Mathematics Companion

WORKBOOK 2

Marilyn Buchanan, *et al.*

GRADE

9

CAPS

Terms 3 & 4

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1 Exercises

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Gr 9 Maths Companion - Schedule of work

WORKBOOK 1

TERM 1

UNIT	TOPIC	CAPS TOPIC NR.	WEEKS	PAGE
1	The Number System	1.1	1	1
2	Rate, Ratio and Proportion	1.1	1	22
3	Financial Maths	1.1	1	51
4	Integers	1.3	1	65
5	Common Fraction Revision	1.4	0,5	73
6	Decimals Revision	1.5	0,5	78
7	Algebra: Exponents	1.2	2	82
8	Numeric and Geometric Patterns	2.1	1	102
9	Functions and Relations Part 1	2.2	1	112
10	Algebraic Expressions Part 1	2.3	1	121
11	Equations Part 1	2.4	1	141
			11 weeks	

TERM 2

UNIT	TOPIC	CAPS TOPIC NR.	WEEKS	PAGE
12	Geometry Part 1: Lines and Angles	3.3	2	155
13	Constructions Part 1: Angles and Triangles	3.5	1	164
14	Constructions Part 2: Quadrilaterals	3.5	1	181
15	Congruency	3.1	1	192
16	Similarity	3.1	1	202
17	The Theorem of Pythagoras	4.3	1	215
18	2D Shapes: Perimeter and Area	4.1	1	228
			8 weeks	

WORKBOOK 2

TERM 3

UNIT	TOPIC	CAPS TOPIC NR.	WEEKS	PAGE
19	Functions and Relations Part 2	2.2	0,5	244
20	Algebraic Expressions Part 2	2.3	1	252
21	Factorisation	2.3	2	260
22	Equations Part 2	2.4	1	273
23	Graphs	2.5	2,5	279
24	3D Shapes: Surface Area and Volume	4.2	1	308
			8 weeks	

TERM 4

UNIT	TOPIC	CAPS TOPIC NR.	WEEKS	PAGE
25	Transformations: Translations, Reflections and Enlargements	3.4	2	321
26	Geometry of 3D Objects	3.2	1	336
27	Data Handling (Statistics)	5.1, 5.2 & 5.3	2,5	345
28	Probability	5.4	1,5	362
			7 weeks	

EXAM PAPERS

	PAPERS	MEMOS
Paper A	Q1	M1
Paper B1	Q8	M10
Paper B2	Q16	M15

7. Consider the following set of diagrams (created using matchsticks), which together form the beginning of a pattern.

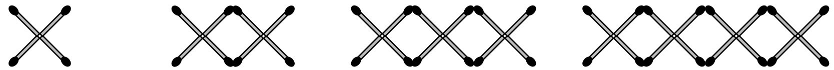


Figure 1 **Figure 2** **Figure 3** **Figure 4**

7.1 Consider the number of matchsticks in each figure.

7.1.1 Complete the following table.

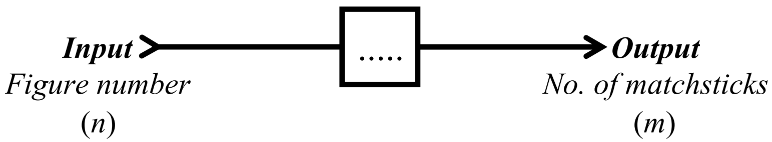
Figure number (n)	1	2	3	4
No. of matchsticks (m)	4	8	12	

7.1.2 Describe in words the relationship between the figure number and the number of matchsticks.

....

....

7.1.3 Complete the following flow diagram for this pattern.



7.1.4 Determine a formula for finding m (the number of matchsticks) when you are given n (the figure number). Write your formula in the form $m = ...$

....

7.1.5 How many matchsticks will appear in Figure 25?

....

7.1.6 Determine a formula for finding n (the figure number) when you are given m (the number of matchsticks). Write your formula in the form $n = ...$

....

7.1.7 Determine the number of the figure which will consist of exactly 120 matchsticks.

....

7.2 Continuing with the same figures, we now consider the number of squares that are formed in each figure.

7.2.1 Complete the following table.

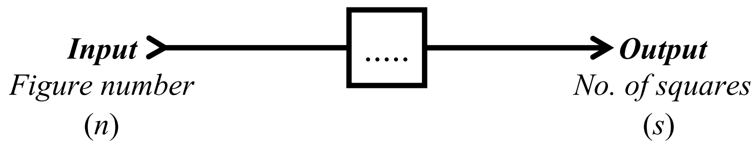
Figure number (n)	1	2	3	4
No. of squares (s)	0	1		

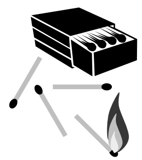
7.2.2 Describe in words the relationship between the figure number and the number of squares.

....

....

7.2.3 Complete the following flow diagram for this pattern.





1.7 $(a-3)(a+3)$

= ....

1.8 $(k+5)(k-5)$

= ....

1.9 $(3x+2)(3x-2)$

= ....

1.10 $(4p-3)(4p+3)$

= ....

1.11 $(2x-3y)(2x+3y)$

= ....

1.12 $(3x+5y)(3x-5y)$

= ....

1.13 $(x+y)(x+y)$

= ....

= ....

1.14 $(3x-2)(3x-2)$

= ....

= ....

1.15 $(a+3)^2$

= ....

= ....

= ....

1.16 $(a+5)^2$

= ....

1.17 $(x-7)^2$

= ....

1.18 $(2x-1)^2$

= ....

1.19 $(5x+3)^2$

= ....

1.20 $(2a+5b)^2$

= ....

2. Simplify each of the following expressions.

2.1 $(3x+2)(x-1)-3x(x+1)$

= ....

= ....

2.2 $(5p+3)(p-2)-p(2p-7)$

= ....

= ....

2.3 $(x+y)^2-x(x-y)$

= ....

= ....

2.4 $(3x+2y)^2-(3x+2y)(3x-2y)$

= ....

= ....

= ....

2.5 $3(x+1)(x+2)-(x-1)^2$

= ....

= ....

= ....

EXERCISE 21.9

Some of the following trinomials have a positive constant term, while others are negative. Be careful to take note of this before factorising each one into the product of two binomials.

1. $a^2 - 2a - 8$

 = ...

2. $b^2 + 2b - 8$

 = ...

3. $m^2 - 7m - 8$

 = ...

4. $n^2 + 7n - 8$

 = ...

5. $p^2 + 6p + 8$

 = ...

6. $q^2 - 7q + 12$

 = ...

7. $x^2 + 13x + 12$

 = ...

8. $y^2 - 13y + 12$

 = ...

9. $a^2 + 8a + 12$

 = ...

10. $b^2 - 8b + 12$

 = ...

11. $m^2 - 11m + 24$

 = ...

12. $n^2 + 11n + 24$

 = ...

13. $p^2 - 2p - 24$

 = ...

14. $q^2 + 2q - 24$

 = ...

15. $x^2 - 5x - 24$

 = ...

16. $y^2 + 5y - 24$

 = ...

17. $a^2 - 10a - 24$

 = ...

18. $b^2 + 10b - 24$

 = ...

FACTORISATION SUMMARY

We have practised three methods of factorisation:

1. TAKE OUT HCF
2. THE DIFFERENCE OF SQUARES
3. TRINOMIALS

and used ***the Golden Rule of Factorisation*** that says

> ***Always look for a common factor first, and take out the highest common factor.***

This also applies when working with trinomials.

Example 1

$2a^2 - 2a - 12$
$= 2(a^2 - a - 6)$
$= 2(a - 3)(a + 2)$

Example 2

$m^2n^2 + 2m^2n - 8m^2$
$= m^2(n^2 + 2n - 8)$
$= m^2(n + 4)(n - 2)$

Example 3

$-2p^2 + 12p - 10$
$= -2(p^2 - 6p + 5)$
$= -2(p - 5)(p - 1)$

EXERCISE 21.10

1. First take out the highest common factor before factorising each of the following trinomials.

1.1 $2a^2 - 6a + 4$

 = ...

 = ...

1.2 $2b^2 + 6b - 20$

 = ...

 = ...

1.3 $3m^2 - 12m + 9$

 = ...

 = ...

1.4 $5n^2 + 30n - 35$

 = ...

 = ...

UNIT 22
EQUATIONS PART 2

This first exercise is revision of the various types of equations that were solved in Unit 11.

EXERCISE 22.1

1. Solve the following linear equations:

1.1 $6x - 4 = 26$

1.2 $20 - 4x = 12$

1.3 $3x - 2 = x + 6$

1.4 $7x + 1 = x + 5$

1.5 $\frac{x}{2} + \frac{1}{4} = \frac{3}{4}$

1.6 $\frac{2x}{5} + 10 = 12$

1.7 $\frac{x+1}{12} + \frac{3x-2}{4} = \frac{4x-1}{3}$

1.8 $\frac{x}{4} + 3 = \frac{2-x}{3}$

1.9 $4(x + 1) = 3(x + 5)$

1.10 $4(x + 1) = 5 - 2(x - 3)$

2. Amongst the following equations there are some that have many, and some that have no solution. Consider these options when reaching your solution:

2.1 $4(x + 3) - 2 = 1 - 5(x - 2)$

2.2 $7(x + 1) - 2 = 5(x - 3)$

2.3 $7(x + 1) - 2 = 5(1 - x)$

2.4 $3(x + 2) - 1 = 3(x + 1) + 2$

2.5 $2x + 3(x - 2) = 5(x + 1) - 2$

2.6 $\frac{2}{x} + \frac{1}{4x} = 9$

- ***Given any two points on the Cartesian Plane***

We need to calculate the gradient using the formula, then substitute the coordinates of one of the given points to find the y-intercept.

Example 5

Determine the equation of the straight line which passes through the points (–2 ; 0) and (4 ; 3).

Solution

The equation will be of the form $y = mx + c$.

We first calculate the gradient.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-3}{-2-4} = \frac{-3}{-6} = \frac{1}{2}$$

The equation of the line is thus $y = \frac{1}{2}x + c$.

Into this equation we substitute the coordinates of either of the points which lie on the line.

Substitute (–2 ; 0) into the equation $y = \frac{1}{2}x + c$.

$$0 = \tfrac{1}{2}(-2) + c$$
$$\therefore\ 0 = -1 + c$$
$$\therefore\ 1 = c$$

Therefore the equation of the line is $y = \frac{1}{2}x + 1$.

Example 6

Determine the equation of the straight line passing through the points (3; 4) and (5; 2).

Solution

$$m = \frac{4-2}{3-5} = \frac{2}{-2} = -1$$

$y = -x + c$

(3; 4): $4 = -3 + c$

$7 = c$

$\therefore\ y = -x + 7$

EXERCISE 23.5

1. Write down the equation of each of the following straight line graphs, given the gradient and y-intercept.

1.1 $m = 3,\ \ c = 5$

....

1.2 $m = \frac{2}{5},\ \ c = -4$

....

1.3 $m = -\frac{1}{3},\ \ c = 2$

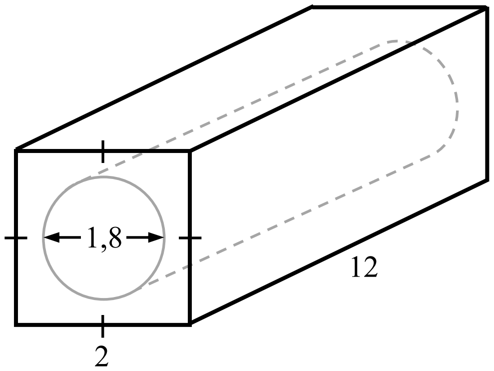
....

1.4 $m = -4,\ \ c = -1$

....

QUESTION 3

Concrete is cast having a circular pipe through it. The pipe has a diameter of 1,8 m and length = 12 m. The concrete around the pipe forms a square with sides of 2 m and is also 12 m long.



Calculate the volume of concrete to the nearest cubic metre.

[6]

QUESTION 4

A family has a rainwater tank in the shape of a cylinder with diameter of 2 m and height 3 m. The water in the tank is at the 80% mark.

4.1 Calculate the height of the water in the tank. (2)

4.2 Calculate the volume of water in the tank in cubic metres correct to 4 decimal places. (2)

4.3 Water from the rainwater tank can be moved to a filtration tank which is a rectangular prism 1,2 m long, 0,5 m wide and 1 m high.

4.3.1 Calculate the number of litres that this tank can filter at a time. (3)

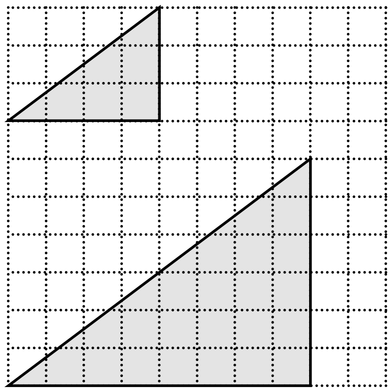
4.3.2 The family plans to use 1 load of the water from the filtration tank over a week.

Assuming that they consume an equal amount of water each day, calculate their allocation per day. (2)

4.3.3 If there is no rain during the dry season, determine in which week their rainwater tank will run out of water. (3)

[12]

TOTAL: [40]



Base: 4 units
Height: 3 units
Perimeter: 3 + 4 + 5 = 12 units
Area: 6 units^2

Base: 8 units
Height: 6 units
Perimeter: 6 + 8 + 10 = 24 units
Area: 24 units^2

The perimeter has increased by a factor of 2 and the area has increased by a factor of 4.

Example 4

Consider a right-angled triangle, with its base 10 cm in length and its height 24 cm. Using the theorem of Pythagoras, we can calculate the length of the hypotenuse to be 26 cm. The perimeter of this triangle is 60 cm and the area is 120 cm^2.

If we halve the length of each of the short sides, the length of the base is 5 cm and the height of the triangle is 12 cm. Using the theorem of Pythagoras, we can calculate the length of the hypotenuse to be 13 cm. The perimeter of the smaller (reduced) triangle is 30 cm and the area of the smaller triangle is 30 cm^2.

Original triangle
Sides: 10 cm, 24 cm and 26 cm
Perimeter: 60 cm
Area: 120 cm^2

Smaller triangle
Sides: 5 cm, 12 cm and 13 cm
Perimeter: 30 cm (half of original)
Area: 30 cm^2 (quarter of original)

Perimeters: $\dfrac{\textit{Original}}{\textit{Smaller}} = \dfrac{60\text{ cm}}{30\text{ cm}} = 2$

Areas: $\dfrac{\textit{Original}}{\textit{Smaller}} = \dfrac{120\text{ cm}^2}{30\text{ cm}^2} = 4$

Example 5

Consider a rectangle, with length 12 cm and breadth 9 cm. The perimeter of this rectangle is 42 cm and the area is 108 cm^2.

If we halve the length, the perimeter of the smaller (reduced) rectangle is 30 cm and the area of the smaller rectangle is 54 cm^2.

Original rectangle
Sides: 12 cm and 9 cm
Perimeter: 42 cm
Area: 108 cm^2

Smaller rectangle
Sides: 6 cm and 9 cm
Perimeter: 30 cm
Area: 54 cm^2

Perimeters: $\dfrac{\textit{Original}}{\textit{Smaller}} = \dfrac{42\text{ cm}}{30\text{ cm}} = \dfrac{7}{5}$

Areas: $\dfrac{\textit{Original}}{\textit{Smaller}} = \dfrac{108\text{ cm}^2}{54\text{ cm}^2} = 2$ We halved only one dimension.

Example 6

Consider a rectangle, with length 12 cm and breadth 9 cm. The perimeter of this rectangle is 42 cm and the area is 108 cm^2.

If we halve the breadth, the perimeter of the smaller (reduced) rectangle is 33 cm and the area of the smaller rectangle is 54 cm^2.

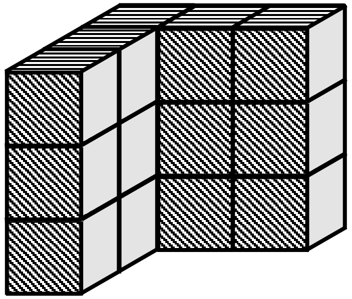
Original rectangle
Sides: 12 cm and 9 cm
Perimeter: 42 cm
Area: 108 cm^2

Smaller rectangle
Sides: 12 cm and 4,5 cm
Perimeter: 33 cm
Area: 54 cm^2

Perimeters: $\dfrac{\textit{Original}}{\textit{Smaller}} = \dfrac{42\text{ cm}}{33\text{ cm}} = \dfrac{14}{11}$ Compare this with **Example 5**.

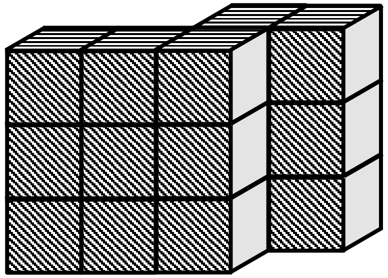
Areas: $\dfrac{\textit{Original}}{\textit{Smaller}} = \dfrac{108\text{ cm}^2}{54\text{ cm}^2} = 2$ We halved only one dimension.

1.7



Volume: ....

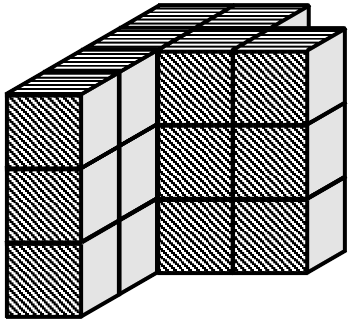
1.8



Volume: ....

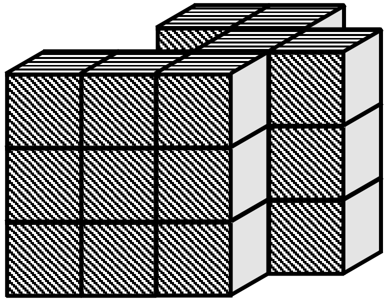
Hint:
Top face and bottom face are identical

1.9



Volume: ....

1.10



Volume: ....

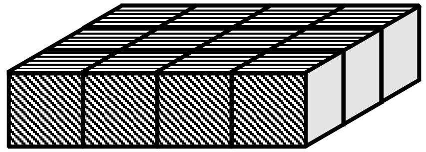
Stop and think!

Right Prisms

We are working with ***right prisms***, where the ***top face*** and the ***bottom face*** are ***identical*** and lie ***parallel*** to each other. The ***vertical faces*** lie at ***right angles*** to the ***bottom face*** (base) and the ***top face***.

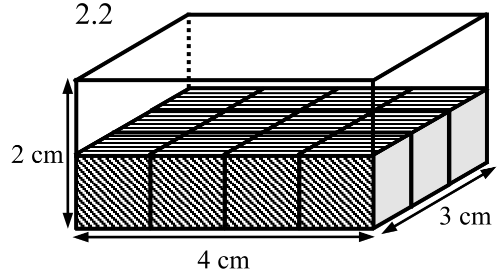
2. Determine the volume of each of the following 3D solid objects, given that each cube is a cubic centimetre. In each case you may consider the top face and the bottom face to be identical to each other, because these are right prisms. In each diagram, only the first layer of cubes (forming the base) is shown.

2.1



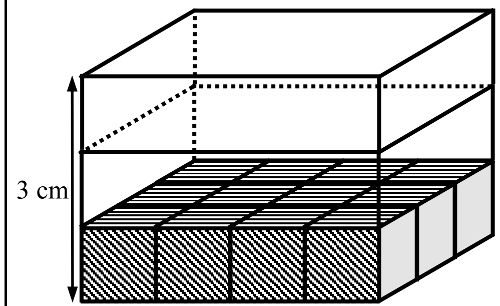
Volume: ....

2.2



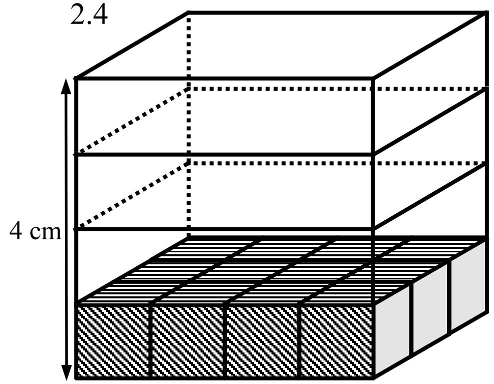
Volume: ....

2.3



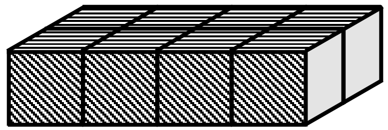
Volume: ....

2.4



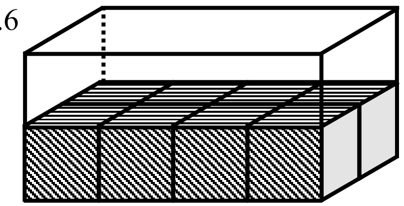
Volume: ....

2.5



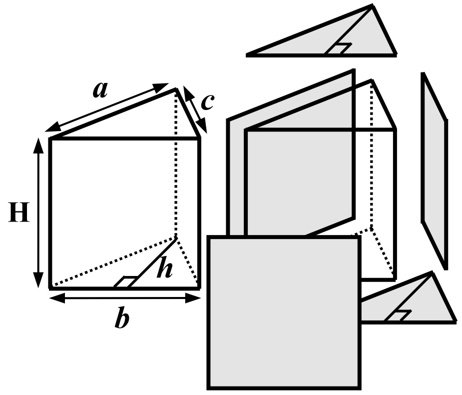
Volume: ....

2.6



Volume: ....

3. Consider the diagram of a triangular right prism shown alongside.



Calculate the total surface area of each of the following triangular right prisms.

3.1 A triangular prism with **H** = 25 cm, ***a*** = 17 cm, ***b*** = 21 cm, ***c*** = 10 cm and ***h*** = 8 cm.

3.2 A triangular prism with **H** = 30 cm, ***a*** = 25 cm, ***b*** = 28 cm, ***c*** = 17 cm and ***h*** = 15 cm.

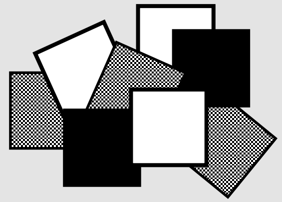
3.3 A triangular prism with **H** = 10 cm, ***a*** = 26 cm, ***b*** = 17 cm, ***c*** = 25 cm and ***h*** = 24 cm.

3.4 A triangular prism with **H** = 40 cm, ***a*** = 37 cm, ***b*** = 51 cm, ***c*** = 20 cm and ***h*** = 12 cm.

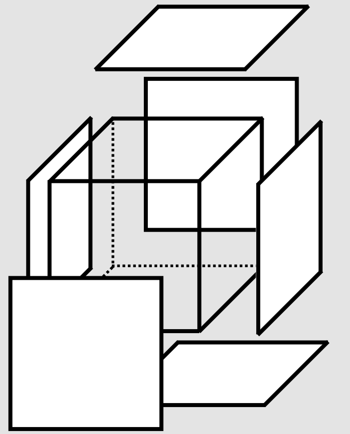
NETS of 3D SOLIDS

Net: a 2D model of a 3D object

Nets: representing 3D in 2D



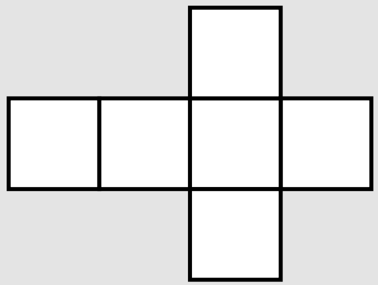
When we consider the total surface area of a three-dimensional object, we are effectively considering it as a collection of two-dimensional surfaces, arranged in three dimensions.



This is the principle which underlies the creation of nets, which are two-dimensional, plane figures, that can be folded to create a three-dimensional object.

When you consider the total surface area of a 3D object, imagine covering the outside with square stickers, or wrapping it in paper. This wrapping is effectively the net.

As we have said before now, when we draw a 3D object on paper, we create the illusion of depth, the third dimension, going into the flat page.



In creating a net, you face the challenge of unfolding a 3D object and flattening it into two dimensions. The diagram alongside shows an example of a net that can be folded to create a cube: a three-dimensional object with six faces, all identical squares.

Example 2

Consider the following set of data.

29 20 21 27 28 24 23 29 24

We may arrange these values in ascending order and indicate their rank as follows.

Rank	1	2	3	4	5	6	7	8	9
Value	**20**	**21**	**23**	**24**	**24**	**27**	**28**	**29**	**29**

For this set of data, the mean, median and mode are as follows.

Mean: $\bar{x} = \frac{20+21+23+24+...+29}{9} = \frac{225}{9} = 25$

Median: 24 Given that $n = 9$, x_5 is the middle value.

Modes: 24 and 29 The set of data is bimodal (with 2 modes). Each value occurs twice.

Example 3

Consider the following set of data.

9 16 25 36 49 64 81 100

We may arrange these values in ascending order and indicate their rank as follows.

Rank	1	2	3	4	5	6	7	8
Value	**9**	**16**	**25**	**36**	**49**	**64**	**81**	**100**

For this set of data, the mean, median and mode are as follows.

Mean: $\bar{x} = \frac{9+16+25+...+100}{8} = \frac{380}{8} = 47{,}5$

Median: 42,5 Given that $n = 8$, $\frac{x_4 + x_5}{2} = \frac{36+49}{2}$

Mode: Each value occurs only once.

EXERCISE 5.1.4

1.1 Consider the following set of nine values.

1 6 2 1 5 4 3 4 1

1.1.1 Rewrite the values, arranging them in ascending order.

....

1.1.2 Write down the mode of this set of values. ....

1.1.3 Determine the median of this set of values. ....

1.1.4 Calculate the mean of this set of values. ....

....

1.2 Consider the following set of twelve values.

4 7 3 1 6 2 5 4 3 4 2 3

1.2.1 Rewrite the values, arranging them in ascending order.

....

1.2.2 Determine the mode(s) of this set of values. ....

1.2.3 Calculate the median of this set of values. ....

1.2.4 Calculate the mean of this set of values, rounded off to two decimal places. ....

....

1.3 After writing five Maths tests, your mean mark is 65%. What percentage must you achieve for your sixth test, in order to have a mean of 70% for the six tests?

....

....

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