Mathematics IEB

PAPERS & ANSWERS

Marilyn Buchanan, *et al.*

GRADE

11

IEB

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The Answer Series would like to acknowledge the huge contribution made by Bonita Morgan and Judy Crowster, who typeset the material in this book with the utmost dedication and expertise.

The Answer Series
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Note:
The information sheet is only provided for the Gr 12 exam.

ABOUT THIS BOOK

The examination papers compiled in this book are an attempt by The Answer Series to provide teachers and learners with practice material in preparation for the end-of-year examinations. They are an interpretation of the CAPS curriculum and should not be taken to indicate the only type of questions that could be asked, but rather as possible examples.

There are 10 paper 1's and 10 paper 2's. The first 5 of each are newly compiled, while the second 5 have been compiled by adapting the papers from the previous edition of this book. All 20 papers have been set according to the requirements of the CAPS curriculum. The allocation of marks to topics has occasionally been influenced by the need to provide more practice where deemed necessary.

All 10 paper 1's have been compiled by Marilyn Buchanan (current IEB examiner) – the first 5 created; the second 5 adapted. The 5 new paper 2's have been compiled by Praveshen Iyer (future IEB examiner) and a team of senior teachers from leading schools. The second 5 paper 2's have been adapted by Anne Eadie, coordinator of this project. We are indebted to Janet Aird and Gail Hallet who made a valuable contribution by checking sections of this book.

We trust that experiencing this comprehensive compendium of questions and answers will place learners in a strong position to succeed in the CAPS examinations.

We will welcome constructive comments from both teachers and learners.

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EXAM PAPER 1D

Approved non-programmable and non-graphical calculators may be used, unless otherwise stated.

*Round off your answers to **ONE** decimal digit where necessary, unless otherwise stated.*

SECTION A

QUESTION 1

1.1 Write down the first four terms of the following sequences:

1.1.1 $T_n = \frac{24}{n}$ 1.1.2 $T_n = 5^n + 2$ (1)(1)

1.2 Simplify:

1.2.1 $\sqrt[4]{\frac{16x^{12}}{y^8}}$ 1.2.2 $\frac{a + b}{a^{-1} + b^{-1}}$ (3)(4)

1.3 Solve for x:

1.3.1 $2x(4x - 1) = 15$ (4)

1.3.2 $x^{\frac{3}{4}} = 27$ (3)

1.3.3 $\sqrt{10 - 3x} = x - 2$ (5)

1.3.4 $\frac{8}{x^2 - 4} + \frac{x}{2 - x} + \frac{1}{x + 2} = 0$ (7)

1.3.5 $px^2 - 6x = q$ by completing the square. (6)

1.4 Given: $2x^2 + mx + 18 = 0$
Determine the values of m so that the equation has real roots. (4)

[38 marks]

QUESTION 2

2.1 Given the quadratic sequence: 59 ; 48 ; 39 ; 32 . . .

Determine:

2.1.1 the constant second difference. (2)

2.1.2 a formula for the n^{th} term of the sequence. (4)

2.2 Given: 1 ; 3 ; 5 ; 7 ; 9 ; 1 ; 3 ; 5 ; 7 ; 9 ; 1 ; 3 . . .
Determine the value of T_{2014}. (3)

[9 marks]

QUESTION 3

3.1 On her 18^{th} birthday, Emma received a new car valued at R265 000.

Cars depreciate in value by 20% per year. Determine the value of Emma's car on her 21^{st} birthday. (2)

3.2 Nomfundo has a bank account earning interest at 8,5% p.a. compounded monthly.

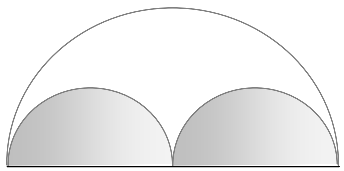
On her 16^{th} birthday she already has R8 500 in the account and decides to invest all her birthday gift money into the account as she hopes to have R20 000 available on her 21^{st} birthday.

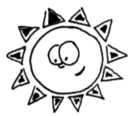
Assuming no further deposits are made into the account, calculate how much money Nomfundo will need to receive on her 16^{th} birthday. (5)

[7 marks]

QUESTION 4

4.1 Determine the probability that a point selected at random within the large semi-circle will also be within one of the equal sized small semi-circles. (4)

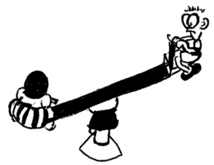




4.2 The probability that at 10 a.m. Shelby will go to the gym is 0,35 and the probability that she will go to a coffee shop is 0,16.

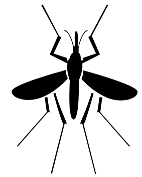
Determine the probability that she will neither go to gym nor go to a coffee shop. (4)

[8 marks]



QUESTION 5

The number of mosquitoes in a certain region in Africa depends on the rainfall in January of a given year.



The function $N(x) = 250x - x^2$ gives an approximate number, $N(x)$, of thousands of mosquitoes when the rainfall is x mm in January.

5.1 Calculate the predicted number of mosquitoes after a January rainfall of 20 mm. (2)

5.2 Determine how much rain will cause 15 million mosquitoes. (5)

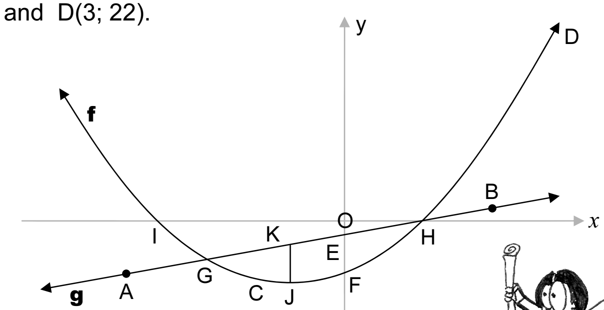
5.3 Determine the maximum number of mosquitoes that can be predicted using this model. (5)

[12 marks]

SECTION B

QUESTION 6

Refer to the figure showing a straight line passing through A(-3; -4) and B(2; 1), and a parabola of the form $f(x) = ax^2 + bx - 5$, passing through C(-1; -6) and D(3; 22).



6.1 Determine the equation of the straight line $y = g(x)$ passing through A and B. (4)

6.2 Showing all working, prove that a = 2 and b = 3. (6)

6.3 Calculate the length of EF. (2)

6.4 Calculate the length of IH. (5)

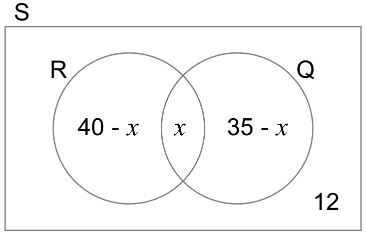
6.5 Determine the coordinates of G. (5)

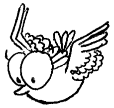
6.6 Calculate the length of KJ, where J is the turning point of the parabola. (7)

[29 marks]

QUESTION 7

7.1 Refer to the Venn diagram below showing information about a sample space S and two events R and Q.





It is given that n(R or Q) = 68.

Determine:

7.1.1 the value of x. 7.1.2 n(S). (3)(1)

7.1.3 The probability of an item chosen at random:

(i) not being in R nor Q. (1)

(ii) being in R but not Q. (2)

7.2 The probability of Isabella passing a driving test on the first appointment is $\frac{3}{7}$.

For each subsequent attempt after failing, the probability of her passing the test is $\frac{3}{5}$.

Determine the probability of Isabella passing the test in:

7.2.1 2 attempts 7.2.2 3 attempts (2)(2)

7.2.3 4 or more attempts (4)

[15 marks]

QUESTION 8

A doctor must decide on which antibiotic to prescribe for Pontsho.

Antibiotic A causes bacteria to decrease at a rate of 3% every 15 minutes.

For antibiotic B, the bacteria decreases at 2,5% every 10 minutes.

Taking t as the number of hours since Pontsho's first dose of medicine, and P_0 as the initial mass of bacteria:

8.1 Set up formulae to indicate the amount of bacteria in Pontsho's body t hours after the first dose of each antibiotic. (4)

8.2 On the same set of axes, draw graphs showing the effect of each antibiotic over a 24 hour period, using $P_0 = 1\,000$. (5)

8.3 Calculate the difference in mass of the bacteria after 12 hours. (1)

[10 marks]

QUESTION 9

9.1 Given $2x - 4$; $2x + 1$; $3x - 5$ as the first three terms of a linear sequence.

9.1.1 Determine the value of x. (4)

9.1.2 Hence calculate the constant difference. (3)

9.2.1 Solve for p: $3 - \frac{9}{p^2} = \frac{26}{p}$ (4)

9.2.2 Hence solve for x: $3^0 - 3^{2-2x} + 2 = \frac{26}{3^x}$ (6)

9.3.1 Rationalise the denominator in the expression $\frac{1}{\sqrt{n} + \sqrt{n+1}}$ where n is a natural number. (3)

9.3.2 Using your result in part 9.3.1, evaluate WITHOUT USING A CALCULATOR:

$$\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \ldots + \frac{1}{\sqrt{99}+\sqrt{100}}$$

(2)

[22 marks]

Decimal digits

The instruction in **IEB** exams is for answers to be rounded off to **ONE decimal digit** where necessary, unless otherwise stated, whereas National exams require **TWO digits**.

EXAM PAPER 1E

Approved non-programmable and non-graphical calculators may be used, unless otherwise stated.

*Round off your answers to **ONE** decimal digit where necessary, unless otherwise stated.*

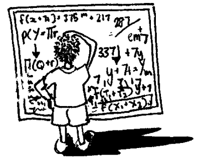
SECTION A

QUESTION 1

1.1 Simplify:

1.1.1 $\frac{\sqrt{9x^3} + \sqrt{x^5} - \sqrt{16x^3}}{\sqrt{x}}$ (5)

1.1.2 $\frac{1^{ab}}{a^{-1} + b^{-1}}$ (4)



1.2 Solve for x:

1.2.1 $x^2 + 8x = 0$ (2)

1.2.2 $\sqrt{x+5} - x - 3 = 0$ (6)

1.2.3 $9^{27x} \times 27^{9x} = 729$ (4)

1.3 The equation $x^2 + (2p - 5)x + p^2 = 0$ needs to have real roots.

1.3.1 Use the discriminant to determine the values of p. (5)

1.3.2 Given that p is an integer, determine the greatest possible value of p. (1)

1.3.3 Using the integer value of p from above, solve the equation. (3)

[30 marks]

QUESTION 2

2.1 Given: $T_n = 2n$ if n is odd

& $T_n = n + 1$ if n is even

Write down the first six terms of this sequence. (2)

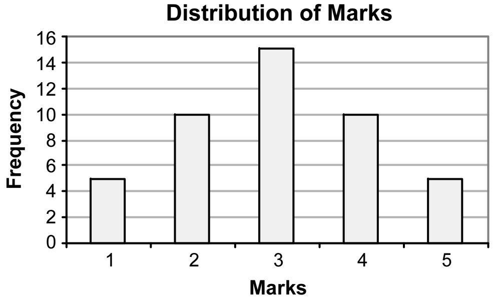
EXAM PAPER 2G

Approved non-programmable and non-graphical calculators may be used, unless otherwise stated.

*Round off your answers to **ONE** decimal digit where necessary, unless otherwise stated.*

QUESTION 1

1.1 The bar chart below shows the distribution of the marks obtained by a class in a particular test question.



1.1.1 Calculate the mean mark. (3)

1.1.2 Complete the table below. (5)

x_i	f_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$f_i \times (x_i - \bar{x})^2$
1				
2				
3				
4				
5				
	Σ :			Σ :

1.1.3 Calculate the variance and standard deviation for the distribution of marks. (3)

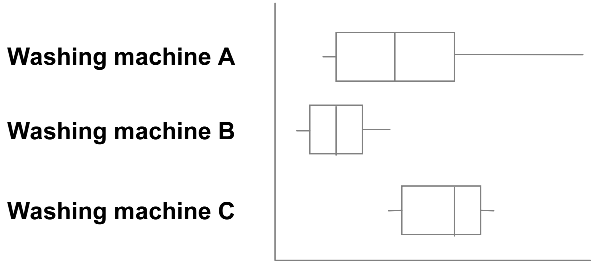
Decimal digits

The instruction in **IEB** exams is for answers to be rounded off to **ONE decimal digit** where necessary, unless otherwise stated, whereas National exams require **TWO digits**.

1.2 A consumer testing company studied three brands of washing machines to see how much water was used during each wash.

Each washing machine was tested 25 times.

The box and whisker plots below show the results of this study.



Number of litres used by washing machine

1.2.1 Which brand machine (A, B or C) frequently uses the most water? (1)

1.2.2 Explain why the mode is not a good measure of central tendency in this situation. (1)

1.2.3 Which brand machine (A, B or C) is the most predictable? (1)

1.2.4 Explain how you can tell from a box and whisker diagram whether the range will be a good measure of dispersion or not. (1)

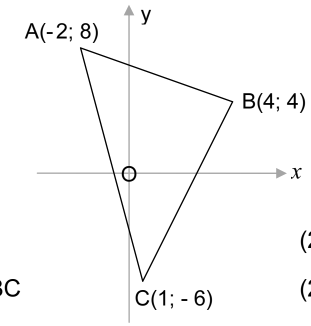
1.2.5 If the interquartile range for the machine A data is 57 litres and the median is 161 litres, estimate the lower quartile litres used and the upper quartile litres used. (2)

1.2.6 In one of the sets of data above the outlier results have not been ignored. State which set (A, B or C), giving reasons. (2)

[19 marks]

QUESTION 2

2.1 The diagram shows triangle ABC with vertices A(-2; 8), B(4; 4) and C(1; -6).



Determine:

2.1.1 the length of AB (2)

2.1.2 the midpoint of BC (2)

2.1.3 the gradient of AC (2)

2.1.4 the angle of inclination of AC (2)

2.1.5 the size of $A\hat{C}B$ (4)

2.2 Given the points A(1; 3) and B(0; 2). Find the equation of the straight line through B and perpendicular to AB. (4)

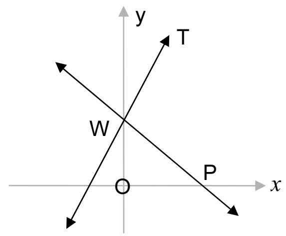
2.3 The equation of a line is given by $y - a = 2(x - b)$.

Determine:

2.3.1 the gradient of the line (1)

2.3.2 a pair of possible values of a and b if the line passes through C(1; -6) (3)

2.4 In the diagram alongside, straight lines TW and PW intersect at W.



The equation of TW is $y = 2x + 5$.

Point W lies on the y-axis.

Point P lies on the x-axis.

2.4.1 Give the coordinates of W. (1)

2.4.2 If TW ⊥ WP, determine the equation of straight line, WP. (4)

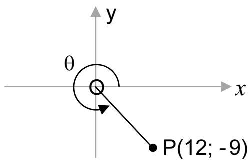
2.4.3 Determine the coordinates of P. (2)

2.4.4 Calculate the area of ΔWOP. (3)

[30 marks]

QUESTION 3

3.1 In the diagram, P(12; -9) is a point on OP.



3.1.1 Determine, without a calculator, the value of $\sin\theta$. (3)

3.1.2 Use a calculator to determine the size of θ. (2)

3.1.3 If R(4; a) is a point on OP, find the value of a. (1)

3.2.1 Simplify $\dfrac{\sin A \cdot \cos A \cdot \tan A}{1 - \cos^2 A}$ (3)

3.2.2 Hence solve the equation for $\theta \in [0°; 360°]$ if

$\tan\theta = \dfrac{\sin A \cdot \cos A \cdot \tan A}{1 - \cos^2 A}$ (3)

3.3 If $\sin x = a$, express the following in terms of a:

3.3.1 $\sin(180° - x)$ 3.3.2 $\sin(180° + x)$ (1)(1)

3.3.3 $\sin(-x)$ (1)

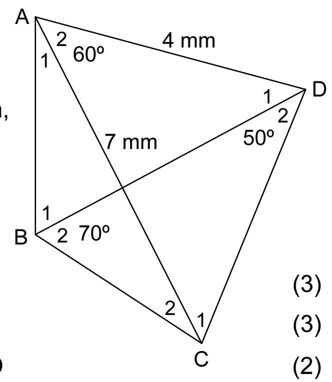
3.4 Evaluate without using a calculator:

$\dfrac{\tan 300° + \cos(90° + x)}{\sin x + 2\cos(-30°)}$ (6)

3.5 Determine the general solution to

$1 - \cos\theta = \cos 44°$ (5)

3.6 In the diagram alongside, quadrilateral ABCD with diagonals AC and BD is such that AD = 4 mm, AC = 7 mm, $\hat{A}_2 = 60°$, $\hat{B}_2 = 70°$ and $\hat{D}_2 = 50°$.



Determine:

3.6.1 the length of DC (3)

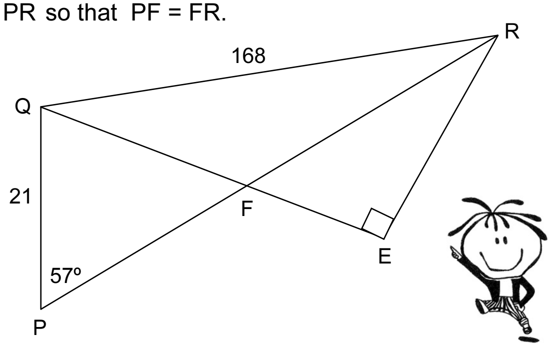
3.6.2 the length of BC (3)

3.6.3 the area of ΔBCD (2)

[34 marks]

QUESTION 4

4.1 In the diagram, QR = 168 units, QP = 21 units, $\hat{P} = 57°$ and PR = x units. E is a point on QF produced so that QE = k(FE) with F a point on PR so that PF = FR.



4.1.1 Show that $x^2 - 22{,}9x - 27\,783 = 0$ (3)

4.1.2 Show that $x = 179$ units, rounded off to the nearest whole number. (2)

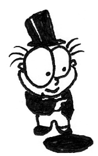
4.1.3 If it is further given that $R\hat{F}E = 12{,}8°$, find the value of k. (5)

4.2.1 Determine the value(s) of $\theta \in [0°; 90°]$ for which $\dfrac{\sin^m\theta - \cos^m\theta}{\tan^m\theta - 1}$ is undefined for all real values of m. (2)

4.2.2 Show that $\dfrac{\sin^m\theta - \cos^m\theta}{\tan^m\theta - 1} = \cos^m\theta$ (3)

4.2.3 Hence simplify as far as possible

$\dfrac{\sin\theta - \cos\theta}{\tan\theta - 1} \times \dfrac{\tan^2\theta - 1}{\sin^2\theta - \cos^2\theta} \times \dfrac{\sin^3\theta - \cos^3\theta}{\tan^3\theta - 1} \times \dfrac{\tan^4\theta - 1}{\sin^4\theta - \cos^4\theta} \times \ldots \times \dfrac{\sin^{2007}\theta - \cos^{2007}\theta}{\tan^{2007}\theta - 1}$ (2)



[17 marks]

Paper 2 THEORY

Grade 11 (and 12) Paper 2 could require **proofs** of theorems and/or trigonometric formulae up to a maximum of **12 marks**.

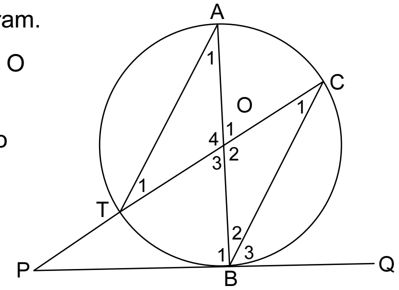
QUESTION 5

5.1 Refer to the diagram.

Circle with centre O is given.

PQ is a tangent to the circle at B.

$\hat{C}_1 = 30°$.



Calculate the size of the following angles:

5.1.1 $\hat{O}_3$ (2)

5.1.2 $\hat{A}_1$ (2)

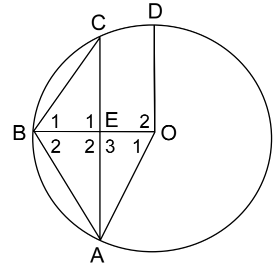
5.1.3 $\hat{P}$ (4)

5.2 Refer to the diagram.

In circle with centre O,

$\hat{E}_1 = 90° = \hat{O}_2$.

$\hat{O}_1 + \hat{O}_2 = 120°$.



Calculate the size of:

5.2.1 $\hat{C}$ (3)

5.2.2 $\hat{B}_1 + \hat{B}_2$ (5)

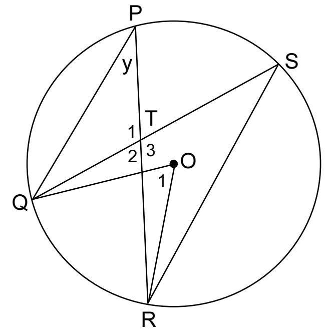
[16 marks]

QUESTION 6

Refer to the figure.

O is the centre of the circle.

PQ || RS and $\hat{P} = y$



6.1 Determine the following angles in terms of y:

6.1.1 $\hat{O}_1$ (2)

6.1.2 T_2 (5)

6.2 From your results in 6.1, what conclusion(s) can you draw about quad QROT?

Motivate your answer. (3)

[10 marks]

MEMO PAPER 1D

1.1.1 **24 ; 12 ; 8 ; 6** ◂

1.1.2 **7 ; 27 ; 127 ; 627** ◂

1.2.1 $\sqrt[4]{\frac{16x^{12}}{y^8}}$

$= \mathbf{\frac{2x^3}{y^2}}$ ◂

1.2.2 $\frac{a+b}{a^{-1}+b^{-1}}$

$= (a+b) \div \left(\frac{1}{a}+\frac{1}{b}\right)$

$= (a+b) \div \frac{b+a}{ab}$

$= (a+b) \times \frac{ab}{a+b}$

$=$ **ab** ◂

1.3.1 $2x(4x-1) = 15$

$\therefore\ 8x^2 - 2x - 15 = 0$

$\therefore\ (4x+5)(2x-3) = 0$

$\therefore\ \mathbf{x = -\frac{5}{4}}$ or $\mathbf{x = \frac{3}{2}}$ ◂

1.3.2 $x^{\frac{3}{4}} = 27$

$\therefore\ \left(x^{\frac{3}{4}}\right)^{\frac{4}{3}} = (3^3)^{\frac{4}{3}}$

$\therefore\ x = 3^4$

$=$ **81** ◂

1.3.3 $\sqrt{10-3x} = x-2$

$\therefore\ 10-3x = x^2-4x+4$

$\therefore\ x^2-x-6 = 0$

$\therefore\ (x-3)(x+2) = 0$

$\therefore\ x = 3$ or $x = -2$

Check $x = 3$: LHS = 1 = RHS

$\therefore\ \mathbf{x = 3}$ ◂

Check $x = -2$: LHS = 4 ; RHS = -4

$\therefore\ \mathbf{x = -2}$ **is not valid**

1.3.4 $\frac{8}{x^2-4} + \frac{x}{2-x} + \frac{1}{x+2} = 0$

$\therefore\ \frac{8}{(x+2)(x-2)} - \frac{x}{x-2} + \frac{1}{x+2} = 0 \qquad x \neq \pm 2$

$\therefore\ 8 - x(x+2) + x - 2 = 0$

$\therefore\ 8 - x^2 - 2x + x - 2 = 0$

$\therefore\ -x^2 - x + 6 = 0$

$\therefore\ x^2 + x - 6 = 0$

$\therefore\ (x+3)(x-2) = 0$

$\therefore\ x = -3$ or $x = 2$

But $x \neq 2$ . . . see restrictions in line 2

$\therefore$ **Only** $\mathbf{x = -3}$ ◂

1.3.5 $px^2 - 6x = q$

$\therefore\ x^2 - \frac{6x}{p} = \frac{q}{p}$

$\therefore\ x^2 - \frac{6x}{p} + \frac{9}{p^2} = \frac{q}{p} + \frac{9}{p^2}$

$\therefore\ \left(x - \frac{3}{p}\right)^2 = \frac{pq+9}{p^2}$

$\therefore\ x - \frac{3}{p} = \pm\sqrt{\frac{pq+9}{p^2}}$

$\therefore\ x = \frac{3}{p} \pm \sqrt{\frac{pq+9}{p^2}}$

$= \mathbf{\frac{3 \pm \sqrt{pq+9}}{p}}$ ◂

1.4 $2x^2 + mx + 18 = 0$

$\Delta = m^2 - 4 \times 2 \times 18$

$= m^2 - 144$

Real roots ➡ $\Delta \geq 0$

$\therefore\ \mathbf{m \leq -12}$ or $\mathbf{m \geq 12}$

2.1

2.1	72	59	48	39	32
2.1.1 1st diff.		−13	−11	−9	−7
2nd diff.			2	2	2

$\therefore$ **The constant 2nd difference = 2** ◂

2.1.2 $\therefore$ The formula for the nth term is a quadratic expression.

$\therefore\ T_n = an^2 + bn + c$

2nd difference, $2a = 2$ ➡ $\mathbf{a = 1}$

$\mathbf{c} = T_0 = \mathbf{72}$

& $T_1 = a + b + c = 59$

$\therefore\ 1 + b + 72 = 59$

$\therefore\ \mathbf{b = -14}$ ◂

$\therefore\ \mathbf{T_n = n^2 - 14n + 72}$ ◂

OR, using the formula (as per Paper 1A Q2.3.2):

$T_n = 59 + (n-1)(-11) + \frac{(n-1)(n-2)}{2} \times 2$

$= 59 - 11n + 11 + n^2 - 3n + 2$

$= \mathbf{n^2 - 14n + 72}$ ◂

2.2 1 ; 3 ; 5 ; 7 ; 9 ; 1 ; 3 ; 5 ; 7 ; 9 ; 1 ; 3 . . .

Repetition of 5 digits.

$\frac{2014}{5}$ Quotient = 402 and Remainder = 4

$\therefore\ \mathbf{T_{2014} = 7}$ ◂

3.1 $265\,000(1-0{,}2)^3 =$ **R135 680** ◂

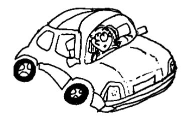
3.2 $(8\,500 + x)\left(1 + \frac{0{,}085}{12}\right)^{60} = 20\,000$

$\therefore\ 8\,500 + x = \frac{20\,000}{1{,}5273...}$

$= 13\,094{,}99913$

$\therefore\ x = 4\,594{,}99913$

$=$ **R4 595** ◂



4.1 Area of 2 semi-circles $= \pi r^2$

Area of big semi-circle $= \frac{1}{2} \times \pi(2r)^2 = 2\pi r^2$

$\therefore$ Probability $= \mathbf{\frac{1}{2}}$ ◂

4.2 P(Gym or Coffee) = 0,35 + 0,16 = 0,51

$\therefore$ P(neither) = **0,49** ◂

5.1 $N(20) = 250 \times 20 - 20^2 =$ **4 600 thousand** ◂

5.2 $250x - x^2 = 15 \times 10^6$

$= 15 \times 10^3$ thousand

$\therefore\ x^2 - 250x + 15\,000 = 0$

$\therefore\ (x-100)(x-150) = 0$

$\therefore\ x = 100$ or $x = 150$

i.e. **100 mm** ◂ or **150 mm** ◂

5.3 $N(x) = -x^2 + 250x$

$x_{TP} = \frac{-250}{2(-1)}$

$= 125$

$\therefore\ y_{TP} = -125^2 + 250 \times 125$

$= 15\,625$ thousand

$=$ **15 625 000** ◂

6.1 $m_{AB} = \frac{1-(-4)}{2-(-3)} = \frac{5}{5} = 1$

Eqn.: $y - 1 = 1(x-2)$

$\therefore\ \mathbf{y = x - 1}$ ◂

6.2 $f(x) = ax^2 + bx - 5$

C: $-6 = a(-1)^2 + b(-1) - 5$
$\therefore -6 = a - b - 5$
$\therefore b = a + 1$ . . . ❶

D: $22 = a \times 3^2 + b \times 3 - 5$
$\therefore 27 = 9a + 3b$
$\therefore 9 = 3a + b$ . . . ❷
$\therefore 9 = 3a + a + 1$
$\therefore 8 = 4a$

$\therefore$ **a = 2** and **b = 3** ➤

6.3 $EF = y_E - y_F$
$= -1 - (-5)$
$=$ **4 units** ➤

6.4 I & H are x-ints. of f:
$\therefore 2x^2 + 3x - 5 = 0$
$\therefore (2x + 5)(x - 1) = 0$
$\therefore x = -\frac{5}{2}$ or $x = 1$
$\therefore$ **IH =** $\mathbf{\frac{7}{2}}$ **units** ➤

6.5 At G: $f(x) = g(x)$
$\therefore 2x^2 + 3x - 5 = x - 1$
$\therefore 2x^2 + 2x - 4 = 0$
$\therefore x^2 + x - 2 = 0$
$\therefore (x + 2)(x - 1) = 0$
$\therefore x = -2$ or $x = 1$
$\therefore x_G = -2$
& $y_G = -2 - 1$
$= -3$

i.e. **G(-2; -3)** ➤

6.6 $x_J = -\frac{3}{4}$
$\therefore y_J = 2\left(-\frac{3}{4}\right)^2 + 3\left(-\frac{3}{4}\right) - 5$
$= -\frac{49}{8}$

& $y_K = -\frac{3}{4} - 1$
$= -\frac{7}{4}$

$\therefore KJ = -\frac{7}{4} + \frac{49}{8}$
$= \mathbf{\frac{35}{8}}$ **units** ➤

7.1.1 $n(R \text{ or } Q) = 68$
$\therefore 40 + 35 - x = 68$
$\therefore -x = 68 - 75$
$\therefore$ $\mathbf{x = 7}$ ➤

7.1.2 $n(S)$
$= 68 + 12$
$=$ **80** ➤

7.1.3 (i) $\frac{12}{80} = \mathbf{\frac{3}{20}}$ ➤ (ii) $\mathbf{\frac{33}{80}}$ ➤

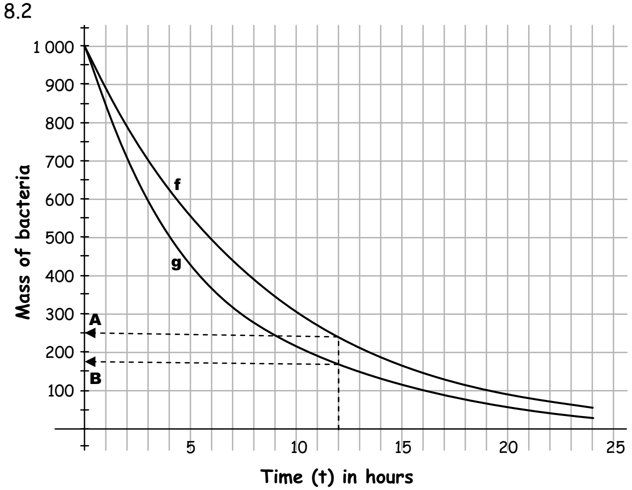
7.2.1 $\frac{4}{7} \times \frac{3}{5} = \mathbf{\frac{12}{35}}$ ➤

7.2.2 $\frac{4}{7} \times \frac{2}{5} \times \frac{3}{5} = \mathbf{\frac{24}{175}}$ ➤

7.2.3 $1 - \left(\frac{3}{7} + \frac{12}{35} + \frac{24}{175}\right) = \mathbf{\frac{16}{175}}$ ➤ . . . $1 - [P(1) + P(2) + P(3)]$

8.1 For A: $\mathbf{P_0(1 - 0{,}03)^{4t}}$ ➤ For B: $\mathbf{P_0(1 - 0{,}025)^{6t}}$

8.2



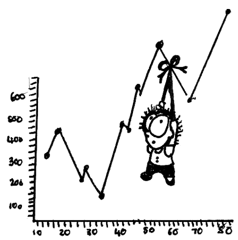
8.3 The difference in the mass of bacteria
$= f(12) - g(12)$
$= 1\,000(1 - 0{,}03)^{48} - 1\,000(1 - 0{,}025)^{72}$
$=$ **70,2** ➤ $= 231{,}762 - 161{,}555$

Confirm that this answer is feasible by referring to **A** and **B** on the graph in 8.2 as the values of f(t) and g(t) for t = 12.

Note: **A** - **B** $\simeq$ 250 - 180 $\simeq$ 70

9.1.1 For a linear sequence, the first differences are equal.
$\therefore T_2 - T_1 = T_3 - T_2$
$\therefore (2x + 1) - (2x - 4) = (3x - 5) - (2x + 1)$
$\therefore 2x + 1 - 2x + 4 = 3x - 5 - 2x - 1$
$\therefore 5 = x - 6$
$\therefore$ $\mathbf{x = 11}$ ➤

9.1.2 Seq.: 18 ; 23 ; 28
d = 5 ➤



9.2.1 $3 - \frac{9}{p^2} = \frac{26}{p}$

$\times p^2)$ $\therefore 3p^2 - 9 = 26p$
$\therefore 3p^2 - 26p - 9 = 0$
$\therefore (3p + 1)(p - 9) = 0$
$\therefore$ $\mathbf{p = -\frac{1}{3}}$ or $\mathbf{p = 9}$ ➤

9.2.2 $3^0 - 3^{2-2x} + 2 = \frac{26}{3^x}$
$\therefore 1 - \frac{3^2}{3^{2x}} + 2 = \frac{26}{3^x}$
$\therefore 3 - \frac{9}{3^{2x}} = \frac{26}{3^x}$
$\therefore p = 3^x$
$\therefore 3^x = -\frac{1}{3}$ or $3^x = 9$
not valid $\therefore x = 2$

9.3.1 $\frac{1}{\sqrt{n} + \sqrt{n+1}}$

$= \frac{1}{\sqrt{n} + \sqrt{n+1}} \times \frac{\sqrt{n} - \sqrt{n+1}}{\sqrt{n} - \sqrt{n+1}}$

$= \frac{\sqrt{n} - \sqrt{n+1}}{(\sqrt{n})^2 - (\sqrt{n+1})^2}$

$= \frac{\sqrt{n} - \sqrt{n+1}}{n - (n+1)}$

$= \frac{\sqrt{n} - \sqrt{n+1}}{-1} \times \frac{(-1)}{(-1)}$

$= \mathbf{\sqrt{n+1} - \sqrt{n}}$ ➤

9.3.2 $\frac{1}{\sqrt{1} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{4}} + \ldots + \frac{1}{\sqrt{99} + \sqrt{100}}$

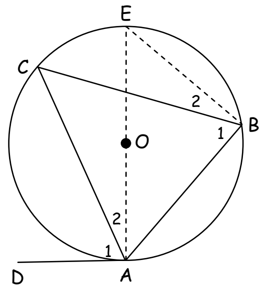
$= \sqrt{2} - \sqrt{1} + \sqrt{3} - \sqrt{2} + \ldots + \sqrt{100} - \sqrt{99}$

$= +\sqrt{1} - \sqrt{100}$

$=$ **9** ➤

9.1 Construction:

Diameter AE. Join EB.



Proof:

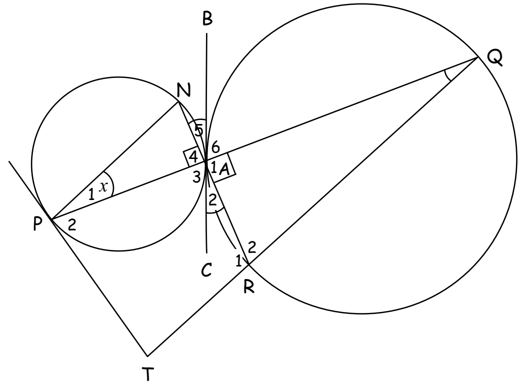
$\hat{A}_1 + \hat{A}_2 = 90°$ ... *diam. ⊥ tang.*

& $\hat{B}_1 + \hat{B}_2 = 90°$ ... *∠ in semi-⊙*

But $\hat{A}_2 = \hat{B}_2$ ... *CE subtends*

$\therefore$ **$\hat{A}_1 = \hat{B}_1$** ◄

9.2



9.2.1 $\hat{Q}$ ◄ = $\hat{A}_2$ ◄ ... *tang. BAC; chord AR - thm. in 9.1*

= $\hat{A}_5$ ◄ ... *vertically opposite $\angle^s$*

= $\hat{P}_1$ **(= x)** ... *tang. BAC; chord AN - thm. in 9.1*

9.2.2 **$\hat{Q} = \hat{P}_1$ (= x) above, i.e. alternate $\angle^s$ are equal** ◄

9.2.3 (a) $\hat{A}_1 = 90°$ ... *diameter QR in bigger ⊙*

$\therefore$ $\hat{A}_4 = 90°$ ... *vertically opposite $\angle^s$*

$\therefore$ **PN is a diameter of the smaller ⊙** ◄

... *converse of '∠ in semi-⊙' thm.*)

(b) $N\hat{P}T = 90°$ ... *diameter PN ⊥ tangent PT*

$\therefore$ $\hat{T} = 90°$ ... *co-int. $\angle^s$; PN || RG*

$\therefore$ $\hat{A}_1 = \hat{T}$ ... *both = 90°*

$\therefore$ **APTR is a cyclic quadrilateral** ◄

... *converse of 'ext. ∠ of c.q.' theorem*

MEMO PAPER 2G

1.1.1 Mean, $\bar{x} = \dfrac{1(5) + 2(10) + 3(15) + 4(10) + 5(5)}{5 + 10 + 15 + 10 + 5} = \dfrac{135}{45}$ **= 3** ◄

The symmetry of the distribution allows us to give the mean from the graph.

1.1.2

x_i	f_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$f_i \times (x_i - \bar{x})^2$
1	5	- 2	4	20
2	10	- 1	1	10
3	15	0	0	0
4	10	1	1	10
5	5	2	4	20
	Σ: 45			Σ: 60

1.1.3 Variance = $\sigma^2 = \dfrac{20 + 10 + 0 + 10 + 20}{5 + 10 + 15 + 10 + 5} = \dfrac{60}{45} = \dfrac{4}{3} \simeq$ **1,3** ◄

Standard Deviation = $\sigma = \sqrt{\dfrac{4}{3}}$ = 1,1547... ≃ **1,2** ◄

1.2.1 **Machine C**. (Look at the position of the median) ◄

1.2.2 **It is likely that all 25 results will be different.** ◄
or: **Mode cannot be seen on B & W plots.** ◄

1.2.3 **Machine B**. (Look at the length of the box) ◄

1.2.4 If the whiskers are small, the range is "close" to the inter-quartile range. ◄

1.2.5 The box plot for machine A is symmetric around the median.
$\therefore$ 161 ± 28,5 ... *57 ÷ 2 = 28,5*

Q₁ ≃ 132,5 ◄ & **Q₃ ≃ 189,5** ◄

1.2.6 **Set A.** The length of the whisker is longer than the length of the box ◄.

A common rule is:
1,5 times the IQR to the extremes on either side.

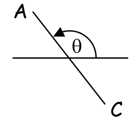
2.1.1 $AB^2 = (-2 - 4)^2 + (8 - 4)^2 = 36 + 16 = 52$

$\therefore$ AB = $\sqrt{52} \simeq$ **7,2 units** ◄

2.1.2 Midpoint of BC = $\left(\dfrac{4+1}{2}; \dfrac{4-6}{2}\right) = \left(\mathbf{\dfrac{5}{2}; -1}\right)$ ◄

2.1.3 $m_{AC} = \dfrac{8 + 6}{-2 - 1} = \dfrac{14}{-3} = \mathbf{-\dfrac{14}{3}}$ ◄

2.1.4 m = tan θ



$\therefore$ tan θ = $-\dfrac{14}{3}$ ... *m_{AC}*

$\therefore$ θ = 180° - 77,9° = **102,1°** ◄

2.1.5 tan α = m_{CB}

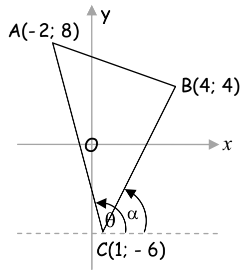
$= \dfrac{4 - (-6)}{4 - 1}$

$= \dfrac{10}{3}$

$\therefore$ α = 73,3°

$\therefore$ $A\hat{C}B$ = θ - α

= **28,8°** ◄



2.2 y-int.; c = 2 ... *B(0; 2) on the y-axis*

& $m_{AB} = \dfrac{2 - 3}{0 - 1} = 1$

$\therefore$ gradient = -1

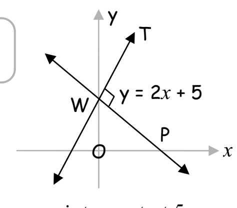
$\therefore$ **Eqn.: y = $-x$ + 2** ◄

2.3.1 y - a = 2(x - b)
$\therefore$ y = 2x - 2b + a
$\therefore$ **Gradient, m = 2** ◄

2.3.2 Many answers are possible.
C(1; -6) on graph y - a = 2(x - b)
➡ -6 - a = 2(1 - b)
$\therefore$ -a = 2 - 2b + 6
$\therefore$ a = 2b - 8

$\therefore$ Any pair of values for a & b as long as a = 2b - 8
e.g. **a = -6 & b = 1; a = -8 & b = 0, etc.** ◄

2.4.1 **W(0; 5)** ◄ ...
substitute x = 0
in y = 2x + 5



2.4.2 $m_{TW} = 2$

$\therefore$ $m_{WP} = -\dfrac{1}{2}$ ... *TW ⊥ WP*

$\therefore$ Equation of WP: **y = $-\dfrac{1}{2}x$ + 5** ◄ ... *y-intercept at 5*

2.4.3 At P, y = 0 $\therefore$ 0 = $-\dfrac{1}{2}x + 5$ ... *use equation of WP*

$\therefore$ $\dfrac{1}{2}x = 5$

× 2) $\therefore$ x = 10

$\therefore$ **P(10; 0)** ◄

2.4.4 Area of $\Delta WOP = \frac{1}{2} OP.OW$ ... $A = \frac{1}{2}bh$

$= \frac{1}{2}(10)(5)$

$=$ **25 square units** ◄

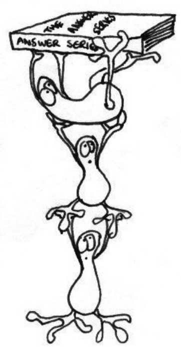
3.1.1 $r^2 = 12^2 + (-9)^2$

$= 144 + 81$

$= 225$

$\therefore r = 15$

$\therefore \sin\theta = \frac{-9}{15} = -\frac{3}{5}$ ◄



3.1.2 $\sin\theta = -\frac{3}{5}$

$\therefore \theta = 360° - 36{,}9°$

$=$ **323,1°** ◄

3.1.3 $\frac{x_P}{x_R} = \frac{12}{4} = 3$

$\therefore \frac{y_P}{y_R} = \frac{-9}{a} = 3$

$\therefore$ **a = -3** ◄

3.2.1 $\frac{\sin A \cdot \cos A \cdot \tan A}{1 - \cos^2 A} = \frac{\sin A \cdot \cos A \cdot \frac{\sin A}{\cos A}}{\sin^2 A} = \frac{\sin^2 A}{\sin^2 A} = 1$ ◄

3.2.2 $\tan\theta = \frac{\sin A \cdot \cos A \cdot \tan A}{1 - \cos^2 A}$

$\therefore \tan\theta = 1$

$\therefore \theta =$ **45°** ◄

or $\theta = 180° + 45° =$ **225°** ◄

3.3.1 **$\sin x = a$** ◄ 3.3.2 **$-\sin x = -a$** ◄ 3.3.3 **$-\sin x = -a$** ◄

3.4 $\frac{\tan 300° + \cos(90° + x)}{\sin x + 2\cos(-30°)}$

$= \frac{(-\tan 60°) + (-\sin x)}{\sin x + 2\cos 30°}$

$= \frac{-\sqrt{3} - \sin x}{\sin x + 2\left(\frac{\sqrt{3}}{2}\right)}$

$= \frac{-(\sin x + \sqrt{3})}{\sin x + \sqrt{3}}$

$=$ **-1** ◄

3.5 $1 - \cos\theta = \cos 44°$

$\therefore \cos\theta = 0{,}280660199$

$\therefore \theta =$ **$\pm 73{,}7° + 360°k, \ k \in \mathbb{Z}$** ◄

OR: $\theta =$ **$73{,}7° + 360°k$** ◄

or $\theta =$ **$286{,}3° + 360k$** ◄

3.6.1 In ΔACD: $DC^2 = 4^2 + 7^2 - 2(4)(7).\cos 60°$

$= 37$

$\therefore$ **DC = 6,1 mm** ◄

3.6.2 In ΔBCD: $\frac{BC}{\sin 50°} = \frac{6{,}1}{\sin 70°}$

$\therefore BC = \frac{6{,}1 \sin 50°}{\sin 70°}$

$\therefore$ **BC ≃ 5,0 mm** ◄

3.6.3 $B\hat{C}D = 60°$

$\therefore$ Area of $\Delta BCD = \frac{1}{2} \times 5 \times 6{,}1 \times \sin 60°$

$=$ **13,2 mm²** ◄ ... *but 13,1 mm² with "stored" values*

4.1.1 In ΔPQR, $21^2 + x^2 - 2 \times 21 \times x \times \cos 57° = 168^2$

$\therefore 441 + x^2 - 22{,}8748x = 28\ 224$

$\therefore x^2 - 22{,}9x - 27\ 783 = 0$

4.1.2 $x = \frac{-(-22{,}9) \pm \sqrt{22{,}9^2 - 4(1)(-27\ 783)}}{2(1)}$

$= 178{,}5$ or $-155{,}6$ n/a

$\simeq$ **179** ◄

4.1.3 $QE = k(FE)$

$\therefore$ **$k = \frac{QE}{FE}$** ◄

In ΔRFE: $FR = \frac{1}{2}x = 89{,}5$

& $\frac{FE}{FR} = \cos 12{,}8°$

$\therefore FE = 89{,}5 \cos 12{,}8°$

$= 87{,}275...$ **STO A**

OR: One can find QF in ΔQPF by using the sine rule: $QF = 79{,}795...$

Also: $\frac{RE}{FR} = \sin 12{,}8°$ ➡ $RE = 89{,}5 \sin 12{,}8°$

$= 19{,}828...$

& In ΔQRE: $QE^2 = 168^2 - RE^2$

$\therefore QE = 166{,}825...$

$\therefore$ **k ≃ 1,9** ◄

4.2.1 Undefined when . . .

$\tan^m\theta = 1$ ... denom $= 0$ **OR** $\tan\theta$ undefined

$\therefore \tan\theta = \pm 1$ $\therefore \theta =$ **90°** ◄

$\therefore \theta =$ **45°** ◄

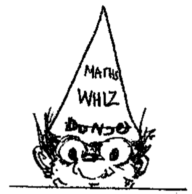
4.2.2 **LHS** $= \frac{\sin^m\theta - \cos^m\theta}{\tan^m\theta - 1}$

$= \frac{\sin^m\theta - \cos^m\theta}{\frac{\sin^m\theta}{\cos^m\theta} - 1}\left(\times \frac{\cos^m\theta}{\cos^m\theta}\right)$

$= \frac{\cos^m\theta(\sin^m\theta - \cos^m\theta)}{\sin^m\theta - \cos^m\theta}$

$= \cos^m\theta$

$=$ **RHS** ◄



4.2.3 $\frac{\sin\theta - \cos\theta}{\tan\theta - 1} \times \frac{\tan^2\theta - 1}{\sin^2\theta - \cos^2\theta} \times \frac{\sin^3\theta - \cos^3\theta}{\tan^3\theta - 1} \times \frac{\tan^4\theta - 1}{\sin^4\theta - \cos^4\theta} \times ... \times \frac{\sin^{2007}\theta - \cos^{2007}\theta}{\tan^{2007}\theta - 1}$

$= \cos\theta \times \frac{1}{\cos^2\theta} \times \cos^3\theta \times \frac{1}{\cos^4\theta} \times ... \times \cos^{2007}\theta$

$= \left(\cos\theta \times \frac{1}{\cos^2\theta}\right) \times \left(\cos^3\theta \times \frac{1}{\cos^4\theta}\right) \times ... \times \cos^{2007}\theta$

$= \left(\frac{1}{\cos\theta}\right)\left(\frac{1}{\cos\theta}\right)$ ...... *1003 times* $\times \cos^{2007}\theta$

$= (\cos\theta)^{-1003} \times \cos^{2007}\theta$

$= (\cos\theta)^{-1003 + 2007}$

$=$ **$(\cos\theta)^{1004}$, i.e. $\cos^{1004}\theta$** ◄

OR: $\cos\theta \times \left(\frac{1}{\cos^2\theta} \times \cos^3\theta\right) \times \left(\frac{1}{\cos^4\theta} \times \cos^5\theta\right) \times ... \times \left(\frac{1}{\cos^{2006}\theta} \times \cos^{2007}\theta\right)$

$= \cos\theta \cdot \cos\theta \cdot \cos\theta \cdot ...... \cdot \cos\theta$

$= (\cos\theta)^{1 + \frac{2006}{2}}$

$=$ **$\cos^{1004}\theta$** ◄

5.1.1 $\hat{O}_3 = 2\hat{C}_1$ ... *∠ at centre = 2 × ∠ at circumference*

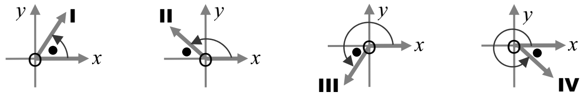
$\therefore \hat{O}_3 =$ **60°** ◄

TRIG SUMMARY (Grade 11)

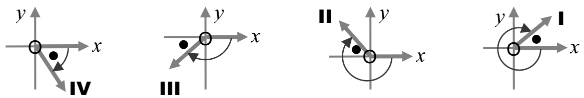
THE ANSWER SERIES *Your Key to Exam Success*

▶ ANGLES IN STANDARD POSITIONS

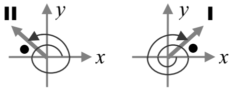
- **Positive ∠s** (anticlockwise from 0º to 360º):



- **Negative ∠s** (clockwise from 0º to –360º):



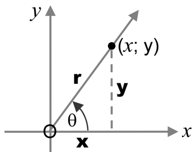
Also possible:



▶ THE RATIOS

& their

- **Definitions:**



- **Signs:**
  sin θ is positive in **I** & **II**
  cos θ is positive in **I** & **IV**
  tan θ is positive in **I** & **III**

- **Critical values:**

	sin θ	cos θ	tan θ
Definitions	$\frac{y}{r}$ $\frac{O}{H}$	$\frac{x}{r}$ $\frac{A}{H}$	$\frac{y}{x}$ $\frac{O}{A}$
Signs	II, I	I, IV	I, III
Critical values	1; 0, 0; –1	0; –1, 1; 0	∞; –1, – +, 1; 0, 0; 1, + –, –1; ∞

- **Minimum & Maximum values** of **sin θ & cos θ**:
  The values of **sin** θ & **cos** θ range from **–1** to **1**.

minimum value is **–1**

$$-1 \leq \begin{matrix}\sin\theta \\ \cos\theta\end{matrix} \leq 1$$

maximum value is **1**

All values are **proper fractions** or **0** or **±1**.

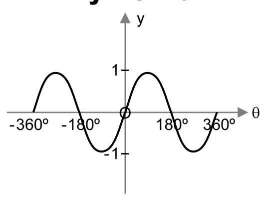
tan θ

Note that tan θ has **no minimum or maximum** values.

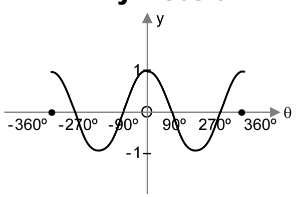
The **range of values** of tan θ is from **–∞** to **∞**.

- **Graphs:**

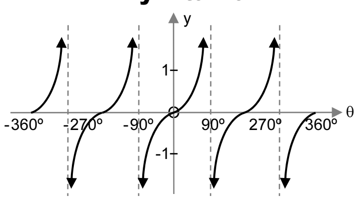
y = sin θ



y = cos θ



y = tan θ



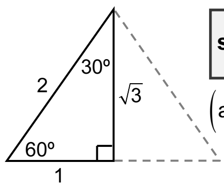
▶ IDENTITIES

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

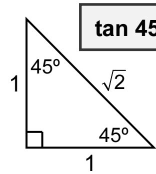
∴ $\sin^2\theta = 1 - \cos^2\theta$
& $\cos^2\theta = 1 - \sin^2\theta$

▶ SPECIAL ∠s

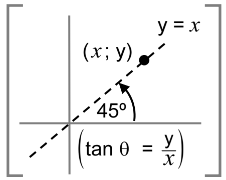


$\sin 30º = \frac{1}{2}$

(and $\cos 60º = \frac{1}{2}$)



$\tan 45º = 1$



$\left(\tan\theta = \frac{y}{x}\right)$

& THEIR "FAMILIES":

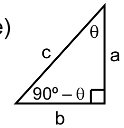
	30º "family"	**60º** "family"	**45º** "family"	
II	150º	120º	135º	◄ 180º – ●
III	210º	240º	225º	◄ 180º + ●
IV	330º	300º	315º	◄ 360º – ●

▶ GENERAL FORMS

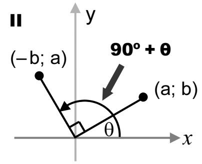
ANY ratio $\begin{pmatrix}180º \pm \theta \\ 360º - \theta \\ -\theta\end{pmatrix}$ = **±** that **SAME** ratio of **θ**

▶ CO-RATIOS (sine and **co**sine)

- **90º– θ** (an **acute** angle)
  sin(90º– θ) = cos θ
  cos(90º– θ) = sin θ



- **90º+ θ** (an **obtuse** angle)
  sin(90º+ θ) = cos θ
  cos(90º+ θ) = –sin θ



The ratio **CHANGES** to the **CO**-ratio.

▶ SOLUTION OF Δs

In Right-angled Δs, we use:

- Regular trig. ratios
- the Theorem of Pythagoras
- Area = $\frac{1}{2}$bh

In Non-Right-angled Δs, we use:

- **Sine Rule:** $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
- **Cosine Rule:** $c^2 = a^2 + b^2 - 2ab\cos C$
- **Area Rule:** **AREA** = $\frac{1}{2}$**ab sin C**

But also: Area of Δ = $\frac{1}{2}$bh

TRIG SUMMARY