Welcome!

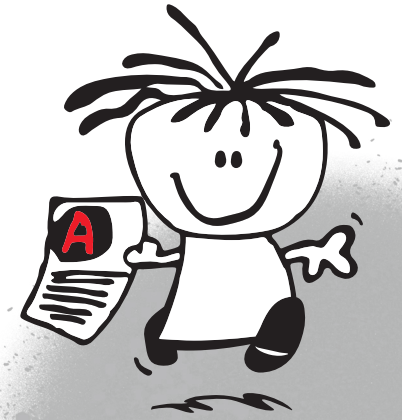
GAUTENG
SUBJECT ADVISORS
Wednesday 8 June

THE ANSWER SERIES

LEVEL 3 & 4 QUESTIONS

PAPER 1

8 JUNE 2022



GAUTENG
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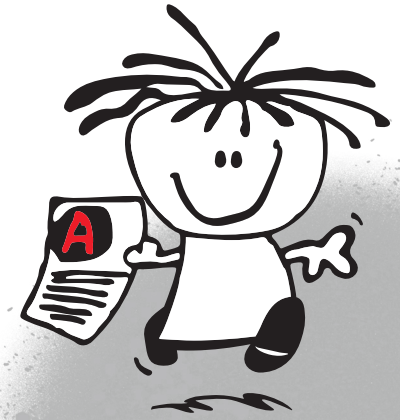
While you wait ...

Glance at these examples to whet your appetite!

Ask yourself ...

How would your learners respond?



DETERMINE:

$$D_x \left[\sum_{n=3}^{5} (n+2)\, x^n \right]$$

Sigma? **Calculus?**

How would your learners respond?

Just observe these examples . . .

Use the rules of differentiation to determine f'(x):

(1) $f(x) = 3x^5 + \frac{1}{2}x^2$

(2) $f(x) = (x^3 - 1)^2$

(3) $f(x) = \frac{x^3 - 5x^2 + 6x}{x - 5}$

Calculate $\frac{dy}{dx}$ if:

(4) $xy = 5$

(5) $y = \frac{x^2 - 25}{x + 5}$

(6) $y = \frac{1}{2x^3} + \sqrt{x}$

How would your learners respond?

CONCAVITY & THE POINT OF INFLECTION . . .

Draw a sketch graph of f, indicating ALL relevant points,

if it is given that f is a cubic function with:

- $f(3) = f'(3) = 0$
- $f(0) = 27$
- $f''(x) > 0$ when $x < 3$ and $f''(x) < 0$ when $x > 3$. (3)

How would your learners respond?

Gr 12 Maths 2 in 1
Challenging Questions Booklet
p. 8 (Q8)

FUN QUESTION

Determine the value of

$$\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\left(1+\frac{1}{5}\right)\ldots$$

up to 98 factors. (4)

How would your learners respond?

MATHS TEACHER SUPPORT:

PAPER 1: Level 3 & 4 Questions

8 June 2022

Hosted by Pumla Ntsele

Presented by Anne Eadie

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This comprehensive package promotes the special skills required to master Level 3 & 4 Questions.

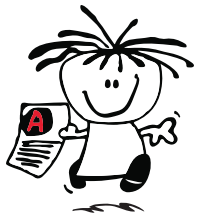
PAPER 1

LEVEL 3 & 4 QUESTIONS

PILLARS OF THINKING

GAPS & INDEPENDENT LEARNING

- Research/Misconceptions/Cognitive levels
- Sequencing
- Language/Notation
- Prior knowledge
- Concept Development
- Deep understanding
- Visualisation
- Summaries
- Strategies
- Integration of topics



PAPER 1

CHALLENGING QUESTIONS

Teaching Documents

by The TAS Maths Team

THE ANSWER SERIES *Your Key to Exam Success*

WWW.THEANSWER.CO.ZA

TEACHING DOCUMENTS

- The 2022 ATP (Proposed)
- Exam Mark Distribution (Gr 11 & Gr 12 Maths Paper 1)
- The most Challenging Topics
- Cognitive Levels
- Research:
  - % performance per topic since 2014
  - a list of challenging questions from 2014 to 2021
  - % performance and diagnostic commentary for DBE 2021 Paper 1 Questions

From the DBE Diagnostic Reports

- Exemplar Questions and Detailed Solutions

A PROPOSED 2022 ATP FOR FET MATHS

	Grade 10		Grade 11		Grade 12	
		No. of weeks		No. of weeks		No. of weeks
Term 1	**Algebraic Expressions, Numbers & Surds**	**4**	**Exponents & Surds**	**1**	**Patterns, Sequences and Series**	**3**
			Equations	**1**	Euclidean Geometry	3
	Exponents, Equations & Inequalities	**2**	**Equations & Inequalities**	**2**	Trigonometry	4
	Equations & Inequalities	**1**	Euclidean Geometry	4	**(Algebra)**	
	Euclidean Geometry (#1)	3	Trig functions & Revision of Gr 10 Trig	1		
			Trig identities & Reduction formulae	1		
Term 2	Trigonometry (#1)	3	Trig eqn. & Gen. sol's	1	Analytical Geometry	2
	Number Patterns	**1**	Quadrilaterals	1	**Functions & Inverse Functions & Exp & Log Functions**	**2**
	Functions (including Trig Functions (#2))	**6**	Analytical Geometry	2		
			Number Patterns	**2**	**Calculus, including Polynomials**	**5**
	Measurement	2	**Functions**	**5**		
			Trig – sin/cos/area rules	1	**Finance**	**3**
Term 3	Statistics	2	Trig – sin/cos/area rules	1	**Finance**	**1**
	Probability	**2**	Measurement	2	Statistics (regression & correlation)	3
	Finance (Growth)	**2**	Statistics	3	**Counting and Probability**	**3**
	Analytical Geometry	2	**Probability**	**4**	**INTERNAL EXAMS**	**4**
	Euclidean Geometry (#2)	3	**Finance (Growth & Decay)**	**1**		
Term 4	Revision	4	**Finance (Growth & Decay)**	**3**	**Revision (Paper 1)**	**1**
	FINAL EXAMS	3	Revision	1	Revision (Paper 2)	1
	Reporting	1 ½	**FINAL EXAMS**	**4**	Revision (Exam Techniques?)	1
			Reporting	1½	**EXTERNAL EXAMS**	6½

FET EXAM: Mark distribution

PAPER 1		
Description	**GR 11**	**GR 12**
Algebra and Equations (and inequalities); *Exponents*	45	25
Patterns & Sequences	25	25
Finance, growth and decay *(Financial Maths)*	15	15
Functions & Graphs	45	35
Differential Calculus		35
Probability	20	15
TOTAL	**150**	**150**

NOTE:

- Questions will not necessarily be compartmentalised in sections, as this table indicates. Various topics can be integrated in the same question.
- A formula sheet will be provided for the final examinations in Grades 10, 11 and 12.

THE MOST CHALLENGING PAPER 1 TOPICS

- **Functions & Graphs (35 marks)**
- **Calculus (35 marks)**
- **Probability (15 marks)**

COGNITIVE LEVELS

Level 1	Level 2	Level 3	Level 4
Knowledge **20%** (30 marks)	Routine procedures **35%** (52 – 53 marks)	Complex procedures **30%** (45 marks)	Problem-solving **15%** (22 – 23 marks)

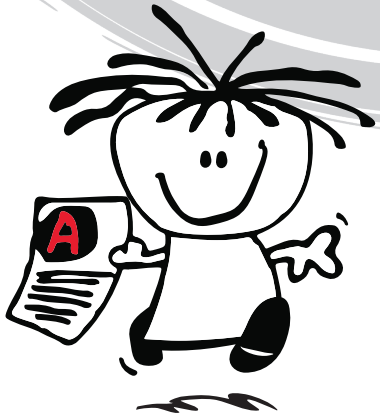
45% of each exam

THE COGNITIVE LEVELS SKILLS

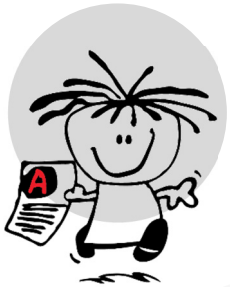
COGNITIVE LEVELS		DESCRIPTION OF SKILLS TO BE DEMONSTRATED
LEVEL 1	**KNOWLEDGE** 20% **(30 marks per paper)**	• Recall • Identification of correct formula on the information sheet (no changing of the subject) • Use of mathematical facts • Appropriate use of mathematical vocabulary • Algorithms • Estimation and appropriate rounding of numbers
LEVEL 2	**ROUTINE PROCEDURES** 35% **(52 - 53 marks per paper)**	• Proofs of prescribed theorems and derivation of formulae • Perform well-known procedures • Simple applications and calculations which might involve a few steps • Derivation from given information may be involved • Identification and use (after changing the subject) of correct formula • Generally similar to those encountered in class
LEVEL 3	**COMPLEX PROCEDURES** 30% **(45 marks per paper)**	• Problems involve complex calculations and/or higher-order reasoning • There is often not an obvious route to the solution • Problems need not be based on a real-world context • Could involve making significant connections between different representations • Require conceptual understanding • Learners are expected to solve problems by integrating different topics
LEVEL 4	**PROBLEM-SOLVING** 15% **(22 - 23 marks per paper)**	• Non-routine problems (which are not necessarily difficult) • Problems are mainly unfamiliar • Higher-order reasoning and processes are involved • Might require the ability to break the problem down into its constituent parts • Interpreting and extrapolating from solutions obtained by solving problems in unfamiliar contexts

RESEARCH:

Results & Diagnostic Reports



How 2021 compared with the previous 7 years ...



PAPER 1:	2014 – 2020 (average)	2021	
Algebra	70%	70%	—
Patterns & Sequences	59%	61%	—
Functions & Graphs	51%	58%	↑
Finance	50%	49%	—
Calculus	**45%**	**43%**	↓
Probability	30%	27%	↓

Some worrying facts ...

- Only 35% of all matrics did Core Maths in the 2021 exams.

CORE MATHS:

- **57,6%** ≥ 30%
- **35,6%** ≥ 40%

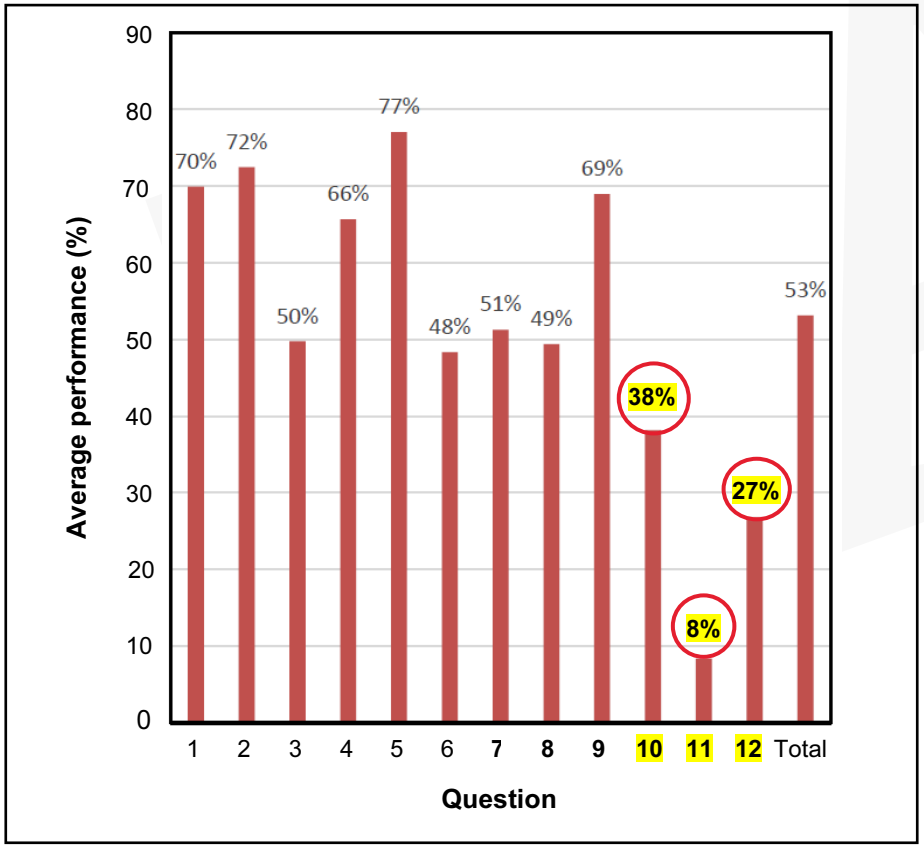
But, that means: 13,7% of all matrics achieved > 40% for Core Maths

Still problematic . . .

1. **Basic Concepts**
2. **Over-reliance on past examination papers**
3. The need for a **deeper understanding** of definitions and concepts.

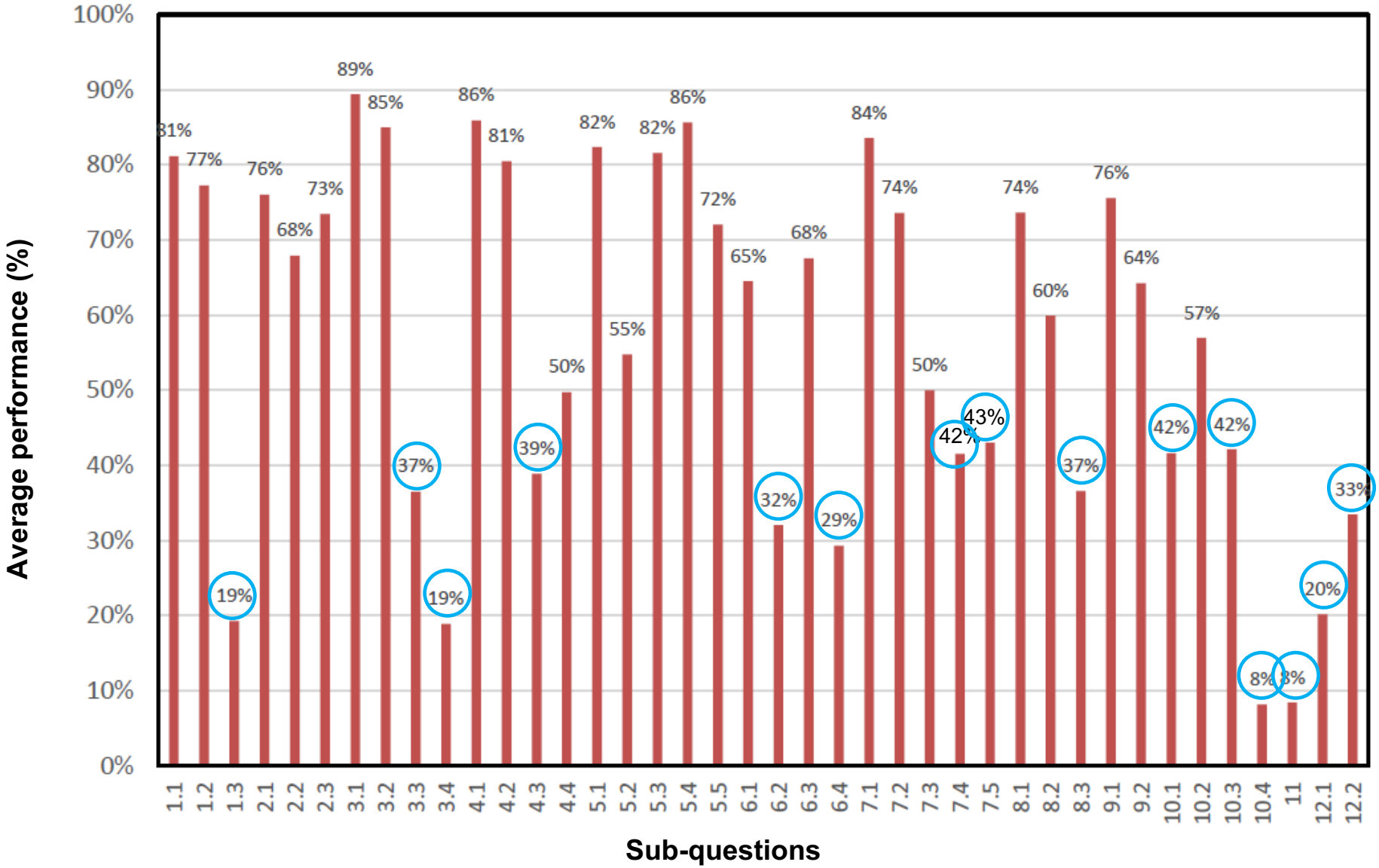
2021: Paper 1

Average % performance per question



Q1	Equations, Inequalities and Algebraic Manipulation
Q2	Number Patterns & Sequences
Q3	Number Patterns & Sequences
Q4	Number Patterns & Sequences
Q5	Functions and Graphs
Q6	Functions and Graphs
Q7	Functions & Graphs
Q8	Finance
Q9	**Calculus**
Q10	**Calculus**
Q11	**Calculus**
Q12	**Probability and Counting**

and, per sub-question



PAPER 1: Challenging Questions

Average mark (as a %) mostly < 40%

NB: Inverse functions to be seen as extension to functions

	2014	2015	2016	2017	2018	2019	2020	2021
FUNC. & GRAPHS								
Exp. & Log graphs	5 (44%)		6 (39%)			5.4 (37%) & 5.5 (31%)		6.4 (29%)
Parabola & Inv. parabola	6 (37%)				6 (45%)			
Parabola, Hyp. & . . .	4 (49%)		5 (27%)		5.4 (17%)			
Inv. functions		5 (26%)			4.4 (41%)		5.3 (34%) & 5.4 (11%)	
CALCULUS								
Derivative concept			8.2 (4%) & 8.4 (32%)			7.4 (40%)		7.4 (42%) & 7.5 (43%)
Cubic function	9 (53%)	9 (29%)	9 (42%)	8 (40%)	9 (38%)	9.2 (19%) & 9.4 (37%)	8.3 (30%); 8.5 (15%) & 9 (25%)	10 (38%)
Applications	10 (32%)	10 (22%)	10 (38%)	9 (9%)	10 (18%)	8 (39%)		11 (8%)
PROBABILITY								
Definition				10 (41%)				
Mut. excl. & Indep. events	11 (39%)	11.1 (53%)	11 (65%)		12.1 (69%)	11.1 (26%)	Tree diagram: 11 (18%)	12.1 (20%)
Counting principles	12 (29%)	11.2 (40%)	12 (2%)	11 (25%)	11 (34%)			12.2 (33%)
Patterns & Seq.			2.4 (38%) & 3.1 (33%)	3 (18%)	3.4 (14%)	3.1 (33%)	3.2 (27%) & 11.3 (1%)	3.3 (37%) & 3.4 (19%)
Finance		7.4 (43%)	7 (33%)	6.2 (29%)	7.1 (36%)		6.3 (36%)	8.3 (37%) & 4.3 (39%)
Algebra		1.2 (39%) & 1.3 (38%)		1.3 (23%)			1.3 (25%)	1.3 (19%)
Exponents					1.3 (19%)	1.3 (6%)		

DIAGNOSTIC REPORT: DBE 2021 Exam Paper 1

QUESTION 1 70%

81% 1.1 Solve for x:

1.1.1 $x^2 - 2x - 24 = 0$ (3)

1.1.2 $2x^2 - 3x - 3 = 0$
(correct to TWO decimal places) (3)

Memo

1.1.1 $x^2 - 2x - 24 = 0$

$\therefore (x - 6)(x + 4) = 0$

$\therefore x - 6 = 0$ or $x + 4 = 0$

$\therefore \boldsymbol{x = 6}$ ◄ $\therefore \boldsymbol{x = -4}$ ◄

1.1.2 $2x^2 - 3x - 3 = 0$:

$$\therefore x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-3)}}{2(2)} \quad \ldots \quad \boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

$$= \frac{3 \pm \sqrt{33}}{4}$$

$\approx$ **2,19 or −0,69** ◄

DIAGNOSTIC REPORT

Common Errors and Misconceptions

(a) Some candidates still factorised incorrectly in Q1.1.1.

(b) Rounding off the answers to two decimal places is still a problem for some candidates. For example, in Q1.1.2 some candidates rounded to −0,68 instead of −0.69. Others simply rounded to −0,6 despite the question stating explicitly to TWO decimal places.

Candidates made the following errors when entering the values into the calculator:

- Omitting brackets around the −3, i.e. $x = \dfrac{-3 \pm \sqrt{(-3)^2 - 4(2)(-3)}}{2(2)}$. This resulted in the following incorrect answers: $x = 0{,}22$ or $x = -1{,}72$.

Brackets!

- Creating the fraction for the part under the square root only, i.e. $x = \dfrac{-3 \pm \sqrt{-3^2 - 4(2)(-3)}}{2(2)}$. This led to the following incorrect answers: $x = 4{,}44$ or $x = 1{,}56$.

1.1.3 $x^2 + 5x \le -4$ (4)

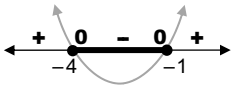
1.1.4 $\sqrt{x+28} = 2 - x$ (4)

Memo

1.1.3 $x^2 + 5x \le -4$

$\therefore\ x^2 + 5x + 4 \le 0$

$\therefore\ (x+4)(x+1) \le 0$



$\therefore\ \mathbf{-4 \le x \le -1}$ ◄ **OR:** $\mathbf{x \in [-4; -1]}$ ◄

1.1.4 $\sqrt{x+28} = 2 - x$

$\therefore\ \left(\sqrt{x+28}\right)^2 = (2-x)^2$ . . . *squaring both sides*

$\therefore\ x + 28 = 4 - 4x + x^2$

$\therefore\ -x^2 + 5x + 24 = 0$

$\therefore\ x^2 - 5x - 24 = 0$

$\therefore\ (x+3)(x-8) = 0$

$\therefore\ x = -3$ or $x = 8$

NOW, CHECK . . .

For $x = -3$: **LHS** $= \sqrt{25} = +5$
& **RHS** $= 2 - (-3) = +5$ ✓

For $x = 8$: **LHS** $= \sqrt{36} = 6$
& **RHS** $= 2 - 8 = -6$ ✗

$\therefore$ Only $\mathbf{x = -3}$ ◄

(c) In answering Q1.1.3 many candidates treated the inequality as an equation. Their answer would read: $(x+1)(x+4) \le 0$ followed by $x \le -1$ or $x \le -4$. These candidates did not realise that the question dealt with the product of two numbers and that the product of two negative numbers does not yield a negative result. In addition, the difference in the solutions: $x \ge -4$ or $x \le -1$ and $x \ge -4$ and $x \le -1$ were not understood by a number of candidates.

Intervals

Many candidates struggled to interpret the correct answer from the inequality.

$x^2 + 5x + 4 \le 0$

$(x+1)(x+4) \le 0$

$\therefore\ x = -1$ or $x = -4$

$\therefore\ x \le -4$ or $x \ge -1$

or

$x^2 + 5x + 4 \le 0$

$(x+1)(x+4) \le 0$

$\therefore\ x = -1$ or $x = -4$

$\therefore\ -1 \le x \le -4$

Inequalities

Some candidates drew a sketch but were unable to use it to write down the required answer.

Intervals

(d) Most candidates had some idea that they had to square both sides of the equation in Q1.1.4. **Few** candidates were unable to **square the binomial** on the RHS correctly, for example, they wrote $x + 28 = 4 + x^2$ or $x + 28 = 4 - x^2$ instead of $x + 28 = 4 - 4x + x^2$. **Very few** candidates **checked** if the **solutions** obtained were valid in the original equation and consequently failed to reject $x = 8$ as a solution.

Surd Equation

77% 1.2 Solve simultaneously for x and y in:

$2y = 3 + x$ and $2xy + 7 = x^2 + 4y^2$ (6)

Memo

1.2 $2y = 3 + x \Rightarrow 2y - 3 = x$ . . . ❶

$2xy + 7 = x^2 + 4y^2$ . . . ❷

❶ in ❷:

$\therefore\ 2y(2y - 3) + 7 = (2y - 3)^2 + 4y^2$

$\therefore\ 4y^2 - 6y + 7 = 4y^2 - 12y + 9 + 4y^2$

$\therefore\ -4y^2 + 6y - 2 = 0$

$\div(-2)$ $\therefore\ 2y^2 - 3y + 1 = 0$

$\therefore\ (2y - 1)(y - 1) = 0$

$\therefore\ y = \frac{1}{2}$ or 1

$\therefore\ y = \frac{1}{2}$ in ❶: $x = 2\left(\frac{1}{2}\right) - 3$

$\therefore\ x = -2$

$\therefore\ y = 1$ in ❶: $x = 2(1) - 3$

$\therefore\ x = -1$

$\therefore$ Solutions: $\left(-2; \frac{1}{2}\right)$ **or** $(-1; 1)$ ◄

(e) In Q1.2 some candidates made the following error when rewriting the linear equation in terms of one variable: $x = 3 - 2y$. Other candidates overlooked the factor of y in the first term when substituting into the quadratic equation. They would write $2(2y - 3) + 7 = (2y - 3)^2 + 4y^2$ instead of $2(2y - 3) + 7 = (2y - 3)^2 + 4y^2$.

Some candidates used the quadratic formula to solve the equation $4y^2 - 6y + 2 = 0$.

However, they wrote their answer as $x = \frac{1}{2}$ or $x = 1$ instead of $y = \frac{1}{2}$ or $y = 1$.

19% 1.3 The roots of an equation are

$$x = \frac{-n \pm \sqrt{n^2 - 4mp}}{2m}$$

where m, n and p are positive real numbers.

The numbers m, n and p, in that order, form a geometric sequence.

Prove that x is a non-real number. (4)

[24]

Memo

1.3 m; n; p a Geom. Seqn. ➡ $\frac{p}{n} = \frac{n}{m}$ . . . *the definition of a G.S.*

$\therefore\ n^2 = mp$

$\therefore\ \Delta = n^2 - 4mp = n^2 - 4n^2 = -3n^2$ ✱

But $n^2 > 0$ . . . *$n \neq 0$ (given that $n > 0$)*

$\therefore\ -3n^2 < 0$

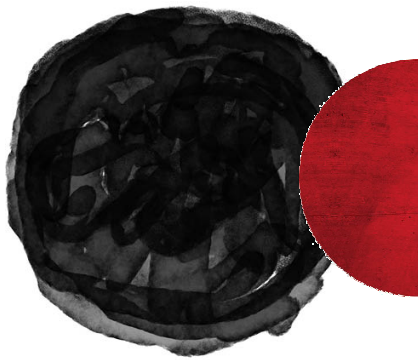
i.e. $\Delta < 0$

$\therefore$ **x is a non-real number** ◄

✱ [OR: $\Delta = mp - 4mp = -3mp$
which is negative $\because$ m & $p > 0$]

(f) Many candidates did not know how to answer Q1.3. **Few** candidates managed to arrive at $\Delta = -3n^2$, but **could not explain** why the roots were non-real. A fair number of candidates took arbitrary values for m, n and p and proved that $n^2 - 4mp$ was negative. This was not acceptable.

Nature of roots



Suggestions for Improvement

(a) Much of the work in this question is covered in Grade 11. It is therefore important for teachers to set revision tasks in these sections of work throughout the Grade 12 year.

(b) More thorough teaching of factorisation in Grades 9 and 10 is needed. Emphasis should be placed on how to identify the type of factorisation that is applicable to the given expression. Encourage weaker learners to use the quadratic formula instead of factorising.

(c) It is unacceptable for learners to write down the quadratic formula incorrectly. Therefore, they should be encouraged to copy the formula from the information sheet. Correct substitution, especially using **brackets** for negative values, should be emphasised in Grade 11. If this is done correctly, then learners should enter the values exactly as they have written it into their calculators to obtain the answers.

(d) Teachers should not take for granted that learners know how to round off a number to the required number of places. Where necessary, this skill should be retaught in Grades 11 and 12.

(e) Teachers should take some time, preferably in Grade 10, to focus on teaching learners how to represent **inequalities** (e.g. $-2 < x < 1$; $x < -2$ or $x > 1$) on a number line and also how to write an inequality from the illustration on a number line. This will benefit learners as they are required to write inequality solutions for a number of questions in both examination papers. Emphasise that correct notation is essential when writing down the solutions to inequalities.

(f) Teachers should explain the difference between *and* and *or* in the context of inequalities. Learners cannot use these words interchangeably as they have different meanings.

(g) When dealing with surd equations, learners should be reminded that they need to **square both sides** of the equation in order **to maintain the** balance. They should not square the radical parts of the equation only. Teachers must emphasise that implicit **restrictions** are placed on surd equations and that learners should continue to test whether their answers satisfy the original equation.

(h) Teachers should emphasise the difference between non-real and undefined numbers as these are two different groups of numbers.

QUESTION 2 72%

Given the geometric series: $x + 90 + 81 + \ldots$

76% 2.1 Calculate the value of x. (2)

68% 2.2 Show that the sum of the first n terms is $S_n = 1\,000\left(1 - (0{,}9)^n\right)$. (2)

73% 2.3 Hence, or otherwise, calculate the sum to infinity. (2)

[6]

Memo

2. G.S.: $x + 90 + 81 + \ldots$

2.1 $\frac{81}{90} = \frac{90}{x}$ $\ldots$ = *the common ratio*

$\therefore\ 81x = 8\,100$

$\therefore\ \boldsymbol{x = 100}$ ◄

2.2 $a = 100$; $r = \frac{81}{90} = \frac{9}{10} = 0{,}9$

$$\mathbf{S_n = \frac{a(1 - r^n)}{1 - r}}$$

$$\therefore\ S_n = \frac{100\left[1 - (0{,}9)^n\right]}{1 - 0{,}9}$$

$$= \frac{100\left[1 - (0{,}9)^n\right]}{\frac{1}{10}} \times \frac{10}{10}$$

$$= \mathbf{1\,000\left[1 - (0{,}9)^n\right]}$$ ◄

2.3 'Hence': $\lim\limits_{n \to \infty} (0{,}9)^n = 0$ $\ldots$

i.e. As $n \to \infty$, so $(0{,}9)^n \to 0$

$\therefore\ S_\infty = 1\,000[1 - 0]$

$= \mathbf{1\,000}$ ◄

OR: 'Otherwise': $\mathbf{S_\infty = \frac{a}{1 - r}}$

$$= \frac{100}{1 - 0{,}9}$$

$$= \frac{100}{\frac{1}{10}}\left(\times \frac{10}{10}\right)$$

$$= \mathbf{1\,000}$$ ◄

DIAGNOSTIC REPORT

Common Errors and Misconceptions

(a) The question indicated that the given sequence was geometric. Despite this, when answering Q2.1, some candidates incorrectly assumed that it was arithmetic and calculated the value of x using common difference between the terms:

$$T_2 - T_1 = T_3 - T_2 = d$$
$$90 - x = 90 - 81$$
$$x = 81$$

Definition of A.S. & G.S.

(b) Q2.2 required candidates to calculate the sum of the first n terms of a geometric series. This is a well-known concept. However, many candidates found difficulty in answering the question because they had to show that $\mathbf{S_n} = 1000(1 - 0{,}9n)$. Some candidates used the $\mathbf{T_n}$ formula for a geometric sequence, whilst others used the sum formula for an arithmetic series despite the question indicating otherwise.

Symbols

(c) Candidates who assumed that the series was arithmetic, calculated the value of r to be –9 and subsequently used this value when calculating the sum to infinity. These candidates failed to realise that this value of r violated the condition for which a geometric series converges, namely $\mathbf{-1 < r < 1}$. Other candidates used the incorrect value of $\frac{10}{9}$ for r.

Convergence

Suggestions for Improvement

(a) At some stage it is advisable to give learners an exercise that contains a mixture of quadratic, arithmetic and geometric sequences and series. Learners should analyse the type of sequence they are working with and which formulae are applicable to it.

(b) Teach learners how to identify whether the question requires them to calculate the value of the nth term **or** the sum of the first n terms.

(c) While covering this section, teachers should emphasise the difference between the **position** and the **value** of a term in a sequence. Learners must read the questions carefully so that they know what is required of them.

(d) Remind learners that **n** cannot be a negative number, zero or a fraction. When solving for n, learners should arrive at a **natural number** solution. If this is not the case, then they should know that they have made a mistake in their working.

(e) Make learners acutely aware of which formulae in the information sheet apply to which type of sequence. It is good practice for learners to use the information sheet in class so that they become familiar with it.

(f) It is important to demonstrate, by way of example, the concept of a convergent geometric series, first by taking a value of $r > 1$ and then taking a value of $\mathbf{-1 < r < 1}$. This should alert learners to **the condition** for which a geometric series will converge.

QUESTION 3 50%

Consider the quadratic number pattern:

$$-145\ ;\ -122\ ;\ -101\ ;\ \ldots$$

89% 3.1 Write down the value of T_4. (1)

85% 3.2 Show that the general term of this number pattern is $T_n = -n^2 + 26n - 170$. (3)

37% 3.3 Between which TWO terms of the quadratic number pattern will there be a difference of -121? (4)

19% 3.4 What value must be added to each term in the number pattern so that the value of the maximum term in the new number pattern formed will be 1? (3)

[11]

Memo

3.

	-145	-122	-101	-82
1st differences:	23	21	19	
2nd differences:	-2	-2		

3.1 $T_4 = -101 + 19 = -82$ ◄

3.2 The 2nd common difference: $2a = -2$

$\therefore a = -1$

The first 1st difference: $3a + b = 23$

$\therefore -3 + b = 23$

$\therefore b = 26$

& The general term, $T_n = an^2 + bn + c$

$\therefore$ The 1st term, $T_1 = a + b + c = -145$

$\therefore -1 + 26 + c = -145$

$\therefore c = -170$

$\therefore$ **$T_n = -n^2 + 26n - 170$** ◄

3.3 The 1st differences are a linear sequence:

$T_n = a + (n - 1)d$

$\therefore T_n = 23 + (n - 1)(-2) = 23 - 2n + 2 = -2n + 25$

Put $-2n + 25 = -121$

$\therefore -2n = -146$

$\therefore n = 73$

$\therefore$ The 73rd 1st difference is -121

$\therefore$ Between the 73rd & 74th terms of the quadratic pattern ◄

OR: Consider $T_{n+1} - T_n = -121$

where $T_{n+1} = -(n + 1)^2 + 26(n + 1) - 170$

$= -(n^2 + 2n + 1) + 26n + 26 - 170$

$= -n^2 + 24n - 145$

$\therefore -n^2 + 24n - 145 - (-n^2 + 26n - 170) = -121$

$\therefore -2n + 25 = -121$

$\therefore -2n = -146$

$\therefore n = 73$

$\therefore$ Between the 73rd & 74th terms ◄

3.4 The maximum of $T_n = -n^2 + 26n - 170$ occurs when the derivative, $-2n + 26 = 0$ ✱

$\therefore -2n = -26$

$\therefore n = 13$

$\therefore$ The maximum of the current number pattern

$= -13^2 + 26(13) - 170$

$= -1$

For the maximum to be 1, we need to add 2.

$\therefore$ **Add 2** ◄

✱ (OR: $n = -\frac{b}{2a}$ or complete the square)

DIAGNOSTIC REPORT

Common Errors and Misconceptions

(a) In answering Q3.3, many candidates incorrectly assumed that -121 was a term in the quadratic sequence instead of it being a term in the sequence of first differences. Consequently, they tried to solve the equation $-n^2 + 26n - 170 = -121$. This was viewed as a breakdown. A fair number of candidates created this equation:

$$\frac{-26 \pm \sqrt{(26)^2 - 4(-1)(-170)}}{2(-1)} = 0.$$

These candidates had no clue that the value of n in a sequence cannot be 0.

(b) The crux to answering Q3.4 was to compare the value of the maximum terms in the given sequence and the new sequence. Many candidates failed to link quadratic number patterns with the quadratic function. Hence, this question was not answered by a large majority of candidates.

Suggestions for Improvement

(a) Remind learners that n cannot be a negative number, zero or a fraction. When solving for n, learners should arrive at a natural number solution. If this is not the case, then they have made a mistake in their working.

Meaning of symbols & formulae

(b) When teaching quadratic number patterns, it is essential to show learners how the formulae: $\mathbf{T_1 = a + b + c}$, the first term of the first differences = $\mathbf{3a + b}$ and the second difference = $\mathbf{2a}$, are deduced.

(c) **The sequence of first differences of a quadratic number pattern form an arithmetic pattern**. This implies that an arithmetic sequence is embedded within a quadratic number pattern. Learners must read the question very carefully in order to establish which pattern the question is making reference to. Glossing over words in the question leads to learners making incorrect statements.

Quadratic vs Linear Sequences

QUESTION 4 66%

Consider the linear pattern: 5 ; 7 ; 9 ; ...

86% 4.1 Determine T_{51}. (3)

81% 4.2 Calculate the sum of the first 51 terms. (2)

39% 4.3 Write down the expansion of $\sum_{n=1}^{5\,000}(2n+3)$.

Show only the first 3 terms and the last term of the expansion. (1)

50% 4.4 Hence, or otherwise, calculate

$$\sum_{n=1}^{5\,000}(2n+3) + \sum_{n=1}^{4\,999}(-2n-1).$$

ALL working details must be shown. (4)

[10]

Memo

4. Linear pattern: 5 ; 7 ; 9 ; . . .

Note: A linear pattern is also an Arithmetic sequence.

4.1 $\mathbf{T_n = a + (n-1)d}$ where $a = 5$; $d = 2$; $n = 51$

$\therefore T_{51} = 5 + (51-1)(2)$

$= \mathbf{105}$ ◄

OR: $T_n = 2n + 3$

$\therefore T_{51} = 2(51) + 3$

$= \mathbf{105}$ ◄

4.2 $\mathbf{S_n = \frac{n}{2}(a + T_n)}$ where $\mathbf{a} = 5$; $\mathbf{n} = 51$; $\mathbf{T_{51}} = 105$ *(in 4.1)*

$= \frac{51}{2}(5 + 105)$

$= \mathbf{2\,805}$ ◄

OR: $\mathbf{S_n = \frac{n}{2}[2a + (n-1)d]}$ where $\mathbf{a} = 5$; $\mathbf{n} = 51$; $\mathbf{d} = 2$

$= \frac{51}{2}[2(5) + (51-1)(2)]$

$= \mathbf{2\,805}$ ◄

4.3 $\sum_{n=1}^{5\,000}(2n+3)$

$= [2(1)+3] + [2(2)+3] + [2(3)+3] + \ldots + [2(5\,000)+3]$

$= \mathbf{5 + 7 + 9 + \ldots + 10\,003}$ ◄

NB: If you used $T_n = 2n + 3$ in 4.1, you would have known this was the given linear pattern:

5 ; 7 ; 9 ; . . . ; 10 003.

4.4 **Method 1:**

The sum $= 10\,003 + \sum_{n=1}^{4\,999}(2n+3) + \sum_{n=1}^{4\,999}(-2n-1)$

$= 10\,003 + \sum_{n=1}^{4\,999}\left[(2n+3) + (-2n-1)\right]$

$= 10\,003 + \sum_{n=1}^{4\,999} 2$

$= 10\,003 + 4\,999 \times 2$

$= \mathbf{20\,001}$ ◄

Method 2: Use the formula $\mathbf{S_n = \frac{n}{2}(a + T_n)}$

For $\sum_{n=1}^{5\,000}(2n+3)$: $\mathbf{n} = 5\,000$; $\mathbf{a} = 5$; $\mathbf{T_n} = 10\,003$

$\therefore \mathbf{S_n} = \frac{5\,000}{2}(5 + 10\,003) = 25\,020\,000$

& For $\sum_{n=1}^{4\,999}(-2n-1)$: $\mathbf{n} = 4\,999$; $\mathbf{a} = -3$; $\mathbf{T_n} = -9\,999$

$\therefore \mathbf{S_n} = \frac{4\,999}{2}[(-3) + (-9\,999)] = -24\,999\,999$

$\therefore$ The sum = **20 001** ◄

Method 3:

$\sum_{n=1}^{4\,999}(-2n-1)$

$= [-2(1)-1] + [-2(2)-1] + [-2(3)-1] + \ldots$

$\ldots + [-2(4\,999)-1]$

$= \mathbf{(-3)} + -5 + -7 + \ldots + -9\,999$

& $\sum_{n=1}^{5\,000}(2n+3)$

$= 5 + 7 + \ldots + 9\,999 + \mathbf{10\,001} + \mathbf{10\,003}$

Now add:

$-3 - (5 + 7 + \ldots + 9\,999) + (5 + 7 + \ldots + 9\,999) + \mathbf{10\,001} + \mathbf{10\,003}$

$= \mathbf{20\,001}$ ◄

Method 4:

$\sum_{n=1}^{5\,000}(2n+3) + \sum_{n=1}^{4\,999}(-2n-1)$

$= (5 + 7 + 9 + \ldots + 10\,001 + \mathbf{10\,003}) + \ldots$

$\ldots + (-3 - 5 - 7 \ldots - 9\,999)$

$= 2 + 2 + 2 + \ldots + 2 + \mathbf{10\,003}$

$= 4\,999(2) + 10\,003$

$= \mathbf{20\,001}$ ◄

DIAGNOSTIC REPORT

Common Errors and Misconceptions

(a) A small minority of candidates used **incorrect formulae** in Q4.1 and Q4.2.

(b) In Q4.3 a number of candidates successfully calculated the first three terms of the series but forgot to calculate the last term. However, the vast majority of the candidates did not write their answer as **a sum** of these terms. They wrote their answer as 5 ; 7 ; 9 ; … ; 10 003 instead of 5 + 7 + 9 + … + 10 003. Candidates failed to realise that **sigma notation** is a compact form of a series of terms.

(c) Many candidates failed to **interpret the sigma notation** correctly in Q4.4. They failed to see that some terms in the second expansion would cancel some terms in the first expansion. Further, candidates failed to realise that the question could have been solved as two separate sums.

4 Methods!

Suggestions for Improvement

(a) Teachers need to clarify that the sigma notation is a short-hand notation of a series of terms. Give learners enough examples where they have to expand the sigma notation. Use simple ones to start with, probably containing only a few terms. Also give them examples that do not represent arithmetic and geometric series.

(b) Learners should also be exposed to writing a series in sigma notation.

QUESTION 5 77%

Given: $f(x) = \frac{-1}{x-3} + 2$

82% 5.1 Write down the equations of the asymptotes of f. (2)

55% 5.2 Write down the domain of f. (1)

82% 5.3 Determine the coordinates of the x-intercept of f. (2)

86% 5.4 Write down the coordinates of the y-intercept of f. (2)

72% 5.5 Draw the graph of f. Clearly show ALL the asymptotes and intercepts with the axes. (3)

[10]

Memo

5. $f(x) = \frac{-1}{x-3} + 2$

5.1 Vertical asymptote: $x = 3$ ◄

Horizontal asymptote: $y = 2$ ◄

5.2 Domain: $x \in \mathbb{R};\ x \neq 3$

5.3 **x-int.** ($y = 0$): $\frac{-1}{x-3} + 2 = 0$

$\therefore \frac{-1}{x-3} = -2$

$\therefore -1 = -2x + 6$

$\therefore 2x = 7$

$\therefore x = \frac{7}{2}$

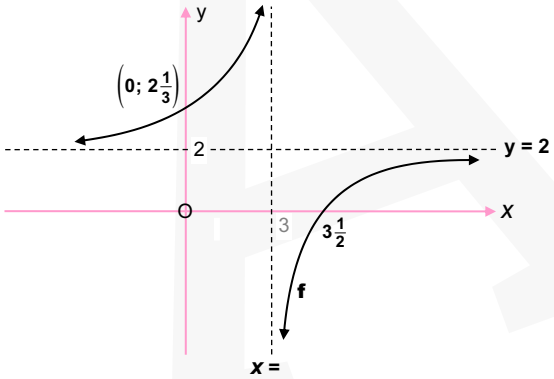
$\therefore \left(\frac{7}{2}; 0\right)$ ◄

5.4 **Y-int.** ($x = 0$): $f(0) = \frac{-1}{0-3} + 2$

$= 2\frac{1}{3}$

$\therefore \left(0; 2\frac{1}{3}\right)$ ◄

5.5



DIAGNOSTIC REPORT

Common Errors and Misconceptions

(a) In Q5.1, instead of the correct answer of $x = 3$ and $y = 2$, some candidates gave as the answer: p = 3 and q = 2, or $x \neq 3$ and $y \neq 2$. None of these were accepted as correct. Some candidates incorrectly wrote the **equation** of the vertical **asymptote** as $x = -3$.

(b) Candidates still confuse the **domain** with the **range** and consequently gave the incorrect answer of y = 2. Many candidates gave their answer as $x \in R$. This was not accepted as it is incorrect.

(c) Candidates were unable to correctly solve the equation $\frac{-1}{x-3} + 2 = 0$ on account of poor simplification skills. Hence, they could not calculate the x-intercept correctly.

(d) Many candidates were able to sketch the hyperbola having the correct increasing shape. However, they failed to label the asymptotes and the intercepts with the axes on their sketch graphs. They were not awarded marks for the asymptotes and intercepts with the axes because their sketches were not drawn to scale.

Suggestions for Improvement

(a) Teachers should pay attention to the **concepts and definitions** when teaching functions.

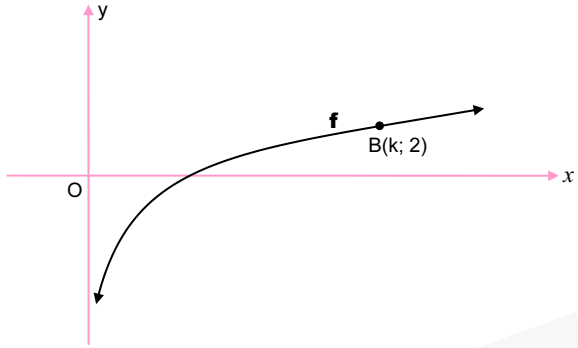
(b) Teachers should spend some time discussing that all points on the x-axis have a y-coordinate of 0 and all points on the y-axis have a x-coordinate of 0. The domain is always a set of x-values and the range is always a set of y-values.

(c) When teaching the hyperbola, start with the **'basic graph'** $y = \frac{a}{x}$ and develop **the general hyperbola** $y = \frac{a}{x + p} + q$.

This will enable learners to understand the effect of the changes in the variables a, p and q on the graph, its asymptotes and axes of symmetry.

QUESTION 6 48%

The graph of $f(x) = \log_4 x$ is drawn below.
B(k; 2) is a point on f.



65% 6.1 Calculate the value of k. (2)

32% 6.2 Determine the values of x for which $-1 \le f(x) \le 2$. (2)

Memo

6.1 **Equation of f:** $y = \log_4 x$

B(k; 2) on f: $\therefore\ 2 = \log_4 k$

$\therefore\ k = 4^2$

$\therefore\ \mathbf{k = 16}$ ◄

6.2 $f(x) = -1$ ➡ $\log_4 x = -1$

$\therefore\ x = 4^{-1}$

$\therefore\ x = \frac{1}{4}$

$\therefore\ \mathbf{\frac{1}{4} \le x \le 16}$ ◄ . . . $f(x) = 2$ for $x = 16$

DIAGNOSTIC REPORT

Common Errors and Misconceptions

(a) Many candidates were unable to solve the logarithmic equation correctly in Q6.1.

Some incorrect answers were:

$2 = \log_4 k$	$2 = \log_4 k$	$2 = \log_4 k$
$\therefore k = 2^4$	$\therefore 2 = 4^k$	$\therefore 4 = 2^k$

Definition of a log

(b) In Q6.2 many candidates failed to interpret the question correctly, i.e. to determine the values of x when the value of y lies from -1 to 2. They did not realise that they had to determine an x-value when $y = -1$. Consequently, they were unable to state the correct **interval** in terms of x. A common incorrect answer was $0 \le x \le 16$.

Intervals

68% 6.3 Write down the equation of f^{-1}, the inverse of f, in the form y = . . . (2)

29% 6.4 For which values of x will $x \cdot f^{-1}(x) < 0$? (2)

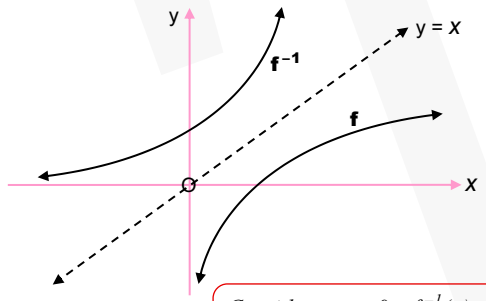
[8]

Memo

6.3 **Equation of f^{-1}:** $x = \log_4 y$

$\therefore$ **$y = 4^x$** ✓

6.4



$\therefore$ **$x < 0$** ✓ . . .

Consider: $x < 0$: $f^{-1}(x) > 0$
$x = 0$: —
$x > 0$: $f^{-1}(x) > 0$

(c) Many candidates understood that they had to swop x and y in order to obtain the inverse of the function f in Q6.3. However, **poor conversion** from logarithmic form to exponential form resulted in an incorrect answer in y-form.

Definition of log

(d) Candidates could not visualise the answer to Q6.4 because the sketch of the inverse of f was not given. Many candidates resorted to calculating the answer algebraically, but their solutions were incorrect.

Suggestions for Improvement

(a) **Teachers should spend some time discussing logarithms as a topic**.

The skill of changing from the exponential form to the logarithmic form and vice versa must be emphasised. This skill is required for determining the equation of the inverse of an exponential graph as well as solving for n in financial questions that observe an exponential pattern.

(b) Teachers should discuss the **meaning of mathematical statements:** $x < 0$, $x > 0$, $y < 0$, $y > 0$, etc. and show where these regions are represented in the Cartesian plane.

(c) Teachers should remind learners that the product of two numbers is negative when one of the numbers is positive and the other is negative. Similarly, the product of two numbers is positive when both numbers are negative or when both numbers are positive.

(d) **Basic interpretation of graphs** should start in Grade 10. Learners should then be able to approach questions in Grades 11 and 12 with a little more confidence.

Deep Understanding

QUESTION 7 51%

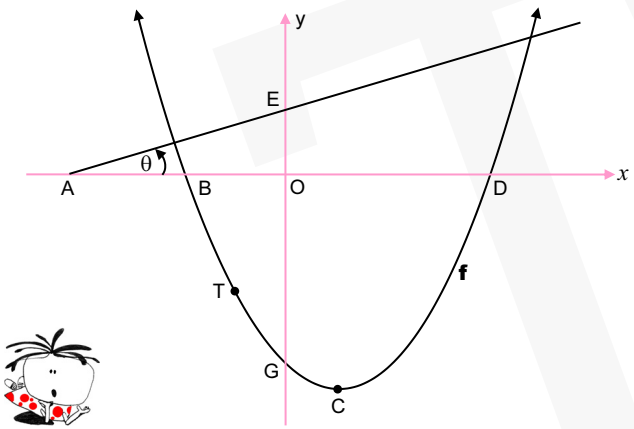
The graph of $f(x) = (x + 4)(x - 6)$ is drawn below.

The parabola cuts the x-axis at B and D and y-axis at G.

C is the turning point of f.

Line AE has an angle of inclination of θ and cuts the x-axis and y-axis at A and E respectively.

T is a point on f between B and G.



84% 7.1 Write down the coordinates of B and D. (2)

74% 7.2 Calculate the coordinates of C. (2)

50% 7.3 Write down the range of f. (1)

42% 7.4 Given that $\theta = 14{,}04°$ and the tangent to f at T is perpendicular to AE.

7.4.1 Calculate the gradient of AE, correct to TWO decimal places. (1)

7.4.2 Calculate the coordinates of T. (5)

43% 7.5 A straight line, g, parallel to AE, cuts f at K(–3; –9) and R. Calculate the x-coordinate of R. (6)

[17]

Memo

7.1 **B(–4; 0)** ◄ & **D(6; 0)** ◄

7.2 $x_C = \frac{-4+6}{2} = 1$

& $f(1) = (1 + 4)(1 - 6) = -25$

$\therefore$ **C(1; –25)** ◄

7.3 Range of f: **$y \geq -25$; $y \in \mathbb{R}$** ◄

7.4.1 $m_{AE} = \tan 14{,}04° \simeq$ **0,25** ◄

7.4.2 The gradient of AE, $m_{AE} = \frac{1}{4}$

$\therefore$ The grad. of the tangent at T $= -4$ . . . *the tangent at $T \perp AE$*

$f(x) = x^2 - 2x - 24$

$f'(x) = 2x - 2$ . . . *the gradient of the tangent at T*

$\therefore 2x - 2 = -4$

$\therefore 2x = -2$

$\therefore x = -1$

& $f(-1) = (-1)^2 - 2(-1) - 24 = 1 + 2 - 24 = -21$

$\therefore$ **T(–1; –21)** ◄

7.5 The gradient of g $= m_{AE} = \frac{1}{4}$

The equation of g:

Subst. K(–3; –9) & $m = \frac{1}{4}$ in:

$y - y_1 = m(x - x_1)$

$\therefore y + 9 = \frac{1}{4}(x + 3)$

$\therefore y = \frac{1}{4}x + \frac{3}{4} - 9$

$\therefore y = \frac{1}{4}x - 8\frac{1}{4}$

At R: $f(x) = g(x)$

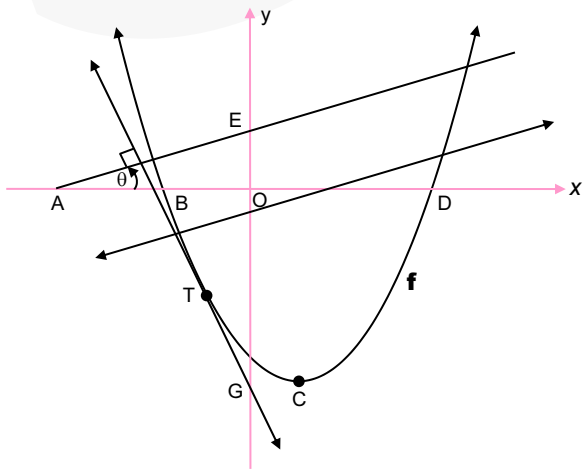
$\therefore x^2 - 2x - 24 = \frac{1}{4}x - 8\frac{1}{4}$

$(\times 4)$ $\therefore 4x^2 - 8x - 96 = x - 33$

$\therefore 4x^2 - 9x - 63 = 0$

$\therefore (4x - 21)(x + 3) = 0$

$\therefore$ **$x_R = \frac{21}{4}$** ◄ . . . *$x > 0$ at R*



DIAGNOSTIC REPORT

(CALCULUS)

2021

QUESTION 7

Common Errors and Misconceptions

(a) While many candidates were able to determine the answers to Q7.1, they did not give their answers in coordinate form as required.

Basic parabola

(b) In Q7.2 many candidates failed to use the most direct method of calculating the x-coordinate of the turning point C, i.e. using the x-intercepts calculated in Q7.1. Instead they performed additional calculations to arrive at this answer.

(c) When answering Q7.3 some candidates gave their answer in terms of x instead of y. These candidates confused the range with the domain. A number of candidates excluded the turning point in their answer. They gave the answers as $y > -25$ instead of $y \geq -25$. Some mistook G to be the turning point and gave the answer as $y \geq -24$.

Range & Domain

(d) In Q7.4.1 some candidates confused the angle of inclination with the gradient. They incorrectly calculated the gradient of AE as $m = \tan^{-1}(14{,}04°) = 85.93$.

Analytical Geometry

(e) Many candidates incorrectly assumed that T was the midpoint of B and C when answering Q7.4.2. They were unable to make the link between the gradient of the tangent and the derivative of the function f. Of those candidates who used the derivative in their answer, some equated the derivative to 0 instead of equating it to the gradient of the tangent.

Concept of Derivative

(f) In Q7.5 many candidates were unable to determine the equation of the straight line passing through K correctly. The challenge in this instance was that they were unable to establish the gradient of the line correctly. Consequently, they were unable to determine the x-coordinate of R by solving a set of equations simultaneously. Some candidates were able to calculate the equation of the line passing through K correctly but then took R to be the x-intercept of this line.

Basic straight line graph

Question 7: Suggestions for Improvement

(a) Teachers should spend some time discussing the basic concepts of functions: all points on the x-axis have a y-coordinate of 0 and all points on the y-axis have a x-coordinate of 0. The domain is always a set of x-values and the range is always a set of y-values.

Ready graph knowledge

(b) Teachers should integrate the findings of the gradient of a tangent to a cubic function to a parabola. They should ensure that learners understand that the gradient of the tangent through the turning point of a parabola is zero.

Understanding that the **derivative** is the **gradient** of the tangent . . .

QUESTION 8 49%

74% 8.1 A farmer bought a tractor for R980 000.

- The value of the tractor depreciates annually at a rate of 9,2% p.a. on the reducing-balance method.
- Calculate the book value of the tractor after 7 years. (3)

60% 8.2 How many years will it take for an amount of R75 000 to accrue to R116 253,50 in an account earning interest of 6,8% p.a., compounded quarterly? (4)

Memo

8.1 **A?** ; **P** = 980 000 ; **i** = 9,2% = 0,092 ; **n** = 7

$\mathbf{A = P(1 - i)^n}$ ➡ $A = 980\,000(1 - 0{,}092)^7$

$= \mathbf{R498\,685{,}82}$ ◄

8.2 **n?** ; **P** = 75 000 ; **A** = 116 253,50 ; $\mathbf{i} = \frac{6{,}8\%}{4} = \frac{0{,}068}{4}$;

n = the number of years

Note: 'accrue' is financial terminology for 'accumulate'

$\mathbf{A = P(1 + i)^n}$ ➡ $116\,253{,}50 = 75\,000\left(1 + \frac{0{,}068}{4}\right)^{4n}$

$\therefore \left(1 + \frac{0{,}068}{4}\right)^{4n} = \frac{116\,253{,}50}{75\,000}$

$\therefore 1{,}017^{4n} = 1{,}55$

$\therefore 4n = \log_{1{,}017} 1{,}55$

$\left[= \frac{\log 1{,}55}{\log 1{,}017}\right]$

$= 25{,}99\ldots$

$\therefore \mathbf{n} \approx 6{,}5$

$\therefore \mathbf{6\frac{1}{2}}$ **years** ◄

DIAGNOSTIC REPORT

Common Errors and Misconceptions

(a) In Q8.1 some candidates used the straight-line depreciation formula instead of the reducing-balance depreciation formula.

Understanding formulae

(b) It was evident in Q8.2 that the candidates were struggling with the application of logarithms in solving questions. In instances where candidates used n as the number of compounding periods, some of them had difficulty in interpreting the final answer.

$116\,253{,}50 = 75\,000\left(1 + \frac{0{,}068}{4}\right)^n$, was followed by:

$n = 25{,}99$

$\therefore n = 26$ years

The calculation is correct but n represented the number of quarters and not years. Some candidates rounded off their answers too early. This resulted in an error in the answer. A few candidates swopped the values of A and P when substituting into the formula.

8.3 Thabo wanted to save R450 000 as a deposit to buy a house on 30 June 2018.

8.3.1
- He deposited a fixed amount of money at the end of every month into an account earning interest of 8,35% p.a., compounded monthly.
- His first deposit was made on 31 July 2013 and his 60th deposit on 30 June 2018.
- Calculate the amount he deposited monthly. (3)

8.3.2
- Thabo bought a house costing R1 500 000 and used his savings as the deposit.
- He obtained a home loan for the balance of the purchase price at an interest of 12% p.a., compounded monthly over 25 years.
- He made his first monthly instalment of R11 058,85 towards the loan on 31 July 2018.

(a) What will the balance outstanding on the loan be on 30 June 2039, 21 years after the loan was granted? (3)

37% (b) Calculate the interest Thabo will have paid over the first 21 years of the loan. (3)

[16]

Memo

8.3.1 $\mathbf{F_v} = 450\,000$; x**?** ; $\mathbf{i} = \frac{8{,}35\%}{12} = \frac{0{,}0835}{12}$; $\mathbf{n} = 60$

$$\mathbf{F_v} = \frac{\mathbf{x}\left[(1+\mathbf{i})^{\mathbf{n}} - 1\right]}{\mathbf{i}}$$

$$\therefore \frac{x\left[\left(1+\frac{0{,}0835}{12}\right)^{60} - 1\right]}{\frac{0{,}0835}{12}} = 450\,000$$

$$\therefore x = \frac{450\,000 \times \frac{0{,}0835}{12}}{\left[\left(1+\frac{0{,}0835}{12}\right)^{60} - 1\right]}$$

$$= \mathbf{R6\,068{,}69}$$

(c) The most common error in Q8.3.1 was that candidates incorrectly selected the present value formula to answer this question. It would seem that candidates immediately use the present value formula to any question in which the purchase of a house is mentioned. Some candidates left out the '– 1' in the future value annuity formula.

Incorrect formula selection (as in Q8.1)

8.3.2 (a) The loan = 1 500 000 – 450 000
= R1 050 000

The Balance Outstanding **(BO)** equals the Present Value of the remaining payments

$\therefore$ **BO** = $\mathbf{P_V}$; **x** = 11 058,85 ; **i** = $\frac{12\%}{12} = \frac{0{,}12}{12}$;
n = $4 \times 12 = 48$

$$\mathbf{P_V} = \frac{\mathbf{x}\left[\mathbf{1 - (1 + i)^{-n}}\right]}{\mathbf{i}}$$

$$\therefore \mathbf{BO} = \frac{11\,058{,}85\left[1 - \left(1 + \frac{0{,}12}{12}\right)^{-48}\right]}{\frac{0{,}12}{12}}$$

$\approx$ **R419 948,32** ◄

OR: **BO** = F_V of the loan, i.e. its accrued value over the 21 years – the future value of the annuity by the end of the 21 years.

$$\mathbf{BO} = (1\,500\,000 - 450\,000)\left(1 + \frac{0{,}12}{12}\right)^{21 \times 12} - \frac{11\,058{,}85\left[\left(1 + \frac{0{,}12}{12}\right)^{21 \times 12} - 1\right]}{\frac{0{,}12}{12}}$$

= **R419 952,39** ◄

(b) The amount paid off = 1 050 000 – 419 948,32
= R630 051,68

$\therefore$ Interest paid = Total amount paid monthly over 21 years – the amount of the loan paid off

= $21 \times 12 \times 11\,058{,}85 - 630\,051{,}68$

= **R2 156 778,52** ◄

(d) In Q8.3.2(a), where candidates used the Pv formula to calculate the outstanding balance, they used the incorrect value of n. They used n = 252, the number of payments made, instead of n = 48, the number of payments outstanding. In the case where candidates used the alternate formula to calculate the outstanding balance, they only calculated the value of the payments made inclusive of interest. They omitted to subtract this amount from the value of the loan inclusive of interest.

Understanding

(e) Very few candidates had any idea how to respond to Q8.3.2(b). Many calculated the balance after 252 payments and subtracted this amount from the original loan amount. They failed to take into consideration the total amount repaid over the period.

Suggestions for Improvement

(a) Learners should be exposed to an exercise in which they **select the correct formula** to each question.

(b) Teachers should explain the difference in meaning between the rate of interest and the amount of interest paid.

Logs

(c) It is essential for learners to be able to accurately change from exponential form to logarithmic form. Teachers should teach this concept thoroughly.

(d) Learners need deeper insight into the relevance of each of the formulae and under which **circumstances** each can be used.

The variables in each formula must be explained. More practice in Financial Mathematics is necessary so that learners can distinguish among the different formulae.

1

(e) Discuss the **two ways** of calculating the outstanding balance of a loan. The first is when the number of payments made is known and the second is when the number of payments outstanding is known.

2

Outstanding balance

(f) Teachers should demonstrate all the steps required when using a **calculator**. Learners should be penalised in formal assessment tasks at school for rounding off early.

Calculator competence

QUESTION 9 69%

76% 9.1 Determine $f'(x)$ from first principles if it is given that $f(x) = 2x^2 - 3x$. (5)

Memo

9.1 $f(x) = 2x^2 - 3x$

$\therefore f(x+h) = 2(x+h)^2 - 3(x+h)$

$= 2(x^2 + 2xh + h^2) - 3x - 3h$

$= 2x^2 + 4xh + 2h^2 - 3x - 3h$

$\therefore f(x+h) - f(x) = 4xh + 2h^2 - 3h$

$\therefore \dfrac{f(x+h) - f(x)}{h} = 4x + 2h - 3$

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}} = \lim_{h \to 0} (4x + 2h - 3)$$

the definition of a derivative

$= \mathbf{4x - 3}$

DIAGNOSTIC REPORT

Conceptual Understanding

Common Errors and Misconceptions

(a) In Q9.1 many candidates made the following notational errors:

$$\boxed{f(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}} \quad \textbf{OR} \quad \boxed{\lim_{h \to 0} = \frac{f(x+h) - f(x)}{h}} \quad \textbf{OR} \quad \boxed{\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

They lost a mark for these errors.

Algebra

Some candidates made the following mistakes when removing brackets:

$2(x+h)^2 = (2x + 2h)^2$, $\quad -(2x^2 - 1) = -2x^2 - 1$,

$2(x+h)^2 = 2x^2 + 2xh + 2h^2$ **OR** $f(x+h) = 2(x+h)^2$.

In other instances, candidates did not use brackets in the numerator:

$$f'(x) = \lim_{h \to 0} \frac{2x^2 + 4xh + 2h^2 - 3x - 3h - 2x^2 - 3x}{h}$$

This lead to a breakdown in the answer.

Suggestions for Improvement

(a) Emphasis should be placed on the use of the correct notation when determining the derivative, either when using first principles or the rules.

64% 9.2 Determine:

9.2.1 $\frac{dy}{dx}$ if $y = 4x^5 - 6x^4 + 3x$ (3)

9.2.2 $D_x\left[-\frac{\sqrt[3]{x}}{2} + \left(\frac{1}{3x}\right)^2\right]$ (4)

[12]

Memo

9.2.1 $y = 4x^5 - 6x^4 + 3x$

$\therefore \frac{dy}{dx} = \mathbf{20x^4 - 24x^3 + 3}$ ◄

9.2.2 $D_x\left[-\frac{x^{\frac{1}{3}}}{2} + \frac{1}{9x^2}\right]$

$= D_x\left[-\frac{1}{2}x^{\frac{1}{3}} + \frac{1}{9} \cdot x^{-2}\right]$

$= -\frac{1}{2} \cdot \frac{1}{3}x^{\frac{1}{3}-1} + \frac{1}{9} \cdot -2x^{-3}$

$= \mathbf{-\frac{1}{6} \cdot x^{-\frac{2}{3}} - \frac{2}{9} \cdot x^{-3}}$ ◄

$\left[= \mathbf{-\frac{1}{6\sqrt[3]{x^2}} - \frac{2}{9x^3}} \text{ ◄}\right]$

DIAGNOSTIC REPORT

Common Errors and Misconceptions

(b) The common error in Q9.2.2 was that candidates were unable to convert the two terms to the differentiable form, **a.x^n**, on account of the fractions.

Candidates wrote $-\frac{\sqrt[3]{x}}{2}$ as $-2(x)^{\frac{1}{3}}$ or $-2(x)^{\frac{2}{3}}$ or $\frac{(x)^{\frac{2}{3}}}{2}$ instead of $-\frac{(x)^{\frac{1}{3}}}{2}$.

Algebra

They also wrote $\left(\frac{1}{3x}\right)^2$ as $3x^{-2}$ or $9x^{-2}$ instead of $\frac{1}{9}x^2$.

Exponents

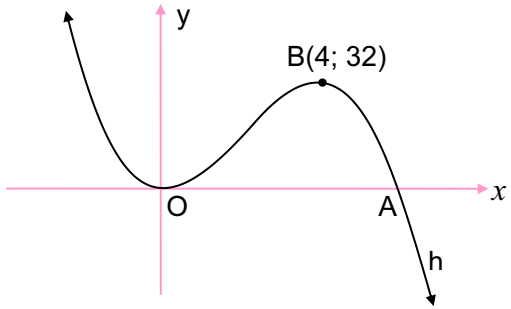
The 'rule' concept

Suggestions for Improvement

(b) Teachers should explain the need for brackets when determining the derivative from first principles. This prevents the incorrect simplification that follows.

QUESTION 10 38%

The graph of $h(x) = ax^3 + bx^2$ is drawn.
The graph has turning points at the origin, O(0; 0) and B(4; 32). A is an x-intercept of h.



42% 10.1 Show that a = –1 and b = 6. (5)

Memo

10. $h(x) = ax^3 + bx^2$

10.1 B(4; 32) on h: $\therefore\ 32 = 64a + 16b$

(÷16) $\therefore\ 2 = 4a + b$ . . . ❶

& $h'(x) = 3ax^2 + 2bx = 0$ at (4; 32)

$\therefore\ 48a + 8b = 0$

(÷8) $\therefore\ 6a + b = 0$ . . . ❷

& $4a + b = 2$ . . . ❶

❷ – ❶: $\therefore\ 2a = -2$

$\therefore\ \mathbf{a = -1}$ ◄

❷: $\therefore\ -6 + b = 0$

$\therefore\ \mathbf{b = 6}$ ◄

DIAGNOSTIC REPORT

Common Errors and Misconceptions

Algebra

(a) In determining a and b in Q10.1, candidates had to derive two linear equations and solve them simultaneously. Most candidates managed to substitute the coordinates of the turning point into the given expression and obtain the first equation. They were unable to derive the second equation because it required them to make use of the derivative.

Concept of derivative

Some candidates took a = –1 and b = 6 as given and used these values in the given expression. They then calculated that the turning points of the function were (0; 0) and (4; 32). This is considered a circular argument and is not acceptable.

Logic

Suggestions for Improvement

Graph Interpretation

(a) The focus when teaching cubic functions should not only be on calculating the critical points but also on **interpreting** the critical points on the graph. For example, what does it mean when we know that the x-coordinate of a turning point on a graph is 4?

Nice question

57% 10.2 Calculate the coordinates of A. (3)

Memo

10.2 At A, $h(x) = 0$

$\therefore\ -x^3 + 6x^2 = 0$

$\therefore\ -x^2(x - 6) = 0$

$\therefore\ x_A = 6 \qquad \ldots\ x_A > 0$

$\therefore$ **A(6; 0)** ◄

DIAGNOSTIC REPORT

Common Errors and Misconceptions

(b) Some candidates experienced difficulty in factorising the expression in order to calculate the coordinates of A.

CONCAVITY

42% 10.3 Write down the values of x for which h is:

10.3.1 increasing (2)

10.3.2 concave down (2)

Memo

10.3.1 $\mathbf{0 < x < 4}$ ◄

$\ldots\ 0 \leq x \leq 4$ *will also be accepted*

10.3.2 At the point of inflection: $h''(x) = 0$

$h'(x) = -3x^2 + 12x$

$\therefore\ h''(x) = -6x + 12 = 0$

$\therefore\ -6x = -12$

$\therefore\ x = 2$

$\therefore\ \mathbf{x > 2}$ ◄

DIAGNOSTIC REPORT

Common Errors and Misconceptions

(c) Many candidates failed to translate the words in Q10.3 into mathematical language. They were unable to link an increasing function to where the value of y increases when moving from left to right on the x-axis. Candidates had little idea that the change in concavity occurs at the point of inflection on the graph. Many did not calculate the x-coordinate of the point of inflection when answering Q10.3.2.

Concept of point of inflection

Suggestions for Improvement

(b) When teaching graphs of cubic functions, teachers should inform learners of both methods of determining the x-coordinate of the point of inflection: solving for x in $f'(x) = 0$ as well as determining the x-value midway between the two turning points.

(c) Teachers should teach concavity in such a way that learners can visually identify where a graph is concave up or concave down. In this way, learners should deduce that the point of inflection is critical to establishing the concavity of a cubic graph.

GRAPH INTERPRETATION

8% 10.4 For which values of k will

$-(x-1)^3 + 6(x-1)^2 - k = 0$

have one negative and two distinct positive roots? (3)

[15]

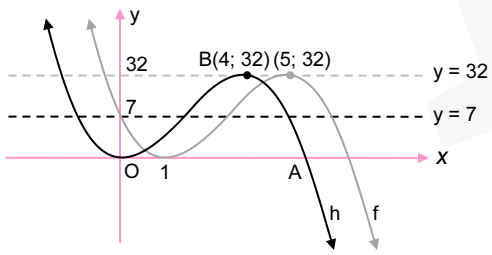
Memo

10.4 $-(x-1)^3 + 6(x-1)^2 = k$

$\therefore\ h(x-1) = k$

say $f(x) = k$

The graph f is the translation of h 1 unit to the right.



The y-intercept of f:

$f(0) = -(0-1)^3 + 6(0-1)^2$

$= 1 + 6$

$= 7$

The roots of the equation are the x-values for which the line y = k intersects f.

$\therefore$ **7 < k < 32** ◄

DIAGNOSTIC REPORT

Common Errors and Misconceptions

(d) Many candidates did not realise that they had to translate the given graph by 1 unit to the right to solve the question. A number of them attempted to solve the question algebraically but were unsuccessful in doing so because the value of $h(x-1)$ was not known.

Graphical concept

QUESTION 11 8%

After travelling a distance of 20 km from home, a person suddenly remembers that he did not close a tap in his garden. He decides to turn around immediately and return home to close the tap.

The cost of the water, at the rate at which water is flowing out of the tap, is R1,60 per hour.

The cost of petrol is $\left(1{,}2 + \dfrac{x}{4\,000}\right)$ rands per km, where x is the average speed in km/h.

Calculate the average speed at which the person must travel home to keep his cost as low as possible. **[7]**

Memo

11.

The cost of the return trip
= the cost of the water + the cost of the petrol

- The cost of the water:

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{20}{x}$$

$\therefore$ The cost of the water

$$= \frac{20}{x} \times 1{,}60 = \frac{32}{x}$$

- The cost of the petrol $= 20\left(1{,}2 + \dfrac{x}{4\,000}\right) = 24 + \dfrac{x}{200}$

$$\therefore \text{ Total cost, C} = \frac{32}{x} + \left(24 + \frac{x}{200}\right)$$

$$= 32x^{-1} + 24 + \frac{1}{200}x$$

Min. Cost when: $\dfrac{dC}{dx} = -32x^{-2} + \dfrac{1}{200} = 0$

$(\times 200x^2)$ $\quad \therefore -6\,400 + x^2 = 0$

$\therefore x^2 = 6\,400$

$\therefore \boldsymbol{x = 80}$ **km/h** ◄

DIAGNOSTIC REPORT

Common Errors and Misconceptions

(a) The vast majority of the candidates did not attempt this question because they were unable to derive the cost function from the given information.

Suggestions for Improvement

(a) Learners appear to be dependent on the formulae being given when solving optimisation problems. It is advisable that learners interrogate the optimum function even when it is given in a question. This should help their conceptual development.

(b) Teachers should ensure that there is enough time for learners to understand the application of Calculus fully.

(c) Reading for understanding should be ongoing if learners are to improve their responses to word problems.

PROBABILITY

QUESTION 12 27%

20% 12.1 A and B are independent events.
It is further given that:

P(A and B) = 0,3 and
P(only B) = 0,2

12.1.1 Are A and B mutually exclusive?
Motivate your answer. (1)

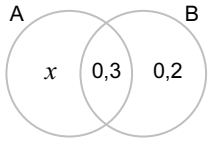
12.1.2 Determine:

(a) P(only A) (4)

(b) P(not A or not B) (2)

Memo

12.1



12.1.1 **No** ◄ P(A and B) $\neq$ 0

12.1.2 (a) A and B are independent events

➡ P(A) $\times$ P(B) = P(A and B)

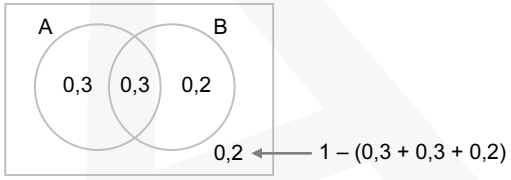
$\therefore$ $(x + 0{,}3)(0{,}5) = 0{,}3$

$\therefore$ $x + 0{,}3 = \frac{3}{5}$

$\therefore$ $x = 0{,}3$

$\therefore$ P(only A) = **0,3** ◄

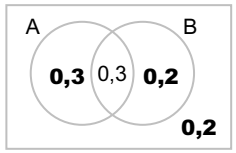
(b)



For any 2 events R & S:
P(R or S) = P(R) + P(S) – P(R and S)

P(A' or B')
= P(A') + P(B') – P(A' and B')
= 0,4 + 0,5 – 0,2
= **0,7** ◄

OR: P(A' or B')
= 0,3 + 0,2 + 0,2
= **0,7** ◄



OR: P(A' or B') = 1 – P(A and B) = 1 – 0,3 = **0,7** ◄

DIAGNOSTIC REPORT

Common Errors and Misconceptions

(a) Many candidates could not provide a reason why events A and B were not mutually exclusive in Q12.1.1.

(b) In Q12.2.1(a) many candidates overlooked the fact that the events A and B were independent. In addition, candidates were not familiar with the concepts 'only A' and 'only B'.

(c) Many candidates could not visualise which region was represented by 'not A and not B'.

33% 12.2 A teacher has 5 different poetry books, 4 different dramas and 3 different novels. She must arrange these 12 books from left to right on a shelf.

12.2.1 Write down the probability that a novel will be the first book placed on the shelf. (1)

12.2.2 Calculate the number of different ways these 12 books can be placed on the shelf if any book can be placed in any position. (2)

12.2.3 Calculate the probability that a poetry book is placed in the first position, the three novels are placed next to each other and a drama is placed in the last position. (4)

[14]

Memo

12.2.1 P(a novel the 1st book)

$= \frac{3}{12}$ . . . the number of novels / . . . the total number of books

$= \frac{1}{4}$ ◄

12.2.2 12 11 10 9 8 7 6 5 4 3 2 1 12 slots

The 12 books are all different.

∴ The number of ways = **12!** = **479 001 600** ◄

12.2.3 **5** **3** **2** **1** 7 6 5 4 3 2 1 **4**

P D

8 slots: N N N

The 3 novels could be in any 1 of the 8 slots.

∴ The number of ways = $(\mathbf{5} \times \mathbf{3!} \times 7! \times \mathbf{4}) \times 8$

∴ The probability = $\frac{(5 \times 3! \times 7! \times 4) \times 8}{12!} = \frac{1}{99}$ ◄

5 _ _ _ _ _ _ _ _ _ _ 4

8 slots

The 3 novels can occupy any **one** of the **8 slots** but, themselves, can be arranged in **3!** different ways.

OR: No. of ways = $5 \times 8! \times 3! \times 4$

= 4 838 400

∴ Probability = $\frac{4\,838\,400}{12!} = \frac{4\,838\,400}{479\,001\,600}$

$= \frac{1}{99}$ ◄

OR: No of ways the 8 slots can be arranged = $8! \times 3!$

P(3 novels together) = $\frac{8! \times 3!}{10!} = \frac{1}{15}$

∴ Probability = $\frac{5}{12} \times \frac{1}{15} \times \frac{4}{11}$* $= \frac{1}{99}$ ◄

∴ P(a poetry book to start **and** a drama at the end **and** 3 novels next to each other)

* P(A and B and C) = P(A) × P(B) × P(C)

DIAGNOSTIC REPORT

Common Errors and Misconceptions

(d) Some candidates were confused about which books were being referred to in Q12.2.2, despite the question explicitly stating 'these 12 books'.

(e) Many candidates were able to calculate the options for the first and last place and how the three novels could be arranged together. However, they were unable to calculate how the novels together with the remaining books could be arranged in the 10 places between the first and last places.

Suggestions for Improvement

(a) Teaching basic concepts cannot be overlooked. When learners understand the basic concepts well enough, then the more complex concepts are easier to grasp.

(b) Use Venn diagrams to teach probability. It helps with the understanding of the different areas that make up the events, e.g. only A, only B, A and B, A or B, not A, not B, not A and not B and not A or not B.

(c) Teach learners the Fundamental Counting Principle in such a way that they will be able to reason answers, instead of trying to remember rules.

CALCULUS

SCARY PAGES FROM 2020

WHAT NOT TO DO

(Binomial)^2

$(x-2)^2 = x^2 + 4$: ✗

Inequality

$(x-1)(x+2) < 0 \Rightarrow x - 1 < 0$ or $x + 2 < 0$ ✗

or $2 > x > -1$ ✗

Intervals

$x^2 + 5x - 6 = (x-6)(x+1)$ ✗ or $(x-3)(x+2)$ ✗

Factorisation

$f(x) = 2x^2 - 1$

Brackets

$2(x+h)^2 = (2x+2h)^2$ ✗ ; $-(2x^2-1) = -2x^2 - 1$ ✗

or $= 2x^2 + 2xh + 2h^2$ ✗

$\sqrt[5]{x^2} = x^{\frac{5}{2}}$

Surds ➡ Exponential form

The meaning of the derivative symbol $\frac{d}{dx}$

$$\frac{d}{dx}\left(\sqrt[5]{x^2} + x^3\right) = \frac{d}{dx}\left(x^{\frac{2}{5}} + 3x^2\right) = \frac{2}{5}x^{-\frac{3}{5}} + 3x^2$$

Remember: **WHAT NOT TO DO**

Factorisation & Fractions

$$\frac{4x^2 - 9}{4x + 6} = \frac{4x^2}{4x} - \frac{9}{6} \quad ✗$$

$$= x - \frac{3}{2}\ ; \quad ✗$$

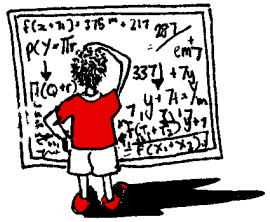
OR

Some differentiated: (the quotient)

$$\frac{4x^2 - 9}{4x + 6} = \frac{8x}{4} \quad ✗$$

$$= 2 \quad ✗$$

This constituted a **'mathematical breakdown'** and candidates were not credited for arriving at the correct answer.



$$f(x) = 2x^2 - 1$$

$$f(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad ✗$$

OR

$$\lim_{h \to 0} = \frac{f(x+h) - f(x)}{h} \quad ✗$$

OR

$$\frac{\lim f(x+h) - f(x)}{h \to 0 \quad h} \quad ✗$$

Notation Errors

'They lose a mark for these errors.'

Differentiate w.r.t. ONE variable only

$$C(w) = 90w^2 + 48wh$$

$$\Rightarrow \quad C'(w) = 180w + 48h \quad ✗$$

$$V = 3w^2h = 5$$

$$\therefore\ h = \frac{5}{3}w^2 \quad ✗ \quad \textbf{or} \quad 5 - 3w^2 \quad ✗$$

DESTROY THESE PAGES!!!

A Way of Thinking

Make sense of Maths . . .

Speak Maths . . .

LANGUAGE

Vocabulary
Notation
Symbols

THEORY

Definitions
Rules
Procedures

Knowledge

CONCEPTS

Integration

CONTEXTS

Summaries

SIMPLICITY

Recognition

COMPLEXITY

& Ultimately . . .

SELF-DIRECTED LEARNING

as we impart
the Gift of Confidence
through our study guides

Gr 10 – 12 Exemplar Papers: Sequencing

Paper 1

	Grade 10	Grade 11	Grade 12
Patterns	linear	quadratic	APs & GPs
Algebra	equations	inequalities/Δ	3rd deg. equations
Graphs	domain & range	average gradient	derivative
Probability	definition of probability & mutually excl. events	independent events	fundamental counting principle

Sequencing, for CALCULUS

Algebra	linear → quadratic → cubic e.g. expressions equations inequalities
	intervals
Graphs	lines → parabolas → cubic graphs
	gradient → average gradient → derivative (& increasing/decreasing)
	intervals

Graph Sketching

Graph	Positive/ Negative	Y-intercept (x = 0)	&	X-intercept (y = 0)	Turning point/ Stationary pt(s)	Point of Inflection
Straight line	✓	✓		✓	—	—
Parabola	✓	✓		✓	✓	—
Cubic	✓	✓		✓	✓	✓

Note: Domain & Range

Interval Notation and Meaning

< means **'is less than'** **≤** means **'is less than or equal to'**

> means **'is greater than'** **≥** means **'is greater than or equal to'**

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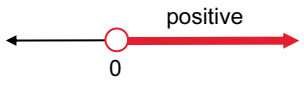
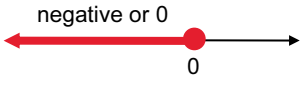
x > 0 means **'x is positive'**



x ≤ 0 means **'x is negative or 0'**



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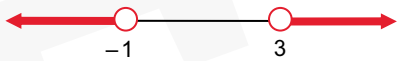
x < –1 **or** **x > 3** means:

Either x is < -1 **or** x is > 3



(2 sections)

(No one number can be both smaller than –1 **and** *bigger than 3 at the same time.)*

---

x > –1 **and** **x < 3** means?

x > –1:

and

x < 3:

∴

x must be both > -1 as well as < 3

Both conditions must be applied. So, we need the **overlap**.

We write: –1 < x < 3

same direction

We say: x lies between –1 and 3

All numbers between –1 and 3 **are** *bigger than –1* **and** *smaller than 3.*

Interval Notations

( ) Round brackets indicate the **exclusion** of the limits

[ ] Square brackets indicate the **inclusion** of the limits

If **–1 < x < 3**, then the interval can be given as **(–1; 3)**

If **–1 ≤ x < 3**, then the interval can be given as **[–1; 3)**

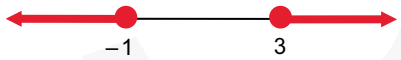
If **–1 ≤ x ≤ 3**, then the interval can be given as **[–1; 3]**

For **±** ∞ (which is undefined), always use **(** or **)**

If **x ≤ –1**, then the interval can be given as **(–∞; –1]**

If **x < –1**, then the interval can be given as **(–∞; –1)**

If **x ≤ –1** **or** **x ≥ 3**, then the interval can be given as **(–∞; –1] or [3; ∞]**



Solving Quadratic Inequalities using a number line

Worked Examples

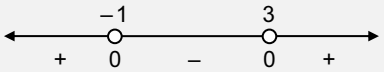
Solve for x:

(a) $(x + 1)(x - 3) > 0$ (b) $(x + 1)(x - 3) < 0$

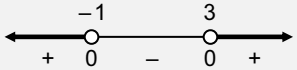
(c) $(x + 1)(x - 3) \geq 0$ (d) $(x + 1)(x - 3) \leq 0$

Answer

$(x + 1)(x - 3)$:

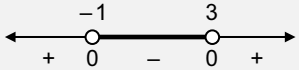


(a)



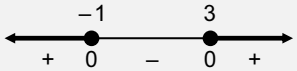
∴ $x < -1$ or $x > 3$

(b)



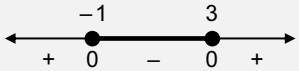
∴ $-1 < x < 3$

(c)



∴ $x \leq -1$ or $x \geq 3$

(d)



∴ $-1 \leq x \leq 3$

An Exercise

1. Between which two integers do ALL PROPER FRACTIONS, negative and positive lie?

2. Express each of the following in interval notation (where possible) and represent each on a number line:

   2.1 $\{x \mid x > -1;\ \ x \text{ a real number}\}$

   2.2 $\{x \mid -4 \le x < 1;\ \ x \in \mathbb{R}\}$

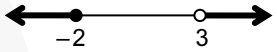
   2.3 $\{x \mid x < 3;\ \ x \in \mathbb{N}_0\}$

   2.4 $\{x \mid -2 < x \le 2;\ \ x \in \mathbb{Z}\}$

3. Express the following in interval notation and algebraically:

   3.1 

   3.2 

4. Express the following algebraically: 

NB: In Q 3.2: x is > -7 **AND** x is ≤ 4: . . . ***1 piece*** *on the no. line*
whereas, in Q 4: x is ≤ -2 **OR** x is > 3: . . . ***2 pieces*** *on the no. line*

5. If x is an integer and lies in the interval $[-4; 3)$, write down:

   5.1 the minimum value of x 5.2 the maximum value of x

6. In which intervals do ALL IMPROPER FRACTIONS, negative and positive, lie?

7. Solve for x:

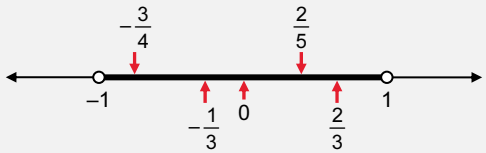
   7.1 $x^2 - 2x - 15 \ge 0$

   7.2 $x^2 - x < 6$

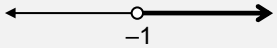
THE ANSWER SERIES *Your Key to Exam Success*

ANSWERS

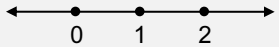
1. Between –1 and 1

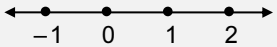


A very important example of this (which includes ± 1) is:
For ALL values of θ: $-1 \le \sin\theta \le 1$ and $-1 \le \cos\theta \le 1$

2.1 $(-1; \infty)$ 

2.2 $[-4; 1)$ 

2.3 

2.4 

3. Interval notation is only used for x real, i.e. for $x \in \mathbb{R}$

   3.1 $[-3; \infty)$; $x \ge -3$; $x \in \mathbb{R}$ 3.2 $(-7; 4]$; $-7 < x \le 4$; $x \in \mathbb{R}$

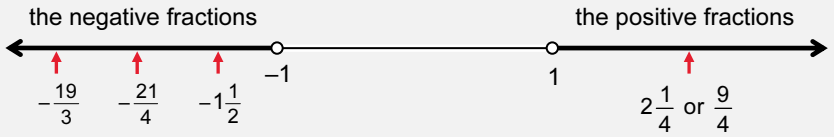
4. $x \le -2$ or $x > 3$; $x \in \mathbb{R}$ . . . There are two separate intervals, indicated separately.
   $\therefore$ **Either** $x \le -2$ **or** $x > 3$

5. x an integer and $x \in [-4; 3)$ ➡ $x = -4; -3; -2; -1; 0; 1$ or 2

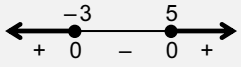
   5.1 The minimum value of x is -4 . . . *the smallest*

   5.2 The maximum value of x is 2 . . . *the biggest*

6. The negative improper (and mixed) fractions lie in the interval $(-\infty; -1)$, i.e. $x < -1$
   & the positive improper (and mixed) fractions lie in the interval $(1; \infty)$, i.e. $x > 1$



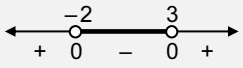
7.1 $(x + 3)(x - 5) \ge 0$



$\therefore\ x \le -3$ or $x \ge 5$

7.2 $x^2 - x - 6 < 0$

$\therefore\ (x + 2)(x - 3) < 0$



$\therefore\ -2 < x < 3$

The Most Fundamental Algebra

KNOW THESE PRODUCTS!!!

$$(x + y)^2 = (x + y)(x + y)$$
$$= x^2 + xy + xy + y^2$$
$$= \mathbf{x^2 + 2xy + y^2}$$

$$(x - y)^2 = (x - y)(x - y)$$
$$= x^2 - xy - xy + y^2$$
$$= \mathbf{x^2 - 2xy + y^2}$$

. . . *The* **same signs** *in the brackets cause a 'doubling up' of the middle term.*

. . . *The* **different signs** *in the brackets make the middle terms fall away.*

whereas $\mathbf{(x + y)(x - y)} = x^2 - xy + xy - y^2$
$$= \mathbf{x^2 - y^2}$$

DIFFERENCE OF SQUARES

And backwards . . .

$$\mathbf{x^2 + 2xy + y^2 = (x + y)^2}$$
$$\mathbf{x^2 - 2xy + y^2 = (x - y)^2}$$
$$\textbf{and} \quad \mathbf{x^2 - y^2 = (x + y)(x - y)}$$

NOW, THE NEXT 'LEVEL' ...

$$\begin{aligned}
(x + y)^3 &= (x + y)(x + y)(x + y) \\
&= (x + y)(x + y)^2 \\
&= \mathbf{(x + y)(x^2 + 2xy + y^2)} \quad \blacktriangleleft \\
&= x^3 + 2x^2y + xy^2 + x^2y + 2xy^2 + y^3 \\
&= \mathbf{x^3 + 3x^2y + 3xy^2 + y^3}
\end{aligned}$$

$$\begin{aligned}
(x - y)^3 &= (x - y)(x - y)(x - y) \\
&= (x - y)(x - y)^2 \\
&= \mathbf{(x - y)(x^2 - 2xy + y^2)} \quad \blacktriangleleft \\
&= x^3 - 2x^2y + xy^2 - x^2y + 2xy^2 - y^3 \\
&= \mathbf{x^3 - 3x^2y + 3xy^2 - y^3}
\end{aligned}$$

So, what product will give us an answer of just $\mathbf{x^3 + y^3}$ or $\mathbf{x^3 - y^3}$ with NO MIDDLE TERMS?

We know that **a linear** × **a quadratic = a cubic!**

So:

$$\begin{aligned}
(x + y)(\ ?\quad ?\quad ?) &= x^3 + y^3 \\
\therefore\ (x + y)(\mathbf{x^2\ ?\ xy + y^2}) &= x^3 + y^3 \\
\therefore\ (x + y)(x^2 - xy + y^2) &= x^3 + y^3
\end{aligned}$$

$$\begin{aligned}
(x - y)(\ ?\quad ?\quad ?) &= x^3 - y^3 \\
\therefore\ (x - y)(\mathbf{x^2\ ?}\ xy + \mathbf{y^2}) &= x^3 - y^3 \\
\therefore\ (x - y)(x^2 + xy + y^2) &= x^3 - y^3
\end{aligned}$$

And backwards . . .

$$\boxed{\begin{aligned}
\mathbf{a^3 + b^3 = (a + b)(a^2 - ab + b^2)} \\
\mathbf{a^3 - b^3 = (a - b)(a^2 + ab + b^2)}
\end{aligned}}$$

SUM OF CUBES

DIFFERENCE OF CUBES

A useful tool for products and factors ...

PASCAL'S TRIANGLE

➡ $(a + b)^1$	**1** **1**	$\mathbf{1}a + \mathbf{1}b$
➡ $(a + b)^2$	**1** **2** **1**	$\mathbf{1}a^2 + \mathbf{2}ab + \mathbf{1}b^2$
➡ $(a + b)^3$	**1** **3** **3** **1**	$\mathbf{1}a^3 + \mathbf{3}a^2b + \mathbf{3}ab^2 + \mathbf{1}b^3$
➡ $(a + b)^4$	**1** **4** **6** **4** **1**	.....

And now, $(a - b)^1$, $(a - b)^2$, $(a - b)^3$, etc.

REMAINDER FACTOR THEOREM

Factorising Expressions and Solving Equations

Gr 12 Maths 2 in 1
p. 27 (Q1)

1. Observe the following:

- the quadratic expression: $x^2 - x - 6$
- the quadratic equation: $x^2 - x - 6 = 0$

Factorise the expression: $x^2 - x - 6 = (x - 3)(x + 2)$

The factors

Solve the equation:

$$x^2 - x - 6 = 0$$
$$\therefore\ (x - 3)(x + 2) = 0$$
$$\therefore\ x - 3 = 0 \quad \textbf{or} \quad x + 2 = 0$$
$$\therefore\ x = 3 \qquad\qquad \therefore\ x = -2$$

The roots

Write down:

1.1 the *factors* of the *expression* $x^2 - x - 6$. (1)

Compare

1.2 the *roots* of the *equation* $x^2 - x - 6 = 0$. (1)

1.3 Compare 1.1 and 1.2. (1)

1.4 If $f(x) = x^2 - x - 6$, find the value of $f(3)$ and $f(-2)$. (2)

1.5 Compare 1.2 and 1.4. Explain. (2)

Compare

THE LAWS OF EXPONENTS

. . . with $a > 0$ & $b > 0$

The multiplication of powers

1 $a^m \times a^n = a^{m+n}$

e.g. $x^3 \times x^2 = x \times x \times x \times x \times x = x^5$

$2^x \times 2^y = 2^{x+y}$; $s^p \times s^q = s^{p+q}$

When we multiply powers with the same bases, we add the exponents.

The power of a power

3 $(a^m)^n = a^{mn}$

e.g. $(x^2)^3 = x \times x \times x \times x \times x \times x = x^6$

$(5^a)^b = 5^{ab}$; $(s^p)^q = s^{pq}$

In equations: $x^3 = 8$

$\therefore x = \sqrt[3]{8} = \mathbf{2}$

When we find the power of a power, we multiply the exponents.

The power of a product

5 $(ab)^n = a^n b^n$

e.g. $(xy)^4 = x^4y^4$

$(x^2y^3)^2 = (x^2)^2(y^3)^2 = x^4y^6$

$(2x^2)^5 = (2)^5(x^2)^5 = 32x^{10}$

The exponent of the product is the exponent of each factor.

The division of powers

2 $\dfrac{a^m}{a^n} = a^{m-n}$

e.g. $\dfrac{x^5}{x^2} = \dfrac{x \times x \times x \times \cancel{x} \times \cancel{x}}{\cancel{x} \times \cancel{x}} = x^3$

$\dfrac{2^x}{2^y} = 2^{x-y}$; $\dfrac{s^p}{s^q} = s^{p-q}$

When we divide powers with the same bases, we subtract the exponents.

The root of a power

4 $\sqrt[n]{a^m} = a^{\frac{m}{n}}$

e.g. $\sqrt{x^6} = x^{\frac{6}{2}} = x^3$;

$\because \sqrt{x^6} = \sqrt{x \times x \times x \times x \times x \times x} = x^3$

$\sqrt[3]{2^{12}} = 2^{\frac{12}{3}} = 2^4$; $\sqrt[a]{5^b} = 5^{\frac{b}{a}}$

In equations: $\sqrt[3]{x} = 2$

$\therefore x = 2^3 = \mathbf{8}$

When we find the root of a power, we divide the exponents.

The power of a quotient

6 $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$, $b \neq 0$

e.g. $\left(\dfrac{p}{q}\right)^6 = \dfrac{p^6}{q^6}$; $\left(\dfrac{-2}{x}\right)^5 = \dfrac{(-2)^5}{(x)^5} = \dfrac{-32}{x^5}$

$\left(\dfrac{3x^3}{y^2}\right)^3 = \dfrac{(3)^3(x^3)^3}{(y^2)^3} = \dfrac{27x^9}{y^6}$

The exponent of the quotient is the exponent of the factors in the numerator and the denominator.

THE LAWS REVERSED

1 $a^{m+n} = a^m . a^n$

e.g. $2^{x+3} = 2^x . 2^3$; $2^{x-1} = 2^x . 2^{-1}$

2 $a^{m-n} = \dfrac{a^m}{a^n}$

e.g. $3^{2-a} = \dfrac{3^2}{3^a}$

3 $a^{mn} = (a^m)^n$ or $(a^n)^m$

e.g. $3^{2x} = (3^2)^x$ or $(3^x)^2$;

$x^{\frac{1}{2}} = \left(x^{\frac{1}{4}}\right)^2$; $x^{\frac{2}{3}} = \left(x^{\frac{1}{3}}\right)^2$

4 $a^{\frac{m}{n}} = \left(\sqrt[n]{a^m}\right)$ or $\left(\sqrt[n]{a}\right)^m$

e.g. $8^{\frac{2}{3}} = \sqrt[3]{8^2}$ or $8^{\frac{2}{3}} = \left(\sqrt[3]{8}\right)^2$

$= \sqrt[3]{64}$ $\quad = 2^2$

$= 4$ $\quad = 4$

5 $a^n b^n = (ab)^n$

e.g. $5^x . 3^x = (5.3)^x = 15^x$

6 $\dfrac{a^n}{b^n} = \left(\dfrac{a}{b}\right)^n$

e.g. $\dfrac{6^n}{2^n} = \left(\dfrac{6}{2}\right)^n = 3^n$

THE MEANING OF . . .

▶ A ZERO exponent: a^0 ?

$a^0 \times a^3 = a^{0+3} = a^3 \Rightarrow a^0 = 1$

$\therefore\ a^0 = 1$ $\quad a \in \mathbb{R};\ a \neq 0$

▶ A NEGATIVE exponent: a^{-n} ?

$a^{-3} \times a^3 = a^{-3+3} = a^0 = 1 \Rightarrow a^{-3} = \frac{1}{a^3}$

$\therefore\ a^{-n} = \frac{1}{a^n}$ or $\left(\frac{1}{a}\right)^n$ $\quad a \in \mathbb{R};\ a \neq 0$

▶ A FRACTION exponent: $a^{\frac{p}{q}}$?

$8^{\frac{2}{3}} = (8^2)^{\frac{1}{3}} = 64^{\frac{1}{3}} = 4 \Rightarrow 8^{\frac{2}{3}} = \sqrt[3]{8^2}$

or $8^{\frac{2}{3}} = \left(8^{\frac{1}{3}}\right)^2 = 2^2 = 4 \Rightarrow 8^{\frac{2}{3}} = \left(\sqrt[3]{8}\right)^2$

$\therefore\ a^{\frac{p}{q}} = \sqrt[q]{a^p}$ or $\left(\sqrt[q]{a}\right)^p$

Popular powers to know

Powers of 2	Powers of 3	Powers of 4	Powers of 5
$2^2 = 4$	$3^2 = 9$	$4^2 = 16$	$5^2 = 25$
$2^3 = 8$	$3^3 = 27$	$4^3 = 64$	$5^3 = 125$
$2^4 = 16$	$3^4 = 81$		
$2^5 = 32$			
$2^6 = 64$			

Note: $64 = 2^6$ & $64 = 4^3$ & $64 = 8^2$

EXPONENTS FOR CALCULUS

LAWS

$$a^0 = 1 \qquad a^{-n} = \frac{1}{a^n} \qquad a^{\frac{p}{q}} = \sqrt[q]{a^p}$$

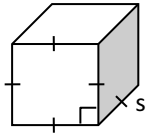
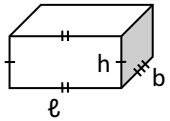
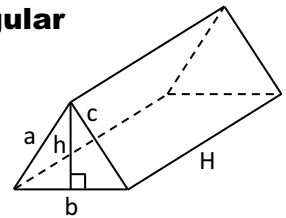
Examples in Calculus

$$5x^0 = 5 \times 1 = 5 \quad ; \quad x^{-3} = \frac{1}{x^3} \quad ; \quad \frac{1}{x} = x^{-1}$$

$$\sqrt{x} = x^{\frac{1}{2}} \quad ; \quad x^{-\frac{1}{2}} = \frac{1}{x^{\frac{1}{2}}} = \frac{1}{\sqrt{x}} \quad ; \quad \sqrt[3]{x^4} = x^{\frac{4}{3}}$$

$$2x^{-3} = 2 \times \frac{1}{x^3} = \frac{2}{x^3} \quad ; \quad \frac{1}{2}x^{-5} = \frac{1}{2} \times \frac{1}{x^5} = \frac{1}{2x^5}$$

VOLUME & SURFACE AREA OF 3D OBJECTS: FORMULAE

3D Shapes	TSA = sum of areas of all surfaces	Volume = Area of base × Height
	The **total exterior area** of all the exposed surfaces of a 3D shape.	The **3D space** that a 3D object occupies.
Cube s = side  	Total Surface Area = (side × side) × 6 = 6(side)^2 ∴ **TSA = 6s^2**	Volume = side × side × side = (side)^3 ∴ **V = s^3**
Rectangular Prism ℓ = length b = breadth h = height  	Total Surface Area = 2 (length × breadth) + 2 (length × height) + 2 (breadth × height) ∴ **TSA = 2ℓb + 2ℓh + 2bh**	Volume = length × breadth × height ∴ **V = ℓ x b x h**
Triangular Prism  a = side$_1$ b = side$_2$ (base) c = side$_3$ h = ⊥ height of Δ H = height of prism	Total Surface Area $= 2\left(\frac{b \times h}{2}\right)$ + (side$_1$ × prism height) + (side$_2$ × prism height) + (side$_3$ × prism height) ∴ **TSA** $= 2\left(\frac{b \times h}{2}\right)$ **+ (a × H) + (b × H)** **+ (c × H)**	Volume $= \frac{1}{2}$ base × ⊥height × prism height ∴ **V** $= \left(\frac{1}{2}b \times \perp h\right) \times H$ **Remember:** Area of Δ $= \frac{1}{2}b \times \perp h$ OR $= \frac{b \times h}{2}$ 

SI Units & Conversions

Small unit ➡ *big unit:* ÷
Big unit ➡ *small unit:* ×

Total Surface Area (TSA)

divide ⬅

÷ (1 000)^2 ÷ (100)^2 ÷ (10)^2

km^2 hm^2 dam^2 **m^2** dm^2 **cm^2** **mm^2**

× (1 000)^2 × (100)^2 × (10)^2

multiply ➡

Since the area is the product of **2D** *lengths, we need to × or ÷ by the* ***(conversion factor)^2***.

Volume

divide ⬅

÷ (100)^3 ÷ (10)^3

m^3 dm^3 **cm^3** **mm^3**

× (100)^3 × (10)^3

multiply ➡

Since volume is the product of **3D** *lengths, we need to × or ÷ by the* ***(conversion factor)^3***.

Gr 10, Gr 11 & Gr 12 Mathematics

EXEMPLAR PAPER 1s

(memos follow)

GRADE 10 EXEMPLAR PAPER 1

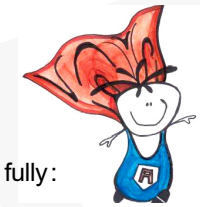
Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.

Answers only will NOT necessarily be awarded full marks.

You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

*If necessary, round off answers to **TWO** decimal places, unless stated otherwise.*

▶ ALGEBRA [32]



QUESTION 1

1.1 Simplify the following expressions fully:

1.1.1 $(m - 2n)(m^2 - 6mn - n^2)$ (3)

1.1.2 $\dfrac{x^3 + 1}{x^2 - x + 1} - \dfrac{4x^2 - 3x - 1}{4x + 1}$ (5)

1.2 Factorise the following expressions fully:

1.2.1 $6x^2 - 7x - 20$ (2)

1.2.2 $a^2 + a - 2ab - 2b$ (3)

1.3 Determine, **without the use of a calculator**, between which two consecutive integers $\sqrt{51}$ lies. (2)

1.4 Prove that $0,2\dot{4}\dot{5}$ is rational. (4) [19]

QUESTION 2

2.1 Determine, **without the use of a calculator**, the value of x in each of the following:

2.1.1 $x^2 - 4x = 21$ (3)

2.1.2 $96 = 3x^{\frac{5}{4}}$ (3)

2.1.3 $R = \dfrac{2\sqrt{x}}{3S}$ (2)

2.2 Solve for p and q simultaneously if:

$6q + 7p = 3$
$2q + p = 5$ (5) [13]

▶ NUMBERS & NUMBER PATTERNS [11]

QUESTION 3

3.1 $3x + 1$; $2x$; $3x - 7$ ..... are the first three terms of a linear number pattern.

3.1.1 If the value of x is three, write down the FIRST THREE terms. (3)

3.1.2 Determine the formula for T_n, the general term of the sequence. (2)

3.1.3 Which term in the sequence is the first to be less than -31? (3)

3.2 The multiples of three form the number pattern:
3 ; 6 ; 9 ; 12 ; . . .

Determine the 13th number in this pattern that is even. (3) [11]

▶ FINANCE & GROWTH [14]

QUESTION 4

4.1 Thando has R4 500 in his savings account. The bank pays him a compound interest rate of 4,25% p.a. Calculate the amount Thando will receive if he decides to withdraw the money after 30 months. (3)

4.2 The following advertisement appeared with regard to buying a bicycle on a hire-purchase agreement loan:

Purchase price	*R5 999*
Required deposit	*R600*
Loan term	*Only 18 months, at 8% p.a. simple interest*

4.2.1 Calculate the monthly amount that a person has to budget for in order to pay for the bicycle. (6)

4.2.2 How much interest does one have to pay over the full term of the loan? (1)

4.3 The following information is given:

1 ounce = 28,35 g
$1 = R8,79

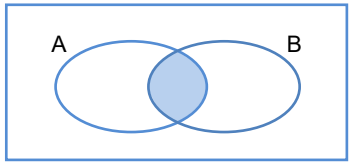
Calculate the rand value of a 1 kg gold bar, if 1 ounce of gold is worth $978,34. (4) [14]

▶ PROBABILITY [13]

QUESTION 5

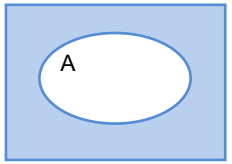
5.1 What expression BEST represents the shaded area of the following Venn diagrams?

5.1.1



(1)

5.1.2



(1)

5.2 State which of the following sets of events is mutually exclusive:

A Event 1: The learners in Grade 10 in the swimming team
Event 2: The learners in Grade 10 in the debating team

B Event 1: The learners in Grade 8
Event 2: The learners in Grade 12

C Event 1: The learners who take Mathematics in Grade 10
Event 2: The learners who take Physical Sciences in Grade 10 (1)

5.3 In a class of 40 learners the following information is TRUE:

- 7 learners are left-handed
- 18 learners play soccer
- 4 learners play soccer and are left-handed
- All 40 learners are either right-handed or left-handed

Let L be the set of all left-handed people and S be the set of all learners who play soccer.

5.3.1 How many learners in the class are right-handed and do NOT play soccer? (1)

5.3.2 Draw a Venn diagram to represent the above information. (4)

5.3.3 Determine the probability that a learner is:
(a) left-handed or plays soccer (3)
(b) right-handed and plays soccer (2) [13]

▶ FUNCTIONS & GRAPHS [30]

QUESTION 6

Given: $f(x) = \frac{3}{x} + 1$ and $g(x) = -2x - 4$

6.1 Sketch the graphs of f and g on the same set of axes. (4)

6.2 Write down the equations of the asymptotes of f. (2)

6.3 Write down the domain of f. (2)

6.4 Solve for x if $f(x) = g(x)$. (5)

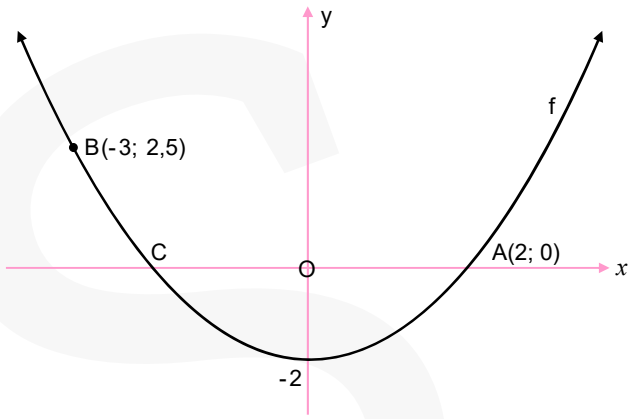
6.5 Determine the values of x for which $-1 \leq g(x) < 3$. (3)

6.6 Determine the y-intercept of k if $k(x) = 2g(x)$ (2)

6.7 Write down the coordinates of the x- and y-intercepts of h if h is the graph of g reflected about the y-axis. (2) [20]

QUESTION 7

The graph of $f(x) = ax^2 + q$ is sketched below.
Points A(2; 0) and B(-3; 2,5) lie on the graph of f.
Points A and C are x-intercepts of f.



7.1 Write down the coordinates of C. (1)

7.2 Determine the equation of f. (3)

7.3 Write down the range of f. (1)

7.4 Write down the range of h, where $h(x) = -f(x) - 2$. (2)

7.5 Determine the equation of an exponential function, $g(x) = b^x + q$, with range $y > -4$ and which passes through the point A. (3) [10]

TOTAL: 100

We trust that working through these exam papers and following our detailed answers and comments will help you prepare thoroughly for your final exam.

The Answer Series Maths study guides offer a key to exam success. In particular, Gr 10 Maths 3 in 1 provides superb foundation in the major topics in Senior Maths.

GRADE 11 EXEMPLAR PAPER 1

You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

*If necessary, round off answers to **TWO** decimal places, unless stated otherwise.*

▶ ALGEBRA AND EQUATIONS AND INEQUALITIES [47]

QUESTION 1

1.1 Solve for x:

1.1.1 $(2x - 1)(x + 5) = 0$ (2)

1.1.2 $2x^2 - 4x + 1 = 0$ (Leave your answer in simplest surd form.) (3)

1.2 Simplify, without the use of a calculator, the following expressions fully:

1.2.1 $125^{\frac{2}{3}}$ (2)

1.2.2 $(3\sqrt{2} - 12)(2\sqrt{2} + 1)$ (3)

1.3 Given: $\dfrac{x^2 - x - 6}{3x - 9}$

1.3.1 For which value(s) of x will the expression be undefined? (2)

1.3.2 Simplify the expression fully. (3) [15]

Algebra includes exponents & surds

QUESTION 2

2.1 Given: $(x + 2)(x - 3) < -3x + 2$

2.1.1 Solve for x if: $(x + 2)(x - 3) < -3x + 2$ (4)

2.1.2 Hence or otherwise, determine the sum of all the integers satisfying the inequality $x^2 + 2x - 8 < 0$. (3)

2.2 Given: $\dfrac{4^{x-1} + 4^{x+1}}{17.12^x}$

2.2.1 Simplify the expression fully. (4)

2.2.2 If $3^{-x} = 4t$, express $\dfrac{4^{x-1} + 4^{x+1}}{17.12^x}$ in terms of t. (1)

2.3 Solve for x and y from the given equations:

$3^y = 81^x$ and $y = x^2 - 6x + 9$ (7) [19]

QUESTION 3

3.1 The solution to a quadratic equation is

$x = \dfrac{3 \pm \sqrt{4 - 8p}}{4}$ where $p \in Q$.

Determine the value(s) of p such that:

3.1.1 The roots of the equation are equal. (2)

3.1.2 The roots of the equation are non-real. (2)

3.2 Given: $\sqrt{5 - x} = x + 1$

3.2.1 Without solving the equation, show that the solution to the above equation lies in the interval $-1 \le x \le 5$. (3)

3.2.2 Solve the equation. (5)

3.2.3 Without any further calculations, solve the equation $-\sqrt{5 - x} = x + 1$. (1) [13]

▶ FINANCE, GROWTH AND DECAY [18]

QUESTION 4

4.1 Melissa has just bought her first car. She paid R145 000 for it. The car's value depreciates on the straight-line method at a rate of 17% per annum. Calculate the value of Melissa's car 5 years after she bought it. (2)

4.2 An investment earns interest at a rate of 8% per annum compounded quarterly.

4.2.1 At what rate is interest earned each quarter of the year? (1)

4.2.2 Calculate the effective annual interest rate on this investment. (2)

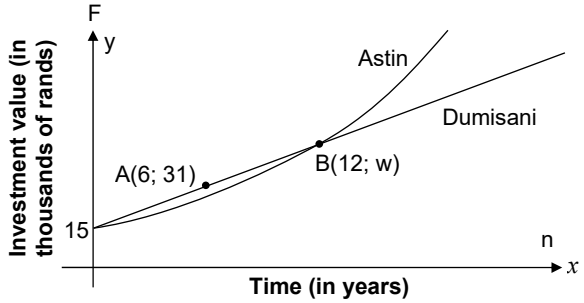
4.3 R14 000 is invested in an account.

The account earns interest at a rate of 9% per annum compounded semi-annually for the first 18 months and thereafter 7,5% per annum compounded monthly.

How much money will be in the account exactly 5 years after the initial deposit? (5) [10]

QUESTION 5

The graphs below represent the growth of two investments, one belonging to Dumisani and one belonging to Astin. Both investments earn interest annually (only).



5.1 What is the value of both initial investments? (1)

5.2 Does Dumisani's investment earn simple or compound interest? (1)

5.3 Determine Dumisani's interest rate. (2)

5.4 Hence or otherwise, calculate the interest rate on Astin's investment. Give your answer correct to ONE decimal place. (4) [8]

▶ PATTERNS AND SEQUENCES [23]

QUESTION 6

6.1 Given: $\frac{1}{2}; \frac{1}{4}; \frac{1}{8}; \ldots; \frac{1}{1\,024}$

6.1.1 Explain how you will determine the 4th term of the sequence. (2)

6.1.2 Write a formula for the nth term of the sequence. (2)

6.1.3 Determine the number of terms in the sequence. (2)

6.2 Given the linear pattern: 156 ; 148 ; 140 ; 132 ; . . .

6.2.1 Write down the 5th term of this number pattern. (1)

6.2.2 Determine a general formula for the nth term of this pattern. (2)

6.2.3 Which term of this linear number pattern is the first term to be negative? (3)

6.2.4 The given linear number pattern forms the sequence of first differences of a quadratic number pattern $T_n = an^2 + bn + c$ with $T_5 = -24$. Determine a general formula for T_n. (5) [17]

Higher order

QUESTION 7

A question asked differently

A quadratic pattern $T_n = an^2 + bn + c$ has $T_2 = T_4 = 0$ and a second difference of 12.

Determine the value of the 3rd term of the pattern. [6]

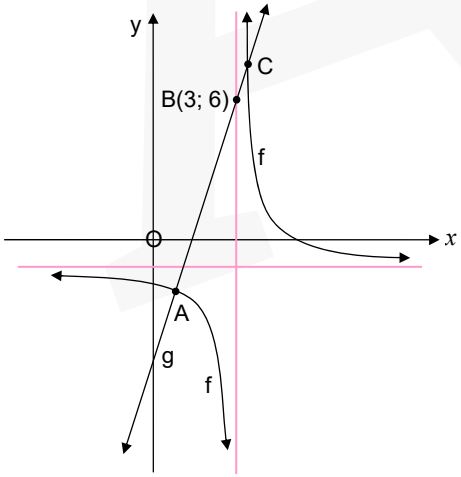
▶ FUNCTIONS AND GRAPHS [43]

QUESTION 8

The sketch below represents the graphs of

$f(x) = \frac{2}{x - 3} - 1$ and $g(x) = dx + e$.

Point B(3; 6) lies on the graph of g and the two graphs intersect at points A and C.



8.1 Write down the equations of the asymptotes of f. (2)

8.2 Write down the domain of f. (2)

8.3 Determine the values of d and e, correct to the nearest integer, if the graph of g makes an angle of 76° with the x-axis. (3)

8.4 Determine the coordinates of A and C. (6)

8.5 For what values of x is $g(x) \geq f(x)$? (3)

8.6 Determine an equation for the axis of symmetry of f which has a positive slope. (3) [19]

QUESTION 9

Given: $f(x) = -x^2 + 2x + 3$ and $g(x) = 1 - 2^x$

9.1 Sketch the graphs of f and g on the same set of axes. (9)

9.2 Determine the average gradient of f between $x = -3$ and $x = 0$. (3)

9.3 For which value(s) of x is $f(x).g(x) \geq 0$? (3)

9.4 Determine the value of c such that the x-axis will be a tangent to the graph of h, where $h(x) = f(x) + c$. (2)

9.5 Determine the y-intercept of t if $t(x) = -g(x) + 1$. (2)

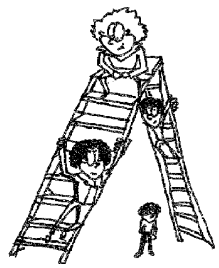
9.6 The graph of k is a reflection of g about the y-axis. Write down the equation of k. (1) [20]

QUESTION 10

Also asked differently

Sketch the graph of $f(x) = ax^2 + bx + c$ if it is also given that:

- the range of f is $(-\infty; 7]$
- $a \neq 0$
- $b < 0$
- one root of f is positive and the other root of f is negative. [4]



▶ PROBABILITY [19]

QUESTION 11

Given: P(W) = 0,4 P(T) = 0,35 P(T and W) = 0,14

11.1 Are the events W and T mutually exclusive? Give reasons for your answer. (2)

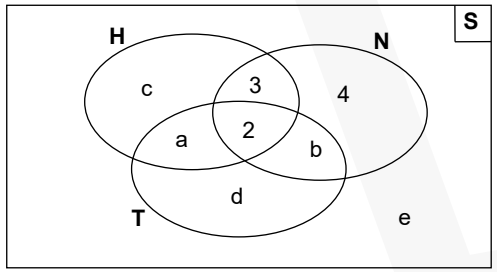
11.2 Are the events W and T independent? Give reasons for your answer. (3) [5]

QUESTION 12

12.1 A group of 33 learners was surveyed at a school. The following information from the survey is given:

- 2 learners play tennis, hockey and netball
- 5 learners play hockey and netball
- 7 learners play hockey and tennis
- 6 learners play tennis and netball
- A total of 18 learners play hockey
- A total of 12 learners play tennis
- 4 learners play netball ONLY

12.1.1 A Venn diagram representing the survey results is given below. Use the information provided to determine the values of a, b, c, d and e. (5)



12.1.2 How many of these learners do not play any of the sports on the survey (that is netball, tennis or hockey)? (1)

12.1.3 Write down the probability that a learner selected at random from this sample plays netball ONLY. (1)

12.1.4 Determine the probability that a learner selected at random from this sample plays hockey or netball. (1)

12.2 In all South African schools, EVERY learner must choose to do either Mathematics or Mathematical Literacy.

At a certain South African school, it is known that 60% of the learners are girls. The probability that a randomly chosen girl at the school does Mathematical Literacy is 55%. The probability that a randomly chosen boy at the school does Mathematical Literacy is 65%.

Determine the probability that a learner selected at random from this school does Mathematics. (6) [14]

TOTAL: 150

NATIONAL GRADE 11 EXAMINATIONS

Recommended weighting for Paper 1 & Paper 2

Description	Grade 11
PAPER 1	
Algebra and Equations (and inequalities)	**45 ± 3**
Patterns and Sequences	**25 ± 3**
Finance, Growth and Decay	**15 ± 3**
Functions and Graphs	**45 ± 3**
Probability	**20 ± 3**
TOTAL	**150**
PAPER 2: Theorems and/or trigonometric proofs: maximum 12 marks	
Statistics	**20 ± 3**
Analytical Geometry	**30 ± 3**
Trigonometry	**50 ± 3**
Euclidian Geometry and Measurement	**50 ± 3**
TOTAL	**150**

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GRADE 12 EXEMPLAR PAPER 1

You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

*If necessary, round off answers to **TWO** decimal places, unless stated otherwise.*

▶ ALGEBRA AND EQUATIONS AND INEQUALITIES [23]

QUESTION 1

1.1 Solve for x:

1.1.1 $3x^2 - 4x = 0$ (2)

1.1.2 $x - 6 + \frac{2}{x} = 0;\ x \neq 0.$ (Leave your answer correct to TWO decimal places.) (4)

1.1.3 $x^{\frac{2}{3}} = 4$ (2)

1.1.4 $3^x(x - 5) < 0$ (2)

1.2 Solve for x and y simultaneously:

$y = x^2 - x - 6$ **and** $2x - y = 2$ (6)

1.3 Simplify, without the use of a calculator:

$\sqrt{3}.\sqrt{48} - \frac{4^{x+1}}{2^{2x}}$ (3)

1.4 Given: $f(x) = 3(x - 1)^2 + 5$ and $g(x) = 3$

1.4.1 Is it possible for $f(x) = g(x)$? Give a reason for your answer. (2)

1.4.2 Determine the value(s) of k for which $f(x) = g(x) + k$ has TWO unequal real roots. (2) [23]

▶ PATTERNS AND SEQUENCES [26]

QUESTION 2

2.1 Given the arithmetic series: 18 + 24 + 30 + . . . + 300

2.1.1 Determine the number of terms in this series. (3)

2.1.2 Calculate the sum of this series. (2)

2.1.3 Calculate the sum of all the whole numbers up to and including 300 that are NOT divisible by 6. (4)

2.2 The first three terms of an infinite geometric sequence are 16, 8 and 4 respectively.

2.2.1 Determine the nth term of the sequence. (2)

2.2.2 Determine all possible values of n for which the sum of the first n terms of this sequence is greater than 31. (3)

2.2.3 Calculate the sum to infinity of this sequence. (2) [16]

QUESTION 3

3.1 A quadratic number pattern $T_n = an^2 + bn + c$ has a first term equal to 1. The general term of the first differences is given by $4n + 6$.

3.1.1 Determine the value of a. (2)

3.1.2 Determine the formula for T_n. (4)

3.2 Given the series:
$(1 \times 2) + (5 \times 6) + (9 \times 10) + (13 \times 14) + \ . . . \ + (81 \times 82)$
Write the series in sigma notation. (It is not necessary to calculate the value of the series.) (4) [10]

▶ FUNCTIONS AND GRAPHS [37]

QUESTION 4

4.1 Given: $f(x) = \frac{2}{x+1} - 3$

4.1.1 Calculate the coordinates of the y-intercept of f. (2)

4.1.2 Calculate the coordinates of the x-intercept of f. (2)

4.1.3 Sketch the graph of f, showing clearly the asymptotes and the intercepts with the axes. (3)

4.1.4 One of the axes of symmetry of f is a decreasing function. Write down the equation of this axis of symmetry. (2)

4.2 The graph of an increasing exponential function with equation $f(x) = a.b^x + q$ has the following properties:

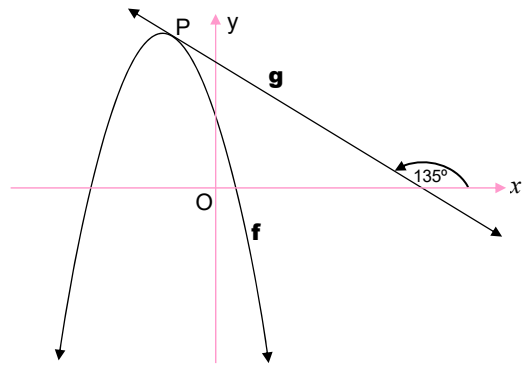
- Range: $y > -3$
- The points (0; -2) and (1; -1) lie on the graph of f.

4.2.1 Determine the equation that defines f. (4)

4.2.2 Describe the transformation from $f(x)$ to $h(x) = 2.2^x + 1$ (2) [15]

QUESTION 5

The sketch below shows the graphs of $f(x) = -2x^2 - 5x + 3$ and $g(x) = ax + q$. The angle of inclination of graph g is 135° in the direction of the positive x-axis. P is the point of intersection of f and g such that g is a tangent to the graph of f at P.



5.1 Calculate the coordinates of the turning point of the graph of f. (3)

5.2 Calculate the coordinates of P, the point of contact between f and g. (4)

5.3 Hence or otherwise, determine the equation of g. (2)

5.4 Determine the values of d for which the line $k(x) = -x + d$ will not intersect the graph of f. (1) [10]

QUESTION 6

The graph of g is defined by the equation $g(x) = \sqrt{ax}$.
The point (8; 4) lies on g.

6.1 Calculate the value of a. (2)

6.2 For what values of x will g be defined? (1)

6.3 Determine the range of g. (1)

6.4 Write down the equation of g^{-1}, the inverse of g, in the form y = . . . (2)

6.5 If $h(x) = x - 4$ is drawn, determine ALGEBRAICALLY the point(s) of intersection of h and g. (4)

6.6 Hence, or otherwise, determine the values of x for which $g(x) > h(x)$. (2) [12]

▶ FINANCE, GROWTH AND DECAY [16]

QUESTION 7

Siphokazi bought a house. She paid a deposit of R102 000, which is equivalent to 12% of the selling price of the house. She obtained a loan from the bank to pay the balance of the selling price. The bank charges her interest of 9% per annum, compounded monthly.

7.1 Determine the selling price of the house. (1)

7.2 The period of the loan is 20 years and she starts repaying the loan one month after it was granted. Calculate her monthly instalment. (4)

7.3 How much interest will she pay over the period of 20 years? Round your answer correct to the nearest rand. (2)

7.4 Calculate the balance of her loan immediately after her 85th instalment. (3)

7.5 She experienced financial difficulties after the 85th instalment and did not pay any instalments for 4 months (that is months 86 to 89). Calculate how much Siphokazi owes on her bond at the end of the 89th month. (2)

7.6 She decides to increase her payments to R8 500 per month from the end of the 90th month. How many months will it take to repay her bond after the new payment of R8 500 per month? (4) [16]

▶ DIFFERENTIAL CALCULUS [32]

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x) = 3x^2 - 2$. (5)

8.2 Determine $\frac{dy}{dx}$ if $y = 2x^{-4} - \frac{x}{5}$. (2) [7]

QUESTION 9

Given: $f(x) = x^3 - 4x^2 - 11x + 30$

9.1 Use the fact that $f(2) = 0$ to write down a factor of $f(x)$. (1)

9.2 Calculate the coordinates of the x-intercepts of f. (4)

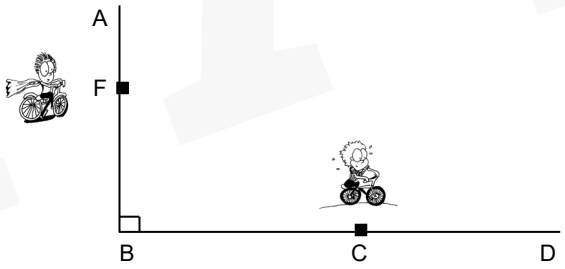
9.3 Calculate the coordinates of the stationary points of f. (5)

9.4 Sketch the curve of f. Show all intercepts with the axes and turning points clearly. (3)

9.5 For which value(s) of x will $f'(x) < 0$? (2) [15]

QUESTION 10

Two cyclists start to cycle at the same time. One starts at point B and is heading due north towards point A, whilst the other starts at point D and is heading due west towards point B. The cyclist starting from B cycles at 30 km/h while the cyclist starting from D cycles at 40 km/h. The distance between B and D is 100 km. After time t (measured in hours), they reach points F and C respectively.



10.1 Determine the distance between F and C in terms of t. (4)

10.2 After how long will the two cyclists be closest to each other? (4)

10.3 What will the distance between the cyclists be at the time determined in Question 10.2? (2) [10]

▶ PROBABILITY [16]

QUESTION 11

11.1 Events A and B are mutually exclusive. It is given that:

- P(B) = 2P(A)
- P(A or B) = 0,57

Calculate P(B). (3)

11.2 Two identical bags are filled with balls. Bag A contains 3 pink and 2 yellow balls. Bag B contains 5 pink and 4 yellow balls. It is equally likely that Bag A or Bag B is chosen. Each ball has an equal chance of being chosen from the bag. A bag is chosen at random and a ball is then chosen at random from the bag.

11.2.1 Represent the information by means of a tree diagram. Clearly indicate the probability associated with each branch of the tree diagram and write down all the outcomes. (4)

11.2.2 What is the probability that a yellow ball will be chosen from Bag A? (1)

11.2.3 What is the probability that a pink ball will be chosen? (3) [11]

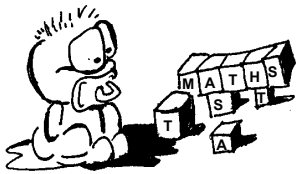
QUESTION 12

Consider the word M A T H S.

12.1 How many different 5-letter arrangements can be made using all the above letters? (2)

12.2 Determine the probability that the letters S and T will always be the first two letters of the arrangements in Question 12.1. (3) [5]

TOTAL: 150



EXEMPLAR MEMOS

Gr 10, 11 & 12

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GRADE 10 EXEMPLAR PAPER 1 MEMO

1.1.1 $(m - 2n)(m^2 - 6mn - n^2)$

$= m^3 - 6m^2n - mn^2$
$\quad - 2m^2n + 12\,mn^2 + 2n^3$

$= m^3 - 8m^2n + 11\,mn^2 + 2n^3$ ◄

1.1.2 $\dfrac{x^3 + 1}{x^2 - x + 1} - \dfrac{4x^2 - 3x - 1}{4x + 1}$

$= \dfrac{(x + 1)\cancel{(x^2 - x + 1)}}{\cancel{(x^2 - x + 1)}} - \dfrac{\cancel{(4x + 1)}(x - 1)}{\cancel{(4x + 1)}}$

$= (x + 1) - (x - 1)$

$= x + 1 - x + 1$

$= 2$ ◄

1.2.1 $6x^2 - 7x - 20$

$= (2x - 5)(3x + 4)$ ◄

1.2.2 $a^2 + a - 2ab - 2b$

$= a(a + 1) - 2b(a + 1)$

$= (a + 1)(a - 2b)$ ◄

1.3 $49 < 51 < 64$ . . . *i.e. 51 lies between 49 and 64*

$\therefore\ 7 < \sqrt{51} < 8$ . . . *taking the square root*

i.e. $\sqrt{51}$ lies between 7 and 8 ◄

1.4 Let $x = 0{,}2\dot{4}\dot{5}$

$\therefore\ x = 0{,}245\,245\ldots$ . . . ❶

$\times 1\,000)\ \therefore\ 1\,000x = 245{,}\,245\,245\ldots$ . . . ❷

❷ - ❶: $\therefore\ 999x = 245$

$\therefore\ x = \dfrac{245}{999}$

. . . *i.e. x can be expressed as $\dfrac{a}{b}$ where $a\ \&\ b \in \mathbb{Z}$*

$\therefore\ x$ is a rational number

2.1.1 $x^2 - 4x = 21$

$\therefore\ x^2 - 4x - 21 = 0$

$\therefore\ (x + 3)(x - 7) = 0$

$\therefore\ x + 3 = 0$ or $x - 7 = 0$

$\therefore\ x = -3$ ◄ $\quad \therefore\ x = 7$ ◄

2.1.2 $3x^{\frac{5}{4}} = 96$

$\div 3)\ \therefore\ x^{\frac{5}{4}} = 32$

$\therefore\ \left(x^{\frac{5}{4}}\right)^{\frac{4}{5}} = (2^5)^{\frac{4}{5}}$

$\therefore\ x = 2^4$

$\therefore\ x = 16$ ◄

2.1.3 $\dfrac{2\sqrt{x}}{3S} = R$

$\times 3S)\ \therefore\ 2\sqrt{x} = 3SR$

$\div 2)\ \therefore\ \sqrt{x} = \dfrac{3SR}{2}$

Square: $\therefore\ x = \dfrac{9\,S^2R^2}{4}$ ◄

2.2 $6q + 7p = 3$ . . . ❶

$2q + p = 5$ . . . ❷

❷ $\times 3$: $6q + 3p = 15$ . . . ❸

❶ - ❸: $\therefore\ 4p = -12$

$\therefore\ p = -3$ ◄

❷: $\therefore\ 2q - 3 = 5$

$\therefore\ 2q = 8$

$\therefore\ q = 4$ ◄

3.1.1 The 1st 3 terms:

$3(3) + 1\ ;\ 2(3)\ ;\ 3(3) - 7$

$\therefore\ 10\ ;\ 6\ ;\ 2$ ◄

3.1.2 The difference is -4

$\therefore$ In $T_n = an + b$: $a = -4$

& $T_0 = b = 14$ . . . *the term before the 1st term*

$\therefore\ T_n = -4n + 14$ ◄

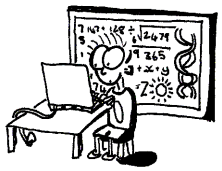
3.1.3 n? if $T_n < -31$

$\therefore -4n + 14 < -31$

$\therefore -4n < -45$

$\div (-4) \quad \therefore n > 11\frac{1}{4}$

$\therefore$ The 12th term ◄



3.2 The even numbers: 6 ; 12 ; 18 . . .

$\therefore$ The 13th even number $= 13 \times 6 = 78$ ◄

OR: The 13th even number
= the 26th term of the pattern
$= 26 \times 3$
$= 78$

4.1 **P** = 4 500; **i** $= \frac{4{,}25}{100} = 0{,}0425$; **n** $= \frac{30}{12} = 2\frac{1}{2}$; **A**?

$\mathbf{A = P(1 + i)^n}$

$= 4\,500(1 + 0{,}0425)^{2{,}5}$

$= R4\,993{,}47$ ◄

4.2.1 The loan amount = R5 999 - R600 = R5 399

The accumulated amount, $\mathbf{A = P(1 + in)}$

where **P** = 5 399; **i** = 8% = 0,08; **n** $= 1\frac{1}{2}$ years; **A**?

$\therefore \mathbf{A} = 5\,399\left[1 + (0{,}08)\left(\frac{3}{2}\right)\right]$

$= R6\,046{,}88$

$\therefore$ The monthly amount to be paid $= \frac{6\,046{,}88}{18}$

$= R335{,}94$ ◄

4.2.2 **The amount of interest**

= **The total amount paid over the 18 months – the loan amount**

= R6 046,88 – R5 399

= R647,88 ◄

4.3 28,35 g is worth \$978,34 = R978,34 × 8,79
= R8 599,61

$\therefore$ 1 g is worth $\frac{R8\,599{,}61}{28{,}35}$

$\therefore$ 1 kg is worth $R\frac{8\,599{,}61}{28{,}35} \times 1\,000$ … *1 kg = 1 000 g*

$\approx R303\,337{,}16$ ◄

5.1.1 A∩B ◄ OR: A and B ◄

5.1.2 A′ ◄ OR: not A ◄

5.2 Set B ◄

5.3.1 Of the 40 learners, 7 are left-handed

$\therefore$ 40 - 7 = 33 are right-handed

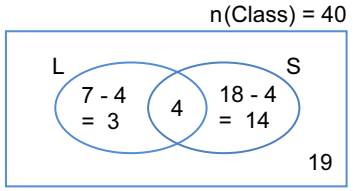
Of the 18 learners who play soccer, 4 are left-handed

$\therefore$ 14 learners who play soccer are right-handed

$\therefore$ The number of learners who are right-handed and DON'T play soccer

= 33 - 14 = 19 ◄

5.3.2



5.3.3 (a) n(L or S) = 3 + 4 + 14 = 21

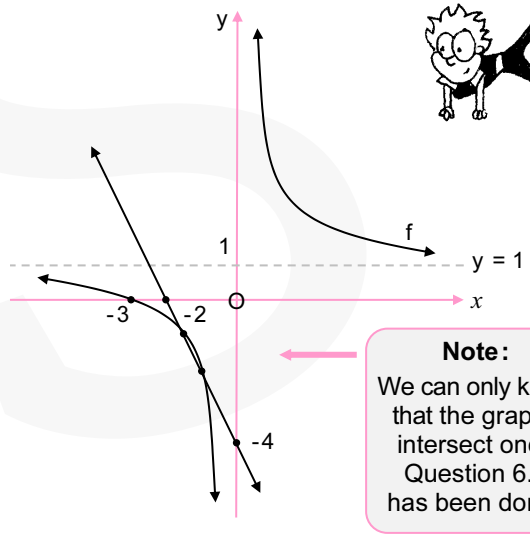
$\therefore$ P(L or S) $= \frac{21}{40}$ ◄

(b) n(R and S) = 14 . . . *where R is the set of all right-handed people*

$\therefore$ P(R and S) $= \frac{14}{40}$

$= \frac{7}{20}$ ◄

6.1



Note:
We can only know that the graphs intersect once Question 6.4 has been done!

f: $y = \frac{3}{x} + 1$

y-intercept ($x = 0$): none

x-intercept (y = 0): $\frac{3}{x} + 1 = 0$

$\therefore \frac{3}{x} = -1$

$\therefore x = -3$

g: $y = -2x - 4$

y-intercept ($x = 0$): y = -4

x-intercept (y = 0): $-2x - 4 = 0$

$\therefore -2x = 4$

$\therefore x = -2$

6.2 Asymptotes: $y = 1$ ◄

& $x = 0$ (the y-axis) ◄

6.3 Domain of f: $x \neq 0;\ x \in \mathbb{R}$ ◄

. . . $(-\infty; 0) \cup (0; \infty)$

6.4 $f(x) = g(x)$ ➡ $\frac{3}{x} + 1 = -2x - 4$

$\times x)\ \therefore\ 3 + x = -2x^2 - 4x$

$\therefore\ 2x^2 + 5x + 3 = 0$

$\therefore\ (2x + 3)(x + 1) = 0$

$\therefore\ 2x + 3 = 0$ or $x + 1 = 0$

$\therefore\ 2x = -3$ $\quad\therefore\ x = -1$ ◄

$\therefore\ x = -\frac{3}{2}$ ◄

Note: These are the x-coordinates of the points of intersection of f and g: $\left(-1\frac{1}{2}; -1\right)$ & $(-1; -2)$

6.5 $-1 \leq g(x) < 3$

$\therefore\ -1 \leq -2x - 4 < 3$ . . . $g(x) = -2x - 4$

add 4: $\therefore\ 3 \leq -2x < 7$

$\div(-2)$: $\therefore\ -\frac{3}{2} \geq x > -\frac{7}{2}$ ... *When one divides by a negative number, the direction of the 'inequality' changes.*

$\therefore\ -\frac{7}{2} < x \leq -\frac{3}{2}$ ... *The inequality has been rewritten with the smaller value on the left.*

i.e. $-3\frac{1}{2} < x \leq -1\frac{1}{2}$ ◄ OR: $\left(-3\frac{1}{2}; -1\frac{1}{2}\right]$ ◄

(means excluding;] means including

6.6 $k(x) = 2g(x) = 2(-2x - 4) = -4x - 8$

$\therefore$ The equation of k: $y = -4x - 8$

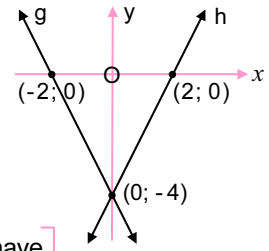
$\therefore$ The y-intercept of k: $(0; -8)$ ◄ . . . *substitute $x = 0$*

6.7 x-intercept of g: $(-2; 0)$

& x-intercept of h: $(2; 0)$ ◄

y-intercept of g: $(0; -4)$

& y-intercept of h: $(0; -4)$ ◄



Notice: The reflected points have the same y-coordinate, but the x-coordinates are opposite in sign.

7.1 C(-2; 0) ◄ . . . *symmetrical about the y-axis*

7.2 The equation of f: $y = a(x + 2)(x - 2)$ ... *roots -2 & 2*

$\therefore\ y = a(x^2 - 4)$

Subst. $B\left(-3; \frac{5}{2}\right)$: $\therefore\ \frac{5}{2} = a\left[(-3)^2 - 4\right]$

$\therefore\ \frac{5}{2} = a(5)$

$\div 5)\ \therefore\ a = \frac{1}{2}$

$\therefore$ The equation of f: $y = \frac{1}{2}(x^2 - 4)$

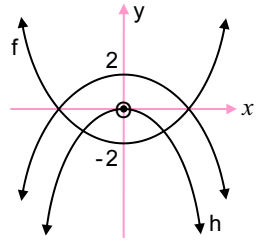
$\therefore\ y = \frac{1}{2}x^2 - 2$ ◄

7.3 The y-intercept of f is (0; -2)

$\therefore$ The range of f: $y \geq -2$ ◄ OR: $[-2; \infty)$ ◄

7.4 The graph of h is obtained by flipping f . . . $-f(x)$

then, shifting down 2 units . . . -2



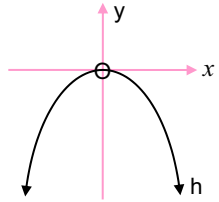
$\therefore$ The range of h: $y \leq 0$ ◄

OR: $(-\infty; 0]$ ◄

OR: $h(x) = -\left(\frac{1}{2}x^2 - 2\right) - 2$

$\therefore\ h(x) = -\frac{1}{2}x^2 + 2 - 2$

$\therefore\ h(x) = -\frac{1}{2}x^2$

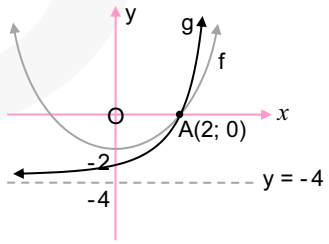


$\therefore$ The range of h: $y \leq 0$ ◄

7.5 $q = -4$ . . . range $y > -4$ ➡ $y = -4$ is an asymptote

$\therefore$ Equation of g:

$y = b^x - 4;\ b > 0$



Substitute A(2; 0):

$0 = b^2 - 4$

$\therefore\ b^2 = 4$

$\therefore\ b = 2$ . . . $b \neq -2\ \because\ b > 0$

$\therefore$ Equation of g:

$y = 2^x - 4$ ◄

$\therefore$ means therefore

$\because$ means because

GRADE 11 EXEMPLAR PAPER 1 MEMO

▶ ALGEBRA AND EQUATIONS AND INEQUALITIES [47]

1.1.1 $(2x - 1)(x + 5) = 0$

➡ $2x - 1 = 0$ **or** $x + 5 = 0$

$\therefore 2x = 1$ $\quad \therefore x = -5$ ◀

$\therefore x = \frac{1}{2}$ ◀

$a \times b = 0$
➡ $a = 0$
or $b = 0$

1.1.2 $2x^2 - 4x + 1 = 0$

$\therefore x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(1)}}{2(2)}$

$= \frac{4 \pm \sqrt{16 - 8}}{4}$

$= \frac{4 \pm \sqrt{8}}{4}$

$= \frac{4 \pm 2\sqrt{2}}{4}$ . . . $\sqrt{8} = \sqrt{4 \times 2} = \sqrt{4} \times \sqrt{2} = 2\sqrt{2}$

$= \frac{\mathbf{2 \pm \sqrt{2}}}{\mathbf{2}}$ ◀ . . . $\frac{\cancel{2}(2 \pm \sqrt{2})}{\cancel{4}\,2}$

The formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Note: No formula sheet is supplied for the Grade 11 exam.

1.2.1 $125^{\frac{2}{3}}$

$= (5^3)^{\frac{2}{3}}$

$= 5^2$ . . . $(a^m)^n = a^{mn}$

$= \mathbf{25}$ ◀

OR: $125^{\frac{2}{3}}$

$= 5^2$ . . . *cubed root of 125 is 5*

$= \mathbf{25}$ ◀

1.2.2 $(3\sqrt{2} - 12)(2\sqrt{2} + 1)$ . . .

'FOIL': Firsts, Outers, Inners, Lasts!

$= 6.\mathbf{2} + 3\sqrt{2} - 24\sqrt{2} - 12$

$= 12 - 21\sqrt{2} - 12$

$= \mathbf{-21\sqrt{2}}$ ◀

$\sqrt{2}\sqrt{2} = \mathbf{2}$

1.3.1 The expression is undefined for $3x - 9 = 0$

$\therefore 3x = 9$

$\therefore \mathbf{x = 3}$ ◀

Division by zero is undefined.

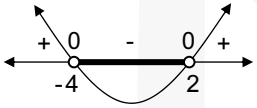
1.3.2 $\frac{x^2 - x - 6}{3x - 9} = \frac{(x - 3)(x + 2)}{3(x - 3)}$

$= \frac{\mathbf{x + 2}}{\mathbf{3}}$ **for** $\mathbf{x \neq 3}$ ◀

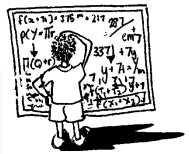
2.1.1 $x^2 - x - 6 < -3x + 2$

$\therefore x^2 + 2x - 8 < 0$

$\therefore (x + 4)(x - 2) < 0$



$\therefore \mathbf{-4 < x < 2}$ ◀



2.1.2 The integers between -4 and 2 are:

-3 ; -2 ; -1 ; 0 and 1

See $x^2 + 2x - 8 < 0$ in line 2 in 2.1.1.

$\therefore$ The sum of the integers $= (-3) + (-2) + (-1) + 0 + 1$

$= \mathbf{-5}$ ◀

2.2.1 $\frac{4^x.4^{-1} + 4^x.4}{17.4^x.3^x}$ . . . $a^{m+n} = a^m.a^n$, $(ab)^n = a^n.b^n$

$= \frac{\cancel{4^x}\left(\frac{1}{4} + 4\right)}{17.\cancel{4^x}.3^x}$ . . .

$= \frac{\frac{17}{4}}{17.3^x}$

$= \frac{\cancel{17}}{4} \times \frac{1}{\cancel{17}.3^x}$

$= \frac{\mathbf{1}}{\mathbf{4.3^x}}$ . . . $\frac{1}{4}.3^{-x}$ ◀

OR: $= \frac{4^{x-1}(1 + 4^2)}{17.4^x.3^x}$

$= \frac{4^{x-1-x}.17}{17.3^x}$

2.2.2 The expression $= \frac{1}{4}.3^{-x} = \frac{1}{4}.4t = \mathbf{t}$ ◀

2.3 $3^y = (3^4)^x$ & $y = x^2 - 6x + 9$ . . . ❷

$\therefore 3^y = 3^{4x}$

$\therefore y = 4x$ . . . ❶

Equating ❷ & ❶: $\therefore x^2 - 6x + 9 = 4x$

$\therefore x^2 - 10x + 9 = 0$

$\therefore (x - 1)(x - 9) = 0$

$\therefore x = 1$ or $x = 9$

If $x = 1$: $y = 4(1) = 4$

If $x = 9$: $y = 4(9) = 36$

$\therefore$ The solutions: **(1; 4)** or **(9; 36)** ◀

3.1 The roots of a quadratic equation: $x = \frac{3 \pm \sqrt{4 - 8p}}{4}$

i.e. The roots are $\frac{3 + \sqrt{4 - 8p}}{4}$ and $\frac{3 - \sqrt{4 - 8p}}{4}$

3.1.1 The roots will be **EQUAL** if

$4 - 8p = 0$

$\therefore -8p = -4$

$\div (-8)$ $\quad \therefore \mathbf{p = \frac{1}{2}}$ ◀

Compare the roots . . . + √ and − √ is the only part that is different.

3.1.2 The roots will be **NON-REAL** if

$4 - 8p < 0$

$\therefore -8p < -4$

$\div (-8)$ $\quad \therefore \mathbf{p > \frac{1}{2}}$ ◀

√a negative no. *is non-real*

3.2.1 Both the following conditions must hold:

▶ $5 - x \geq 0$ . . . *$\sqrt{a}$ only real if $a \geq 0$*

$\therefore -x \geq -5$

$\times (-1)$ $\quad \therefore x \leq 5$

AND

▶ $x + 1 \geq 0$ . . .

$\therefore x \geq -1$

$\therefore \mathbf{-1 \leq x \leq 5}$ ◀

$\sqrt{a}$ defined as $+\sqrt{a}$ for all $a \geq 0$

∴ For equation to be true RHS ≥ 0

3.2.2 $\sqrt{5-x} = x+1$

$\therefore (\sqrt{5-x})^2 = (x+1)^2$

$\therefore 5-x = x^2+2x+1$

$\therefore 0 = x^2+3x-4$

$\therefore (x+4)(x-1) = 0$

$\therefore x = -4$ or 1

But $-1 \le x \le 5$ . . . *see 3.2.1*

$\therefore$ **Only $x = 1$** ◄

OR: Test . . .

For $x = -4$:

LHS $= \sqrt{9} = 3$ & RHS $= -3$ $\therefore x \ne -4$

For $x = 1$: LHS = RHS = 2 $\therefore x = 1$ ✓

3.2.3 The solution: $\boldsymbol{x = -4}$ ◄

Note: This is the rejected answer in 3.2.2!
Squaring the equation $-\sqrt{5-x} = x+1$
will yield the identical calculation as in 3.2.2
except, when we test, $x + 1$ must be *negative*.

► FINANCE, GROWTH AND DECAY [18]

4.1 **A = P(1 - in)** . . . Formula for **depreciation** on the **straight-line** method.

A? ; **P** = R145 000 ; **i** = 17% = $\frac{17}{100}$ = 0,17 ; **n** = 5

$\therefore$ A = 145 000[1 - (0,17)(5)]
= **R21 750** ◄

4.2.1 The rate earned quarterly, **i** = $\frac{8\%}{4}$ = **2%** = **0,02** ◄

4.2.2 $\mathbf{1 + i_{eff}} = \left(\mathbf{1 + \frac{i_{nom}}{4}}\right)^4$ . . . *Note: $i_{nom} = 8\%$*

$= (1 + 0{,}02)^4$

$= (1{,}02)^4$

$= 1{,}08243\ldots$

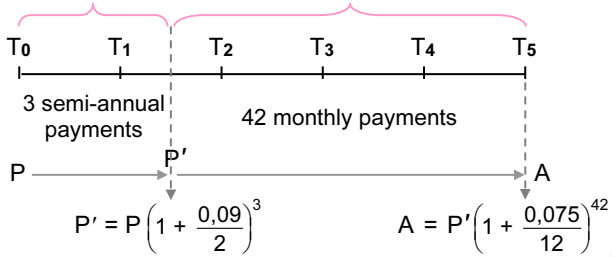
$\therefore \mathbf{i_{eff}} = 0{,}08243\ldots$

$\approx$ **8,24% per annum** ◄

Note:
A = P$\mathbf{(1 + i_{eff})^1}$
and
A = P$\left(\mathbf{1 + \frac{i_{nom}}{4}}\right)^4$

4.3

semi-annually	monthly
i = $\frac{9\%}{2} = \frac{0{,}09}{2}$	**i** = $\frac{7{,}5\%}{12} = \frac{0{,}075}{12}$
n = 3	**n** = 42



P = R14 000

$\therefore$ The accumulated amount, A

$= \text{R}14\,000\left(1 + \frac{0{,}09}{2}\right)^3\left(1 + \frac{0{,}075}{12}\right)^{42}$

$\approx$ **R20 755,08** ◄

5.1 The value (of both investments) at the start (i.e. at $x = 0$) = **R15 000** ◄

5.2 **Simple interest** ◄ . . . *straight-line appreciation*

5.3 **i?** ; **P** = R15 000 ; **n** = 6 ; **A** = R31 000 See point A

A = P(1 + in)

$\therefore$ 31 000 = 15 000[1 + (i)(6)]

÷ 15 000) $\therefore 1 + 6i = 2{,}0\dot{6}$

$\therefore 6i = 1{,}0\dot{6}$

$\therefore i = 0{,}1\dot{7}$

$\therefore$ ***i* = 17,78%** ◄

5.4 Determine w:

(12; w) is a point on Dumisani's graph.

$\therefore$ Substitute n = 12 ; P = R15 000 ; i = 17,777... in

A = P(1 + in) . . . *Dumisani's formula*

$\therefore$ w = 15[1 + (0,1$\dot{7}$)(12)]
$\approx$ 47

Note:
A, P and w represent 'thousands of rands'

Substitute point B(12; 47) in

A = P(1 + i)^n . . . *Astin's formula*

$\therefore 47 = 15(1 + i)^{12}$

$\therefore (1 + i)^{12} = 3{,}1\dot{3}$

$\therefore 1 + i = 1{,}09985\ldots$

$\therefore i = 0{,}09985\ldots$

= **10,0%** ◄

► PATTERNS AND SEQUENCES [23]

6.1 $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{8}$; . . . ; $\frac{1}{1\,024}$

6.1.1 Multiply $\frac{1}{8}$ by $\frac{1}{2}$:

$\mathbf{T_4 = \frac{1}{16}}$ ◄ . . . $\left(\frac{1}{2}\right)^1$; $\left(\frac{1}{2}\right)^2$; $\left(\frac{1}{2}\right)^3$; . . .

OR: The terms are: 2^{-1} ; 2^{-2} ; 2^{-3} ; . . . ; 2^{-10}

$\therefore \mathbf{T_4} = 2^{-4}$ . . . *the fourth term = 2^{-four}*

$= \mathbf{\frac{1}{16}}$ ◄

6.1.2 The nth term, $\mathbf{T_n = \left(\frac{1}{2}\right)^n}$ or $\mathbf{2^{-n}}$ ◄ . . . *see 6.1.1*

6.1.3 $1\,024 = 2^{10}$ . . . *trial and error!*

$\therefore \frac{1}{1\,024} = \left(\frac{1}{2}\right)^{10}$ or 2^{-10}

$\therefore$ The number of terms in the sequence, **n = 10** ◄

6.2 156 ; 148 ; 140 ; 132 ; . . .

6.2.1 The 5th term, $\mathbf{T_5}$ = 132 - 8 = **124** ◄

6.2.2 The general term of a linear pattern is $T_n = an + b$

This sequence has a common 1st difference of -8

$\therefore$ a = -8

and T_1 = a + b = 156 . . . $T_1 = a(1) + b$

$\therefore$ -8 + b = 156

$\therefore$ b = 164

$\therefore$ A general formula: $\mathbf{T_n = -8n + 164}$ ◄

6.2.3 T_n negative, i.e. $T_n < 0$

➡ $-8n + 164 < 0$

$\therefore -8n < -164$

÷ (-8) $\therefore n > 20\frac{1}{2}$

$\therefore$ **The 1st term to be negative is the 21st term** ◄

6.2.4 ▸ 1st difference (between T_1 and T_2 of the quadratic pattern)

$= 3a + b = 156$

156 148 140 132

-8 -8 -8

▸ The 2nd difference,

$2a = -8$

$\therefore a = -4$

$\therefore 3(-4) + b = 156$

$\therefore b = 168$

▸ $T_5 = a(5)^2 + b(5) + c = -24$ . . . *given $T_5 = -24$*

$\therefore 25a + 5b + c = -24$

$\therefore 25(-4) + 5(168) + c = -24$

$\therefore -100 + 840 + c = -24$

$\therefore c = -764$

$\therefore T_n = -4n^2 + 168n - 764$

There are various other methods!

---

7. $T_n = an^2 + bn + c$

$T_2 = a(2)^2 + b(2) + c = 4a + 2b + c = 0$ . . . ❶

$T_4 = a(4)^2 + b(4) + c = 16a + 4b + c = 0$ . . . ❷

❷ - ❶: $12a + 2b = 0$

$\therefore 6a + b = 0$

& 2nd difference, $2a = 12$

$\therefore a = 6$

$\therefore 36 + b = 0$

$\therefore b = -36$

❶: $4(6) + 2(-36) + c = 0$

$\therefore c = -24 + 72$

$\therefore c = 48$

$T_3 = a(3)^2 + b(3) + c$

$= 9a + 3b + c$

$= 9(6) + 3(-36) + 48$

$= \mathbf{-6}$ ◂

There are various other methods!

▸ FUNCTIONS AND GRAPHS [43]

8.1 $\mathbf{x = 3}$ *(vertical asymptote)* ◂

& $\mathbf{y = -1}$ *(horizontal asymptote)* ◂

8.2 $\mathbf{x \in \mathbb{R}\ ;\ x \neq 3}$ ◂

8.3 $\mathbf{d} = \tan 76^\circ \approx \mathbf{4}$ ◂

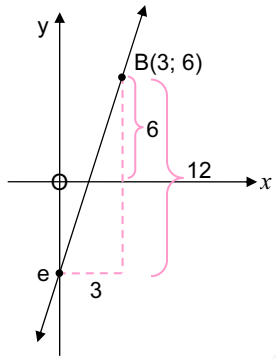
& $\mathbf{e = -6}$ ◂

OR: Find e by substituting $d = 4$ and B(3; 6) into $g(x) = dx + e$:

$6 = (4)(3) + e$

$6 = 12 + e$

$-6 = e$



8.4 At A & C:

$$\frac{2}{x-3} - 1 = 4x - 6$$

$\times (x - 3)$ $\therefore 2 - (x - 3) = (4x - 6)(x - 3)$

$\therefore 2 - x + 3 = 4x^2 - 18x + 18$

$\therefore 0 = 4x^2 - 17x + 13$

$\therefore 0 = (4x - 13)(x - 1)$

$\therefore x = 1$ at A and $x = \frac{13}{4}$ at C

$g(1) = 4(1) - 6 = -2$ and $g\left(\frac{13}{4}\right) = 4\left(\frac{13}{4}\right) - 6 = 7$

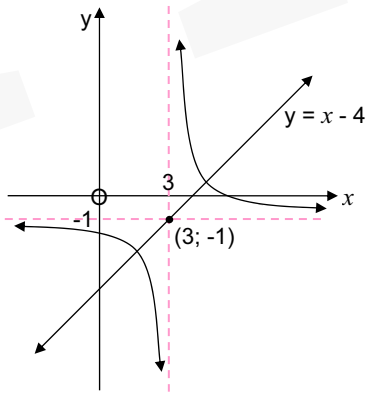
$\therefore$ **A(1; -2) and C$\left(\frac{13}{4}; 7\right)$** ◂

8.5 $\mathbf{1 \le x < 3}$ or $\mathbf{x \ge \frac{13}{4}}$ ◂ . . . *g is above or on f*

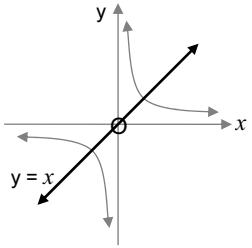
[Note: $x \neq 3$ $\because$ f(x) is undefined at $x = 3$]

8.6 $y = (x - 3) - 1$ . . .

$\therefore$ $\mathbf{y = x - 4}$ ◂



The axis of symmetry, $y = x$, of $y = \frac{k}{x}$



moves 3 units right & 1 unit down

OR: Axis of symmetry: $y = x + c$

Substitute (3; -1): $-1 = 3 + c$

$\therefore -4 = c$

$\therefore$ Equation: $\mathbf{y = x - 4}$ ◂

9.1 ▸ $f(x) = -x^2 + 2x + 3$

› y-intercept: (0; 3) . . . *$x = 0$*

› x-intercepts: Substitute $y = 0$

$-x^2 + 2x + 3 = 0$

$\times (-1)$ $\therefore x^2 - 2x - 3 = 0$

$\therefore (x - 3)(x + 1) = 0$

$\therefore x = 3$ or -1

› Turning point: Axis of symmetry: $x = 1$

(Halfway between the roots)

& Maximum $y = -(1)^2 + 2(1) + 3 = 4$

$\therefore$ Turning point is (1; 4)

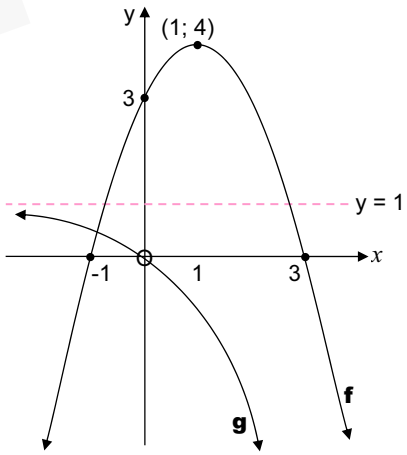
▸ $g(x) = 1 - 2^x$

› y-intercept: Substitute $x = 0$

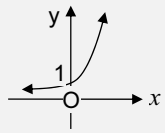
$\therefore y = 1 - 2^0 = 1 - 1 = 0$

$\therefore$ (0; 0) . . . $\therefore$ *x-int. too!*

› equation of asymptote: $y = 1$

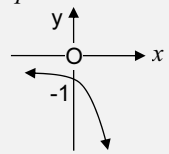


Consider $\mathbf{y = 2^x}$



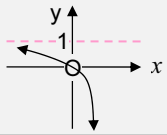
then, $y = \mathbf{-2}^x$

. . . *flip*



then, $y = \mathbf{+1} - 2^x$:

. . . *move 1 unit up*



9.2 $f(-3) = -(-3)^2 + 2(-3) + 3 = -9 - 6 + 3 = -12$

& $f(0) = -(0)^2 - 2(0) + 3 = 3$

$\therefore$ Average gradient between $x = -3$ and $x = 0$

$= \dfrac{f(0) - f(-3)}{0 - (-3)}$

$= \dfrac{3 - (-12)}{3}$

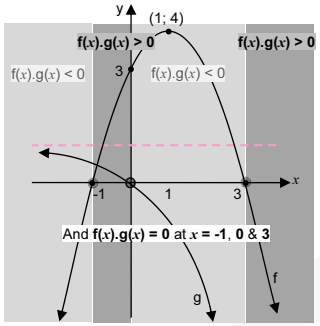
$= \dfrac{15}{3}$

$= 5$ ◄

Average gradient $= \dfrac{\text{change in } y}{\text{change in } x}$

Also: $\dfrac{y_2 - y_1}{x_2 - x_1}$

9.3 $-1 \le x \le 0$ or $x \ge 3$ ◄

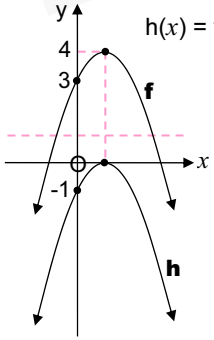


Observe f(x) and g(x), the y-values of f and g.

The question is asking for which values of x, moving from left to right, is the product of the graphs positive or zero?

i.e. for which values of x do the graphs have the same sign, either both positive or both negative and for which values of x are either of the graphs zero.

9.4 **c = -4** ◄



h(x) = f(x) + a number.

For f to just *touch* the x-axis, it must move down 4 units.

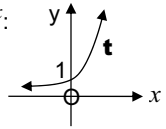
$\therefore$ 'the number' must be -4.

9.5 $t(x) = -(1 - 2^x) + 1$

$= -1 + 2^x + 1$

$= 2^x$

$y = 2^x$:



At the y-intercept, $x = 0$

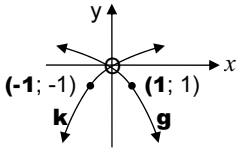
$\therefore y = 2^0 = 1$

$\therefore$ **(0; 1)** ◄

9.6 **$k(x) = 1 - 2^{-x}$** ◄

When points (or graphs) are reflected about the y-axis, x is replaced by $-x$.

e.g. (**1**; -1) on g becomes (**-1**; -1) on k.



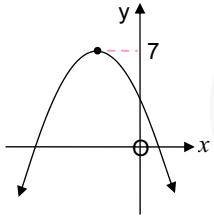
10. The range, $(-\infty; 7]$, indicates the y-values.

➡ Max f(x) = 7 and a < 0 ;

Axis of symmetry:

$x = -\dfrac{b}{2a} = -\dfrac{-}{-}$ is negative;

(. . . it is given that b < 0 and concluded that a < 0.)



One root positive & one negative

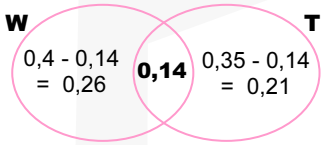
➡ roots on opposite sides of y-axis.

Note: Range notation:

(means excluding &] means including

► PROBABILITY [19]

11.



Note:

W includes the part that overlaps with T (just as T includes the part that overlaps with W).

$\therefore$ The overlap needs to be subtracted to find W without T and T without W.

► Events A and B are mutually exclusive if:

- **P(A∩B) = 0, or** if:
- **P(A or B) = P(A) + P(B)**

► Events A and B are independent if:

- **P(A and B) = P(A) × P(B)**

11.1 **Method 1**

P(W and T) = 0,14 . . . *given*

$\therefore$ P(W∩T) ≠ 0

$\therefore$ **W and T are not mutually exclusive events** ◄

Method 2

P(W or T) = 0,26 + 0,14 + 0,21 = 0,61

P(W) + P(T) = 0,4 + 0,35 = 0,75 . . . *≠ 0,61*

$\therefore$ P(W or T) ≠ P(W) + P(T)

$\therefore$ **W and T are not mutually exclusive events** ◄

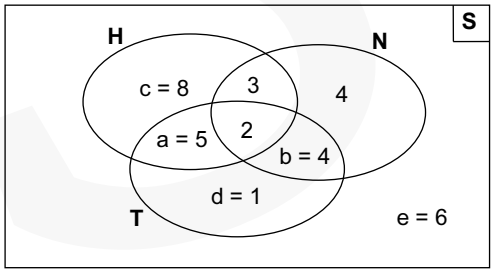
11.2 P(W and T) = 0,14 . . . *given*

P(W) × P(T) = (0,4)(0,35) = 0,14

$\therefore$ P(W and T) = P(W) × P(T)

$\therefore$ **W and T are independent events** ◄

12.



Lines 1, 2 and 7 were not required for finding values a to e.

12.1.1 **a = 5** ◄ . . . *line 3*

b = 4 ◄ . . . *line 4*

c = 8 ◄ . . . *line 5, but after a is determined*

d = 1 ◄ . . . *line 6*

e = 6 ◄ . . . *e = n(S) - n(H∪T∪N)*

= 33 - 27 . . . *33 learners were surveyed*

Note: n(H∪T∪N) = 18 + 1 + 4 + 4 = 27

12.1.2 **6** ✓ . . . *the value of e, the number not in H, T or N*

Note: In Question 12.1.2, the **number** of learners is required.

In Question 12.1.3 & 12.1.4, the **probability** is required.

NB: The probability of an event (E) occurring

$$= \frac{\text{the number of ways E can occur}}{\text{the total number of outcomes}}$$

12.1.3 The **number** of learners playing netball ONLY = 4

∴ The **probability** that a learner plays netball only

$$= \frac{\text{the number that play netball only}}{\text{the total number of learners}}$$

$$= \frac{4}{33} \checkmark$$

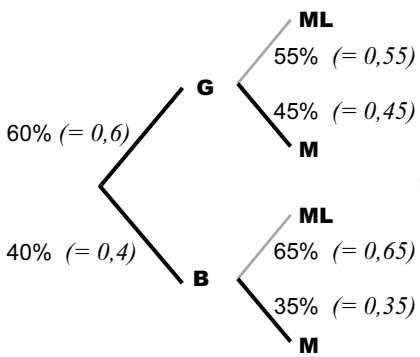
($\simeq$ 0,12)

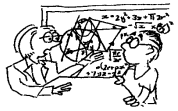
12.1.4 The **number** of learners playing hockey or netball (or both) = 26 . . . *n(H∪N)*

∴ The **probability** that a learner plays hockey or netball (or both)

$$= \frac{n(H \cup N)}{n(S)} = \frac{26}{33} \ (\approx 0{,}78) \checkmark$$

12.2





P(a learner does Maths)

= P(a girl doing Maths) + P(a boy doing Maths)

= (60% × 45%) + (40% × 35%)

= 0,27 + 0,14

= **0,41** ✓ . . . *= 41%*

OR: Using decimals only:

P(M) = P(G and M) + P(B and M)

= (0,6 × 0,45) + (0,4 × 0,35)

= 0,27 + 0,14

= **0,41** ✓

GRADE 12 EXEMPLAR PAPER 1 MEMO

▶ ALGEBRA AND EQUATIONS AND INEQUALITIES [23]

1.1.1 $3x^2 - 4x = 0$

$\therefore x(3x - 4) = 0$

$\therefore \mathbf{x = 0}$ ◀ **or** $3x - 4 = 0$

$\therefore 3x = 4$

$\therefore \mathbf{x = \frac{4}{3}}$ ◀

1.1.2 $x - 6 + \frac{2}{x} = 0$

$\times x)\ \therefore x^2 - 6x + 2 = 0$

$\therefore x = \frac{-(-6) \pm \sqrt{(1-6)^2 - 4(1)(2)}}{2(1)}$

$\therefore x = \frac{6 \pm \sqrt{28}}{2}$

$\therefore \mathbf{x \simeq 5{,}65}$ **or** $\mathbf{0{,}35}$ ◀

1.1.3 $x^{\frac{2}{3}} = 4$

Take $\sqrt{\ }$: $\therefore x^{\frac{1}{3}} = \pm 2$

Raise to power 3: $\therefore \mathbf{x = \pm 8}$ ◀

OR: $\left(x^{\frac{2}{3}}\right)^{\frac{3}{2}} = 4^{\frac{3}{2}}$

$\therefore x = 2^3$

$\therefore x = 8$

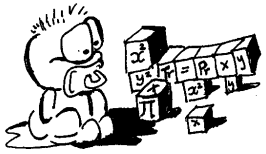
But, **remember** that an *eventh* root was taken

$\therefore \mathbf{x = \pm 8}$ ◀

1.1.4 $3^x(x - 5) < 0$

➡ $x - 5 < 0$ . . . because 3^x is positive for all real values of x.

$\therefore \mathbf{x < 5}$ ◀ . . . (+)(-) = (-)



1.2 $2x - y = 2$ ➡ $2x - 2 = y$ . . . ❶

$y = x^2 - x - 6$ . . . ❷

Equate ❶ and ❷:

$\therefore x^2 - x - 6 = 2x - 2$

$\therefore x^2 - 3x - 4 = 0$

$\therefore (x + 1)(x - 4) = 0$

$\therefore x = -1$ or $x = 4$

OR: Substitute ❷ in ❶

$\therefore 2x - (x^2 - x - 6) = 2$, etc.

❶: If $x = -1$: $y = 2(-1) - 2 = -4$

If $x = 4$: $y = 2(4) - 2 = 6$

$\therefore$ The solution: **(-1; -4)** **or** **(4; 6)** ◀

1.3 $\sqrt{3}\sqrt{16 \times 3} - \frac{(2^2)^{x+1}}{2^{2x}}$

$= \sqrt{3}\sqrt{16}\sqrt{3} - \frac{2^{2x+2}}{2^{2x}}$

$= (\sqrt{3})^2 . 4 - 2^{2x+2-2x}$

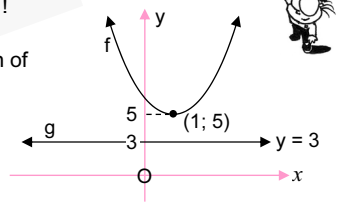
$= 3 . 4 - 2^2$

$= 12 - 4$

$= \mathbf{8}$ ◀

1.4 **Note:** Each of the 2 questions requires a 2 mark answer only! Lengthy algebraic calculations (see the alternative methods) would not be appropriate!

A rough sketch of f and g:



1.4.1 **No** ◀ ; The MINIMUM value of $f(x) = 5$

$\therefore$ **f and g have no points of intersection** ◀

1.4.2 **k > 2** ◀ . . . $g(x) + k$ must be > 5 so that a line $y = g(x) + k$ (parallel to the x-axis) will cut f twice.

OR: Algebraic methods, requiring more time!

1.4.1 **No** ◀ ; $f(x) = g(x)$ ➡ $3(x - 1)^2 + 5 = 3$

$\therefore 3(x - 1)^2 = -2$

$\therefore (x - 1)^2 = -\frac{2}{3}$,

which is impossible because a square cannot be negative.

OR: $3(x^2 - 2x + 1) + 5 = 3$

$\therefore 3x^2 - 6x + 3 + 5 = 3$

$\therefore 3x^2 - 6x + 5 = 0$

$\therefore x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(5)}}{2(3)}$

$= \frac{6 \pm \sqrt{\mathbf{-24}}}{6}$ ⟵ $\Delta = -24$, $\therefore \sqrt{\Delta}$ is non-real

$\therefore$ There are no solutions to the equation $f(x) = g(x)$.

1.4.2 $f(x) = g(x) + k$ ➡ $3(x - 1)^2 + 5 = 3 + k$

$\therefore 3(x^2 - 2x + 1) + 5 - 3 - k = 0$

$\therefore 3x^2 - 6x + (5 - k) = 0$

$\Delta = (-6)^2 - 4(3)(5 - k)$

$= 36 - 60 + 12k$

$= 12k - 24$

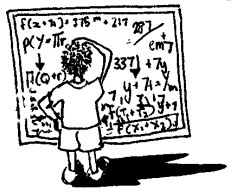
If we want 2 (real & unequal) roots, then Δ must be positive:

$\therefore 12k - 24 > 0$

$\therefore 12k > 24$

$\therefore \mathbf{k > 2}$ ◀

The sketch is much easier.



▶ PATTERNS AND SEQUENCES [26]

2.1 $18 + 24 + 30 + \ldots + 300$

2.1.1 The series is arithmetic: $a = 18$; $d = 6$; n?

$T_n = a + (n - 1)d$ ➡ $300 = 18 + (n - 1)(6)$

$\therefore 282 = 6(n - 1)$

$\div 6)$ $\therefore n - 1 = 47$

$\therefore n = 48$

$\therefore$ **48 terms** ◄

OR: This is a linear series

$\therefore$ The general term, $T_n = an + b$ where

a = the 1st difference = 6 & $b = T_0 = 12$

$\therefore T_n = 6n + 12$

$\therefore$ Let $6n + 12 = 300$

$\therefore 6n = 288$

$\therefore n = 48$

$\therefore$ **48 terms** ◄

2.1.2 The sum, $S_n = \frac{n}{2}(a + T_n)$

where $n = 48$ (from 2.1.1) ; $a = 18$ & $T_{48} = 300$

$\therefore S_{48} = \frac{48}{2}(18 + 300)$

$= \mathbf{7\ 632}$ ◄

OR: $S_n = \frac{n}{2}[2a + (n - 1)d]$

where $n = 48$; $a = 18$ & $d = 6$

$\therefore S_n = \frac{48}{2}[2(18) + (48 - 1)(6)]$

$= \mathbf{7\ 632}$ ◄

2.1.3 The sum of all the whole numbers up to and including 300

$= (0 +) 1 + 2 + 3 + \ldots + 300$

$= \frac{300}{2}(1 + 300)$ . . . $S_n = \frac{n}{2}(a + T_n)$

$= 45\ 150$

from 2.1.2

$\therefore$ The required sum $= 45\ 150 - (6 + 12 + 7\ 632)$

$= \mathbf{37\ 500}$ ◄

2.2 G.S.: 16; 8; 4; . . .

2.2.1 $T_n = ar^{n-1}$ where $a = 16$ & $r = \frac{8}{16}$ or $\frac{4}{8} = \frac{1}{2}$

$\therefore T_n = 16.\left(\frac{1}{2}\right)^{n-1} = 2^4.(2^{-1})^{n-1}$

$= 2^4.2^{-n+1}$

$= 2^{4-n+1}$

$= \mathbf{2^{5-n}}$ ◄

2.2.2 Consider $16 + 8 + 4 + 2 + 1 = 31$

i.e. $S_5 = 31$

$\therefore S_n > 31$ ➡ $\mathbf{n > 5}$ ◄

OR:

$S_n = \frac{a(1 - r^n)}{1 - r}$ where $a = 16$ & $r = \frac{1}{2}$

$= \frac{16\left[1 - \left(\frac{1}{2}\right)^n\right]}{1 - \frac{1}{2}}$

$= \frac{16\left[1 - \left(\frac{1}{2}\right)^n\right]}{\frac{1}{2}}$

$= 32$

$S_n > 31$ ➡ $32\left[1 - \left(\frac{1}{2}\right)^n\right] > 31$

$\therefore 1 - \left(\frac{1}{2}\right)^n > \frac{31}{32}$

$\therefore -\left(\frac{1}{2}\right)^n > -\frac{1}{32}$

$\times(-1)$ $\therefore \left(\frac{1}{2}\right)^n < \left(\frac{1}{2}\right)^5$

$\therefore \mathbf{n > 5}$ ◄

Note: It is acceptable to write: $n \geq 6$ because $n \in \mathbb{N}$.

2.2.3 $S_\infty = \frac{a}{1 - r} = \frac{16}{1 - \frac{1}{2}} = \frac{16}{\frac{1}{2}} = \mathbf{32}$ ◄

3.1.1 The terms: -5 1 11

1st differences: $4(0) + 6 = 6$ $4(1) + 6 = 10$ $4(2) + 6 = 14$ ⬅ $4n + 6$

2nd differences: 4 4

$\therefore 2a = 4$

$\therefore \mathbf{a = 2}$ ◄

3.1.2 $T_n = an^2 + bn + c$

$\therefore T_0 = c = -5$

$T_1 = a + b + c = 1$ . . . OR: First 1st diff:

$\therefore 2 + b - 5 = 1$ $3a + b = 10$

$\therefore b = 4$ $\therefore 3(2) + b = 10$

$\therefore b = 4$

$\therefore T_n = 2n^2 + 4n - 5$ ◄

3.2 ▶ The first factors of each term:

1 ; 5 ; 9 ; 13 ; . . . ; 81

is a linear sequence . . . OR: A.S.

$T_n = an + b$ $\therefore T_n = a + (n - 1)d$, etc.

where $a = 4$ and $b = T_0 = -3$

$\therefore$ General term: $T_n = 4n - 3$

The nth term, $T_n = 81$

$\therefore 4n - 3 = 81$

$\therefore 4n = 84$

$\therefore n = 21$

▶ The second factors of each term:

2 ; 6 ; 10 ; 14 ; . . .

Each term is just 1 more than the above sequence

$\therefore T_n = 4n - 2$ up to $n = 21$

$\therefore$ Sigma notation: $\sum_{n=1}^{21}(4n - 3)(4n - 2)$

This question could have been done entirely by inspection!

▶ FUNCTIONS AND GRAPHS [37]

4.1 $f(x) = \frac{2}{x + 1} - 3$

4.1.1 **y-int.**:

Substitute $x = 0$

then $y = \frac{2}{0 + 1} - 3 = -1$ . . . $y = f(0)$

$\therefore$ **(0; -1)** ◄

4.1.2 **x-int.**:

Substitute $y = 0$ . . . $f(x) = 0$

then $0 = \frac{2}{x + 1} - 3$

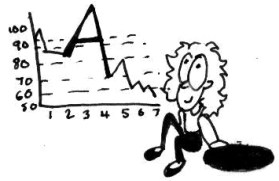
$\therefore 3 = \frac{2}{x + 1}$

$\therefore 3x + 3 = 2$

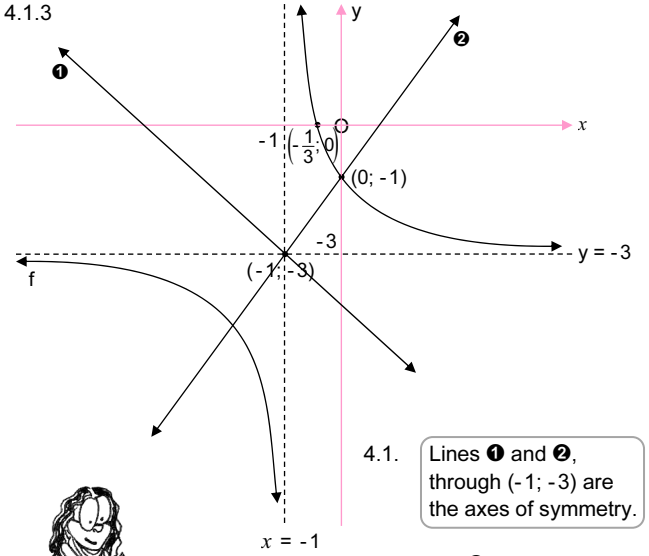
$\therefore 3x = -1$

$\therefore x = -\frac{1}{3}$

$\therefore \mathbf{\left(-\frac{1}{3}; 0\right)}$ ◄



4.1.3



4.1. Lines ❶ and ❷, through (-1; -3) are the axes of symmetry.

Line ❶ is a decreasing function.

Equation of line ❶:

$y = -(x + 1) - 3$

$\therefore$ **$y = -x - 4$** ◄

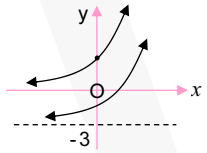
OR: Substitute (-1; -3) in $y = -x + c$

The equation of line ❷ is

$y = x + 1 - 3$

$\therefore y = x - 2$

4.2 $f(x) = a.b^x + q$



4.2.1 $q = -3$ . . . range: $y > -3$

$\therefore$ Equation: $y = a.b^x - 3$

Substitute (0; -2):

$\therefore -2 = a.b^0 - 3$

$\therefore 1 = a$

$\therefore$ Equation: $y = b^x - 3$

Substitute (1; -1):

$\therefore -1 = b^1 - 3$

$\therefore 2 = b$

$\therefore$ Equation: **$y = 2^x - 3$** ◄

Note:
There are 3 unknowns to be determined.
The order of the process is important: asymptote, y-intercept, then the other point.

4.2.2 $h(x) = 2^{x+1} + 1$ . . . $2^1.2^x = 2^{x+1}$

$\therefore$ **Shift f 1 unit left and 4 units up** ◄

OR: $h(x) = 2.2^x + 1$

$= 2(2^x - 3) + 7$ . . . To relate h to f, the whole f(x) must be dilated.

$\therefore$ **Dilate f by a factor of 2, then shift it 7 units up** ◄

5.1 $f(x) = -2x^2 - 5x + 3$

Max occurs when $x = -\frac{b}{2a} = -\frac{-5}{2(-2)} = -\frac{5}{4}$

OR: when $f'(x) = 0$, i.e. $-4x - 5 = 0$

$\therefore -4x = 5$

$\therefore x = -\frac{5}{4}$

$\therefore$ Maximum $= -2\left(-\frac{5}{4}\right)^2 - 5\left(-\frac{5}{4}\right) + 3 = \frac{49}{8}$

$\therefore$ Turning point: $\left(\mathbf{-\frac{5}{4}; \frac{49}{8}}\right)$ ◄

OR: $f(x) = -2\left(x^2 + \frac{5}{2}x - \frac{3}{2}\right)$

$= -2\left[x^2 - \frac{5}{2}x + \left(\frac{5}{4}\right)^2 - \frac{3}{2} - \frac{25}{16}\right]$

$= -2\left[\left(x - \frac{5}{4}\right)^2 - \frac{49}{16}\right]$

$= -2\left(x - \frac{5}{4}\right)^2 + \frac{49}{8}$

$\therefore$ Turning point $\left(\mathbf{-\frac{5}{4}; \frac{49}{8}}\right)$ ◄

5.2 At P, the gradient of f, $f'(x)$, equals the gradient of the tangent (g)

$\therefore f'(x) = \tan 135°$

$\therefore -4x - 5 = -1$

$\therefore -4x = 4$

$\therefore x = -1$

& $f(-1) = -2(-1)^2 - 5(-1) + 3$

$= -2 + 5 + 3$

$= 6$

$\therefore$ **P(-1; 6)** ◄

5.3 Equation of g: $y = ax + q$

a = the gradient of g = -1

$\therefore y = -x + q$

Substitute P(-1; 6):

$\therefore 6 = -(-1) + q$

$\therefore 5 = q$

$\therefore$ **Eqn. of g: $y = -x + 5$** ◄

OR: Substitute the gradient = -1 and the point (-1; 6) in $y - y_1 = m(x - x_1)$:

$\therefore y - 6 = -1(x + 1)$

$\therefore y - 6 = -x - 1$

$\therefore$ **$y = -x + 5$** ◄

5.4 **d > 5** ◄ . . . *y-intercept, d must be > 5*

6.1 Equation of g: $y = \sqrt{ax}$

(8; 3) on g ➡ $4 = \sqrt{a(8)}$ . . . If a point lies on a graph, the coordinates of the point satisfy the equation of the graph.

Square both sides: $\therefore 16 = 8a$

$\therefore$ **a = 2** ◄

6.2 **$g(x) = \sqrt{2x}$ is defined for $x \geq 0$** ◄

6.3 **$y \geq 0$** ◄

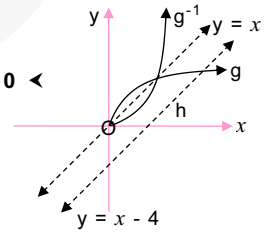
6.4 Equation of g: $y = \sqrt{2x}$

$\therefore$ Eqn. of g^{-1}: $x = \sqrt{2y}$

$\therefore x^2 = 2y$

$\therefore$ **$y = \frac{1}{2}x^2$; $x \geq 0$** ◄

See the sketches of g and g^{-1} below:



It is important to understand the reflections of the inverse functions, g and g^{-1}, in the line $y = x$.

6.5 $h(x) = g(x)$ ➡ $x - 4 = \sqrt{2x}$ . . . **Note:** $x - 4 \geq 0$ $\therefore x \geq 4$

$\therefore (x - 4)^2 = 2x$

$\therefore x^2 - 8x + 16 = 2x$

$\therefore x^2 - 10x + 16 = 0$

$\therefore (x - 2)(x - 8) = 0$

$\therefore x = 2$ or $x = 8$

BUT, for $x = 2$: LHS $= h(x) = -2$ and RHS $= g(x) = +2$

$\therefore$ Only $x = 8$ . . . See the sketch: The point (2; -4) cannot lie on g.

and $y = 8 - 4$ or $\sqrt{2(8)} = 4$

$\therefore$ **The point of intersection is (8; 4)** ◄

6.6 $0 \le x < 8$ ◄

Although you obtained the point of intersection algebraically, it is important to understand this entire Q6 graphically too.

► FINANCE, GROWTH AND DECAY [16]

7.1 12% of the selling price = R102 000

∴ 1% of the selling price = R102 000 ÷ 12

∴ 100% of the selling price = (R102 000 ÷ 12) × 100

= **R850 000** ◄

7.2 The balance of the selling price = R748 000 (= the loan)

Method 1: Present value

This is the quicker method!

$$\mathbf{P_v} = \frac{\mathbf{x}\left[\mathbf{1 - (1+i)^{-n}}\right]}{\mathbf{i}}$$ where $\mathbf{P_v}$ = R748 000; x?

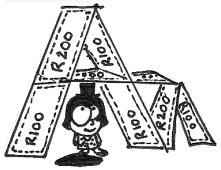
$\mathbf{i} = \frac{9\%}{12} = \frac{0{,}09}{12}$; $\mathbf{n} = 20 \times 12 = 240$

$$\therefore\ 748\,000 = \frac{x\left[1 - \left(1 + \frac{0{,}09}{12}\right)^{-240}\right]}{\frac{0{,}09}{12}} = x\,.\mathbf{A}$$

STOre 111,144954 in **A**

$$\therefore\ x = \frac{748\,000}{\mathbf{A}}$$

$$\simeq \mathbf{R6\,729{,}25}$$ ◄



Method 2: Future value

The Future value of the loan:

$$\therefore\ \mathbf{F_v = P_v(1+i)^n}$$ where $\mathbf{P_v}$ = R748 000; $\mathbf{n} = 20 \times 12 = 240$

$$= 748\,000\left(1 + \frac{0{,}09}{12}\right)^{240}$$ and $\mathbf{i} = \frac{9\%}{12} = \frac{0{,}09}{12}$

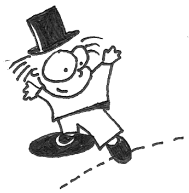
= R4 494 845,34 → **STO**re in **A**

and $$\mathbf{F_v} = \frac{\mathbf{x}\left[\mathbf{(1+i)^n - 1}\right]}{\mathbf{i}}$$

$$= \frac{x\left[\left(1 + \frac{0{,}09}{12}\right)^{240} - 1\right]}{\frac{0{,}09}{12}}$$

$= x\,.\mathbf{B}$

$$\therefore\ x = \frac{\mathbf{A}}{\mathbf{B}} \simeq \mathbf{R6\,729{,}05}$$ ◄



7.3 The amount of interest

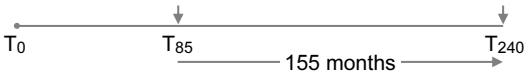
= The amount paid over 20 years - the original amount

= (240 × R6 729,95) - R748 000

= R1 615 188 - R748 000

= **R867 188** ◄

7.4



⬆ **The 'present'**

Method 1: Present value

After the 85th instalment,
the number of instalments remaining = 240 - 85 = 155
& the balance of the loan, then

$$= \frac{6\,729{,}95\left[1 - \left(1 + \frac{0{,}09}{12}\right)^{-155}\right]}{\frac{0{,}09}{12}}$$

= R615 509,74 ◄

Method 2: Future value

At this stage:
The value of the loan,

- The amount owed ➡ . . . $A = 748\,000\left(1 + \frac{0{,}09}{12}\right)^{85}$

  = 1 411 663,73 STOre in A

whereas:
The value of the annuity,

- The amount paid ➡ . . . $F_v = \dfrac{6\,729{,}95\left[\left(1 + \frac{0{,}09}{12}\right)^{85} - 1\right]}{\frac{0{,}09}{12}}$

  = R796 153,96 STOre in B

The balance of the loan = A - F_v = R615 509,77 ◄

= **A - F_v = R615 509,77** ◄ . . . *the remaining amount to be paid*

7.5 The amount owed after month 89

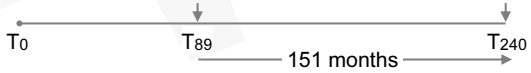
= The accrued amount for the months after month 85

$$= \text{R}615\,509{,}74\left(1 + \frac{0{,}09}{12}\right)^{4}$$

Note: No payments were made, so there was nothing to subtract.

= **R634 183,81** ◄ . . . OR: R634 183,84 if the amount from Method 2 in 7.4 was used.

7.6



The present value of the annuity following month 89 must equal the amount owed at that stage.

$$\therefore\ \frac{8\,500\left[1 - \left(1 + \frac{0{,}09}{12}\right)^{-n}\right]}{\frac{0{,}09}{12}} = 634\,183{,}81$$

$$\mathbf{P_v} = \frac{\mathbf{x}\left[\mathbf{1 - (1+i)^{-n}}\right]}{\mathbf{i}}$$ where x = 8 500

$\times \frac{0{,}09}{12}$ and $\div$ 8 500:

$$\therefore\ 1 - \left(1 + \frac{0{,}09}{12}\right)^{-n} = 0{,}55957\ldots$$

$$\therefore\ 0{,}44042605 = \left(1 + \frac{0{,}09}{12}\right)^{-n}$$

$$✱\ \therefore\ -n = \frac{\log 0{,}44042605}{\log\left(1 + \frac{0{,}09}{12}\right)}$$

= -109,744 . . .

∴ **n ≃ 110 months** ◄

$a = b^x$

➡ $x = \log_b a$

$\therefore\ x = \dfrac{\log a}{\log b}$

✱ OR:

$$\log 0{,}44042605 = \log\left(1 + \frac{0{,}09}{12}\right)^{-n}$$ . . . A = B ➡ log A = log B

$$\therefore\ \log 0{,}44042605 = -n\log\left(1 + \frac{0{,}09}{12}\right)$$ . . . $\log A^x = x \log A$

$$\therefore\ \frac{\log 0{,}44042605}{\log\left(1 + \frac{0{,}09}{12}\right)} = -n$$

etc.

▶ DIFFERENTIAL CALCULUS [32]

8.1 $f(x) = 3x^2 - 2$

$\therefore f(x+h) = 3(x+h)^2 - 2$

$= 3(x^2 + 2xh + h^2) - 2$

$= 3x^2 + 6xh + 3h^2 - 2$

$\therefore f(x+h) - f(x) = 6xh + 3h^2$

$\therefore \dfrac{f(x+h) - f(x)}{h} = 6x + 3h$

$\therefore \mathbf{f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}}$

$= \lim\limits_{h \to 0} (6x + 3h)$

$= \mathbf{6x}$ ◄

OR:

$\mathbf{f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}}$

$= \lim\limits_{h \to 0} \dfrac{3(x+h)^2 - 2 - (3x^2 - 2)}{h}$

$= \lim\limits_{h \to 0} \dfrac{3(x^2 + 2xh + h^2) - 2 - 3x^2 + 2)}{h}$

$= \lim\limits_{h \to 0} \dfrac{3x^2 + 6xh + 3h^2 - 3x^2}{h}$

$= \lim\limits_{h \to 0} \dfrac{6xh + 3h^2}{h}$

$= \lim\limits_{h \to 0} 6x + 3h$

$= \mathbf{6x}$ ◄

You must choose one or the other of these layouts.

Either you determine the components required for the definition of a derivative first and then apply the definition.

OR: Start with the definition, remembering to repeat $\lim\limits_{h \to 0}$ on every line until you find the limit in the last line.

The most important thing is to understand the definition.

8.2 $y = 2x^{-4} - \frac{1}{5}x$

$\therefore \dfrac{dy}{dx} = 2 \cdot -4x^{-5} - \frac{1}{5} \cdot x^0$ . . .

$= \mathbf{-8x^{-5} - \frac{1}{5}}$ ◄ . . . $x^0 = 1$

$\left[= -\dfrac{8}{x^{-5}} - \dfrac{1}{5}\right]$

If $y = kx^n$; then $\dfrac{dy}{dx} = k \cdot nx^{n-1}$

---

9. $f(x) = x^3 - 4x^2 - 11x + 30$

9.1 **$f(2) = 0$ ➡ $x - 2$ is a factor of $f(x)$** ◄

9.2 $\therefore f(x) = (x - 2)(x^2 \ldots\ldots x - 15)$ . . . $[-2x^2 - 2x^2 = -4x^2]$

$= (x - 2)(x^2 - 2x - 15)$ . . . [Check: $-15x + 4x = -11x$ ✓]

$= (x - 2)(x - 5)(x + 3)$

$f(x) = 0$ ➡ $x = -3$ or 2 or 5

$\therefore$ **Coordinates of x-intercepts: (-3; 0), (2; 0) & (5; 0)** ◄

9.3 At the stationary points: $f'(x) = 0$

$\therefore 3x^2 - 8x - 11 = 0$

$\therefore (3x - 11)(x + 1) = 0$

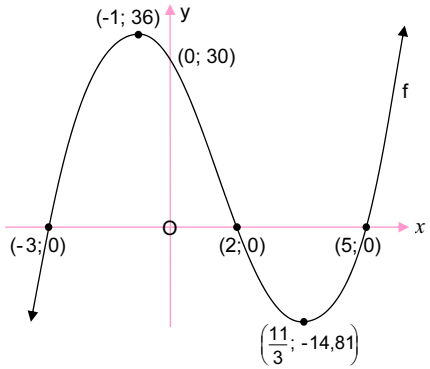
$\therefore x = \dfrac{11}{3}$ or -1

$f\left(\dfrac{11}{3}\right) = \left(\dfrac{11}{3}\right)^3 - 4\left(\dfrac{11}{3}\right)^2 - 11\left(\dfrac{11}{3}\right) + 30 \simeq -14{,}81$

& $f(-1) = (-1)^3 - 4(-1)^2 - 11(-1) + 30 = 36$

$\therefore$ Coordinates of stationary points: $(-1; 36)$ and $\left(\dfrac{11}{3}; -14{,}81\right)$

9.4



9.5 $\mathbf{-1 < x < \frac{11}{3}}$ ◄ . . . *for these values of x, the gradient of f is negative*

10.1 After t hours:

DC = 40t ; $\therefore$ BC = 100 - 40t ; BF = 30t

$FC^2 = BF^2 + BC^2$

$= (30t)^2 + (100 - 40t)^2$

$= 900t^2 + 10\,000 - 8\,000t + 1\,600t^2$

$= 2\,500t^2 - 8\,000t + 10\,000$

$\therefore$ **$FC = \sqrt{2\,500t^2 - 8\,000t + 10\,000}$** ◄

10.2 Min FC occurs when FC^2 is a minimum

$\therefore t = -\dfrac{b}{2a} = -\dfrac{-8\,000}{2(2\,500)}$

$= 1{,}6$ . . .

$\therefore$ **After 1 hr and 36 min** ◄

[OR: the derivative (of FC^2) = 0

$\therefore 5\,000t - 8\,000 = 0$, etc.]

10.3 Min FC $= \sqrt{2\,500(1{,}6)^2 - 8\,000(1{,}6) + 10\,000}$

$=$ **60 km** ◄

▶ PROBABILITY [16]

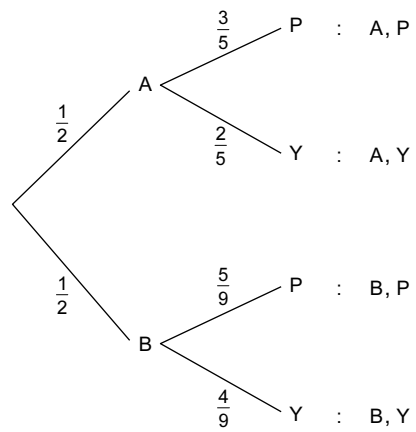
11.1 P(A or B) = P(A) + P(B) . . . *A & B are mutually exclusive*

$\therefore 0{,}57 = \frac{1}{2}P(B) + P(B)$ . . . $2P(A) = P(B)$ ➡ $P(A) = \frac{1}{2}P(B)$

$\therefore 1{,}5\,P(B) = 0{,}57$

$\therefore$ **P(B) = 0,38** ◄

11.2.1



11.2.2 $P(A, Y) = \frac{1}{2} \times \frac{2}{5} = \mathbf{\frac{1}{5}}$ ◄

11.2.3 $P(\text{Pink}) = P(A, P) + P(B, P)$

$= \frac{1}{2} \times \frac{3}{5} + \frac{1}{2} \times \frac{5}{9}$

$= \frac{3}{10} + \frac{5}{18}$

$= \mathbf{\frac{26}{45}}$ ◄

12.1 <u>5 choices</u> <u>4 choices</u> <u>3 choices</u> <u>2 choices</u> <u>1 choice</u>

∴ The number of different 5-letter arrangements

$= 5 \times 4 \times 3 \times 2 \times 1$

$= 5!$

$= \mathbf{120}$ ◄

12.2 <u>2 ways</u> <u>1 way</u> <u>3 ways</u> <u>2 ways</u> <u>1 way</u>
S or T

∴ The number of 5-letter arrangements starting ST _ _ _ or TS _ _ _

$= 2! \times 3!$

∴ The PROBABILITY of this

$= \frac{2! \times 3!}{120}$ . . . $P(E) = \frac{n(E)}{n(S)}$

$= \mathbf{\frac{1}{10}}$ ◄

CALCULUS

CALCULUS

FROM FIRST PRINCIPLES

If $f(x) = x^2$, determine

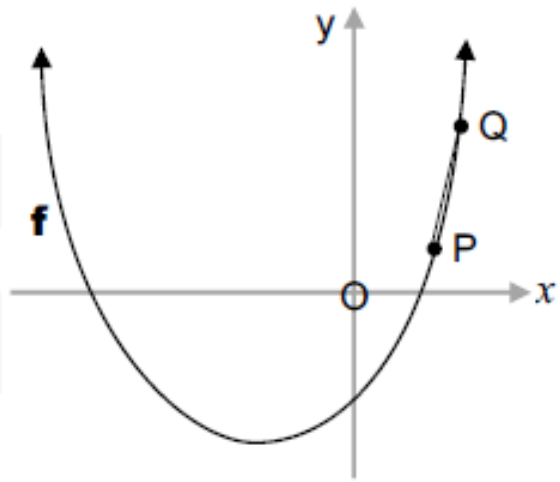
1. $f(x + h)$
2. $f(x + h) - f(x)$
3. $\dfrac{f(x + h) - f(x)}{h}$
4. $\lim\limits_{x \to 0} \dfrac{f(x + h) - f(x)}{h}$
5. $f'(x)$, the gradient of a tangent to the function
6. $f'(3)$, the gradient of the tangent to the function at $x = 3$
7. the gradient of the tangent to f when $x = 3$ (7)

Gr 12 Maths 2 in 1
p. 29 (Q1)

The diagram alongside shows the graph of y = f(x).

P(x; f(x)) & Q(x + h; f(x + h)) are points on the graph.

The gradient of the straight line through P and Q is given by $m = \dfrac{f(x + h) - f(x)}{(x + h) - x}$.



1. What line has a gradient given by $\lim\limits_{h \to 0} \dfrac{f(x + h) - f(x)}{h}$? (2)

2. Calculate the gradient of PQ in terms of h and x if $f(x) = x^2 + 2x$. (4)

3. Hence, determine the value of $\lim\limits_{h \to 0} \dfrac{f(x + h) - f(x)}{h}$. (2)

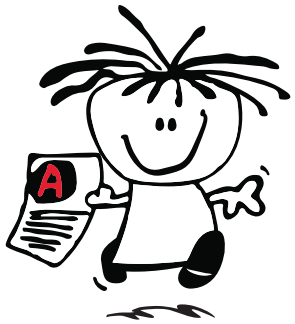
4. Hence, find the value of

   4.1 $f'(x)$ 4.2 $f'(2)$ 4.3 $\lim\limits_{h \to 0} \dfrac{f(2 + h) - f(2)}{h}$ (3)

Gr 12 Maths 2 in 1
p. 29 (Q2.6)

CALCULUS CONTENT FRAMEWORK

- Concepts:
  - limit
  - average gradient
  - gradient of a tangent
  - limit of average gradient
  - derivative
- Rules of differentiation
- Tangents: Gradient & Equation
- Cubic graphs (Remainder & Factor Theorems)
- f, f′, f″ & Concavity
- Optimisation / Maximum & Minimum



WHAT IS CALCULUS ABOUT?

In Grade 12 it is about the **DERIVATIVE**

What is a derivative?

Practically …

It is the **gradient** of a curve (at a point)

Theoretically …

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

How do you calculate it?

Practically …

- from rules

OR

Theoretically …

- From 1st principles (using the mathematical definition)

Where will you use it?

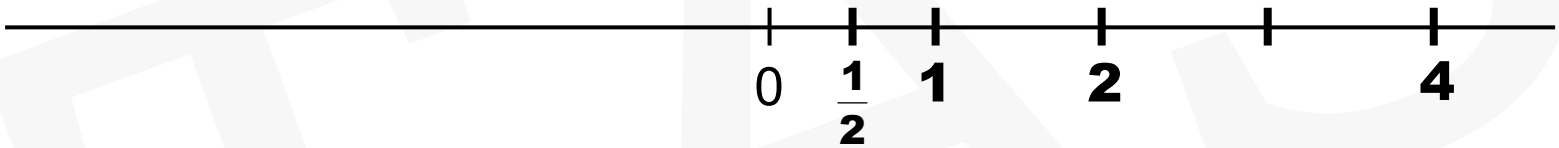
Practically & Theoretically …

To determine:

- the gradient/equation of tangents to curves
- the maximum & minimum turning points of graphs
- the maximum & minimum values in practical applications

The LIMIT concept

$$4 \;;\; 2 \;;\; 1 \;;\; \frac{1}{2} \;;\; \frac{1}{4} \;;\; \ldots \quad T_n$$



Zero is the **target/limit** of the values of the terms in the sequence as the number of terms 'tends to ∞'

We write: $\lim_{n \to \infty} T_n = 0$

The LIMIT concept, cont.

$$4 \;;\; -2 \;;\; 1 \;;\; -\frac{1}{2} \;;\; -\frac{1}{4} \;;\; \ldots \quad T_n$$

We write: $\lim\limits_{n \to \infty} T_n = 0$

GRADIENT

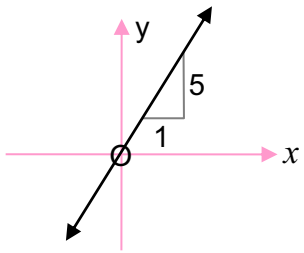
The DERIVATIVE **is** the GRADIENT . . . .

Just focus on this fact for now.

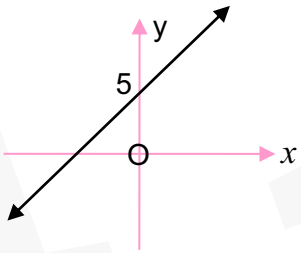
Straight line graphs

$y = 5x$ $\qquad$ $y = x + 5$ $\qquad$ $y = -5x$ $\qquad$ $y = 5$

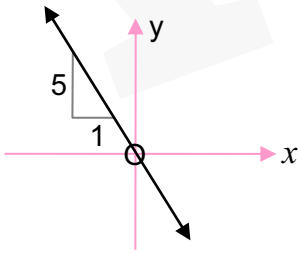
What are the **gradients** of these graphs?



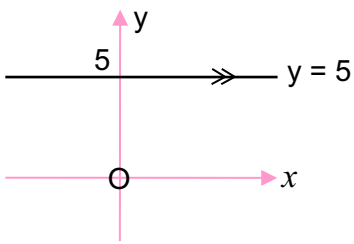
m = 5



m = 1



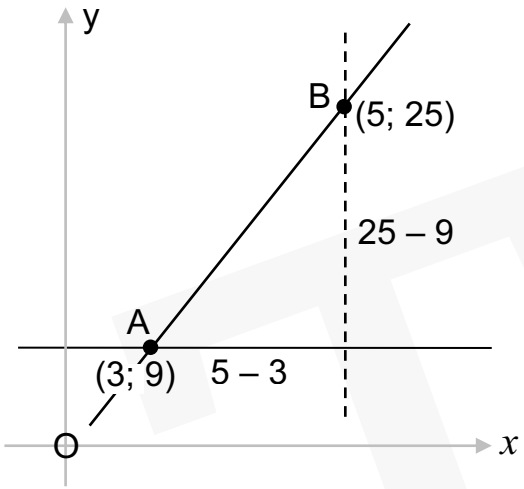
m = –5



m = 0

So, what are the **derivatives** of these graphs? . . . . . *the answers would be the same !*

Remember, the derivative is the gradient!

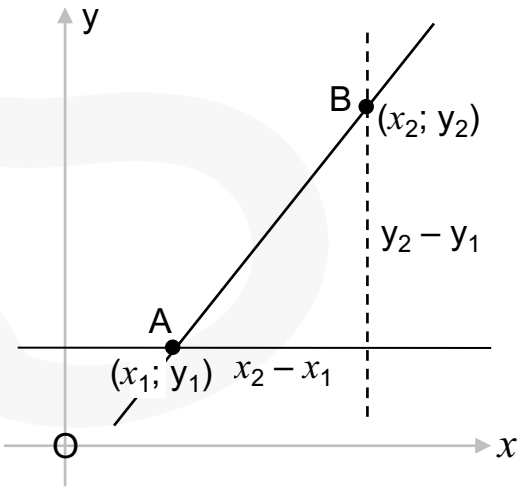


$$\boxed{m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}}$$

$$= \frac{25 - 9}{5 - 3}$$

$$= \frac{16}{2}$$

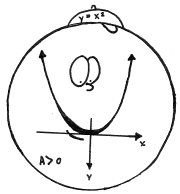
$$= 8$$

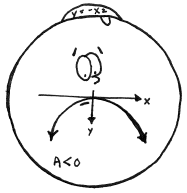


The **gradient** of a straight line graph is **constant**,

i.e. it is the same at each point, no matter where or how you read it.

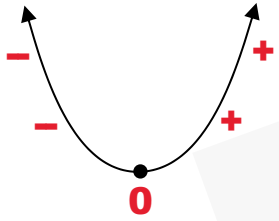
Parabolas and Cubic graphs

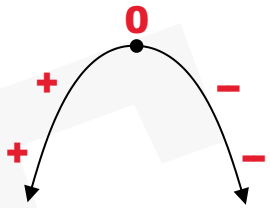


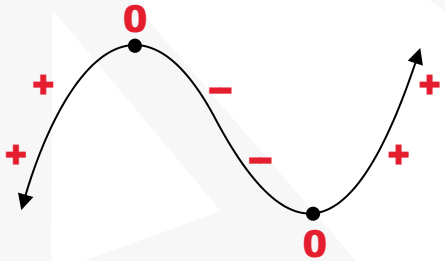


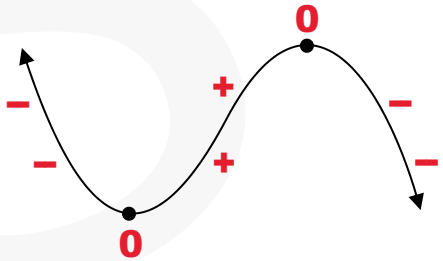
The **gradient** of a curve **changes** at each point . . .

Either Increasing **(+ve)** or Decreasing **(–ve)** or Stationary **(0)**

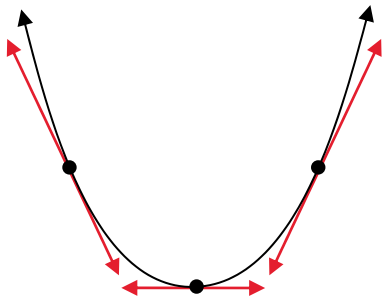




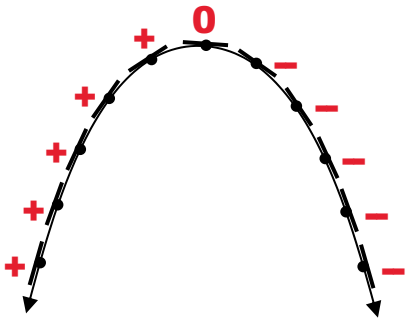




Consider **tangents** drawn at each point:



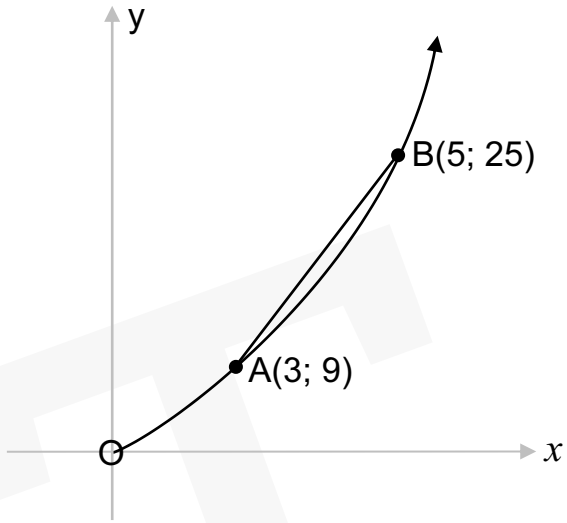
- Note the **gradient** of successive tangents at points on the curve . . .
- Note the **signs** of the gradients.

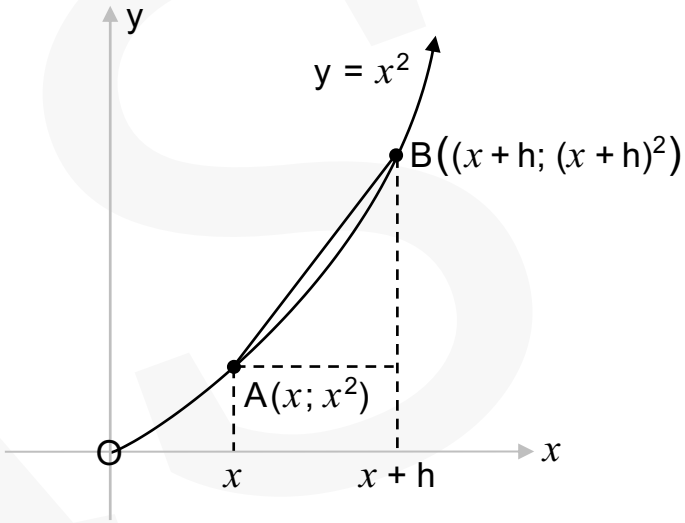


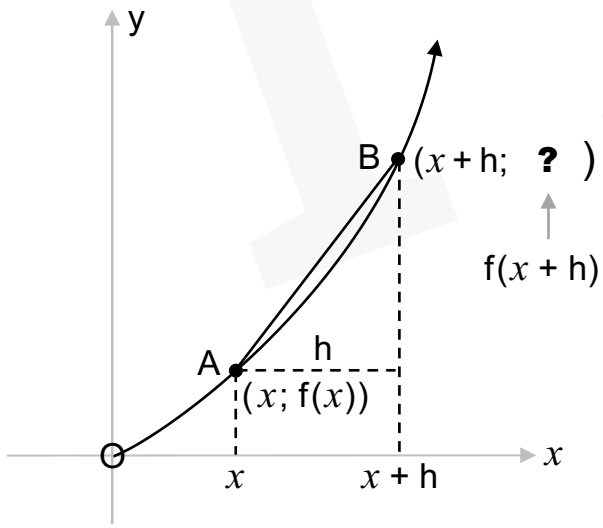
Average Gradient

Consider graph $f(x) = x^2$ (or $y = x^2$):

Graph f:







The **Average Gradient of f**

from A to B

$= m_{AB} \quad \ldots \left(= \dfrac{25 - 9}{5 - 3} = 8 \right)$

$= \dfrac{\mathbf{f(x + h) - f(x)}}{\mathbf{h}}$

Now consider:

The **Average Gradient** between two points, A and B, on curve f.

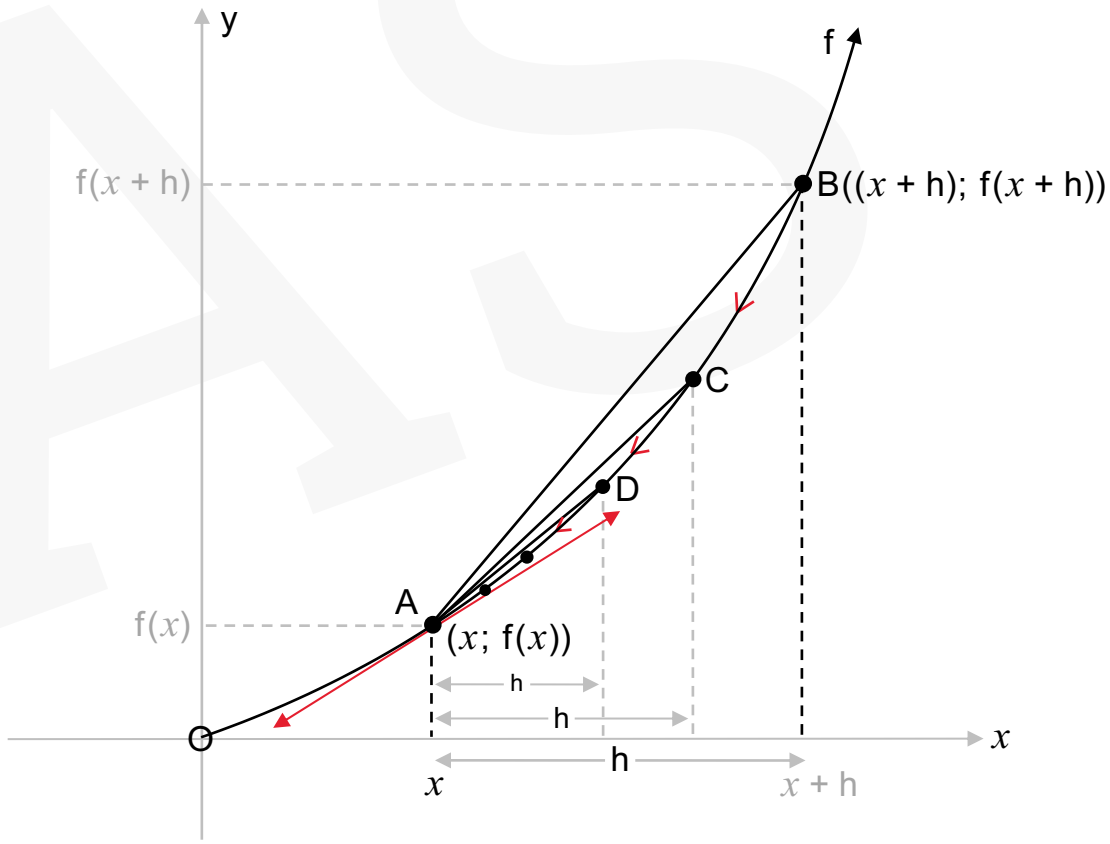
& then, successively, the average gradient of curves AC, AD, ...

As the 2nd point moves closer to point A,
notice that h is getting smaller and smaller . . .

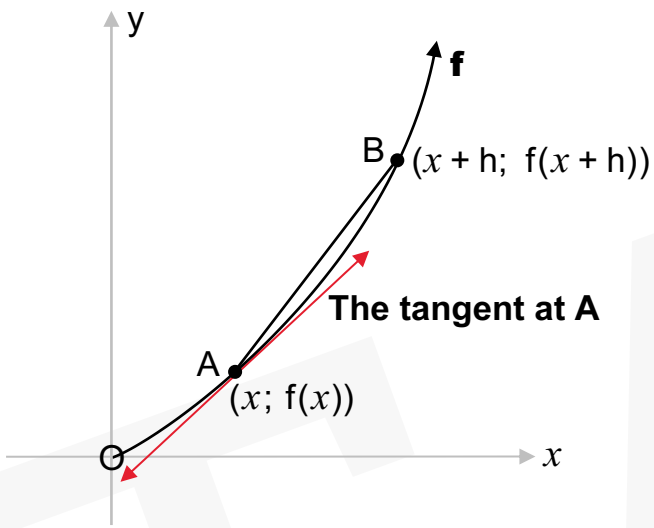
And that as these lines *approach* the position of the TANGENT to the curve at POINT A,

– the 'target' position! –

so, their gradient approaches the gradient of the tangent at A.



The Gradient of the Tangent at a point on the curve



The gradient of curve **f**
at point A

= the 'target value'/limit
of m_{AB} (as B → A)
i.e. as h → 0

$$= \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

This is the DERIVATIVE

$\therefore$ The **Average Gradient** between 2 points $= \dfrac{\mathbf{f(x + h) - f(x)}}{\mathbf{h}}$

BUT the **DERIVATIVE** is the **GRADIENT OF THE TANGENT AT A POINT** on the curve:

$$\mathbf{f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}}$$

FIRST PRINCIPLES

Question 1

Determine $f'(x)$ from first principles if $f(x) = 3x^2$.

Solution

$$
\begin{aligned}
f(x) &= 3x^2 \\
\therefore\ f(x+h) &= 3(x+h)^2 \\
&= 3(x^2 + 2xh + h^2) \\
&= 3x^2 + 6xh + 3h^2 \\
\therefore\ f(x+h) - f(x) &= 6xh + 3h^2 \\
\therefore\ \frac{f(x+h) - f(x)}{h} &= 6x + 3h
\end{aligned}
$$

$$
\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}} = \lim_{h \to 0} (6x + 3h)
$$

$$
= \mathbf{6x}
$$

the definition of $f'(x)$

Just observe these examples . . .

Remember this 'fun' question?

Use the rules of differentiation to determine f'(x):

(1) $f(x) = 3x^5 + \frac{1}{2}x^2$

(2) $f(x) = (x^3 - 1)^2$

(3) $f(x) = \frac{x^3 - 5x^2 + 6x}{x - 5}$

Calculate $\frac{dy}{dx}$ if:

(4) $xy = 5$

(5) $y = \frac{x^2 - 25}{x + 5}$

(6) $y = \frac{1}{2x^3} + \sqrt{x}$

How would your learners respond?

Answers

Determine $f'(x)$:

(1) $f(x) = 3x^5 + \frac{1}{2}x^2$

$f'(x) = 3.5x^4 + \frac{1}{2}.2x$

$= 15x^4 + x$ ◄

(2) $f(x) = (x^3 - 1)^2$

$= x^6 - 2x^3 + 1x^0$

$\therefore f'(x) = 6x^5 - 6x^2$ ◄

(3) $f(x) = \frac{x^3 - 5x^2 + 6x}{x - 2}$

$= \frac{x(x^2 - 5x + 6)}{x - 2}$

$= \frac{x\cancel{(x - 2)}(x - 3)}{\cancel{x - 2}}$

$= x^2 - 3x \quad \ldots \; x \neq 2$

$\therefore f'(x) = 2x - 3$ ◄

Calculate $\frac{dy}{dx}$:

Here you have to express y in terms of x.

(4) $xy = 5$

$\therefore y = \frac{5}{x}$

$\therefore y = 5x^{-1}$

$\therefore \frac{dy}{dx} = 5(-x^{-2})$

$= -\frac{5}{x^2}$ ◄

(5) $y = \frac{x^2 - 25}{x + 5}$

$\therefore y = \frac{\cancel{(x + 5)}(x - 5)}{\cancel{(x + 5)}}$

$\therefore y = x - 5 \quad \ldots \; x \neq -5$

$\therefore \frac{dy}{dx} = 1$ ◄

(6) $y = \frac{1}{2x^3} + \sqrt{x}$

$\therefore y = \frac{1}{2}x^{-3} + x^{\frac{1}{2}}$

$\therefore \frac{dy}{dx} = \frac{1}{2}(-3x^{-4}) + \frac{1}{2}x^{-\frac{1}{2}}$

$= \frac{1}{2}\left(-\frac{3}{x^4}\right) + \frac{1}{2} \cdot \frac{1}{x^{\frac{1}{2}}}$

$= -\frac{3}{2x^4} + \frac{1}{2\sqrt{x}}$ ◄

The Rules of Differentiation

The Rules

1 The Power Rule:

$$y = x^n \quad \Rightarrow \quad \frac{dy}{dx} \quad \text{or} \quad \frac{d}{dx}(x^n) = ?$$

2 The Constant Rule:

$$f(x) = kx^n \quad \Rightarrow \quad f'(x) = ?$$

3 The Sum & Difference Rule:

$$D_x[f(x) \pm g(x)] = ?$$

Get to know all the notations!

1 The POWER rule

$f(x) = x^2$

$$\therefore\ f(x+h) = (x+h)^2$$
$$= x^2 + 2xh + h^2$$

$$\therefore\ f(x+h) - f(x) = \cancel{x^2} + 2xh + h^2 - \cancel{x^2}$$
$$= 2xh + h^2$$

$$\therefore\ \frac{\mathbf{f(x+h) - f(x)}}{\mathbf{h}} = \frac{2xh + h^2}{h}$$

The AVERAGE GRADIENT !

$$= \mathbf{2x + h}$$

$$\boxed{\mathbf{f'(x)} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

*The definition of a **derivative** !*

$$= \lim_{h \to 0} \mathbf{(2x + h)}$$

$$= \mathbf{2x}$$

$f(x) = x^3$

$\mathbf{f'(x) = 3x^2}$ **?**

$f(x) = x^4$

$$\mathbf{f(x) = x^n \longrightarrow f'(x) = nx^{n-1}}$$

2 The CONSTANT rule

$$f(x) = 5x^2$$

$$\therefore\ f(x+h) = 5(x+h)^2$$
$$= 5x^2 + 2xh + h^2$$

$$\therefore\ f(x+h) - f(x) = \cancel{5x^2} + 10xh + 5h^2 - \cancel{5x^2}$$
$$= 10xh + 5h^2$$

$$\therefore\ \frac{f(x+h) - f(x)}{h} = \frac{10xh + 5h^2}{h}$$
$$= \mathbf{10x + 5h}$$

The AVERAGE GRADIENT !

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

*The definition of a **derivative** !*

$$= \lim_{h \to 0} (10x + 5h)$$
$$= \mathbf{10x}$$

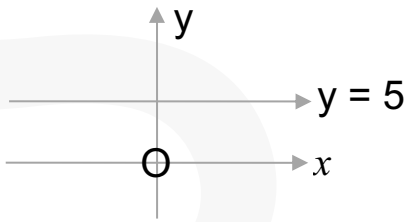
$$f(x) = 5x^3$$

$$f'(x) = ?$$

$$f(x) = 5 \quad ?$$

Consider:

- The graph y = 5



The gradient?
The derivative?

- The rule

$f(x) = 5 \Rightarrow f(x) = 5x^0$

$$f'(x) = ?$$

A SPECIAL CASE

$$f(x) = kx^n \longrightarrow f'(x) = k.nx^{n-1}$$

Special case:

$$f(x) = k \longrightarrow f'(x) = 0$$

Differentiating more than one term ...

Differentiate $f(x) = x^2 + x^3$ from first principles:

$$\therefore\ f(x) = x^2 + x^3$$

$$\therefore\ f(x+h) = (x+h)^2 + (x+h)^3$$

$$= (x^2 + 2xh + h^2) + (x^3 + 3x^2h + 3xh^2 + h^3)$$

$$\therefore\ f(x+h) - f(x) = 2xh + h^2 + 3x^2h + 3xh^2 + h^3$$

$$\therefore\ \frac{f(x+h) - f(x)}{h} = \frac{2xh + h^2 + 3x^2h + 3xh^2 + h^3}{h}$$

$$= \mathbf{2x + h + 3x^2 + 3xh + h^2}$$

The AVERAGE GRADIENT

The definition of **the DERIVATIVE**

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

$$= \lim_{h \to 0} (2x + h + 3x^2 + 3xh + h^2)$$

$$= \mathbf{2x + 3x^2}$$

So: The derivative of the **sum** of terms

= The **sum** of the derivatives of the terms

3 The SUM and DIFFERENCE rule

Similarly: $f(x) = x^2 - x^3 \quad \Rightarrow \quad f'(x) = 2x - 3x^2$

$$\mathbf{D_x[f(x) \pm g(x)] = D_x[f(x)] \pm D_x[g(x)]}$$

i.e. The derivative of the **sum** of terms = The **sum** of the derivatives of the terms.

& the derivative of the **difference** of terms = The **difference** of the derivatives of the terms.

But

There are NO rules for the derivative of
- PRODUCTS or
- QUOTIENTS

So, always convert expressions to separate TERMS before differentiating . . .

Finding the derivative using the rules of differentiation

1 The Power Rule:

$$y = x^n \Rightarrow \frac{dy}{dx} = nx^{n-1} \quad \text{or} \quad \frac{d}{dx}(x^n) = nx^{n-1}$$

2 The Constant Rule:

$$f(x) = kx^n \Rightarrow f'(x) = k.nx^{n-1}$$

Special case: $f(x) = k \longrightarrow f'(x) = 0$

3 The Sum & Difference Rule:

$$D_x[f(x) \pm g(x)] = D_x[f(x)] \pm D_x[g(x)]$$

Get to know all the notations !

PROOFS OF THE RULES OF DIFFERENTIATION

BEWYSE VAN DIE DIFFERENSIASIEREËLS

NB Definition of a derivative / Definisie van 'n afgeleide: $D_x[f(x)] = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

I $$\boxed{D_x[f(x)+g(x)] = D_x[f(x)] + D_x[g(x)]}$$

i.e. The derivative of the sum equals the sum of the derivatives!

d.w.s. Die afgeleide van die som is gelyk aan die som van die afgeleides.

In the proof we'll use:
- the definition of a derivative (both ways!), &
- the fact that the LIMIT of the sum equals the sum of the LIMITS.

In die beweys gebruik ons:
- die definisie van 'n afgeleide (albei rigtings), &
- die feit dat die LIMIET van die som wel gelyk is aan die som van die LIMIETE!

Proof / Bewys: $D_x[f(x)+g(x)]$

$$= \lim_{h \to 0} \frac{[f(x+h)+g(x+h)] - [f(x)+g(x)]}{h}$$

$$= \lim_{h \to 0} \frac{[f(x+h)-f(x)] + [g(x+h)-g(x)]}{h}$$

$$= \lim_{h \to 0} \left[\frac{f(x+h)-f(x)}{h} + \frac{g(x+h)-g(x)}{h}\right]$$

$$= \lim_{h \to 0} \frac{f(x+h)-f(x)}{h} + \lim_{h \to 0} \frac{g(x+h)-g(x)}{h}$$

$$= f'(x) + g'(x)$$

eg. / bv. $D_x[x^3 + 4x] = D_x[x^3] + D_x[4x] = 3x^2 + 4$

II $$\boxed{D_x[f(x)-g(x)] = D_x[f(x)] - D_x[g(x)]}$$

Proof / Bewys: (similar to I above / Netsoos I hierbo.)

III $$\boxed{D_x[k\,f(x)] = k.\,D_x[f(x)]}$$

Proof / Bewys: $D_x[k\,f(x)]$

$$= \lim_{h \to 0} \frac{k\,f(x+h) - k\,f(x)}{h}$$

$$= \lim_{h \to 0} \frac{k[f(x+h)-f(x)]}{h}$$

$$= k.\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

$$= k.\,D_x[f(x)]$$

[or / of $k.\,f'(x)$]

(Also similar to I!
Ook soortgelyk aan I!)

Examples

Determine $f'(x)$ for the following:

(1) $f(x) = 3x^5 + \frac{1}{2}x^2$

(2) $f(x) = (x^3 - 1)^2$

(3) $f(x) = \frac{x^3 - 5x^2 + 6x}{x - 2}$

Calculate $\frac{dy}{dx}$ if:

(4) $xy = 5$

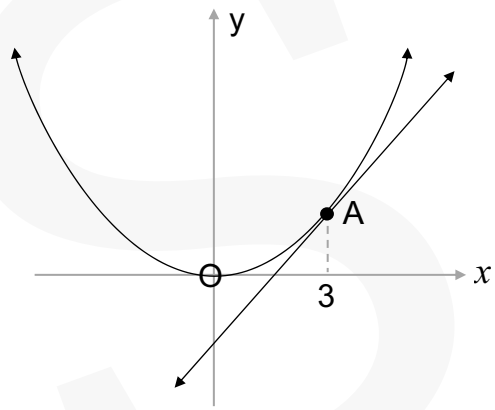
(5) $y = \frac{x^2 - 25}{x + 5}$

(6) $y = \frac{1}{2x^3} + \sqrt{x}$

TANGENTS

Finding the gradient/equation

Scaffolded Questions . . .



1. Alongside is the graph y = x^2 and the tangent to this graph at $x = 3$.

   1.1 Write down the coordinates of A. (2)

   1.2 Find

   (a) the **gradient** of the tangent at A. (2)

   (b) the **equation** of the tangent at A. (2)

   1.3 Find the coordinates of the point on this graph where the gradient of the tangent is:

   (a) –6 (b) 10 (2)(2)

2. Given: $f(x) = 2x^2 - 6x$. Calculate:

   2.1 the average gradient between the points with $x = 2$ and $x = 5$. (4)

   2.2 the gradient of the tangent to the curve where $x = 3$. (3)

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Derivative and a Tangent

Consider the graph of $g(x) = -2x^2 - 9x + 5$.

1. Determine the equation of the tangent to the graph of g at $x = -1$. (4)

2. For which values of q will the line $y = -5x + q$ not intersect the parabola? (3)

Gr 12 Maths 2 in 1
p. 150 (Q9.2)

QUESTION

Consider the graph of $g(x) = -2x^2 - 9x + 5$.

1. Determine the equation of the tangent to the graph of g at $x = -1$. (4)

2. For which values of q will the line $y = -5x + q$ not intersect the parabola? (3)

MEMO

1. $g(x) = -2x^2 - 9x + 5$

The gradient of the tangent to g at any x: $g'(x) = -4x - 9$

$\therefore$ The gradient of the tangent to g at $x = -1$:

$g'(-1) = -4(-1) - 9$

$= -5 \quad \ldots = m$

& $g(-1) = -2(-1)^2 - 9(-1) + 5$

$= -2 + 9 + 5$

$= 12$

$\therefore$ The point of contact is $(-1; 12)$

Substitute $m = -5$ and $(-1; 12)$ in:

$\mathbf{y - y_1 = m(x - x_1)}$ . . .

$\therefore y - 12 = -5(x + 1)$

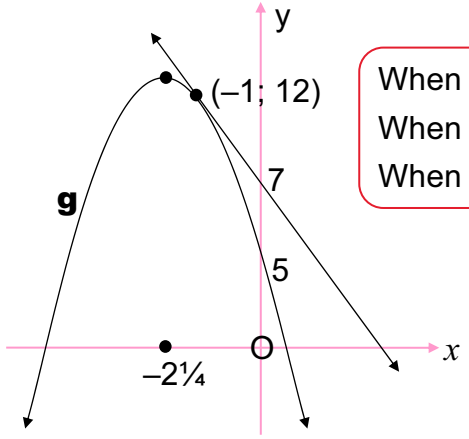
$\therefore \mathbf{y = -5x + 7}$ ◄

OR: in $\mathbf{y = mx + c}$

$\therefore 12 = (-5)(-1) + c$

$\therefore 7 = c$, etc.

2. **q > 7** ◄



When q = 7, the line *touches* g
When q > 7, the line won't intersect g
When q < 7, the line will cut g twice

Gradient of a Tangent

Given: $h(x) = 4x^3 + 5x$

Explain if it is possible to draw a tangent to the graph of h that has a negative gradient.

Show ALL your calculations. (3)

Gr 12 Maths 2 in 1
p. 150 (Q9.3)

QUESTION

Given: $h(x) = 4x^3 + 5x$

Explain if it is possible to draw a tangent to the graph of h that has a negative gradient.

Show ALL your calculations. (3)

MEMO

The gradient of the tangent to h at any x:

$h'(x) = 12x^2 + 5$

x^2 is ≥ 0 for all $x \in \mathbb{R}$ . . . *a square*

$\therefore$ $12x^2$ is ≥ 0 for all $x \in \mathbb{R}$

$\therefore$ $12x^2 + 5 > 0$ for all $x \in \mathbb{R}$

In fact, the gradient is ≥ 5

i.e. The gradient of the tangent is always positive.

$\therefore$ It is impossible to draw a tangent to h which has a negative gradient.

OR: A negative gradient would require

$12x^2 + 5 < 0$

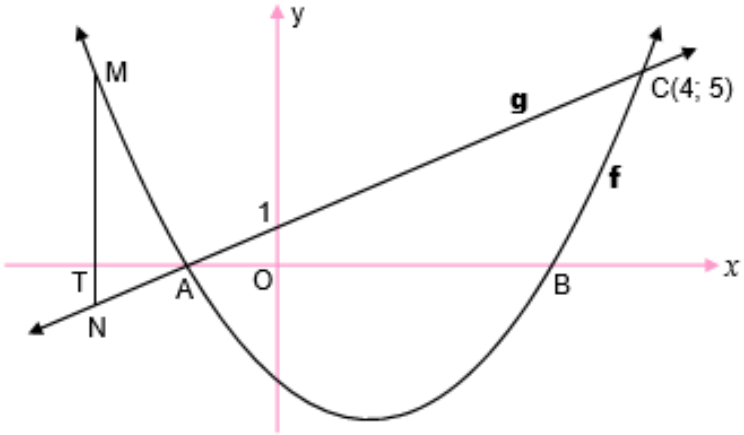
$\therefore$ $12x^2 < -5$

$\therefore$ $x^2 < -\frac{5}{2}$

which is impossible! . . . *a square is always ≥ 0*

GRAPHS & FUNCTIONS (& CALCULUS)

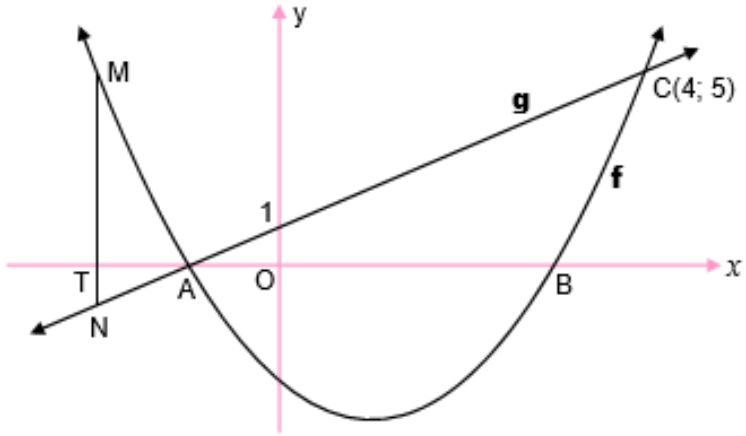
In the diagram alongside, A and B are the x-intercepts of the graph of $f(x) = x^2 - 2x - 3$. A straight line, g, through A cuts f at C(4; 5) and the y-axis at (0; 1). M is a point on f and N is a point on g such that MN is parallel to the y-axis. MN cuts the x-axis at T.

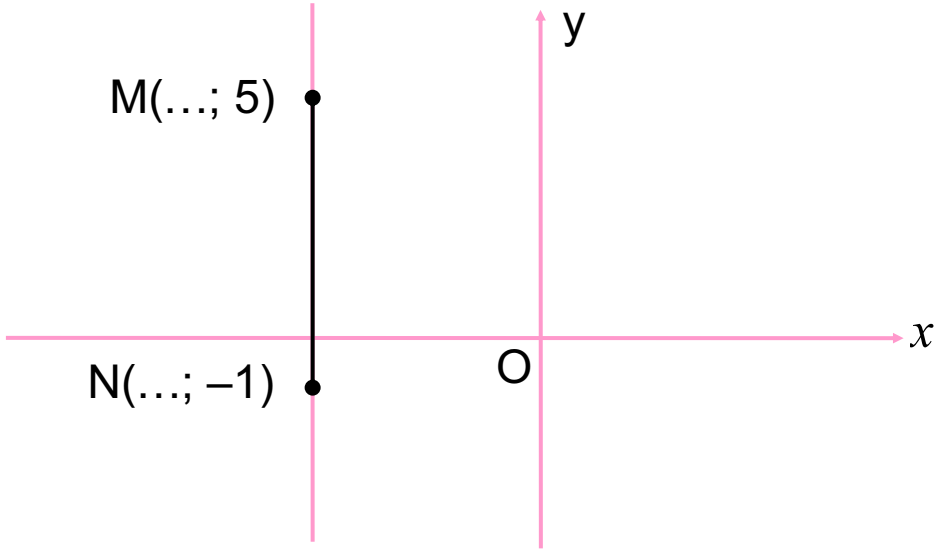


1. Show that $g(x) = x + 1$. (2)

2. Calculate the coordinates of A and B. (3)

3. Determine the range of f. (3)





MN = $y_M - y_N$

4. If MN = 6:

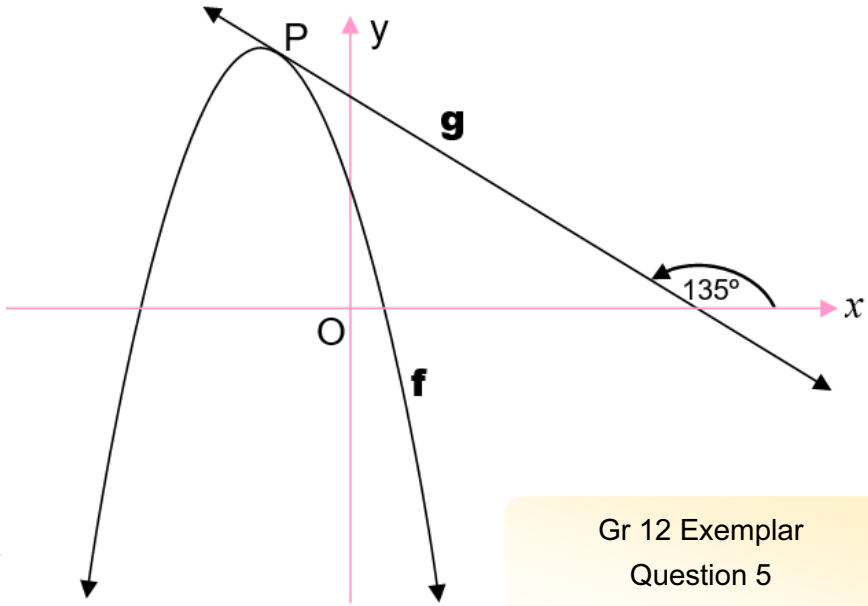
4.1 Determine the length of OT if T lies on the negative x-axis. Show ALL your working. (4)

4.2 Hence, write down the coordinates of N. (2)

5. Determine the equation of the tangent to f drawn parallel to g. (5)

6. For which value(s) of k will $f(x) = x^2 - 2x - 3$ and $h(x) = x + k$ NOT intersect? (1) **[20]**

The sketch alongside shows the graphs of $f(x) = -2x^2 - 5x + 3$ and $g(x) = ax + q$.
The angle of inclination of graph g is 135º in the direction of the positive x-axis.
P is the point of intersection of f and g such that g is a tangent to the graph of f at P.



Gr 12 Exemplar
Question 5

1. Calculate the coordinates of the turning point of the graph of f. (3)

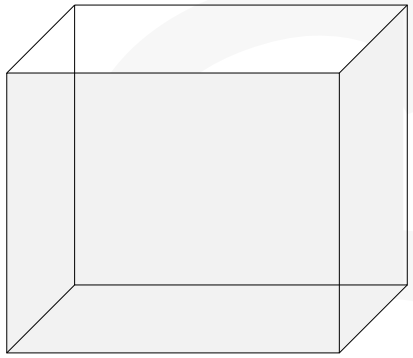
2. Calculate the coordinates of P, the point of contact between f and g. (4)

3. Hence or otherwise, determine the equation of g. (2)

4. Determine the values of d for which the line $k(x) = -x + d$ will not intersect the graph of f. (1) [10]

OPTIMISATION IN CALCULUS . . .

The **volume** of a certain rectangular box, which is open at the top, is given by the equation $f(x) = x^3 - 8x^2 + 5x + 50$.



1. If the **height** of the box is $(5 - x)$ units, determine an algebraic expression for the **area of the base** of the box. (3)

2. Calculate the value of x for which the **volume** is a maximum. (6) [9]

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Challenging Questions Booklet
p. 13 (Q10)

SOLUTIONS

1. Volume = area of base $\times$ height

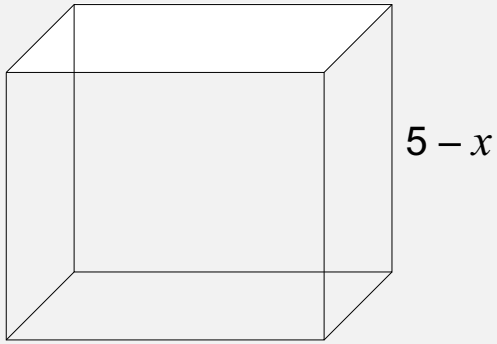
$\therefore\ x^3 - 8x^2 + 5x + 50$ = area of base $\times\ (5 - x)$

$= (?\quad ?\quad ?)(5 - x)$

$= (10\ ?\ - x^2)(5 - x)$ . . . *by inspection*

$= (10 + 3x - x^2)(5 - x)$ . . . $-5x^2 - \mathbf{3x^2}$

$= -8x^2$



$\therefore$ Area of the base = **$(10 + 3x - x^2)$ square units** ◄

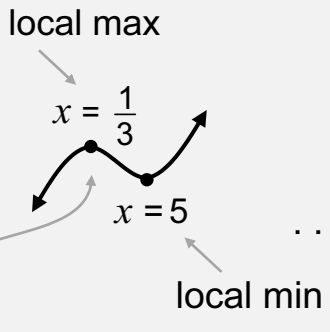
2. Volume is a max when $\dfrac{dV}{dx} = 0$

$\therefore\ 3x^2 - 16x + 5 = 0$

$\therefore\ (3x - 1)(x - 5) = 0$

$\therefore\ x = \dfrac{1}{3}$ or 5

But, the shape of the curve of f is



. . . $\boxed{\because\ a > 0}$

$\therefore$ **Maximum volume occurs at $x = \dfrac{1}{3}$** ◄

Note: Minimum & maximum values occur when the derivative equals 0.

ADVICE

The derivative of a function is zero at the turning point(s).

Whenever you have to find when (for which x or t or h, etc.) an expression has a **maximum** or a **minimum** value:

1. Determine the algebraic expression (in terms of x, or t, or h, etc.)
2. Find the **derivative**
3. **Put the derivative equal to 0**
4. Solve

Calculus of motion example

A particle moves along a straight line. The distance, s, (in metres) of the particle from a fixed point on the line at time t seconds ($t \geq 0$) is given by $s(t) = 2t^2 - 18t + 45$.

1. Calculate the particle's initial velocity. (Velocity is the rate of change of distance with respect to time.) (3)

2. Determine the rate at which the velocity of the particle is changing at t seconds. (1)

3. After how many seconds will the particle be closest to the fixed point? (2) [6]

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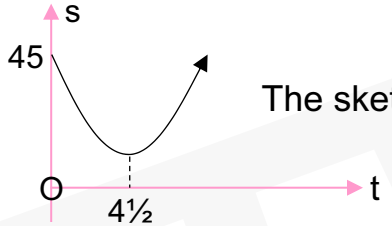
p. 150 (Q10)

QUESTION

A particle moves along a straight line. The distance, s, (in metres) of the particle from a fixed point on the line at time t seconds ($t \geq 0$) is given by $s(t) = 2t^2 - 18t + 45$.

1. Calculate the particle's initial velocity. (Velocity is the rate of change of distance with respect to time.) (3)

MEMO



The sketch will be very useful!

1. The velocity is the speed in a particular direction.

$s(t) = 2t^2 - 18t + 45$ . . . s(t) is the distance of the particle from a fixed point.

The **velocity** at time t $= s'(t) = 4t - 18$

'Initial' means: 'at the start', i.e. $t = 0$

$\therefore$ The **initial** velocity $= 4(0) - 18 = -18$ m/s

$\therefore$ **18 m/s towards the fixed point** ◄

QUESTION

2. Determine the rate at which the velocity of the particle is changing at t seconds. (1)

MEMO

2. Velocity is measured in m per sec — rate of change of distance w.r.t. time

Acceleration is measured in m per sec per sec — rate of change of velocity w.r.t.

$\therefore$ The 'rate of change of the velocity' $= s''(t)$
$= \mathbf{4\ m/s^2}$ ◄

QUESTION

3. After how many seconds will the particle be closest to the fixed point? (2) [6]

MEMO

3. The particle will be closest when the **distance**, s, of the particle is a **minimum**

$\therefore$ when the derivative is zero — **OR:** when $t = -\frac{b}{2a}$

$\therefore 4t - 18 = 0$ . . . $s'(t) = 4t - 18$ above

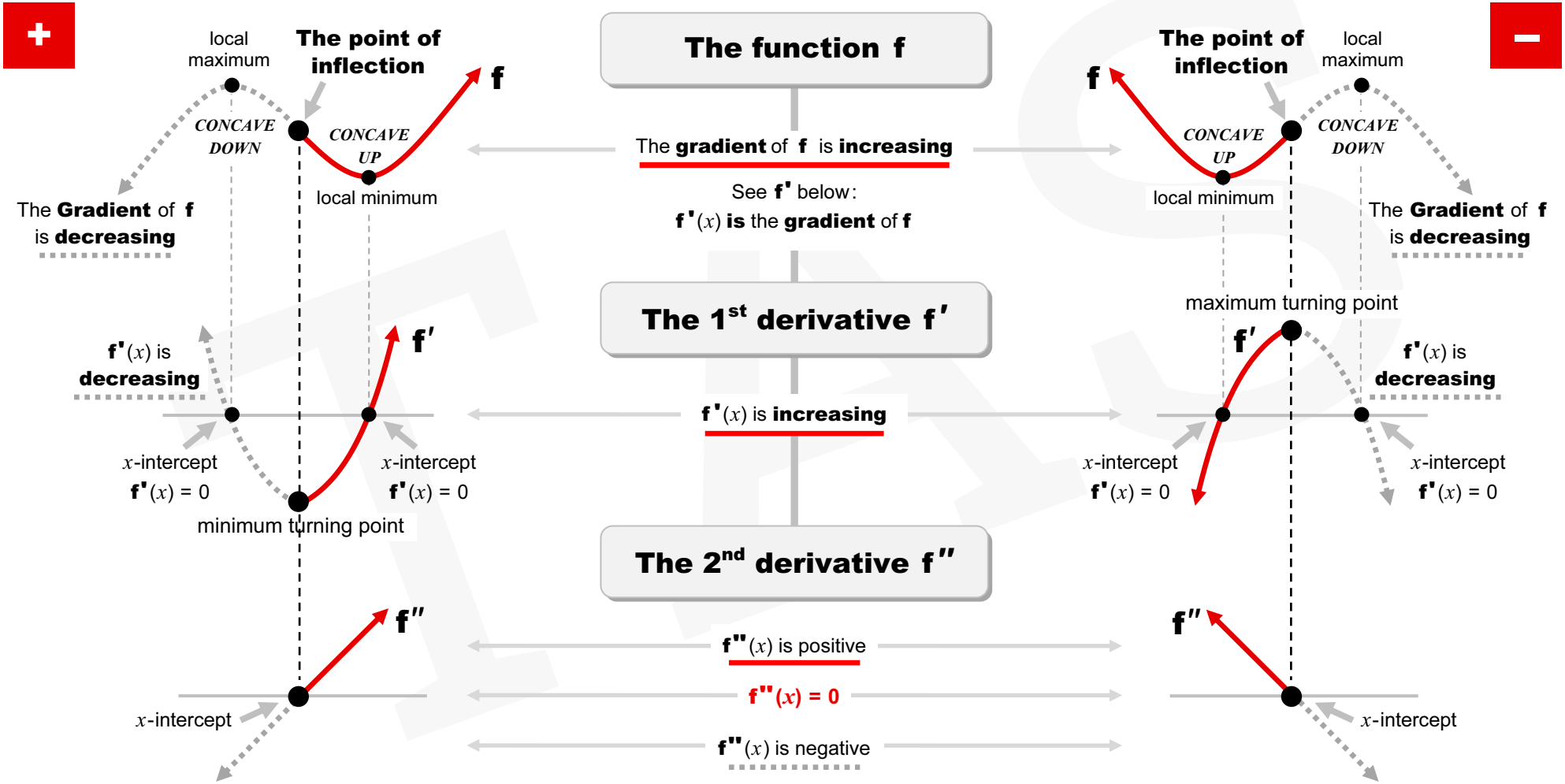
$\therefore 4t = 18$

$\therefore t = \frac{9}{2}$

CONCAVITY & THE POINT OF INFLECTION

The **Concavity** of cubic graphs: **Concave up** or **Concave down** , changes at the point of inflection:

As x increases (i.e. from left to right) . . .



Note: For cubic graphs, the midpoint between the x-coordinates of the turning points (when there are 2) can be used to determine the x-coordinate of the point of inflection of cubic graphs.

$f''(x) = 0$ at the point of inflection of f

$f''(x) < 0$ where f is concave down

$f''(x) > 0$ where f is concave up

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CONCAVITY & THE POINT OF INFLECTION cont.

Only 1 Stationary Point

f

$$y = x^3 - 9x^2 + 27x$$

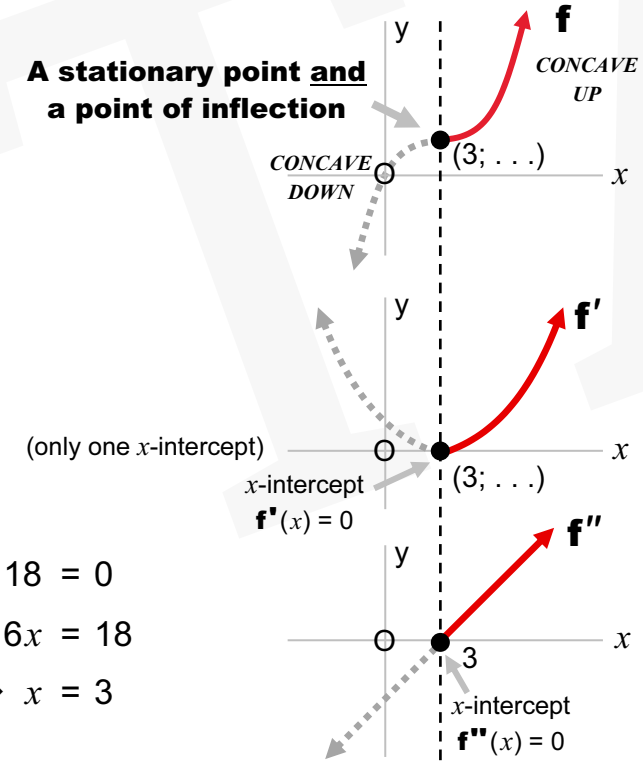
$$\therefore \frac{dy}{dx} = 3x^2 - 18x + 27$$

$$= 3(x^2 - 6x + 9)$$

$$= 3(x - 3)^2$$

Derivative = 0 only once, when $x = 3$

$\therefore$ **only one stationary point**



$$f''(x) = 6x - 18 = 0$$

$$\Rightarrow 6x = 18$$

$$\Rightarrow x = 3$$

No Stationary Points

g

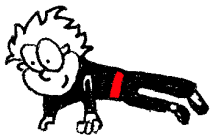
$$g(x) = -x^3 - 3x^2 - 9x$$

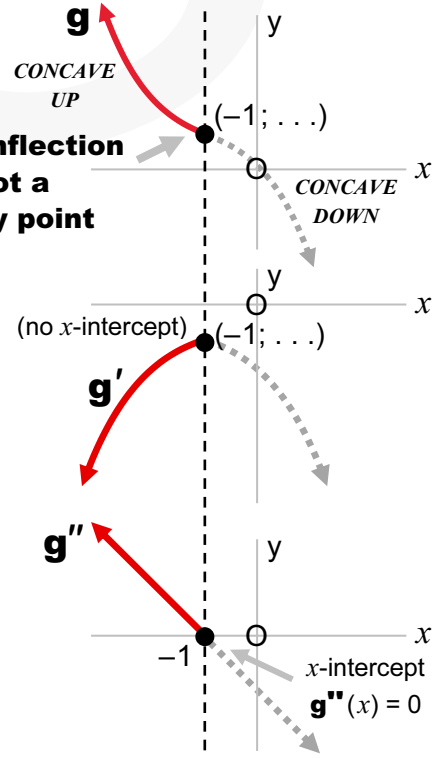
$$\therefore g'(x) = -3x^2 - 6x - 9$$

$$= -3(x^2 + 2x + 3)$$

Derivative $\neq 0$ because $\Delta = 2^2 - 4(1)(3) = 4 - 12 = -8$

$\therefore$ **no stationary points**





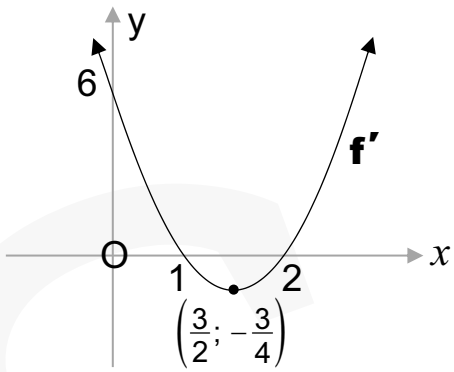
$$g''(x) = -6x - 6 = 0$$

$$\Rightarrow -6x = 6$$

$$\Rightarrow x = -1$$

Some Examples ...

1. The graph of the parabola $y = f'(x)$ is shown.



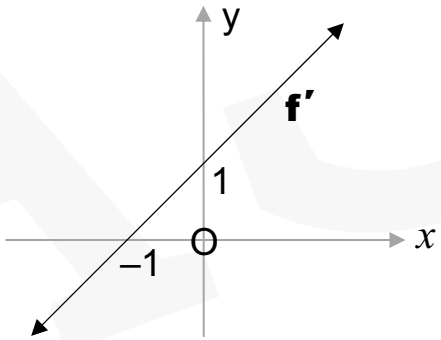
   (a) Write down the x-coordinate of the local minimum of $y = f(x)$. (2)

   (b) For which values of x will $f(x)$ be decreasing? (2)

   (c) What is the gradient of the tangent to f when $x = 0$? (2)

   (d) At which value of x will there be a tangent to f parallel to the one in 1(c)? (2)

2. The graph of $y = f'(x)$ is shown.



   (a) Explain why $f(x)$ is a quadratic function (having the form $ax^2 + bx + c$). (2)

   (b) What is the value of $f'(-1)$? (1)

   (c) Use the graph to solve the inequalities:

   (i) $f'(x) < 0$ (ii) $f'(x) > 0$ (1)(1)

   (d) (i) For which values of x is $f(x)$ decreasing? (1)

   (ii) For which values of x is $f(x)$ increasing? (1)

   (e) Does $f(x)$ have a maximum or minimum turning point? Justify your answer. (2)

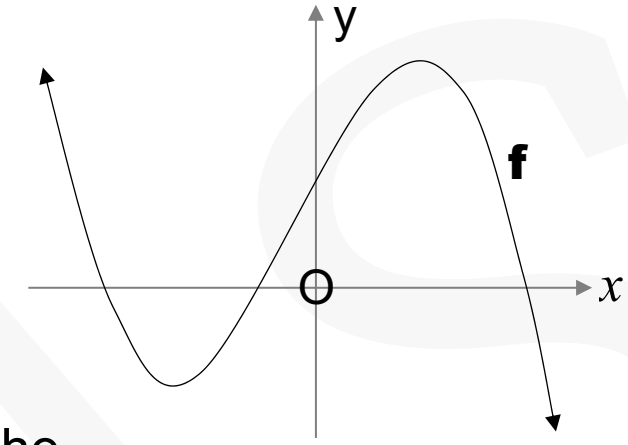
   (f) Write down the equation of the axis of symmetry of $f(x)$. (2)

f / f′ / f″

The graph of the function

$f(x) = -x^3 - x^2 + 16x + 16$

is sketched alongside.



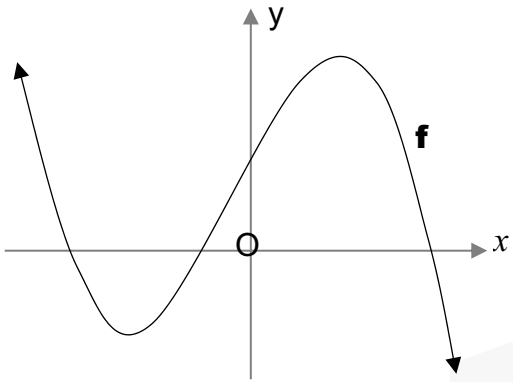
1. Calculate the x-coordinates of the turning points of f. (4)

2. Calculate the x-coordinate of the point at which $f'(x)$ is a maximum. (3)

Gr 12 Maths 2 in 1
p. 150 (Q9.1)

QUESTION

The graph of the function $f(x) = -x^3 - x^2 + 16x + 16$ is sketched alongside.



1. Calculate the x-coordinates of the turning points of f. (4)

2. Calculate the x-coordinate of the point at which $f'(x)$ is a maximum. (3)

MEMO

$f(x) = -x^3 - x^2 + 16x + 16$

1. At the turning points: $f'(x) = 0$

$\therefore -3x^2 - 2x + 16 = 0$

$\times(-1)$ $\therefore 3x^2 + 2x - 16 = 0$

$\therefore (3x + 8)(x - 2) = 0$

$\therefore x = -\frac{8}{3}$ **or** 2 ◄

2. **Note:** The graph $y = f'(x)$ is a parabola

$f'(x)$ a maximum ➡ $f''(x) = 0$

$\therefore -6x - 2 = 0$

$\therefore -6x = 2$

This is the x-coordinate of the pt. of inflection.

$\therefore x = -\frac{1}{3}$ ◄

Remember this 'fun' question?

CONCAVITY & THE POINT OF INFLECTION ...

Draw a sketch graph of f, indicating ALL relevant points, if it is given that f is a cubic function with:

- $f(3) = f'(3) = 0$
- $f(0) = 27$
- $f''(x) > 0$ when $x < 3$ and $f''(x) < 0$ when $x > 3$. (3)

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Challenging Questions Booklet
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Draw a sketch graph of f, indicating ALL relevant points, if it is given that f is a cubic function with:

- $f(3) = f'(3) = 0$
- $f(0) = 27$
- $f''(x) > 0$ when $x < 3$ and $f''(x) < 0$ when $x > 3$. (3)

SOLUTION

$f(3) = 0$ ➡ x-intercept is at 3

$f'(3) = 0$ ➡ stationary pt. is at 3

$f(0) = 27$ ➡ y-intercept is at 27

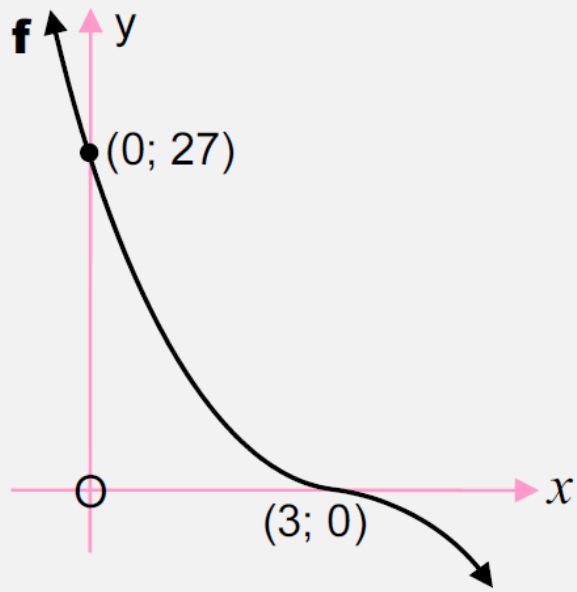
$f''(x)$ positive for $x < 3$

➡ f is concave up for $x < 3$

$f''(x)$ negative for $x > 3$

➡ f is concave down for $x > 3$

$\therefore$ The point of inflection is at $x = 3$.



Note: Point (3; 0) is the x-intercept, stationary point and horizontal point of inflection.

For a certain function f, the first derivative is given as

$f'(x) = 3x^2 + 8x - 3$

1. Calculate the x-coordinates of the stationary points of f. (3)

2. For which values of x is f concave down? (3)

3. Determine the values of x for which f is strictly increasing. (2)

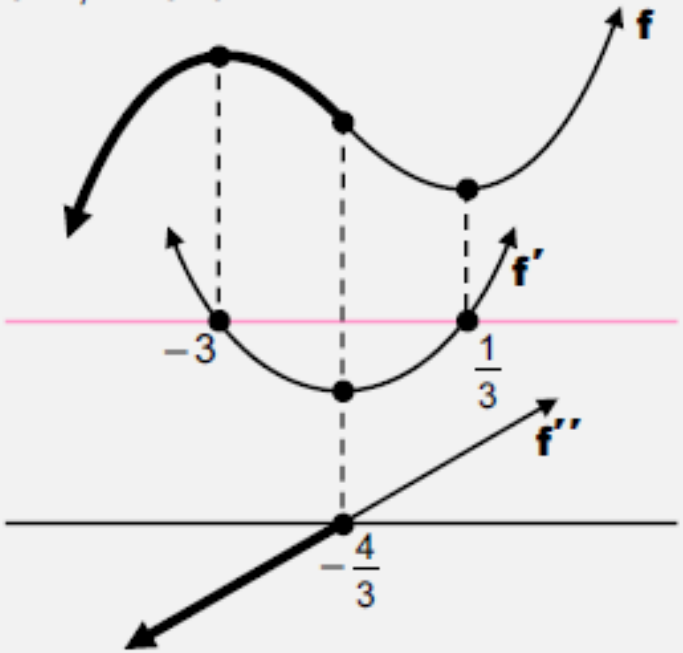
4. If it is further given that $f(x) = ax^3 + bx^2 + cx + d$ and $f(0) = -18$, determine the equation of f. (5) [13]

SOLUTIONS

For a certain function f, the first derivative is given as $f'(x) = 3x^2 + 8x - 3$

1. Calculate the x-coordinates of the stationary points of f. (3)

Sketches of f, f′ and f″:

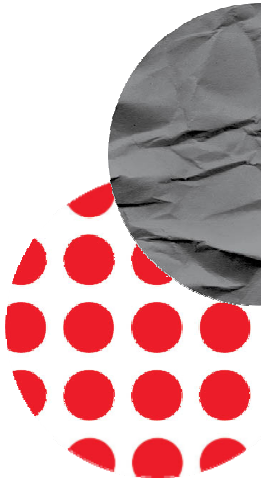


1. At the stationary points of f:

$f'(x) = 0 \Rightarrow 3x^2 + 8x - 3 = 0$

$\therefore (3x - 1)(x + 3) = 0$

$\therefore x = \frac{1}{3}$ or -3



2. For which values of x is f concave down? (3)

2. At the point of inflection:

$$f''(x) = 0$$
$$\therefore\ 6x + 8 = 0$$
$$\therefore\ 6x = -8$$
$$\therefore\ x = -\frac{4}{3}$$

f is concave down for $\boldsymbol{x < -\frac{4}{3}}$ ◄ . . . *See the sketch of f and f''.*

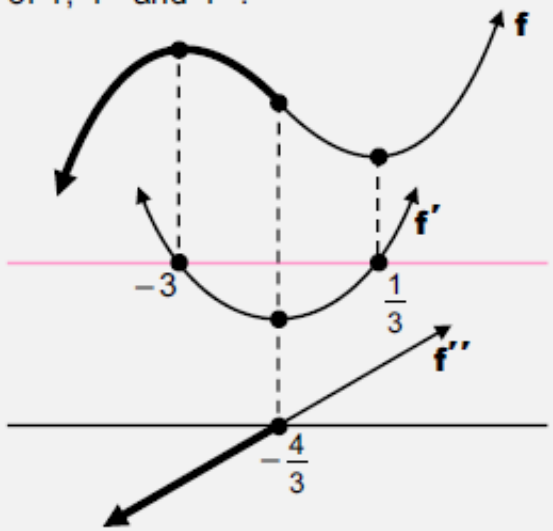
OR: x is halfway between $\frac{1}{3}$ & -3

$$\therefore\ x = \frac{\frac{1}{3} + (-3)}{2}$$
$$= \frac{-2\frac{2}{3}}{2}$$
$$= \mathbf{-1\frac{1}{3}}$$ ◄

OR: $f''(x) < 0$

$$\therefore\ 6x + 8 < 0$$
$$\therefore\ 6x < -8$$
$$\therefore\ \boldsymbol{x < -\frac{4}{3}}$$ ◄

Sketches of f, f′ and f″:



3. Determine the values of x for which f is strictly increasing. (2)

3. f strictly increasing ➡ $f'(x) > 0$

$\therefore\ x < -3$ or $x > \frac{1}{3}$ . . . *See the sketch of f and f''.*

OR: $3x^2 + 8x - 3 > 0$

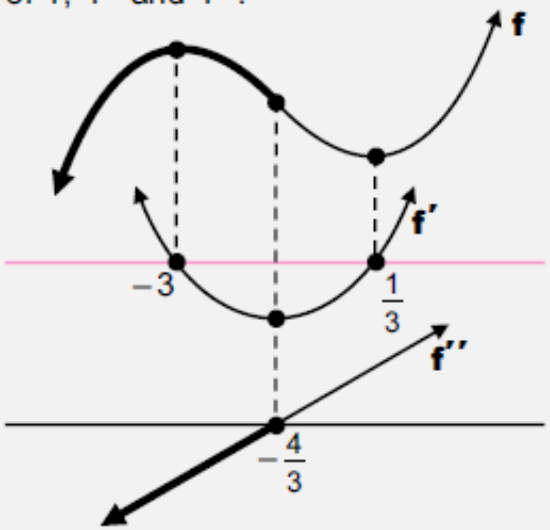
$\therefore\ (3x - 1)(x + 3) > 0$

$f'(x)$: + − +

x: -3 $\frac{1}{3}$

$\therefore\ x < -3$ or $x > \frac{1}{3}$

Sketches of f, f′ and f″:



4. If it is further given that $f(x) = ax^3 + bx^2 + cx + d$ and $f(0) = -18$, determine the equation of f. (5) **[13]**

4. $f(x) = ax^3 + bx^2 + cx + d$

$\therefore\ f(0) = -18 \quad \Rightarrow \quad d = -18$

& $f'(x) = 3ax^2 + 2bx + c$

But, $f'(x) = 3x^2 + 8x - 3 \quad \ldots\ given$

$\therefore\ 3a = 3 \quad ; \quad 2b = 8 \quad ; \quad c = -3$

$\therefore\ a = 1 \qquad \therefore\ b = 4$

$\therefore\ \mathbf{f(x) = x^3 + 4x^2 - 3x - 18}$

A selection Of Challenging Questions & Solutions (Paper 1)

THE ANSWER SERIES

Your Key to Exam Success

ALGEBRA

1. Solve for x: $3^x(x-5) < 0$ (2)

Gr 12 Maths 2 in 1
Challenging Questions Booklet
p. 2 Exponents (Q1)

2. Solve for x and y: $(3x-y)^2 + (x-5)^2 = 0$ (4)

Gr 12 Maths 2 in 1
Challenging Questions Booklet
p. 2 Algebra (Q4)

3. Recall:

Solve for x: $(x-2)(x-3) = 0$ (2)

4. Solve for y: $(x-2)(y-3) = 0$

If (a) $x = 3$ (b) $x = 2$ (2)

Gr 12 Maths 2 in 1
p. 2 (Q2.1)

A SURD QUESTION

Given: $\sqrt{5 - x} = x + 1$

1. Without solving the equation, show that the solution to the above equation lies in the interval $-1 \leq x \leq 5$. (3)

2. Solve the equation. (5)

3. Without any further calculations, solve the equation $-\sqrt{5 - x} = x + 1$. (1) [13]

ALGEBRA OR GRAPHS/FUNCTIONS?

1. Given: $f(x) = x^2 + 8x + 16$

   1.1 Solve for x if $f(x) > 0$. (3)

   1.2 For which values of p will $f(x) = p$ have TWO unequal negative roots? (4)

Gr 12 Maths 2 in 1
Challenging Questions Booklet
p. 1 (Q2)

1. Given: $f(x) = x^2 + 8x + 16$

1.1 Solve for x if $f(x) > 0$. (3)

SOLUTION

Gr 12 Maths 2 in 1
Challenging Questions Booklet
p. 1 (Q2)

$$f(x) = x^2 + 8x + 16$$

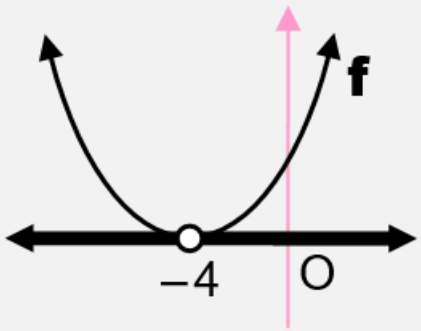
$$\therefore\ f(x) = (x + 4)^2 \qquad \ldots \textit{ a perfect square}$$

Algebraically ...

For **all** values of x, $(x + 4)^2 \geq 0$

$\therefore$ Solution: $\boldsymbol{x \in \mathbb{R};\ \ x \neq -4}$ ◄

Graphically ...



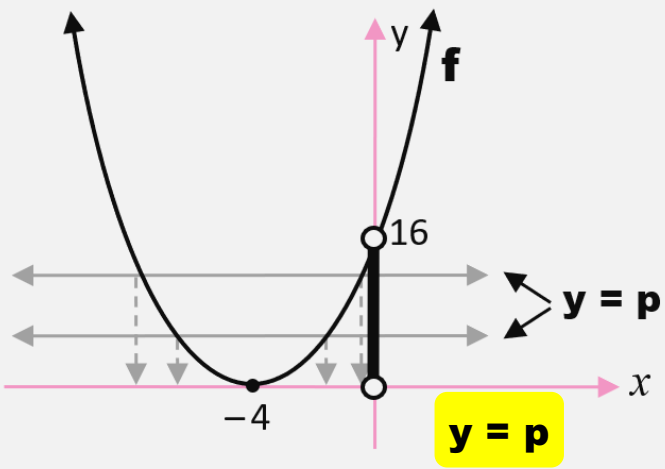
$\therefore$ Solution: $\boldsymbol{x \in \mathbb{R};\ \ x \neq -4}$ ◄

1.2 For which values of p will $f(x) = p$ have TWO unequal negative roots? (4)

SOLUTION

Graphically ...

$y = f(x)$



0 < p < 16

So, all lines $y = p$ which lie between $y = 0$ and $y = 16$

2. Given: $f(x) = 3(x - 1)^2 + 5$ and $g(x) = 3$

2.1 Is it possible for $f(x) = g(x)$?

Give a reason for your answer. (2)

2.2 Determine the value(s) of k for which

$f(x) = g(x) + k$ has TWO unequal real roots. (2)

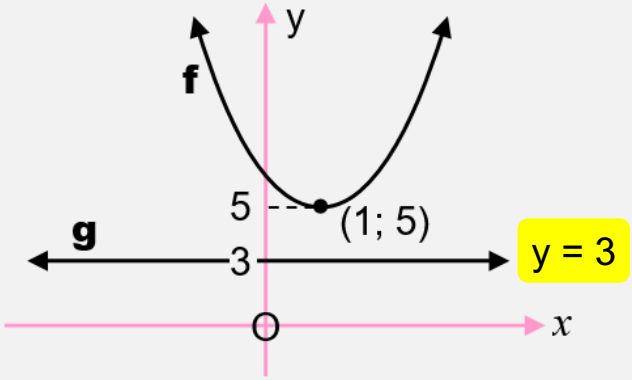
2. Given: $f(x) = 3(x-1)^2 + 5$ and $g(x) = 3$

2.1 Is it possible for $f(x) = g(x)$? Give a reason for your answer. (2)

2.2 Determine the value(s) of k for which $f(x) = g(x) + k$ has TWO unequal real roots. (2)

Gr 12 Maths 2 in 1
Challenging Questions Booklet
p. 1 (Q3)

SOLUTIONS



2.1 **No** ∴ **f and g have no points of intersection**

2.2 **k > 2**

OR: ALGEBRAIC METHODS, requiring more time!

OR: Algebraic methods, requiring more time!

2.1 **No** ; $f(x) = g(x) \Rightarrow 3(x-1)^2 + 5 = 3$

$$\therefore\ 3(x-1)^2 = -2$$

$$\therefore\ (x-1)^2 = -\frac{2}{3},$$

which is impossible because a square cannot be negative.

OR: $3(x^2 - 2x + 1) + 5 = 3$

$$\therefore\ 3x^2 - 6x + 3 + 5 = 3$$

$$\therefore\ 3x^2 - 6x + 5 = 0$$

$$\therefore\ x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(5)}}{2(3)}$$

$$= \frac{6 \pm \sqrt{-24}}{6}$$

$\Delta = -24$

$\therefore\ \sqrt{\Delta}$ is non-real

$\therefore$ There are no solutions to the equation $f(x) = g(x)$.

2.2 $f(x) = g(x) + k \Rightarrow 3(x-1)^2 + 5 = 3 + k$

$$\therefore\ 3(x^2 - 2x + 1) + 5 - 3 - k = 0$$

$$\therefore\ 3x^2 - 6x + (5 - k) = 0$$

$$\Delta = (-6)^2 - 4(3)(5 - k)$$
$$= 36 - 60 + 12k$$
$$= 12k - 24$$

If we want 2 (real & unequal) roots, then Δ must be positive:

$$\therefore\ 12k - 24 > 0$$

$$\therefore\ 12k > 24$$

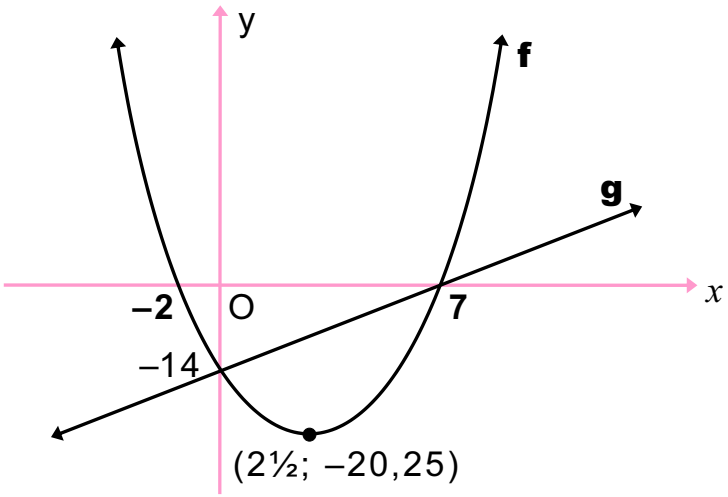
$$\therefore\ \mathbf{k > 2}$$

The sketch is much easier.

3. $f(x) = -x^2 - 5x - 14$ & $g(x) = 2x - 14$

3.1 The equation of the tangent to f at $x = 2\frac{1}{2}$ (1)

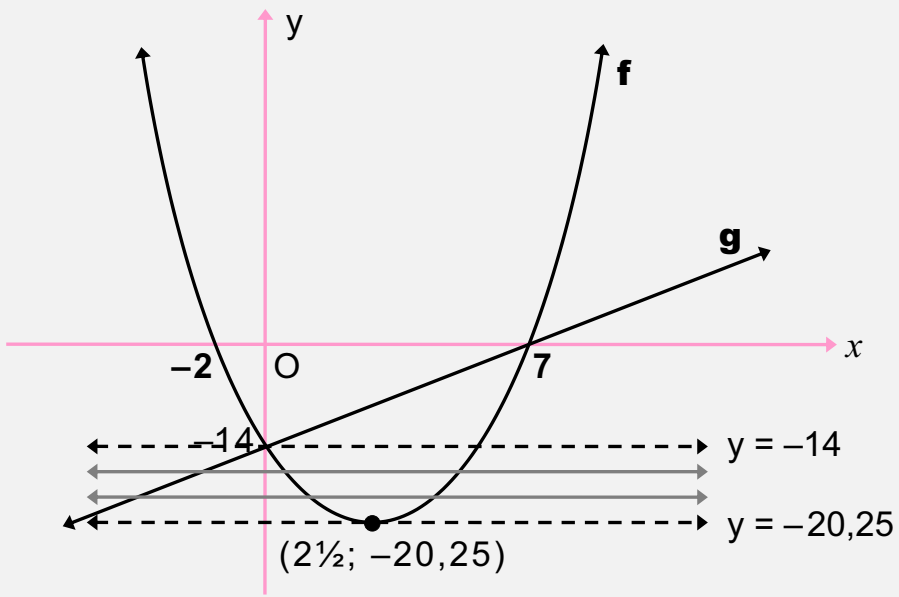
3.2 For which values of k will $f(x) = k$ have 2 unequal positive real roots? (3)



SOLUTIONS

2.1 **y = –20,25** ◄

2.2 **–20,25 < k < –14** ◄



QUADRATIC SEQUENCES

QUESTION

1. Given the quadratic sequence: 2 ; 3 ; 10 ; 23 ; . . .

   1.1 Write down the next term of the sequence. (1)

   1.2 Determine the nth term of the sequence. (4)

   1.3 Calculate the 20th term of the sequence. (2)

Given the quadratic sequence: 2 ; 3 ; 10 ; 23 ; . . .

1.1 Write down the next term of the sequence. (1)

SOLUTION

	T_0		T_1		T_2		T_3		T_4		T_5
	7		2		3		10		23		**42**
1st differences:		**−5**		1		7		13		**19**	
2nd differences:					6		6		**6**		

Next term, T_5 = **42**

Given the quadratic sequence: 2 ; 3 ; 10 ; 23 ; . . .

1.2 Determine the nth term of the sequence. (4)

SOLUTION

Method 1:

A Quadratic sequence:

$T_n = an^2 + bn + c$ . . . *the* **general** *term*

$\therefore T_1 = a + b + c$ . . . *the* **first** *term*

$T_2 = 4a + 2b + c$ . . . *the* **second** *term*

$T_3 = 9a + 3b + c$ . . . *the* **third** *term*

The structure of the terms (& 1st & 2nd differences):

	T_1	T_2	T_3
	$\mathbf{a + b + c}$	$4a + 2b + c$	$9a + 3b + c$
1st differences:	$\mathbf{3a + b}$	$5a + b$	
2nd differences:	$\mathbf{2a}$ ← **The constant common difference**		

Finding the nth term (i.e. solving for a, b and c):

$\mathbf{2a} = 6$ . . . *the common 2nd difference*

$\therefore a = 3$

$\mathbf{3a + b} = 1$ . . . *the first first difference*

$\therefore 9 + b = 1$

$\therefore b = -8$

$\mathbf{a + b + c} = 2$ . . . *T_1, the first term*

$\therefore 3 - 8 + c = 2$

$\therefore c = 7$

$\therefore \mathbf{T_n = 3n^2 - 8n + 7}$ ◄

Method 2:

$\mathbf{T_n = an^2 + bn + c}$

$\therefore T_1 = a + b + c = 2$

$T_0 = c = 7$ and $2a = 6$ . . . *2nd difference*

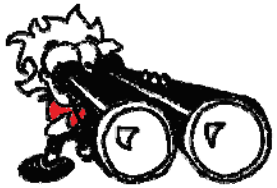
$\therefore a = 3$

$\therefore 3 + b + 7 = 2$

$\therefore b = -8$

$\therefore$ The nth term, $\mathbf{T_n = 3n^2 - 8n + 7}$ ◄

1.3 Calculate the 20th term of the sequence. (2)



SOLUTION

$$T_n = 3n^2 - 8n + 7$$

$\therefore$ The 20th term,

$$\mathbf{T_{20}} = 3(20)^2 - 8(20) + 7$$

$$= \mathbf{1\ 047}$$

QUESTION

2. A quadratic pattern $T_n = an^2 + bn + c$ has $T_2 = T_4 = 0$ and a second difference of 12.

Determine the value of the 3rd term of the pattern. (6)

2. A quadratic pattern $T_n = an^2 + bn + c$ has $T_2 = T_4 = 0$ and a second difference of 12.

Determine the value of the 3rd term of the pattern. (6)

SOLUTION

Let the 3rd term be x

T_1	T_2	T_3	T_4
	0	x?	0

T_1	T_2	T_3	T_4
	0	x?	0

$x \qquad -x$

$-2x$

$\therefore\ -2x = 12$

$\therefore\ x = -6$

i.e. The 3rd Term is **-6** ◄

ALTERNATIVE SOLUTION

$T_n = an^2 + bn + c$

$T_2 = a(2)^2 + b(2) + c = 4a + 2b + c = 0 \quad \ldots ❶$

$T_4 = a(4)^2 + b(4) + c = 16a + 4b + c = 0 \quad \ldots ❷$

❷ - ❶: $12a + 2b = 0$

$\therefore 6a + b = 0$

& 2nd difference, $2a = 12$

$\therefore a = 6$

$\therefore 36 + b = 0$

$\therefore b = -36$

❶: $4(6) + 2(-36) + c = 0$

$\therefore c = -24 + 72$

$\therefore c = 48$

$T_3 = a(3)^2 + b(3) + c$

$= 9a + 3b + c$

$= 9(6) + 3(-36) + 48$

$= \mathbf{-6}$ ◄

There are various other methods!

QUESTION

3. The nth term of a sequence is given by $T_n = -2(n-5)^2 + 18$.

3.1 Write down the first THREE terms of the sequence. (3)

3.2 Which term of the sequence will have the greatest value? (1)

3.3 What is the second difference of this quadratic sequence? (2)

3.4 Determine ALL values of n for which the terms of the sequence will be less than –110. (6)

Gr 12 Maths 2 in 1
p. 149 (Q3.2)

QUESTION

The nth term of a sequence is given by $T_n = -2(n-5)^2 + 18$.

3.1 Write down the first THREE terms of the sequence. (3)

3.2 Which term of the sequence will have the greatest value? (1)

3.3 What is the second difference of this quadratic sequence? (2)

3.4 Determine ALL values of n for which the terms of the sequence will be less than –110. (6)

MEMO

$T_n = -2(n-5)^2 + 18$

3.1 $T_1 = -2(1-5)^2 + 18 = -32 + 18 = \mathbf{-14}$ ◄

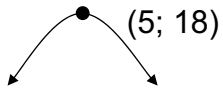
$T_2 = -2(2-5)^2 + 18 = -18 + 18 = \mathbf{0}$ ◄

$T_3 = -2(3-5)^2 + 18 = -8 + 18 = \mathbf{10}$ ◄

3.2 If one drew a graph of $T_n = -2(n-5)^2 + 18$,

Compare to: $y = -2(x-5)^2 + 18$

then the turning point would be (5; 18)



∴ The maximum value of T_n (which is 18) would occur when n = 5.

∴ **The 5th term** ◄

3.3

	T_1	T_2	T_3
	–14	0	10
1st differences:	14	10	
2nd differences:	-4		

∴ **The second difference = –4** ◄

3.4 $T_n < -110$ ➡ $-2(n-5)^2 + 18 < -110$

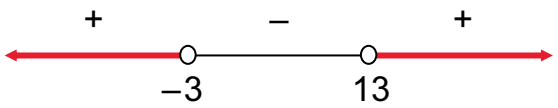
$\therefore -2(n^2 - 10n + 25) + 128 < 0$

$\therefore -2n^2 + 20n - 50 + 128 < 0$

$\therefore -2n^2 + 20n + 78 < 0$

$\div (-2) \quad \therefore n^2 - 10n - 39 > 0$

$\therefore (n+3)(n-13) > 0$



$\therefore n < -3$ or $n > 13$

n is the number of terms $\therefore n \geq 0$ *and* $n \in \mathbb{N}_0$

∴ **n > 13 ; n∈ℕ** ◄

Remember this 'fun' question?

DETERMINE:

$$D_x \left[\sum_{n=3}^{5} (n+2)\, x^n \right]$$

Sigma? **Calculus?**

How would your learners respond?

DETERMINE: $D_x \left[\sum_{n=3}^{5} (n+2)\, x^n \right]$

SOLUTION

$$\sum_{n=3}^{5} (n+2)\, x^n = (3+2)x^3 + (4+2)x^4 + (5+2)x^5$$

$$= 5x^3 + 6x^4 + 7x^5$$

$$D_x \left[\sum_{n=3}^{5} (n+2)\, x^n \right] = 15x^2 + 24x^3 + 35x^4$$

SYMBOLS . . .

Sigma

1. Given the series:

   $(1 \times 2) + (5 \times 6) + (9 \times 10) + (13 \times 14) + \ \ldots \ + (81 \times 82)$

   Write this series in sigma notation. (4)

Gr 12 Maths 2 in 1
Challenging Questions Booklet
p. 3 (Q3)

Factors

2. Determine the value of

   $\left(1 + \frac{1}{2}\right)\left(1 + \frac{1}{3}\right)\left(1 + \frac{1}{4}\right)\left(1 + \frac{1}{5}\right) \ldots$ up to 98 factors. (4)

Gr 12 Maths Toolkit
p. 3 (Q3.3)

S_n: The sum of n terms

3. If $S_n = 4n^2 + 1$, find the 2nd term. (4)

Solutions

1. Given the series: $(1 \times 2) + (5 \times 6) + (9 \times 10) + (13 \times 14) + \ldots + (81 \times 82)$

   Write this series in sigma notation. (4)

$$\sum_{n=1}^{21} (4n - 3)(4n - 2)$$

2. Determine the value of $\left(1 + \frac{1}{2}\right)\left(1 + \frac{1}{3}\right)\left(1 + \frac{1}{4}\right)\left(1 + \frac{1}{5}\right)$ . . . up to 98 factors. (4)

$$\left(\frac{\cancel{3}}{2}\right)\left(\frac{4}{\cancel{3}}\right)\left(\frac{\cancel{5}}{4}\right)\left(\frac{6}{\cancel{5}}\right) \ldots\ldots \left(\frac{100}{\cancel{99}}\right) = \left(\frac{100}{2}\right) = \mathbf{50}$$

3. If $S_n = 4n^2 + 1$, find the 2nd term. (4)

$$T_2 = S_2 - S_1 \quad \text{where}$$

$$S_2 = 4(2)^2 + 1 = 17 \quad \& \quad S_1 = 4(1)^2 + 1 = 5$$

$$\therefore\ T_2 = 17 - 5 = \mathbf{12}$$

Hyperbola

QUESTION

1. The graph of a hyperbola with equation $y = f(x)$ has the following properties:

   - Domain: $x \in \mathbb{R}$, $x \neq 5$
   - Range: $y \in \mathbb{R}$, $y \neq 1$
   - Passes through the point (2; 0)

   Determine $f(x)$. (4)

Gr 12 Maths 2 in 1
p. 149 (Q6)

QUESTION

1. The graph of a hyperbola with equation $y = f(x)$ has the following properties:

   - Domain: $x \in \mathbb{R}, x \neq 5$
   - Range: $y \in \mathbb{R}, y \neq 1$
   - Passes through the point (2; 0)

   Determine $f(x)$. (4)

MEMO

1. The equation of the hyperbola: $y = \dfrac{a}{x - p} + q$

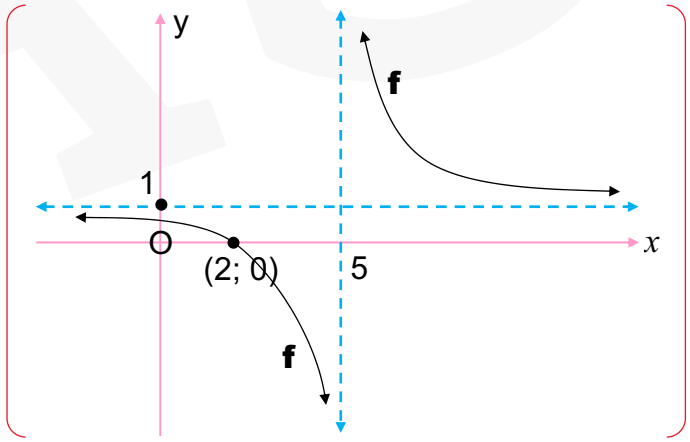
$$\therefore\ y = \frac{a}{x - 5} + 1$$

Substitute (2; 0): $\therefore\ 0 = \dfrac{a}{(2 - 5)} + 1$

$$\therefore\ -1 = \frac{a}{-3}$$

$\times(-3)$ $\therefore\ 3 = a$

$$\therefore\ \mathbf{f(x) = \frac{3}{x - 5} + 1}$$



QUESTION

2. Given: $f(x) = \dfrac{x + 3}{x + 1}$

2.1 Calculate the x- and y-intercepts of f. (4)

2.2 Show that $f(x) = \dfrac{2}{x + 1} + 1$. (2)

2.3 Write down the equations of the vertical and horizontal asymptotes of f. (2)

2.4 Draw a sketch graph of f showing clearly the intercepts and asymptotes on the axes provided alongside. (4)

2.5 Use your graph to solve: $\dfrac{2}{x + 1} \geq -1$. (3) [15]

Gr 12 Nat. November 2009 (leaked)

Question 7

SOLUTIONS

2.1 Equation of f: $y = \dfrac{x+3}{x+1}$

x-intercept: When $y = 0$, $x = -3$ $\therefore$ **(–3; 0)** ◄

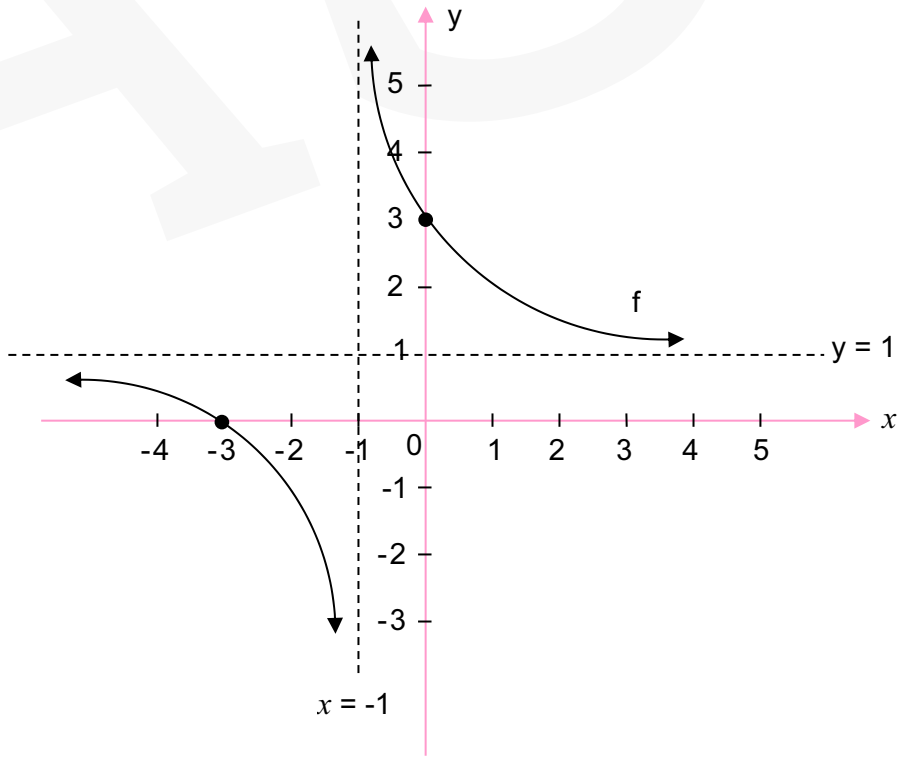
y-intercept: When $x = 0$, $y = \dfrac{3}{1} = 3$ $\therefore$ **(0; 3)** ◄

2.2 $$f(x) = \frac{x+1+2}{x+1}$$

$$= \frac{x+1}{x+1} + \frac{2}{x+1}$$

$$= 1 + \frac{2}{x+1}$$

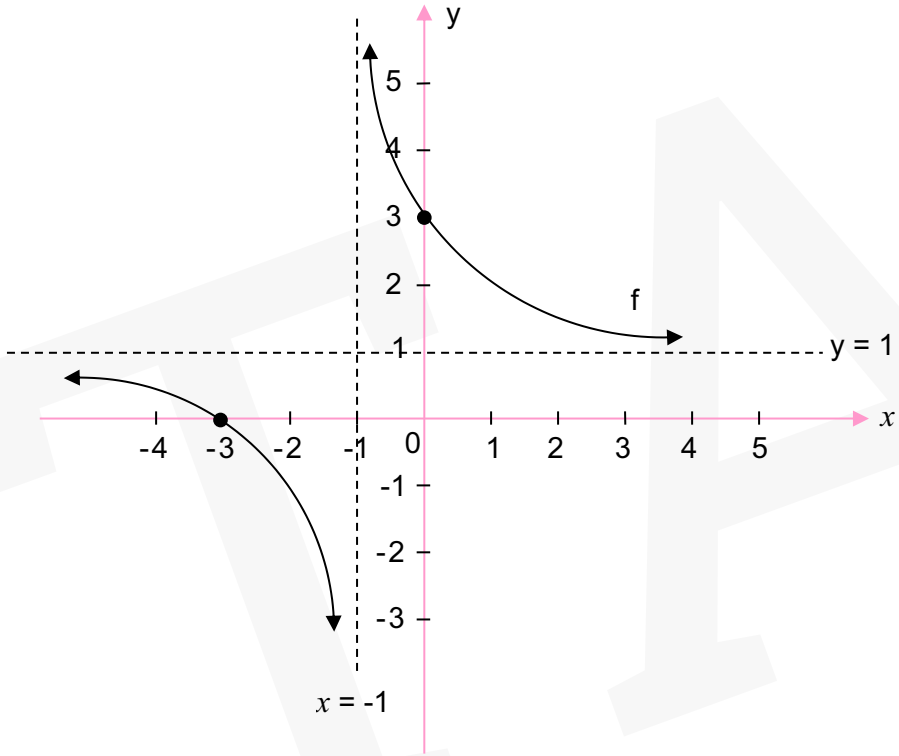
$$\therefore\ f(x) = \boldsymbol{\frac{2}{x+1} + 1}$$ ◄



2. Given: $f(x) = \dfrac{x + 3}{x + 1}$

2.3 Vertical asymptote: $\boldsymbol{x = -1}$ ◄ Horizontal asymptote: $\mathbf{y = 1}$ ◄

2.4



2.5 $\dfrac{2}{x + 1} \geq -1$ ➡ $\dfrac{2}{x + 1} + 1 \geq 0$, i.e. $f(x) \geq 0$

$\therefore$ $\boldsymbol{x \leq -3}$ or $\boldsymbol{x > -1}$ ◄

FUNCTION LANGUAGE

Sketch the graph of $f(x) = ax^2 + bx + c$ if it is also given that:

- the range of f is $(-\infty; 7]$
- $a \neq 0$
- $b < 0$
- one root of f is positive and the other root of f is negative. (4)

Gr 11 Maths 3 in 1
p. Q2
(Gr 11 Exemplar Q10)

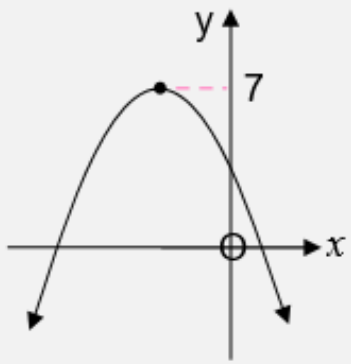
FUNCTION LANGUAGE

Sketch the graph of
$f(x) = ax^2 + bx + c$
if it is also given that:

- the range of f is $(-\infty; 7]$
- $a \neq 0$
- $b < 0$
- one root of f is positive and the other root of f is negative.

The range, $(-\infty; 7]$, indicates the y-values.

➡ Max $f(x) = 7$ and $a < 0$;



Axis of symmetry:

$x = -\frac{b}{2a} = -\frac{-}{-}$ is negative;

(. . . it is given that $b < 0$ and concluded that $a < 0$.)

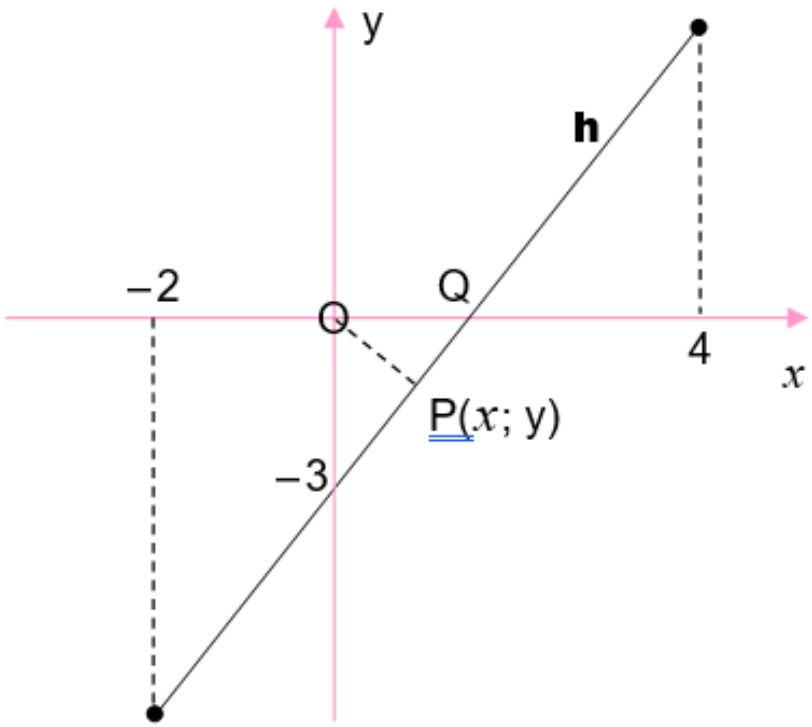
One root positive & one negative

➡ roots on opposite sides of y-axis.

Note: Range notation:
(means excluding &] means including

INVERSE FUNCTIONS

1. Given: $h(x) = 2x - 3$ for $-2 \leq x \leq 4$.

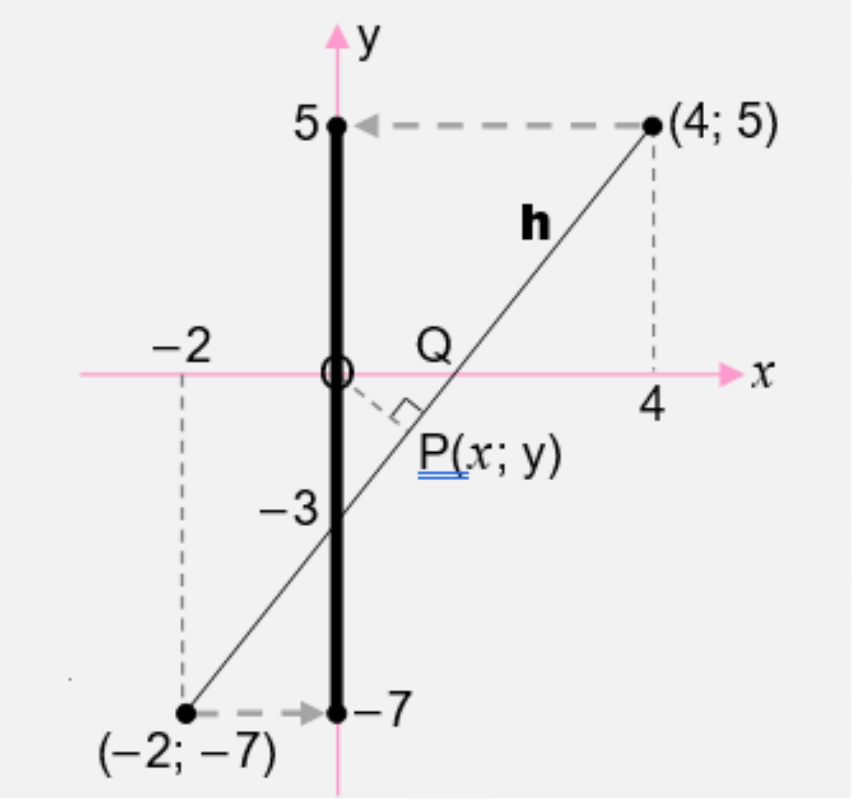


The x-intercept of h is at Q.

Write down the domain of $\mathbf{h^{-1}}$. (3)

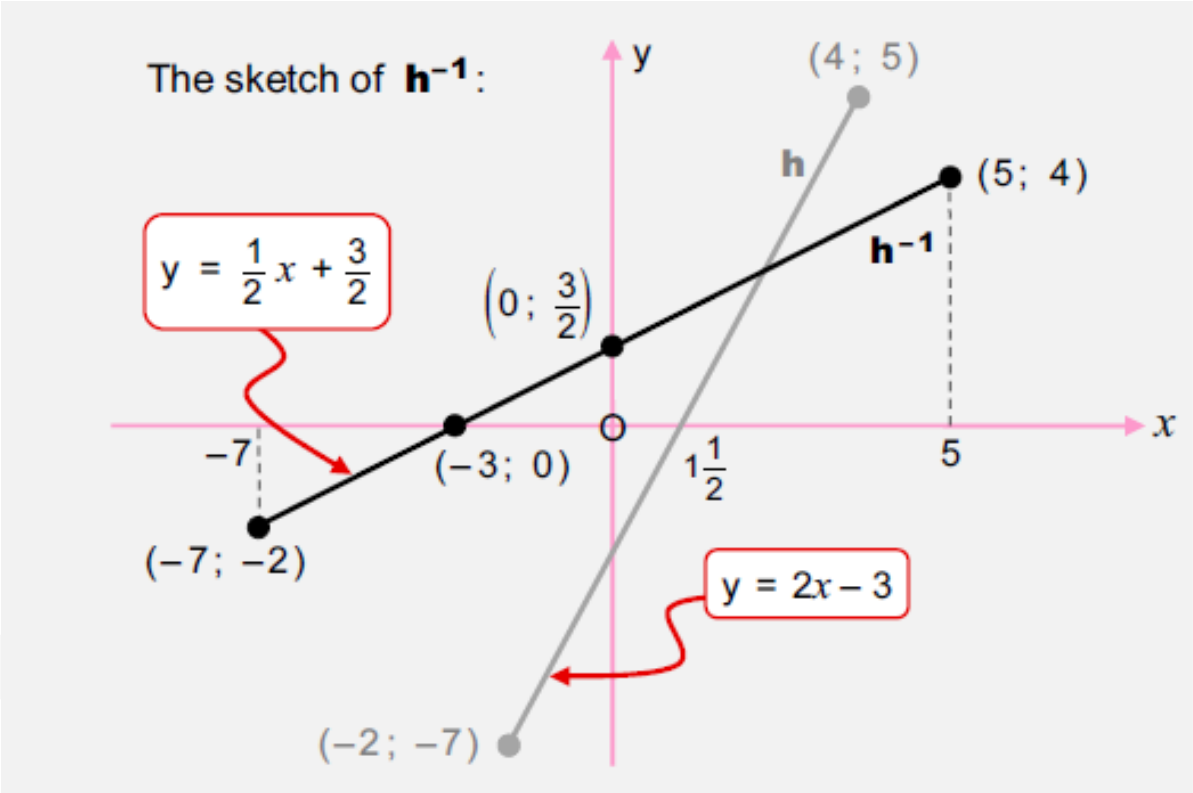
Gr 12 Maths 2 in 1
Challenging Questions Booklet
p. 8 (Q9 [26%]

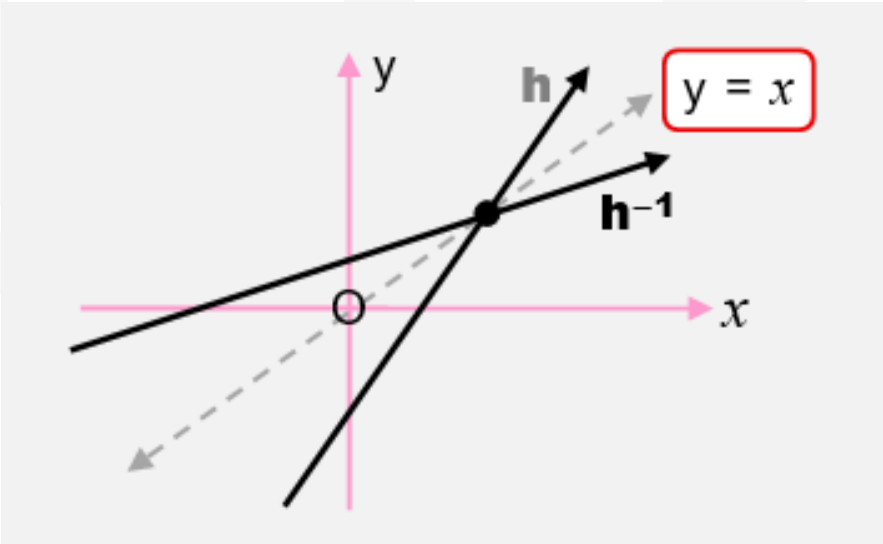
SOLUTION



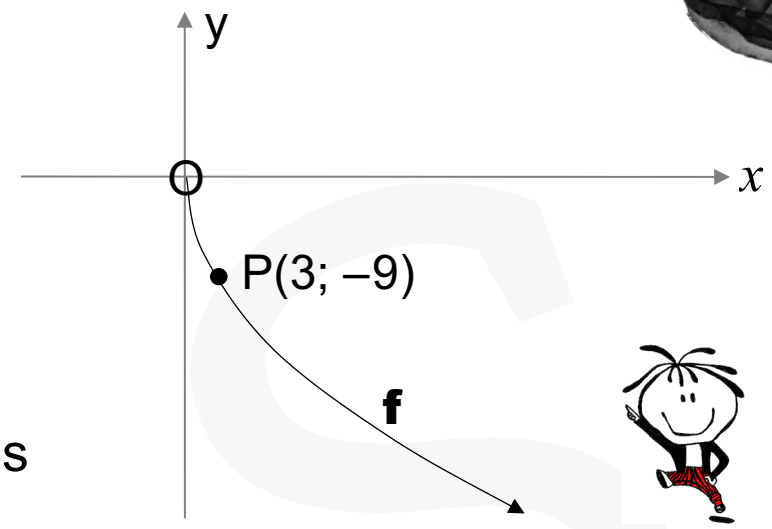
Range of h: [–7; 5]

∴ **Domain of h^{-1}?**


The sketch of $\mathbf{h^{-1}}$:
y
(4; 5)
h
(5; 4)
$\mathbf{h^{-1}}$
$y = \frac{1}{2}x + \frac{3}{2}$
$\left(0; \frac{3}{2}\right)$
x
O
−7
(−3; 0)
$1\frac{1}{2}$
5
(−7; −2)
y = 2x − 3
(−2; −7)


y
h
y = x
$\mathbf{h^{-1}}$
O
x

2. The graph of $f(x) = -\sqrt{27x}$ for $x \geq 0$ is sketched alongside.



The point P(3; –9) lies on the graph of f.

2.1 Use the graph to determine the values of x for which $f(x) \geq -9$. (2)

2.2 Write down the equation of f^{-1} in the form y = . . . Include ALL restrictions. (3)

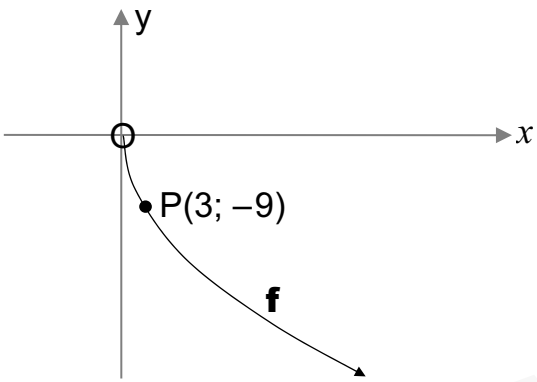
2.3 Sketch f^{-1}, the inverse of f on the graph above. Indicate the intercept(s) with the axes and coordinates of ONE other point. (3)

2.4 Describe the transformation from f to g if $g(x) = \sqrt{27x}$, where $x \geq 0$. (1) [9]

Gr 12 Maths 2 in 1
p. 149 (Q5)

QUESTION

2. The graph of $f(x) = -\sqrt{27x}$ for $x \geq 0$ is sketched alongside.



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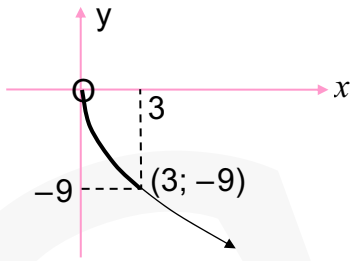
2.4 Describe the transformation from f to g if $g(x) = \sqrt{27x}$, where $x \geq 0$. (1) [9]

MEMO

2.1 **$0 \leq x \leq 3$** ◄



2.2 The equation of f: $y = -\sqrt{27x}$ for $x \geq 0$

$\therefore$ The equation of f^{-1}: $x = -\sqrt{27y}$ for $y \geq 0$ . . .

we swop x and y

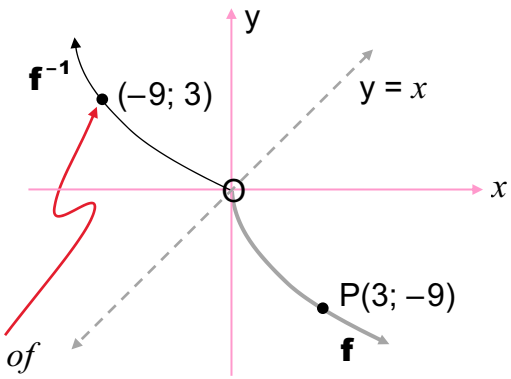
$\therefore x^2 = 27y$; but remember: $x \leq 0$

$\div 27)$ $\therefore y = \dfrac{x^2}{27}$ . . . *or* $y = \dfrac{1}{27}x^2$

$\therefore$ **$y = \dfrac{x^2}{27}$ for $x \leq 0$** ◄

2.3

Note: f^{-1} is the reflection of f in the line $y = x$



This point is a reflection of (3; –9) in the dashed line

2.4 **A reflection in the x-axis** ◄ . . . $(x; y) \rightarrow (x; -y)$

Finance

Gr 10 & Gr 11 SIMPLE and COMPOUND GROWTH and DECAY

$$A = P(1 \pm in) \qquad A = P(1 \pm i)^n$$

Gr 11 NOMINAL and EFFECTIVE INTEREST RATES

The formula: $1 + i_{eff} = \left(1 + \frac{i_{nom}}{m}\right)^m$

Gr 12 FUTURE AND PRESENT VALUE ANNUITIES

$$F_v = \frac{x\left[(1+i)^n - 1\right]}{i} \qquad P_v = \frac{x\left[1 - (1+i)^{-n}\right]}{i}$$

Finance

Gr 12 Maths 2 in 1
p. 150 (Q7.1)

1. A business buys a machine that costs R120 000. The value of the machine depreciates at 9% per annum according to the diminishing-balance method.

   1.1 Determine the scrap value of the machine at the end of 5 years. (3)

   1.2 After five years the machine needs to be replaced. During this time, inflation remained constant at 7% per annum. Determine the cost of the new machine at the end of 5 years. (3)

   1.3 The business estimates that it will need R90 000 by the end of five years.
   A sinking fund for R90 000, into which equal monthly instalments must be paid, is set up. Interest on this fund is 8,5% per annum, compounded monthly. The first payment will be made immediately and the last payment will be made at the end of the 5-year period.

   Calculate the value of the monthly payment into the sinking fund. (5)

2. Lorraine receives an amount of R900 000 upon her retirement.
She invests this amount immediately at an interest rate of 10,5% per annum, compounded monthly.

She needs an amount of R18 000 per month to maintain her current lifestyle. She plans to withdraw the first amount at the end of the first month.

For how many months will she be able to live from her investment? (6) [17]

QUESTION

1. A business buys a machine that costs R120 000.
   The value of the machine depreciates at 9% per annum according to the diminishing-balance method.

   1.1 Determine the scrap value of the machine at the end of 5 years. (3)

   1.2 After five years the machine needs to be replaced. During this time, inflation remained constant at 7% per annum. Determine the cost of the new machine at the end of 5 years. (3)

   1.3 The business estimates t hat it will need R90 000 by the end of five years. A sinking fund for R90 000, into which equal monthly instalments must be paid, is set up. Interest on this fund is 8,5% per annum, compounded monthly. The first payment will be made immediately and the last payment will be made at the end of the 5-year period.

   Calculate the value of the monthly payment into the sinking fund. (5)

MEMO

1.1 **P** = 120 000 ; **i** = 9% = 0,09 ; **n** = 5 ; **A**?

A = P(1 – i)^n ➡ A = 120 000(1 – 0,09)^5

= 120 000(0,91)^5

≃ **R74 883,86** ◄

OR: $F_V = P_V(1 - i)^n$

The 'reducing' (diminishing)-balance method.

1.2 **P** = 120 000 ; **i** = 7% = 0,07 ; **n** = 5 ; **A**?

A = P(1 + i)^n ➡ A = 120 000(1 + 0,07)^5

= 120 000(1,07)^5

≃ **R168 306,21** ◄

OR: $F_V = P_V(1 + i)^n$

1.3 **Note:** From 1.1 & 1.2 the (estimated) value of the sinking fund = R168 306,21 – R74 883,86 ≃ R93 422,35. So, R90 000 is close to that value!

n = 60

But we use n + 1 . . . *'beginning of first month and end of last month'.*

$$F_V = \frac{x[(1+i)^{n+1} - 1]}{i} = 90\,000$$

$$\therefore x.\frac{\left[\left(1 + \frac{0,085}{12}\right)^{61} - 1\right]}{\frac{0,085}{12}} = 90\,000$$

$\therefore x$ = R1 184,682...

$\therefore$ **Monthly payment = R1 184,68** ◄

OR: Store **i** and **1 + i** for convenience.

i = 8,5% ÷ 12 = 0,00708... . . . **STO**re in **A**

$\therefore$ **1 + I** = 1,00708... . . . **STO**re in **B**

$$\therefore F_V = x\frac{[\mathbf{B}^{61} - 1]}{\mathbf{A}} = 90\,000$$

$$\therefore x = \frac{90\,000\,\mathbf{A}}{\mathbf{B}^{61} - 1}$$

= R1 184,68

QUESTION

2. Lorraine receives an amount of R900 000 upon her retirement. She invests this amount immediately at an interest rate of 10,5% per annum, compounded monthly.

   She needs an amount of R18 000 per month to maintain her current lifestyle. She plans to withdraw the first amount at the end of the first month.

   For how many months will she be able to live from her investment? (6) [17]

MEMO

2. $\mathbf{P_V} = 900\,000$; $\mathbf{i} = 10{,}5\% \div 12 = 0{,}105 \div 12$;
   $x = 18\,000$; $\mathbf{n} = \mathbf{?}$

$$\mathbf{P_V} = \frac{\mathbf{x}\left[\mathbf{1 - (1+i)^{-n}}\right]}{\mathbf{i}} = 900\,000$$

$$\therefore \frac{18\,000\left[1 - \left(1 + \frac{0{,}105}{12}\right)^{-n}\right]}{\frac{0{,}105}{12}} = 900\,000$$

$$\div 18\,000) \quad \therefore \frac{\left[1 - \left(1 + \frac{0{,}105}{12}\right)^{-n}\right]}{\frac{0{,}105}{12}} = 50$$

$$\therefore 1 - \left(1 + \frac{0{,}105}{12}\right)^{-n} = \frac{7}{16}$$

$$\therefore 1 - \frac{7}{16} = \left(1 + \frac{0{,}105}{12}\right)^{-n}$$

$$\therefore \frac{9}{16} = \left(1 + \frac{0{,}105}{12}\right)^{-n}$$

Invert: $$\therefore \left(1 + \frac{0{,}105}{12}\right)^{n} = \frac{16}{9}$$

$$\therefore n \log\left(1 + \frac{0{,}105}{12}\right) = \log\frac{16}{9}$$

$$\therefore n = 66{,}043...$$

$\therefore$ **For 66 months** ◄

OR: Store **i** and **1 + i** for a neater calculation.

$\mathbf{i} = 10{,}5\% \div 12 = 0{,}008\,75$ . . . **STO**re in **A**

$\therefore \mathbf{1 + i} = 1{,}008\,75$ . . . **STO**re in **B**

$$\therefore \mathbf{P_V} = \frac{\mathbf{x}\left[\mathbf{1 - (1+i)^{-n}}\right]}{\mathbf{i}} = 900\,000$$

$$\therefore 18\,000 . \frac{\left[1 - \mathbf{B}^{-n}\right]}{\mathbf{A}} = 900\,000$$

$$\times \frac{\mathbf{A}}{18\,000}) \quad \therefore 1 - \mathbf{B}^{-n} = 50\mathbf{A}$$

$$\therefore 1 - 50\mathbf{A} = \mathbf{B}^{-n}$$

$$\therefore -n = \log_B(1 - 50\mathbf{A}) \quad ...$$

$N = b^x$
$\Rightarrow \log_b N = x$

$$= \frac{\log(1 - 50\mathbf{A})}{\log \mathbf{B}}$$

$$= -66{,}043...$$

$$\therefore n \simeq 66$$

$\therefore$ **For 66 months** ◄

FINANCE: GRADE 10

▶ FINANCE & GROWTH [14]

QUESTION 4

4.1 Thando has R4 500 in his savings account. The bank pays him a compound interest rate of 4,25% p.a. Calculate the amount Thando will receive if he decides to withdraw the money after 30 months. (3)

4.2 The following advertisement appeared with regard to buying a bicycle on a hire-purchase agreement loan:

Purchase price	*R5 999*
Required deposit	*R600*
Loan term	*Only 18 months, at 8% p.a. simple interest*

4.2.1 Calculate the monthly amount that a person has to budget for in order to pay for the bicycle. (6)

4.2.2 How much interest does one have to pay over the full term of the loan? (1)

4.3 The following information is given:

1 ounce = 28,35 g

$1 = R8,79

Calculate the rand value of a 1 kg gold bar, if 1 ounce of gold is worth $978,34. (4) [14]

MEMO

4.1 **P** = 4 500; **i** = $\frac{4,25}{100}$ = 0,0425;

n = $\frac{30}{12} = 2\frac{1}{2}$; **A**?

A = P(1 + i)^n

= $4\,500(1 + 0,0425)^{2,5}$

= **R4 993,47** ◄

4.2.1 The loan amount = R5 999 – R600

= R5 399

The accumulated amount, **A = P(1 + in)**

where **P** = 5 399; **i** = 8% = 0,08;

n = $1\frac{1}{2}$ years; **A**?

$\therefore$ **A** = $5\,399\left[1 + (0,08)\left(\frac{3}{2}\right)\right]$

= R6 046,88

$\therefore$ The monthly amount to be paid

= $\frac{6\,046,88}{18}$

= **R335,94** ◄

4.2.2 **The amount of interest**

= The total amount paid over the 18 months – the loan amount

= R6 046,88 – R5 399

= **R647,88** ◄

4.3 28,35 g is worth $978,34 = R978,34 × 8,79

= R8 599,61

$\therefore$ 1 g is worth $\frac{R8\,599,61}{28,35}$

$\therefore$ 1 kg is worth R$\frac{8\,599,61}{28,35} \times 1\,000$

… 1 kg = 1 000 g

≈ **R303 337,16** ◄

▶ FINANCE, GROWTH AND DECAY [18]

QUESTION 4

4.1 Melissa has just bought her first car. She paid R145 000 for it. The car's value depreciates on the straight-line method at a rate of 17% per annum.

Calculate the value of Melissa's car 5 years after she bought it. (2)

4.2 An investment earns interest at a rate of 8% per annum compounded quarterly.

4.2.1 At what rate is interest earned each quarter of the year? (1)

4.2.2 Calculate the effective annual interest rate on this investment. (2)

4.3 R14 000 is invested in an account. The account earns interest at a rate of 9% per annum compounded semi-annually for the first 18 months and thereafter 7,5% per annum compounded monthly.

How much money will be in the account exactly 5 years after the initial deposit? (5) [10]

MEMO

4.1 **A = P(1 - in)** . . .

Formula for **depreciation** on the **straight-line** method.

A? ; **P** = R145 000 ; **i** = 17% = $\frac{17}{100}$ = 0,17 ;

n = 5

$\therefore$ A = 145 000[1 - (0,17)(5)]

= **R21 750** ◄

4.2.1 The rate earned quarterly,

$\mathbf{i} = \frac{8\%}{4} = \mathbf{2\%} = \mathbf{0{,}02}$ ◄

4.2.2 $\mathbf{1 + i_{eff}} = \left(\mathbf{1 + \frac{i_{nom}}{4}}\right)^{4}$ . . . *Note:* $i_{nom} = 8\%$

$= (1 + 0{,}02)^4$

$= (1{,}02)^4$

$= 1{,}08243\ldots$

$\therefore$ $\mathbf{i_{eff}}$ = 0,08243…

≈ **8,24% per annum** ◄

Note:

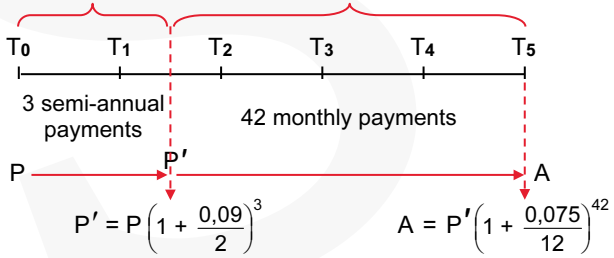
$A = P\mathbf{(1 + i_{eff})^1}$

and

$A = P\left(\mathbf{1 + \frac{i_{nom}}{4}}\right)^{4}$

4.3

semi-annually	monthly
$\mathbf{i} = \frac{9\%}{2} = \frac{0{,}09}{2}$	$\mathbf{i} = \frac{7{,}5\%}{12} = \frac{0{,}075}{12}$
n = 3	**n** = 42



P = R14 000

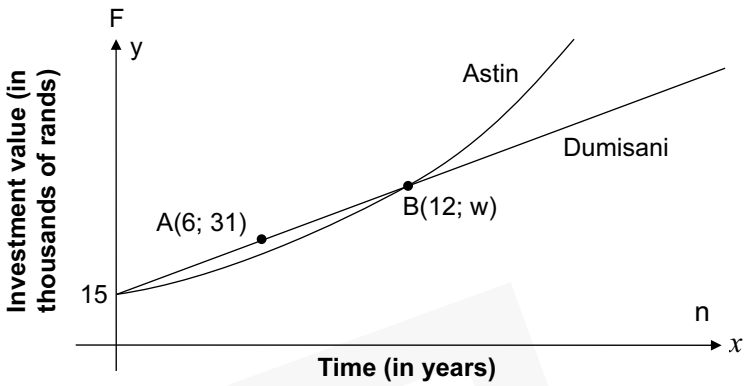
$\therefore$ The accumulated amount, A

$= \text{R}14\,000\left(1 + \frac{0{,}09}{2}\right)^{3}\left(1 + \frac{0{,}075}{12}\right)^{42}$

≈ **R20 755,08** ◄

QUESTION 5

The graphs below represent the growth of two investments, one belonging to Dumisani and one belonging to Astin. Both investments earn interest annually (only).



5.1 What is the value of both initial investments? (1)

5.2 Does Dumisani's investment earn simple or compound interest? (1)

5.3 Determine Dumisani's interest rate. (2)

5.4 Hence or otherwise, calculate the interest rate on Astin's investment. Give your answer correct to ONE decimal place. (4) [8]

MEMO

5.1 The value (of both investments) at the start (i.e. at $x = 0$) = **R15 000** ◄

5.2 **Simple interest** ◄ . . . *straight-line appreciation*

5.3 **i?** ; **P** = R15 000 ; **n** = 6 ; **A** = R31 000 See point A

A = P(1 + in)

$\therefore$ 31 000 = 15 000[1 + (i)(6)]

÷ 15 000) $\therefore$ $1 + 6i = 2{,}0\dot{6}$

$\therefore$ $6i = 1{,}0\dot{6}$

$\therefore$ $i = 0{,}1\dot{7}$

$\therefore$ $\boldsymbol{i}$ **= 17,78%** ◄

5.4 Determine w:

(12; w) is a point on Dumisani's graph.

$\therefore$ Substitute n = 12 ; P = R15 000 ; i = 17,777... in

A = P(1 + in) . . . *Dumisani's formula*

$\therefore$ w = $15[1 + (0{,}1\dot{7})(12)]$

$\approx$ 47

Note: A, P and w represent 'thousands of rands'

Substitute point B(12; 47) in

A = P(1 + i)^n . . . *Astin's formula*

$\therefore$ $47 = 15(1 + i)^{12}$

$\therefore$ $(1 + i)^{12} = 3{,}1\dot{3}$

$\therefore$ $1 + i = 1{,}09985...$

$\therefore$ $i = 0{,}09985...$

= **10,0%** ◄

▶ FINANCE, GROWTH AND DECAY [16]

QUESTION 7

Siphokazi bought a house. She paid a deposit of R102 000, which is equivalent to 12% of the selling price of the house. She obtained a loan from the bank to pay the balance of the selling price. The bank charges her interest of 9% per annum, compounded monthly.

7.1 Determine the selling price of the house. (1)

7.2 The period of the loan is 20 years and she starts repaying the loan one month after it was granted. Calculate her monthly instalment. (4)

7.3 How much interest will she pay over the period of 20 years? Round your answer correct to the nearest rand. (2)

MEMO

7.1 12% of the selling price = R102 000

∴ 1% of the selling price = R102 000 ÷ 12

∴ 100% of the selling price = (R102 000 ÷ 12) × 100

= **R850 000** ◄

7.2 The balance of the selling price = R748 000 (= the loan)

Method 1: Present value

This is the quicker method!

$$\mathbf{P_v} = \frac{\mathbf{x}\left[\mathbf{1-(1+i)^{-n}}\right]}{\mathbf{i}} \quad \text{where } \mathbf{P_v} = \text{R748 000};\ x?$$

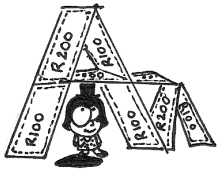
$$\mathbf{i} = \frac{9\%}{12} = \frac{0{,}09}{12};\ \mathbf{n} = 20 \times 12 = 240$$

$$\therefore\ 748\,000 = \frac{x\left[1-\left(1+\frac{0{,}09}{12}\right)^{-240}\right]}{\frac{0{,}09}{12}} = x.\mathbf{A}$$

STOre 111,144954 in **A**

$$\therefore\ x = \frac{748\,000}{\mathbf{A}}$$

$$\simeq \mathbf{R6\ 729{,}25}$$ ◄



Method 2: Future value

The Future value of the loan:

$$\therefore\ \mathbf{F_v} = \mathbf{P_v(1+i)^n} \quad \text{where } \mathbf{P_v} = \text{R748 000};$$

$$\mathbf{n} = 20 \times 12 = 240$$

$$= 748\,000\left(1+\frac{0{,}09}{12}\right)^{240} \quad \text{and } \mathbf{i} = \frac{9\%}{12} = \frac{0{,}09}{12}$$

= R4 494 845,34 → STOre in **A**

$$\text{and } \mathbf{F_v} = \frac{\mathbf{x}\left[\mathbf{(1+i)^n - 1}\right]}{\mathbf{i}}$$

$$= \frac{x\left[\left(1+\frac{0{,}09}{12}\right)^{240} - 1\right]}{\frac{0{,}09}{12}}$$

$$= x.\mathbf{B}$$

$$\therefore\ x = \frac{\mathbf{A}}{\mathbf{B}}$$

$$\simeq \mathbf{R6\ 729{,}05}$$ ◄

7.3 The amount of interest

= The amount paid over 20 years – the original amount

= (240 × R6 729,95) – R748 000

= R1 615 188 – R748 000

= **R867 188** ◄

QUESTION 7 cont.

Siphokazi bought a house. She paid a deposit of R102 000, which is equivalent to 12% of the selling price of the house. She obtained a loan from the bank to pay the balance of the selling price. The bank charges her interest of 9% per annum, compounded monthly.

7.3 How much interest will she pay over the period of 20 years? Round your answer correct to the nearest rand. (2)

7.4 Calculate the balance of her loan immediately after her 85th instalment. (3)

MEMO, cont.

7.3 The amount of interest

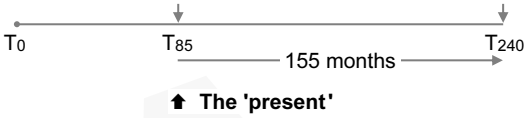
= The amount paid over 20 years – the original amount

= (240 × R6 729,95) – R748 000

= R1 615 188 – R748 000

= **R867 188** ◄

7.4



Method 1: Present value

After the 85th instalment,

the number of instalments remaining = 240 - 85 = 155

& the balance of the loan, then

$$= \frac{6\,729{,}95\left[1-\left(1+\frac{0{,}09}{12}\right)^{-155}\right]}{\frac{0{,}09}{12}}$$

= **R615 509,74** ◄

Method 2: Future value

At this stage:

The value of the loan,

- The amount owed ➡ . . . $A = 748\,000\left(1+\frac{0{,}09}{12}\right)^{85}$

  = 1 411 663,73 STOre in A

whereas:

The value of the annuity,

- The amount paid ➡ . . . $F_v = \dfrac{6\,729{,}95\left[\left(1+\frac{0{,}09}{12}\right)^{85}-1\right]}{\frac{0{,}09}{12}}$

  = R796 153,96 STOre in B

The balance of the loan = A – F_v = R615 509,77 ◄

= A – F_v = R615 509,77 ◄ . . . *the remaining amount to be paid*

QUESTION 7 cont.

Siphokazi bought a house. She paid a deposit of R102 000, which is equivalent to 12% of the selling price of the house. She obtained a loan from the bank to pay the balance of the selling price. The bank charges her interest of 9% per annum, compounded monthly.

7.5 She experienced financial difficulties after the 85th instalment and did not pay any instalments for 4 months (that is months 86 to 89).

Calculate how much Siphokazi owes on her bond at the end of the 89th month. (2)

7.6 She decides to increase her payments to R8 500 per month from the end of the 90th month.

How many months will it take to repay her bond after the new payment of 8 500 per month? (4)

[16]

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MEMO, cont.

7.5 The amount owed after month 89

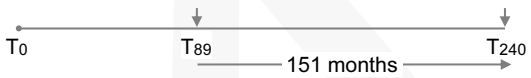
= The accrued amount for the months after month 85

$= R615\,509{,}74\left(1+\frac{0{,}09}{12}\right)^{4}$

Note: No payments were made, so there was nothing to subtract.

= **R634 183,81** ◄ . . . OR: R634 183,84 if the amount from Method 2 in 7.4 was used.

7.6



The present value of the annuity following month 89 must equal the amount owed at that stage.

$$\therefore \frac{8\,500\left[1-\left(1+\frac{0{,}09}{12}\right)^{-n}\right]}{\frac{0{,}09}{12}} = 634\,183{,}81$$

$$P_v = \frac{x\left[1-(1+i)^{-n}\right]}{i}$$ where $x = 8\,500$

$\times \frac{0{,}09}{12}$ and $\div 8\,500$:

$$\therefore 1-\left(1+\frac{0{,}09}{12}\right)^{-n} = 0{,}55957\ldots$$

$$\therefore 0{,}44042605 = \left(1+\frac{0{,}09}{12}\right)^{-n}$$

$$* \therefore -n = \frac{\log 0{,}44042605}{\log\left(1+\frac{0{,}09}{12}\right)}$$

$$= -109{,}744\ldots$$

$\therefore$ **n ≃ 110 months** ◄

$a = b^x$

➡ $x = \log_b a$

$\therefore x = \frac{\log a}{\log b}$

* OR:

$$\log 0{,}44042605 = \log\left(1+\frac{0{,}09}{12}\right)^{-n}$$ . . . A = B ➡ log A = log B

$$\therefore \log 0{,}44042605 = -n\log\left(1+\frac{0{,}09}{12}\right)$$ . . . $\log A^x = x\log A$

$$\therefore \frac{\log 0{,}44042605}{\log\left(1+\frac{0{,}09}{12}\right)} = -n$$

etc.

Probability Theory

The Definition of Probability:

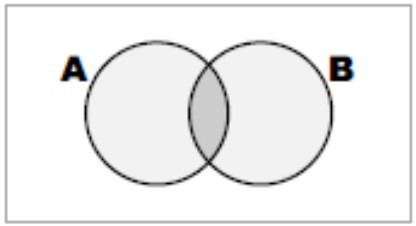
$$P(E) = \frac{n(E)}{n(S)}$$

i.e. Probability of an event E = $\frac{\text{the no. of ways E can occur}}{\text{total no. of possible outcomes}}$

THE PROBABILITY RULES

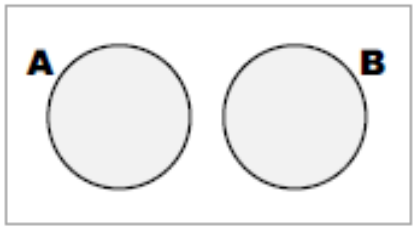
GENERAL RULE

- For **ANY 2 events** A and B:



$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

MUTUALLY EXCLUSIVE EVENTS



There is no overlap of events A & B.

- For 2 **mutually exclusive events** A and B:

$$P(A \text{ or } B) = P(A) + P(B) \quad \ldots \text{ THE SUM RULE}$$

Note: Since A and B do not intersect, **P(A and B) = 0** in this case.

INDEPENDENT EVENTS

- For 2 **independent** events A and B:

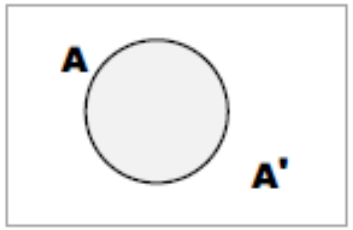
$$P(A \text{ and } B) = P(A) \times P(B) \quad \ldots \text{ THE PRODUCT RULE}$$

THE COMPLEMENTARY RULE

$$P(\text{not } A) = 1 - P(A)$$

Note: The sum of the probabilities

$$P(A) + P(A') = 1$$



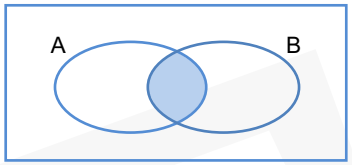
PROBABILITY: GRADE 10

▶ PROBABILITY [13]

QUESTION 5

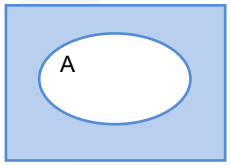
5.1 What expression BEST represents the shaded area of the following Venn diagrams?

5.1.1



(1)

5.1.2



(1)

5.2 State which of the following sets of events is mutually exclusive:

A Event 1: The learners in Grade 10 in the swimming team

Event 2: The learners in Grade 10 in the debating team

B Event 1: The learners in Grade 8

Event 2: The learners in Grade 12

C Event 1: The learners who take Mathematics in Grade 10

Event 2: The learners who take Physical Sciences in Grade 10 (1)

MEMO

5.1.1 $A \cap B$ ◄ [OR: A and B ◄]

5.1.2 A' ◄ [OR: not A ◄]

5.2 Set B ◄

QUESTION 5 cont.

5.3 In a class of 40 learners the following information is TRUE:

- 7 learners are left-handed
- 18 learners play soccer
- 4 learners play soccer and are left-handed
- All 40 learners are either right-handed or left-handed

Let L be the set of all left-handed people and S be the set of all learners who play soccer.

5.3.1 How many learners in the class are right-handed and do NOT play soccer? (1)

5.3.2 Draw a Venn diagram to represent the above information. (4)

5.3.3 Determine the probability that a learner is:

(a) left-handed or plays soccer (3)

(b) right-handed and plays soccer (2)

[13]

MEMO, cont.

5.3.1 Of the 40 learners, 7 are left-handed

$\therefore$ 40 – 7 = 33 are right-handed

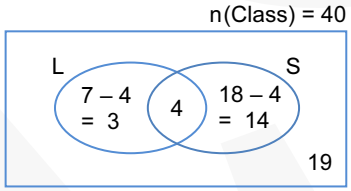
Of the 18 learners who play soccer, 4 are left-handed

$\therefore$ 14 learners who play soccer are right-handed

$\therefore$ The number of learners who are right-handed and DON'T play soccer

= 33 – 14 = 19 ◄

5.3.2



5.3.3 (a) n(L or S) = 3 + 4 + 14 = 21

$\therefore$ P(L or S) = $\frac{21}{40}$ ◄

(b) n(R and S) = 14 . . . *where R is the set of all right-handed people*

$\therefore$ P(R and S) = $\frac{14}{40}$

= $\frac{7}{20}$ ◄

▶ PROBABILITY [19]

QUESTION 11

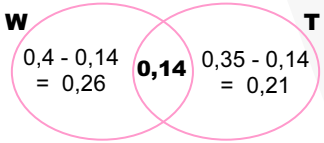
Given: P(W) = 0,4 P(T) = 0,35 P(T and W) = 0,14

11.1 Are the events W and T mutually exclusive?
Give reasons for your answer. (2)

11.2 Are the events W and T independent?
Give reasons for your answer. (3) [5]

MEMO

11.



Note:
W includes the part that overlaps with T (just as T includes the part that overlaps with W).
∴ The overlap needs to be subtracted to find W without T and T without W.

▶ Events A and B are mutually exclusive if:
- **P(A∩B) = 0, or** if:
- **P(A or B) = P(A) + P(B)**

▶ Events A and B are independent if:
- **P(A and B) = P(A) × P(B)**

11.1 **Method 1**

P(W and T) = 0,14 . . . *given*

∴ P(W∩T) ≠ 0

∴ **W and T are not mutually exclusive events** ◄

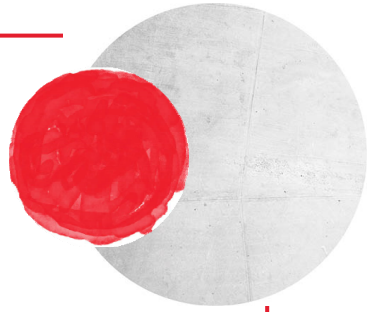
Method 2

P(W or T) = 0,26 + 0,14 + 0,21 = 0,61

P(W) + P(T) = 0,4 + 0,35 = 0,75 . . . *≠ 0,61*

∴ P(W or T) ≠ P(W) + P(T)

∴ **W and T are not mutually exclusive events** ◄



11.2 P(W and T) = 0,14 . . . *given*

P(W) × P(T) = (0,4)(0,35) = 0,14

∴ P(W and T) = P(W) × P(T)

∴ **W and T are independent events** ◄

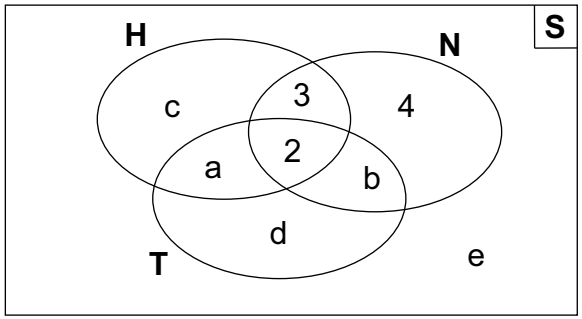
QUESTION 12

12.1 A group of 33 learners was surveyed at a school.
The following information from the survey is given:

- 2 learners play tennis, hockey and netball
- 5 learners play hockey and netball
- 7 learners play hockey and tennis
- 6 learners play tennis and netball
- A total of 18 learners play hockey
- A total of 12 learners play tennis
- 4 learners play netball ONLY

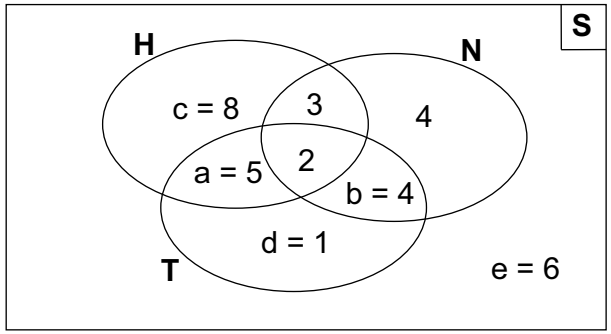
12.1.1 A Venn diagram representing the survey results is given below.

Use the information provided to determine the values of a, b, c, d and e. (5)



MEMO

12.



12.1.1 **a = 5** ◄ . . . line 3

b = 4 ◄ . . . line 4

Lines 1, 2 and 7 were not required for finding values a to e.

c = 8 ◄ . . . line 5, but after a is determined

d = 1 ◄ . . . line 6

e = 6 ◄ . . . e = n(S) – n(H∪T∪N)
= 33 – 27 . . . 33 learners were surveyed

Note: n(H∪T∪N) = 18 + 1 + 4 + 4 = 27

QUESTION 12 cont.

12.1.2 How many of these learners do not play any of the sports on the survey (that is netball, tennis or hockey)? (1)

MEMO, cont.

12.1.2 **6** ➤ . . . the value of e, the number not in H, T or N

Note:

In Question 12.1.2, the **number** of learners is required.

In Question 12.1.3 & 12.1.4, the **probability** is required.

NB: The probability of an event (E) occurring

$$= \frac{\text{the number of ways E can occur}}{\text{the total number of outcomes}}$$

12.1.3 Write down the probability that a learner selected at random from this sample plays netball ONLY. (1)

12.1.4 Determine the probability that a learner selected at random from this sample plays hockey or netball. (1)

MEMO, cont.

12.1.3 The **number** of learners playing netball ONLY = 4

∴ The **probability** that a learner plays netball only

$$= \frac{\text{the number that play netball only}}{\text{the total number of learners}}$$

$$= \frac{4}{33} \text{ ➤}$$

($\simeq$ 0,12)

12.1.4 The **number** of learners playing hockey or netball (or both) = 26 . . . n(H∪N)

∴ The **probability** that a learner plays hockey or netball (or both)

$$= \frac{n(H \cup N)}{n(S)}$$

$$= \frac{26}{33} (\approx 0{,}78) \text{ ➤}$$

QUESTION 12 cont.

12.2 In all South African schools, EVERY learner must choose to do either Mathematics or Mathematical Literacy.

At a certain South African school, it is known that 60% of the learners are girls. The probability that a randomly chosen girl at the school does Mathematical Literacy is 55%.

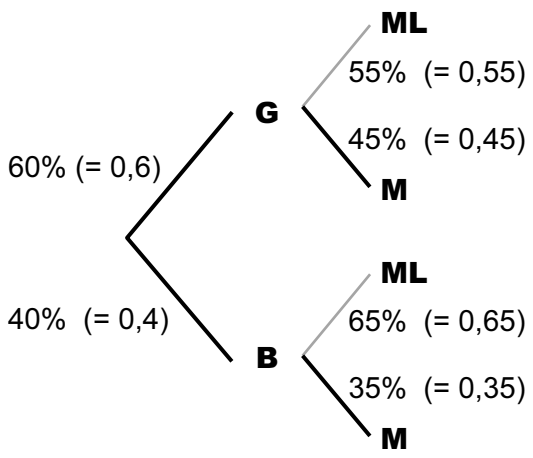
The probability that a randomly chosen boy at the school does Mathematical Literacy is 65%.

Determine the probability that a learner selected at random from this school does Mathematics. (6)

[14]

MEMO, cont.

12.2



P(a learner does Maths)

= P(a girl doing Maths) + P(a boy doing Maths)

= (60% × 45%) + (40% × 35%)

= 0,27 + 0,14

= **0,41** ✓ . . . = 41%

OR: Using decimals only:

P(M) = P(G and M) + P(B and M)

= (0,6 × 0,45) + (0,4 × 0,35)

= 0,27 + 0,14

= **0,41** ✓

Fundamental Counting Principle

QUESTION 1

Consider the word M A T H S.

1.1 How many different arrangements of the 5 letters (not excluding the given one) are possible if the letters may be repeated. (2)

1.2 How many different 5-letter arrangements can be made using all the above letters? (i.e. letters are not repeated) (2)

1.3 Determine the **probability** that the letters S and T will always be the first two letters of the arrangements in Question 1.2. (3) [7]

Resources:
Page Q7 (adapted) in
Gr 12 Exemplar I (Question 12)

SOLUTIONS

1.1 There are 5 (different) letters in the word MATHS $\therefore$ 5 slots to be filled:

_____ _____ _____ _____ _____

and, if they MAY be repeated, then there are 5 possibilities for each slot

5	5	5	5	5

$\therefore$ The number of ways the letters could be arranged is $5 \times 5 \times 5 \times \ldots$ 5 times

$= 5^5$

$=$ **3 125** ➤ . . . the power button on your calculator

1.2

5 choices	4 choices	3 choices	2 choices	1 choice

$\therefore$ The number of different 5-letter arrangements

$= 5 \times 4 \times 3 \times 2 \times 1$

$= 5!$

$=$ **120** ➤ . . . the factorial (x!) button on your calculator

1.3

2 ways	1 way	3 ways	2 ways	1 way
S or T				

$\therefore$ The number of different 5-letter arrangements

starting ST _ _ _ or TS _ _ _

$= 2! \times 3!$

$\therefore$ The PROBABILITY of this

$$= \frac{2! \times 3!}{120} \quad \ldots \quad \mathbf{P(E) = \frac{n(E)}{n(S)}}$$

$$= \frac{\mathbf{1}}{\mathbf{10}}$$

The Definition of Probability:

$$\mathbf{P(E) = \frac{n(E)}{n(S)}}$$

i.e. Probability of an event E $= \dfrac{\text{the no. of ways E can occur}}{\text{total no. of possible outcomes}}$

Fundamental Counting Principle, continued

QUESTION 2

Every client of CASHSAVE Bank has a personal identity number (PIN) which is made up of 5 digits chosen from the digits 0 to 9.

2.1 How many personal identity numbers (PINs) can be made if:

2.1.1 Digits can be repeated (2)

2.1.2 Digits cannot be repeated (2)

2.2 Suppose a PIN can be made up by selecting digits at random and that the digits can be repeated. What is the probability that such a PIN will contain at least one 9? (4) [8]

Gr 12 Maths 2 in 1
p. 149 (Q12)

2.1 How many personal identity numbers (PINs) can be made if:

2.1.1 Digits can be repeated (2)

2.1.2 Digits cannot be repeated (2)

MEMO

2.1.1 There are 10 digits from 0 to 9!

10 choices	10 choices	10 choices	10 choices	10 choices
❶	❷	❸	❹	❺

Because digits may be repeated, you have 10 choices for each position.

∴ The number of different PINS

$= 10 \times 10 \times 10 \times 10 \times 10 = 10^5 =$ **100 000** ◄

2.1.2

10 choices	9 choices	8 choices	7 choices	6 choices
❶	❷	❸	❹	❺

Because digits cannot be repeated, there is one less to choose from, each time.

∴ The number of different PINS

$= 10 \times 9 \times 8 \times 7 \times 6 =$ **30 240** ◄

Use the factorial on your calculator to check your answer.

Or, expressed in factorial notation:

$$\frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{5 \times 4 \times 3 \times 2 \times 1} = \frac{10!}{5!} \quad \ldots = \frac{10!}{(10-5)!}$$

the number of places

2.2 Suppose a PIN can be made up by selecting digits at random and that the digits can be repeated. What is the probability that such a PIN will contain at least one 9? (4) [8]

MEMO

2.2 The probability of a digit not being 9 $= \frac{9}{10}$

∴ The probability of no digits being 9 $= \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10}$

$= (0{,}9)^5$

$= 0{,}59$

with repeats

∴ The probability of at least one digit being 9

$= 1 - 0{,}59 =$ **0,41** ◄

OR: The number of PINs where no digit is 9

$= 9 \times 9 \times 9 \times 9 \times 9$

$= 59\ 049$

with repeats

The total number of PINs, with repeats, is 100 000 (see 2.1.1)

∴ $P(\text{no 9's}) = \frac{\text{The no. of PINs where no digit is 9}}{\text{The total no. of PINs}}$

$= \frac{59\ 049}{100\ 000}$

$\simeq 0{,}59$

∴ $P(\text{at least one 9}) = 1 - P(\text{no 9's})$

$= 1 - 0{,}59 =$ **0,41** ◄

Patterns & Sequences

- **Gr 10 Maths 3-in-1:**

  Module 1: pp 1.7 – 1.14 & Exam p. E1

- **Gr 11 Maths 3-in-1:**

  Module 4: pp 4.1 – 4.10 & Exam Q2

  NB: Note page 4.8 !

- **Gr 12 Maths 2-in-1:**

  Module 2: pp. 4 – 7

  Topic guide: p. 147

  & Level 3 & 4 Challenging Questions booklet: pp. 2 & 3

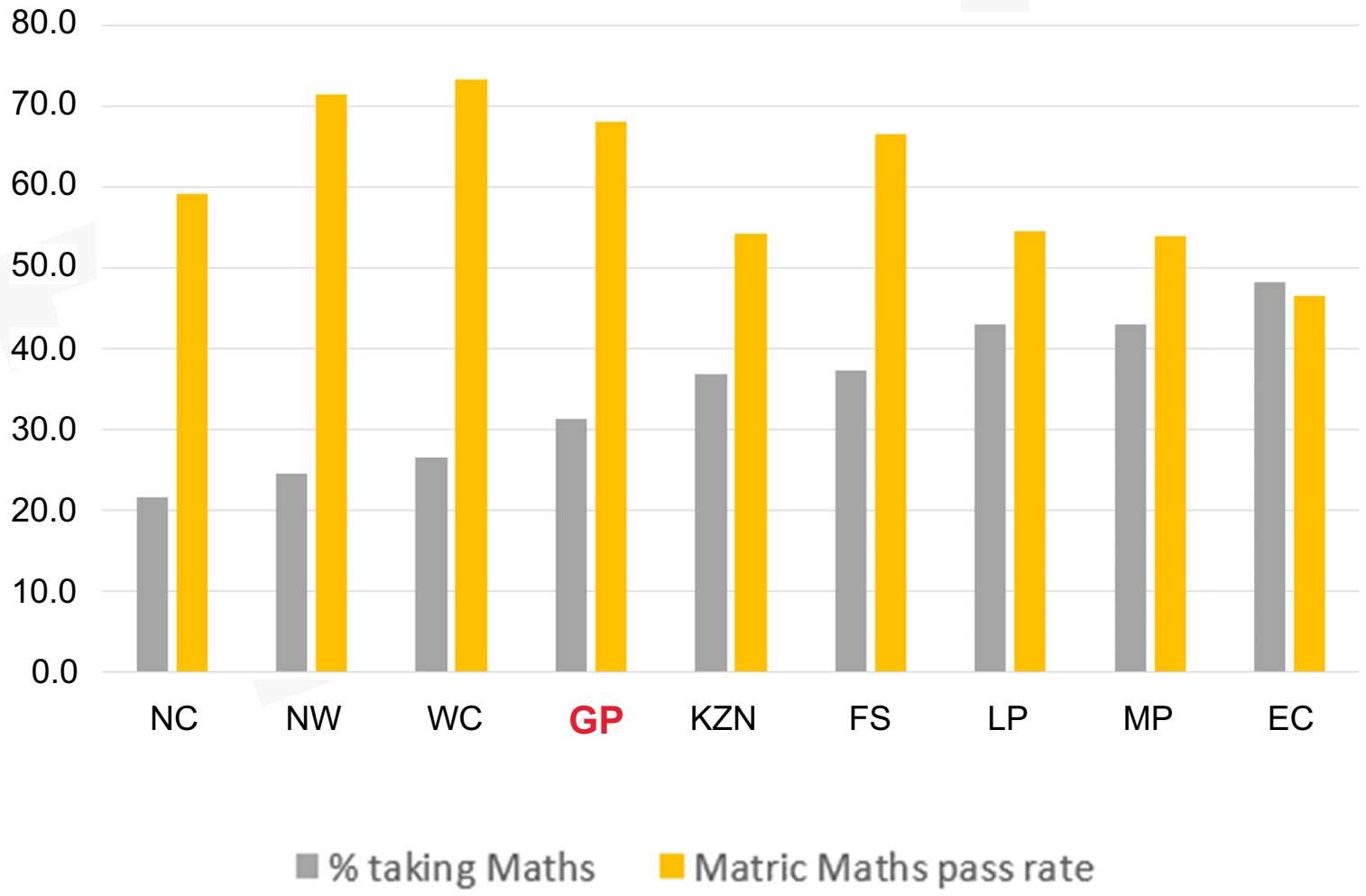
- **Gr 12 Maths PAST PAPERS TOOLKIT:**

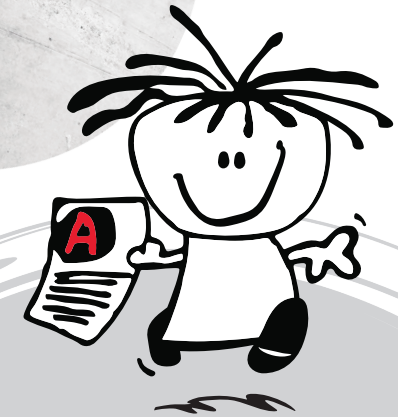
  DBE Paper 1s Topic Guide: p. 1

  DBE Paper 1s Topic Guide: p. 39

  Bookwork on APs & GPs: page i

GRAPH SHOWING THE HIGH PERCENTAGE OF LEARNERS WHO ARE INEPT AT MATHS OR OUR FAILURE TO TEACH THE SUBJECT SUCCESSFULLY (2021).





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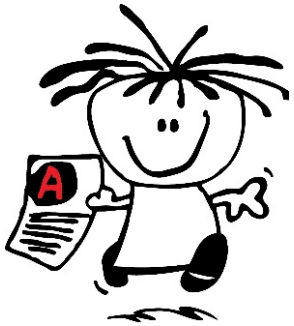
- engage learners eagerly as they participate and thrive on their maths journey
- accommodate all cognitive levels

The questions and detailed solutions have been provided in



SECTION 1: Separate topics

It is important that learners focus on and master one topic at a time BEFORE attempting 'past papers' which could be bewildering and demoralising. In this way they can develop confidence and a deep understanding.

SECTION 2: Exam Papers

When learners have worked through the topics and grown fluent, they can then move on to the exam papers to experience working through a variety of questions in one session, and to perfect their skills.

There are **TOPIC GUIDES** which enable learners to continue mastering one topic at a time, even when working through the exam papers.

PLUS, . . .

NEW

EXTENSION SECTION

CHALLENGING QUESTIONS & MEMOS

These questions are Cognitive Level 3 & 4 questions, diagnosed as such following poor performance of learners in recent examinations.

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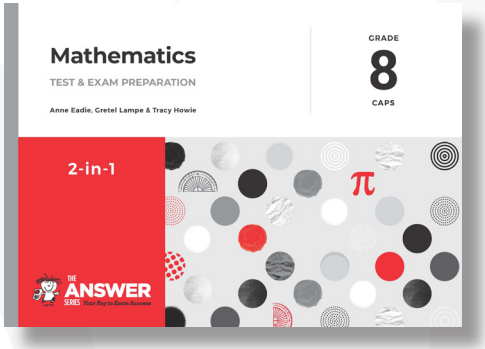
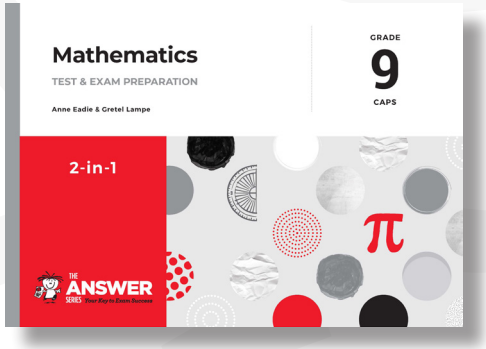
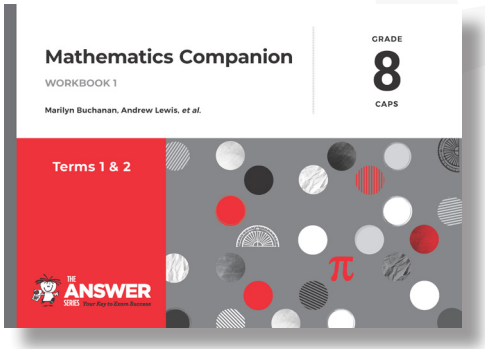
MATHEMATICS: Senior Phase

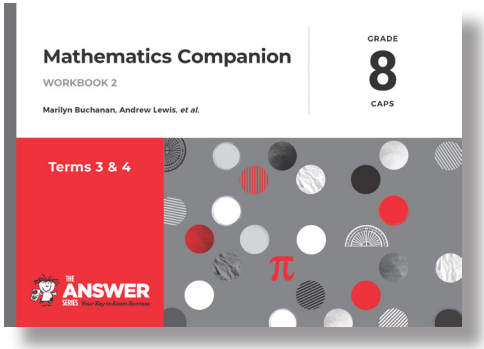
GRADE 8 & GRADE 9 '2 IN 1' MATHS

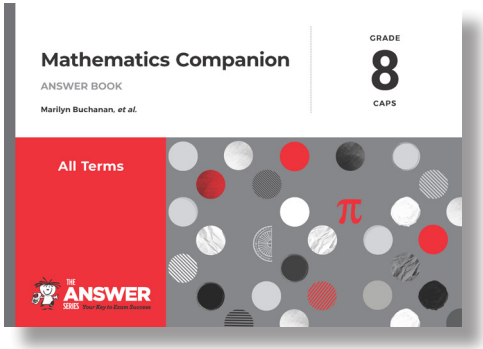




GRADE 8 & GRADE 9 MATHS COMPANION







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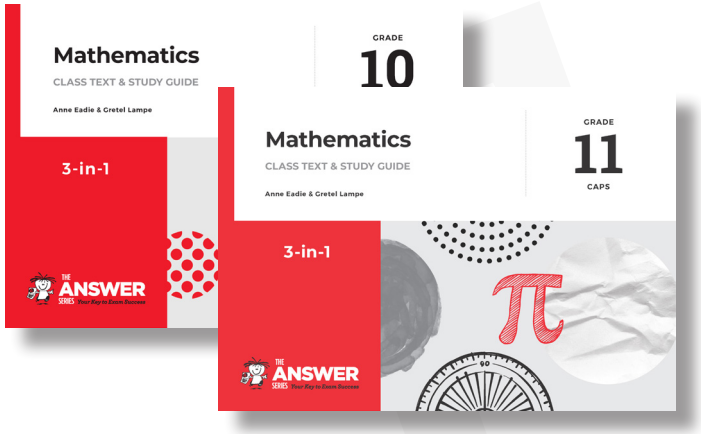
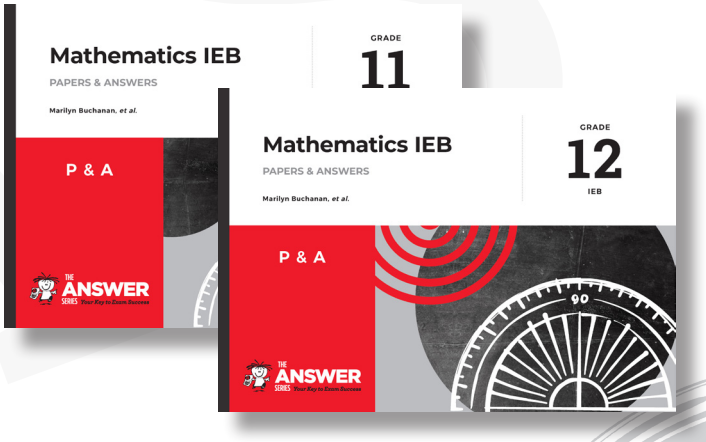
MATHEMATICS: FET

GRADE 10 & GRADE 11 MATHS '3 IN 1'

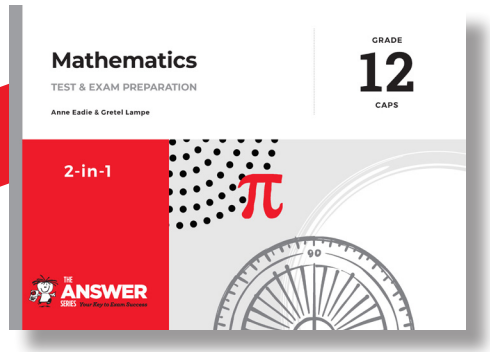


GRADE 11 & GRADE 12 MATHS 'P & A'



GRADE 12 MATHS '2 IN 1' (Extended)

EXTENDED



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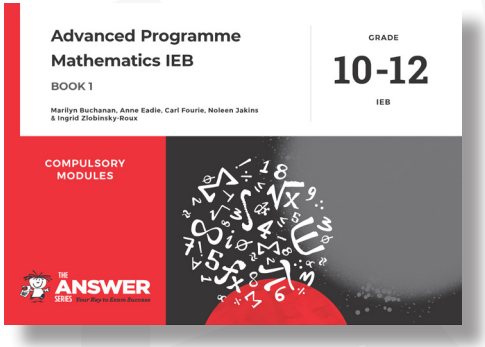
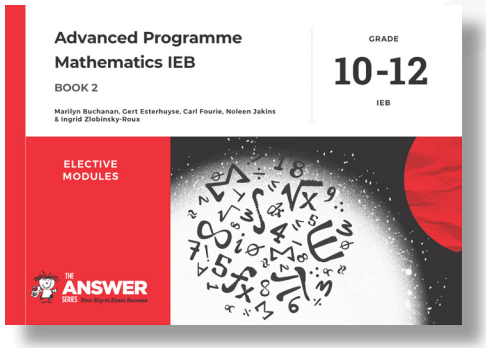
FURTHER STUDIES MATHEMATICS

STANDARD LEVEL BOOK 1



EXTENDED LEVEL BOOK 2



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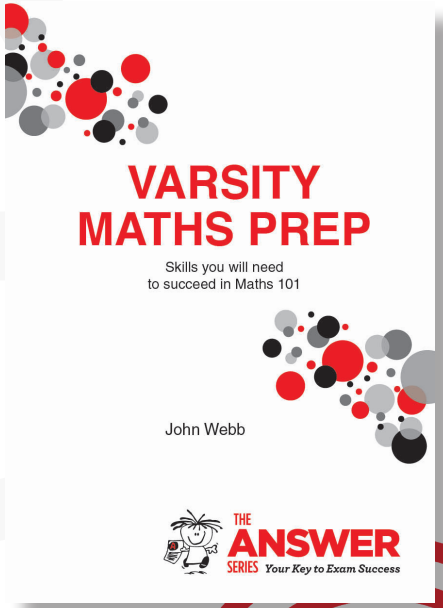
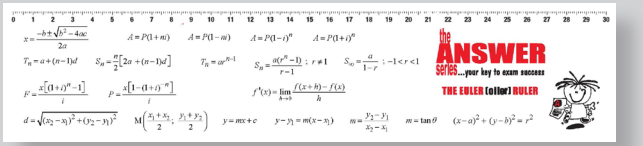


GRADE 12 EULER RULER



This 30 cm × 7 cm ruler includes all the information provided on the formula sheet in the National or IEB matric exam. Ready access to this information will ensure that learners become familiar with applying it successfully.

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Please submit your feedback by clicking on the link in the chat.

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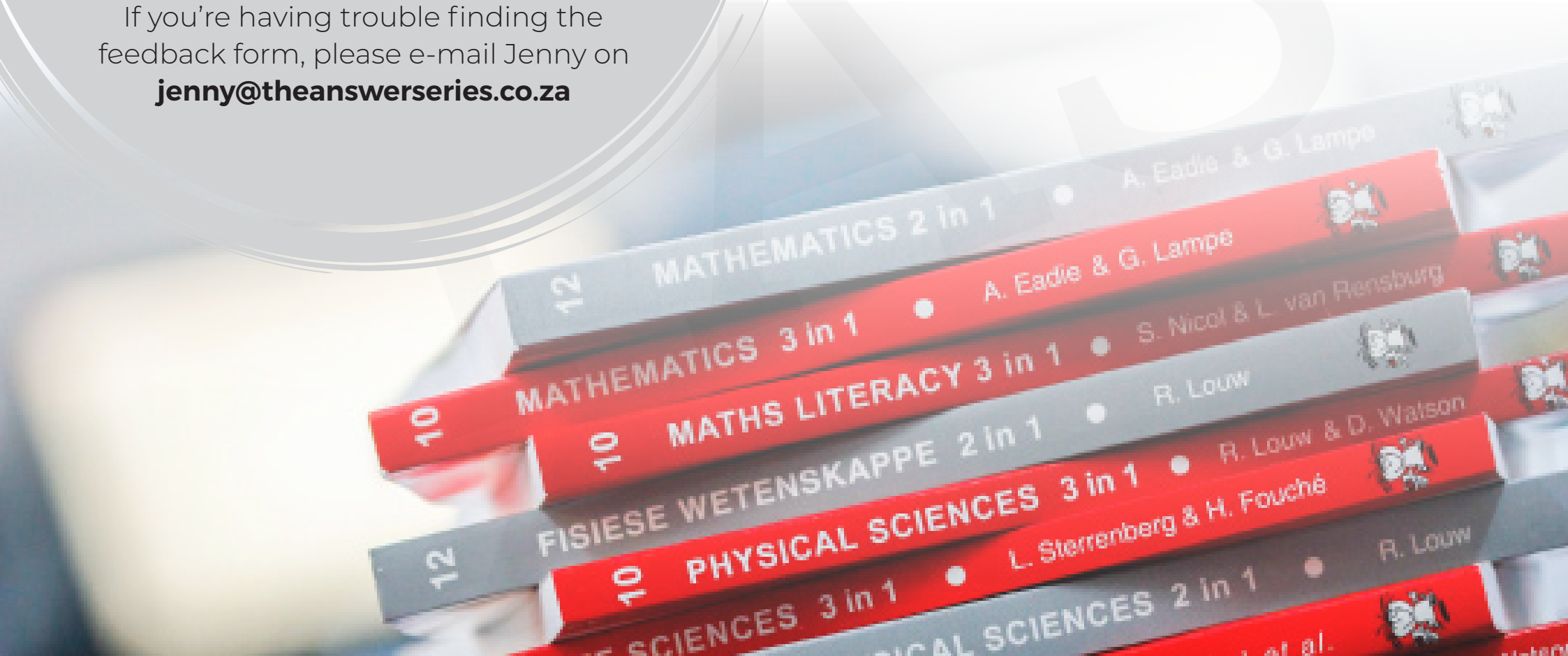
THANK YOU



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