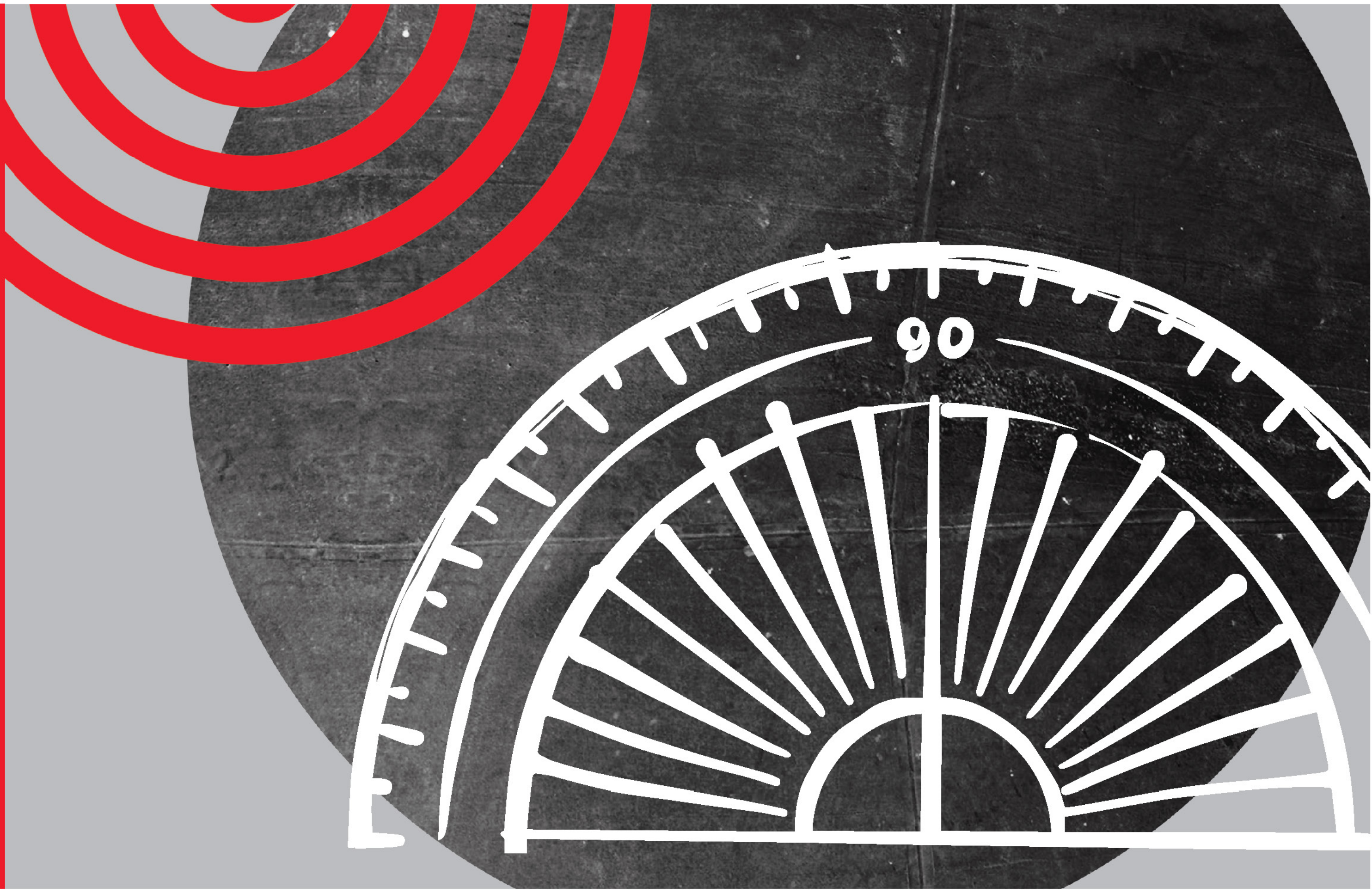
Mathematics IEB

PAPERS & ANSWERS

Marilyn Buchanan, ***et al.***

GRADE
12
IEB

P & A



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The Answer Series would like to acknowledge the huge contribution made by Bonita Morgan and Judy Crowster, who typeset the material in this book with the utmost dedication and expertise.

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CONTENTS

Note:
The information sheet is only provided for the Gr 12 exam.

ABOUT THIS BOOK

The examination papers compiled in this book are an attempt by The Answer Series to provide teachers and learners with practice material in preparation for the end-of-year examinations. They are an interpretation of the CAPS curriculum and should not be taken to indicate the only type of questions that could be asked, but rather as possible examples.

There are 10 paper 1's and 10 paper 2's. The first 5 of each are newly compiled, while the second 5 have been compiled by adapting the papers from the previous edition of this book. All 20 papers have been set according to the requirements of the CAPS curriculum. The allocation of marks to topics has occasionally been influenced by the need to provide more practice where deemed necessary.

All 10 paper 1's have been compiled by Marilyn Buchanan (current IEB examiner) – the first 5 created; the second 5 adapted. The 5 new paper 2's have been compiled by Praveshen Iyer (future IEB examiner) and a team of senior teachers from leading schools. The second 5 paper 2's have been adapted by Anne Eadie, coordinator of this project. We are indebted to Janet Aird and Gail Hallet who made a valuable contribution by checking sections of this book.

We trust that experiencing this comprehensive compendium of questions and answers will place learners in a strong position to succeed in the CAPS examinations.

We will welcome constructive comments from both teachers and learners.

Marilyn Buchanan: mabkjb@telkomsa.net Anne Eadie: anne@theanswerseries.co.za

7.2 Debbie needs to create 3-digit codes as follows:

The digits must be selected from the digits 0 to 9, and the codes must be 3-digit numbers.

Note that numbers such as 7 or 24 are one- and two-digit numbers and therefore not acceptable, i.e. 007 and 024 are not valid codes.

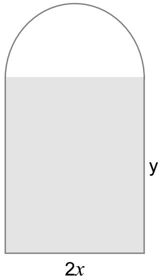
Calculate the number of unique codes that Debbie can create if:

7.2.1 repetition is allowed. (2)

7.2.2 repetition is not allowed. (2)

[15 marks]

QUESTION 8



A bathroom window is in the form of a rectangle (width $2x$ and height y) surmounted by a semicircle.

The semicircle is of clear glass while the rectangle is made of coloured glass which transmits only half as much light per square metre as clear glass does.

The total perimeter of the window is fixed at 6 metres.

The dimensions need to be determined so that the maximum amount of light is transmitted.

8.1 Derive a formula in terms of x for the amount of light that is transmitted through the window. (7)

8.2 Determine the values for x and y (in terms of π) when this light is maximised. (5)

8.3 Hence calculate the maximum amount of light. (2)

[14 marks]

QUESTION 9

A manufacturer of a product finds that they make a profit of R450 for each of the first 500 units that are produced and sold. Beyond 500 units, the profit decreases by R2 per additional unit produced. For example, when 504 units are produced and sold, the total profit is R$(500 \times 450 + 4 \times 442)$.

Calculate the maximum profit. **[10 marks]**

TOTAL: 150

EXAM PAPER 1C

Approved non-programmable and non-graphical calculators may be used, unless otherwise stated.

*Round off your answers to **ONE** decimal digit where necessary, unless otherwise stated.*

SECTION A

QUESTION 1

1.1 Solve for x:

1.1.1 $\sqrt{x - 2} + 4 = x$ (5)

1.1.2 $(x - 3)(x - 4) \geq 6$ (5)

1.2 Without using a calculator, show that $3\sqrt{5} - 2\sqrt{2}$ is a square root of $53 - 12\sqrt{10}$. (3)

1.3 Given $f(x) = (x^3 - 2)(5 - x)$, determine $f'(2)$. (5)

[18 marks]

QUESTION 2

2.1 Given $T_k = 5k - 2$

Determine:

2.1.1 T_{21} (1)

2.1.2 which term equals 1173. (2)

2.1.3 the sum of the first 300 terms. (3)

2.2 Given the sequence: 1 ; 7 ; 17 ; 31 . . .

2.2.1 Write down the next two terms. (2)

2.2.2 Determine a formula for the n^{th} term of the sequence. (5)

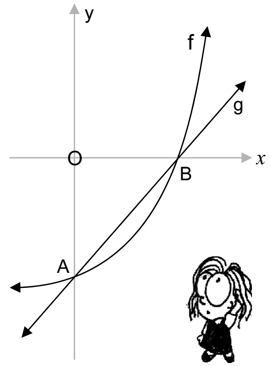
2.3 The third term of a geometric series is $\frac{2}{9}$ and the eighth term is $\frac{-2}{2\ 187}$.

Find the constant ratio and the first term. (6)

[19 marks]

QUESTION 3

3.1 Refer to the figure, showing a sketch of $f(x) = 2^{x-1} - 4$ and straight line $y = g(x)$.



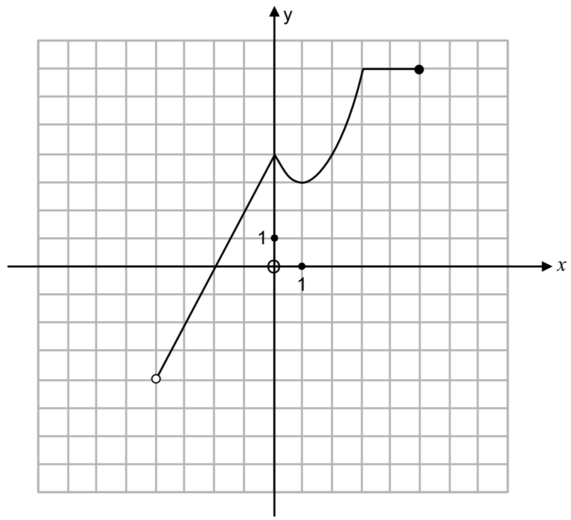
3.1.1 Write down the range of f. (1)

3.1.2 Determine the coordinates of A and B, the intercepts of the graphs with the axes. (4)

3.1.3 Determine the equation of the straight line. (3)

3.1.4 Use the figure and your answers above to write down the values of x for which $f(x) > g(x)$. (2)

3.2 Refer to the figure, showing a sketch of $y = f(x)$.



3.2.1 State the domain and range of f. (2)

3.2.2 On a copy of the grid, draw the inverse of f. (3)

3.2.3 On a copy of the grid, draw $y = 2 - f(x)$. (3)

3.2.4 On a copy of the grid, draw $y = 2f(x)$. (4)

[22 marks]

QUESTION 4

4.1 The population of a sea-side resort has increased by 11% per year for the last 10 years and is now 14 000.

Determine the size of the population 10 years ago. (2)

4.2 Mrs Brown was granted a loan. She has to repay the loan in monthly instalments of R9 089,13 for 20 years with interest charged on a reducing balance at 12,5% p.a. compounded monthly.

Calculate:

4.2.1 the amount of the loan (to the nearest R100). (3)

4.2.2 the effective annual interest rate of the loan. (3)

4.2.3 the outstanding balance on the loan at the end of the first two years. (4)

4.2.4 how much interest Mrs Brown had paid in these two years. (4)

[16 marks]

SECTION B

QUESTION 5

5.1 Anne has been monitoring the operation of the traffic light on the corner of her street for the last 120 days. There were thunder storms on 54 days, and the lights failed to work properly on 48 of these days. They also failed to work properly on 12 of the days when there was fine weather. It is predicted that there is a 70% probability of a thunder storm in Anne's area on Monday.

5.1.1 Draw a tree diagram to represent the information. (6)

5.1.2 Determine the probability that the traffic lights will not operate correctly on Monday. (2)

5.2 In a bookshop, 13 different travel books need to be placed on a shelf by an assistant. There are 7 different books about Europe and 6 different books about America.

5.2.1 Calculate the number of different ways the books can be arranged if they are randomly placed on the shelf. (1)

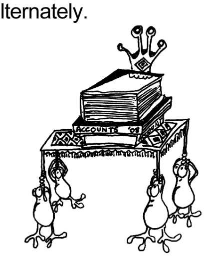
5.2.2 Determine the probability that:

(a) the European books are together and the American books are together. (3)

(b) only the European books are together. (3)

(c) the European and American books are arranged alternately. (3)

[18 marks]

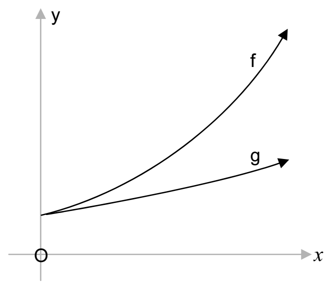


QUESTION 6

A city council was informed that the population of the city (currently at 3 million) was growing at a rate of 4% per year.

The figure below shows the graphs $y = f(x)$ and $y = g(x)$ where the function f is the predicted exponential model for the present growth and g is a suggested retarded exponential growth model.

x represents the number of years from the present time and y is the population of the city, in millions.



6.1 Calculate the predicted population of the city if the current growth rate continues for the next 10 years. (3)

6.2 Determine the percentage increase of the population over the 10 years. (2)

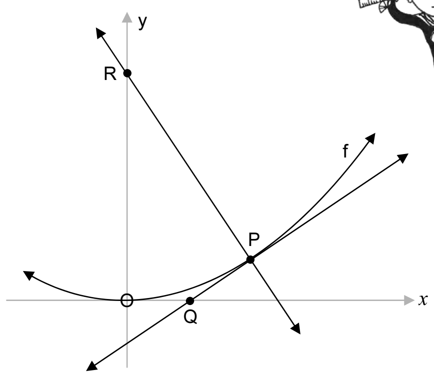
6.3 Determine the number of years it would take for the population to double in size. (4)

6.4 The council will start construction to cope with a population of 4 460 000 in 20 years' time. Calculate the required growth rate (correct to two decimal digits). (4)

[13 marks]

QUESTION 7

Refer to the figure showing part of a parabola where $f(x) = \dfrac{x^2}{9k}$. The point P on f has x-coordinate 3kp. PQ is a tangent to the curve.



7.1 Show that the equation of the tangent PQ is $y = \dfrac{2px}{3} - kp^2$. (6)

7.2 The line through PR is known as a normal to the curve as $PQ \perp PR$. Determine the equation of this normal. (3)

7.3 Determine the coordinates (in terms of k and p) of:

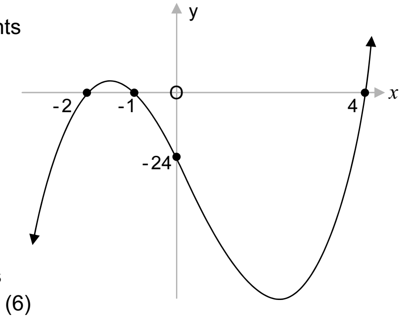
7.3.1 Q, the x-intercept of PQ. (2)

7.3.2 R, the y-intercept of PR. (2)

[13 marks]

QUESTION 8

The graph which represents the function $f(x) = ax^3 + bx^2 + cx + d$ intersects the x-axis at -2, -1 and 4, and intersects the y-axis at -24.



Determine:

8.1 the numerical values of a, b, c and d. (6)

8.2 the equation of the tangent to the curve at the y-intercept. (4)

8.3 P is a point on the curve with x-coordinate -1,5. Show that the equation of the tangent to the curve at P is: $y = -\dfrac{3x}{4} + 3$. (5)

8.4 Show that the x-intercept of this tangent is also an x-intercept of the cubic curve. (2)

[17 marks]

QUESTION 9

9.1 Given the adult dose, d, of a medicine, the child dose, c, can be determined using a, the age of the child by various formulae.

Young's Rule: $c = \dfrac{a}{a+12} \times d$

Cowling's Rule: $c = \dfrac{a+1}{24} \times d$

Determine the age when both formulae gives the same child dose.

Give your answer to the nearest year. (5)

9.2 The equation $x^2 + px + 1 = 0$ has equal roots. Determine the nature of the roots of the equation $x^2 + px + 6 = 0$. (5)

9.3 The shortest side of a quadrilateral is 1 cm. The 4 sides form a geometric sequence and the perimeter of the quadrilateral is 15 cm. Determine the lengths of the other sides. (4)

[14 marks]

TOTAL: 150

EXAM PAPER 2B

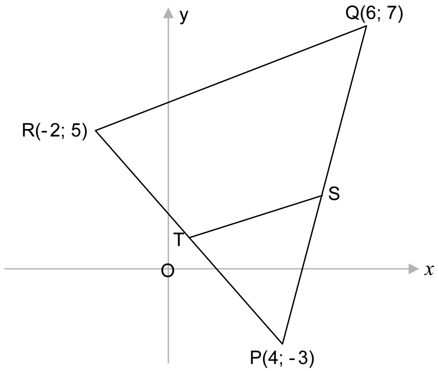
Approved non-programmable and non-graphical calculators may be used, unless otherwise stated.

*Round off your answers to **ONE** decimal digit where necessary, unless otherwise stated.*

QUESTION 1

Refer to the diagram:

P(4; -3) ; Q(6; 7) and R(-2; 5) are vertices of ΔPQR.



1.1 Determine the coordinates of S, the midpoint of PQ, and T, the midpoint of PR. (2)

1.2 Determine the gradient of RQ and hence show that ST || RQ. (3)

1.3 Determine $R\hat{P}Q$. (4)

1.4 If the perpendicular distance between the lines QR and ST is 4 units, determine the area of RQST, in simplest surd form. (5)

[14 marks]

QUESTION 2

A circle is defined by the equation: $x^2 + 6x + y^2 + 4y = 4$

2.1 Express the above equation in the form $(x - a)^2 + (y - b)^2 = r^2$, and hence write down the coordinates of the centre of the circle. (4)

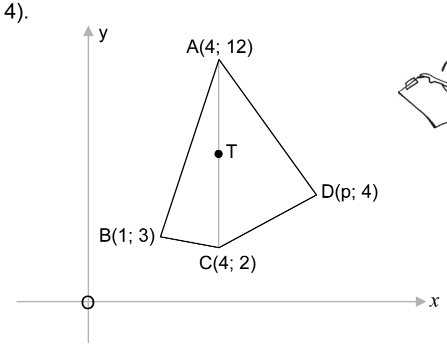
2.2 Find the equation of the tangent to the circle at the point H(-4; 2). (4)

[8 marks]

QUESTION 3

Refer to the diagram:

ABCD is a quadrilateral with vertices A(4; 12) ; B(1; 3) ; C(4; 2) and D(p; 4).



3.1 Prove that AB $\perp$ BC. (3)

3.2 Determine the coordinates of T, the midpoint of AC. (2)

3.3 Hence, or otherwise, determine the equation of the circle in the form $(x - a)^2 + (y - b)^2 = r^2$ which passes through the points A, B and C. (4)

3.4 Determine the value of p for ABCD to be a cyclic quadrilateral. (4)

3.5 Determine the size of angle $B\hat{T}D$, if ABCD is a cyclic quadrilateral and p = 8. (4)

[17 marks]

QUESTION 4

The frequency table below gives the masses of the first 60 babies born at a hospital in Johannesburg.

Mass (kg)	Frequency *f*
$1{,}0 \le w < 1{,}5$	4
$1{,}5 \le w < 2{,}0$	7
$2{,}0 \le w < 2{,}5$	12
$2{,}5 \le w < 3{,}0$	14
$3{,}0 \le w < 3{,}5$	10
$3{,}5 \le w < 4{,}0$	9
$4{,}0 \le w < 4{,}5$	4
$4{,}5 \le w$	0

4.1 Calculate the estimated mean mass of the babies. (3)

4.2 Calculate the estimated standard deviation of the mass of the babies. (2)

[5 marks]

QUESTION 5

A group of learners was investigating whether there is a relationship between foot length and height in their class.

They randomly selected 11 learners from whom to collect data.

Length of left foot x (in cm)	27,4	26,3	25	25,4	27,6	22,5	27	27,5	24,6	28	26,4
Height y (in cm)	175	172	167	169	173	162	168	170	158	172	180

5.1 Calculate the equation of the line of best fit to 4 decimal digits. (4)

5.2 Use your answer to predict the height of a learner with foot length of 26 cm to 1 decimal digit. (2)

5.3 Prove that the point with coordinates $(\overline{x}; \overline{y})$ to 4 decimal digits lies on the line of best fit. (4)

5.4 Calculate the correlation coefficient for the data to one decimal digit. (3)

5.5 What conclusions, based on your answer in 5.4 above, can you draw about the relationship between the foot size and the height of the 11 learners in the data set? (2)

[15 marks]

QUESTION 6

6.1 If sin 40º = k, express each of the following in terms of k:

6.1.1 $\frac{1}{\cos 50^\circ}$ 6.1.2 sin 80º 6.1.3 cos 20º (1)(4)(4)

6.2 Given $\sin\alpha = -\frac{4}{5}$ and $\cos\beta = \frac{3}{5}$ with $0^\circ < \alpha < 270^\circ$ and $\beta > 90^\circ$. Find the value of $\sin(\alpha - \beta)$ (5)

6.3 Simplify:

$$\frac{\sin(90^\circ - x) \, . \, \sin(90^\circ + x)}{(\sin 90^\circ - \sin x)(\sin 90^\circ + \sin x)}$$ (5)

6.4 Simplify without using a calculator:

$$\frac{\cos 330^\circ \, . \, \cos(-200^\circ)}{\tan 405^\circ \, . \, \sin(-150^\circ) \, . \, \sin 290^\circ}$$ (6)

6.5 Solve each of the following equations for $\alpha \in [0^\circ; 180^\circ]$:

6.5.1 $(\tan\alpha - 1)(\tan\alpha + 1) = 0$ (3)

6.5.2 $\frac{2\sin\alpha \, . \, \cos\alpha}{\cos^2\alpha - \sin^2\alpha} = 1$ (4)

6.6 Determine the general solution of the following equation:

$\cos(x + 30°) = \left(\frac{\sqrt{3} - 1}{2}\right)\sin x$ (6)

[38 marks]

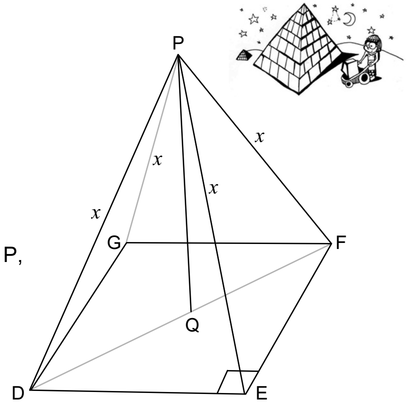
QUESTION 7

A pyramid stands on a square base DEFG.

Each of its side faces is an equilateral triangle with length x units.

How high, in terms of x, is the top of the pyramid P, above Q, the centre of the base?

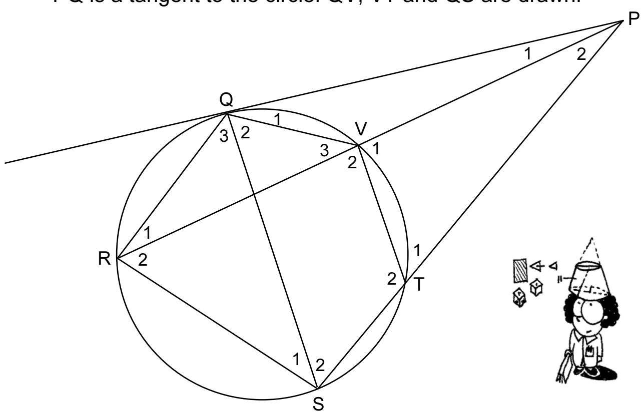
Leave your answer in simplest surd form.



[6 marks]

QUESTION 8

8.1 Refer to the diagram: RV and ST are produced to meet at P. PQ is a tangent to the circle. QV, VT and QS are drawn.



Refer to the statements below and state whether they are true or false.

8.1.1 $\hat{Q}_1 = \hat{R}_1$ (1)

8.1.2 $\hat{R}_1 = \hat{S}_2$ (1)

8.1.3 $\hat{V}_1 = \hat{S}_1 + \hat{S}_2$ (1)

8.1.4 RQVS is a cyclic quadrilateral (1)

8.1.5 $\hat{R}_1 + \hat{R}_2 = 90°$ (1)

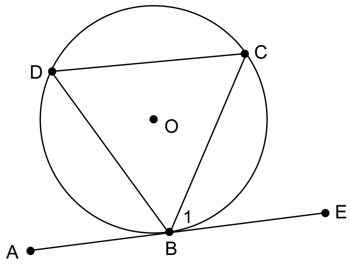
8.1.6 $\hat{R}_1 + \hat{R}_2 + \hat{T}_2 = 180°$ (1)

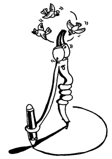
8.2 In a cyclic quadrilateral PQRS, $\hat{S} = 120°$, PQ = 5 units and QR = 8 units. Calculate the length of PR. (4)

[10 marks]

QUESTION 9

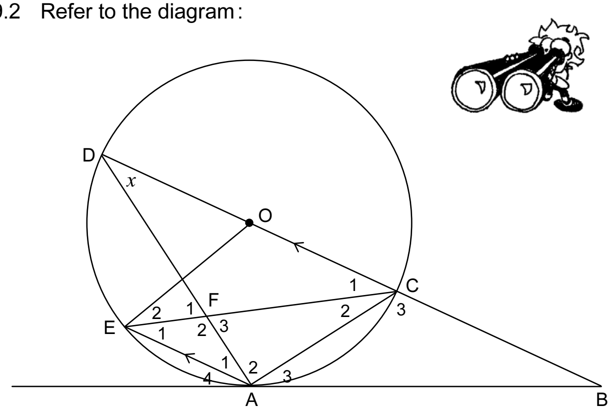
9.1 Prove the theorem that states that the angle formed between a tangent and a chord is equal to the angle in the alternate segment. (i.e. Prove that $\hat{B}_1 = \hat{D}$)





(6)

9.2 Refer to the diagram:



AB is a tangent to the circle with centre O.
BCOD is a straight line. AE || BD and $\hat{D} = x$.

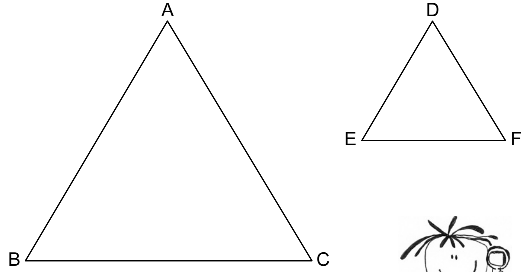
9.2.1 Find, with reasons, five other angles each equal to x. (8)

9.2.2 Prove that EC is a tangent to the circle passing through points A, B and C. (5)

[19 marks]

QUESTION 10

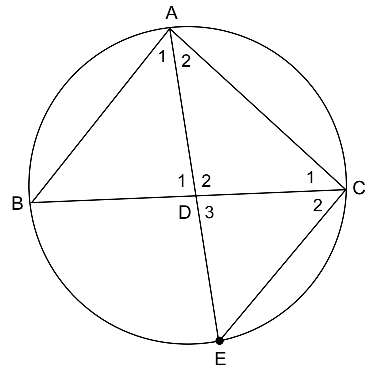
10.1 Prove the theorem that states:
If two triangles are equiangular, then the triangles are similar.



(6)

10.2 Refer to the diagram:

A, B, E and C are points on a circle.
AE bisects $B\hat{A}C$. BC and AE intersect at D.



Prove that:

10.2.1 $\Delta ABD \mathbin{|||} \Delta AEC$ (4)

10.2.2 $\Delta ABD \mathbin{|||} \Delta CED$ (3)

10.2.3 $AB.AC = AD^2 + BD.DC$ (5)

[18 marks]

TOTAL: 150

8.2 Maximum light when $\frac{d\,\text{Light}}{dx} = 0$

$\therefore\ 3 - 2x = 0$

$\therefore\ \boldsymbol{x = \frac{3}{2}}$ ◄

$y = 3 - \frac{3}{2} - \frac{\pi}{2} \times \frac{3}{2}$

$= \boldsymbol{\frac{3}{2}\left(1 - \frac{\pi}{2}\right)}$ ◄

8.3 Maximum light $= 3 \times \frac{3}{2} - \left(\frac{3}{2}\right)^2$

$= \boldsymbol{\frac{9}{4}}$ ◄

---

9. Let n be the total number of units produced.

Profit

$= 225\,000 + (n - 500)(450 - 2n + 1\,000)$

$= 225\,000 + (n - 500)(1\,450 - 2n)$

$= 225\,000 + 1\,450n - 2n^2 - 725\,000 + 1\,000n$

$= -2n^2 + 2\,450n - 500\,000$

Maximum profit when $\frac{d\,\text{Profit}}{dn} = 0$

$\therefore\ -4n + 2\,450 = 0$

$\therefore\ n = 612{,}5$

Using 612 or 613 gives Maximum profit = **R250 312** ◄

OR: Let n be the number of units above 500.

$P = 500 \times 450 + n(450 - 2n)$

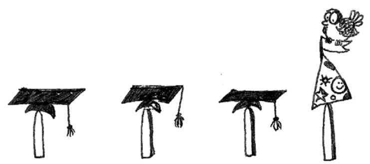
$= 225\,000 - 2n^2 + 450n$

$\frac{dP}{dn} = -4n + 450 = 0$

$\therefore\ n = 112{,}5$

Using 112 or 113:

Maximum profit, P = **R250 312** ◄



MEMO PAPER 1C

1.1.1 $\sqrt{x+2} + 4 = x$

$\therefore\ \sqrt{x+2} = x - 4$

$\therefore\ x + 2 = x^2 - 8x + 16$

$\therefore\ x^2 - 9x + 14 = 0$

$\therefore\ (x - 2)(x - 7) = 0$

$\therefore\ x = 2$ or $x = 7$

Check: $x = 2$:

LHS $= \sqrt{2+2} + 4$

$= 6 \neq$ **RHS**

$x = 7$:

LHS $= \sqrt{7+2} + 4$

$= 7 =$ **RHS**

$\therefore\ \boldsymbol{x = 7}$ ◄

1.1.2 $(x - 3)(x - 4) \geq 6$

$\therefore\ x^2 - 7x + 12 - 6 \geq 0$

$\therefore\ x^2 - 7x + 6 \geq 0$

$\therefore\ (x - 1)(x - 6) \geq 0$

expr.: + 0 - 0 +

x: 1 6

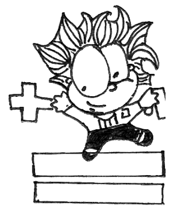
$\boldsymbol{x \leq 1}$ or $\boldsymbol{x \geq 6}$ ◄

1.2 $\left(3\sqrt{5} - 2\sqrt{2}\right)^2$

$= 9 \times 5 - 12\sqrt{10} + 4 \times 2$

$= 45 - 12\sqrt{10} + 8$

$= \boldsymbol{53 - 12\sqrt{10}}$ ◄



1.3 $f(x) = (x^3 - 2)(5 - x)$

$= 5x^3 - x^4 - 10 + 2x$

$f'(x) = 15x^2 - 4x^3 + 2$

$f'(2) = 15 \times 2^2 - 4 \times 2^3 + 2$

$= 30$

2.1 $T_k = 5k - 2$

2.1.1 $T_{21} = 5 \times 21 - 2$

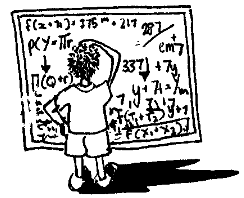
$= \boldsymbol{103}$ ◄

2.1.2 $5k - 2 = 1\,173$

$\therefore\ 5k = 1\,175$

$\therefore\ k = 235$

i.e. $\boldsymbol{T_{235}}$ ◄



2.1.3 A.P.: $a = 3$, $d = 5$

$S_{300} = \frac{300}{2}[2 \times 3 + 299 \times 5]$

$= \boldsymbol{225\,150}$ ◄

2.2.1 **49 ; 71** ◄

2.2.2

	1	7	17	31	49	71
1st difference		6	10	14	18	22
2nd difference			4	4	4	4

Using the formula: $T_n = T_1 + (n - 1)f + \frac{(n-1)(n-2)}{2}s$

$\therefore\ T_n = 1 + (n - 1)6 + \frac{(n-1)(n-2)}{2} \times 4$

$= 1 + 6n - 6 + 2(n^2 - 3n + 2)$

$= 6n - 5 + 2n^2 - 6n + 4$

$= \boldsymbol{2n^2 - 1}$ ◄

Alternatively: $2a = d = 4$ . . . *constant second diff.*

$\therefore\ a = 2$

$T_0 = c = -1$

$T_n = 2n^2 + bn - 1$

$\therefore\ T_1 = 2 + b - 1 = 1$

$\therefore\ b = 0$

$\therefore\ \boldsymbol{T_n = 2n^2 - 1}$ ◄

OR: $2a = 4$ ➡ $a = 2$

$T_0 = c = -1$

& The first term of the first differences:

$3a + b = 6$

$\therefore\ 3(2) + b = 6$

$\therefore\ b = 0$

$\therefore\ \boldsymbol{T_n = 2n^2 - 1}$ ◄

2.3 $T_3 = \frac{2}{9} = ar^2$ .... ❶

$T_8 = \frac{-2}{2\ 187} = ar^7$ .... ❷

❷ ÷ ❶: $\frac{ar^7}{ar^2} = \frac{-2}{2\ 187} \div \frac{2}{9}$

$\therefore\ r^5 = -\frac{1}{243}$

$= \left(-\frac{1}{3}\right)^5$

$\therefore\ \mathbf{r = -\frac{1}{3}}$ ◄

❶: $a\left(-\frac{1}{3}\right)^2 = \frac{2}{9}$

$\therefore\ \mathbf{a} = \frac{2}{9} \div \frac{1}{9}$

$\mathbf{= 2}$ ◄

3.1.1 $\mathbf{y > -4}$ ◄

3.1.2 A: $f(0) = 2^{0-1} = -4$

$\therefore\ \mathbf{A\left(0; -\frac{7}{2}\right)}$ ◄

B: $2^{x-1} - 4 = 0$

$\therefore\ 2^{x-1} = 2^2$

$\therefore\ x - 1 = 2$

$\therefore\ x = 3$

same bases
∴ equal exponents

$\therefore\ \mathbf{B(3;\ 0)}$ ◄

3.1.3 $m_{AB} = \frac{0 + \frac{7}{2}}{3 - 0} = \frac{7}{6}$

∴ Equation of AB:

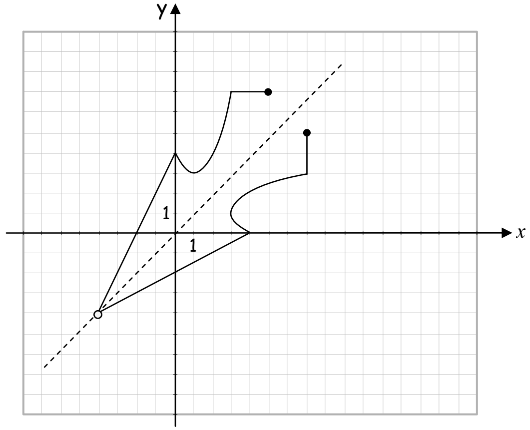
$\mathbf{y = \frac{7x}{6} - \frac{7}{2}}$ ◄

3.1.4 $\mathbf{x < 0}$ or $\mathbf{x > 3}$ ◄

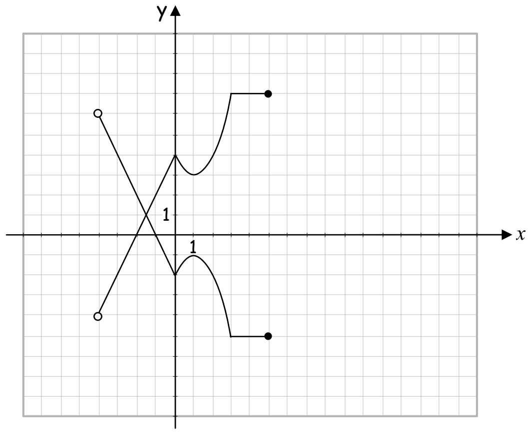
3.2.1 Domain: $\mathbf{x \in (-4;\ 5]}$ ◄

Range: $\mathbf{y \in (-4;\ 7]}$ ◄

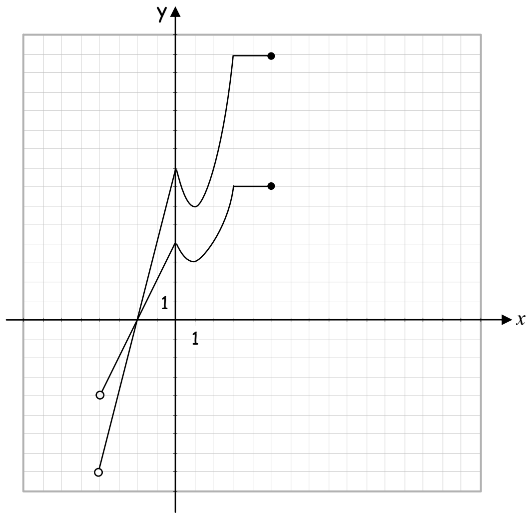
3.2.2



3.2.3



3.2.4



4.1 $P(1{,}11)^{10} = 14\ 000$

$\therefore\ P = 4\ 930{,}5827$

$\simeq \mathbf{4\ 931}$ ◄

4.2.1 $A \leftarrow 1 + \frac{0{,}125}{12}$

$= 1{,}010416666...$

Loan $= \dfrac{9\ 089{,}13\left[1 - A^{-240}\right]}{\frac{0{,}125}{12}}$

$= 800\ 000{,}4931$

$\simeq \mathbf{R800\ 000}$ ◄

4.2.2 $1 + i^{(e)} = A^{12}$

$= 1{,}132416046$

$\therefore\ i^{(e)} = 0{,}132416...$

$\simeq \mathbf{13{,}2\%}$ ◄

4.2.3 Outstanding balance $= 800\ 000A^{24} - \dfrac{9\ 089{,}13\left[A^{24} - 1\right]}{\frac{0{,}125}{12}}$

$= 779\ 512{,}5096$

$\simeq \mathbf{R779\ 512{,}51}$ ◄

Alternatively:

Outstanding balance $= \dfrac{9\ 089{,}13\left[1 - A^{-216}\right]}{\frac{0{,}125}{12}} = \mathbf{R779\ 513{,}14}$ ◄

4.2.4 Total payments $= 24 \times 9\ 089{,}13$

$= 218\ 139{,}12$

Paid off $= 800\ 000 - 779\ 512{,}51$

$= 20\ 487{,}49$

Interest $= 218\ 139{,}12 - 20\ 487{,}49$

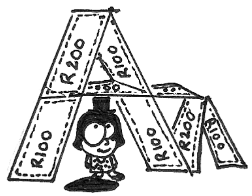
$= \mathbf{R197\ 651{,}63}$ ◄

OR: Paid off $= 800\ 000 - 779\ 513{,}14$

$= 20\ 486{,}86$

Interest $= 218\ 139{,}12 - 20\ 486{,}86$

$= \mathbf{R197\ 652{,}26}$ ◄



MEMO PAPER 2B

1.1 **S(5; 2)** ◄ **T(1; 1)** ◄

1.2 $m_{RQ} = \frac{7-5}{6+2} = \frac{2}{8} = \frac{1}{4}$

$m_{ST} = \frac{1-2}{1-5} = \frac{1}{4}$

$\therefore$ **ST || RQ** ◄ . . . *equal gradients*

1.3 $m_{RP} = -\frac{4}{3}$

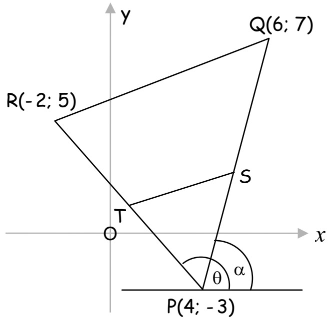
$\therefore \tan\theta = -\frac{4}{3}$

$\therefore \theta = -53{,}1° + 180°$
$= 126{,}9°$

$m_{PQ} = 5$

$\therefore \tan\alpha = 5$
$\alpha = 78{,}7°$

$R\hat{P}Q = \theta - \alpha$
$= \mathbf{48{,}2°}$ ◄



1.4 Area of RQST $= \frac{(RQ + ST).h}{2}$ . . . *trapezium*

$RQ = \sqrt{(8)^2 + (2)^2} = \sqrt{68} = 2\sqrt{17}$

$ST = \sqrt{17}$ . . . *midpoint theorem; ST || RQ in ΔPQR*

$\therefore$ Area $= \frac{3\sqrt{17}.4}{2} = \mathbf{6\sqrt{17}\ units^2}$ ◄

2.1 $x^2 + 6x \quad + y^2 + 4y \quad = 4$

$\therefore x^2 + 6x + \mathbf{9} + y^2 + 4y + \mathbf{4} = 4 + \mathbf{9} + \mathbf{4}$

$\therefore \mathbf{(x+3)^2 + (y+2)^2 = 17}$ ◄

The centre: **C(-3; -2)** ◄

2.2 $m_{rad} = \frac{2-(-2)}{-4-(-3)} = \frac{4}{-1} = -4$

$\therefore m_{TAN} = \frac{1}{4}$ . . . *tan ⊥ rad*

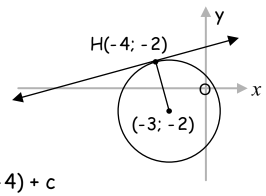
$\therefore$ Eqn. of tangent: $y = \frac{1}{4}x + c$

Substitute (-4; 2): $\therefore 2 = \frac{1}{4}(-4) + c$

$\therefore 2 = -1 + c$

$\therefore 3 = c$

Eqn.: $\mathbf{y = \frac{1}{4}x + 3}$ ◄



3.1 $m_{AB} = \frac{3-12}{1-4} = \frac{-9}{-3} = 3$

& $m_{BC} = \frac{2-3}{4-1} = \frac{-1}{3} = -\frac{1}{3}$

$\therefore m_{AB} \times m_{BC} = -1$, i.e. **AB ⊥ BC** ◄

3.2 Midpoint AC $\left(4;\ \frac{12+2}{2}\right)$

$\therefore$ **T(4; 7)** ◄

Note: $x_A = x_C = x_T$

3.3 AC is the diameter of ⊙ABC . . . $A\hat{B}C = 90°$

& T, midpoint of AC, is the centre.

$\therefore$ Radius $= \frac{1}{2}AC = \frac{1}{2}(10) = 5$ units & centre is T(4; 7)

$\therefore$ Equation of ⊙ABC: $\mathbf{(x-4)^2 + (y-7)^2 = 25}$ ◄

3.4 For ABCD to be a cyclic quadrilateral,

$A\hat{D}C = 90°$ . . . *opposite ∠s of cyclic quadrilateral*

$m_{AD} = \frac{4-12}{p-4} = \frac{-8}{p-4}$ & $m_{CD} = \frac{4-2}{p-4} = \frac{2}{p-4}$

$A\hat{D}C = 90° \Rightarrow m_{AD} \times m_{CD} = -1$ . . . *prod. of gradients = -1*

$\therefore \frac{-8}{p-4} \times \frac{2}{p-4} = -1$

$\times (p-4)^2$: $\therefore -16 = -1(p-4)^2$

$\therefore (p-4)^2 = 16$

$\therefore p - 4 = \pm 4$

$\therefore p = 4 \pm 4$

$\therefore$ **p = 8** ◄ or p = 0 *(not valid)*

p ≠ 0 because if D is on the y-axis, we do not have a quadrilateral ABCD, but ADBC.

3.5 $\tan\theta = m_{BT} = \frac{7-3}{4-1} = \frac{4}{3}$

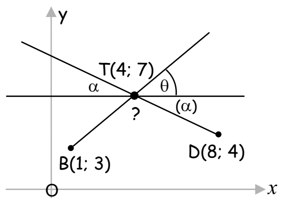
$\therefore \theta = 53{,}13°$

$\tan(180° - \alpha) = \frac{7-4}{4-8} = \frac{3}{-4}$

$\therefore 180° - \alpha = 180° - 36{,}87°$

$\therefore \alpha = 36{,}87°$

$B\hat{T}D = 180° - (53{,}13° + 36{,}87°) = \mathbf{90°}$ ◄



4.1 $\bar{X}$ **= 2,8** ◄ . . .*

See calculator instructions at the back of this book.

***Note:** When working with raw data, the mean is represented by the notation $\bar{x}$.
But for grouped data, the estimated mean is $\bar{X}$.

4.2 **SD, σ = 0,8** ◄

5.1 $y = a + bx$

$\therefore$ $\mathbf{y = 105{,}1114 + 2{,}4671x}$ ◄

5.2 $y = 105{,}1114 + 2{,}4671(26)$

$\simeq$ **169,3 cm** ◄

5.3 $\bar{x} = 26{,}1545\ldots$ & $\bar{y} = 169{,}6364\ldots$

Substitute $\bar{x} = 26{,}1545\ldots$

$y = 105{,}1114 + 2{,}4671(26{,}1545)$

$= 169{,}64$

$= \bar{y}$

$\therefore (\bar{x}; \bar{y})$ **lies on the line of best fit** ◄

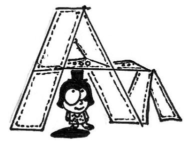
5.4 The correlation coefficient, **r ≃ 0,7** ◄

5.5 **Fairly strong positive correlation** ◄

6.1.1 $\frac{1}{\cos 50°}$

$= \frac{1}{\sin 40°}$

$= \mathbf{\frac{1}{k}}$ ◄

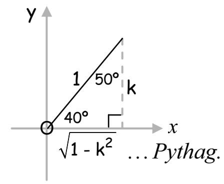


6.1.2 $\sin 80°$

$= \sin 2(40°)$

$= 2\sin 40°.\cos 40°$

$= \mathbf{2k\left(\sqrt{1-k^2}\right)}$ ◄



6.1.3 $\cos 20°$

$= \cos(60° - 40°)$

$= \cos 60°.\cos 40° + \sin 60° \sin 40°$

$= \mathbf{\frac{1}{2}\left(\sqrt{1-k^2}\right) + \frac{\sqrt{3}}{2}(k)}$ ◄

6.2 $\sin\alpha = -\frac{4}{5}$

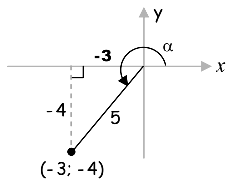
& $0° < \alpha < 270°$

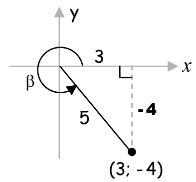
$\therefore \alpha$ in 3rd quad.

$\cos\beta = \frac{3}{5}$

& $\beta > 90°$

$\therefore \beta$ in 4th quad.





$\sin(\alpha - \beta) = \sin\alpha \cdot \cos\beta - \cos\alpha \cdot \sin\beta$

$= \left(-\frac{4}{5}\right)\left(\frac{3}{5}\right) - \left(\frac{-3}{5}\right)\left(\frac{-4}{5}\right)$

$= -\frac{12}{25} - \frac{12}{25}$

$= \mathbf{-\frac{24}{25}}$ ➤

6.3 Expression $= \dfrac{\cos x \cdot \cos x}{(1 - \sin x)(1 + \sin x)}$

$= \dfrac{\cos^2 x}{1 - \sin^2 x}$

$= \dfrac{\cos^2 x}{\cos^2 x}$

$= \mathbf{1}$ ➤

6.4 $\dfrac{(\cos 30°)\cdot(-\cos 20°)}{(\tan 45°)(-\sin 30°)(-\sin 70°)}$

$= -\dfrac{(\cos 30°)}{(\tan 45°)(\sin 30°)}$ ... *$\cos 20° = \sin 70°$*

$= \dfrac{-\left(\frac{\sqrt{3}}{2}\right)}{(1)\left(\frac{1}{2}\right)} \times \dfrac{2}{2}$

$= \mathbf{-\sqrt{3}}$ ➤

6.5.1 $\tan\alpha = \pm 1$

$\therefore\ \alpha = \mathbf{45°}$ or $\mathbf{135°}$ ➤

6.5.2 $\dfrac{2\sin\alpha \cdot \cos\alpha}{\cos^2\alpha - \sin^2\alpha} = 1$

$\therefore\ \dfrac{\sin 2\alpha}{\cos 2\alpha} = 1$

$\therefore\ \tan 2\alpha = 1$

$\therefore\ 2\alpha = 45°$ or $225°$ ... $\alpha \in [0°; 180°]$

$\therefore\ \alpha = \mathbf{22{,}5°}$ or $\mathbf{112{,}5°}$ ➤ ➡ $2\alpha \in [0°; 360°]$

6.6 $2[\cos x \cdot \cos 30° - \sin x \cdot \sin 30°] = (\sqrt{3} - 1)\sin x$

$\therefore\ 2\left[\cos x\left(\frac{\sqrt{3}}{2}\right) - \sin x\left(\frac{1}{2}\right)\right] = \sqrt{3}\sin x - \sin x$

$\therefore\ \sqrt{3}\cos x - \sin x = \sqrt{3}\sin x - \sin x$

$\div\sqrt{3}\cos x)$ $\therefore\ \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{\sin x}{\cos x}$

$\therefore\ \tan x = 1$

$\therefore\ \mathbf{x = 45° + k.180°;\ k \in \mathbb{Z}}$ ➤

7. $(DF)^2 = x^2 + x^2$ ... *Pythagoras*

$= 2x^2$

$\therefore\ DF = \sqrt{2x^2} = \sqrt{2}x$

$\therefore\ DQ = \frac{1}{2}\sqrt{2}x$

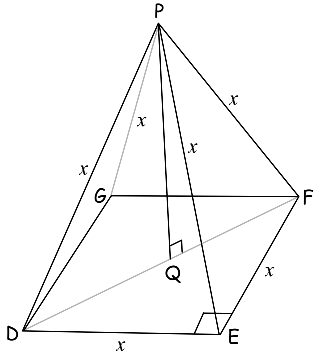
$\therefore\ (PQ)^2 = x^2 - \left(\frac{1}{2}\sqrt{2}x\right)^2$

$= x^2 - \dfrac{2x^2}{4}$

$= \frac{1}{2}x^2$

$\therefore\ PQ = \dfrac{x}{\sqrt{2}}\left(\times\dfrac{\sqrt{2}}{\sqrt{2}}\right)$

$= \mathbf{\dfrac{\sqrt{2}x}{2}}$ **units** ➤



8.1.1 **true** ➤ ... *tangent (PQ) - chord (QV) theorem*

8.1.2 **false** ➤ ... *subtended by different arcs*

8.1.3 **true** ➤ ... *exterior ∠ of cyclic quad. VRST*

8.1.4 **true** ➤ ... *all vertices on the circumference of the ⊙*

8.1.5 ✱**false** ➤ ... *it has not been given that QS is a diameter*
✱[It would be true **if** QS is a diameter.]

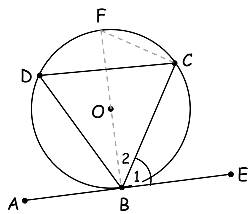
8.1.6 **false** ➤ ... *not opp. $\angle^s$ of a cyclic quadrilateral*

8.2 $\hat{Q} = 60°$ ... *opp. $\angle^s$ of cyclic quad.*

In $\triangle PQR$: $(PR)^2 = (5)^2 + (8)^2 - 2(5)(8)\cdot\cos 60°$

$\therefore$ **PR = 7 units** ➤

9.1 Prove: $\hat{B}_1 = \hat{D}$



Construct:

Diameter FB and draw FC

Proof:

$\hat{B}_1 + \hat{B}_2 = 90°$ ... *tan ⊥ diam.*

$F\hat{C}B = 90°$ ... *∠ in semi-⊙*

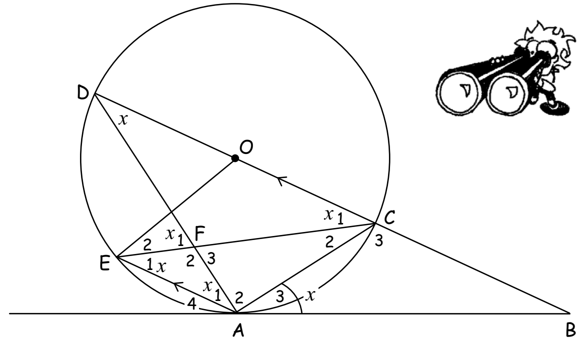
$\hat{F} + \hat{B}_2 = 90°$ ... *int. $\angle^s$ of ΔFBC*

$\therefore\ \hat{B}_1 = \hat{F}$

but $\hat{F} = \hat{D}$ ... *$\angle^s$ in same segment*

$\therefore\ \mathbf{\hat{B}_1 = \hat{D}}$ ➤

9.2



9.2.1 $\hat{A}_1 = x$ ➤ ... *alternate $\angle^s$; AE || BD*

$\hat{E}_1 = x$ ➤ ... *$\angle^s$ in same segment (as $\hat{D}$)*

$\hat{C}_1 = \hat{A}_1 = x$ ➤ ... *alternate $\angle^s$; AE || BD*

$\hat{E}_2 = \hat{C}_1 = x$ ➤ ... *∠ opp. = sides - radii*

$\hat{A}_3 = \hat{D} = x$ ➤ ... *tan-chord theorem*

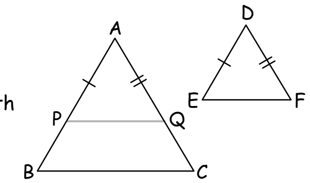
9.2.2 Need to prove: $\hat{C}_2 = \hat{B}$
& then apply the converse of the tan-chord theorem.

$\hat{C}_2 = \hat{A}_4$ ... *tan-chord theorem*

$= \hat{B}$ ... *corresponding $\angle^s$; AE || BD*

$\therefore$ **EC is a tangent to the circle passing through A, B and C** ➤ ... *converse tan-chord theorem*

10.1 Prove: $\triangle ABC\ |||\ \triangle DEF$



Construction:

Mark AP = DE & AQ = DF with P & Q on AB and AC. Join PQ.

Proof: In ΔAPQ and ΔDEF

AP = DE ... *construction*

$\hat{A} = \hat{D}$ ... *given*

AQ = DF ... *construction*

$\therefore\ \triangle APQ \equiv \triangle DEF$ ... *S∠S*

$\therefore\ A\hat{P}Q = \hat{E}$

$\therefore\ A\hat{P}Q = \hat{B}$ ... *$\hat{E} = \hat{B}$*

$\therefore$ PQ || BC ... *corresponding $\angle^s$ equal*

$\therefore\ \dfrac{AP}{AB} = \dfrac{AQ}{AC}$ ... *proportion theorem*

But AP = DE and AQ = DF

$\therefore\ \dfrac{DE}{AB} = \dfrac{DF}{AC}$

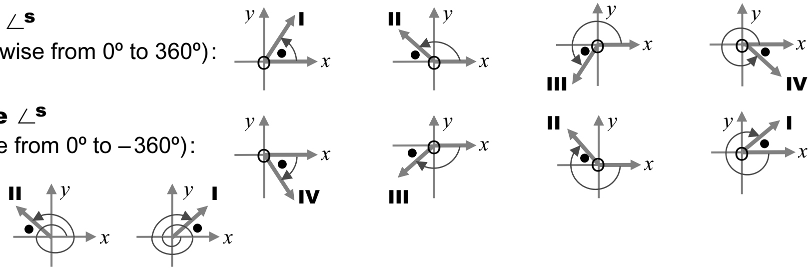
TRIG SUMMARY (Grade 12)

THE ANSWER SERIES *Your Key to Exam Success*

▸ ANGLES IN STANDARD POSITIONS

- **Positive** $\angle^s$
  (anticlockwise from 0º to 360º):
- **Negative** $\angle^s$
  (clockwise from 0º to –360º):

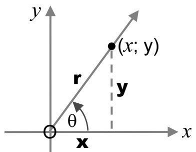
Also possible:



▸ THE RATIOS

& their

- **Definitions:**



- **Signs:**
  sin θ is positive in **I** & **II**
  cos θ is positive in **I** & **IV**
  tan θ is positive in **I** & **III**

- **Critical values:**

	sin θ	cos θ	tan θ
Definitions	$\frac{y}{r}$ (O/H)	$\frac{x}{r}$ (A/H)	$\frac{y}{x}$ (O/A)
Signs	II, I	I, IV	I, III
Critical values	1; 0, 0; –1	0; –1, 1; 0	∞; –1, 1; 0, 0; 1, –1; ∞

- **Minimum & Maximum values** of **sin θ & cos θ**:
  The values of **sin** θ & **cos** θ range from **–1** to **1**.

minimum value is **–1**

$$-1 \le \begin{matrix}\sin\theta \\ \cos\theta\end{matrix} \le 1$$

maximum value is **1**

All values are **proper fractions** or **0** or **±1**.

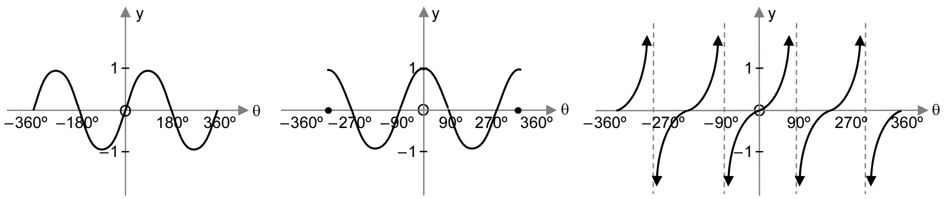
tan θ

Note that tan θ has **no minimum or maximum** values.

The **range of values** of tan θ is from **–∞** to **∞**.

- **Graphs:**

y = sin θ **y = cos θ** **y = tan θ**



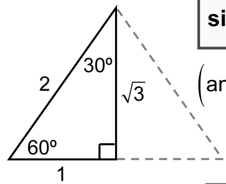
▸ IDENTITIES

$$\boxed{\tan\theta = \frac{\sin\theta}{\cos\theta}}$$

$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$

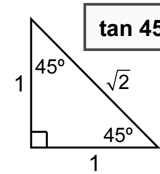
$\therefore\ \sin^2\theta = 1 - \cos^2\theta$
& $\cos^2\theta = 1 - \sin^2\theta$

▸ SPECIAL $\angle^s$

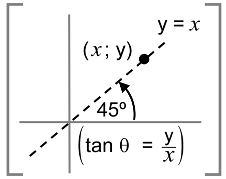


$$\boxed{\sin 30º = \frac{1}{2}}$$

$\left(\text{and } \cos 60º = \frac{1}{2}\right)$



$$\boxed{\tan 45º = 1}$$



$\left(\tan\theta = \frac{y}{x}\right)$

& their 'families':

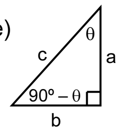
	30º 'family'	60º 'family'	45º 'family'	
II	150º	120º	135º	180º – ●
III	210º	240º	225º	180º + ●
IV	330º	300º	315º	360º – ●

▸ GENERAL FORMS

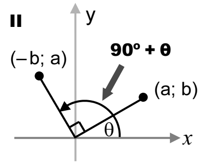
ANY ratio $\begin{pmatrix}180º \pm \theta \\ 360º - \theta \\ -\theta\end{pmatrix}$ = **±** that **SAME** ratio of **θ**

▸ CO-RATIOS (sine and **co**sine)

- **90º – θ** (an **acute** angle)
  sin(90º – θ) = cos θ
  cos(90º – θ) = sin θ



- **90º + θ** (an **obtuse** angle)
  sin(90º + θ) = cos θ
  cos(90º + θ) = –sin θ



The ratio **CHANGES** to the **CO**-ratio.

▸ SOLUTION OF Δ^s

In Right-angled Δ^s, we use:

- Regular trig. ratios
- the Theorem of Pythagoras
- Area = $\frac{1}{2}$bh

In Non-Right-angled Δ^s, we use:

- **Sine Rule:** $\boxed{\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}}$
- **Cosine Rule:** $\boxed{c^2 = a^2 + b^2 - 2ab\cos C}$
- **Area Rule:** $\boxed{\textbf{AREA} = \frac{1}{2}\textbf{ab sin C}}$

But also: Area of Δ = $\frac{1}{2}$ bh

▸ COMPOUND $\angle^s$

- $\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$
- $\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

▸ DOUBLE $\angle^s$

- $\sin 2x = 2\sin x\cos x$
- $\cos 2x = \cos^2 x - \sin^2 x$
  $= 1 - 2\sin^2 x$
  $= 2\cos^2 x - 1$

TRIG SUMMARY