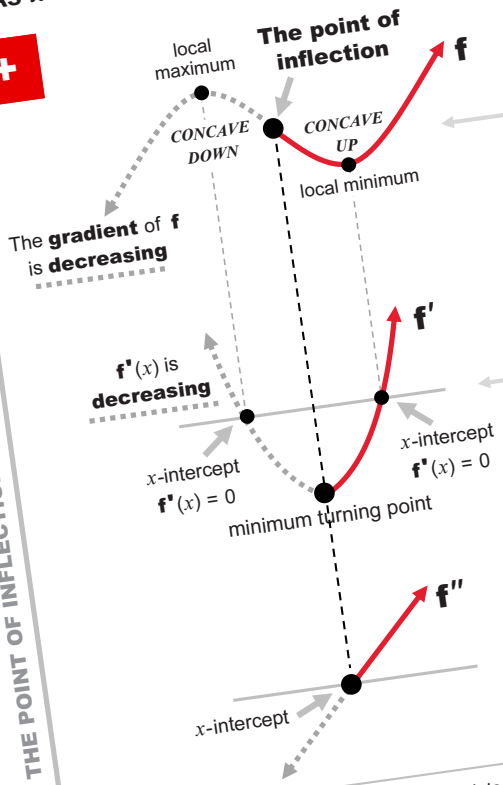
Comprehensive NOTES

CONCAVITY & THE POINT OF INFLECTION

The **Concavity** of cubic graphs: **Concave up** ∪ or **Concave down** ∩, changes at the point of inflection:

As x increases (i.e. from left to right) . . .

+



The function f

The **gradient** of **f** is **increasing**

See **f'** below:

f'(x) is the gradient of f

The 1st derivative f'

f'(x) is increasing

The 2nd derivative f''

f''(x) is positive

f''(x) = 0

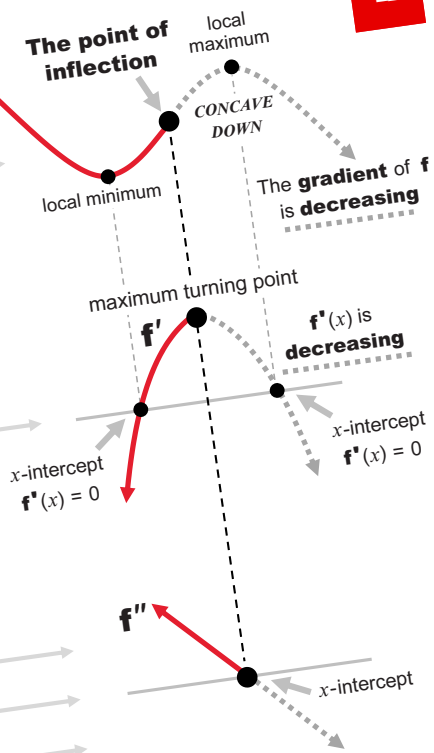
f''(x) is negative

f''(x) = 0 at the point of inflection of f

f''(x) < 0 where f is concave down

f''(x) > 0 where f is concave up

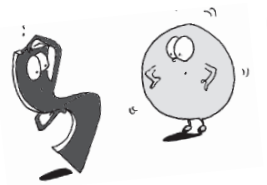
−



Note: For cubic graphs with 2 stationary points, the coordinates of the point of inflection are the averages of the x- and y-coordinates of the stationary points.

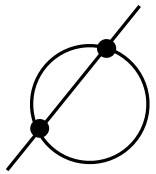
Explanatory DIAGRAMS

Point(s) of intersection of a Line and a Circle

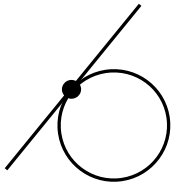


A line and a circle **either**

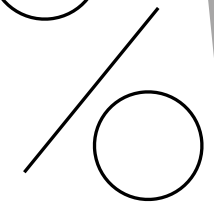
1. **'cut'** (twice!) [secant] (2 points in common)



or 2. **'touch'** (once!) [tangent] (1 point in common)



or 3. **don't cut or touch** (no points in common)



If we substitute $y = mx + c$ into the equation of the ⊙,

there will **either** be:

1. **2 solutions**

or 2. **1 solution**

or 3. **no solutions**

for x, resulting in one of the above scenarios.

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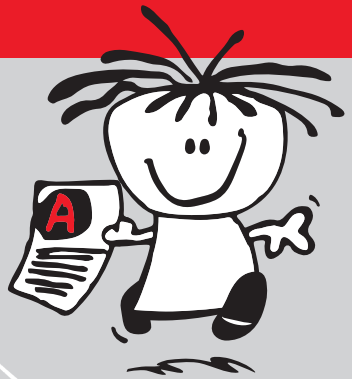

If you learn or memorise something you are likely to eventually forget it.

If you understand something you will know it forever.

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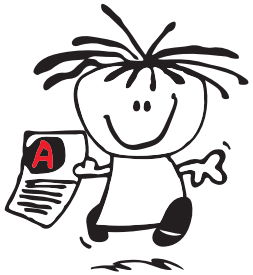

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NOTE

The questions m
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Level 3 & 4 qu
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1. **Bookwork: Examinable Proofs** i
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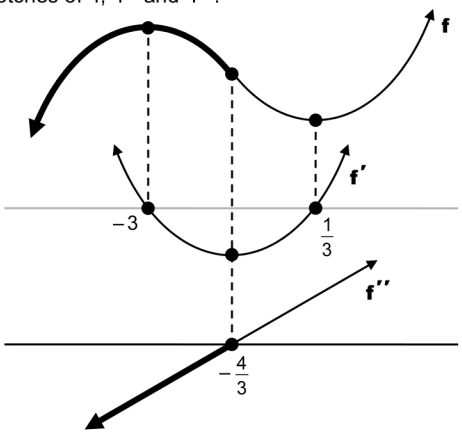
We are grateful to the Department of Basic Education and the IEB for granting their permission for the inclusion of these exam papers.

DBE P1: TOPIC GUIDE

DBE P1: TOPIC GUIDE	2014	2015	2016	2019	2020
➤ **Algebra:** [25]					
Quadratic equations & theory	1.1.1, 1.1.2, 1.4	1.1.1, 1.1.2, 1.3*	1.1.1, 1.1.2, 1.2.1	1.1.1, 1.1.2	1.1.1, 1.1.2
Quadratic inequalities	1.3	1.1.5	1.2.2	1.1.3	1.1.3,
Simultaneous equations	1.2	1.2*	1.3	1.2	1.2
Expressions					
➤ **Exponents:**					
Expressions				1.3*	1.3*
Equations & inequalities	1.1.3	1.1.3	1.1.4		
➤ **Surds:**				1.1.4	1.1.4
Expressions					
Equations		1.1.4	1.1.3	2.1	2.2
➤ **Logs (Application):**					2.1
				2.2	11.3*
➤ **Patters & Sequences:** [25]				3.1*	3.1, 3.2*
Quadratic	3.1		3.1*	3.2	
Arithmetic	2.1, 2.2, 2.4, 2.5	3.1 – 3.3, 3.4*	2.1 – 2.3, 2.4*	6.1	6.2
Geometric	3.2	2.1 – 2.4	3.2*	6.2	6.1, 6.3*
➤ **Functions & Graphs:** [35]					
Straight line and/or parabola		5.1, 6.1.1 – 6.1.3		1.3*, 4.1 – 4.4, 4.5*, 4.6, 4.7*	6.1 – 6.3, 6.4*, 6.6*
Hyperbola	4	6.2			5.1 – 5.3,
Exponent. & log function (incl. Inverses)	5	4.1 – 4.3, 5.2*, 5.3, 5.4	4.1 – 4.4, 4.5*		
Inverse functions				4.1 – 4.3, 4.4*	5.1 – 5.3, 5.4*, 5.5*
Mixed	6*	5.5*	5.1, 5.2*, 5.3, 5.4*, 5.5*, 6.1, 6.2, 6.3*, 6.4	5.1 – 5.5, 5.6*	4.1 – 4.6, 4.7*
➤ **Differential Calculus:** [35]					
Finding the derivative: 1st principles	8.1	8.1	8.1, 8.2*	7.1	8.1
Finding the derivative: using the rules	8.2, 8.3	8.2	8.3	7.2	8.2
(or) Finding the average gradient		9.2			
Tangent: the gradient & the equation		9.5*	8.4*		7.4*
Curve sketching & f'' & concavity	8.4, 9.1 – 9.3	5.6*, 6.1.4, 9.1, 9.3*, 9.4*	5.6*, 9.1, (9.2 – 9.4)*	8*	5.4*, 6.5*, 9.1*, 9.2
Practical application (incl. Max/min)	10*	10*	1.2.3, 10.1, 10.2*, 10.3*	9*	10*
➤ **Probability:** [15]					
Probability rules		11.1			12.1
Venn diagrams				10.1, 10.2*, 10.3*	11.1*
Tree diagrams		11.3*			12.2*
2-way contingency tables	11*		11.1, 11.2*, 11.3		
Fundamental Counting Principle	12*	11.2*	12*	11*	11*

Questions marked with an asterisk (*) are **Level 3 & 4**/Challenging Questions.

9.1 Sketches of f, f′ and f″:



At the stationery points of f:

$f'(x) = 0 \quad \Rightarrow \quad 3x^2 + 8x - 3 = 0$

9.2 At the point of inflection:

$f''(x) = 0$

$\therefore \; 6x + 8 = 0$

$\therefore \; 6x = -8$

$\therefore \; x = -\frac{4}{3}$

f is concave down for $x < -\frac{4}{3}$ ◄ . . . *See the sketch of f and f″.*

OR: x is halfway between $\frac{1}{3}$ & -3

$$\therefore \; x = \frac{\frac{1}{3} + (-3)}{2}$$

$$= \frac{-2\frac{2}{3}}{2}$$

$$= -1\frac{1}{3} \; \blacktriangleleft$$

OR: $f''(x) < 0$

$\therefore \; 6x + 8 < 0$

$\therefore \; 6x < -8$

$\therefore \; x < -\frac{4}{3}$ ◄

9.3 f strictly increasing ➡ $f'(x) > 0$

$\therefore \; x < -3 \quad \text{or} \quad x > \frac{1}{3}$ . . . *See the sketches of f and f′.*

OR: $3x^2 + 8x - 3 < 0$

$\therefore \; (3x - 1)(x + 3) < 0$

$f'(x)$: + (−3) − ($\frac{1}{3}$) +

(x):

$\therefore \; x < -3 \quad \text{or} \quad x > \frac{1}{3}$ ◄

9.4 $f(x) = ax^3 + bx^2 + cx + d$

$\therefore$ f(0) = −18 ➡ d = −18

$x) = 3ax^2 + 2bx + c$

$'(x) = 3x^2 + 8x - 3$ . . . *given*

= 3 ; 2b = 8 ; c = −3

= 1 ∴ b = 4

$) = x^3 + 4x^2 - 3x - 18$ ◄

he information very carefully, so that you know t

= the **number of molecules** after time t hou

= the **number of hours** after the drug has be taken

$= -t^3 + 3t^2 + 72t, \quad 0 < t < 10$

hours (t = 3), the **number of molecules**:

$= -3^3 + 3(3)^2 + 72(3)$

$= -27 + 27 + 216$

= 216 molecules ◄

10.2 The **'rate of change'** of M(t) vs t **at** time t = 2 is the derivative:

as opposed to the **'average rate of change'** which would be $\frac{M(2) - M(0)}{2 - 0}$ **during** the first 2 hours

$M'(t) = -3t^2 + 6t + 72$

$\therefore \; M'(2) = -3(2)^2 + 6(2) + 72$

$= -12 + 12 + 72$

= 72 molecules per hour ◄

10.3 The **rate** at which the number of molecules, M(t), is changing is: $M'(t) = -3t^2 + 6t + 72$

. . . *a quadratic expression*

& it will be a maximum at the turning point, i.e. when

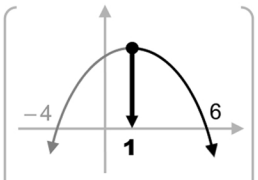
$t = \frac{-b}{2a}$ or $M''(t) = 0$

$= \frac{-6}{2(-3)}$ $\therefore \; -6t + 6t = 0$

$= 1$ $\therefore \; -6t = -6t$

$\therefore \; t = 1$

$\therefore$ **After 1 hour** ◄



OR:

t = the average of −4 & 6

$= \frac{-4+6}{2}$

$= 1$

	DURING EXAMINATIONS	TV DURING EXAMINATIONS	TOTALS
Males	80	a = 20	100
Females	48	12	60
Total	b = 128	32	160

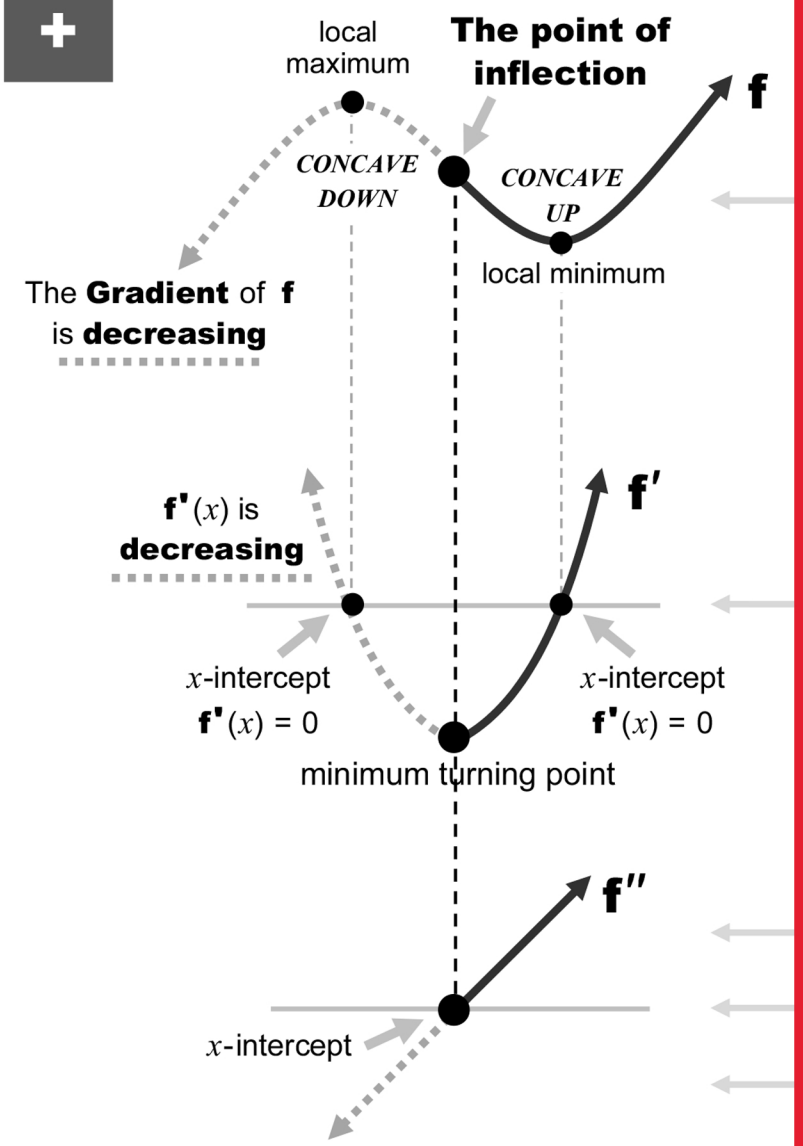
11.1 **a** = 100 − 80 = **20** ◄

& **b** = 80 + 48 or 160 − 32 = **128** ◄

CONCAVITY & THE POINT OF INFLECTION

up ∪ or **Concave down** ∩ , changes at the point of inflection:

+



The function f

The **gradient** of **f** is **increasing**

See **f'** below:
f'(x) is the **gradient** of **f**

The 1st derivative f'

f'(x) is **increasing**

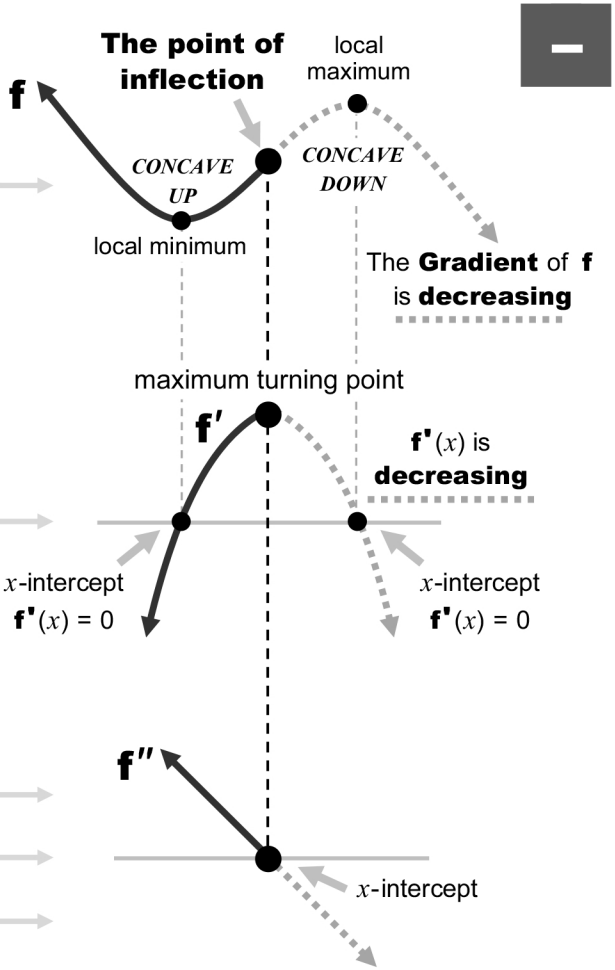
The 2nd derivative f''

f''(x) is positive

f''(x) = 0

f''(x) is negative

−



Note: For cubic graphs with 2 stationary points, the coordinates of the point of inflection are the averages of the x- and y-coordinates of the stationary points.

f''(x) = 0 at the point of inflection of f
f''(x) < 0 where f is concave down
f''(x) > 0 where f is concave up

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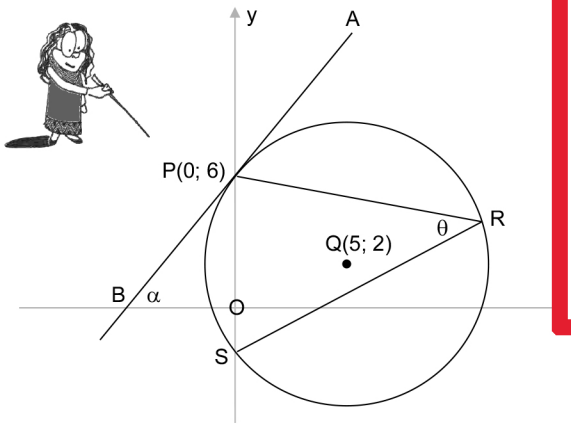
QUESTION 4

Answers o

In the diagram below, Q(5; 2) is the centre of a circle th
intersects the y-axis at P(0; 6) and S. The tangent APB
at P intersects the x-axis at B and makes the angle α w
the positive x-axis. R is a point on the circle and $P\hat{R}S$ =



4.1 Determine the equation of the circle in the form $(x - a)^2 + (y - b)^2 = r^2$. (3)

4.2 Calculate the coordinates of S. (3)

4.3 Determine the equation of the tangent APB in the form $y = mx + c$. (4)

4.4 Calculate the size of α. (2)

4.5 Calculate, with reasons, the size of θ. (4)

Need help – go to pp. v & vi to master Compound and Double Angle Formulae.

5.2* Simplify the following expression to a single trigonometric function:

$$\frac{4\cos(-x).\cos(90^\circ + x)}{\sin(30^\circ - x).\cos x \;+\; \cos(30^\circ - x).\sin x} \qquad (6)$$

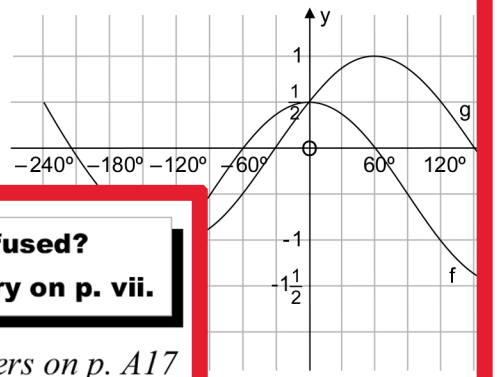
5.3 Determine the general solution of

$\cos 2x - 7\cos x - 3 = 0$. (6)

5.4* Given that $\sin\theta = \frac{1}{3}$, calculate the numerical value of $\sin 3\theta$, WITHOUT using a calculator. (5) **[24]**

In the diagram below, the graphs of $f(x) = \cos x + q$ and $g(x) = \sin(x + p)$ are drawn on the same system of axes for $-240^\circ \le x \le 240^\circ$.

The graphs intersect at $\left(0^\circ; \frac{1}{2}\right)$, $(-120^\circ; -1)$ and $(240^\circ; -1)$.



values of p and q.

values of x in the interval
40° for which $f(x) > g(x)$.

sformation that the graph o
to form the graph of **h**, wh

TION 7*

Answers on p. A18

of a rectangular block of wood is cut off and shown
gram below.

ned plane, that is, ΔACD, is an isosceles triangle
$A\hat{D}C = A\hat{C}D = \theta$.

$\hat{C}B = \frac{1}{2}\theta$, AC = $x + 3$ and CD = $2x$.

Feeling rusty or confused?
Refer to the Trig Summary on p. vii.

QUESTION 5

Answers on p. A17

5.1 Given that $\sin 23^\circ = \sqrt{k}$, determine, in its simplest form, the value of each of the following in terms of k, WITHOUT using a calculator:

5.1.1 $\sin 203^\circ$ 5.1.2 $\cos 23^\circ$ (2)(3)

5.1.3 $\tan(-23^\circ)$ (2)

Your tools . . .

RIGHT ANGLED ΔS	NON-RIGHT ANGLED ΔS
❶ Regular trig ratios	❶ Sine rule
❷ Theorem of Pythagoras	❷ Cos rule

Also: Area of a $\Delta = \frac{1}{2}bh$ *or* $\frac{1}{2}ab\sin C$

See the Paper 2 Topic Guides (on pp. 2 & 40) to select and practice more examples.

Also see p. 23 of the **EXTENSION Booklet** on CHALLENGING QUESTIONS accompanying our **Gr 12 Maths 2-in-1** study guide (the booklet also forms part of the Gr 12 Maths 2-in-1 E-book).

USEFUL REMINDERS

A helpful reference for what to study before a test or exam

PAPER 1

Linear & Quadratic Equations

Solve using . . .

- Factorising
- Substitution method or the k-method
- Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Nature of Roots

- Use Δ (the discriminant) to classify roots: $x = \frac{-b \pm \sqrt{\Delta}}{2a}$, where $\Delta = b^2 - 4ac$

Simultaneous Equations

Linear & Quadratic Inequalities

- Number lines
- Interval and inequality notation

Fractions

- Denominators and/or numerators may need to be factorised
- Check for zero denominators & invalid solutions

Exponents & Surds & Logs

Quadratic Patterns: $T_n = an^2 + bn + c$

- constant second difference

Sigma: $\sum_{k=1}^{n} T_k = S_n$ . . . Note: $T_n = S_n - S_{n-1}$

Simple Interest Growth & Decay $A = P(1 \pm in)$

- Application of SI Growth involving hire purchase: Find interest rate, no. of years or principle amount
- Simple Interest Decay = Straight line Depreciation

Compound Interest Growth & Decay $A = P(1 \pm i)^n$

- Applications involving inflation, population growth, exchange rates
- Find P, i, or n (using logs)
- The effect of different compounding intervals

= Depreciation on a Reducing Balance

st Rates

minal and **effective** interest rates for:

y/semi-annual compounding periods

$= \frac{x\left[1 - (1+i)^{-n}\right]}{i}$

$= \frac{x\left[(1+i)^{n} - 1\right]}{i}$

nmences 1 time period from the present and ends at n.

ded at the same rate as the payments

of the variables in the above formulae except i

d payments, early payments, missed payments

Patterns & Sequences

See Sum Formulae on p. i

Linear Patterns (APs): $T_n = an + b$ or $T_n = a + (n-1)d$ & $S_n = \frac{n}{2}(a + T_n)$;

$S_n = \frac{n}{2}[2a + (n-1)d]$

think **+** and **–** for **APs**

- constant first difference: $d = T_n - T_{n-1}$ . . . Def: $T_2 - T_1 = T_3 - T_2$

Exponential Patterns (GPs): $T_n = ar^{n-1}$ & $S_n = \frac{a(r^n - 1)}{r - 1}$; $S_n = \frac{a(1 - r^n)}{1 - r}$;

- Sum to infinity: $S_\infty = \frac{a}{1 - r}$ for $-1 < r < 1$
- constant ratio: $r = \frac{T_n}{T_{n-1}}$ . . . Def: $\frac{T_2}{T_1} = \frac{T_3}{T_2}$

think **×** and **÷** for **GPs**

Compound Angle Formulae

1. $\sin(A + B) = \sin A \cos B + \cos A \sin B$

2. $\sin(A - B) = \sin A \cos B - \cos A \sin B$

Sign stays the same sine & cosine of A and B mixed

3. $\cos(A + B) = \cos A \cos B - \sin A \sin B$

4. $\cos(A - B) = \cos A \cos B + \sin A \sin B$

Sign changes cosine of A and B first, then sine of A & B

We will prove formula no. 4 (see alongside) and then derive the other 3 from it.

5. $\sin 2A = 2 \sin A \cos A$ . . . This formula will be derived from the formula no. 1.

6. $\cos 2A = \cos^2 A - \sin^2 A$ . . . This formula will be derived from the formula no. 3.

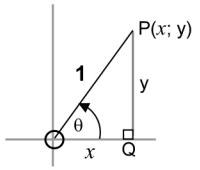
or $\cos 2A = 1 - 2\sin^2 A$

or $\cos 2A = 2\cos^2 A - 1$

Proof of the Formula:

$\cos(A - B) = \cos A \cos B + \sin A \sin B$

First, important concept!



NOTE: **If OP = 1 unit!**

then: $\frac{x}{1} = \cos\theta$ and $\frac{y}{1} = \sin\theta$

i.e. $x = \cos\theta$ and $y = \sin\theta$

i.e. **P is the point (cos θ; sin θ)**

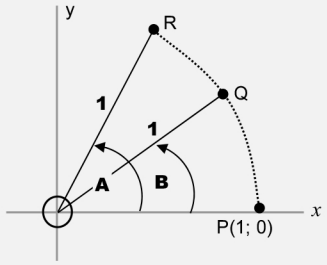
ketch alongside, **Â** and **B̂** have been placed in standard position.

$R\hat{O}Q = \hat{A} - \hat{B}$.

he coordinates of the points **R** and **Q**, both **1 unit** from the origin, are:

os A ; sin A) & **Q(cos B ; sin B)**

. . . See NOTE above



ne 2 expressions for RQ^2

$= 1^2 + 1^2 - 2(1)(1)\cos(A - B)$ . . . **COSINE RULE**

$= 2 - 2\cos(A - B)$ . . . ❶

& $\mathbf{RQ^2} = (\cos A - \cos B)^2 + (\sin A - \sin B)^2$ . . . **DISTANCE FORMULA**

$= \cos^2 A - 2\cos A \cos B + \cos^2 B + \sin^2 A - 2\sin A \sin B + \sin^2 B$

$= 2 - 2\cos A \cos B - 2\sin A \sin B$ ◄ . . . ❷ . . . $\sin^2\theta + \cos^2\theta = 1$

▶ **Equate the two expressions for RQ^2 above:**

❶ = ❷ $\therefore\ 2 - 2\cos(A - B) = 2 - 2\cos A \cos B - 2\sin A \sin B$

▶ **Subtract 2:** $\therefore\ -2\cos(A - B) = -2\cos A \cos B - 2\sin A \sin B$

▶ **Divide by −2** $\left(\text{or} \times \text{by} -\frac{1}{2}\right)$: $\therefore$ **$\cos(A - B) = \cos A \cos B + \sin A \sin B$** ◄

BOOKWORK: EXAMINABLE PROOFS

What does TAS offer in Mathematics?

Further Studies Mathematics

- Further Studies Maths (IEB) Bk 1 – Compulsory Modules
- Further Studies Maths (IEB) Bk 2 – Elective Modules
  - *Elective modules also available separately*

Grade 10 Complex Numbers Exam Questions

(Solutions on p. 19 in the Answ

1. Factorise $x^3 - 1$, and hence solve $x^3 - 1 = 0$ for $x \in C$.

2. Calculate the values of a and b so that $\frac{a+3i}{2-5i}.bi = -11 - 13i$

3. Given the complex numbers $z = 5 - 2i$ and $w = 6i - 1$.
   Determine in simplest form: $2z - iw$.

4. Determine, in terms of a and b, the real part of the complex expression $\frac{a+bi}{a-bi}$.

5. The quadratic equation $x^2 - 2x + p = 0$ has a root $x = q + \sqrt{3}i$.
   Find the rational values of p and q. *(IEB 2016)*

6. (a) It is given that $px^2 + px + 1 = 0$.
   Determine real values of p such that the solutions of the equation are of the form $a + bi$ where a and b are rational and $b \neq 0$.

7. Thabo is practising division of complex numbers of the form $a + bi$, where $a, b \in \mathbb{R}$. He notices that:

   $$\frac{3+2i}{-2+3i} = -i, \quad \frac{5-7i}{7+5i} = -i \text{ and } \frac{4+5i}{-5+4i} = -i.$$

   Prove that $\frac{a+bi}{-b+ai} = -i$ for all $a, b \in \mathbb{R}$. *(IEB 2018)*

8. Given that $m = 4 + 2i$ and $n = -2 - i$.
   Simplify the following expressions; show all calculations:
   (a) $m - 2n^*$ (b) $\frac{m}{n}$

9. (a) Factorise $x^2 + 8x + 25$ with complex numbers.

   (b) Find a quadratic equation that has a solution of $2 + 3i$.

10. Consider the following equation: $x^2 - 4x - 8 = 0$
    (a) Calculate the value of the discriminant.
    (b) Comment on the nature of the roots.
    (c) What constant must be added to the left hand side of the equation, so that the equation has one double real root? (Remember that 1 double root is the same as 2 equal roots.)

Now sketch the number in the Argand Plane.

12. Given that $z = -1 + 4i$, calculate the value of the following expressions. Show how these values are obtained and represented on the Argand plane.
    (a) $z.i^3$ (b) $z + 1$
    (c) $2z + z^*$ (d) $z.z^*$

for p and q if $(3 + i)(p + qi) = -4 + 2i$.

bi is a root of the quadratic equation $x^2 + kx + t = 0$,
eta's Formulae to show that $a^2 + b^2 = t$ and $2a + k = 0$.

RULES FOR DERIVATIVES

1. The Constant rule

$f(x) = k$ where k is a constant, then $f'(x) = 0$.

2. The Power rule

$f(x) = x^n$ where $n \in \mathbb{R}$, then $f'(x) = nx^{n-1}$.

3. The Constant-Power rule

$D_x[k.f(x)] = k.f'(x)$

The derivative of a constant multiplied by a function is equal to the constant multiplied by the derivative of the function.

Thus we have:

$f(x) = x = x^1 \quad \Rightarrow \quad f'(x) = 1x^0 = 1$

$f(x) = x^2 \quad \Rightarrow \quad f'(x) = 2x^1 = 2x$

$f(x) = x^3 \quad \Rightarrow \quad f'(x) = 3x^2$

$f(x) = \frac{1}{x} = x^{-1} \quad \Rightarrow \quad f'(x) = -1x^{-2} = \frac{-1}{x^2}$

$f(x) = \sqrt{x} = x^{\frac{1}{2}} \quad \Rightarrow \quad f'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

$f(x) = x^{\pi} \quad \Rightarrow \quad f'(x) = \pi x^{\pi - 1}$

4. The Sum (Difference) rule

$D_x[f(x) \pm g(x)] = f'(x) \pm g'(x)$

The derivative of a sum (difference) of functions is equal to the sum (difference) of the derivatives of the functions.

(c) $f(x) = \frac{5}{x} + \sqrt{x}$ (NB: first change the expression to powers of x)

(d) $f(x) = \frac{3 + x - 3x^2 + x^3}{x^3}$

ions

$x) = 6x$ (b) $f'(x) = -6x^2 + 5$

$f(x) = 5x^{-1} + x^{\frac{1}{2}}$,

$f'(x) = -5x^{-2} + \frac{1}{2}x^{-\frac{1}{2}} = \frac{-5}{x^2} + \frac{1}{2\sqrt{x}}$

$f(x) = 3x^{-3} + x^{-2} - 3x^{-1} + 1$

$f'(x) = -9x^{-4} - 2x^{-3} + 3x^{-2} = \frac{-9}{x^4} - \frac{2}{x^3} + \frac{3}{x^2}$

cise 10.3

(Solutions on p. 63 in the Answer book)

termine the derivatives of the following functions:

$f(x) = x^2 + 3$ (b) $f(x) = 5x^2 + 2x$

$f(x) = 4x^2 - x + 7$ (d) $f(x) = \sqrt{x} + 4$

$f(x) = 3x - \frac{1}{\sqrt{x}}$ (f) $f(x) = x^3 - 6x^2 + 9x - 4$

$f(x) = \frac{x^3}{3} + x^2 - 5x + 1$ (h) $f(x) = \frac{x^2 - 4x}{x}$

$f(x) = \frac{3x^2 + x - 1}{x}$

Find $\frac{dy}{dx}$ given $y = 3x^3 + 5x^2 - 4x - 3$

Find $g'(x)$ given $g(x) = \frac{4x^2 - 1}{2x + 1}$

d the following derivatives. Leave answers with positive exponents:

(a) $D_x\left[x^2 - \frac{1}{x^3}\right]$ (b) $\frac{d}{dx}\left(\frac{1 + x^{\frac{3}{2}}}{\sqrt{x}}\right)$ (c) $D_t\left[\frac{\sqrt{t} - 3t}{\sqrt{t}}\right]$

(d) $\frac{d}{ds}\left(\frac{2s - s^2 + 3s^3}{s^2}\right)$ (e) $f'(x)$ if $f(x) = \frac{2x^3 - x^2 - 8x + 4}{x - 2}$

CASE 8: Integration of rational functions with degree of numerator equal or one higher than degree of denominator.

In Chapter 17: Further Derivatives, we manipulated rational functions when considering asymptotes for graphs. This process can also be used in integration.

Worked Example 27

Given: $\int \frac{x^2 + x + 1}{x^2 + 1} dx$

We first manipulate the expression:

$$\frac{x^2 + x + 1}{x^2 + 1} = \frac{x^2 + 1 + x}{x^2 + 1}$$

$$= 1 + \frac{x}{x^2 + 1}$$

This can also be done using Long Division.

$$\int \left(1 + \frac{x}{x^2 + 1}\right) dx = \int 1 dx + \int \frac{x}{x^2 + 1} dx = x + \int \frac{2x}{2} \cdot \frac{1}{x^2 + 1} dx$$

$x^2 + 1 \geq 1 > 0$

$$= \boldsymbol{x + \frac{\ln(x^2 + 1)}{2} + c}$$

Given: $\int \frac{x^2 + x - 6}{x^2 - 5x + 6} dx$

$$\frac{x^2 + x - 6}{x^2 - 5x + 6} = \frac{(x + 3)(x - 2)}{(x - 3)(x - 2)} = \frac{x + 3}{x - 3}, \; x \neq 2$$

$$= \frac{x - 3 + 6}{x - 3} = 1 + \frac{6}{x - 3}$$

$$\int \left(1 + \frac{6}{x - 3}\right) dx = \boldsymbol{x + 6 \ln|x - 3| + c}$$

...d Example 29

$$\int \frac{x^2 - 2}{x + 1} dx$$

... degree of the numerator is one more than that of the denominator.

$$= \frac{x(x + 1) - x - 2}{x + 1}$$

$$= \frac{x(x + 1) - (x + 1) + 1 - 2}{x + 1}$$

$$= x - 1 - \frac{1}{x + 1}$$

We may prefer to do Long Division.

$$\left(1 - \frac{1}{x + 1}\right) dx$$

$$\ldots - \ln|x + 1| + c$$

...se 18.12

(Solutions on p. 131 in the Answer book)

...f the following questions, first manipulate the expression before ...ning the integral.

1. $\int \frac{\ldots - 1}{\ldots + 1} dx$

2. $\int \frac{x + 3}{x - 5} dx$

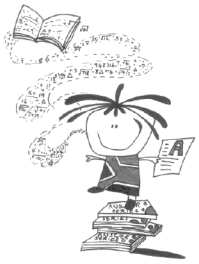
3. $\int \frac{\ldots - 2x + 3}{x} dx$

4. $\int \frac{x^3}{x^2 - 4} dx$

5. $\int \frac{x^2}{x - 2} dx$

6. $\int \frac{x^2 - 5x + 3}{x + 2} dx$

7. $\int \frac{x^3}{x^2 + 1} dx$

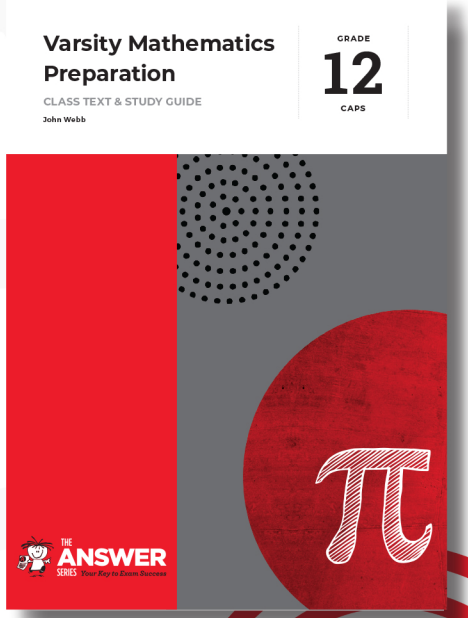


PREPARING FOR UNIVERSITY

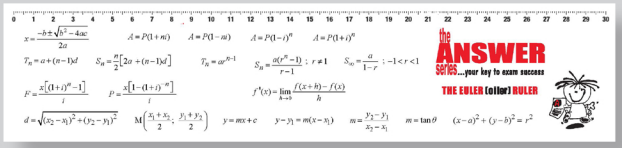
VARSITY MATHS PREP - a self-study book

Compiled by Emeritus Professor John Webb in response to the dire challenges experienced by first year university students.

By working through the problem sets in this self-study book, students will develop and test appropriate higher education skills on their own. Learners preparing for NBTs or Level 4 questions will certainly benefit from the techniques and flexible thinking acquired through dedicated, independent focus on the higher order questions in this book.



GRADE 12 EULER RULER



This 30 cm × 7 cm ruler includes all the information provided on the formula sheet in the National or IEB matric exam. Ready access to this information will ensure that learners become familiar with applying it successfully.

The Answer Series Mathematics publications have been designed to develop …

- conceptual understanding
  - reasoning techniques
    - procedural fluency & adaptability
      - a variety of strategies for problem-solving

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- Problem-solving
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- Flexible thinking

These skills cannot be taught. They are best achieved without assistance, prior to entering university.

The problem-solving techniques which learners could acquire from dedicated, independent use of this outstanding booklet will contribute significantly to their success in the National Benchmark tests (NBTs).

THE ANSWER SERIES *Your Key to Exam Success*

HOW TO USE THIS BOOK

- Start with the Basic Arithmetic, Basic Algebra and Basic Geometry problem sets. The later sets are a little harder, and may contain problems requiring ideas you will have gained from the earlier sets.
- Give yourself at least an hour or two to tackle a problem set, doing the problems in one continuous concentrated session.
- Don't allow yourself to be distracted by tweets or emails. Switch off all mobile devices!
- You don't have to do the problems in order. If a problem looks complicated, look again. It may have a simple solution if viewed from a different perspective. If you are still baffled, don't give up quickly. Come back to it later.
- No diagrams have been given. That is deliberate. It is a useful skill to be able to draw a figure from a written description.
- No calculators! You should be able to do simple arithmetic in your head, and none of the problems requires more than pencil-and-paper calculations.
- No formula sheet! You must have the standard trig and algebra formulas at your fingertips.
- These problems are for you to do by yourself, and certainly not with an "extra lessons tutor". However, working through them with a friend could be useful.
- When you have finished a problem set, check your answers for quick feedback. Before looking at the solutions, go back to any problem you got wrong and see if you can find your mistake. If you can't, look at the full solution.
- Even if you have got a full house of correct answers, read the solutions carefully. You may have got the right answer by luck, or by using a wrong method. The solutions may also give you alternative approaches, quicker methods and extra insights into the problems.
- Every wrong answer indicates a possible weakness in your mathematical background that needs to be fixed before your first Maths I lecture at your chosen university.

BASIC ALGEBRA PROBLEMS

1. When $(3a-2b)(7b-5c)(6c-9a)$ is multiplied out, what is the coefficient of abc?

   (A) 40 (B) 36 (C) 44 (D) 49 (E) 52

2. $(a^{-1}+b^{-1})^{-2}$ is equal to

   (A) a^2+b^2 (B) $a^{-2}+b^{-2}$ (C) $\dfrac{a^2b^2}{(a+b)^2}$ (D) $\dfrac{a^2+b^2}{a^2b^2}$ (E) $2(a+b)$

(A) $\frac{1}{2}(6\pm\sqrt{7})$ (B) -3 and $\frac{1}{3}$ (C) $\frac{1}{3}(4\pm\sqrt{7})$ (D) 1 and $-\frac{1}{9}$
(E) $\frac{1}{6}(4\pm\sqrt{14})$

4. Which of the following is not a factor of $6x^4+5x^3-75x^2+10x+24$?

   (A) $2x+1$ (B) $x-3$ (C) $x+4$ (D) $3x-2$ (E) $x-5$

5. $\dfrac{3}{x-2}-\dfrac{2}{x+3}$ is equal to

   (A) $\dfrac{x+13}{x^2+x-6}$ (B) $\dfrac{x-5}{x^2-x-6}$ (C) $\dfrac{2x+13}{x^2-x-6}$ (D) $\dfrac{x+13}{x^2-x+6}$

   (E) $\dfrac{x-8}{2x^2+x-3}$

6. When $(3x^2-2x+6)(x^2-4x+7)(x^2+3x-1)$ is multiplied out to the form $ax^6+bx^5+cx^4+dx^3+ex+f$, what is the value of $a+b+c+d+e+f$?

   (A) 84 (B) 96 (C) 72 (D) 108 (E) 120

7. The set of all real numbers x such that $x^2<5x+24$ is the interval

   (A) $(-3,8)$ (B) $(1,7)$ (C) $(3,8)$ (D) $(-8,-3)$ (E) $(-2,4)$

8. The sum $\sum_{n=1}^{\infty}(\frac{2}{3})^n$ is equal to

   (A) $\frac{3}{2}$ (B) 2 (C) $\frac{2}{3}$ (D) 3 (E) 6

9. If

$$2x+5y+4z=13$$
$$5x+4y+2z=15$$
$$4x+2y+5z=16$$

   then $x+y+z$ is equal to

   (A) 3 (B) 4 (C) 5 (D) 6 (E) 7

PROBLEMS 5

1. The sides of quadrilateral $ABCD$ are produced:
AB is produced to E, BC is produced to F and CD is produced to G.
If $\angle EBC = 79°$, $\angle FCD = 64°$ and $\angle GDA = 127°$, then $\angle BAD$ is equal to
(A) $61°$ (B) $72°$ (C) $83°$ (D) $90°$ (E) $105°$

2. The graph of $y = (x-3)(1-x)$ is tangent to the graph of $y = kx^2$. Determine k.
(A) $\frac{2}{5}$ (B) $\frac{1}{3}$ (C) -2 (D) $\sqrt{2}$ (E) $-\frac{1}{2}$

3. The solution of the inequality $\log_2 x + \log_2(x-3) < 2$ is
(A) $-1 < x < 4$ (B) $x > 0$ (C) $0 < x < 4$ (D) $3 < x < 4$ (E) $x > 3$

4. The roots of the equation $x^2 - 2x - 7 = 0$ are a and b. Which of the following equations has roots $a+1$ and $b+1$?
(A) $x^2 - 3x - 8 = 0$ (B) $x^2 - x - 6 = 0$ (C) $x^2 - 4x - 4 = 0$
(D) $x^2 + 3x - 8 = 0$ (E) $x^2 + 3x - 5 = 0$

5. In triangle ABC, with $AB = c$, $BC = a$ and $CA = b$,

$$\frac{4 \times \text{Area } ABC}{b^2 + c^2 - a^2}$$

is equal to
(A) $\cos A$ (B) $\tan A$ (C) $\sin 2A$ (D) $\frac{1}{2}\cos A$ (E) $2\sin A$

6. If $f(x) = 3x + 5$ and $g(x) = 4x + 7$, then $g(f(x)) - f(g(x))$ is equal to
(A) x (B) 0 (C) $x + 1$ (D) 1 (E) $x - 1$

tively, so that $BD = BE$ and $CE = CF$. If $\angle A = x$, what is the size of $\angle DEF$?
(A) $90° + x$ (B) $180° - 2x$ (C) $90° + 2x$ (D) $90° - \frac{1}{2}x$ (E) $180° - x$

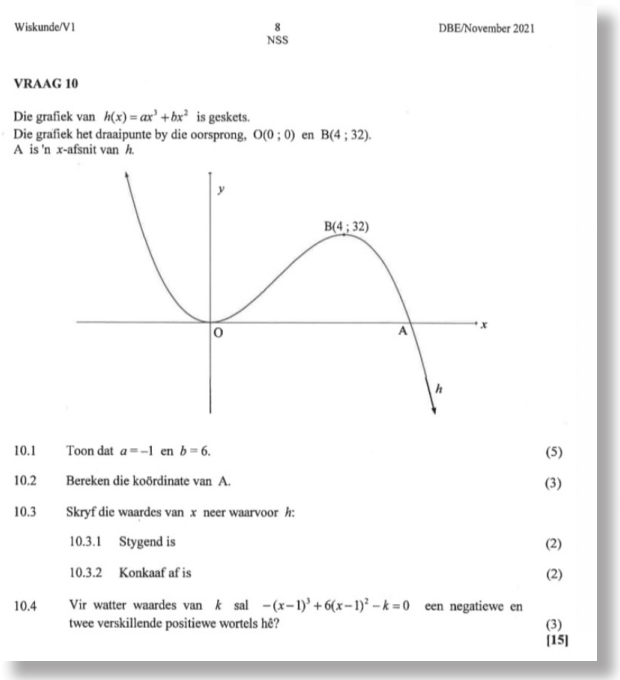
8. Let f be a function defined by the equation $f(x) = 3^x$, where x is a real number. Which of the following is true for all real numbers a and b?
(A) $f(ab) = f(a) + f(b)$ (B) $f(ab) = f(a)f(b)$
(C) $f(ab) = 3f(ab-1)$ (D) $f(a+b) = f(a) + f(b)$
(E) $f(3ab) = f(ab+3)$

Teacher WhatsApp Support Groups

Grade 10 – 12 (Started late in 2021)

- ADMIN Group (386)
- CHAT Group (245)

Posted at 13:44



Wiskunde/V1 8 NSS DBE/November 2021

VRAAG 10

Die grafiek van $h(x) = ax^3 + bx^2$ is geskets.
Die grafiek het draaipunte by die oorsprong, O(0 ; 0) en B(4 ; 32).
A is 'n x-afsnit van h.

10.1 Toon dat $a = -1$ en $b = 6$. (5)

10.2 Bereken die koördinate van A. (3)

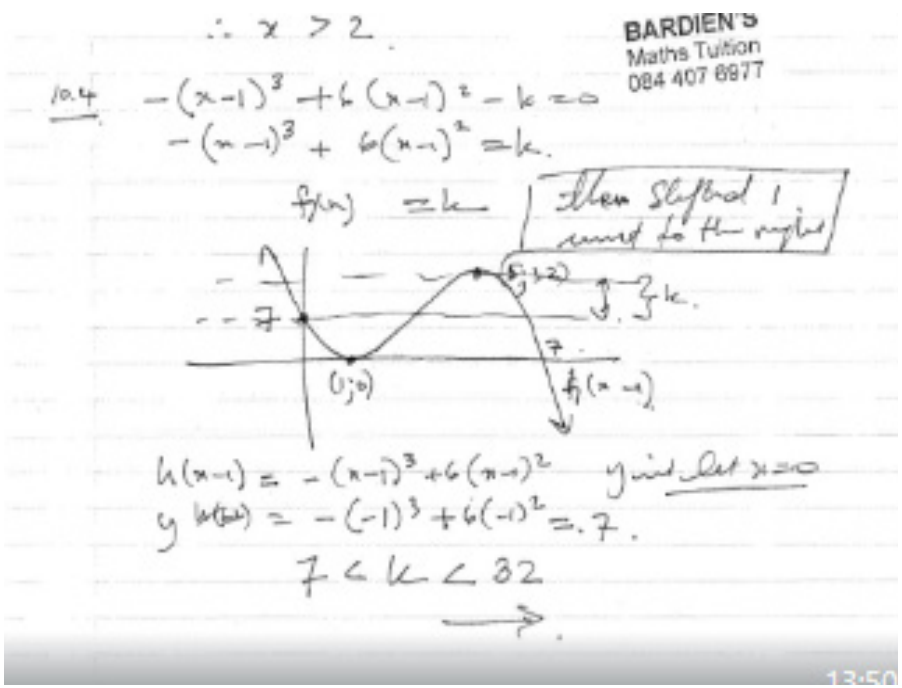
10.3 Skryf die waardes van x neer waarvoor h:

10.3.1 Stygend is (2)

10.3.2 Konkaaf af is (2)

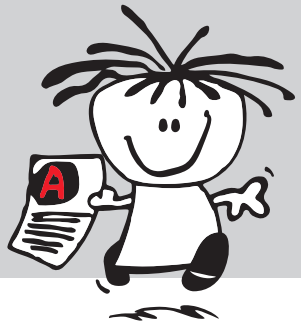
10.4 Vir watter waardes van k sal $-(x-1)^3 + 6(x-1)^2 - k = 0$ een negatiewe en twee verskillende positiewe wortels hê? (3)
[15]

Answered at 13:50



Grade 7 – 9 (Started 15 Sept 2022)

- ○ ADMIN Group (9)
- ○ CHAT Group (15)



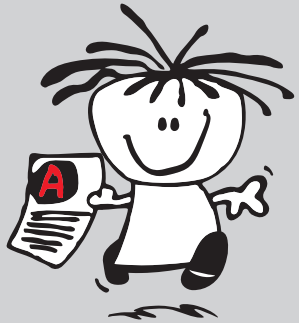
Introduction to Problem Solving

Problem Solving is part of our curriculum, and we need to expose our learners to it on a regular basis. The first strategy we are going to be looking at is drawing a table to help them find the answer. You don't have to fill in all the entries in the table, but you use it to see what is happening. In these five problems below, draw a table in order to find the answer. Answers will be posted in a few days' time.

1. **A boy decides to pick up pieces of litter. On the first day he picks up 10 pieces, the second day 20 pieces, the third day 30 pieces, and so on. Determine the fewest number of days it will take him to pick up at least 500 pieces of litter in total?**
2. Two people go running. Andile runs every Monday, Wednesday and Friday. Sophie always takes two days off between runs. If they both run on a Monday, on what day of the week will they both next run again?
3. A car sets off on a journey at 08h00 and travels at 60 km/h. A second car leaves on the same route an hour later. If the second car travels at 75 km/h, at what time will it catch up to the first car?
4. A girl is 8 years old and her mother is 30 years old. In how many years' time will the daughter be half the mother's age?
5. There are 18 animals on a farm. Some are chickens and some are cows. If there are 50 legs in total, how many chickens are there?

Grade 7 (Started 14 Sept 2022)

- ADMIN Group (55)
- CHAT Group (12)



Susan Carletti

Welcome to this group everybody! We are very excited to see the energy and enthusiasm of you all! This is a chat group - feel free to ask questions, answer each other's questions, and just generally share with other Grade 7 colleagues. From our side, we will be focussing on problem solving. Every assessment is supposed to have 10% of the marks on PS. We owe it to our learners to expose them to it in class. PS can be daunting so we are going to break it down into strategies that you can teach your learners. We encourage you to work through these yourselves, and give them to your learners as well! Solutions will be posted after a few days. We hope you enjoy being on this group! If you find the participation too much, please let Jenny know and she can add you to the Admin only group!

16:28

Thomas Malolek...

Thanks 🙏

18:23

Rose Nedzingahe

👏🏽

18:16

Sandra Hlungwani

🙏

17:26

Elisa Maswanganyi

🙏

18:33

Richard Nyamande

👍

18:18

Rendani Nevondo

Thank you

21:03

*"As promised a photo with some of the **Roedean Academy** girls with your books.*

The girls just love the books - it makes such a huge difference.

Thank you for all your help."

Sarah

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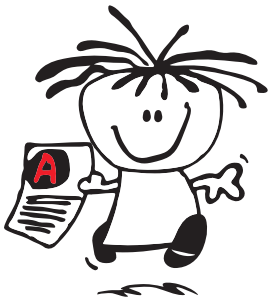

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