Mathematics Companion

WORKBOOK 1

Marilyn Buchanan, *et al.*

GRADE
9
CAPS

Terms 1 & 2

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Grade 9 Maths Companion Workbook 1

TERM 1 & 2

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- Arithmetical concepts move seamlessly into algebraic development
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Mathematics Companion

LEARNER'S WORKBOOK 1

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GRADE 9
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THIS STUDY GUIDE INCLUDES

1. Exercises
2. End-of-unit tests

Book 1 covers Term 1 and 2

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Gr 9 Maths Companion - Schedule of work

WORKBOOK 1

TERM 1

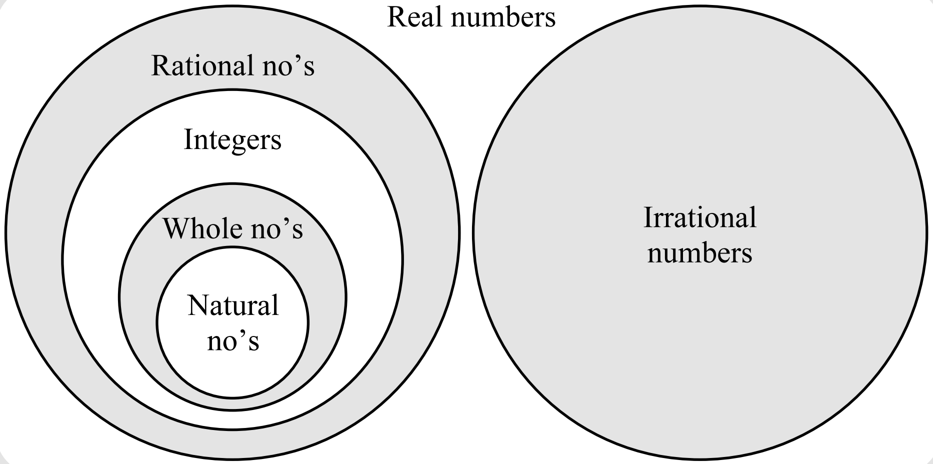
TERM 2

WORKBOOK 2

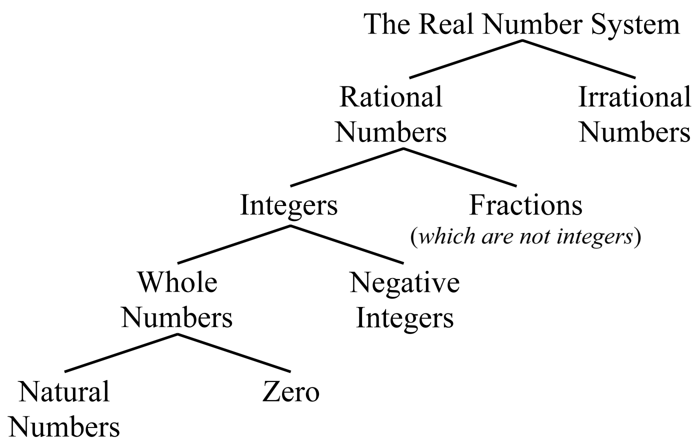
TERM 3

TERM 4

EXAM PAPERS



The following diagram indicates that the set of ***real numbers*** may be divided into two sets of numbers, namely ***irrational numbers*** and ***rational numbers***. The set of ***rational numbers*** may be divided into ***integers*** and ***fractions*** which are not ***integers***. The set of ***integers*** may be divided into ***whole numbers*** and ***negative integers***. The set of ***whole numbers*** consists of ***zero*** and the ***natural numbers***.



EXERCISE 1.3

1. Consider the following numbers, all of which are real numbers.

   Circle all the irrational numbers. All the remaining numbers that you do not circle are rational numbers, each of which can be written in the form $\frac{a}{b}$, where a and b are integers, and $b \neq 0$.

1,5	$\sqrt{49}$	1024	$2\frac{1}{3}$	$-\sqrt{4}$
–3	–0,123	$\frac{2}{3}$	$\sqrt{5}$	1,58740...
3,16227...	$-\sqrt{25}$	$\sqrt{36}$	$-\sqrt{8}$	$-\sqrt{9}$
–0,125	$-\frac{2}{3}$	$-1\frac{3}{4}$	$\frac{5}{9}$	$\sqrt[3]{9}$
2,6666...	1,414213...	$\sqrt[3]{-1}$	$\sqrt{27}$	0,25
–0,444...	0,083333...	$-0,\dot{7}$	$0,\dot{1}\dot{5}$	$\sqrt{15}$
$-\sqrt{16}$	$\sqrt[3]{-8}$	$\sqrt[4]{8}$	$-\sqrt[4]{16}$	$\sqrt[4]{16}$
$\sqrt{3}$	0,252525.....	$\sqrt{9}$	0,181818...	$\sqrt[3]{-9}$

2.1 You are given that $A = \sqrt{x+1}$.

Consider the following list of numbers: $\{0;\ 1;\ 2;\ 3;\ 4\}$

From this list of numbers, write down all the values of x for which A is **rational**.

....

UNIT 1: THE NUMBER SYSTEM

END-OF-UNIT 2 TEST

40 marks
40 minutes

QUESTION 1

1. Vuzi has noticed that when offered a lollipop, for every 5 chocolate flavoured ones selected, his friends select 4 strawberry pops.

   1.1 Given that there are 180 lollipops, calculate how many are chocolate and how many are strawberry, keeping this ratio that Vuzi noticed.

   ....

   ....

   ....

   ....

   .... (3)

   1.2 Suppose that Vuzi has 140 strawberry lollipops. Determine how many chocolate ones he needs in order to have his ratio.

   ....

   ....

   ....

   ....

   ....

   ....

   .... (3)

[6]

QUESTION 2

Sam and Ayanda started a new business with Sam contributing R15 000 and Ayanda contributing R25 000. After a year they had made a profit of R80 000 which they shared in the ratio of what they had originally contributed. Calculate each of their share.

....

....

....

....

....

....

[5]

QUESTION 3

3.1 In the following table, y is **directly proportional** to x. Complete the table, by filling in the two missing values.

x	5	12	
y	20		96

(2)

3.2 In the following table, y is **inversely proportional** to x. Complete the table, by filling in the two missing values.

x	4	3	
y	12		8

(2)

[4]

A formula for calculating compound interest

The following formula allows you to calculate the final amount (***A***) when compound interest is added to a given principal amount (***P***), if you know the interest rate (***r***, expressed as a percentage) and the number of interest periods (***n***).

$$A = P \times \left(1 + \frac{r}{100}\right)^n$$

P: principal amount

n: number of interest periods

r: interest rate per interest period (expressed as a percentage)

If we express the interest rate as a decimal fraction, then the formula looks as follows.

$$A = P \times (1 + i)^n \qquad i = \frac{r}{100}$$

Example 2

Consider an investment of R5 000 which earns interest at a rate of 9% per annum, compounded annually. Calculate the value of the investment after four years.

$A = P \times (1 + i)^n$ $\quad$ $P = \text{R}5\ 000$; $n = 4$ years; $i = \frac{9}{100} = 0{,}09$ p.a.

$= 5\ 000\,(1 + 0{,}09)^4$

$\approx$ R7 057, 91 $\quad$ (*correct to two decimal places*)

2. *In this you are advised to make use of the following formula*

$$A = P \times (1 + i)^n$$

2.1 You open a savings account and deposit R1 000 into the account. You leave the money there for five years, during which time it accumulates interest at a rate of 8% p.a., compounded annually. Calculate how much will be in the account after five years.

....

....

2.2 You open a savings account and deposit R2 500 into the account. You leave the money there for six years, during which time it accumulates interest at a rate of 9% p.a., compounded annually. Calculate how much will be in the account after six years.

....

....

2.3 You take out a loan of R25 000 on which you are charged interest at a rate of 30% p.a., compounded annually, over a period of four years. Calculate how much interest you will pay on this loan.

....

....

....

2.4 You take out a loan of R50 000 on which you are charged interest at a rate of 25% p.a., compounded annually, over a period of four years. Calculate how much interest you will pay on this loan.

....

....

....

SUBSTITUTION

EXERCISE 4.3

1. Without using a calculator, determine the value of each of the following expressions for $\boldsymbol{a = -2}$, $\boldsymbol{b = -3}$, $\boldsymbol{c = -4}$ and $\boldsymbol{d = -5}$.

1.1 $ab - c + 2d$

= ....

= ....

= ....

1.2 $a + b \times c + d$

= ....

= ....

= ....

1.3 $a - b \times c - d$

= ....

= ....

= ....

1.4 $(a - b) \times c - d$

= ....

= ....

= ....

= ....

1.5 $(a - b) \times (c - d)$

= ....

= ....

= ....

1.6 $bc - ad$

= ....

= ....

= ....

1.7 $(a - b)(b - c)(c - d)$

= ....

= ....

= ....

1.8 $a(b + c) - b(a + b)$

= ....

= ....

= ....

= ....

1.9 $\dfrac{(d-a)(b-d)-c(d-b)}{a(b-c)+d}$

= ....

= ....

= ....

= ....

1.10 $(a - c)(b + d) - a(c + d)$

= ....

= ....

= ....

= ....

EXPONENTIAL EQUATIONS

The solution of an exponential equation is based on the fact that, if two **powers are equal**, and the **bases are equal**, then the **exponents must be equal**.

The process of solving an exponential equation involves creating **equal bases** on the left hand side (LHS) and right hand side (RHS) of the equation.

Example 1

$2^a = 8$

$\therefore 2^a = 2^3$ [*We must create equal bases on the LHS and RHS.*]

$\therefore a = 3$ [*The bases are equal, so the exponents must be equal.*]

Example 2

$3^{p+1} = \frac{1}{27}$

$\therefore 3^{p+1} = \frac{1}{3^3}$ [*We must create equal bases on the LHS and RHS.*]

$\therefore 3^{p+1} = 3^{-3}$

$\therefore p + 1 = -3$ [*The bases are equal, so the exponents must be equal.*]

$\therefore p = -4$

Example 3

$5^{2y-1} = 1$

$\therefore 5^{2y-1} = 5^0$ [*We must create equal bases on the LHS and RHS.*]

$\therefore 2y - 1 = 0$ [*The bases are equal, so the exponents must be equal.*]

$\therefore 2y = 1$

$\therefore y = \frac{1}{2}$

EXERCISE 7.11

Solve for x in each of the following equations.

1. $2^x = 32$

$\therefore$ ....

$\therefore$ ....

2. $3^{x+1} = 27$

$\therefore$ ....

$\therefore$ ....

$\therefore$ ....

3. $5^{x-1} = 25$

$\therefore$ ....

$\therefore$ ....

$\therefore$ ....

4. $7^{3-2x} = 49$

$\therefore$ ....

$\therefore$ ....

$\therefore$ ....

$\therefore$ ....

5. $2^x = \frac{1}{8}$

$\therefore$ ....

$\therefore$ ....

$\therefore$ ....

6. $3^{x+1} = \frac{1}{81}$

$\therefore$ ....

$\therefore$ ....

$\therefore$ ....

$\therefore$ ....

7. $5^{x-1} = \frac{1}{125}$

$\therefore$ ....

$\therefore$ ....

$\therefore$ ....

$\therefore$ ....

8. $7^{3-2x} - \frac{1}{7} = 0$

$\therefore$ ....

$\therefore$ ....

$\therefore$ ....

$\therefore$ ....

$\therefore$ ....

3.4

Figure 1 Figure 2 Figure 3 Figure 4

Figure number	1	2	3	4	5	***n***	20	
No. of matchsticks	3	5	7					201

3.5

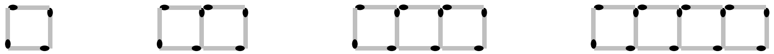


Figure 1 Figure 2 Figure 3 Figure 4

Figure number	1	2	3	4	5	***n***	20	
No. of matchsticks	4	7	10					181

3.6

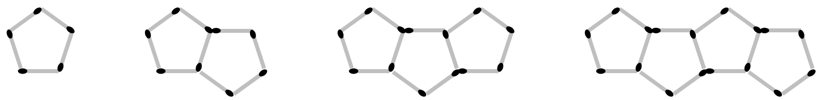


Figure 1 Figure 2 Figure 3 Figure 4

Figure number	1	2	3	4	5	***n***	20	
No. of matchsticks	5	9	13					301

Other Formulae aided by diagrams

Breaking down into parts as a useful technique when determining a formula.

Suppose we are given the following set of figures.

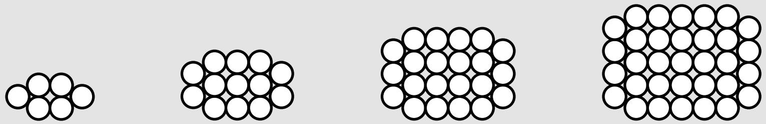


Figure 1 Figure 2 Figure 3 Figure 4

Keeping the square of dots as white and changing the outer dots to black helps to generate a formula.

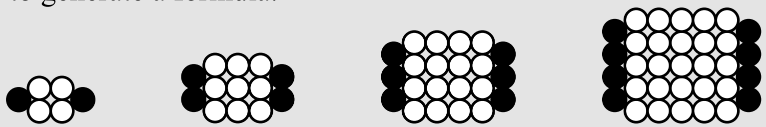


Figure 1 Figure 2 Figure 3 Figure 4

We then summarise the information in a table.

Figure number	n	1	2	3	4
No. of black dots	b	2	4	6	8
No. of white dots	w	4	9	16	25
Total no. of dots	T_n	6	13	22	33

We see that:

1. The black dots are multiples of 2. $b = 2n$
2. The white dots are square numbers. $w = (n+1)^2$
3. The total number of dots is made up of the number of black dots and white dots.

$$T_n = b + w$$
$$= 2n + (n+1)^2$$

Stop and take note!

F-O-I-L

To avoid repeating or leaving out a term,
you may like to remember the process using the mnemonic ***FOIL***.

F	... First × First	(First + ...)(First + ...)
O	... Outer × Outer	(Outer + ...)(... + Outer)
I	... Inner × Inner	(... + Inner)(Inner + ...)
L	... Last × Last	(... + Last)(... + Last)

Example

Expand and simplify: $(2m + 3)(3m - 5)$

F	$(2m + 3)(3m - 5)$	... $6m^2$ (Firsts)
O	$(2m + 3)(3m - 5)$	.... $-10m$ (Outers)
I	$(2m + 3)(3m - 5)$	.... $+9m$ (Inners)
L	$(2m + 3)(3m - 5)$	.... -15 (Lasts)

$(2m + 3)(3m - 5) = 6m^2 - 10m + 9m - 15$

$= 6m^2 - m - 15$

EXERCISE 10.8

Determine the product of each of the following pairs of binomials, using the FOIL method.

1. $(x + 3)(x + 4)$

   = ...

   = ...

2. $(x + 6)(x + 2)$

   = ...

   = ...

3. $(x + 1)(x + 12)$

   = ...

   = ...

4. $(x - 3)(x + 4)$

   = ...

   = ...

5. $(x - 6)(x + 2)$

   = ...

   = ...

6. $(x - 1)(x + 12)$

   = ...

   = ...

7. $(x + 3)(x - 4)$

   = ...

   = ...

8. $(x + 6)(x - 2)$

   = ...

   = ...

9. $(x + 1)(x - 12)$

   = ...

   = ...

10. $(x - 3)(x - 4)$

    = ...

    = ...

11. $(x - 6)(x - 2)$

    = ...

    = ...

12. $(x - 1)(x - 12)$

    = ...

    = ...

EXERCISE 11.4

Solve for x in each of the following equations. **Be sure to check whether or not each of your solutions is valid.** Look out for identities and false statements.

1. $\frac{1}{x} + 3 = 5$

2. $\frac{1}{x} + \frac{2}{3} = 2$

3. $\frac{1}{3x} + \frac{2}{3} = \frac{1}{x}$

4. $\frac{7}{2x} - \frac{2}{3} = \frac{1}{2}$

5. $\frac{3}{2x} - \frac{1}{4} = 1 - \frac{9}{4x}$

6. $1 - \frac{2}{3x} + \frac{1}{2x} = \frac{4}{3}$

7. $\frac{x-2}{2x} + \frac{1}{x} + \frac{3}{2} = 0$

8. $\frac{1}{2x} - \frac{1}{3} = \frac{5}{12x}$

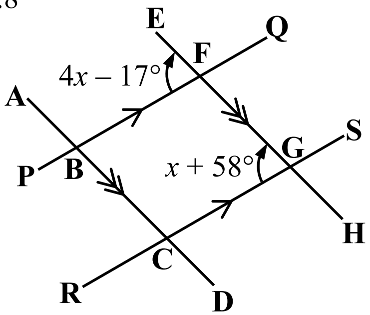
9. $\frac{1}{3} = \frac{2x-1}{5x}$

10. $\frac{1}{2} + \frac{3}{x} = \frac{x+3}{x}$

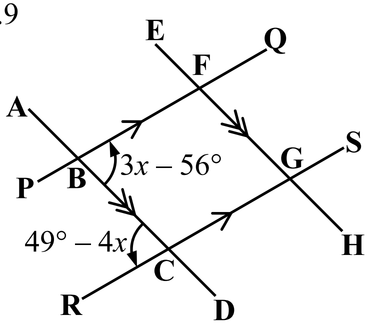
11. $\frac{2}{x} = \frac{6}{3x-5}$

12. $\frac{1}{2} + \frac{3}{x} = \frac{x+6}{2x}$

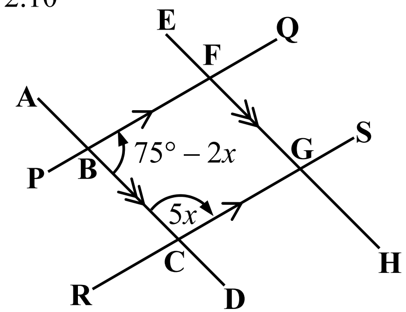
2.8



2.9

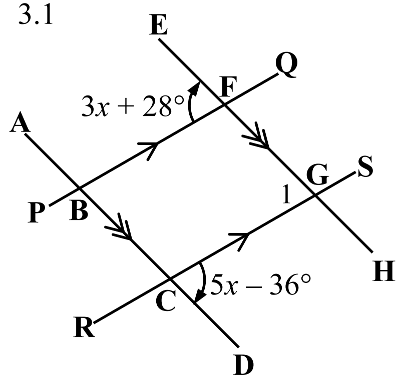


2.10

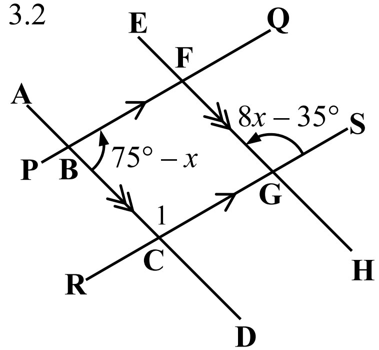


3. In each of the following diagrams, it is not possible to correctly determine the value of x without using more than one rule. Be careful to select the appropriate pairs of parallel lines at each step.

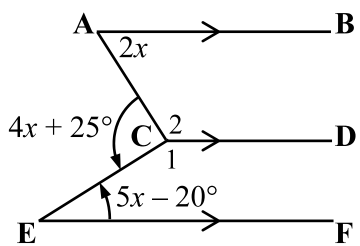
3.1 (Hint: use $\hat{G}_1$)



3.2 (Hint: use $\hat{C}_1$)



3.3



END-OF-UNIT 15 TEST

40 marks
40 minutes

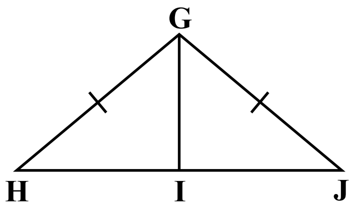
QUESTION 1

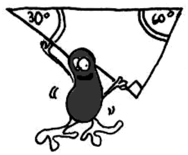
1.1 Refer to the diagrams showing ΔABC and ΔDEF.

Explain why we cannot determine if the triangles are congruent.

.... (1)

1.2 In the diagram, HIJ is a straight line and GH = GJ.





1.2.1 Why is $\hat{H} = \hat{J}$?

.... (1)

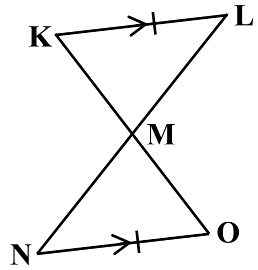
1.2.2 Can we use the common arm (GI), GH = GJ and $\hat{H} = \hat{J}$ to prove that $\Delta GHI \equiv \Delta GJI$?

.... (1)

[3]

QUESTION 2

2.1 Prove that $\Delta KLM \equiv \Delta ONM$



....

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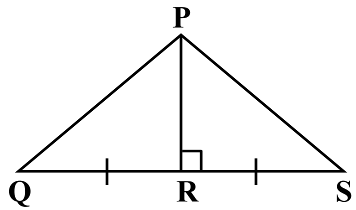
....

....

.... (4)

2.2 Prove that $\Delta PQR \equiv \Delta PSR$



....

....

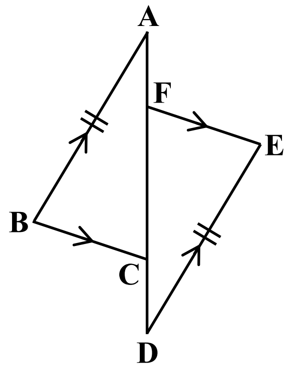
....

....

.... (4)

2.3 In the given diagram, AB = ED, AB || ED and BC || FE.

Prove, by congruency, that BC = FE.



....

....

....

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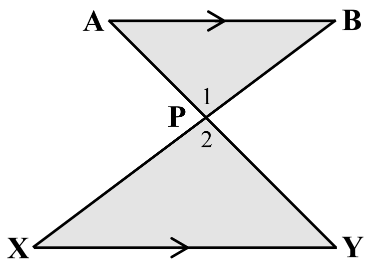
....

.... (6)

Example 3

In the diagram alongside, AB || XY.

1. Prove that ΔPAB ||| ΔPYX.
2. If AB = 8 cm, XY = 12 cm and AP = 6 cm, determine the length of PY.

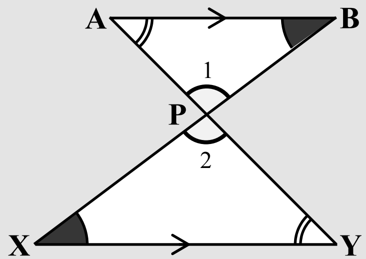


Solution

1. In ΔPAB and ΔPYX

   1. $\hat{P}_1 = \hat{P}_2$ (vert. opp. ∠'s)
   2. $\hat{A} = \hat{Y}$ (alt. ∠'s; AB || XY)
   3. $\hat{B} = \hat{X}$ (alt. ∠'s; AB || XY)

   ∴ ΔPAB ||| ΔPYX (AAA)



Note: *Remember to name the triangles in matching order.*

2. ΔPAB ||| ΔPYX (proven above)

$\therefore \dfrac{PA}{PY} = \dfrac{AB}{YX} = \dfrac{PB}{PX}$ ΔPAB ||| ΔPYX

Δ PAB ||| Δ PYX

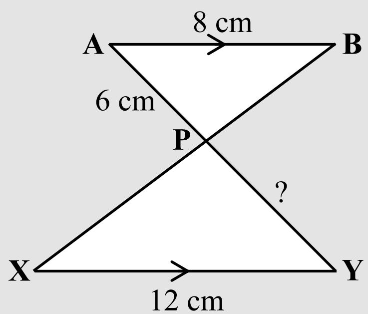
$\dfrac{AB}{YX} = \dfrac{8}{12} = \dfrac{2}{3}$

$\therefore \dfrac{6}{PY} = \dfrac{2}{3}$

$\therefore \dfrac{PY}{6} = \dfrac{3}{2}$

$PY = \dfrac{3}{2} \times \dfrac{6}{1}$

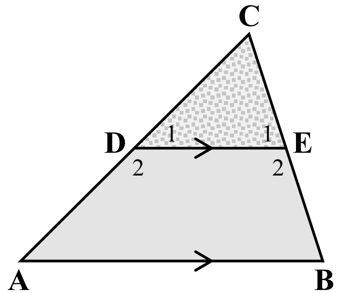
$\therefore PY = 9$ cm



Example 4

In the diagram alongside, AB || DE.

1. Prove that ΔABC ||| ΔDEC.
2. If AB = 16 cm, DE = 12 cm and CD = 9 cm, determine the length of DA.

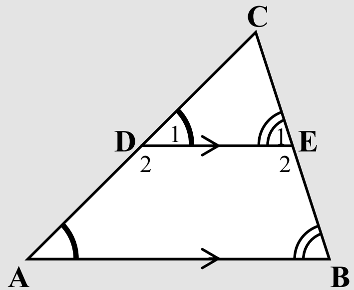


Solution

1. In ΔABC and ΔDEC

   1. $\hat{C}$ is common
   2. $\hat{A} = \hat{D}_1$ (corres. ∠'s; AB || DE)
   3. $\hat{B} = \hat{E}_1$ (corres. ∠'s; AB || DE)

   ∴ ΔABC ||| ΔDEC (AAA)



2. ΔABC ||| ΔDEC (proven above)

$\therefore \dfrac{AB}{DE} = \dfrac{BC}{EC} = \dfrac{AC}{DC}$ ΔABC ||| ΔDEC

Δ ABC ||| Δ DEC

$\dfrac{AB}{DE} = \dfrac{16}{12} = \dfrac{4}{3}$

Let AD $= x$

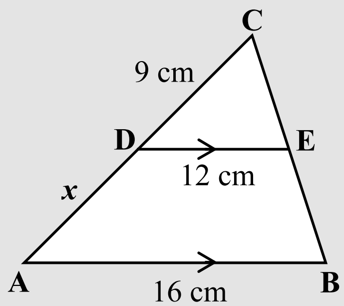
$\therefore \dfrac{x+9}{9} = \dfrac{4}{3}$

$\therefore 3(x+9) = 4(9)$

$\therefore 3x + 27 = 36$

$\therefore 3x = 9$

$\therefore x = 3$ cm



Determining whether a triangle is Right-angled

We have used the theorem of Pythagoras when given a Right-angled triangle.

Now, given the length of the three sides of a triangle, we can determine if it is a right-angled triangle or not.

The CONVERSE of the theorem states that

IF...		THEN...
...the square on the longest side of a triangle is equal to the sum of the squares on the other two sides...	→	*...the triangle is right-angled.* ***In other words, the angle opposite the longest side is a right angle.***

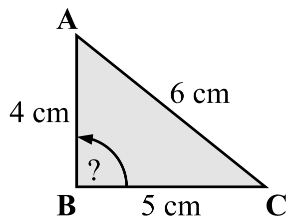
Example 1

Consider ΔABC, with AB = 4 cm, BC = 5 cm and AC = 6 cm.
Determine, by calculation, whether or not ΔABC is a right-angled triangle.

Solution

Longest side: AC = 6 cm

$\therefore\ AC^2 = 36\ cm^2$



$AB^2 + BC^2 = 4^2 + 5^2$

$= 16 + 25$

$= 41\ cm^2$

Equal or not?

$\therefore\ AC^2 \neq AB^2 + BC^2$

Compare the square on the longest side with the sum of the squares on the other two sides.

$\therefore\ \hat{B} \neq 90^\circ$ (converse of Pythag.)

∴ΔABC is not a right-angled triangle.

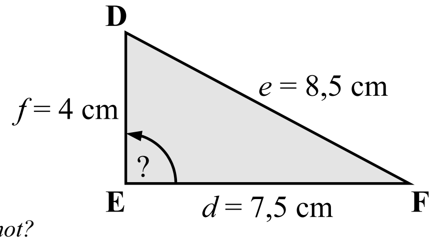
Example 2

Consider ΔDEF, with $d = 7{,}5$ cm, $e = 8{,}5$ cm and $f = 4$ cm. Determine, by calculation, whether or not ΔDEF is a right-angled triangle.

Solution

Longest side: $e = 8{,}5$ cm

$\therefore\ e^2 = 72{,}25\ cm^2$



$f^2 + d^2 = 4^2 + 7{,}5^2$

$= 16 + 56{,}25$

$= 72{,}25\ cm^2$

Equal or not?

$\therefore\ e^2 = d^2 + f^2$

Compare the square on the longest side with the sum of the squares on the other two sides.

$\therefore\ \hat{E} = 90^\circ$ (converse of Pythag.)

∴ΔDEF is a right-angled triangle.

EXERCISE 17.3

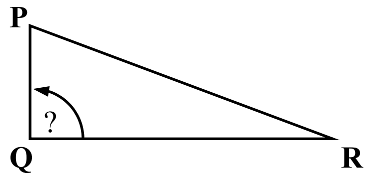
1. Determine, by calculation, whether or not each of the following triangles is right-angled. Show all your working in the spaces provided.

   1.1 In ΔPQR, PQ = 6 cm, QR = 20 cm and PR = 21 cm.

   Longest side: PR = ... $\therefore\ PR^2$ = ...

   $PQ^2 + QR^2$ = ... + ...

   = ...



   $\therefore\ PR^2$ ..... $PQ^2 + QR^2$

   $\therefore\ \hat{Q}$ ..... 90° (converse of Pythag.)

   ∴ ΔPQR ....